UAEU College of Science كلية العلوم
Department of Mathematical Sciences

جامعة الإمارات العربية المتحدة
United Arab Emirates University
قسم علـوم الرياضيـــــات

Math 140
Linear Algebra

Test 2 Fall 2018

Student Name:..... **Instructor: .....**

ID number:..... **Section number:**

Please write your answers in detail and show your work to get full mark. Giving short answers might cause you losing points.

Question	1	2	3	4	5	Total	
Grade	8	8	8	8	8	40	

1. Let P_2 be the vector space of polynomials of degree at most 2, with the usual polynomial addition and scalar multiplication.
   (a) Let $S = \{ \quad a_0 + a_1 x + a_2 x^2 \quad | \quad a_0 + a_1 - a_2 = 0 \quad \}$. Is S a vector subspace?

this question is not covered in Test 2

(b) Let $V = \{ \quad a_0 + a_1 x + a_2 x^2 \quad | \quad a_0, a_1, a_2 \text{ are integers} \quad \}$. Is V a vector subspace?

2. Let R^3 be the usual 3-dimensional Euclidean vector space (with the usual vector addition and scalar multiplication).

   (a) Let $v_1 = (1, 2, 3)$ and $v_2 = (2, 3, 1)$. Are v_1 and v_2 linearly independent?

   (b) Let $v_1 = (2, 0, 1)$, $v_2 = (-1, 3, -1)$, $v_3 = (0, 6, -1)$, $v_4 = (3, 2, 1)$ and $v_5 = (-2, 6, -2)$. Let $S = \{v_1, v_2, v_3, v_4, v_5\}$. Is R^3 spanned by all these vectors? (Or, in other words, is $Span(S) = R^3$ true?)

3. Let P_2 be the vector space of polynomials of degree at most 2, with the usual polynomial addition and scalar multiplication. Determine whether the following sets of vectors form a basis for P_2. Explain your answer briefly please.

   (a) $p_1 = 1 - x + 5x^2$, $p_2 = -3 + 2x + 7x^2$, $p_3 = 6x - x^2$.

   (b) $p_1 = 2 - x + x^2$, $p_2 = 3 - 10x + 6x^2$.

4. Given

$$M = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 6 & 13 \\ -1 & -2 & -2 & -4 \\ 2 & 4 & 6 & 8 \end{bmatrix}$$

(a) Find a basis of the null space. Find also the nullity of the matrix M.

(b) Find a basis of the column space, and a basis for the row space. Find also the rank of the matrix M.

5. Given that matrix A has size 5×6.

(a) What is the maximum possible value of rank(A)? What is the minimum possible value of nullity(A)?

(b) List an example of A such that rank(A)=3 and nullity(A)=3. (Note: You only need to list one matrix A. No further explanation or computation is required.)

(c) What is the minimum possible value of rank(A)? What is the maximum possible value of nullity(A)?