# Optimization

## Assignment 1

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### Q1

Inserting the midpoint rule (2):

Into with eq. (1)

To parameterize the line :

Where and

Where is derived from

### Q2

The relationship between and is linear:

And is calculated using the diagram in figure (1) and the equation (\*) from the previous question in the included matlab script.

### Q3

The derivative defined in the question is the same as matlabs [Gx, Gy] = gradient(X) function.

If the X matrix is small enough calculating matrices Dx, Dy is feasible and is done using the matlab function function [Dx, Dy] = CreateDerivativeOperators(X\_rows, X\_cols)

### Q4

In the following figures we will display the images X1, X2, X3 and the magnitude of their gradients.





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### Q5

The number of unknown parameters is 5x5=25. The number of observations is 8. The rank of A is however 7.

## Q6

Let's take the equation

(\*)

From the KKT conditions we know that the Lagrangian for this is:

This is the dual form. If we set then eq. (\*) is equivalent to

(3)

Likewise by taking the following

(\*\*)

Using KKT conditions the Lagrangian is:

We can set and and then by minimizing the Lagrangian we get the equivalent of eq. (3)

### Q7

TBC

### Q8

A simple closed form solution for eq. (3) can be developed by expressing the equation as

This equation can be minimized by the LS equation:

### Q9

As shown in Q8

Obviously , and

From multivariate calculus we know that

This is all we need for gradient descent.

### Q14

Prove that

Proof:

For a regularization matrix and unknown vector

The of can be written as:

Let's begin with a derivative of

So it follows that

Note:

QED

Prove that and find the entries of :

Proof:

From the previous section we know that:

Where

In order to accommodate for we can define

QED

First order optimality condition for

Is

Begin by opening up the eq. 6

From here we can derive an iterative equation and by fixing W as constant at each step

,

To summarize

Initialization:

For k = 1,2,…

end