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```
%%%%%%%%%%%%%% Main Entrance %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%% By Max Yi Ren and Emrah Bayrak %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Instruction: Please read through the code and fill in blanks
% (marked by ***). Note that you need to do so for every involved
% function, i.e., m files.
```

Optional overhead

```
clear; % Clear the workspace
close all; % Close all windows
```

Optimization settings

Here we specify the objective function by giving the function handle to a variable, for example:

```
objective = @(x) x(1)^2+ (x(2)-3)^2; % replace with your objective function
% In the same way, we also provide the gradient of the
% objective:
% first order derivative(gradient) of objective with respect to x1 and x2
df =@(x) [2*x(1), 2*x(2)-6]; % replace accordingly

%constraints
g =@(x) [x(2)^2-2*x(1);
    (x(2)-1)^2+5*x(1)-15];
%first order derivative(gradient) of objective with respect to x1 and x2
dg =@(x) [-2, 2*x(2);
          5, 2*x(2)-2];


% Note that explicit gradient and Hessian information is only optional.
% However, providing these information to the search algorithm will save
% computational cost from finite difference calculations for them.

% % Specify algorithm
opt.alg = 'matlabqp'; % 'myqp' or 'matlabqp'

% Turn on or off line search. You could turn on line search once other
% parts of the program are debugged.
opt.linesearch = true; % false or true
```

```
% Set the tolerance to be used as a termination criterion:
opt.eps = 1e-3;

% Set the initial guess:
x0 = [1;1];

% Checking Feasibility for the initial point.
if max(g(x0)>0)
    errordlg('Infeasible initial point! You need to start from a feasible
 one!');
    return
end
```

Run optimization

Run your implementation of SQP algorithm. See mysqp.m

```
solution = mysqp(objective, df, g, dg, x0, opt);
```

Report

```
for i = 1:length(solution.x)
    sol(i) = objective(solution.x(:, i));  %to store all instances of the
 solution
end
for i = 1:length(solution.x)
    G = g(solution.x(:, i));  % array to store values of g1 and g2
    constraint1(i) = G(1);  %variable to store value of g1
    constraint2(i) = G(2);  %variable to store value of g2
end

count = 1:length(solution.x); % each x1 and x2 interated value


h1=figure(1);
plot(count, sol,'g','lineWidth',1.5,'Marker','+')
set(h1,'Position',[10 10 500 500])
set(gca,'XGrid','off','YGrid','on')
xlabel('No of Iterations')
ylabel('objective function')
title('objective function vs. No ofIterations')

h2=figure(2);
hold on
plot(count, sol, 'r','lineWidth',1.5,'Marker','*')
plot(count, constraint1,'Marker','*')
plot(count, constraint2,'Marker','*')
set(h2,'Position',[510 10 500 500])
set(gca,'XGrid','off','YGrid','on')
xlabel('No of Iterations')
ylabel('Objective function and constraints')
```

```
title('Objective function & Constraints vs. No of Iterations')
legend('f(x) value', 'g1(x)', 'g2(x)','Location','best')

h3=figure(3);
hold on
plot(solution.x(1, :), solution.x(2, :),'b','lineWidth',1.5,'Marker','*')
set(h3,'Position',[1010 10 500 500])
set(gca,'XGrid','off','YGrid','on')
title('x2 values vs. x1 values')
xlabel('x1 value')
ylabel('x2 value')


disp("The optimized values of x1 and x2 = ");
disp(solution.x(:, end));

disp("The objective function values for the solved x1 and x2 = ");
disp(sol(end));

disp("The first constraint g1  = ");
disp(constraint1(end));

disp("The second constraint g2  = ");
disp(constraint2(end));

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%%%%%%%%%%%%%% Sequential Quadratic Programming Implementation with BFGS %%%%%
%%%%%%%%%%%
%%%%%%%%%%%%%% By Max Yi Ren and Emrah Bayrak %%%%%%%%%%%%%%%%%%%%%%%%%%%%%


function solution = mysqp(f, df, g, dg, x0, opt)
    % Set initial conditions

    x = x0; % Set current solution to the initial guess

    % Initialize a structure to record search process
    solution = struct('x',[]);
    solution.x = [solution.x, x]; % save current solution to solution.x

    % Initialization of the Hessian matrix
    W = eye(numel(x));             % Start with an identity Hessian matrix
    % Initialization of the Lagrange multipliers
    mu_old = zeros(size(g(x)));    % Start with zero Lagrange multiplier
 estimates
    % Initialization of the weights in merit function
    w = zeros(size(g(x)));         % Start with zero weights
```

```
    % Set the termination criterion
    gnorm = norm(df(x) + mu_old'*dg(x)); % norm of Largangian gradient

    while gnorm>opt.eps % if not terminated

        % Implement QP problem and solve
        if strcmp(opt.alg, 'myqp')
            % Solve the QP subproblem to find s and mu (using your own method)
            [s, mu_new] = solveqp(x, W, df, g, dg);
        else
            % Solve the QP subproblem to find s and mu (using MATLAB's solver)
            qpalg = optimset('Algorithm', 'active-set', 'Display', 'off');
            [s,~,~,~,lambda] = quadprog(W,[df(x)]', dg(x), -g(x), [], [], [],
 [], x, qpalg);
            mu_new = lambda.ineqlin;
        end

        % opt.linesearch switches line search on or off.
        % You can first set the variable "a" to different constant values and
 see how it
        % affects the convergence.
        if opt.linesearch
            [a, w] = lineSearch(f, df, g, dg, x, s, mu_old, w);
        else
            a = 0.1;
        end

        % Update the current solution using the step
        dx = a*s;               % Step for x
        x = x + dx;             % Update x using the step

        % Update Hessian using BFGS. Use equations (7.36), (7.73) and (7.74)
        % Compute y_k
        y_k = [df(x) + mu_new'*dg(x) - df(x-dx) - mu_new'*dg(x-dx)]';
        % Compute theta
        if dx'*y_k >= 0.2*dx'*W*dx
            theta = 1;
        else
            theta = (0.8*dx'*W*dx)/(dx'*W*dx-dx'*y_k);
        end
        % Compute  dg_k
        dg_k = theta*y_k + (1-theta)*W*dx;
        % Compute new Hessian
        W = W + (dg_k*dg_k')/(dg_k'*dx) - ((W*dx)*(W*dx)')/(dx'*W*dx);

        % Update termination criterion:
      gnorm = norm(df(x) + mu_new'*dg(x)); % norm of Largangian gradient
      mu_old = mu_new;

        % save current solution to solution.x
      solution.x = [solution.x, x];
    end
end
```

```
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%The following code performs line search on the merit function

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Armijo line search
function [a, w] = lineSearch(f, df, g, dg, x, s, mu_old, w_old)
    t = 0.1; % scale factor on current gradient: [0.01, 0.3]
    b = 0.8; % scale factor on backtracking: [0.1, 0.8]
    a = 1; % maximum step length

    D = s;                  % direction for x

    % Calculate weights in the merit function using eaution (7.77)
    w = max(abs(mu_old), 0.5*(w_old+abs(mu_old)));
    % terminate if line search takes too long
    count = 0;
    while count<100
        % Calculate phi(alpha) using merit function in (7.76)
        phi_a = f(x + a*D) + w'*abs(min(0, -g(x+a*D)));

        % Caluclate psi(alpha) in the line search using phi(alpha)
        phi0 = f(x) + w'*abs(min(0, -g(x)));        % phi(0)
        dphi0 = df(x)*D + w'*((dg(x)*D).*(g(x)>0)); % phi'(0)
        psi_a = phi0 +  t*a*dphi0;                  % psi(alpha)
        % stop if condition satisfied
        if phi_a<psi_a
            break;
        else
            % backtracking
            a = a*b;
            count = count + 1;
        end
    end
end

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%The following code solves the QP subproblem using active set strategy

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

function [s, mu0] = solveqp(x, W, df, g, dg)
    % Implement an Active-Set strategy to solve the QP problem given by
    % min     (1/2)*s'*W*s + c'*s
```

```
   % s.t.    A*s-b <= 0
   %
   % where As-b is the linearized active contraint set

   % Strategy should be as follows:
   % 1-) Start with empty working-set
   % 2-) Solve the problem using the working-set
   % 3-) Check the constraints and Lagrange multipliers
   % 4-) If all constraints are staisfied and Lagrange multipliers are
positive, terminate!
   % 5-) If some Lagrange multipliers are negative or zero, find the most
negative one
   %     and remove it from the active set
   % 6-) If some constraints are violated, add the most violated one to the
working set
   % 7-) Go to step 2


   % Compute c in the QP problem formulation
   c = [df(x)]';

   % Compute A in the QP problem formulation
   A0 = dg(x);

   % Compute b in the QP problem formulation
   b0 = -g(x);

   % Initialize variables for active-set strategy
   stop = 0;           % Start with stop = 0
   % Start with empty working-set
   A = [];         % A for empty working-set
   b = [];         % b for empty working-set
   % Indices of the constraints in the working-set
   active = [];    % Indices for empty-working set

   while ~stop  % Continue until stop = 1
       % Initialize all mu as zero and update the mu in the working set
       mu0 = zeros(size(g(x)));

       % Extact A corresponding to the working-set
       A = A0(active,:);
       % Extract b corresponding to the working-set
       b = b0(active);

       % Solve the QP problem given A and b
       [s, mu] = solve_activeset(x, W, c, A, b);
       % Round mu to prevent numerical errors (Keep this)
       mu = round(mu*1e12)/1e12;

       % Update mu values for the working-set using the solved mu values
       mu0(active) = mu;

       % Calculate the constraint values using the solved s values
       gcheck = A0*s-b0;
```

```
        % Round constraint values to prevent numerical errors (Keep this)
        gcheck = round(gcheck*1e12)/1e12;

        % Variable to check if all mu values make sense.
        mucheck = 0;        % Initially set to 0

        % Indices of the constraints to be added to the working set
        Iadd = [];              % Initialize as empty vector
        % Indices of the constraints to be added to the working set
        Iremove = [];           % Initialize as empty vector

        % Check mu values and set mucheck to 1 when they make sense
        if (numel(mu) == 0)
            % When there no mu values in the set
            mucheck = 1;         % OK
        elseif min(mu) > 0
            % When all mu values in the set positive
            mucheck = 1;         % OK
        else
            % When some of the mu are negative
            % Find the most negaitve mu and remove it from acitve set
            [~,Iremove] = min(mu);  % Use Iremove to remove the constraint
        end

        % Check if constraints are satisfied
        if max(gcheck) <= 0
            % If all constraints are satisfied
            if mucheck == 1
                % If all mu values are OK, terminate by setting stop = 1
                stop = 1;
            end
        else
            % If some constraints are violated
            % Find the most violated one and add it to the working set
            [~,Iadd] = max(gcheck); % Use Iadd to add the constraint
        end
        % Remove the index Iremove from the working-set
        active = setdiff(active, active(Iremove));
        % Add the index Iadd to the working-set
        active = [active, Iadd];

        % Make sure there are no duplications in the working-set (Keep this)
        active = unique(active);
    end
end

function [s, mu] = solve_activeset(x, W, c, A, b)
    % Given an active set, solve QP

    % Create the linear set of equations given in equation (7.79)
    M = [W, A'; A, zeros(size(A,1))];
    U = [-c; b];
    sol = M\U;          % Solve for s and mu
```

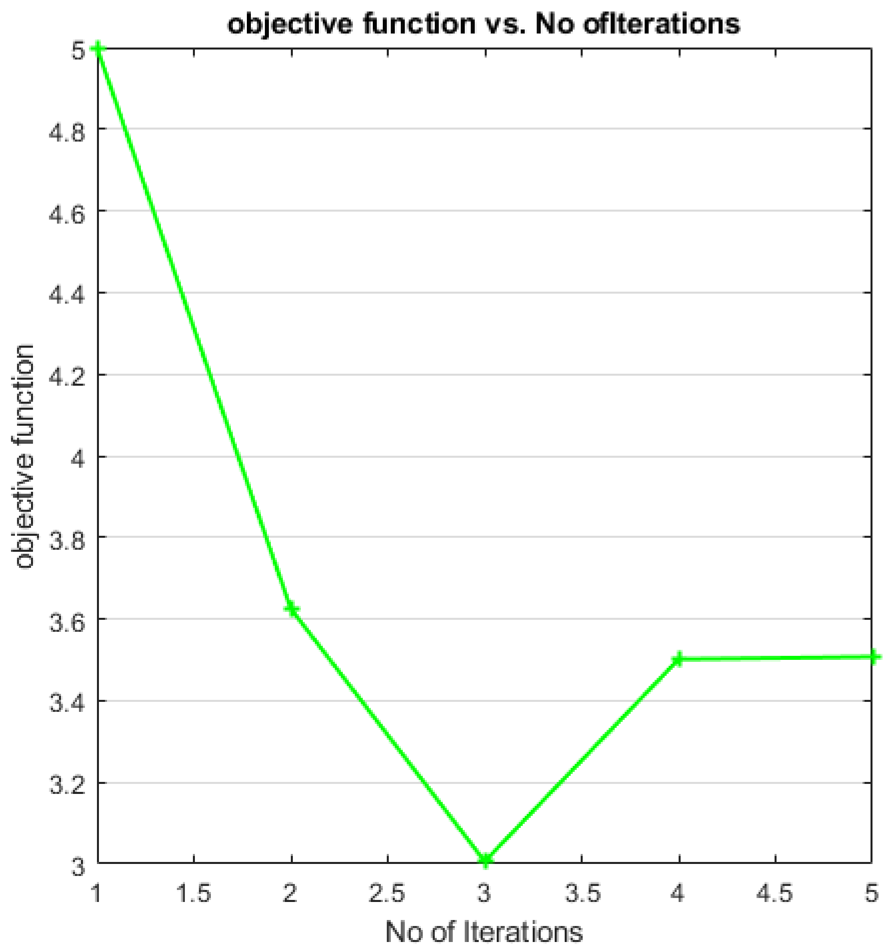
```
    s = sol(1:numel(x));                % Extract s from the solution
    mu = sol(numel(x)+1:numel(sol));    % Extract mu from the solution

end
```

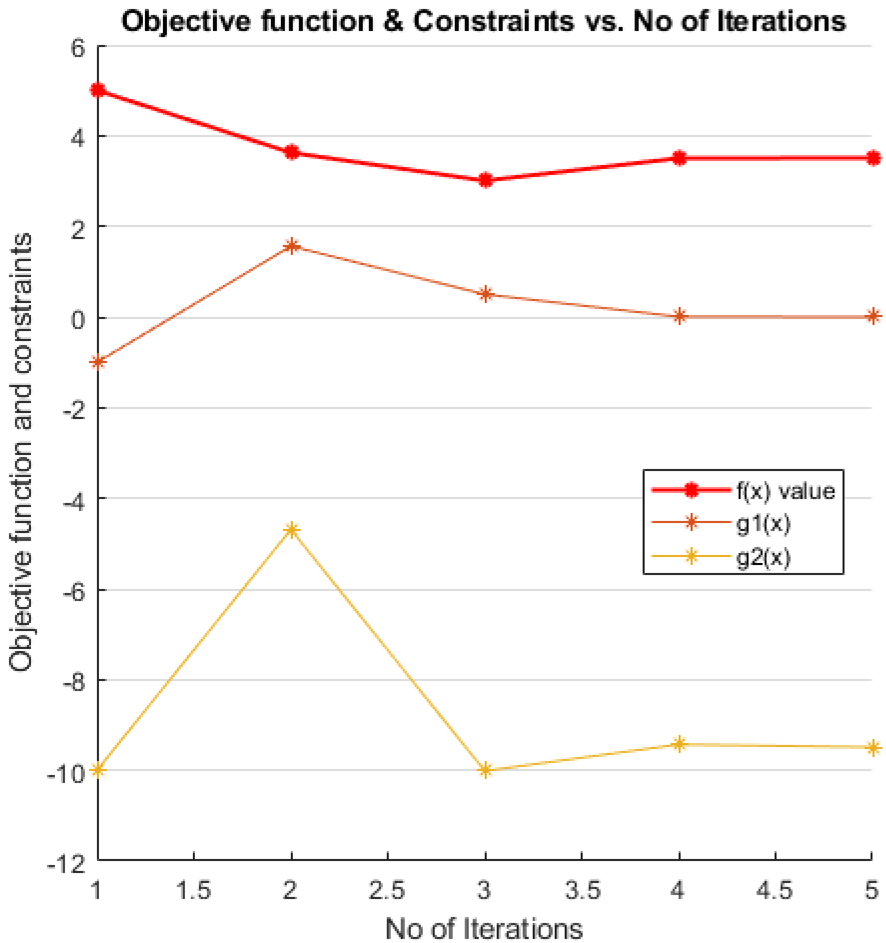
The optimized values of x1 and x2 =
    1.0604
    1.4563

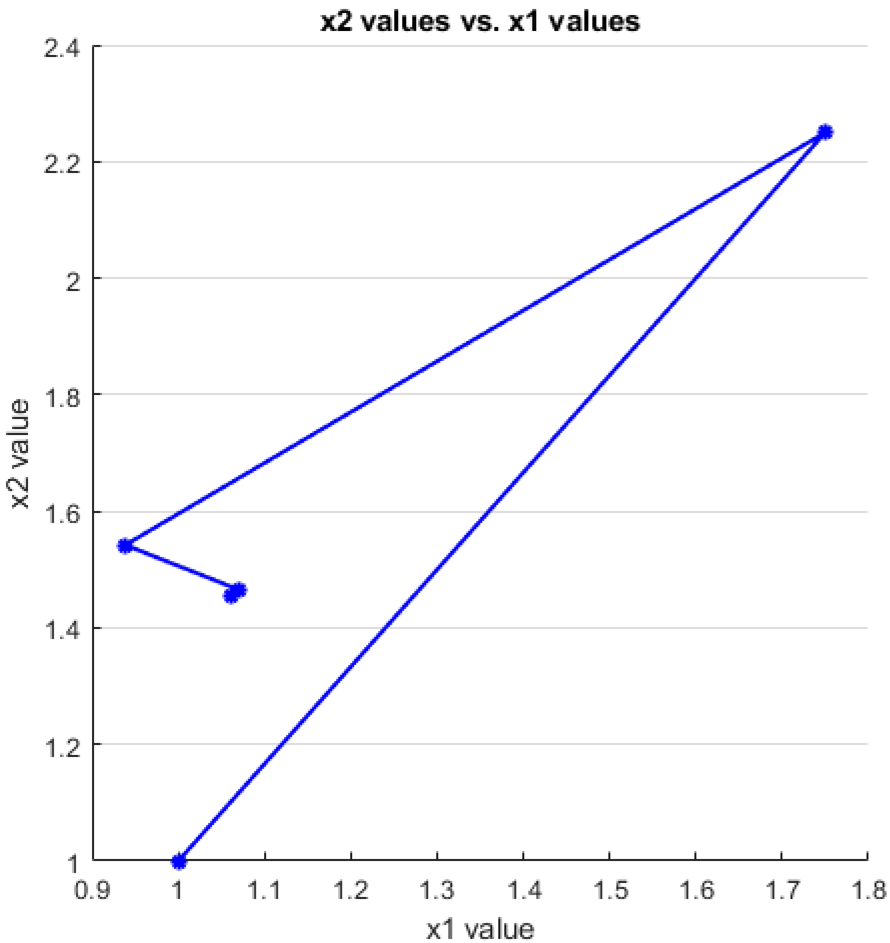
The objective function values for the solved x1 and x2 =
    3.5074

The first constraint g1 =
   7.9687e-05

The second constraint g2 =
   -9.4897




Objective function & Constraints vs. No of Iterations
Objective function and constraints
No of Iterations
f(x) value
g1(x)
g2(x)
6
4
2
0
-2
-4
-6
-8
-10
-12
1
1.5
2
2.5
3
3.5
4
4.5
5



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