Group 3-B GARCH Model For Apple Time Series

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1 Group 3-B R GARCH Model Analysis of Apple Stock Price

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In [20]:
```
#import the necessary libraries
library(tidyverse)
library(quantmod)
library(rugarch)
library(forecast)
library(tseries)
library(lmtest)
library(zoo)
library(fBasics)
```

In [21]:
```
#import the data from Yahoo Finance
getSymbols('AAPL', src='yahoo')
```

'AAPL'

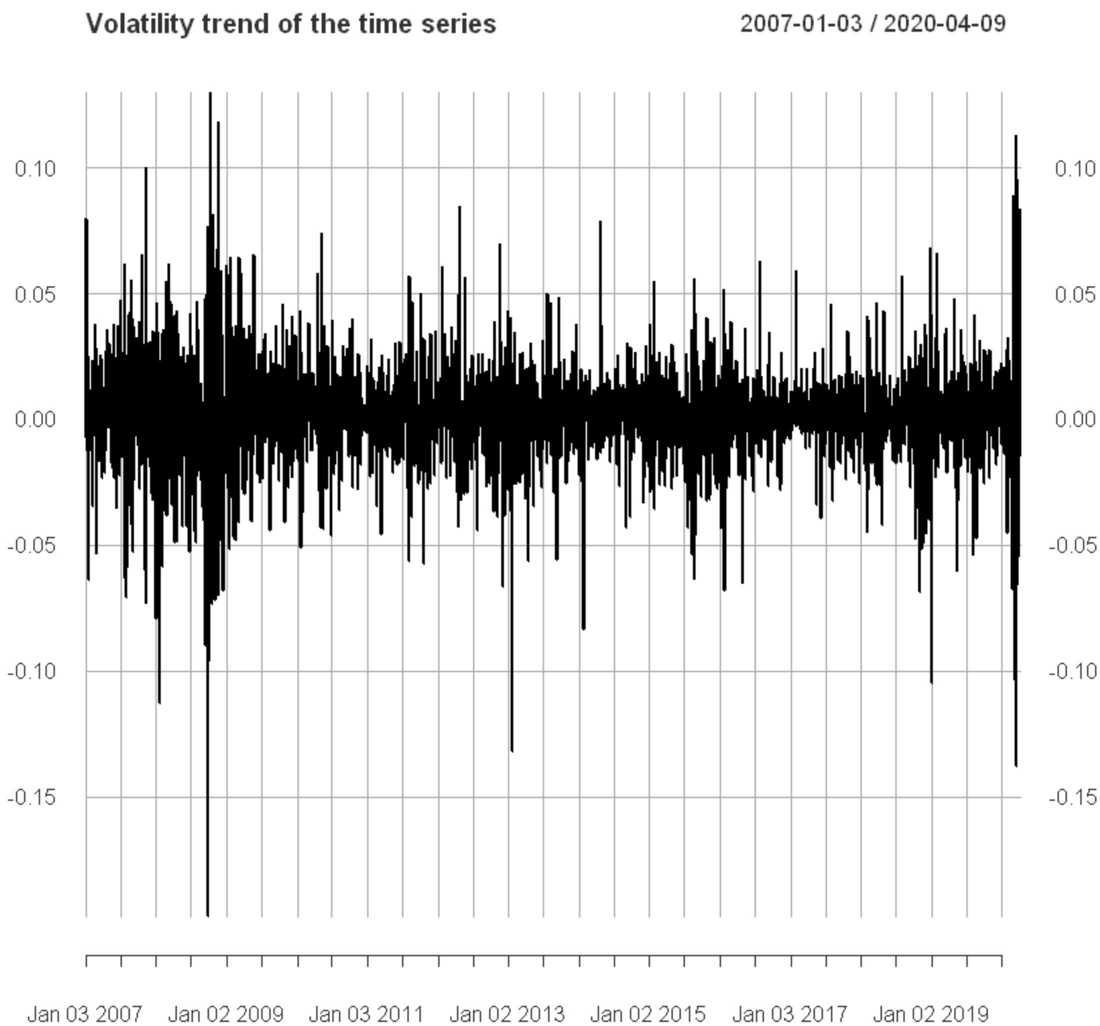
In [3]:
```
#Displays the head of the time series
head(AAPL)
```

```
           AAPL.Open AAPL.High AAPL.Low AAPL.Close AAPL.Volume AAPL.Adjusted
2007-01-03  12.32714  12.36857 11.70000   11.97143   309579900      10.39169
2007-01-04  12.00714  12.27857 11.97429   12.23714   211815100      10.62234
2007-01-05  12.25286  12.31428 12.05714   12.15000   208685400      10.54669
2007-01-08  12.28000  12.36143 12.18286   12.21000   199276700      10.59878
2007-01-09  12.35000  13.28286 12.16429   13.22429   837324600      11.47922
2007-01-10  13.53571  13.97143 13.35000   13.85714   738220000      12.02857
```

```
In [4]: #display a plot of the time series
        plot(AAPL[,'AAPL.Adjusted'], main='Time Series Plot of Apple', ylab='Adjusted Closing I
```

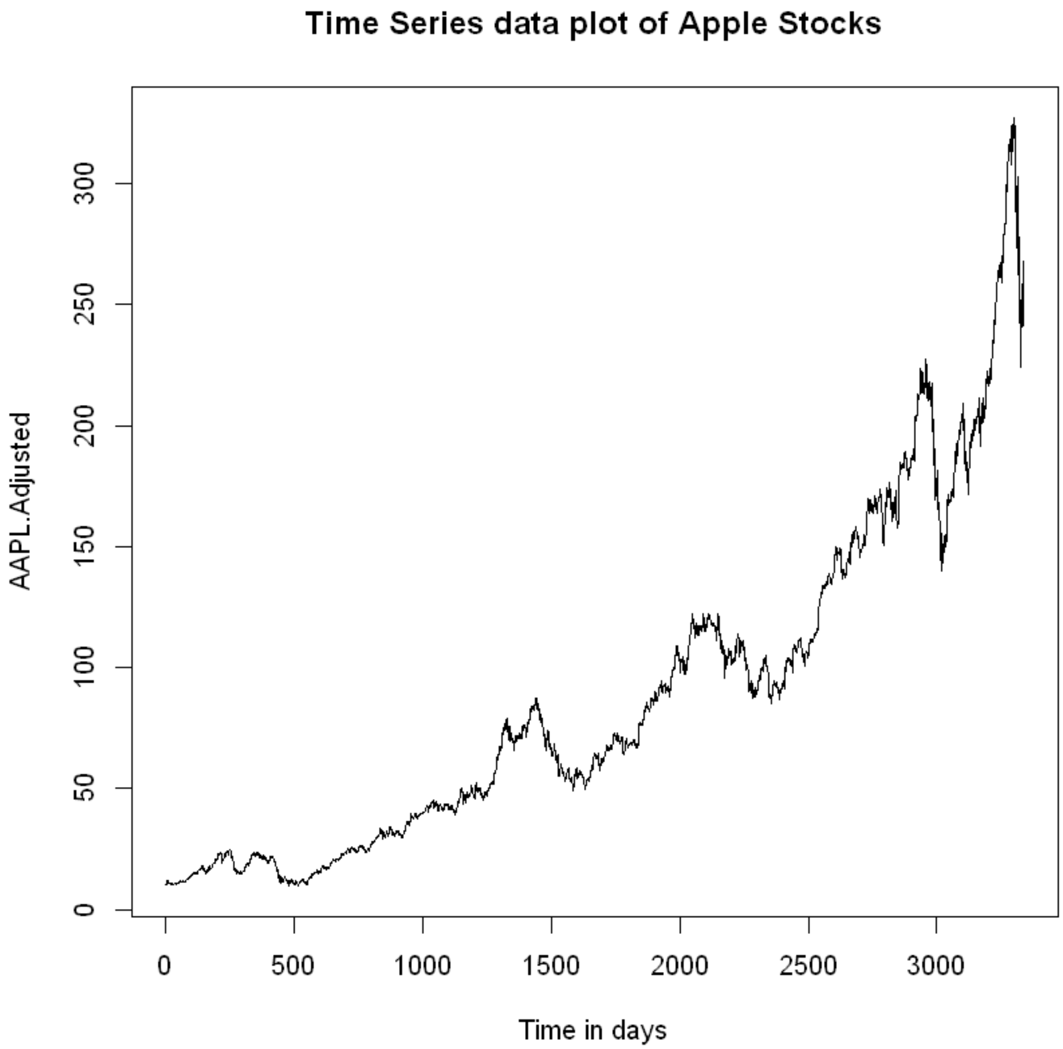
Observing the graph indicates that the time series is non-stationary with varying mean.

```
In [5]: plot(diff(log(AAPL[,'AAPL.Adjusted'])), main="Volatility trend of the time series")
```

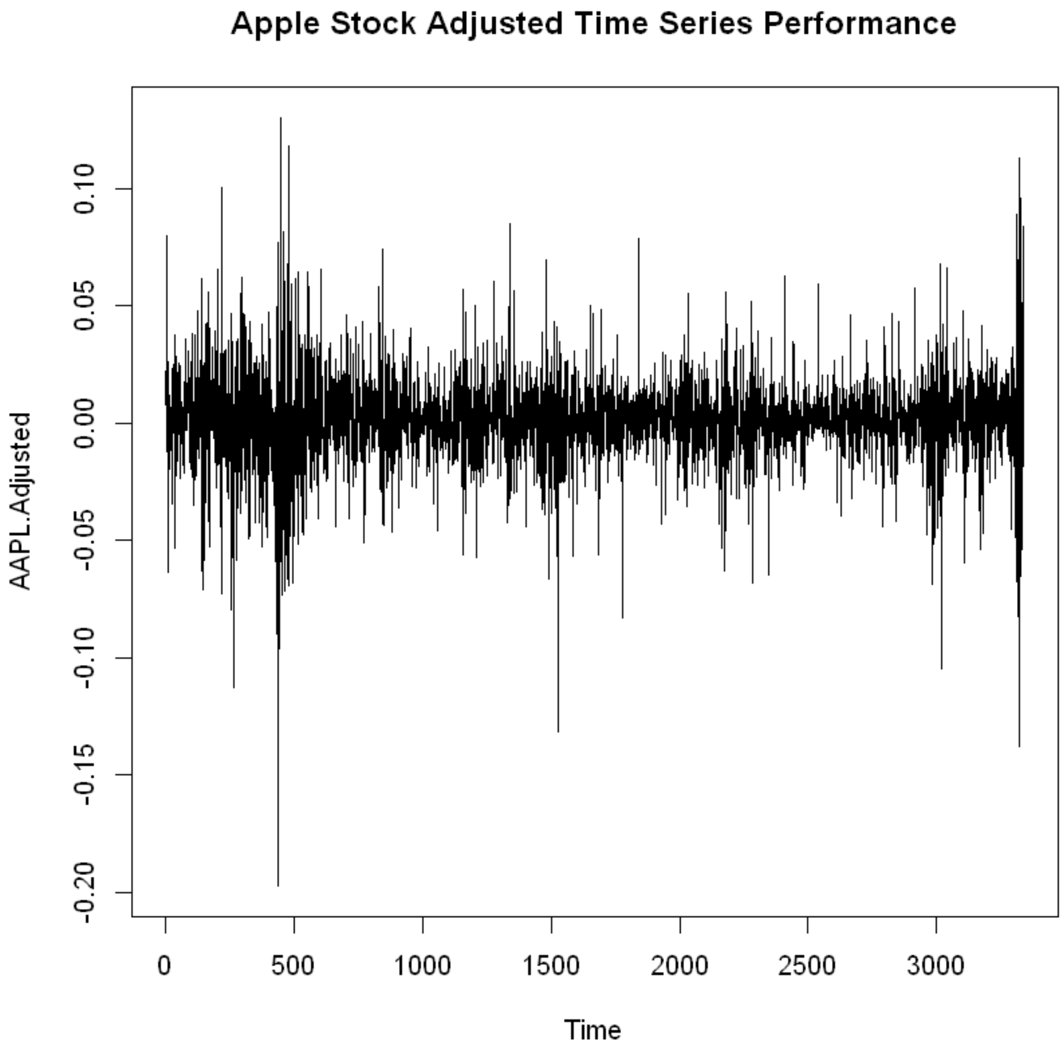


This plot displays that the time series exhibits volatility and persistence

In [7]:
```
#Converting to a time series for ease of dealing with
AAPL_ts=ts(AAPL[,'AAPL.Adjusted'], frequency=1, start=0)
plot(AAPL_ts,  main="Time Series data plot of Apple Stocks", xlab="Time in days")
```



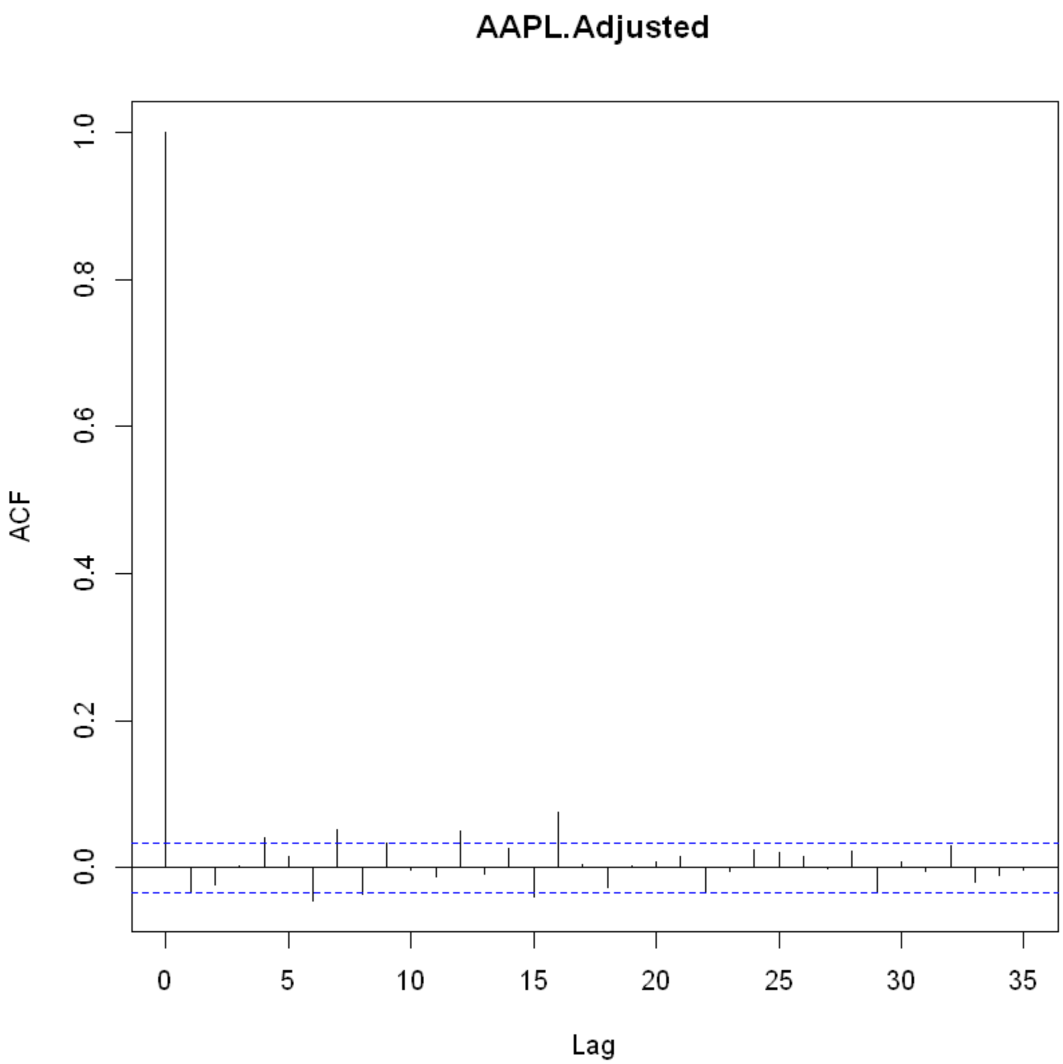

```
In [8]: #Converting to log-Diff since the series is not stationary and plotting
        AAPL_ts_log = diff(log(AAPL_ts))
        plot(AAPL_ts_log, main="Apple Stock Adjusted Time Series Performance")
```



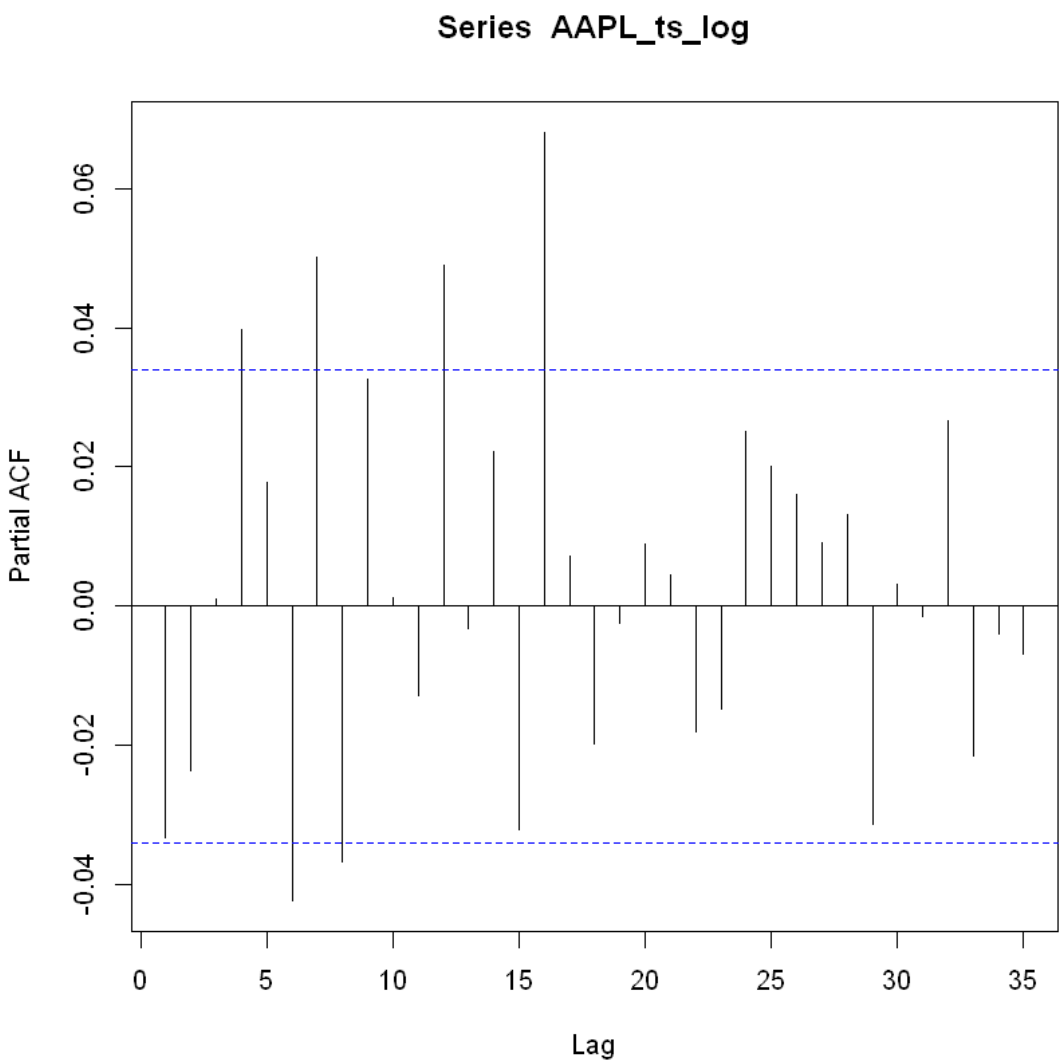
1.0.1 We notice that the time series experiences significant volatility hence, there is a chance it would benefit from a GARCH model

1.0.2 In order to fit an appropriate GARCH model, we need to carry out a ACF and PACF analysis of the time series.

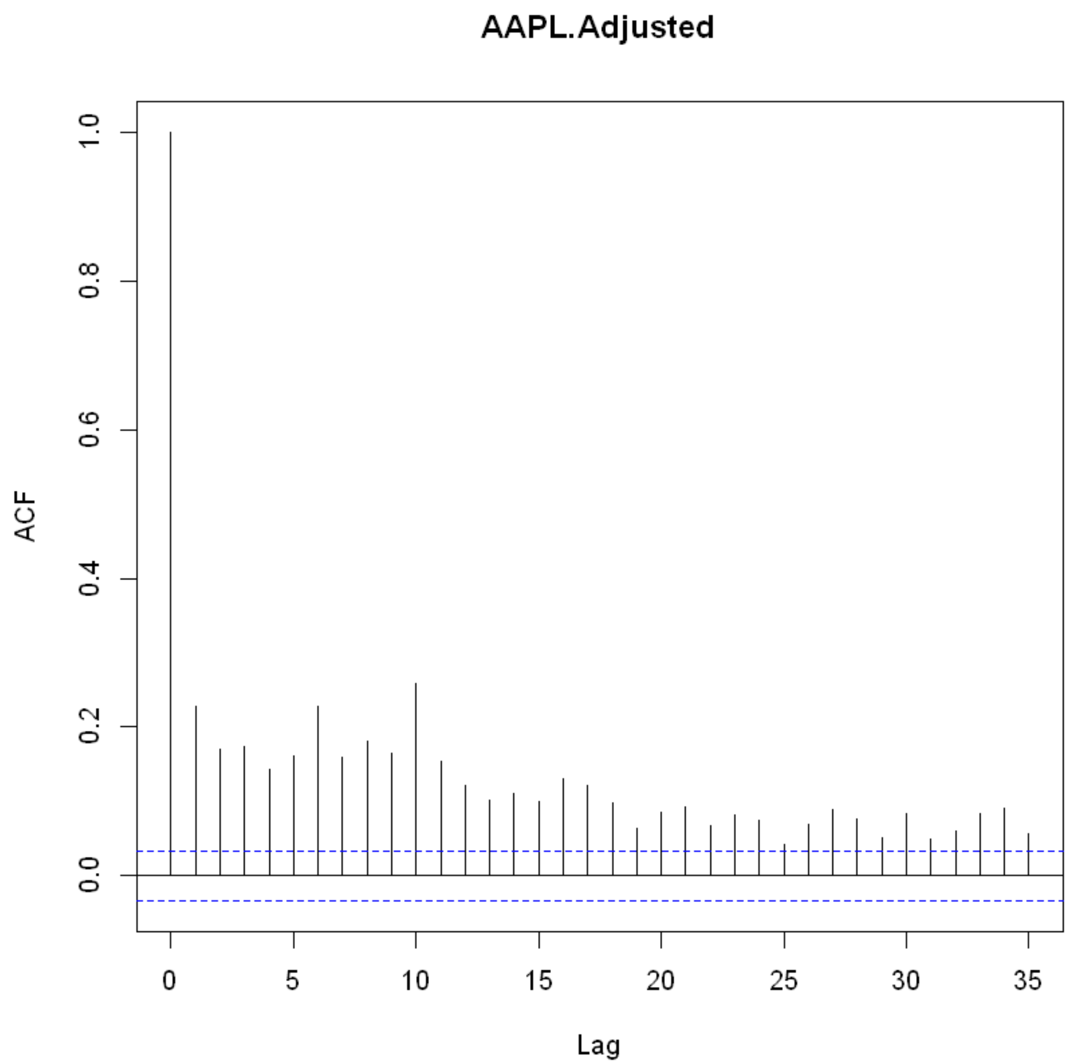
```
In [9]: acf(AAPL_ts_log)
```



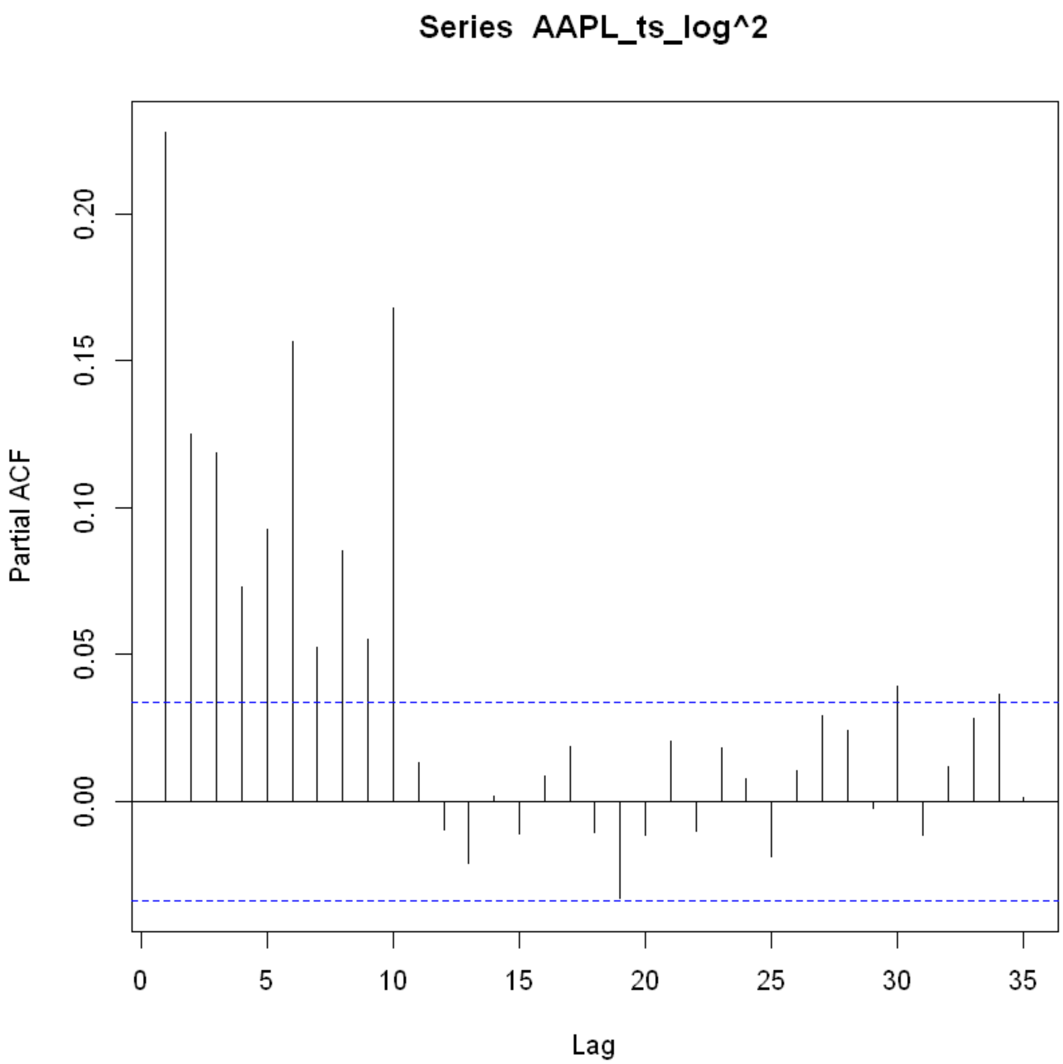

```
In [10]: pacf(AAPL_ts_log)
```




```
In [13]: ### inspection of the squared also yields
         acf(AAPL_ts_log^2)
```




```
In [14]: pacf(AAPL_ts_log^2)
```



The initial huge decay in the ACF curve is indicative that an ARMA model of 1 can capture the variance in the time series.

The Lag in the residuals plot persists which is quite indicative of expected behaviour.

We use the garch module to explore which garch model would be best for decribing the volatility in the time series

In [24]:
```
#Checking for a suitable GARCH model to use
garch(x=AAPL_ts_log,grad="numerical",trace=FALSE)
```

```
Call:
```

```
garch(x = AAPL_ts_log, grad = "numerical", trace = FALSE)

Coefficient(s):
       a0         a1         b1
0.0003421  0.0500001  0.0500000
```

The test confirms that a garch 1,1 model is suitable

2 Model Fitting

2.0.1 1. GARCH (1,1) Model with normally distributed errors

```
In [22]: garch11.spec = ugarchspec(variance.model=list(garchOrder=c(1,1)), mean.model=list(arma
         #Estimate the model
         garch11.fit = ugarchfit(spec=garch11.spec, data=AAPL_ts_log)
         garch11.fit
```

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model : sGARCH(1,1)
Mean Model : ARFIMA(0,0,0)
Distribution : norm

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.001855    0.000270   6.8808        0
omega   0.000014    0.000002   9.0459        0
alpha1  0.113564    0.001821  62.3633        0
beta1   0.853263    0.009360  91.1643        0

Robust Standard Errors:
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.001855    0.000359   5.1624 0.000000
omega   0.000014    0.000005   2.9990 0.002709
alpha1  0.113564    0.022238   5.1067 0.000000
beta1   0.853263    0.015317  55.7060 0.000000

LogLikelihood : 8722.727

Information Criteria
```

```
------------------------------------

Akaike       -5.2208
Bayes        -5.2135
Shibata      -5.2208
Hannan-Quinn -5.2182

Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                        statistic p-value
Lag[1]                      2.166  0.1411
Lag[2*(p+q)+(p+q)-1][2]     2.351  0.2106
Lag[4*(p+q)+(p+q)-1][5]     4.743  0.1748
d.o.f=0
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
                        statistic p-value
Lag[1]                     0.1169  0.7325
Lag[2*(p+q)+(p+q)-1][5]    1.2567  0.7994
Lag[4*(p+q)+(p+q)-1][9]    2.7840  0.7940
d.o.f=2

Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[3]    0.3011 0.500 2.000  0.5832
ARCH Lag[5]    2.0536 1.440 1.667  0.4595
ARCH Lag[7]    2.4581 2.315 1.543  0.6211

Nyblom stability test
------------------------------------
Joint Statistic:  8.5206
Individual Statistics:
mu     0.1835
omega  2.7431
alpha1 0.5676
beta1  0.7298

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:       1.07 1.24 1.6
Individual Statistic:  0.35 0.47 0.75

Sign Bias Test
------------------------------------
                   t-value     prob sig
Sign Bias           1.4851 0.137625
```

```
Negative Sign Bias  1.0991 0.271804
Positive Sign Bias  0.6372 0.524051
Joint Effect       11.7547 0.008272 ***


Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20     140.5    1.455e-20
2    30     152.3    1.122e-18
3    40     160.3    1.215e-16
4    50     166.3    1.118e-14


Elapsed time : 1.601127
```

Residual Diagnostics: Ljung Box tests for white noise behaviour in residuals. Since the residuals have p-values>0.05 and we fail to reject the null hypothesiis, there is no eveidence of autocorrelation in the residuals. Hence, we may conclude that the residuals behave as white noise

Test for ARCH behaviour in residuals: Analysing the standardized squared residuals and ARCH LM tests, the p-values> 0.05 and we fail to reject the null hypothesis. Hence, there is no evidence of serial correlation in squared residuals. This confirms that the residuals behave as a white noise process

Looking at the output for the goodness of fit test, since the p-values>0.05, the normal distribution assumption is strongly rejected

2.0.2 2. GARCH (1,1) Model with t-distribution

```
In [31]: garch11.t.spec = ugarchspec(variance.model=list(garchOrder=c(1,1)), mean.model=list(a
         #Estimate the model
         garch11.t.fit = ugarchfit(spec=garch11.t.spec, data=AAPL_ts_log)
         garch11.t.fit
```

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model : sGARCH(1,1)
Mean Model : ARFIMA(0,0,0)
```

```
Distribution : std

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.001602    0.000246   6.5164 0.000000
omega   0.000007    0.000004   2.0959 0.036095
alpha1  0.100853    0.013905   7.2529 0.000000
beta1   0.887990    0.016249  54.6492 0.000000
shape   4.709780    0.432270  10.8955 0.000000

Robust Standard Errors:
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.001602    0.000251  6.38710  0.00000
omega   0.000007    0.000008  0.95432  0.33992
alpha1  0.100853    0.018734  5.38357  0.00000
beta1   0.887990    0.024746 35.88361  0.00000
shape   4.709780    0.633550  7.43395  0.00000

LogLikelihood : 8875.626

Information Criteria
------------------------------------

Akaike       -5.3118
Bayes        -5.3026
Shibata      -5.3118
Hannan-Quinn -5.3085

Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                        statistic p-value
Lag[1]                      2.041  0.1531
Lag[2*(p+q)+(p+q)-1][2]     2.202  0.2314
Lag[4*(p+q)+(p+q)-1][5]     4.683  0.1803
d.o.f=0
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
                        statistic p-value
Lag[1]                  1.521e-05  0.9969
Lag[2*(p+q)+(p+q)-1][5] 8.426e-01  0.8942
Lag[4*(p+q)+(p+q)-1][9] 2.086e+00  0.8945
d.o.f=2

Weighted ARCH LM Tests
------------------------------------
```

```
            Statistic Shape Scale P-Value
ARCH Lag[3]    0.2083 0.500 2.000  0.6481
ARCH Lag[5]    1.5955 1.440 1.667  0.5676
ARCH Lag[7]    2.0381 2.315 1.543  0.7093

Nyblom stability test
------------------------------------
Joint Statistic:  3.8628
Individual Statistics:
mu     0.2303
omega  1.3510
alpha1 1.3915
beta1  1.6943
shape  2.0416

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:       1.28 1.47 1.88
Individual Statistic:  0.35 0.47 0.75

Sign Bias Test
------------------------------------
                   t-value     prob sig
Sign Bias           1.6239 0.104485
Negative Sign Bias  0.9811 0.326603
Positive Sign Bias  0.7088 0.478516
Joint Effect       12.6976 0.005338 ***


Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20     20.91       0.3418
2    30     34.77       0.2123
3    40     42.11       0.3380
4    50     56.05       0.2275


Elapsed time : 0.522681
```

Analyzing the result of this model displays based on the Ljung-Box test on squared esiduals, there is evidence of serial correlation as the p-values>0.05 and hence the null hypothesis of serial correlation can be rejected and we may conclude that the residuals behave as a white noice process

Looking at the goodness of fit, we observe that the p-values>0.05 hence we can not reject the null hypothesis that this model is adequate for this porcess

2.0.3 3. GARCH (1,1) Model with skewed t-distribution

In [32]:
```
garch11.skt.spec = ugarchspec(variance.model=list(garchOrder=c(1,1)), mean.model=list
#Estimate the model
garch11.skt.fit = ugarchfit(spec=garch11.skt.spec, data=AAPL_ts_log)
garch11.skt.fit
```

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model : sGARCH(1,1)
Mean Model : ARFIMA(0,0,0)
Distribution : sstd

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.001541    0.000271   5.6817  0.00000
omega   0.000007    0.000003   2.0870  0.03689
alpha1  0.100381    0.013830   7.2581  0.00000
beta1   0.888230    0.016304  54.4793  0.00000
skew    0.987384    0.023509  41.9998  0.00000
shape   4.731981    0.435996  10.8533  0.00000

Robust Standard Errors:
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.001541    0.000274  5.61753  0.00000
omega   0.000007    0.000008  0.95013  0.34204
alpha1  0.100381    0.018392  5.45783  0.00000
beta1   0.888230    0.025025 35.49394  0.00000
skew    0.987384    0.022804 43.29917  0.00000
shape   4.731981    0.642142  7.36906  0.00000

LogLikelihood : 8875.768

Information Criteria
------------------------------------

Akaike       -5.3112
Bayes        -5.3003
Shibata      -5.3112
```

```
Hannan-Quinn -5.3073

Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                        statistic p-value
Lag[1]                      2.037  0.1535
Lag[2*(p+q)+(p+q)-1][2]     2.197  0.2321
Lag[4*(p+q)+(p+q)-1][5]     4.681  0.1805
d.o.f=0
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
                        statistic p-value
Lag[1]                  0.0000398  0.9950
Lag[2*(p+q)+(p+q)-1][5] 0.8427702  0.8941
Lag[4*(p+q)+(p+q)-1][9] 2.0856333  0.8945
d.o.f=2

Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[3]    0.2063 0.500 2.000  0.6497
ARCH Lag[5]    1.5910 1.440 1.667  0.5687
ARCH Lag[7]    2.0348 2.315 1.543  0.7099

Nyblom stability test
------------------------------------
Joint Statistic:  3.8472
Individual Statistics:
mu     0.23388
omega  1.34489
alpha1 1.41203
beta1  1.71625
skew   0.05839
shape  2.03664

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:       1.49 1.68 2.12
Individual Statistic:  0.35 0.47 0.75

Sign Bias Test
------------------------------------
                   t-value     prob sig
Sign Bias           1.6473 0.099595   *
Negative Sign Bias  0.9741 0.330064
Positive Sign Bias  0.7034 0.481886
Joint Effect       12.8334 0.005011 ***
```

```
Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20     21.15       0.3286
2    30     32.75       0.2878
3    40     44.91       0.2380
4    50     59.16       0.1517


Elapsed time : 0.8654649
```

Looking at the output, we observe that the skewness value has p-value = 0<0.05 and hence, is significant. Since, the skew value<1(0.98), it indicates that the t-distribution is skewed to the right. The shape value has p-value=0<.05 and is significant. We might be interested in this model for the process looking further into the output. AIC value = -5.3112 and BIC value = -5.3003

Residual diagnostics: Ljung Box test for white noise behaviour in residuals. Since the residuals have p-values>0.05 and we fail to reject the null hypothesis, there is no evidence of autocorrelation in the residuals.Hence, we may conclude that the residuals behave as hite noise.

Test for ARCH beaviour in residuals: Looking at the standaridized squared residuals and ARCH LM Tests, the p-values>0.05 and we fail to reject the null hypothesis hence there is no evidence of serial correlation in squared residuals. This confirms that the residuals behave as a white noise process.

Looking at the output for the goodness of fit test, since the p-values>0.05, the null hypothesis can't be rejected and hence this model is a good fit

2.0.4 4. eGARCH (1,1) Model with t-distribution

In [34]:
```
egarch11.t.spec = ugarchspec(variance.model=list(model='eGARCH', garchOrder=c(1,1)), m
#Estimate the model
egarch11.t.fit = ugarchfit(spec=egarch11.t.spec, data=AAPL_ts_log)
egarch11.t.fit
```

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
```

```
GARCH Model : eGARCH(1,1)
Mean Model : ARFIMA(0,0,0)
Distribution : std

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.001363    0.000261   5.2247        0
omega  -0.228102    0.020033 -11.3862        0
alpha1 -0.093095    0.012394  -7.5112        0
beta1   0.971920    0.002447 397.1779        0
gamma1  0.198029    0.020375   9.7191        0
shape   5.101985    0.447539  11.4001        0

Robust Standard Errors:
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.001363    0.000299   4.5640    5e-06
omega  -0.228102    0.010203 -22.3568    0e+00
alpha1 -0.093095    0.013041  -7.1384    0e+00
beta1   0.971920    0.001257 772.9848    0e+00
gamma1  0.198029    0.022739   8.7088    0e+00
shape   5.101985    0.444911  11.4674    0e+00

LogLikelihood : 8912.915

Information Criteria
------------------------------------

Akaike       -5.3335
Bayes        -5.3225
Shibata      -5.3335
Hannan-Quinn -5.3296

Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                        statistic p-value
Lag[1]                      3.096 0.07849
Lag[2*(p+q)+(p+q)-1][2]     3.271 0.11853
Lag[4*(p+q)+(p+q)-1][5]     5.680 0.10686
d.o.f=0
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
                        statistic p-value
Lag[1]                   0.007769  0.9298
Lag[2*(p+q)+(p+q)-1][5]  0.450007  0.9651
Lag[4*(p+q)+(p+q)-1][9]  1.224844  0.9750
```

```
d.o.f=2

Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[3]    0.3304 0.500 2.000  0.5654
ARCH Lag[5]    0.6916 1.440 1.667  0.8260
ARCH Lag[7]    1.1720 2.315 1.543  0.8841

Nyblom stability test
------------------------------------
Joint Statistic:  3.8695
Individual Statistics:
mu     0.7093
omega  1.7783
alpha1 0.3786
beta1  1.6444
gamma1 0.1963
shape  1.4981

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:       1.49 1.68 2.12
Individual Statistic:  0.35 0.47 0.75

Sign Bias Test
------------------------------------
                   t-value   prob sig
Sign Bias          1.55035 0.1212
Negative Sign Bias 0.43840 0.6611
Positive Sign Bias 0.06576 0.9476
Joint Effect       3.82708 0.2808


Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20     21.51       0.3094
2    30     35.25       0.1964
3    40     41.37       0.3678
4    50     57.01       0.2019


Elapsed time : 0.8189549
```

The above R output displays an AR(0) mean model with standard Egarch(1,1) model for variance with t-distribution.We look at the alpha value and since alpha1 < 0, the leverage effect is significant and we may conclude that the volatility reacts more havily to negative shocks.

The shape parameter is significant as the p-value < 0.05, indicating that the t-distibution is a good choice.

AIC value = -5.3335 and BIC value = -5.225

Residual diagnostics: All the p-values for the Ljung Box Test of residuals are > 0.05, thus indicating that there is no evidence of serial correlation in the squared residuals and hence, they behave as white noise process.

Looking at the test for goodness-of-fit, since all the p-values > 0.05, we cant reject the null hypothesis, and hence we may conclude that the Egarch model with the t-distribution is a good choice.

2.0.5 5. fGARCH (1,1) Model with t-distribution

In [37]:
```
fgarch11.t.spec = ugarchspec(variance.model=list(model='fGARCH', garchOrder=c(1,1), su
#Estimate the model
fgarch11.t.fit = ugarchfit(spec=fgarch11.t.spec, data=AAPL_ts_log)
fgarch11.t.fit
```

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model : fGARCH(1,1)
fGARCH Sub-Model : APARCH
Mean Model : ARFIMA(0,0,0)
Distribution : std

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.001342    0.000245   5.4853  0.00000
omega   0.000651    0.000410   1.5879  0.11231
alpha1  0.116835    0.013614   8.5822  0.00000
beta1   0.881546    0.014553  60.5736  0.00000
eta11   0.501722    0.074898   6.6987  0.00000
lambda  0.978302    0.145954   6.7028  0.00000
shape   5.154519    0.452045  11.4027  0.00000
```

```
Robust Standard Errors:
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.001342    0.000263   5.1058  0.00000
omega   0.000651    0.000374   1.7438  0.08119
alpha1  0.116835    0.016285   7.1744  0.00000
beta1   0.881546    0.018131  48.6219  0.00000
eta11   0.501722    0.075552   6.6408  0.00000
lambda  0.978302    0.133566   7.3245  0.00000
shape   5.154519    0.442366  11.6522  0.00000

LogLikelihood : 8916.532

Information Criteria
------------------------------------

Akaike       -5.3350
Bayes        -5.3222
Shibata      -5.3351
Hannan-Quinn -5.3305

Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                        statistic p-value
Lag[1]                      4.217 0.04001
Lag[2*(p+q)+(p+q)-1][2]     4.441 0.05741
Lag[4*(p+q)+(p+q)-1][5]     6.816 0.05745
d.o.f=0
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
                        statistic p-value
Lag[1]                    0.02268  0.8803
Lag[2*(p+q)+(p+q)-1][5]   0.41915  0.9694
Lag[4*(p+q)+(p+q)-1][9]   0.97239  0.9874
d.o.f=2

Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[3]    0.2199 0.500 2.000  0.6391
ARCH Lag[5]    0.6739 1.440 1.667  0.8314
ARCH Lag[7]    0.8529 2.315 1.543  0.9363

Nyblom stability test
------------------------------------
Joint Statistic:  3.9811
Individual Statistics:
```

```
mu     0.8273
omega  1.7636
alpha1 1.9315
beta1  2.0912
eta11  0.8056
lambda 1.8144
shape  2.1242

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:       1.69 1.9 2.35
Individual Statistic:  0.35 0.47 0.75

Sign Bias Test
------------------------------------
                   t-value   prob sig
Sign Bias           1.1805 0.2379
Negative Sign Bias  0.6998 0.4841
Positive Sign Bias  0.3452 0.7299
Joint Effect        2.6557 0.4478


Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20     27.75      0.08839
2    30     32.75      0.28775
3    40     40.77      0.39267
4    50     53.14      0.31765


Elapsed time : 4.333713
```

The shape parameter is significant as the p-value < 0.05, indicating that the t-distibution is a good choice.

AIC value = -5.3350 and BIC value = -5.3222

Residual diagnostics: All the p-values for the Ljung Box Test of residuals are > 0.05, thus indicating that there is no evidence of serial correlation in the squared residuals and hence, they behave as white noise process.

Looking at the test for goodness-of-fit, since all the p-values > 0.05, we cant reject the null hypothesis, and hence we may conclude that the fgarch model with the t-distribution is a good choice.

2.0.6 6. iGARCH (1,1) Model with t-distribution

In [38]:
```
igarch11.t.spec = ugarchspec(variance.model=list(model='iGARCH', garchOrder=c(1,1)), m
#Estimate the model
igarch11.t.fit = ugarchfit(spec=igarch11.t.spec, data=AAPL_ts_log)
igarch11.t.fit
```

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model : iGARCH(1,1)
Mean Model : ARFIMA(0,0,0)
Distribution : std

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.001593    0.000244   6.5192 0.000000
omega   0.000006    0.000003   2.1939 0.028241
alpha1  0.105945    0.018884   5.6103 0.000000
beta1   0.894055          NA       NA       NA
shape   4.428983    0.309265  14.3210 0.000000

Robust Standard Errors:
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.001593    0.000249   6.3875 0.000000
omega   0.000006    0.000005   1.1378 0.255200
alpha1  0.105945    0.032364   3.2736 0.001062
beta1   0.894055          NA       NA       NA
shape   4.428983    0.348376  12.7132 0.000000

LogLikelihood : 8874.645

Information Criteria
------------------------------------

Akaike       -5.3118
Bayes        -5.3044
Shibata      -5.3118
Hannan-Quinn -5.3091

Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                        statistic p-value
Lag[1]                      2.069  0.1503
```

```
Lag[2*(p+q)+(p+q)-1][2]     2.222  0.2284
Lag[4*(p+q)+(p+q)-1][5]     4.726  0.1764
d.o.f=0
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
                        statistic p-value
Lag[1]                  2.135e-05  0.9963
Lag[2*(p+q)+(p+q)-1][5] 8.594e-01  0.8906
Lag[4*(p+q)+(p+q)-1][9] 2.059e+00  0.8978
d.o.f=2

Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[3]    0.2422 0.500 2.000  0.6226
ARCH Lag[5]    1.5412 1.440 1.667  0.5815
ARCH Lag[7]    2.0667 2.315 1.543  0.7032

Nyblom stability test
------------------------------------
Joint Statistic:  3.2769
Individual Statistics:
mu     0.2278
omega  0.4526
alpha1 1.0978
shape  1.7291

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:       1.07 1.24 1.6
Individual Statistic:  0.35 0.47 0.75

Sign Bias Test
------------------------------------
                   t-value     prob sig
Sign Bias           1.5641 0.117897
Negative Sign Bias  0.7896 0.429795
Positive Sign Bias  0.8984 0.369027
Joint Effect       12.2106 0.006695 ***


Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20     23.65      0.20981
2    30     42.71      0.04845
3    40     52.67      0.07072
```

```
4    50     64.46      0.06835


Elapsed time : 0.445576
```

The shape parameter is significant as the p-value < 0.05, indicating that the t-distibution is a good choice.

AIC value = -5.3118 and BIC value = -5.3044

Residual diagnostics: All the p-values for the Ljung Box Test of residuals are > 0.05, thus indicating that there is no evidence of serial correlation in the squared residuals and hence, they behave as white noise process.

Looking at the test for goodness-of-fit, since all the p-values > 0.05, we cant reject the null hypothesis, and hence we may conclude that the igarch model with the t-distribution is a good choice.

2.1 Model Selection

2.1.1 Analysing the performance of the fitted models. Models 2 to Models 6 perform suitably

2.1.2 Models 4 and 5, the fGARCH and eGARCH models perform best and scores are quite similar.

2.1.3 Based on the AIC score we select fGARCH as the most parsimonious model

2.2 Forecasting

```
In [48]: fgarch11.t.fit = ugarchfit(spec=fgarch11.t.spec, data=AAPL_ts_log, out.sample=100)
         f=ugarchforecast(fgarch11.t.fit, n.ahead=20, n.roll=10)
         f
```

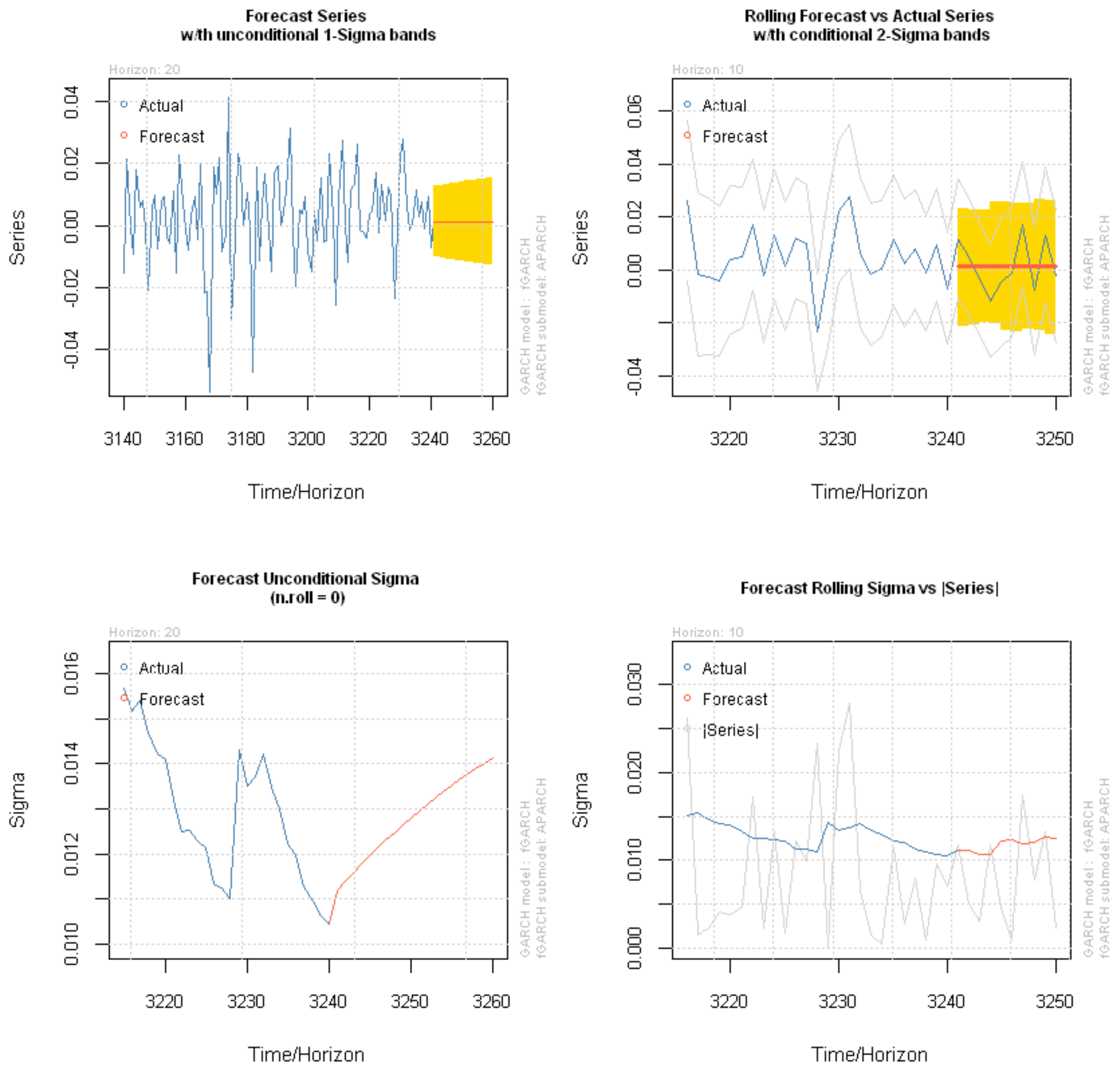
```
*------------------------------------*
*       GARCH Model Forecast         *
*------------------------------------*
Model: fGARCH
fGARCH Sub-Model: APARCH

Horizon: 20
Roll Steps: 10
Out of Sample: 20

0-roll forecast [T0=3240-01-01]:
       Series   Sigma
T+1  0.001302 0.01120
```

```
T+2  0.001302 0.01140
T+3  0.001302 0.01159
T+4  0.001302 0.01178
T+5  0.001302 0.01196
T+6  0.001302 0.01214
T+7  0.001302 0.01231
T+8  0.001302 0.01248
T+9  0.001302 0.01264
T+10 0.001302 0.01279
T+11 0.001302 0.01295
T+12 0.001302 0.01309
T+13 0.001302 0.01324
T+14 0.001302 0.01338
T+15 0.001302 0.01351
T+16 0.001302 0.01364
T+17 0.001302 0.01377
T+18 0.001302 0.01390
T+19 0.001302 0.01402
T+20 0.001302 0.01413
```

```
In [49]: plot(f, which="all")
```


Forecast Series
w/th unconditional 1-Sigma bands
Horizon: 20
Actual
Forecast
Series
Time/Horizon
GARCH model : fGARCH
fGARCH submodel: APARCH
Rolling Forecast vs Actual Series
w/th conditional 2-Sigma bands
Horizon: 10
Actual
Forecast
Series
Time/Horizon
GARCH model : fGARCH
fGARCH submodel: APARCH
Forecast Unconditional Sigma
(n.roll = 0)
Horizon: 20
Actual
Forecast
Sigma
Time/Horizon
GARCH model : fGARCH
fGARCH submodel: APARCH
Forecast Rolling Sigma vs |Series|
Horizon: 10
Actual
Forecast
|Series|
Sigma
Time/Horizon
GARCH model : fGARCH
fGARCH submodel: APARCH