TOPOLOGY OPTIMIZATION

PROJECT 3 REPORT

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MAE 598: Design Optimization (Fall 2021)

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Under the guidance of

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ACKNOWLEDGEMENT

I consider myself highly fortunate for the opportunity to do this project under the guidance of **YI (MAX) REN** who provided us a sample template code and instructions to work on.

ABSTRACT

In this project, you will learn to implement an optimization algorithm for minimizing the compliance of a cantilever beam with a point load in y direction at its equilibrium state with respect to its topology.

Introduction

I have considered similar conditions from the Sigmund's original paper to perform topology optimization for cantilever beam with a point load at the end of the beam in y direction.

I have used both 88 and 99 lines code to perform topology optimization

88 line matlab code

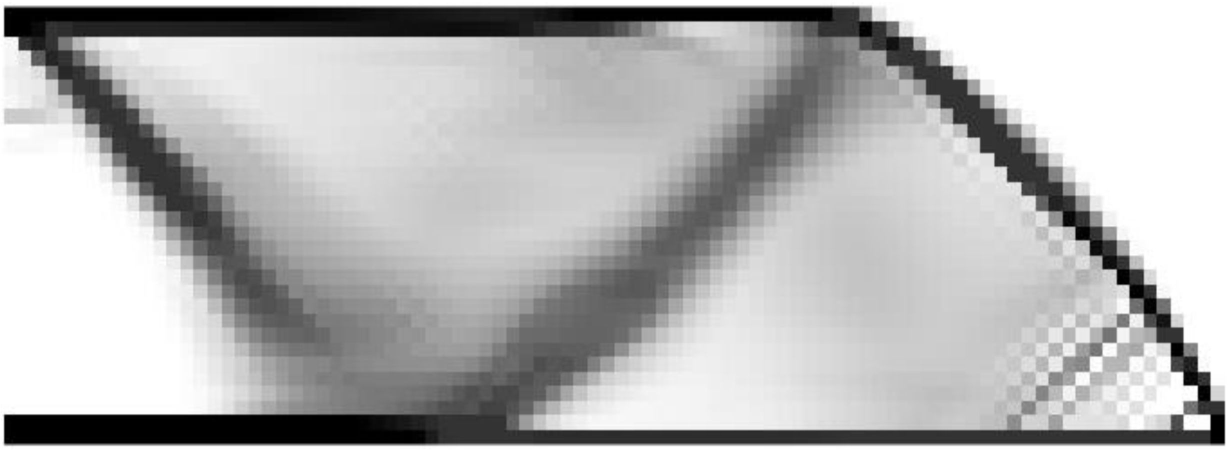
```
%%%% Modified by Max Yi Ren (ASU) %%%%%%%%%%%%%%%%%%%%%%%
%%%% AN 88 LINE TOPOLOGY OPTIMIZATION CODE Nov, 2010 %%%%
nelx=60;
nely=20;
volfrac=0.3;
penal=3;
rmin=1.5;
%function Designproject3(nelx,nely,volfrac,penal,rmin,ft)
%% MATERIAL PROPERTIES
E0 = 1;
Emin = 1e-9;
nu = 0.3;
%% PREPARE FINITE ELEMENT ANALYSIS
A11 = [12  3 -6 -3;  3 12  3  0; -6  3 12 -3; -3  0 -3 12];
A12 = [-6 -3  0  3; -3 -6 -3 -6;  0 -3 -6  3;  3 -6  3 -6];
B11 = [-4  3 -2  9;  3 -4 -9  4; -2 -9 -4 -3;  9  4 -3 -4];
B12 = [ 2 -3  4 -9; -3  2  9 -2;  4  9  2  3; -9 -2  3  2];
KE = 1/(1-nu^2)/24*([A11 A12;A12' A11]+nu*[B11 B12;B12' B11]);
nodenrs = reshape(1:(1+nelx)*(1+nely),1+nely,1+nelx);
edofVec = reshape(2*nodenrs(1:end-1,1:end-1)+1,nelx*nely,1);
edofMat = repmat(edofVec,1,8)+repmat([0 1 2*nely+[2 3 0 1] -2 -1],nelx*nely,1);
iK = reshape(kron(edofMat,ones(8,1))',64*nelx*nely,1);
jK = reshape(kron(edofMat,ones(1,8))',64*nelx*nely,1);
% DEFINE LOADS AND SUPPORTS (HALF MBB-BEAM)
F = sparse(2,1,-1,2*(nely+1)*(nelx+1),1);
U = zeros(2*(nely+1)*(nelx+1),1);
fixeddofs = union((1:2:2*(nely+1)),(2*(nelx+1)*(nely+1)));
alldofs = (1:2*(nely+1)*(nelx+1));
freedofs = setdiff(alldofs,fixeddofs);
%% PREPARE FILTER
iH = ones(nelx*nely*(2*(ceil(rmin)-1)+1)^2,1);
jH = ones(size(iH));
sH = zeros(size(iH));
k = 0;
for i1 = 1:nelx
    for j1 = 1:nely
        e1 = (i1-1)*nely+j1;
        for i2 = max(i1-(ceil(rmin)-1),1):min(i1+(ceil(rmin)-1),nelx)
```

```
            for j2 = max(j1-(ceil(rmin)-1),1):min(j1+(ceil(rmin)-1),nely)
                e2 = (i2-1)*nely+j2;
                k = k+1;
                iH(k) = e1;
                jH(k) = e2;
                sH(k) = max(0,rmin-sqrt((i1-i2)^2+(j1-j2)^2));
            end
        end
    end
end
H = sparse(iH,jH,sH);
Hs = sum(H,2);
%% INITIALIZE ITERATION
x = repmat(volfrac,nely,nelx);
xPhys = x;
loop = 0;
change = 1;
%% START ITERATION
while change > 0.1
    loop = loop + 1;
%% FE-ANALYSIS
    sK= reshape(KE(:)*(Emin+xPhys(:)'.^penal*(E0-Emin)),64*nelx*nely,1);
    K = sparse(iK,jK,sK); K = (K+K')/2;
    U(freedofs) = K(freedofs,freedofs)\F(freedofs);
%% OBJECTIVE FUNCTION AND SENSITIVITY ANALYSIS
    ce = reshape(sum((U(edofMat)*KE).*U(edofMat),2),nely,nelx); % element-wise strain
energy
    c  = sum(sum((Emin+xPhys.^penal*(E0-Emin)).*ce)); % total strain energy
    dc = -penal*(E0-Emin)*xPhys.^(penal-1).*ce; % design sensitivity
    dv = ones(nely,nelx);
%% FILTERING/MODIFICATION OF SENSITIVITIES
ft=heaviside(x);
    if ft == 2
        dc(:) = H*(x(:).*dc(:))./Hs./max(1e-3,x(:));
    elseif ft == 3
        dc(:) = H*(dc(:)./Hs);
        dv(:) = H*(dv(:)./Hs);
    end

%% OPTIMALITY CRITERIA UPDATE OF DESIGN VARIABLES AND PHYSICAL DENSITIES
    l1 = 0; l2 = 1e9; move=0.2;
    while (l2-l1)/(l1+l2) > 1e-3
        lmid = 0.5*(l2+l1);
        xnew = max(0,max(x-move,min(1,min(x+move,x.*sqrt(-dc./dv/lmid)))));
        if ft == 1
            xPhys = xnew;
        elseif ft==2
            xPhys(:) = (H*xnew(:))./Hs;
        end
        if sum(xPhys(:)) > volfrac*nelx*nely, l1 = lmid;
        else
            l2 = lmid;
        end
    end
    change = max(abs(xnew(:)-x(:)));
    x = xnew;
```

```
%% PRINT RESULTS
    fprintf(' It.:%5i Obj.:%11.4f Vol.:%7.3f ch.:%7.3f\n',loop,c, ...
        mean(xPhys(:)),change);
%% PLOT DENSITIES
    colormap(gray); imagesc(1-xPhys); caxis([0 1]); axis equal; axis off; drawnow;
end
```

output:



99 line Matlab code:

The code was borrowed from " A 99 line topology optimization code written in Matlab" – Ole Sigmond.

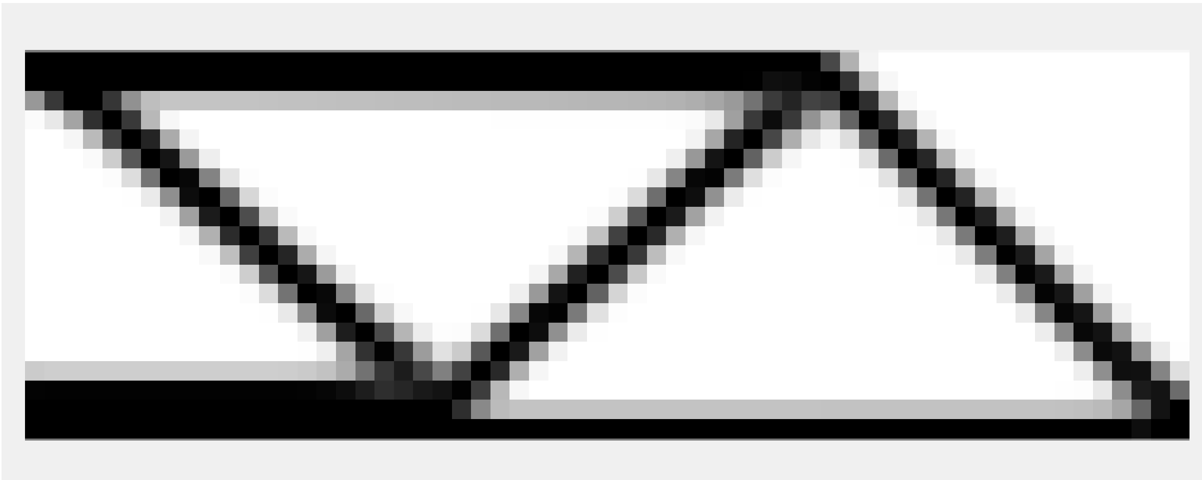
I have used this code since it produces better output.

```
%function (nelx,nely,volfrac,penal,rmin)
% INITIALIZE
nelx=60;
nely=20;
% nelz = 4;
volfrac=0.3;
penal=3;
rmin=1.5;
x(1:nely,1:nelx) = volfrac;
loop = 0;
change = 1.;
% START ITERATION
while change > 0.01
loop = loop + 1;
xold = x;
% FE-ANALYSIS
[U]=FE(nelx,nely,x,penal);
% OBJECTIVE FUNCTION AND SENSITIVITY ANALYSIS
[KE] = lk;
```

```
c = 0.;
for ely = 1:nely
    for elx = 1:nelx
        n1 = (nely+1)*(elx-1)+ely;
        n2 = (nely+1)* elx +ely;
        Ue = U([2*n1-1;2*n1; 2*n2-1;2*n2; 2*n2+1;2*n2+2; 2*n1+1;2*n1+2],1);
        c = c + x(ely,elx)^penal*Ue'*KE*Ue;
        dc(ely,elx) = -penal*x(ely,elx)^(penal-1)*Ue'*KE*Ue;
    end
end
% FILTERING OF SENSITIVITIES
[dc] = check(nelx,nely,rmin,x,dc);
% DESIGN UPDATE BY THE OPTIMALITY CRITERIA METHOD
[x] = OC(nelx,nely,x,volfrac,dc);
% PRINT RESULTS
change = max(max(abs(x-xold)));
fprintf(' It.:%5i Obj.:%11.4f Vol.:%7.3f ch.:%7.3f\n',loop,c, ...
 sum(sum(x))/(nelx*nely),change);
% disp(['It.: 'sprintf('%4i',loop)' Obj.: 'sprintf('%10.4f',c)' ...
% 'Vol.: ' sprintf('%6.3f',sum(sum(x))/(nelx*nely)) ...
% 'ch.: ' sprintf('%6.3f',change)]);
% PLOT DENSITIES
colormap(gray); imagesc(-x); axis equal; axis tight;
 axis off;pause(1e-6);
end
%%%%%%%%%% OPTIMALITY CRITERIA UPDATE %%%%%%%%%
function [xnew]=OC(nelx,nely,x,volfrac,dc)
l1 = 0; l2 = 100000; move = 0.2;
while (l2-l1 > 1e-4)
lmid = 0.5*(l2+l1);
xnew = max(0.001,max(x-move,min(1.,min(x+move,x.*sqrt(-dc./lmid)))));
if sum(sum(xnew)) - volfrac*nelx*nely > 0
 l1 = lmid;
else
 l2 = lmid;
end
end
end
%%%%%%%%%% MESH-INDEPENDENCY FILTER %%%%%%%%%%%
function [dcn]=check(nelx,nely,rmin,x,dc)
dcn=zeros(nely,nelx);
for i = 1:nelx
 for j = 1:nely
 sum=0.0;
 for k = max(i-round(rmin),1):min(i+round(rmin),nelx)
 for l = max(j-round(rmin),1):min(j+round(rmin), nely)
 fac = rmin-sqrt((i-k)^2+(j-l)^2);
 sum = sum+max(0,fac);
 dcn(j,i) = dcn(j,i) + max(0,fac)*x(l,k)*dc(l,k);
 end
 end
 dcn(j,i) = dcn(j,i)/(x(j,i)*sum);
 end
end
end
%%%%%%%%%% FE-ANALYSIS %%%%%%%%%%%%
```

```
function [U]=FE(nelx,nely,x,penal)
 [KE] = lk;
 K = sparse(2*(nelx+1)*(nely+1), 2*(nelx+1)*(nely+1));
 F = sparse(2*(nely+1)*(nelx+1),1);
 U = sparse(2*(nely+1)*(nelx+1),1);
    for ely = 1:nely
        for elx = 1:nelx
            n1 = (nely+1)*(elx-1)+ely;
            n2 = (nely+1)* elx +ely;
            edof = [2*n1-1; 2*n1; 2*n2-1; 2*n2;2*n2+1;2*n2+2;2*n1+1; 2*n1+2];
            K(edof,edof)=K(edof,edof)+x(ely,elx)^penal*KE;
        end
    end
% DEFINE LOADS AND SUPPORTS(HALF MBB-BEAM)
F(2,1) = -1;
fixeddofs = union([1:2:2*(nely+1)],[2*(nelx+1)*(nely+1)]);
alldofs = [1:2*(nely+1)*(nelx+1)];
freedofs = setdiff(alldofs,fixeddofs);
% SOLVING
U(freedofs,:) = K(freedofs,freedofs) \ F(freedofs,:);
U(fixeddofs,:)= 0;
end
%%%%%%%%%% ELEMENT STIFFNESS MATRIX %%%%%%%
function [KE]=lk
 E = 1.;
 nu = 0.3;
 k=[ 1/2-nu/6 1/8+nu/8 -1/4-nu/12 -1/8+3*nu/8 ...
 -1/4+nu/12 -1/8-nu/8 nu/6 1/8-3*nu/8];
 KE = E/(1-nu^2)*...
 [ k(1) k(2) k(3) k(4) k(5) k(6) k(7) k(8)
 k(2) k(1) k(8) k(7) k(6) k(5) k(4) k(3)
 k(3) k(8) k(1) k(6) k(7) k(4) k(5) k(2)
 k(4) k(7) k(6) k(1) k(8) k(3) k(2) k(5)
 k(5) k(6) k(7) k(8) k(1) k(2) k(3) k(4)
 k(6) k(5) k(4) k(3) k(2) k(1) k(8) k(7)
 k(7) k(4) k(5) k(2) k(3) k(8) k(1) k(6)
 k(8) k(3) k(2) k(5) k(4) k(7) k(6) k(1)];
end
```

Output:



Conclusion:

The topology optimization for the cantilever beam, where the volume is effectively reduced by determining the load acting on different places on beam .

References:

1. Efficient topology optimization in MATLAB using 88 lines of code - Erik Andreassen, Anders Clausen, Mattias Schevenels, Boyan S. Lazarov, Ole Sigmund.
2. A 99 line topology optimization code written in Matlab - Ole Sigmond.
3. Topology optimization tutorial – Yi (Max) Ren.