Caballero Meets Bewley: The Permanent-Income Hypothesis in General Equilibrium

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The permanent-income hypothesis (PIH) of Milton Friedman (1957) states that the agent saves in anticipation of possible future declines in labor income (John Y. Campbell, 1987). He also saves for precautionary reasons, and dissaves because of impatience.[1] To justify the PIH in an intertemporal optimization framework, it has been conventional to assume both (i) quadratic utility, to turn off precautionary motives (Robert E. Hall, 1978), and (ii) equality between the subjective discount rate and the interest rate, in order to rule out dissavings for lack of patience.[2] Neither assumption is plausible. Much work on consumption in the past decade has focused on individual's precautionary savings motives and liquidity constraints.[3] Impatience is a standard result in heterogeneous-agents general-equilibrium incomplete-markets models, generally known as Bewley models.[4]

This paper shows that the PIH is in any case the optimal rule, in a Bewley model, in which each agent solves the precautionary-savings model of Caballero (1990, 1991). In addition to the demand for savings for a "rainy day," Caballero's model also predicts a constant precautionary-savings demand and constant dissavings due to impatience. In equilibrium, I show that these two forces must cancel each other. As a result, the agent behaves in accordance with the PIH.

Section I describes the model. Section II concludes. The Appendix provides a heuristic derivation and a proof of the optimal consumption rule.

I. Model

First, I briefly recapitulate Caballero's precautionary-savings model. Second, I introduce a simple Bewley-type heterogeneous-agents equilibrium model. Finally, I show that, in equilibrium, the agent's precautionary savings demand is exactly offset by dissavings due to impatience, making each agent effectively a permanent-income consumer (Friedman, 1957).

A. *Optimal Consumption and Savings*

Two key assumptions of Caballero's model are time-additive separable exponential utility and a stochastic uninsurable autoregressive[5] income process. I fix a probability space $(\Omega, \mathcal{F}, P)$ and an information filtration $\{\mathcal{F}_t\}_{t=0}^{\infty}$, and suppose that the agent receives labor income at time t, at the rate y_t given by

$$(1) \quad y_t = \phi_0 + \phi_1 y_{t-1} + \sigma w_t, \qquad t \geq 1,$$

where $\sigma > 0$, the initial level y_0 of income is given, and $\{w_1, w_2, \cdots\}$ are independent innovations, with a distribution ν having zero mean and unit variance. Since the focus of this note is the property of optimal consumption in stationary equilibrium, I also assume that the income process (1) is stationary, in that $|\phi_1| < 1$.

[1] By "impatience," I mean that the subjective discount rate is higher than the interest rate.

[2] There are no liquidity constraints either.

[3] See Stephen P. Zeldes (1989), Ricardo J. Caballero (1990, 1991), Miles S. Kimball (1990), Angus Deaton (1991), and Christopher D. Carroll (1997).

[4] See Truman Bewley (1986), Mark Huggett (1993), S. Rao Aiyagari (1994), Per Krusell and Anthony A. Smith (1998), and Lars Ljungqvist and Thomas J. Sargent (2000) for standard treatments.

[5] He also studies autoregressive moving-average processes, and includes the unit-root specification as a special case.

The agent can borrow or lend at a constant rate of interest $r > 0$, in that

$$(2) \qquad A_{t+1} = (1 + r)A_t + y_t - c_t,$$

with a given initial asset level A_0, where A_t is the level of wealth at the beginning of period t. He receives income y_t, and consumes c_t, at the end of period t. The agent has time-additive state-separable constant-absolute-risk-averse (CARA) utility,

$$(3) \qquad U(c) = E\left[\sum_{t=0}^{\infty} \left(\frac{1}{1+\delta}\right)^t u(c_t)\right],$$

for any consumption process c, where $\delta > 0$ is the agent's subjective discount rate[6] and $u(\cdot)$ is constant-absolute-risk-averse utility, in that $u(c) = -e^{-\theta c}/\theta$ with $\theta > 0$. His objective is to solve

$$(4) \qquad \sup_{c \in L(A_0, y_0)} U(c),$$

where the admissible set $L(A_0, y_0)$ is defined by a maximum growth rate of debt given in the Appendix.

THEOREM 1: *Suppose that the Laplace transform*[7] $\zeta(\cdot)$ *of the income innovation* w_t *is finite over the range from 0 through* $-\theta \sigma r a$ *(normality of* w_t *suffices[8]). The agent's optimal consumption rule for (4) is then*

$$(5) \qquad c_t^* = r(A_t + ay_t + a_0),$$

where

$$(6) \qquad a_0 = \frac{a\phi_0}{r} - \Gamma(r),$$

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[6] The implied wealth dynamics is not stationary, as shown later. One way to generate a stationary wealth distribution is to let agent die with probability $p > 0$ every period. The optimization problem for such a finitely lived agent is equivalent to (3) with an elevated subjective discount rate to incorporate mortality risk (Menahem E. Yaari, 1965; Olivier Jean Blanchard, 1985).

[7] The Laplace transform $\zeta(\cdot)$ of the innovations distribution v is defined by $\zeta(k) = \int_{\mathbb{R}} e^{kz} \, dv(z)$.

[8] This condition places few restrictions on the type of innovations allowed. Other commonly used distributions, such as uniform, geometric, and Gamma distributions, also meet this condition.

$$(7) \qquad \Gamma(r) = \frac{1}{\theta r^2}[\Pi(r) - \Psi(r)],$$

$$(8) \qquad \Psi(r) = \log\left(\frac{1+\delta}{1+r}\right),$$

$$(9) \qquad \Pi(r) = m(-\theta \sigma r a) > 0,$$

$$(10) \qquad a = \frac{1}{1 + r - \phi_1},$$

and $m(k) = \log \zeta(k)$.

A proof of the theorem is given in the Appendix. Following Friedman (1957) and Hall (1978), I define *human wealth* as the expected present value of future labor income, discounted at the risk-free interest rate, conditioning on $\mathcal{F}_t$, the agent's information set at time t. For (1), human wealth at time t is

$$(11) \qquad h_t \equiv \left(\frac{1}{1+r}\right) E_t\left[\sum_{j=0}^{\infty} \left(\left(\frac{1}{1+r}\right)^j y_{t+j}\right)\right]$$

$$= a\left(y_t + \frac{\phi_0}{r}\right),$$

where a is given by (10), and E_t denotes $\mathcal{F}_t$-conditional expectation. The optimal consumption may also be written, using human wealth, in that

$$(12) \qquad c_t^* = r(A_t + h_t - \Gamma(r)).$$

A key feature of (12) is that the marginal propensities to consume (MPCs) out of financial and out of human wealth are equal. The following is immediate from the theorem.

COROLLARY: *The savings rate may be decomposed into the three components, in that*

$$(13) \qquad s_t^* = rA_t^* + y_t - c_t^*$$

$$= f_t + \frac{1}{\theta r}\Pi(r) - \frac{1}{\theta r}\Psi(r),$$

where

$$(14) \qquad f_t = a(1 - \phi_1)(y_t - \bar{y}),$$

and $\bar{y} = \phi_0/(1 - \phi_1)$ *is the long-run mean of income.*

The term $\Psi(r)$ in (8) captures the agent's dissavings ($\Psi(r) > 0$) for lack of patience ($\delta > r$). The term $\Pi(r)$ in (9) is proportional to the agent's precautionary-savings demand. The term f_t in (14) captures the agent's demand of savings "for a rainy day," which is an equivalent way of phrasing the PIH in terms of savings (Campbell, 1987). When $y_t > \bar{y}$, the agent expects that his income will fall in the long run, and saves a portion of $(y_t - \bar{y})$ in anticipation of future "rainy days."

B. General Equilibrium

The economy is populated by a continuum of *ex ante* identical, but *ex post* heterogeneous agents, of total mass normalized to one, with each agent solving (4). The risk-free asset is the pure-consumption loan, in zero net supply (Huggett, 1993). The initial cross-sectional distribution of income is assumed to be its stationary distribution $\Phi(\cdot)$. By the law of large numbers (LLN) of Yeneng Sun (2000), provided that we construct the space of agents and the probability space appropriately, with pairwise-independent incomes, aggregate income and the cross-sectional distribution of income $\Phi(\cdot)$ is constant over time (almost surely).

PROPOSITION 1: *For any positive interest rate r, the total savings demand "for a rainy day" in the economy is equal to zero. That is, $F_t(r) = \int f_t(r)\, d\Phi(y_t) = 0$, for $r > 0$.*

PROOF:
From (14), the LLN implies that, for $r > 0$,

$$F_t(r) = \int f_t(r)\, d\Phi(y_t)$$

$$= a(1 - \phi_1) \int (y_t - \bar{y})\, d\Phi(y_t)$$

$$= 0.$$

Proposition 1 states that the total savings "for a rainy day" is zero, at *any* positive interest rate.

Therefore, from (13), for $r > 0$, the total savings at time t is

$$(15) \quad S(r) = r\Gamma(r) = \frac{1}{\theta r}\left(\Pi(r) - \Psi(r)\right)$$

$$= \frac{1}{\theta r}\left[m(-\theta\sigma ra) - \log\left(\frac{1 + \delta}{1 + r}\right)\right].$$

An equilibrium is defined by an interest rate r^* satisfying $S_t(r^*) = 0$. The following proposition shows the existence of equilibrium.

PROPOSITION 2: *There exists no equilibrium with interest rate above δ, and at least one equilibrium with an interest rate r^* such that $0 < r^* < \delta$.*

PROOF:
If $r \geq \delta$, then $\Psi(r) \leq 0$, which implies that the aggregate savings $S(r) > 0$; therefore, there is no equilibrium with $r \geq \delta$. Let $D = [0, \delta]$. Given that $\Pi(0) - \Psi(0) = 1 - \log(1 + \delta) < 0$, and $\Pi(\delta) - \Psi(\delta) = m(-\theta\sigma\delta/(1 + \delta - \phi_1)) > 0$, the continuity of $\Pi(r) - \Psi(r)$ on D implies that there exists at least one interest rate $r^* \in (0, \delta)$, solving $S(r^*) = 0$.

The following theorem states the main result of this note.

THEOREM 2: *There is an equilibrium with an interest rate $r^* \in (0, \delta)$. For any such equilibrium, each agent's consumption is described by the PIH, in that*
$$(15) \qquad c_t^* = r(A_t + h_t),$$

where h is the human wealth given in (11).

The proof is immediate from (5) and Proposition 2. The intuition behind the theorem is as follows. With an individual's constant precautionary savings demand $\Pi(r)$, for any $r > 0$, the equilibrium interest rate r^* must be at a level with the property that individual's dissavings demand due to impatience is exactly balanced by their precautionary-savings demand [$\Psi(r^*) = \Pi(r^*)$]. This implies that the agent's consumption satisfies (2), the PIH. An equivalent statement of the PIH is that consumption is a martingale (Hall, 1978), in that $c_t^* = E_t(c_{t+1}^*)$. The implied wealth is not stationary
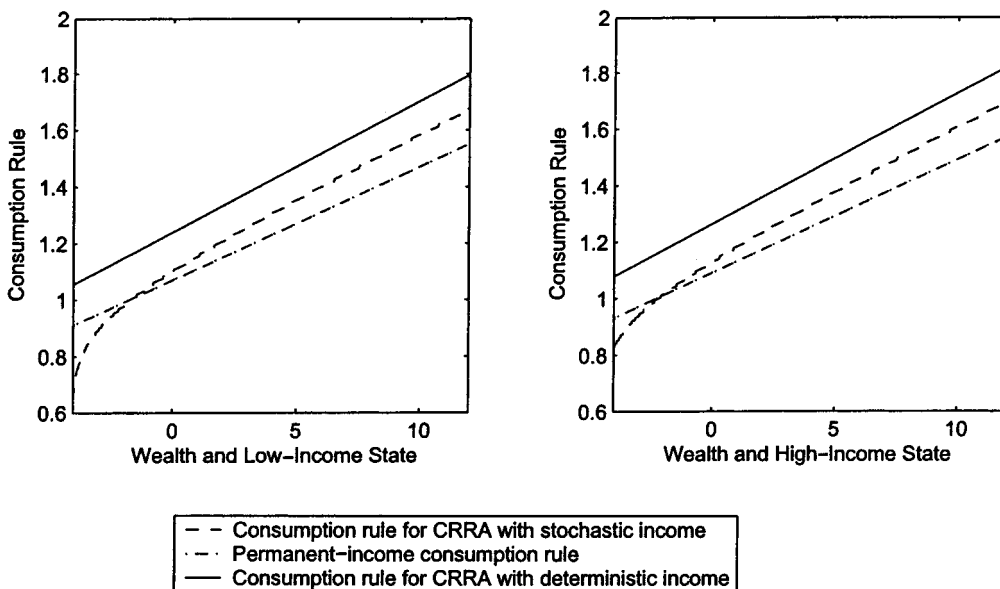
FIGURE 1. COMPARISONS ACROSS CONSUMPTION RULES (HIGH-INCOME VOLATILITY: $\sigma_y = 0.2$)

even if income is stationary. Wealth accumulates at a rate proportional to $(y - \bar{y})$, income in excess of its long-run mean $\bar{y} = \phi_0/(1 - \phi_1)$, in that $A_{t+1} = A_t + b(y_t - \bar{y})$ where $b = (1 - \phi_1)/(1 + r - \phi_1)$.

That the economywide net saving each period is zero is essential for the permanent-income hypothesis result in equilibrium. With endogenous capital accumulation and growth, the total supply of capital is elastic and part of precautionary savings demand leads to an increase in the total capital stock. As a result, the permanent-income hypothesis may no longer hold exactly in that case.[9]

Next, I compare this model with other Bewley models that use the constant-relative-risk-averse (CRRA) utility (Huggett, 1993; Aiyagari, 1994).
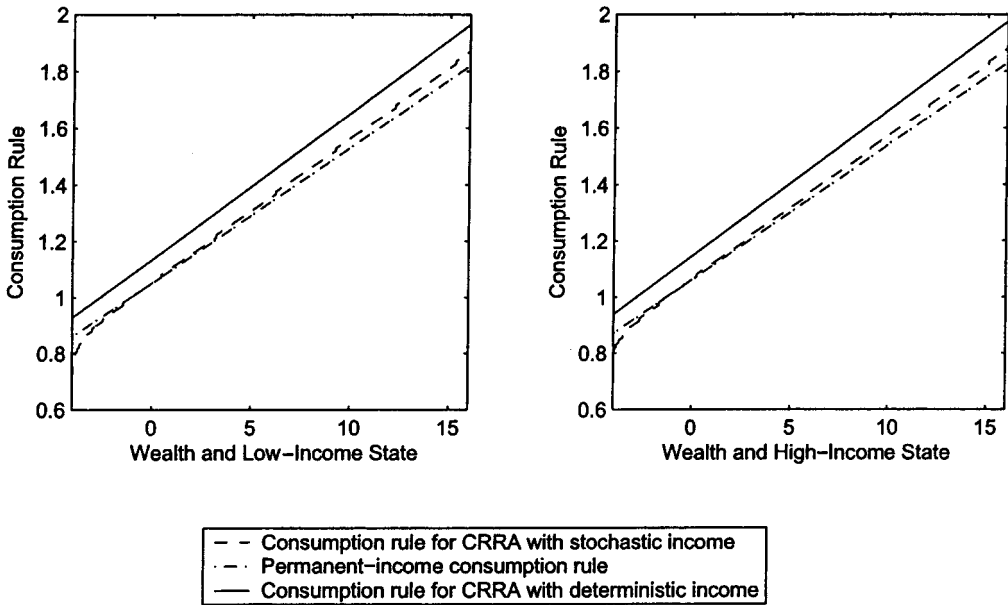
C. Discussions

CRRA-utility-based Bewley models such as those of Huggett (1993) and Aiyagari (1994) predict that the agent behaves as a buffer-stock saver (Carroll, 1997) by targeting a finite level of wealth to smooth consumption. For illustra-

tive purposes, I solve and plot the optimal consumption for a simple CRRA-utility-based model with a two-state income process[10] ($y = (1 - \sigma_y, 1 + \sigma_y)^T$, with $\sigma_y = 0.2$) in Figure 1. One key difference between the consumption rules with and without income uncertainty is that the former (dashed line) is concave, while the latter (solid line) is linear. Carroll and Kimball (1996) provides a proof for the concavity of the consumption rule. The intuition is as follows. Because marginal utility (for CRRA utility) approaches infinity for consumption level close to zero and near-zero consumption is more likely for the poor, the agent naturally behaves more prudently (by saving more for precaution) at the lower end of wealth, implying a concave consumption rule in wealth.

At the higher end of wealth, income uncertainty shall have little effect on consumption, because consumption of the wealthy is mostly financed by wealth rather than income. Figure

[9] I thank the referee for pointing this out.

[10] The two-state model may be viewed as an approximation for an autoregressive process (George Tauchen and Robert Hussey, 1991; Aiyagari, 1994). The coefficient of relative risk aversion is set at 3. The annual subjective discount rate δ is set at 5 percent. Income shocks, for the purpose of this calculation, are assumed to be independently and identically distributed (i.i.d.).

FIGURE 2. COMPARISONS ACROSS CONSUMPTION RULES (LOW-INCOME VOLATILITY: $\sigma_y = 0.1$)

1 shows that for $A \geq 1.9$, consumption for CRRA utility with stochastic income is almost parallel to the consumption rule for CRRA utility with deterministic income,[11] implying that the precautionary savings demand, measured by the gap between the solid and dashed lines, is approximately 0.12, at both low-income and high-income states.[12] Thus, for CRRA utility, the MPC (for $A \geq 1.9$) is approximately equal to $\alpha = 1 + r - [(1 + r)/(1 + \delta)]^\nu$, the MPC out of financial wealth for deterministic income,[13] where ν is the elasticity of intertemporal substitution. Impatience ($r < \delta$) in equilibrium implies $\alpha > r$ and thus a stationary wealth process.[14] Because CARA utility lacks wealth effect, the saving rate s_t^* is independent

of the level of wealth, and therefore wealth is nonstationary.

Briefly summarized, the PIH rule (dash-dotted line) overstates the consumption of the poor and understates the consumption of the wealthy in equilibrium CRRA-utility-based models. However, the optimal consumption rule for CRRA-utility agent and the PIH rule may be quite close numerically. Figure 2 plots the optimal and the PIH rules for the case of $\sigma_y = 0.1$, a smaller income dispersion[15] than that in Figure 1, and supports this insight. The intuition is as follows. A smaller income dispersion σ_y gives a lower precautionary savings, implying that the equilibrium interest rate r^* is closer to the subjective discount rate δ. Therefore, both precautionary savings and dissavings due to impatience are smaller, suggesting a narrower distance between the optimal rule and the PIH rule. In the limit with no income dispersion, the optimal consumption coincides with the PIH rule, because the interest rate is equal to the subjective discount rate and there is no precautionary savings either.

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[11] Of course, other parameters of the model are fixed.

[12] The precautionary savings demand at the low-income state is equal to that at the high-income state is due to i.i.d. assumption of the two-state income process. Calculation results for the income process with other degrees of persistence are available upon request.

[13] The MPC α is solved by using the Euler equation $c_t^{-1/\nu} = [(1 + r)/(1 + \delta)]c_{t+1}^{-1/\nu}$ and the budget equation (2).

[14] From (2), wealth evolution is approximately given by $A_{t+1} = (1 + r - \alpha)A_t + $ a function of y_t, at the higher end of wealth. Note that $1 + r - \alpha = [(1 + r)/(1 + \delta)]^\nu < 1$, if and only if $r < \delta$. Therefore, wealth is stationary if agents are impatient.

[15] All other exogenous parameters of the model are held fixed.

II. Concluding Remarks

This paper shows that the permanent-income hypothesis gives the optimal consumption rule in an equilibrium model, using the precautionary-savings model of Caballero (1991). The equilibrium justification of the PIH rule differs fundamentally from the standard quadratic-utility certainty-equivalence-based argument (Hall, 1978). Equilibrium makes the agent effectively impatient. The resulting dissavings due to impatience partially offsets the precautionary savings demand, making the optimal consumption rule closer to the PIH than otherwise.

APPENDIX: PROOF OF THEOREM 1

The first half of this Appendix conjectures the candidate optimal consumption rule (5), using the Bellman equation. The second half verifies the conjecture.

Heuristic Derivation of Optimal Consumption

Derivations in this subsection assume technical regularity conditions. The Bellman equation for (4) implies that

$$
(A1) \qquad V(A_t, y_t) = \sup_{c_t} \left\{ u(c_t) + \left(\frac{1}{1 + \delta} \right) E_t V(A_{t+1}, y_{t+1}) \right\}.
$$

First, conjecture that the value function for (4) takes the form

$$
(A2) \qquad V(A, y) = -\frac{1}{\theta r} \exp(-\theta r(A + ay + \bar{b})).
$$

The first-order condition with respect to c is

$$
(A3) \qquad u'(c_t) = \left(\frac{1}{1 + \delta} \right) E_t V_1(A_{t+1}, y_{t+1}),
$$

where V_1 denotes the derivative of $V(\cdot, \cdot)$ with respect to wealth, its first argument. Using the Envelope Theorem leaves

$$
(A4) \qquad V_1(A, y) = \left(\frac{1 + r}{1 + \delta} \right) E_t V_1(A_{t+1}, y_{t+1}).
$$

Equations (A3) and (A4) together imply the envelope condition, in that

$$
(A5) \qquad u'(c) = \frac{1}{1 + r} V_1(A, y).
$$

From (A2) and (A5), the candidate optimal consumption is given by

$$
(A6) \qquad c_t^* = r(A_t + ay_t + a_0),
$$

where

$$
a_0 = \bar{b} + \frac{1}{\theta r} \log(1 + r).
$$

Using (A2), and (A4), the Bellman equation (A1) implies that

$$\text{(A7)} \qquad V(A_t, y_t) = \frac{r}{1+r} V(A_t, y_t) - \left(\frac{1}{1+\delta}\right)\left(\frac{1}{\theta r}\right) E_t \exp[-\theta r(A_{t+1} + ay_{t+1} + \bar{b})].$$

Plugging the income process (1) into (A7) and solving for the conjectured consumption rule c gives

$$\text{(A8)} \qquad c_t^* = r\left[A_t + \left(\frac{1-a+a\phi_1}{r}\right)y_t + \frac{a\phi_0}{r} + \frac{1}{\theta r^2}\left(\log\left(\frac{1+\delta}{1+r}\right) - m(-\theta\sigma ra)\right)\right],$$

assuming that $m(\cdot) = \log \zeta(\cdot)$ is well defined at $-\theta\sigma ra$. For the conjecture (A2) to be correct, it is necessary that (A6) and (A8) are the same rules. Matching coefficients in (A6) and (A8) gives the coefficient a, as in (10), and

$$\text{(A9)} \qquad \bar{b} = \frac{\phi_0 a}{r} + \frac{1}{\theta r^2}\left[\log\left(\frac{1+\delta}{1+r}\right) - m(-\theta\sigma ra) - r\log(1+r)\right].$$

The candidate optimal asset holding is[16]

$$\text{(A10)} \qquad A_t^* = A_{t-1}^* + (1-ra)y_{t-1} - ra_0$$

$$= A_0 + r\Gamma(r)t + (1-\phi_1)a\left[\left(\frac{1-\phi_1^t}{1-\phi_1}\right)(y_0 - \bar{y}) + \sigma\sum_{u=1}^{t-1}\sum_{j=0}^{u-1}\phi_1^j w_{u-j}\right]$$

$$= A_0 + r\Gamma(r)t + a[(1-\phi_1^t)(y_0 - \bar{y}) + \sigma\sum_{j=1}^{t-1}(1-\phi_1^j)w_{t-j}],$$

where $\Gamma(r)$ is given in (7). The candidate optimal consumption (5) may also be written as

$$\text{(A11)} \qquad c_{t+1}^* = c_t^* + r^2\Gamma(r) + ra\sigma w_{t+1}$$

$$\text{(A12)} \qquad = c_0^* + r^2\Gamma(r)(t+1) + ra\sigma W_{t+1},$$

where $W_{t+1} = \sum_{u=1}^{t+1} w_u$ is the cumulative shock. From (A11), consumption is expected to grow at a constant rate of $r^2\Gamma(r)$, per period, in that $E_t(c_{t+1}^*) = c_t^* + r^2\Gamma(r)$.

For future reference, let M be the marginal-utility process, evaluated at the candidate optimal consumption process c^* given in (5), in that

$$\text{(A13)} \qquad M_t = (1+\delta)^{-t}u'(c_t^*) = (1+\delta)^{-t}e^{-\theta c_t^*} = (1+r)^{-t}M_0\xi_t,$$

where

$$\xi_t = \left(\frac{1+r}{1+\delta}\right)^t e^{-\theta(c_t^* - c_0^*)} = \Pi_{u=1}^t \psi_u,$$

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[16] Equation (A10) holds for $t \geq 2$. At $t = 1$, the candidate optimal asset holding is $A_1 = A_0 + (1 - \phi_1)a(y_0 - \bar{y}) + r\Gamma$.

$$(A14) \qquad \psi_u = \frac{\exp(-\theta\sigma r a w_u)}{\zeta(-\theta\sigma r a)}.$$

Since w is i.i.d., ψ is also i.i.d., and ξ is thus an exponential martingale.

Verification of Optimality and Transversality

Fixing the initial wealth A_0 and the initial income y_0, on the given probability space $(\Omega, \mathcal{F}, P)$, with information filtration $\{\mathcal{F}_t\}_{t=0}^{\infty}$, an adapted consumption process is defined to be in the set $L(A_0, y_0)$ if the following "transversality condition" is satisfied:

$$(A15) \qquad \lim_{t\uparrow\infty} E[(1 + \delta)^{-t}\exp(-\theta r(A_t^c + ay_t))] = 0.$$

Condition (A15) restricts the rate at which the debt is allowed to grow, and is satisfied for the candidate consumption rule (5) because, from (A10),

$$(A16) \qquad \lim_{t\uparrow\infty} E[(1 + \delta)^{-t}\exp(-\theta r(A_t^* + ay_t))] = \bar{D} \lim_{t\uparrow\infty} (1 + r)^{-t} = 0,$$

where

$$(A17) \qquad \bar{D} = \lim_{t\uparrow\infty} E[\exp(-\theta r(A_0 + ay_0))\Pi_{u=1}^t \psi_u] = \exp[-\theta r(A_0 + ay_0)].$$

Let c be any consumption process from $L(A_0, y_0)$, and A^c be the associated wealth process. For any time t, by the Bellman equation (A1),

$$(A18) \qquad V(A_t^c, y_t) \geq u(c_t) + (1 + \delta)^{-1}E_t[V(A_{t+1}^c, y_{t+1})].$$

Multiplying through by $(1 + \delta)^{-t}$, taking expectations on both sides, using the law of iterated expectations, and rearranging gives

$$(A19) \qquad E[(1 + \delta)^{-t}V(A_t^c, y_t)] - E[(1 + \delta)^{-t-1}V(A_{t+1}^c, y_{t+1})] \geq E[(1 + \delta)^{-t}u(c_t)].$$

Adding the above expression for each t from $t = 0$ to $t = T$, for any $T \geq 1$, leaves

$$(A20) \qquad V(A_0, y_0) - (1 + \delta)^{-T-1}E[V(A_{T+1}^c, y_{T+1})] \geq E\left(\sum_{t=0}^{T} (1 + \delta)^{-t}u(c_t)\right).$$

Without loss of generality, we restrict attention to c in $L(A_0, y_0)$ with $U(c) \geq U(y)$, so that

$$(A21) \qquad E\left[\sum_{t=0}^{\infty} (1 + \delta)^{-t} u(c_t)\right] \geq E\left[\sum_{t=0}^{\infty} (1 + \delta)^{-t} u(y_t)\right] > -\infty.$$

Also, recall that $U(c) \leq 0$, since $u(c) = -e^{-\theta c}/\theta < 0$. Therefore,

$$(A22) \qquad E\left[\sum_{t=0}^{\infty} (1 + \delta)^{-t} |u(c_t)|\right] < \infty.$$

With (A22), the Dominated Convergence Theorem implies that, for any feasible consumption process c with $U(c) \geq U(y)$,

$$(A23) \qquad \lim_{T\uparrow\infty} E\left(\sum_{t=0}^{T} (1 + \delta)^{-t} u(c_t)\right) = E\left(\sum_{t=0}^{\infty} (1 + \delta)^{-t} u(c_t)\right).$$

Taking limits on both sides of (A20), using (A23) and (A15), leaves

$$(A24) \qquad V(A_0, y_0) \geq E\left(\sum_{t=0}^{\infty} (1 + \delta)^{-t} u(c_t)\right) = U(c).$$

All of the above calculations apply to the candidate optimal consumption c^*, given in (5), for which we may replace the inequality in (A20) with equality. This leaves $V(A_0, y_0) = U(c^*)$. Therefore, V is the value function and c^* of (5) is optimal, in that it solves (4).

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