

Adaptive sparse grids and quasi Monte Carlo for option pricing under the rough Bergomi model

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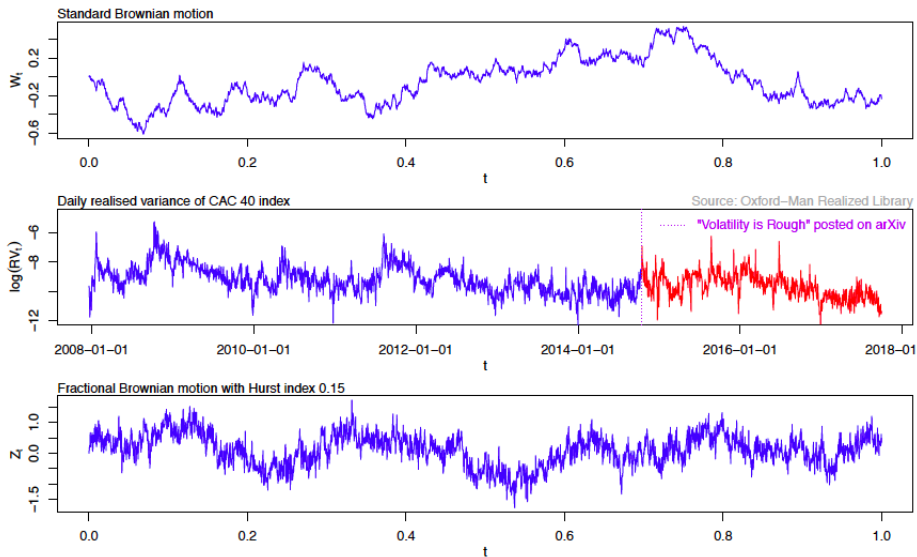
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Outline

- ➊ Option Pricing under the Rough Bergomi Model: Motivation & Challenges
- ➋ Our Hierarchical Deterministic Quadrature Methods
- ➌ Numerical Experiments and Results
- ➍ Conclusions

- 1 Option Pricing under the Rough Bergomi Model: Motivation & Challenges
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Rough volatility [Gatheral et al., 2018]



The rough Bergomi model [Bayer et al., 2016]

This model, under a pricing measure, is given by

$$\begin{cases} dS_t &= \sqrt{v_t} S_t dZ_t, \\ v_t &= \xi_0(t) \exp \left(\eta \widetilde{W}_t^H - \frac{1}{2} \eta^2 t^{2H} \right), \\ Z_t &:= \rho W_t^1 + \bar{\rho} W_t^\perp \equiv \rho W^1 + \sqrt{1 - \rho^2} W^\perp, \end{cases} \quad (1)$$

- (W^1, W^\perp) : two independent standard Brownian motions
- \widetilde{W}^H is **Riemann-Liouville process**, defined by

$$\begin{aligned} \widetilde{W}_t^H &= \int_0^t K^H(t-s) dW_s^1, \quad t \geq 0, \\ K^H(t-s) &= \sqrt{2H}(t-s)^{H-1/2}, \quad \forall 0 \leq s \leq t. \end{aligned}$$

- $H \in (0, 1/2]$ ($H = 1/2$ for Brownian motion): controls the **roughness** of paths, $\rho \in [-1, 1]$ and $\eta > 0$.
- $t \mapsto \xi_0(t)$: forward variance curve, known at time 0.

Model challenges

- **Numerically:**

- ▶ The model is **non-affine** and **non-Markovian** \Rightarrow Standard numerical methods (PDEs, characteristic functions) seem **inapplicable**.
- ▶ The only prevalent pricing method for mere **vanilla options** is **Monte Carlo (MC)** [Bayer et al., 2016, Bayer et al., 2017, McCrickerd and Pakkanen, 2018]: still a **time consuming task**.
- ▶ Discretization methods have **poor behavior of the strong error**, that is the convergence rate is of order of $H \in [0, 1/2]$ [Neuenkirch and Shalaiko, 2016] \Rightarrow Variance reduction methods, such as **multilevel Monte Carlo (MLMC)**, are inefficient for **very small values** of H .

- **Theoretically:**

- ▶ No proper weak error analysis done in the rough volatility context.

Option pricing challenges

The integration problem is **challenging**

- **Issue 1:** Time-discretization of the rough Bergomi process (large N (number of time steps)) $\Rightarrow S$ takes values in a high-dimensional space \Rightarrow **Curse of dimensionality** when using numerical integration methods.
- **Issue 2:** The payoff function g is typically **not smooth** \Rightarrow **low regularity** \Rightarrow slow convergence of deterministic quadrature methods.

⚠ **Curse of dimensionality:** An integration error of order ε requires M function evaluations

$$M \geq c_{\varepsilon} \bar{d}^{-c \log \varepsilon},$$

where \bar{d} depends on d and N .

Methodology

We design a **hierarchical efficient pricing method** based on

- ① **Analytic smoothing** to uncover available regularity (inspired by [Romano and Touzi, 1997] in the context of stochastic volatility models).
- ② Approximating the option price using **deterministic quadrature methods**
 - ▶ **Adaptive sparse grids quadrature (ASGQ).**
 - ▶ **Quasi Monte Carlo (QMC).**
- ③ Coupling our methods with **hierarchical transformations** \Rightarrow **Reduce the dimension** of the problem.
 - ▶ **Brownian bridges** as a path generation method.
 - ▶ **Richardson Extrapolation** \Rightarrow Faster convergence of the weak error $\Rightarrow \searrow$ number of time steps (smaller dimension).

Simulation of the rough Bergomi dynamics

Goal: Simulate jointly $(W_t^1, \widetilde{W}_t^H : 0 \leq t \leq T)$, resulting in $W_{t_1}^1, \dots, W_{t_N}^1$ and $\widetilde{W}_{t_1}^H, \dots, \widetilde{W}_{t_N}^H$ along a given grid $t_1 < \dots < t_N$

❶ Covariance based approach [Bayer et al., 2016]

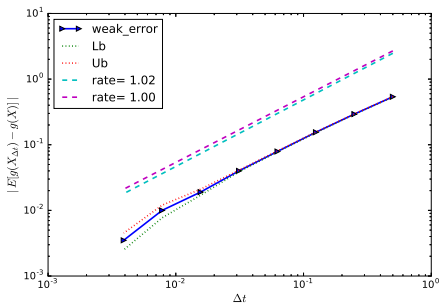
- ▶ Based on Cholesky decomposition of the covariance matrix of the $(2N)$ -dimensional Gaussian random vector $W_{t_1}^1, \dots, W_{t_N}^1, \widetilde{W}_{t_1}^H, \dots, \widetilde{W}_{t_N}^H$.
- ▶ **Exact method but slow.**

❷ The hybrid scheme [Bennedsen et al., 2017]

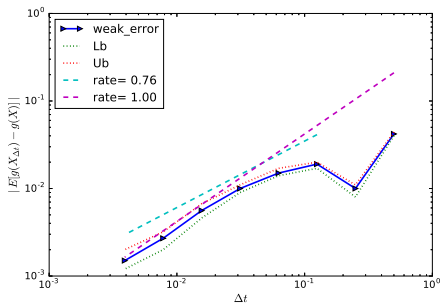
- ▶ Based on **Euler discretization** but crucially **improved by moment matching** for the singular term in the left point rule.
- ▶ **Accurate scheme that is much faster** than the Covariance based approach.

On the choice of the simulation scheme

Figure 1.1: The convergence of the weak error \mathcal{E}_B , using MC with 6×10^6 samples, for **Set 1 parameter in Table 1**. The upper and lower bounds are 95% confidence intervals. a) With **the hybrid scheme** b) With **the exact scheme**.



(a)



(b)

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Analytic smoothing

$$\begin{aligned} C_{RB}(T, K) &= E \left[(S_T - K)^+ \right] \\ &= E \left[E \left[(S_T - K)^+ \mid \sigma(W^1(t), t \leq T) \right] \right] \\ &= E \left[C_{BS} \left(S_0 = \exp \left(\rho \int_0^T \sqrt{v_t} dW_t^1 - \frac{1}{2} \rho^2 \int_0^T v_t dt \right), \right. \right. \\ &\quad \left. \left. k = K, \sigma^2 = (1 - \rho^2) \int_0^T v_t dt \right) \right] \\ &\approx \int_{\mathbb{R}^{2N}} C_{BS} \left(G(\mathbf{w}^{(1)}, \mathbf{w}^{(2)}) \right) \rho_N(\mathbf{w}^{(1)}) \rho_N(\mathbf{w}^{(2)}) d\mathbf{w}^{(1)} d\mathbf{w}^{(2)} \\ &= C_{RB}^N. \end{aligned} \tag{2}$$

- $C_{BS}(S_0, k, \sigma^2)$: the Black-Scholes call price, for initial spot price S_0 , strike price k , and volatility σ^2 .
- G maps $2N$ independent standard Gaussian random inputs to the parameters fed to Black-Scholes formula.
- ρ_N : the multivariate Gaussian density, N : number of time steps.

Numerical integration methods

- **Plain Monte Carlo (MC)**

- ▶ $\varepsilon(M) = \mathcal{O}(M^{-1/2})$
- ▶ (+) insensitive to d , (–) slow convergence, no profit from regularity.

- **Classical Quasi-Monte Carlo (QMC)**

- ▶ $\varepsilon(M) = \mathcal{O}(M^{-1} \log(M)^{d-1})$
- ▶ (+) better convergence, (–) sensitive to d , no profit from regularity.

- **Quadrature based on product approaches**

- ▶ $\varepsilon(M) = \mathcal{O}(M^{-r/d})$
- ▶ (+) profits from regularity, faster than QMC if $r > d$, (–) highly sensitive to d .

- **Sparse grids quadrature (SGQ)**

- ▶ $\varepsilon(M) = \mathcal{O}(M^{-s} \log(M)^{(d-1)(s+1)})$
- ▶ (+) profits from regularity, faster than QMC if $s > 1$, less sensitive to d .

ε : prescribed accuracy, M : the amount of work, d : dimension of problem, r, s : smoothness indices (bounded mixed (total) derivatives up to order $s(r)$).

Sparse grids I

Goal: Given $F : \mathbb{R}^d \rightarrow \mathbb{R}$ and a multi-index $\beta \in \mathbb{N}_+^d$, **approximate**

$$\mathbb{E}[F] \approx Q^{m(\beta)}[F],$$

where $Q^{m(\beta)}$ a Cartesian quadrature grid with $m(\beta_n)$ points along y_n .

Idea: Denote $Q^{m(\beta)}[F] = F_\beta$ and introduce the **first difference operator**

$$\Delta_i F_\beta \begin{cases} F_\beta - F_{\beta - e_i}, & \text{if } \beta_i > 1 \\ F_\beta & \text{if } \beta_i = 1 \end{cases}$$

where e_i denotes the i th d -dimensional unit vector, and **mixed difference operators**

$$\Delta[F_\beta] = \otimes_{i=1}^d \Delta_i F_\beta$$

Sparse grids II

A quadrature estimate of $E[F]$ is

$$\mathcal{M}_{\mathcal{I}_\ell}[F] = \sum_{\beta \in \mathcal{I}_\ell} \Delta[F_\beta], \quad (3)$$

- **Product approach:** $\mathcal{I}_\ell = \{\max\{\beta_1, \dots, \beta_d\} \leq \ell; \beta \in \mathbb{N}_+^d\}$
- **Regular SG:** $\mathcal{I}_\ell = \{|\beta|_1 \leq \ell + d - 1; \beta \in \mathbb{N}_+^d\}$

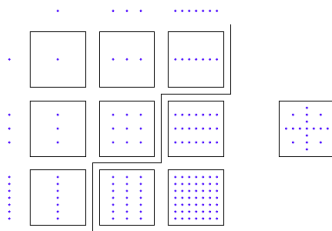


Figure 2.1: Left are product grids $\Delta_{\beta_1} \otimes \Delta_{\beta_2}$ for $1 \leq \beta_1, \beta_2 \leq 3$. Right is the corresponding SG construction.

- **ASGQ:** $\mathcal{I}_\ell = \mathcal{I}^{\text{ASGQ}}$.

ASGQ in practice

- The construction of $\mathcal{I}^{\text{ASGQ}}$ is done by **profit thresholding**

$$\mathcal{I}^{\text{ASGQ}} = \{\beta \in \mathbb{N}_+^d : P_\beta \geq \bar{T}\}.$$

- **Profit of a hierarchical surplus** $P_\beta = \frac{|\Delta E_\beta|}{\Delta \mathcal{W}_\beta}$.
- **Error contribution:** $\Delta E_\beta = |\mathcal{M}^{\mathcal{I} \cup \{\beta\}} - \mathcal{M}^{\mathcal{I}}|$.
- **Work contribution:** $\Delta \mathcal{W}_\beta = \text{Work}[\mathcal{M}^{\mathcal{I} \cup \{\beta\}}] - \text{Work}[\mathcal{M}^{\mathcal{I}}]$.

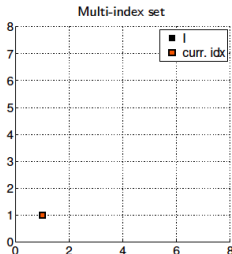


Figure 2.2: **A posteriori, adaptive construction:** Given an index set \mathcal{I}_k , compute the profits of the neighbor indices and select the most profitable one

ASGQ in practice

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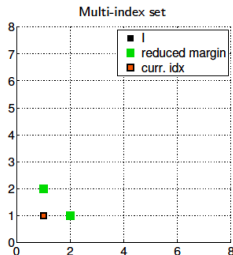


Figure 2.3: **A posteriori, adaptive construction:** Given an index set \mathcal{I}_k , compute the profits of the neighbor indices and select the most profitable one.

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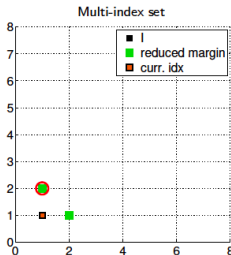


Figure 2.4: **A posteriori, adaptive construction:** Given an index set \mathcal{I}_k , compute the profits of the neighbor indices and select the most profitable one.

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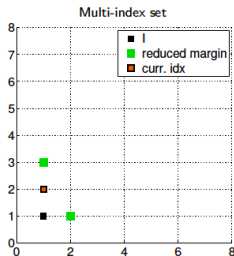


Figure 2.5: **A posteriori, adaptive construction:** Given an index set \mathcal{I}_k , compute the profits of the neighbor indices and select the most profitable one.

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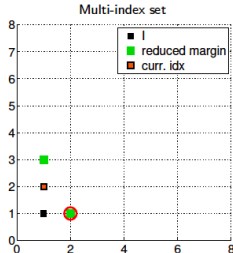


Figure 2.6: **A posteriori, adaptive construction:** Given an index set \mathcal{I}_k , compute the profits of the neighbor indices and select the most profitable one.

ASGQ in practice

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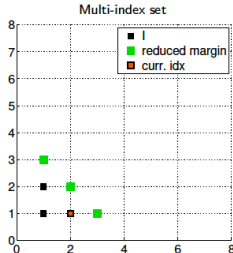


Figure 2.7: **A posteriori, adaptive construction:** Given an index set \mathcal{I}_k , compute the profits of the neighbor indices and select the most profitable one.

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- Work contribution:** $\Delta \mathcal{W}_\beta = \text{Work}[\mathcal{M}^{\mathcal{I} \cup \{\beta\}}] - \text{Work}[\mathcal{M}^{\mathcal{I}}]$.

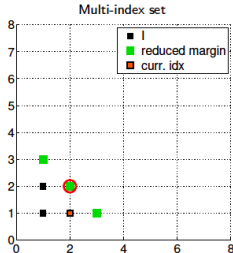


Figure 2.8: **A posteriori, adaptive construction:** Given an index set \mathcal{I}_k , compute the profits of the neighbor indices and select the most profitable one.

Randomized QMC

- A (rank-1) lattice rule [Sloan, 1985, Nuyens, 2014] with n points

$$Q_n(f) := \frac{1}{n} \sum_{k=0}^{n-1} f\left(\frac{kz \bmod n}{n}\right),$$

where $z = (z_1, \dots, z_d) \in \mathbb{N}^d$.

- A randomly shifted lattice rule

$$\overline{Q}_{n,q}(f) = \frac{1}{q} \sum_{i=0}^{q-1} Q_n^{(i)}(f) = \frac{1}{q} \sum_{i=0}^{q-1} \left(\frac{1}{n} \sum_{k=0}^{n-1} f\left(\frac{kz + \Delta^{(i)} \bmod n}{n}\right) \right), \quad (4)$$

where $\{\Delta^{(i)}\}_{i=1}^q$: independent random shifts, and $M^{\text{QMC}} = q \times n$.

- ▶ Unbiased approximation of the integral.
- ▶ Practical error estimate.
- We use a pre-made point generators using latticeseq_b2.py from <https://people.cs.kuleuven.be/~dirk.nuyens/qmc-generators/>.

Path generation methods

$\{t_i\}_{i=0}^N$: Grid of time steps, $\{B_{t_i}\}_{i=0}^N$: Brownian motion increments

- **Random Walk**

- ▶ Proceeds incrementally, given B_{t_i} ,

$$B_{t_{i+1}} = B_{t_i} + \sqrt{\Delta t} z_i, \quad z_i \sim \mathcal{N}(0, 1).$$

- ▶ All components of $\mathbf{z} = (z_1, \dots, z_N)$ have the same scale of importance: **isotropic**.

- **Hierarchical Brownian Bridge** [Glasserman, 2004]

- ▶ Given a past value B_{t_i} and a future value B_{t_k} , the value B_{t_j} (with $t_i < t_j < t_k$) can be generated according to ($\rho = \frac{j-i}{k-i}$)

$$B_{t_j} = (1 - \rho)B_{t_i} + \rho B_{t_k} + \sqrt{\rho(1 - \rho)(k - i)\Delta t} z_j, \quad z_j \sim \mathcal{N}(0, 1). \quad (5)$$

- ▶ The most important values (determine the large scale structure of Brownian motion) are the first components of $\mathbf{z} = (z_1, \dots, z_N)$.
- ▶ \searrow the **effective dimension** (# important dimensions) and \nearrow **anisotropy** between different directions \Rightarrow **Faster** ASGQ and QMC convergence.

Error comparison

\mathcal{E}_{tot} : the total error of approximating the expectation in (2).

- When using ASGQ estimator, Q_N

$$\mathcal{E}_{\text{tot}} \leq |C_{\text{RB}} - C_{\text{RB}}^N| + |C_{\text{RB}}^N - Q_N| \leq \mathcal{E}_B(N) + \mathcal{E}_Q(\text{TOL}_{\text{ASGQ}}, N),$$

where \mathcal{E}_Q is the quadrature error, \mathcal{E}_B is the bias, TOL_{ASGQ} is a user selected tolerance for ASGQ method.

- When using randomized QMC or MC estimator, $Q_N^{\text{MC (QMC)}}$

$$\mathcal{E}_{\text{tot}} \leq |C_{\text{RB}} - C_{\text{RB}}^N| + |C_{\text{RB}}^N - Q_N^{\text{MC (QMC)}}| \leq \mathcal{E}_B(N) + \mathcal{E}_S(M, N),$$

where \mathcal{E}_S is the statistical error, M is the number of samples used for MC or randomized QMC method.

- M^{QMC} and M^{MC} , are chosen so that $\mathcal{E}_{S,\text{QMC}}(M^{\text{QMC}})$ and $\mathcal{E}_{S,\text{MC}}(M^{\text{MC}})$ satisfy

$$\mathcal{E}_{S,\text{QMC}}(M^{\text{QMC}}) = \mathcal{E}_{S,\text{MC}}(M^{\text{MC}}) = \mathcal{E}_B(N) = \frac{\mathcal{E}_{\text{tot}}}{2}.$$

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Numerical experiments

Table 1: Reference solution (using MC with 500 time steps and number of samples, $M = 8 \times 10^6$) of call option price under the rough Bergomi model, for different parameter constellations. The numbers between parentheses correspond to the statistical errors estimates.

Parameters	Reference solution
Set 1: $H = 0.07, K = 1, S_0 = 1, T = 1, \rho = -0.9, \eta = 1.9, \xi_0 = 0.235^2$	0.0791 ($5.6e-05$)
Set 2: $H = 0.02, K = 1, S_0 = 1, T = 1, \rho = -0.7, \eta = 0.4, \xi_0 = 0.1$	0.1246 ($9.0e-05$)
Set 3: $H = 0.02, K = 0.8, S_0 = 1, T = 1, \rho = -0.7, \eta = 0.4, \xi_0 = 0.1$	0.2412 ($5.4e-05$)
Set 4: $H = 0.02, K = 1.2, S_0 = 1, T = 1, \rho = -0.7, \eta = 0.4, \xi_0 = 0.1$	0.0570 ($8.0e-05$)

- The first set is the **closest to the empirical findings** [Gatheral et al., 2018, Bennedsen et al., 2016], suggesting that $H \approx 0.1$. The choice of values $\nu = 1.9$ and $\rho = -0.9$ is justified by [Bayer et al., 2016].
- For the remaining three sets, we wanted to test the potential of our method for a **very rough case**, where variance reduction methods are inefficient.

Relative errors and computational gains

Table 2: In this table, we highlight the computational gains achieved by ASGQ and QMC over MC method to meet a certain error tolerance. We note that the ratios are computed **for the best configuration with Richardson extrapolation for each method.**

Parameters	Relative error	CPU time ratio (MC/ASGQ)	CPU time ratio (MC/QMC)
Set 1	1%	15	10
Set 2	0.2%	21.5	73.2
Set 3	0.4%	26.7	21.3
Set 4	2%	5	10

Complexity of the MC method with different configurations

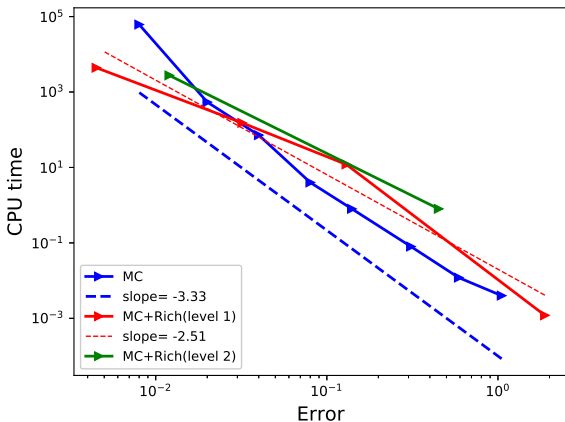


Figure 3.1: Numerical complexity of the MC method with the different configurations in terms of Richardson extrapolation 's level. Case of parameter set 1 in Table 1.

Complexity of the QMC method with different configurations

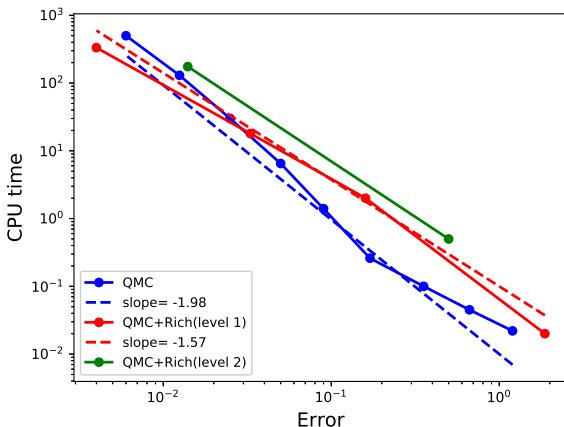


Figure 3.2: Numerical complexity of the QMC method with the different configurations in terms of Richardson extrapolation 's level. Case of parameter set 1 in Table 1.

Complexity of the ASGQ method with different configurations

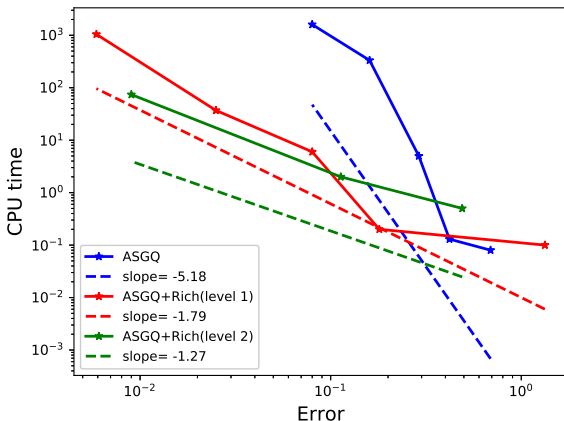


Figure 3.3: Numerical complexity of the QMC method with the different configurations in terms of Richardson extrapolation 's level. Case of parameter set 1 in Table 1.

Complexity of the different methods with their best configurations

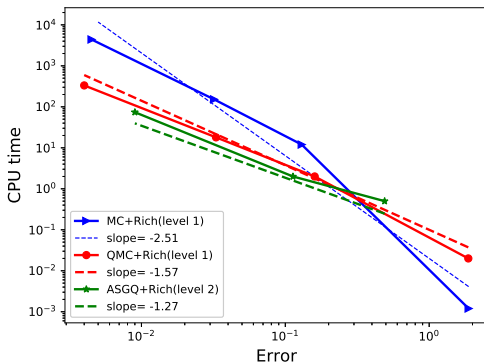


Figure 3.4: Computational work comparison for the different methods with the best configurations concluded from Figures 3.1, 3.2 and 3.3, for the case of parameter set 1 in Table 1.

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Conclusions

- Proposed novel, fast option pricers, based on hierarchical deterministic quadrature methods, for options whose underlyings follow the rBergomi model.
- Given a sufficiently small relative error tolerance, our proposed methods demonstrate substantial computational gains over the standard MC method, for different parameter constellations.
- Accelerating our novel methods can be achieved by using more optimal hierarchical path generation method than Brownian bridge construction, such as PCA or LT transformations.

Thank you for your attention

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