67) Интеграл вида $\int R\left(x;\sqrt[n]{\frac{ax+b}{cx+d}}\right)dx$. Пример.

$$\int R\left(x;\sqrt[n]{\frac{ax+b}{cx+d}}\right)dx = \int R\left(x;\sqrt[n_1]{\frac{ax+b}{cx+d}},\dots,\sqrt[n_k]{\frac{ax+b}{cx+d}}\right)dx =$$

$ad-bc\neq 0$

$y=\sqrt[n]{\frac{ax+b}{cx+d}}$

$Li: y^n=\frac{ax+b}{cx+d}$

$y=t$

$t^n(cx+d)=ax+b$

$x(ct^n-a)=b-dt^n$

$x=\frac{b-dt^n}{ct^n-a}=\varphi(t)$

$y=t=\varphi(t)$

$$=\int R\left(\frac{b-dt^N}{ct^N-a},t^{m_1},\dots t^{m_k}\right)\left(\frac{b-dt^N}{ct^N-a}\right)'dt.$$

$N=\text{НОК}(n_1,\dots n_k)$

$m_1=\frac{N}{n_1},\ \dots\ m_k=\frac{N}{n_k},\ \in\mathbb{N}$

$t=\sqrt[N]{\frac{ax+b}{cx+d}},\ x=\frac{b-dt^N}{ct^N-a}=\varphi(t)$

$\sqrt[n_j]{\frac{ax+b}{cx+d}}=t^{m_j}$

<u>Пример:</u>

$$\int\frac{dx}{\sqrt{x}\,(\sqrt[6]{x}+1)}=\int\frac{6t^5dt}{t^3(t+1)}=6\int\left(t-1+\frac{1}{t+1}\right)dt=$$

$n_1=2\quad n_2=6\qquad t=\sqrt[6]{x}\Rightarrow x=t^6$

$N=6\qquad \sqrt{x}=t^3\qquad dx=6t^5dt$

$m_1=3\quad m_2=1$

$$=3t^2-6t+6\ln|t+1|+C=3\sqrt[3]{x}-6\sqrt[6]{x}+6\ln|\sqrt[6]{x}+1|+C.$$