An Asset Allocation Strategy using Modern Portfolio Theory and CAPM

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This proposal aims to provide a concise overview of Modern Portfolio Theory, the Capital Asset Pricing Model, and our utilization of these concepts to formulate a strategy that fulfills the requirements outlined in the Request for Proposal (RFP). Furthermore, we aim to demonstrate how this strategy not only meets the RFP criteria but also achieves superior performance to the S&P 500 in our backtesting analysis. The proposed algorithm outperforms the S&P 500 by 319% in regards to cumulative returns and more than doubles the Sharpe ratio, while ensuring the investment stays above 80% of the high-water mark.

Strategy

Navigating the ever-changing expanse of financial markets demands an astute ability to allocate assets effectively, balancing optimal returns with risk management. As the global economy evolves with intricacies, the integration of robust theoretical frameworks takes on paramount significance. This study employs two pivotal pillars of modern finance—Modern Portfolio Theory (MPT) and the Capital Asset Pricing Model (CAPM)—to craft an algorithm for strategic asset buying and selling.

Modern Portfolio Theory, introduced by Harry Markowitz in the 1950s, ushered in a paradigm shift in investment strategy by championing diversification as a means to amplify portfolio performance. By recognizing that an asset's risk and return attributes are not standalone determinants but interconnected within a broader portfolio context, MPT lays the groundwork for constructing portfolios that either maximize returns for a given risk level or minimize risk for a desired return level. Fortunately, the optimization of portfolio problems adheres to convexity, encompassing the following form:

$$\begin{aligned} \underset{\omega}{\text{minimize}} \quad & \omega^T \Sigma \omega \\ \text{subject to} \quad & R^T \omega = \mu \\ & A\omega = b \\ & C\omega \geq d. \end{aligned}$$

In this formulation, Σ symbolizes the covariance matrix of asset returns within the portfolio, μ the desired return level, ω denotes a vector of portfolio weights with $\sum_i \omega_i = 1$ (since we don't allow short selling these weights are always positive), and $R = (R_1, R_2, ...)^T$ is a vector of expected returns. We further assume that the set of admissible portfolios is a nonempty polyhedral set and it is represented by $\{\omega : A\omega = b, C\omega \geq d\}$, where A is a $m \times n$ matrix, b is a m-dim vector, C is a $p \times n$ matrix and d is p-dim vector. Resolving this optimization necessitates knowledge of anticipated returns and quantification of asset risk via the covariance matrix. To address the former, we turn to the Capital Asset Pricing Model, which further refines the relationship between risk and return by introducing the systematic risk component, beta. CAPM provides a framework for determining the expected return of an asset relative to its systematic risk:

$$R_i = r_f + \beta_i(R_m - r_f),$$

where r_f denotes the risk-free rate, and R_m encapsulates expected market returns. For the covariance matrix, the sample covariance is the default choice, but because of its susceptibility to extreme errors, we use a method called covariance shrinkage, where we move extreme values toward the center. This technique leads to a much more stable covariance matrix and enhances overall performance. For more information see [1].

In general, our strategy aims to identify market conditions that result in a positive overall market trend. In such situations, the model will invest in a diversified portfolio using Modern Portfolio Theory and the Capital Asset Pricing Model. Conversely, when specific indicators point to negative market trends, the model will take action to minimize losses by allocating a significant portion of the investment to a low-volatility protection asset.

To identify these two market scenarios, we utilize the Volatility Index (VIX) and the Deutsche Bank Multi-Asset Risk Monitor Signal (DBMARSS). We assume that volatility is inversely correlated with the market trend. After conducting rigorous testing and analysis on the provided historical data, we established the following subsequent threshold for the Volatility Index

$$\text{VIX threshold} := \min(23, \text{mean of VIX for the last 9 days})$$

to distinguish between positive and negative expected market trends. Additionally, a DBMARSS value of zero signals a positive market trend, while a DBMARSS value of one signals a negative market trend.

In the context of both VIX and DBMARSS indicating positive market trends, our model will allocate the portfolio with respect to the maximal Sharpe ratio. The convex problem for this has the following form:

$$\begin{aligned} \underset{\omega}{\text{maximize}} \quad & \frac{R^T\omega - r_f}{(\omega^T\Sigma\omega)^{\frac{1}{2}}} \\ \text{subject to} \quad & \sum_i \omega_i = 1. \end{aligned}$$

Should only one of the indicators yield positive trends, we will take a more secure approach and optimize the portfolio for minimal volatility, yielding the following optimization problem.

$$\begin{aligned} \underset{\omega}{\text{minimize}} \quad & \omega^T\Sigma\omega \\ \text{subject to} \quad & \sum_i \omega_i = 1. \end{aligned}$$

In the event of negative market trends, i.e., both values VIX and DBMARSS indicate a downward market trajectory, we further differentiate between short-term and long-term trends and take appropriate defensive measures. We allocate either 50%, or 99.5% on consecutive days, of the portfolio value to the protective asset. We choose 99.5% instead of a flat 100% to ensure that at all times money is invested in any of the Investment Assets. The remaining value is channeled into optimization for minimal volatility. This strategic approach serves to shield the portfolio from excessive market volatility while yielding favorable returns.
The daily asset allocation follows this pattern:

- Assess VIX and DBMARSS values.
- **IF** VIX aligns within the defined threshold and DBMARSS equals zero:
  - Compute expected returns and the covariance matrix, inclusive of the protection asset.
  - Optimize the portfolio for maximum Sharpe ratio.
- **ELIF** Vix strays from the bound or DBMARSS equals one:
  - Compute expected returns and the covariance matrix, inclusive of the protection asset.
  - Optimize for minimal volatility.
- **ELIF** Both indices deviate:
  - Allocate a portion (50%, or 99.5% for consecutive deviations over multiple days) of the current portfolio's value to the protection asset.
  - Compute expected returns and the covariance matrix, exclusive of the protection asset.
  - Allocate the remaining value by optimizing for minimal volatility.

- Using the weights derived from the optimization process, we generate a new optimal portfolio and BUY/SELL orders accordingly.

Results

Our strategy underwent backtesting from January 2007 to August 2023. Addressing the common issue of missing values in time-series analysis, we applied forward-filling to rows containing at least four non-NA values, the remaining rows were dropped. The risk-free rate was set at 2%.

Over the entire analyzed period, we surpassed the benchmark in terms of cumulative returns, 532.38% to 213.14%.

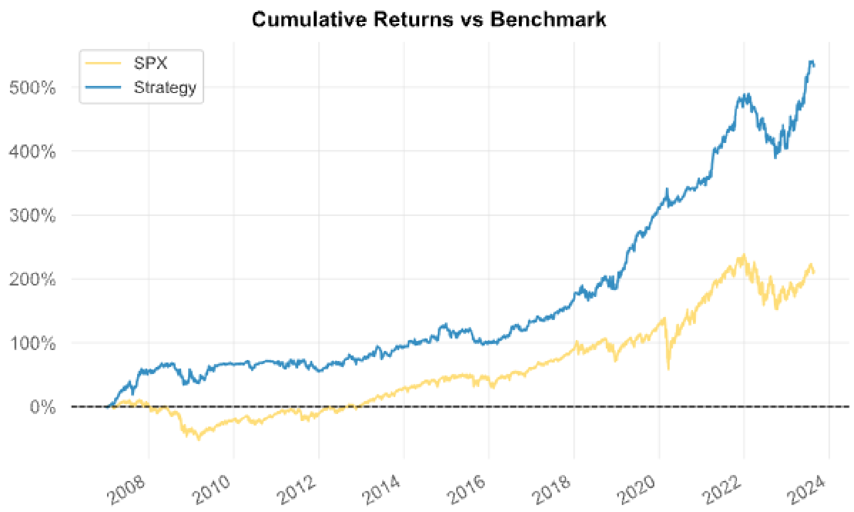

Figure 1: Cumulative Returns

Our strategy showcases its robustness amidst periods of market turbulence, exemplified by its performance during the 2007/2008 financial crisis. Throughout this critical phase, our strategy weathered a substantial maximum drawdown of 19.93%, which also stands as the maximum drawdown observed over the entire tested timeframe. This starkly contrasts with the benchmark's drawdown of 56.78%, as detailed in Table (b). It's worth noting that during this financial crisis, the portfolio encompassed a maximum of four assets (with just two before June 2007), inclusive of the protection asset. We can speculate that with a more extensive array of instruments in our portfolio, the drawdown might have been less severe. In a parallel manner, amid the market downturn in March 2020, our strategy incurred a mere 6.73% maximum drawdown.

Moreover, when evaluating risk-adjusted cumulative returns, our strategy's performance garners even greater distinction. A pronounced upward trajectory becomes evident from 2019 onward, aligning seamlessly with the introduction of the last two investment assets, see Figure 2. Upon reviewing the subsequent tables, it's evident

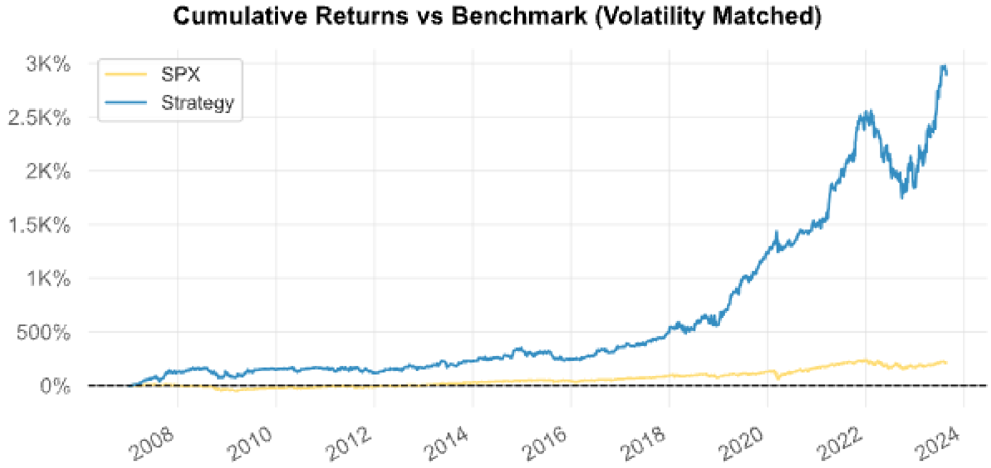

Figure 2: Risk-Adjusted Cumulative Returns

that we surpass the benchmark in key metrics such as the annualized Sharpe ratio and the Sortino ratio. Notably, our model exhibits nearly half the annualized volatility compared to the benchmark. This outperformance persists even during the financial crisis. (left: SPX right: Model)

Sharpe	0.34	0.94
Prob. Sharpe Ratio	68.83%	99.95%
Smart Sharpe	0.32	0.89
Sortino	0.47	1.38
Smart Sortino	0.44	1.29
Sortino/√2	0.33	0.97
Smart Sortino/√2	0.31	0.91
Omega	1.21	1.21

(a) Sharpe Ratio

Max Drawdown	-56.78%	-19.93%
Longest DD Days	1996	708
Volatility (ann.)	20.3%	10.48%
R^2	0.08	0.08
Information Ratio	0.01	0.01
Calmar	0.09	0.41
Skew	-0.26	-0.2
Kurtosis	11.86	5.76

(b) Max Drawdown

Best Day	11.58%	4.12%
Worst Day	-11.98%	-4.6%
Best Month	12.68%	11.1%
Worst Month	-16.94%	-10.52%
Best Year	29.97%	55.24%
Worst Year	-38.49%	-14.17%

(a) Best Day

Expected Daily	0.03%	0.04%
Expected Monthly	0.57%	0.93%
Expected Yearly	6.95%	11.46%
Kelly Criterion	0.34%	7.16%
Risk of Ruin	0.0%	0.0%
Daily Value-at-Risk	-2.07%	-1.04%
Expected Shortfall (cVaR)	-2.07%	-1.04%

(b) Expected Daily Returns

Limitations

Our framework operates under the assumption of no transaction costs. We therefore acknowledge that our model would overestimate rates of returns when applied in a real-world context, where transaction costs are applicable.

Regarding our model's structure, it can primarily be divided into two layers. The outer layer is the identification of specific market conditions, positive or negative market trends. The inner layer focuses on portfolio allocation based on these conditions, primarily centered around positive market trends.
Within the inner layer, we establish two key assumptions:
Firstly, we presume that the covariances among assets, as well as between assets and the benchmark, can be deduced from past historical data. These covariances, which hold a pivotal role at the core of our model, are expected to offer a reasonably accurate projection of near-future covariances. Mathematically speaking, we assume a continuous evolution of covariances. While this assumption is highly plausible, it's conceivable that sudden shifts in covariance may arise, potentially impacting the model's performance.
Secondly, we assert that the Capital Asset Pricing Model presents a robust predictive framework for future asset returns. This assertion finds substantial empirical validation and is widely accepted in the financial community, lending credence to its validity. Nonetheless, it is pivotal to underscore the foundational nature of this assumption in shaping the model's predictive efficacy. Additionally, it is noteworthy that the initial premise of continuous covariance shifts directly influences the CAPM model's predictions, considering that covariance plays a pivotal role in the computation of beta coefficients.
Turning to the outer layer of our analysis, a fundamental premise assumes the predictive power of our two indicators—the VIX index and the DBMAR Signal—in shaping forthcoming market trends. This foundational assumption plays an unequivocal role in underpinning the model's efficacy. Delving into the VIX index, as discussed in the strategy section, we posit an inverse correlation between market volatility and market returns. Similarly, with regard to the DBMARSS, we anticipate it to yield a signal to market risk. While these assumptions are widely accepted and reasonable, improbable scenarios could undermine their validity. In such instances, the model's susceptibility to exceeding the 20% threshold for maximum drawdowns looms as a possible outcome.

Outlook

Numerous avenues for enhancing this model persist. One way involves transitioning the model's binary response to the two market trend indicators to a continuous spectrum. This modification would imbue the optimization objective with a gradual progression from the portfolio with a maximal Sharpe ratio to a portfolio with minimal volatility.

References

[1] Ledoit, O., & Wolf, M.: *Honey, I Shrunk the Sample Covariance Matrix. The Journal of Portfolio Management, 30(4), 110–119.* (2004)