41 Таблица разложений основных ф-ий по формуле Тейлора.

1) $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \ldots + \frac{x^n}{n!} + o(x^n) \ (x \to 0)$

2) $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots + \frac{(-1)^n \cdot x^{2n}}{(2n)!} + o(x^{2n+1}) \ (x \to 0)$

3) $\sin x = 1 - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots + \frac{(-1)^n x^{2n+1}}{(2n+1)!} + o(x^{2n+2}) \ (x \to 0)$

4) $(1+x)^\mu = 1 + \mu x + \frac{\mu(\mu-1)x^2}{2!} + \ldots + \frac{\mu(\mu-1)\ldots(\mu-n+1)x^n}{n!} + o(x^n) \ (x \to 0) \ \mu \in \mathbb{R}$

5) $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots + \frac{(-1)^{n-1} x^n}{n!} + o(x^n) \ (x \to 0)$

Д-во: 1) $f(x) = e^x$, $f^{(n)}(x) = e^x$, $f^{(k)}(0) = e^0 = 1$

$\frac{f^{(k)}(0) x^k}{k!} = \frac{x^k}{k!} \Rightarrow e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \ldots + \frac{x^n}{n!} + o(x^n)$

2) $f(x) = \cos x$, $f^{(k)}(x) = \cos(x + \frac{k\pi}{2})$, $f^{(k)}(0) = \cos\frac{\pi k}{2} = \begin{cases} (-1)^n, & k = 2n \\ 0, & k = 2n+1 \end{cases}$

$\frac{f^{(2n)}(0) x^{2n}}{(2n)!} = \frac{(-1)^n x^{2n}}{(2n)!}$; $\cos x = 1 + 0 \cdot x - \frac{x^2}{2!} + \frac{x^3 \cdot 0}{} + \ldots + \frac{(-1)^n x^{2n}}{(2n)!} + o(x^{2n+1}) \ (x \to 0)$

3) $f(x) = \sin x$; $f^{(k)}(x) = \sin(x + \frac{\pi k}{2})$, $f^{(k)}(0) = \sin\frac{\pi k}{2} = \begin{cases} 0, & k = 2n \\ (-1)^n, & k = 2n+1 \end{cases}$

$\sin x = 0 + \frac{1 \cdot x}{1!} + \frac{0 x^2}{2!} - \frac{x^3}{3!} + \ldots + \frac{(-1)^n \cdot x^{2n+1}}{(2n+1)!} + o(x^{2n+2}) \ (x \to 0)$

4) $f(x) = (1+x)^\mu$, $f^{(k)}(x) = \mu(\mu-1)\ldots(\mu-k+1)(1+x)^{\mu-k}$, $f^{(k)}(0) = \mu(\mu-1)\ldots(\mu-k+1) \Rightarrow$

$\Rightarrow (1+x)^\mu = 1 + \mu x + \frac{\mu(\mu-1)x^2}{2!} + \ldots + \frac{(\mu)(\mu-1)\ldots(\mu-n+1)x^n}{n!} + o(x^n) \ (x \to 0)$

5) $f(x) = \ln(1+x)$, $f^{(k)}(x) = \frac{(-1)^{k+1}(k-1)!}{(1-x)^k}$, $f^{(k)}(0) = \ln 1 = 0$, $k > 1$, $f^{(k)}(0) = (-1)^{k-1}(k-1)!$

$\frac{f^{(k)}(0) x^k}{k!} = \frac{(-1)^{k-1}(k-1)! x^k}{k!} = \frac{(-1)^{k-1} \cdot x^k}{k}$; $\ln(1+x) = 0 + x - \frac{x^2}{2!} + \ldots + \frac{(-1)^{n-1} x^n}{n!} + o(x^n) \ (x \to 0)$