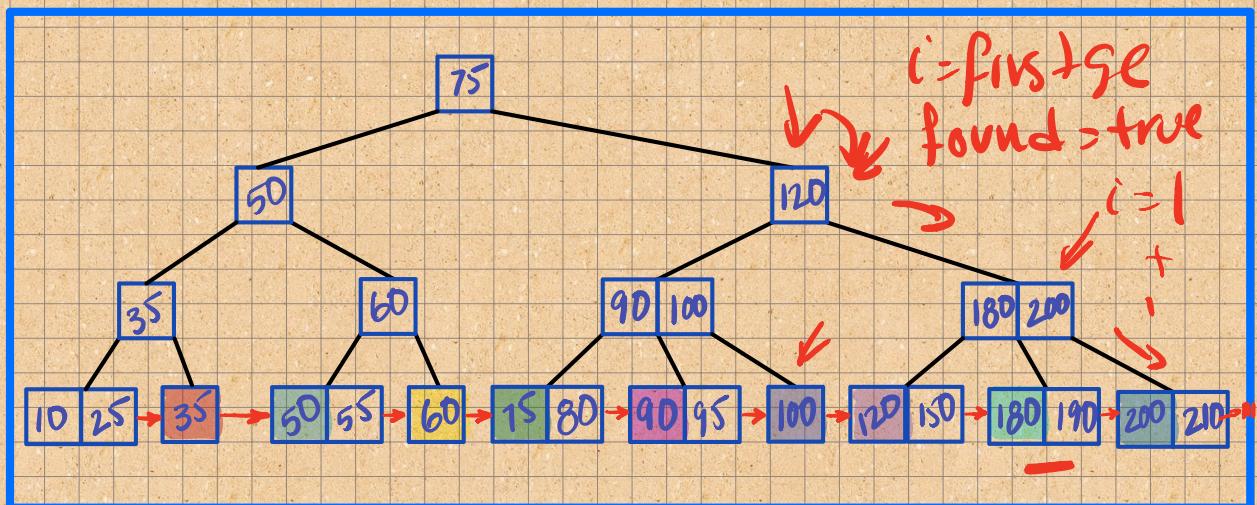
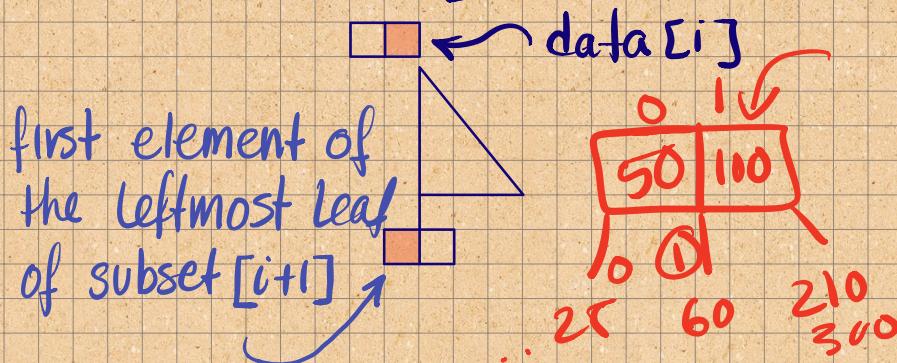


BPlus Tree

- All keys are present at the leaf level.
- A next pointer links each leaf node to the next **and leaf nodes only**
- Each entry in $\text{data}[i]$ is equal to the first element of the leftmost leaf of subset $[i+1]$



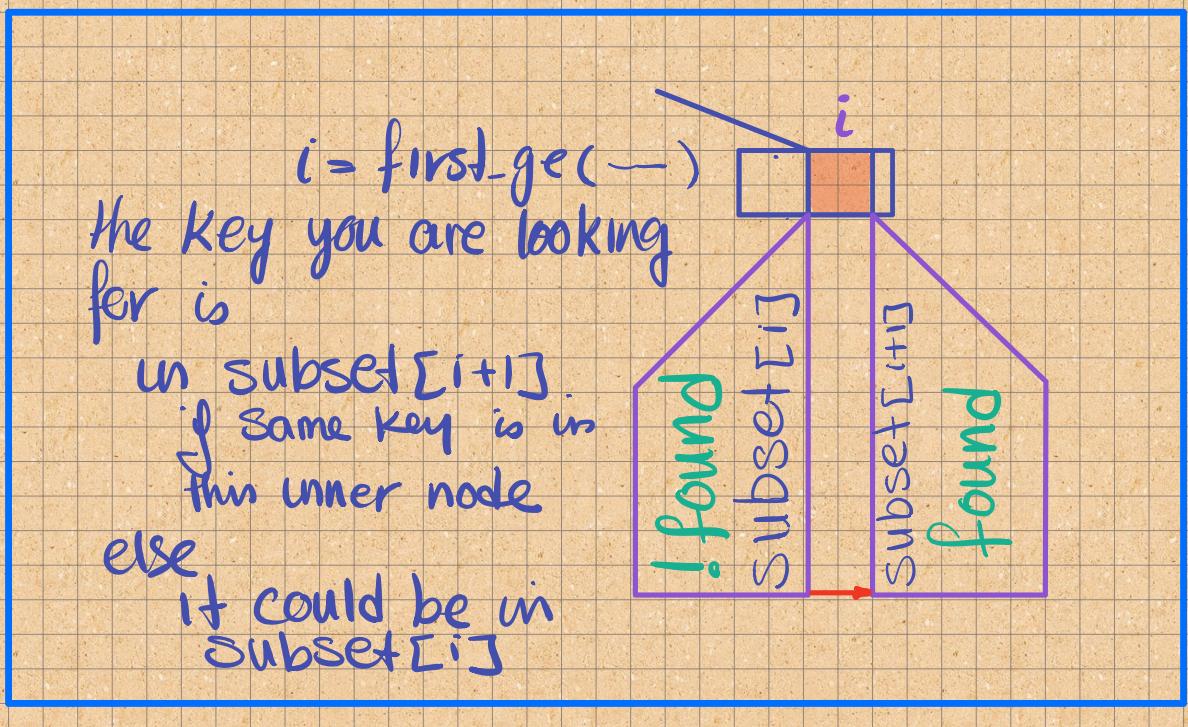
- The inner nodes serve as bread crumbs to find the actual node in the leaves



General approach for b+trees to seek and process entries

index = first_ge(...)
found = ...

	found ✓	! found	
Leaf	found it	it's not there	inner nodes are bread crumbs:
! Leaf	Subset[i:i+1] → recursive fix things ✗	Subset[i:i] → recursive fix things	





for loose-insert:

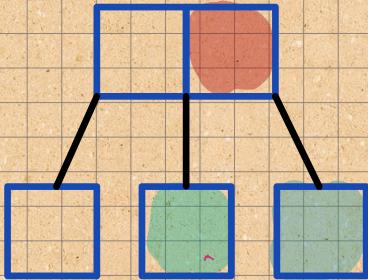
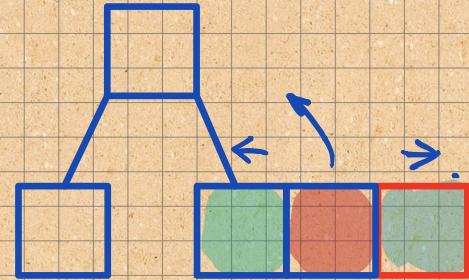
	found	!found	
Leaf	it's a duplicate	insert key here @ pos i	first-ge returns i found is true $i < dc \&$ $data[i] == entry$
!Leaf	subset $[i+1] \rightarrow$ loose-insert fix-excess	subset $[i] \rightarrow$ loose-insert fix-excess	



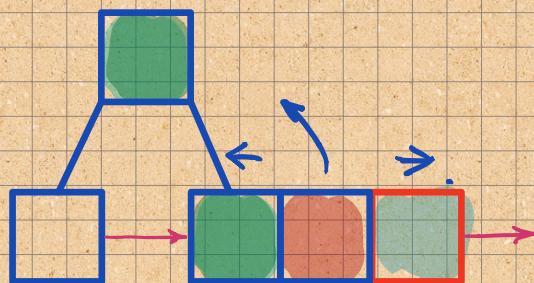
flex-excess

1

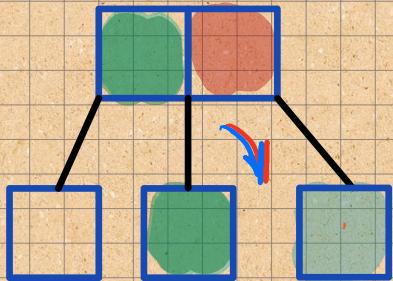
btree



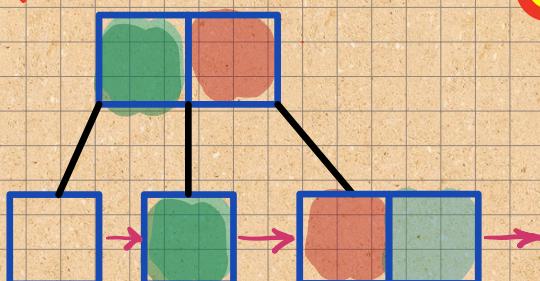
b+tree



b+tree inner nodes
behave exactly as
btree nodes do



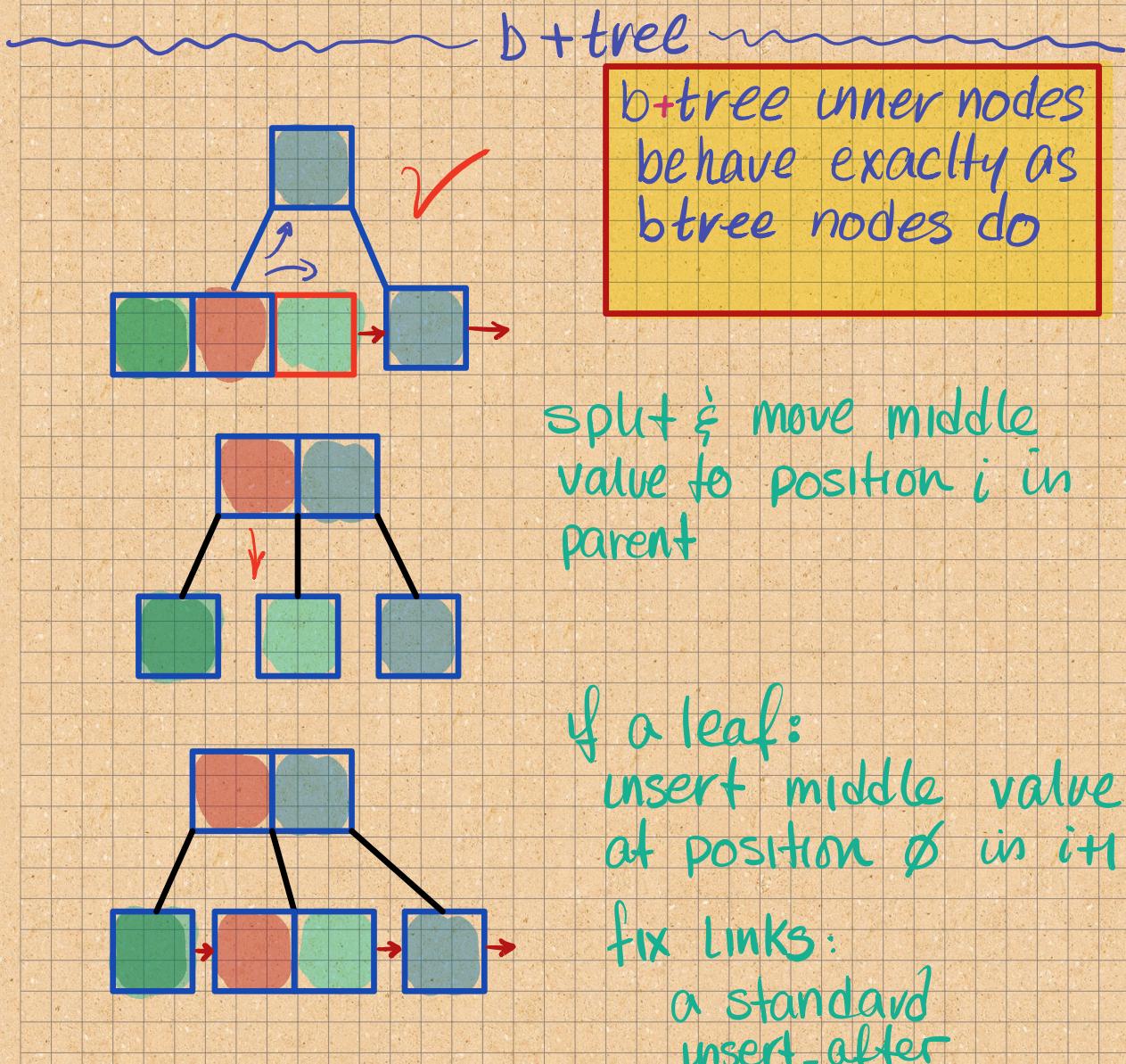
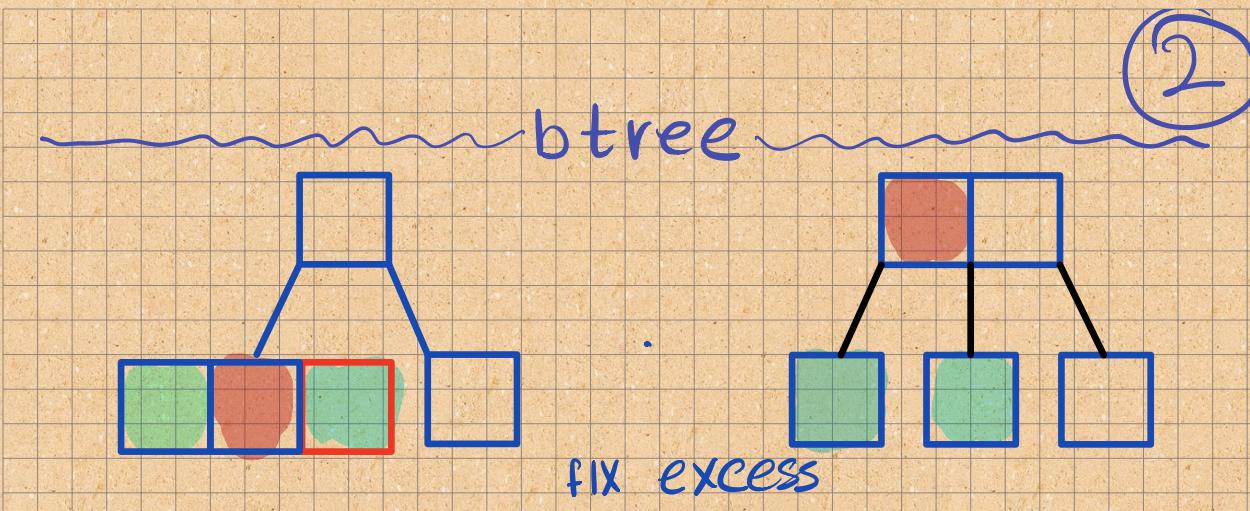
If a leaf:



Split and move half
to $i+1$
Move middle up to Pos
 i

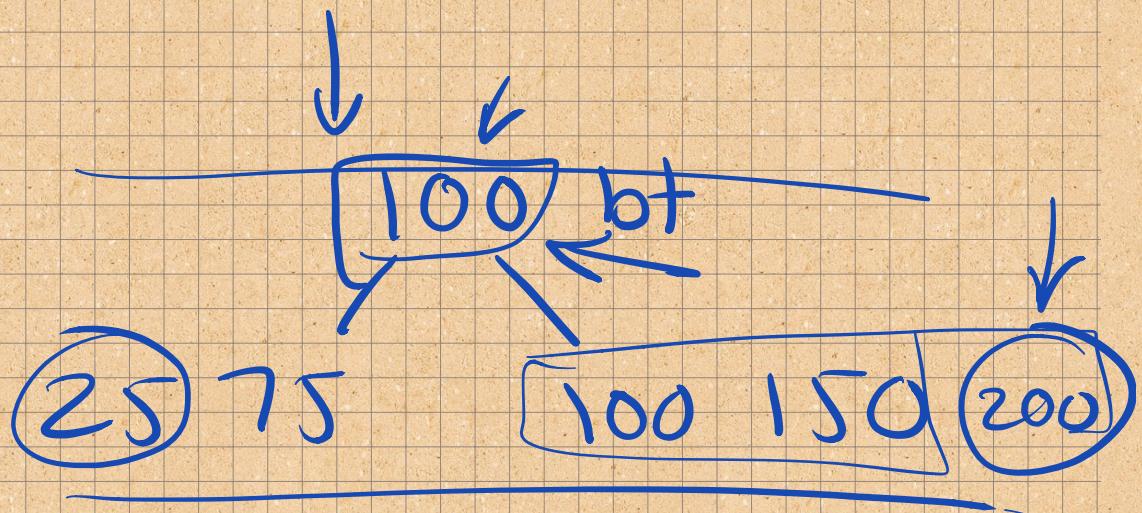
If leaf:

insert middle
value to pos \emptyset
of $i+1$
connect the links



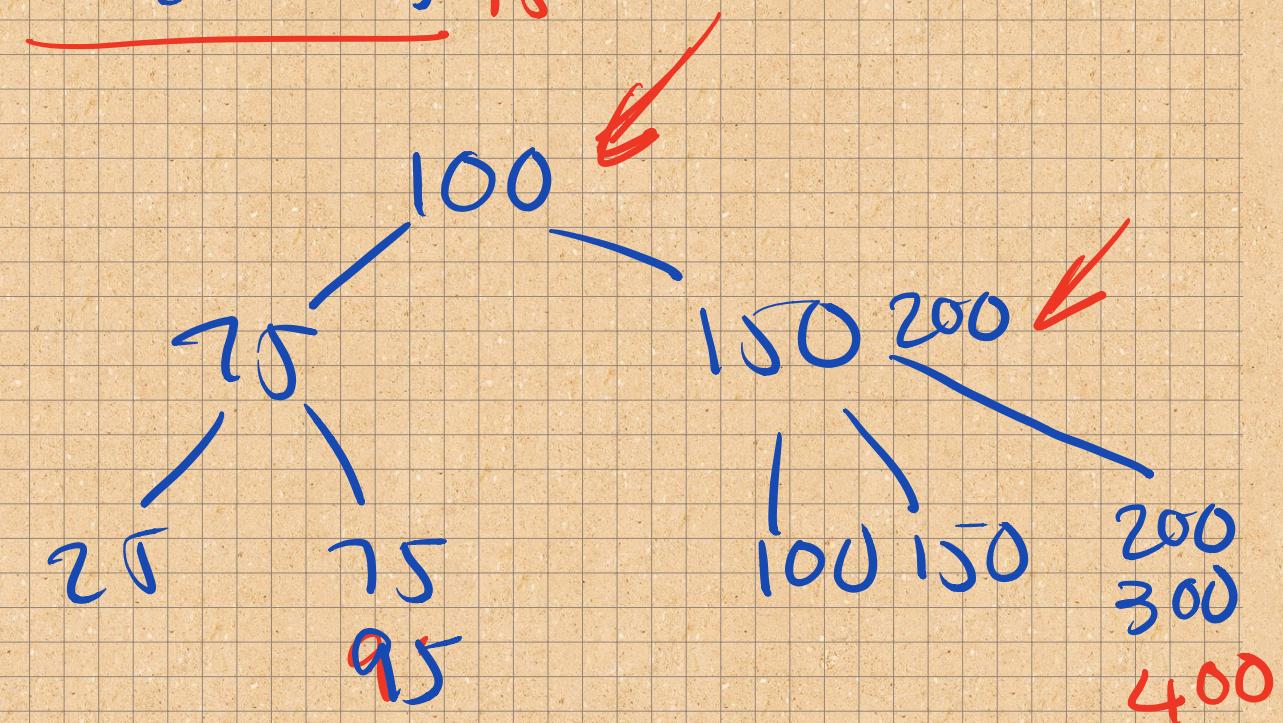
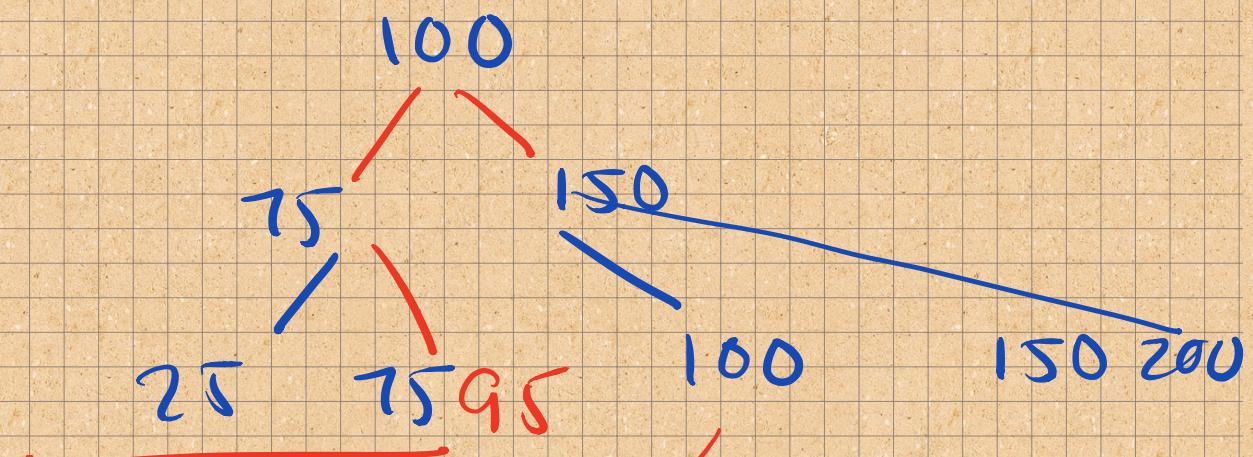
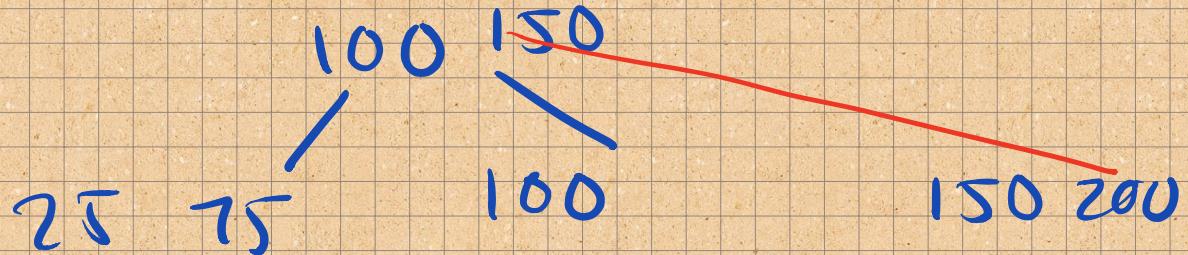


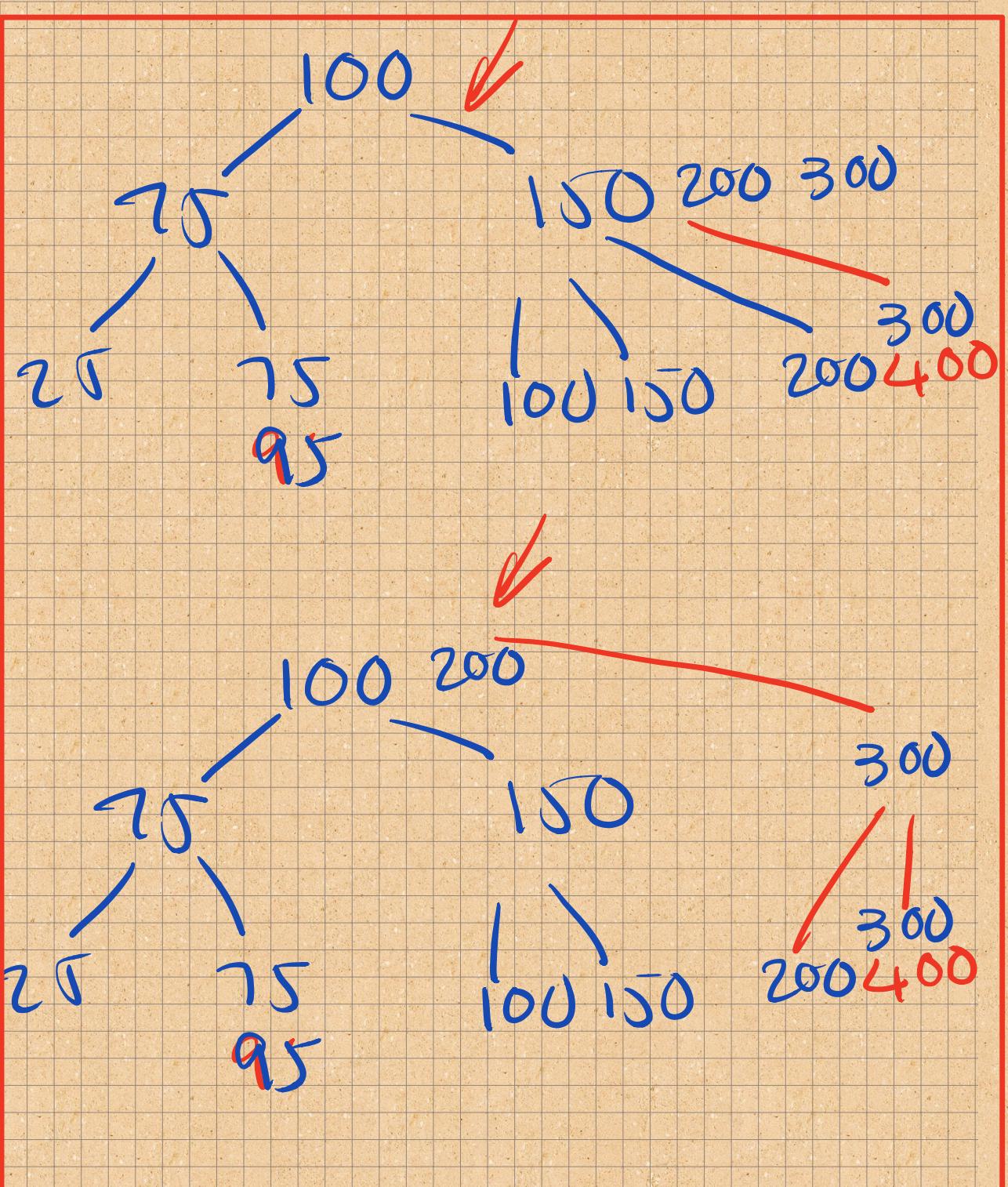
75 100 150



bt.Insert(200)
i-fge()

subset[1].Insert
f.e.





loose - remove

may cause this node with less items than minimum

once loose-remove returns, the caller may need to call fix-shortage

① data to be removed resides in the leaf nodes

② once the data is removed any "sister data" elements (breadcrumbs) must be removed and be replaced with the smallest data item in subset i+1 subtree

③ poorman's version:

leaving the breadcrumb data items on the tree will, in time, clutter the b+ tree, but will not interfere with the proper operations of the b+ tree - I hope.

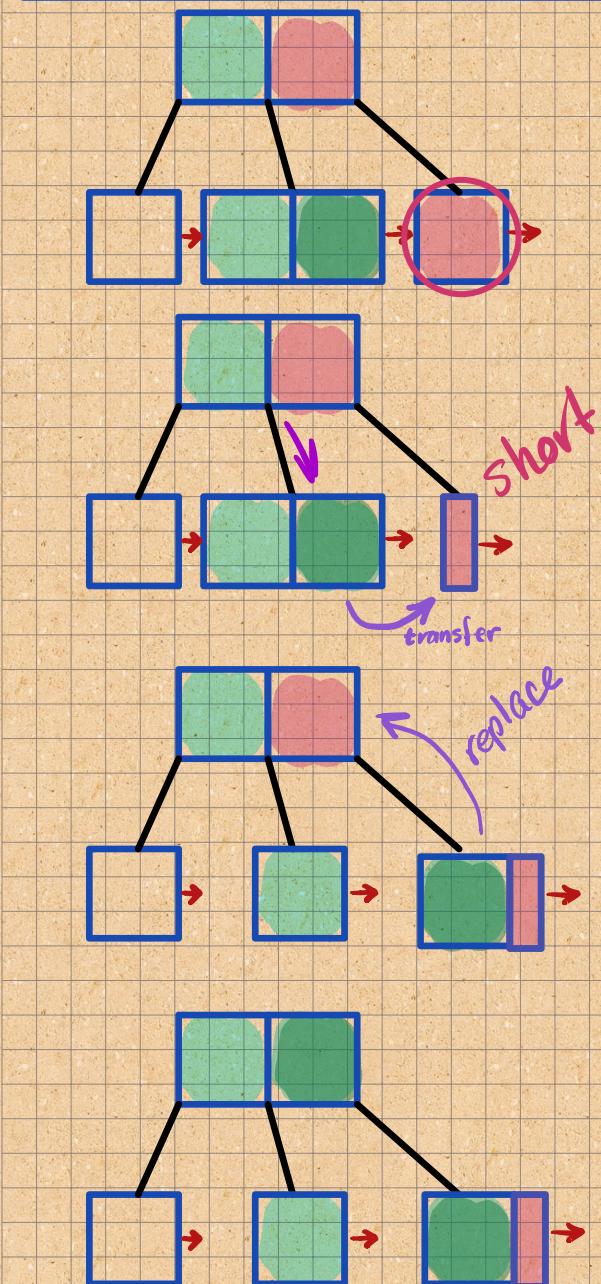
fix-shortage:

fix-shortage is called on the child with less than minimum number of data items, by the parent node.

4 Scenarios:

1. child $i+1$ has more than min
borrow from right:
 transfer / rotate left 
2. child $i-1$ has more than min
borrow from left:
 transfer / rotate right 
3. there is a left sibling
merge with previous:
merge($i-1$)
4. there has to be a right sibling
merge with next:
merge(i)

I. transfer right ($i-1$)

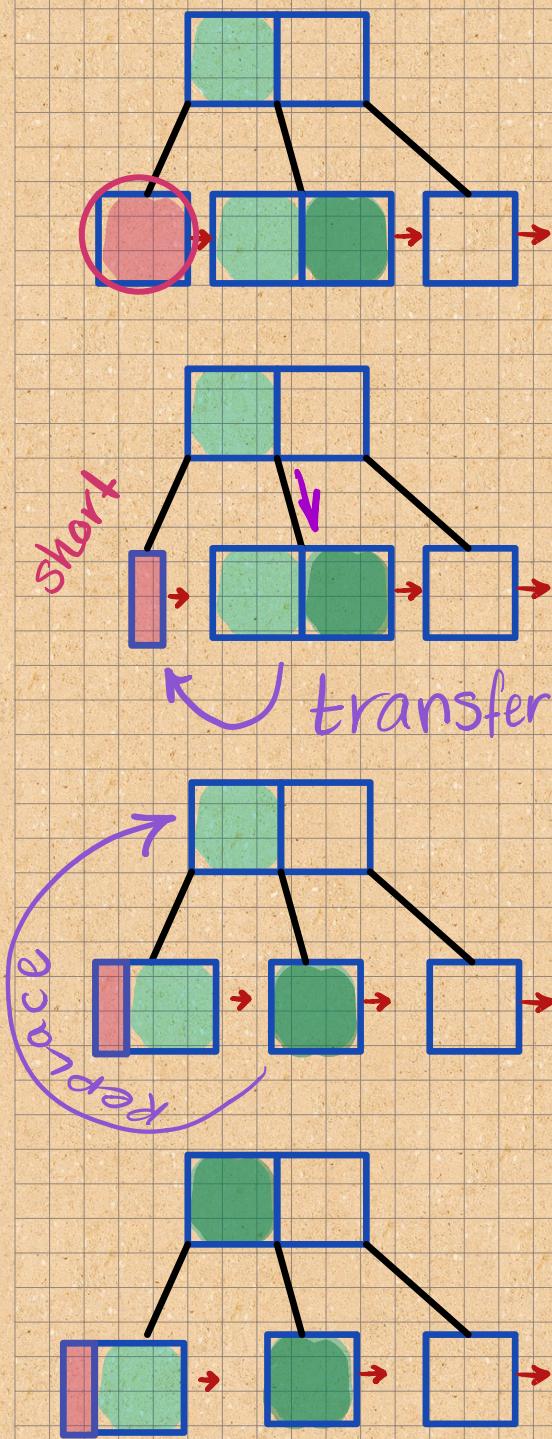


tell the subset to short's left ($i-1$), to transfer right to short

Leaves only

- ① transfer the last data item from $[i-1] \rightarrow \text{data}$ to $[i] \rightarrow \text{data}$
- ② replace $\text{data}[i]$ with this data item.

2. transfer Left ($i+1$)



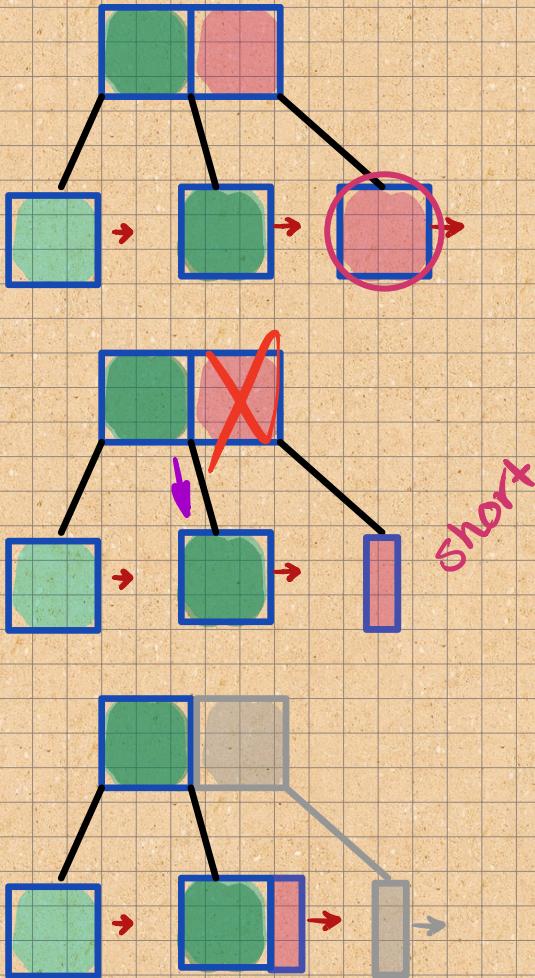
tell the subset to the left of short ($i+1$) to transfer left - to short

Leaves only
Transfer

$[i+1] \rightarrow \text{data}[\emptyset]$
to the end of
 $[i] \rightarrow \text{data}$

- replace $\text{data}[i]$ with
 $[i+1] \rightarrow \text{data}[\emptyset]$

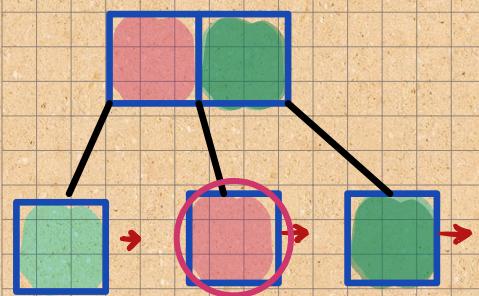
3. merge-with-next($i-1$)



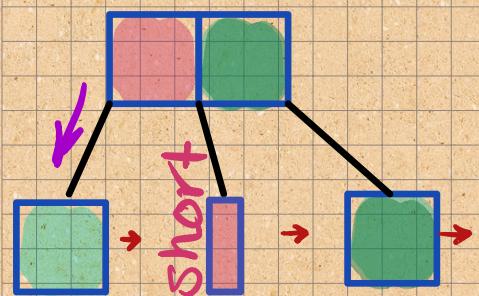
$i > 0$: tell the subset to the left of **short** ($i-1$), to merge with his next subset
- short

for leaves:
delete $\text{data}[i]$, but
do not attach to
 $\text{subset}[i] \rightarrow \text{data}$
then merge $\text{data}[i+1]$
with $\text{data}[i]$

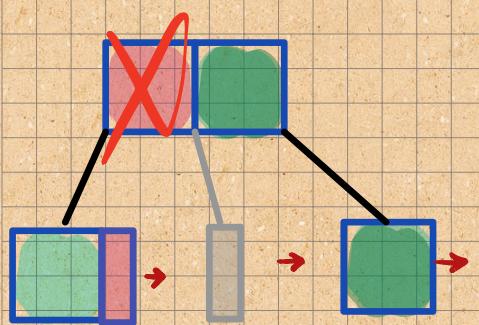
4-merge with-next (i)



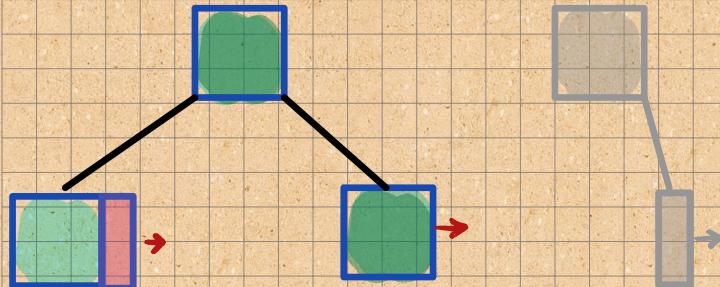
$i < \text{data_count} - 1$:
merge the short subset with his next sibling



delete $\text{data}[i]$, but
do not attach to
 $\text{subset}[i] \rightarrow \text{data}$



merge
 $\text{subset}[i+1] \rightarrow \text{data}$
with $\text{subset}[i] \rightarrow \text{data}$



copy-tree

- A solid copy-tree is necessary for a reliable Big Three
- Challenge : How to connect the next pointers on the leaves
- copy-tree function takes two arguments:

- a btree node : Other
- a btree pointer : (by reference)
"Last_Leaf"

copy-list algorithm

copy the other's data over

if the other guy is not a leaf

for every child of the other

Copy other's child as my own

else, if other is a leaf :

point last-leaf's next to this
node if last-leaf is not null

set last-leaf to this node.

Iterators

A pointer for the nodes
and
an int for the data index in
data array

`++` : increments the int
until it reaches `dc-1`
then set it to zero
& advance the pointer

`*` : returns T at index
of the node being pointed to
by pointer

