# Adaptive sparse grids and quasi Monte Carlo for option pricing under the rough Bergomi model

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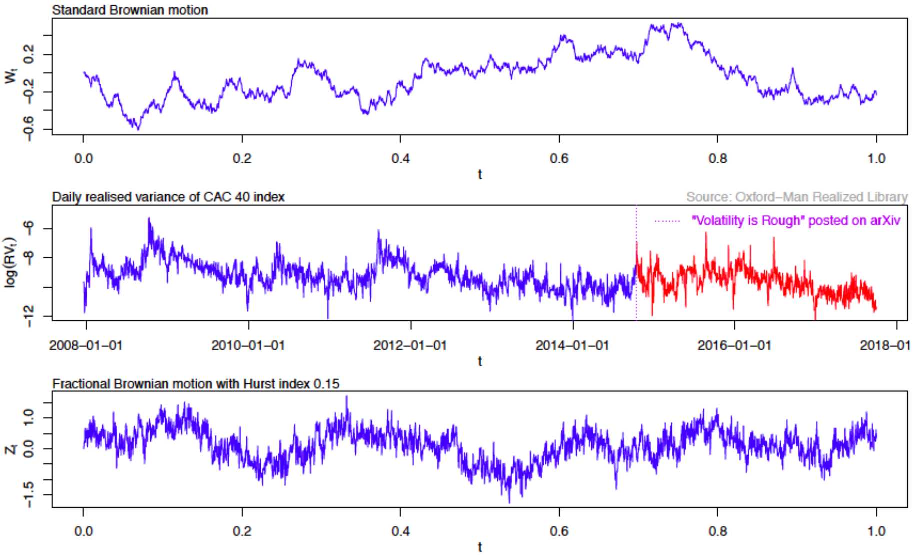
# Outline

1. Option Pricing under the Rough Bergomi Model: Motivation & Challenges
2. Our Hierarchical Deterministic Quadrature Methods
3. Numerical Experiments and Results
4. Conclusions

# Rough volatility [Gatheral et al., 2018]



# The rough Bergomi model [Bayer et al., 2016]

This model, under a pricing measure, is given by

$$\begin{cases} dS_t &= \sqrt{v_t} S_t dZ_t, \\ v_t &= \xi_0(t) \exp\left(\eta \widetilde{W}_t^H - \frac{1}{2}\eta^2 t^{2H}\right), \\ Z_t &:= \rho W_t^1 + \bar{\rho} W_t^{\perp} \equiv \rho W^1 + \sqrt{1-\rho^2} W^{\perp}, \end{cases} \tag{1}$$

- $(W^1, W^{\perp})$: two independent standard Brownian motions
- $\widetilde{W}^H$ is Riemann-Liouville process, defined by

$$\widetilde{W}_t^H = \int_0^t K^H(t-s) dW_s^1, \quad t \geq 0,$$

$$K^H(t-s) = \sqrt{2H}(t-s)^{H-1/2}, \quad \forall\, 0 \leq s \leq t.$$

- $H \in (0, 1/2]$ ($H = 1/2$ for Brownian motion): controls the roughness of paths, , $\rho \in [-1, 1]$ and $\eta > 0$.
- $t \mapsto \xi_0(t)$: forward variance curve, known at time 0.

# Model challenges

- **Numerically:**
  - The model is non-affine and non-Markovian $\Rightarrow$ Standard numerical methods (PDEs, characteristic functions) seem inapplicable.
  - The only prevalent pricing method for mere vanilla options is Monte Carlo (MC) [Bayer et al., 2016, Bayer et al., 2017, McCrickerd and Pakkanen, 2018]: still a time consuming task.
  - Discretization methods have poor behavior of the strong error, that is the convergence rate is of order of $H \in [0, 1/2]$ [Neuenkirch and Shalaiko, 2016] $\Rightarrow$ Variance reduction methods, such as multilevel Monte Carlo (MLMC), are inefficient for very small values of $H$.
- **Theoretically:**
  - No proper weak error analysis done in the rough volatility context.

# Option pricing challenges

The integration problem is challenging

- Issue 1: Time-discretization of the rough Bergomi process (large $N$ (number of time steps)) $\Rightarrow$ $S$ takes values in a high-dimensional space $\Rightarrow$ Curse of dimensionality when using numerical integration methods.
- Issue 2: The payoff function $g$ is typically not smooth $\Rightarrow$ low regularity$\Rightarrow$ slow convergence of deterministic quadrature methods.

⚠ Curse of dimensionality: An integration error of order $\varepsilon$ requires $M$ function evaluations

$$M \geq c_{\varepsilon} \bar{d}^{-c \log \varepsilon},$$

where $\bar{d}$ depends on $d$ and $N$.

# Methodology

We design a hierarchical efficient pricing method based on

1. Analytic smoothing to uncover available regularity (inspired by [Romano and Touzi, 1997] in the context of stochastic volatility models).
2. Approximating the option price using deterministic quadrature methods
   - **Adaptive sparse grids quadrature (ASGQ)**.
   - **Quasi Monte Carlo (QMC)**.
3. Coupling our methods with hierarchical transformations $\Rightarrow$ Reduce the dimension of the problem.
   - **Brownian bridges** as a path generation method.
   - **Richardson Extrapolation** $\Rightarrow$ Faster convergence of the weak error $\Rightarrow \searrow$ number of time steps (smaller dimension).

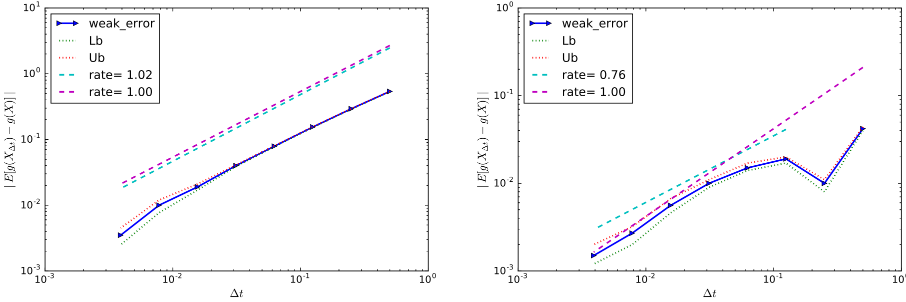
# Simulation of the rough Bergomi dynamics

**Goal:** Simulate jointly $(W_t^1, \widetilde{W}_t^H : 0 \leq t \leq T)$, resulting in $W_{t_1}^1, \dots, W_{t_N}$ and $\widetilde{W}_{t_1}^H, \dots, \widetilde{W}_{t_N}^H$ along a given grid $t_1 < \cdots < t_N$

1. **Covariance based approach** [Bayer et al., 2016]
   - Based on Cholesky decomposition of the covariance matrix of the $(2N)$-dimensional Gaussian random vector $W_{t_1}^1, \dots, W_{t_N}^1, \widetilde{W}_{t_1}^H, \dots, \widetilde{W}_{t_N}$.
   - Exact method but slow.
2. **The hybrid scheme** [Bennedsen et al., 2017]
   - Based on Euler discretization but crucially improved by moment matching for the singular term in the left point rule.
   - Accurate scheme that is much faster than the Covariance based approach.

# On the choice of the simulation scheme

Figure 1.1: The convergence of the weak error $\mathcal{E}_B$, using MC with $6 \times 10^6$ samples, for Set 1 parameter in Table 1. The upper and lower bounds are 95% confidence intervals. a) With the hybrid scheme b) With the exact scheme.




(a)

(b)

# Hybrid scheme [Bennedsen et al., 2017]

$$\widetilde{W}_t^{H} = \int_0^t K^{H}(t-s)dW_s^1, \quad t \geq 0,$$
$$K^{H}(t-s) = \sqrt{2H}(t-s)^{H-1/2}, \quad \forall\, 0 \leq s \leq t.$$

- The hybrid scheme discretizes the $\widetilde{W}^{H}$ process into Wiener integrals of power functions and a Riemann sum, appearing from approximating the kernel by power functions near the origin and step functions elsewhere.

$$\widetilde{W}^{H}_{\frac{i}{N}} \approx \overline{W}^{H}_{\frac{i}{N}} = \sqrt{2H}\left(W_i^2 + \sum_{k=2}^{i}\left(\frac{b_k}{N}\right)^{H-\frac{1}{2}}\left(W^1_{\frac{i-(k-1)}{N}} - W^1_{\frac{i-k}{N}}\right)\right),$$

  - $N$ is the number of time steps
  - $\{W_j^2\}_{j=1}^N$: Artificially introduced $N$ Gaussian random variables that are used for left-rule points in the hybrid scheme.
  - $b_k = \left(\frac{k^{H+\frac{1}{2}} - (k-1)^{H+\frac{1}{2}}}{H+\frac{1}{2}}\right)^{\frac{1}{H-\frac{1}{2}}}$.

# Analytic smoothing

$$
\begin{aligned}
C_{RB}\left(T,K\right) &= E\left[\left(S_T-K\right)^+\right] \\
&= \mathrm{E}\left[\mathrm{E}\left[(S_T-K)^+ \mid \sigma(W^1(t), t \le T)\right]\right] \\
&= E\left[C_{BS}\left(S_0 = \exp\left(\rho\int_0^T \sqrt{v_t}dW_t^1 - \frac{1}{2}\rho^2\int_0^T v_t dt\right),\right.\right. \\
&\qquad\qquad \left.\left. k = K,\ \sigma^2 = (1-\rho^2)\int_0^T v_t dt\right)\right] \\
&\approx \int_{\mathbb{R}^{2N}} C_{BS}\left(G(\mathbf{w}^{(1)},\mathbf{w}^{(2)})\right)\rho_N(\mathbf{w}^{(1)})\rho_N(\mathbf{w}^{(2)})d\mathbf{w}^{(1)}d\mathbf{w}^{(2)} \\
&= C_{RB}^N.
\end{aligned}
\tag{2}
$$

- $C_{\mathrm{BS}}(S_0, k, \sigma^2)$: the Black-Scholes call price, for initial spot price $S_0$, strike price $k$, and volatility $\sigma^2$.
- $G$ maps $2N$ independent standard Gaussian random inputs to the parameters fed to Black-Scholes formula.
- $\rho_N$: the multivariate Gaussian density, $N$: number of time steps.

# Numerical integration methods

- **Plain Monte Carlo (MC)**
  - $\varepsilon(M) = \mathcal{O}\left(M^{-1/2}\right)$
  - (+) insensitive to $d$, (−) slow convergence, no profit from regularity.
- **Classical Quasi-Monte Carlo (QMC)**
  - $\varepsilon(M) = \mathcal{O}\left(M^{-1}\log(M)^{d-1}\right)$
  - (+) better convergence, (−) sensitive to $d$, no profit from regularity.
- **Quadrature based on product approaches**
  - $\varepsilon(M) = \mathcal{O}\left(M^{-r/d}\right)$
  - (+) profits from regularity, faster than QMC if $r > d$, (−) highly sensitive to $d$.
- **Sparse grids quadrature (SGQ)**
  - $\varepsilon(M) = \mathcal{O}\left(M^{-s}\log(M)^{(d-1)(s+1)}\right)$
  - (+) profits from regularity, faster than QMC if $s > 1$, less sensitive to $d$.

$\varepsilon$: prescribed accuracy, $M$: the amount of work, $d$: dimension of problem, $r, s$: smoothness indices (bounded mixed (total) derivatives up to order $s(r)$).

# Sparse grids I

**Goal:** Given $F : \mathbb{R}^d \to \mathbb{R}$ and a multi-index $\boldsymbol{\beta} \in \mathbb{N}_+^d$, approximate

$$\mathrm{E}\left[F\right] \approx Q^{m(\boldsymbol{\beta})}[F],$$

where $Q^{m(\boldsymbol{\beta})}$ a Cartesian quadrature grid with $m(\beta_n)$ points along $y_n$.
**Idea:** Denote $Q^{m(\boldsymbol{\beta})}[F] = F_{\boldsymbol{\beta}}$ and introduce the first difference operator

$$\Delta_i F_{\boldsymbol{\beta}} \begin{cases} F_{\boldsymbol{\beta}} - F_{\boldsymbol{\beta}-e_i}, & \text{if } \beta_i > 1 \\ F_{\boldsymbol{\beta}} & \text{if } \beta_i = 1 \end{cases}$$

where $e_i$ denotes the $i$th $d$-dimensional unit vector, and mixed difference operators

$$\Delta[F_{\boldsymbol{\beta}}] = \otimes_{i=1}^{d} \Delta_i F_{\boldsymbol{\beta}}$$

# Sparse grids II

A quadrature estimate of $E[F]$ is

$$\mathcal{M}_{\mathcal{I}_\ell}[F] = \sum_{\boldsymbol{\beta} \in \mathcal{I}_\ell} \Delta[F_{\boldsymbol{\beta}}], \tag{3}$$

- Product approach: $\mathcal{I}_\ell = \{\max\{\beta_1, \ldots, \beta_d\} \leq \ell;\ \boldsymbol{\beta} \in \mathbb{N}_+^d\}$
- Regular SG: $\mathcal{I}_\ell = \{\mid \boldsymbol{\beta} \mid_1 \leq \ell + d - 1;\ \boldsymbol{\beta} \in \mathbb{N}_+^d\}$

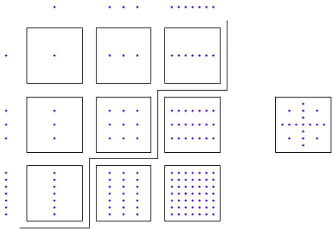


Figure 2.1: Left are product grids $\Delta_{\beta_1} \otimes \Delta_{\beta_2}$ for $1 \leq \beta_1, \beta_2 \leq 3$. Right is the corresponding SG construction.

- ASGQ: $\mathcal{I}_\ell = \mathcal{I}^{\text{ASGQ}}$.

# ASGQ in practice

- The construction of $\mathcal{I}^{\text{ASGQ}}$ is done by profit thresholding

$$\mathcal{I}^{\text{ASGQ}} = \{\boldsymbol{\beta} \in \mathbb{N}_+^d : P_{\boldsymbol{\beta}} \geq \overline{T}\}.$$

- **Profit of a hierarchical surplus** $P_{\boldsymbol{\beta}} = \frac{|\Delta E_{\boldsymbol{\beta}}|}{\Delta \mathcal{W}_{\boldsymbol{\beta}}}$.
- **Error contribution**: $\Delta E_{\boldsymbol{\beta}} = \left| \mathcal{M}^{\mathcal{I} \cup \{\boldsymbol{\beta}\}} - \mathcal{M}^{\mathcal{I}} \right|$.
- **Work contribution**: $\Delta \mathcal{W}_{\boldsymbol{\beta}} = \text{Work}[\mathcal{M}^{\mathcal{I} \cup \{\boldsymbol{\beta}\}}] - \text{Work}[\mathcal{M}^{\mathcal{I}}]$.

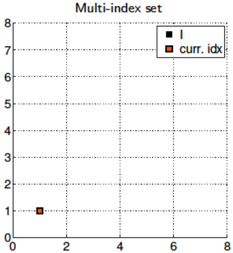



Figure 2.2: A posteriori, adaptive construction: Given an index set $\mathcal{I}_k$, compute the profits of the neighbor indices and select the most profitable one

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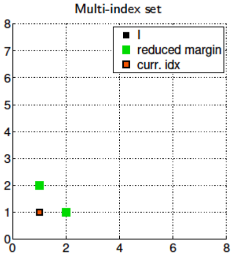


Figure 2.3: A posteriori, adaptive construction: Given an index set $\mathcal{I}_k$, compute the profits of the neighbor indices and select the most profitable one.

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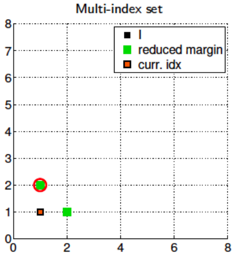


Figure 2.4: A posteriori, adaptive construction: Given an index set $\mathcal{I}_k$, compute the profits of the neighbor indices and select the most profitable one.

# ASGQ in practice

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- **Error contribution**: $\Delta E_{\boldsymbol{\beta}} = \left|\mathcal{M}^{\mathcal{I} \cup \{\boldsymbol{\beta}\}} - \mathcal{M}^{\mathcal{I}}\right|$.
- **Work contribution**: $\Delta \mathcal{W}_{\boldsymbol{\beta}} = \text{Work}[\mathcal{M}^{\mathcal{I} \cup \{\boldsymbol{\beta}\}}] - \text{Work}[\mathcal{M}^{\mathcal{I}}]$.

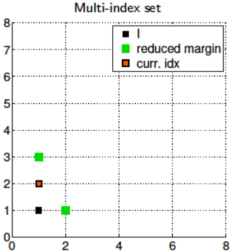


Figure 2.5: A posteriori, adaptive construction: Given an index set $\mathcal{I}_k$, compute the profits of the neighbor indices and select the most profitable one.

# ASGQ in practice

- The construction of $\mathcal{I}^{\text{ASGQ}}$ is done by profit thresholding

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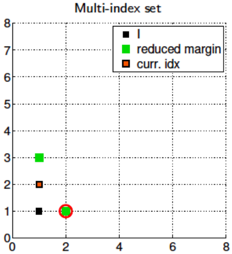


Figure 2.6: A posteriori, adaptive construction: Given an index set $\mathcal{I}_k$, compute the profits of the neighbor indices and select the most profitable one.

# ASGQ in practice

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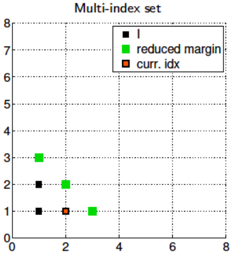


Figure 2.7: A posteriori, adaptive construction: Given an index set $\mathcal{I}_k$, compute the profits of the neighbor indices and select the most profitable one.

# ASGQ in practice

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- **Work contribution**: $\Delta \mathcal{W}_{\boldsymbol{\beta}} = \text{Work}[\mathcal{M}^{\mathcal{I}\cup\{\boldsymbol{\beta}\}}] - \text{Work}[\mathcal{M}^{\mathcal{I}}]$.

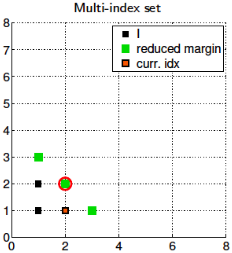



Figure 2.8: A posteriori, adaptive construction: Given an index set $\mathcal{I}_k$, compute the profits of the neighbor indices and select the most profitable one.

# Randomized QMC

- A (rank-1) lattice rule [Sloan, 1985, Nuyens, 2014] with $n$ points

$$Q_n(f) := \frac{1}{n}\sum_{k=0}^{n-1} f\left(\frac{kz \bmod n}{n}\right),$$

where $z = (z_1, \dots, z_d) \in \mathbb{N}^d$.

- A randomly shifted lattice rule

$$\overline{Q}_{n,q}(f) = \frac{1}{q}\sum_{i=0}^{q-1} Q_n^{(i)}(f) = \frac{1}{q}\sum_{i=0}^{q-1}\left(\frac{1}{n}\sum_{k=0}^{n-1} f\left(\frac{kz + \Delta^{(i)} \bmod n}{n}\right)\right), \quad (4)$$

where $\{\Delta^{(i)}\}_{i=1}^{q}$: independent random shifts, and $M^{\mathrm{QMC}} = q \times n$.

  - Unbiased approximation of the integral.
  - Practical error estimate.

- We use a pre-made point generators using latticeseq_b2.py from `https://people.cs.kuleuven.be/~dirk.nuyens/qmc-generators/`.

# Path generation methods

$\{t_i\}_{i=0}^N$: Grid of time steps, $\{B_{t_i}\}_{i=0}^N$: Brownian motion increments

- **Random Walk**
  - Proceeds incrementally, given $B_{t_i}$,

$$B_{t_{i+1}} = B_{t_i} + \sqrt{\Delta t} z_i, \; z_i \sim \mathcal{N}(0,1).$$

  - All components of $\mathbf{z} = (z_1, \ldots, z_N)$ have the same scale of importance: isotropic.
- **Hierarchical Brownian Bridge** [Glasserman, 2004]
  - Given a past value $B_{t_i}$ and a future value $B_{t_k}$, the value $B_{t_j}$ (with $t_i < t_j < t_k$) can be generated according to ($\rho = \frac{j-i}{k-i}$)

$$B_{t_j} = (1-\rho) B_{t_i} + \rho B_{t_k} + \sqrt{\rho(1-\rho)(k-i)\Delta t} z_j, \; z_j \sim \mathcal{N}(0,1). \tag{5}$$

  - The most important values (determine the large scale structure of Brownian motion) are the first components of $\mathbf{z} = (z_1, \ldots, z_N)$.
  - $\searrow$ the effective dimension (# important dimensions) and $\nearrow$ anisotropy between different directions $\Rightarrow$ Faster ASGQ and QMC convergence.

# Error comparison

$\mathcal{E}_{\text{tot}}$: the total error of approximating the expectation in (2).

- When using ASGQ estimator, $Q_N$

$$\mathcal{E}_{\text{tot}} \leq \left|C_{\text{RB}} - C_{\text{RB}}^N\right| + \left|C_{\text{RB}}^N - Q_N\right| \leq \mathcal{E}_B(N) + \mathcal{E}_Q(\text{TOL}_{\text{ASGQ}}, N),$$

  where $\mathcal{E}_Q$ is the quadrature error, $\mathcal{E}_B$ is the bias, $\text{TOL}_{\text{ASGQ}}$ is a user selected tolerance for ASGQ method.

- When using randomized QMC or MC estimator, $Q_N^{\text{MC (QMC)}}$

$$\mathcal{E}_{\text{tot}} \leq \left|C_{\text{RB}} - C_{\text{RB}}^N\right| + \left|C_{\text{RB}}^N - Q_N^{\text{MC (QMC)}}\right| \leq \mathcal{E}_B(N) + \mathcal{E}_S(M, N),$$

  where $\mathcal{E}_S$ is the statistical error, $M$ is the number of samples used for MC or randomized QMC method.

- $M^{\text{QMC}}$ and $M^{\text{MC}}$, are chosen so that $\mathcal{E}_{S,\text{QMC}}(M^{\text{QMC}})$ and $\mathcal{E}_{S,\text{MC}}(M^{\text{MC}})$ satisfy

$$\mathcal{E}_{S,\text{QMC}}(M^{\text{QMC}}) = \mathcal{E}_{S,\text{MC}}(M^{\text{MC}}) = \mathcal{E}_B(N) = \frac{\mathcal{E}_{\text{tot}}}{2}.$$

# Numerical experiments

Table 1: Reference solution (using MC with 500 time steps and number of samples, $M = 8 \times 10^6$) of call option price under the rough Bergomi model, for different parameter constellations. The numbers between parentheses correspond to the statistical errors estimates.

| Parameters | Reference solution |
|---|---|
| Set 1: $H = 0.07, K = 1, S_0 = 1, T = 1, \rho = -0.9, \eta = 1.9, \xi_0 = 0.235^2$ | 0.0791 (5.6$e$−05) |
| Set 2: $H = 0.02, K = 1, S_0 = 1, T = 1, \rho = -0.7, \eta = 0.4, \xi_0 = 0.1$ | 0.1246 (9.0$e$−05) |
| Set 3: $H = 0.02, K = 0.8, S_0 = 1, T = 1, \rho = -0.7, \eta = 0.4, \xi_0 = 0.1$ | 0.2412 (5.4$e$−05) |
| Set 4: $H = 0.02, K = 1.2, S_0 = 1, T = 1, \rho = -0.7, \eta = 0.4, \xi_0 = 0.1$ | 0.0570 (8.0$e$−05) |

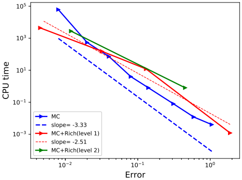
- The first set is the closest to the empirical findings [Gatheral et al., 2018, Bennedsen et al., 2016], suggesting that $H \approx 0.1$. The choice of values $\nu = 1.9$ and $\rho = -0.9$ is justified by [Bayer et al., 2016].
- For the remaining three sets, we wanted to test the potential of our method for a very rough case, where variance reduction methods are inefficient.

# Relative errors and computational gains

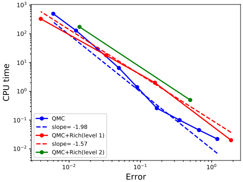
Table 2: In this table, we highlight the computational gains achieved by ASGQ and QMC over MC method to meet a certain error tolerance. We note that the ratios are computed for the best configuration with Richardson extrapolation for each method.

| **Parameter set** | **Relative error** | **CPU time ratio (MC/ASGQ)** | **CPU time ratio (MC/QMC)** |
|---|---|---|---|
| Set 1 | 1% | 15 | 10 |
| Set 2 | 0.2% | 21.5 | 73.2 |
| Set 3 | 0.4% | 26.7 | 21.3 |
| Set 4 | 2% | 5 | 10 |

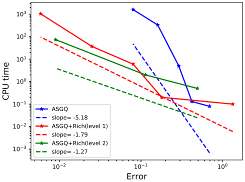
# Complexity of the different methods




(a)




(b)




(c)

Figure 3.1: Numerical complexity of the different methods with the different configurations in terms of Richardson extrapolation 's level. Case of parameter set 1 in Table 1. a) MC methods. b) QMC methods. d) ASGQ methods.

# Complexity of the different methods with their best configurations

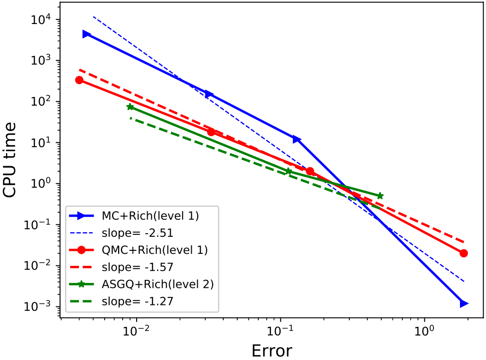



Figure 3.2: Computational work comparison for the different methods with the best configurations concluded from Figure 3.1, for the case of parameter set 1 in Table 1.

# Conclusions

- Proposed novel, fast option pricers, based on hierarchical deterministic quadrature methods, for options whose underlyings follow the rBergomi model.
- Given a sufficiently small relative error tolerance, our proposed methods demonstrate substantial computational gains over the standard MC method, for different parameter constellations.
- Accelerating our novel methods can be achieved by using more optimal hierarchical path generation method than Brownian bridge construction, such as PCA or LT transformations.

Thank you for your attention

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