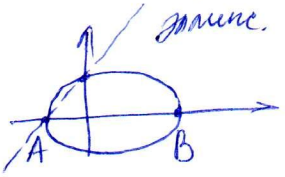
(66) Интеграл вида $\int R(x, \sqrt{ax^2+bx+c})dx$. Случай $a<0$. Пример

$\int R(x, \sqrt{ax^2+bx+c})dx$, $a<0$, $b^2-4ac>0$, $ax^2+bx+c = a(x-\alpha)(x-\beta)$, $\alpha<\beta$

$y^2 = a\left(x+\frac{b}{2a}\right)^2 = \frac{4ac-b^2}{4a} > 0$



$y = t(x-\alpha)$

$y^2 = t^2(x-\alpha)^2 = a(x-\alpha)(x-\beta)$

$t^2x - t^2\alpha = ax - a\beta$

$x = \frac{t^2\alpha - a\beta}{t^2-a} = \varphi(t)$; $y = \frac{t(t^2\alpha-a\beta)}{t^2-a} - \alpha = \psi(t)$.

Пример: ① $\int \sqrt{1-x^2}\,dx = -8\int \frac{t^2dt}{(1+t^2)^3} = \left[\begin{array}{ll} u=t & dv = \frac{2tdt}{(1+t^2)^3} \\ du=dt & v = \frac{-1}{2(1+t^2)^2} \end{array}\right] =$

$y^2 = 1-x^2$

$x^2+y^2=1$

$y = t(x-\alpha)$

$t^2(x+1) = 1-x$

$x = \frac{1-t^2}{1+t^2} = \varphi(t)$

$x+1 = \frac{1-t^2+1+t^2}{1+t^2} = \frac{2}{1+t^2}$

$y = \frac{2t}{1+t^2}$

$dx = -\frac{4t}{(1+t^2)^2}dt$

$= \frac{2t}{(1+t^2)^2} - 2\int \frac{dt}{(1+t)^2}$.