Chapter 11

Spatial Outlier Detection

"Time and space are modes by which we think and not conditions in which we live." – Albert Einstein

11.1 Introduction

Spatial data is a contextual data type in which one can clearly distinguish between two different types of attributes, one of which is spatial. These two types of attributes are as follows:

- **Behavioral attributes:** This is the attribute of interest that is measured for each object. For example, this attribute could correspond to sea-surface temperatures, wind speeds, car speeds, disease outbreak numbers, the color of an image pixel, and so on. It is possible to have more than one behavioral attribute in a given application. Thus, in many applications, this attribute is non-spatial because it measures some quantity of interest at a given location. However, in some data types such as trajectories, it is possible for the behavioral attribute to be spatial.

- **Contextual attributes:** In many spatial data types, the contextual attribute is spatial, although it might not be spatial in some occasional cases (such as trajectories in which the context is temporal). Sea-surface temperatures, wind speeds, and car speeds are often measured in the context of specific spatial locations. Spatial context is often expressed in terms of coordinates, which typically correspond to two or three numerical values. In some cases, the contextual attributes may be expressed at the granularity of a *region of interest*, such as a county, ZIP code, and so on.

Spatial data shares a number of similarities with time-series data in being a contextual data type. In fact, it is often possible for the spatial and temporal attributes to occur in various combinations of behavioral and contextual attributes. Such data is also referred to as *spatiotemporal* data. For example, in some applications, such as hurricane tracking, the contextual attributes are both spatial and temporal. A particularly unusual case is that

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of trajectory data in which the behavioral attributes are spatial, whereas the contextual attributes are temporal. Thus, it is clear that different types of semantics and applications can be captured depending on whether the spatial attributes are contextual or behavioral. In general, there are two key settings for spatial data:

1. **The spatial attribute is contextual:** In this setting, some quantity of interest is measured at various spatial locations. Often there might be other contextual attributes. For example, the time attribute might be temporal. In such cases, one might be interested in determining important spatiotemporal anomalies (or events) based on the underlying dynamics. For example, the dynamics of behavioral attributes such as humidity, wind speeds, sea-surface temperatures, and pressure can be used in order to identify and predict anomalous weather events.

2. **The spatial attribute is behavioral:** The most common example is that of trajectory data. In fact, trajectories can also be viewed as a special case of multivariate temporal data. For example, a 2-dimensional *real-time* trajectory-mining application can be modeled as a bivariate time series in which the X-coordinate and Y-coordinate is each a time series.

   Trajectories are also analyzed in the *offline* setting. In the offline shape-analysis scenario, anomalies may correspond to unusual shapes irrespective of their temporal provenance. In the latter case, the temporal aspects of the problem are limited. Therefore, trajectory-based applications can be modeled in multiple ways, depending on the scenario at hand.

In the setting in which the spatial attributes are contextual, outliers are objects that have very different *behavioral* attribute values from those of their surrounding *spatial* neighbors. Thus, *spatial continuity* plays an important role in the identification of anomalies, just as temporal continuity is important in time-series outlier detection. The fundamental principle of spatial continuity is as follows [550]:

> " Everything is related to everything else, but nearby objects are more related than distant objects."

In the setting in which the spatial attributes are behavioral, the contextual attribute is often temporal. This corresponds to trajectory data, which is a form of multivariate time-series data. Therefore, the techniques of Chapter 9 can be used.

Some examples of spatial applications are as follows:

- **Meteorological data:** Numerous weather parameters are measured at different geographical locations, which may be used in order to predict anomalous weather patterns in the underlying data [615].

- **Traffic data:** Moving objects may be associated with many parameters such as speed, direction, and so on. In many cases, such data is also spatiotemporal, since it has a temporal component. Finding anomalous behavior of moving objects [102] can provide numerous insights. For example, the discovery of anomalous taxi-trajectories can be used to discover greedy and dishonest taxi-drivers [118, 137].

- **Earth science data:** The land-cover types at different spatial locations may be the behavioral attributes. Anomalies in such patterns provide insights about anomalous trends in human activity such as de-forestation or other anomalous vegetation trends [341].

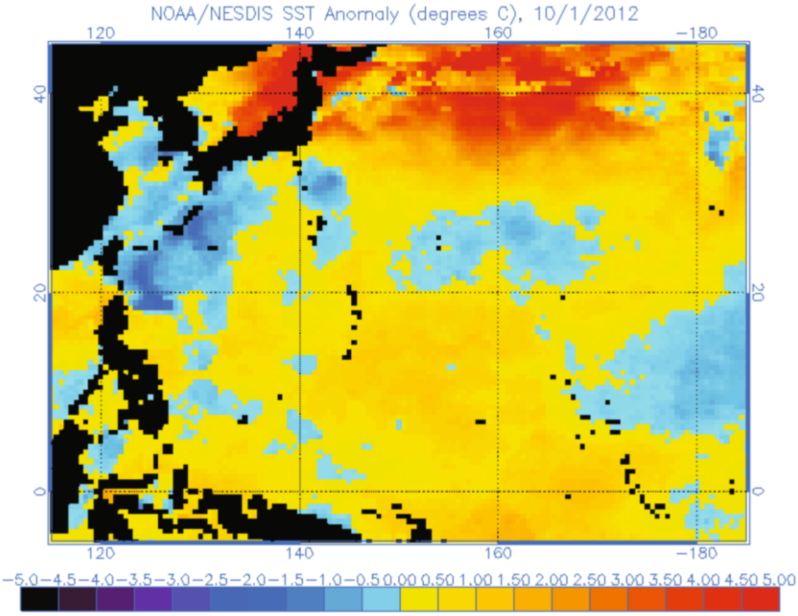


Figure 11.1: Sea surface temperature anomalies. Source: NOAA Satellite and Information Service

- **Disease outbreak data:** Data about disease outbreaks is often aggregated by spatial locations such as ZIP code and county. Anomalous trends in such data [568] can provide information about the causality of the outbreaks.

- **Medical diagnostics:** Magnetic resonance imaging (MRI) and positron emission tomography (PET) scans are spatial data in two or three dimensions. The detection of unusual localized regions in such data can help in detecting diseases such as brain tumors, the onset of Alzheimer disease, and multiple sclerosis lesions [251, 448, 505, 553].

- **Demographic data:** Demographic attributes such as age, gender, race, and salary can be used in order to identify demographic anomalies. Such information can be useful for target-marketing applications.

As in the case of temporal data, *abrupt changes in the behavioral attribute that violate spatial continuity* are used to identify contextual anomalies. For example, consider a meteorological application, in which sea-surface temperatures and pressure are measured. Unusually high sea-surface temperature in a very small localized region is a hot-spot which may be the result of volcanic activity under the surface. In this case, spatial continuity is violated by the attribute of interest. Such attributes are often tracked by meteorologists in real-time. In Figure 11.1, a color-coded map of the sea-surface temperatures on October 1, 2012 from the *NOAA Satellite and Information Service* is illustrated. Unusually high-temperature anomalies are illustrated in red, whereas unusually low-temperature anomalies are illustrated in blue.

In *spatiotemporal* data, both spatial and temporal continuity are used for modeling. For example, a sudden change in the velocity of a few cars in a small localized region may suggest the occurrence of an accident or other anomalous event. Similarly, evolving events such as hurricanes and disease outbreaks are spatiotemporal in nature. Spatiotemporal methods for outlier detection [141, 142] are significantly more challenging because of the additional challenge of modeling the temporal and spatial components jointly.

There are two main characteristics of spatial data that are commonly leveraged in outlier detection algorithms:

- **Spatial autocorrelations:** This corresponds to the fact that behavioral attribute values in spatial neighborhoods are closely correlated with one another. However, unlike temporal data, where future values of the time-series are unknown, the values in all spatial directions of a data point can be used. Note that spatial autocorrelations are *exactly* analogous to the temporal autocorrelations that are leveraged in time-series autoregressive (AR) models. Refer to Chapter 9 for details of AR models.

- **Spatial heteroscedasticity:** This corresponds to the fact that the variances of the behavioral attribute depend on spatial location [523].

Whereas the first property is the primary criterion for outlier analysis and is used universally, the second is used more occasionally. Nevertheless, it has proven to be useful in many scenarios. This is because when certain regions are likely to have greater variance as a matter of expectation, then abrupt changes in those regions are less likely to be significant. Such insights have lead to local methods [523], which are based on ideas derived from local density-based methods (LOF) [96].

Most of the work on spatial outliers is about finding *abrupt changes* which violate spatial auto-correlations. Such outliers are *contextual* outliers, for which numerous methods have been proposed in the literature. The primary ones among them use *variations of the behavioral attribute* within a neighborhood to define outliers. Such outliers use either multidimensional analysis methods or graph-based methods. In addition, many of the temporal autocorrelation methods discussed in the previous chapter can be generalized to the spatial domain. It is sometimes useful to intuitively visualize the key outlier points. The spatial nature of the data also lends itself to more intuitive visualization methodologies. Two examples of such methodologies are *variogram clouds* and *pocket plots* [247, 424]. The former will be described in this chapter.

As in the case of time-series databases, it is also useful to find unusual *shapes of behavioral-attribute patterns* in a database of multiple spatial distributions. For example, the color distribution in an image or MRI scan may correspond to an unusual shape, when compared to other images in the database. Such images may be identified as *collective* outliers in the context of spatial data.

Supervised methods are also very useful in the spatial domain, where it is desirable to determine unusual shapes from multiple spatial patterns. For example, while many conditions such as weather patterns of interest, or brain tumors in MRI scans may be rare on a *relative* basis, a significant amount of training data may be available on an *absolute* basis for modeling purposes. In medical applications, large numbers of pathological examples are sometimes available for modeling purposes. Similarly, many examples of pathological patterns of unusual shapes may be available in meteorological and earth science applications. In such cases, it is useful to utilize supervision for the purposes of outlier detection. Supervised methods are particularly useful in the context of outlier detection in such cases because of the unusually high complexity of a database containing multiple spatial patterns. Such methods are closely related to topics such as image classification.

A close relationship exists between temporal and spatial outlier detection because both methods use concepts of *behavioral attribute continuity with respect to one or more contextual attributes*. The main difference lies in the fact that spatial contextual attributes are often multidimensional, whereas time is a single attribute. Furthermore, time is unidirectional, in which only values in the past are known at a given point, whereas spatial attributes are usually known in the different directions of all axes. Nevertheless, in many applications, these differences are not significant enough to invalidate the applicability of similar methods in both settings. While recent work has adapted temporal techniques to some spatial applications such as anomalous image shape detection [565], many other temporal techniques have the potential for use in the spatial domain. This chapter will point out the different temporal techniques that are adaptable to the spatial domain. In some cases, these temporal methods are not adaptable, especially when the spatial context is not specified in a quantitative way (e.g., coordinates). For example, the spatial attribute may be specified as a categorical attribute such as a county or ZIP code.

This chapter is organized as follows. In the next section, the case in which spatial attributes are contextual will be discussed. Many algorithms such as multidimensional methods, graph-based methods, and autoregressive methods will be studied. Methods for discovering anomalous shapes will be discussed in this section. The setting in which both spatial and temporal attributes are contextual will studied in section 11.3. In section 11.4, we will study the case in which the spatial attributes are behavioral. This setting corresponds to the case of trajectory data. We will also study the connections of this approach with multivariate time-series analysis. The conclusions and summary are presented in section 11.5.

11.2 Spatial Attributes are Contextual

In this section, we will discuss the case in which the spatial attributes are contextual. This setting corresponds to weather data, temperature data, disease outbreaks, and so on. Different types of models, such as neighborhood-based models, graph-based models, and autoregressive models will be discussed.

11.2.1 Neighborhood-Based Algorithms

Neighborhood-based algorithms can be very useful in the context of a wide variety of tasks. In these algorithms, abrupt changes in the spatial neighborhood of a data point are used in order to diagnose outliers. Such algorithms depend on the exact way in which the spatial neighborhood is defined, the function used to combine these neighborhood values into an expected value, and the computation of the deviations from the expected values. The neighborhood may be defined in many different ways [2, 324, 376, 487, 488, 489, 490], depending on the nature of the underlying data:

- **Multidimensional neighborhoods:** In this case, the neighborhoods are defined on the basis of multidimensional distances between data points.

- **Graph-based neighborhoods:** In this case, the neighborhoods are defined by linkage relationships between spatial objects. The linkage relationships might be defined by a spatial domain expert with an understanding of spatial neighborhoods. Such neighborhoods may be more useful in cases in which the location of the spatial objects may not correspond to exact coordinates (e.g., county or ZIP code), and graph-representations provide a more general modeling tool.

This section will study methods for neighborhood-based outlier detection with the use of multidimensional and graph-based methods.

11.2.1.1 Multidimensional Methods

While traditional multidimensional outlier detection methods (e.g., LOF) can also be used to detect outliers in spatial data, such methods do not distinguish between the contextual attributes and the behavioral attribute. Therefore, such methods are not optimized for outlier detection in spatial data, especially in cases in which the outliers are defined on the basis of the behavioral attribute within a specific contextual vicinity.

Many methods define the contextual vicinity (spatial neighborhood) with the use of multidimensional distances on the spatial attributes. Thus, the contextual attributes are used for determining the k nearest neighbors, and the deviations on the behavioral attribute values are used in order to predict outliers. A variety of distance functions can be used on the multidimensional spatial data for determination of proximity. The choice of the distance function is important, because it regulates the choice of the neighborhood for comparative purposes. For a given spatial object o with behavioral attribute value $f(o)$, let $o_1 \ldots o_k$ be its k-nearest neighbors. Then, the predicted value $g(o)$ of the behavioral attribute of object o may be computed using the neighborhood mean:

$$g(o) = \sum_{i=1}^{k} f(o_i)/k$$

Alternatively, one can use the neighborhood median instead of the mean to reduce the impact of extreme values. Then, for each data object o, the value of $f(o) - g(o)$ represents a deviation from predicted values. The extreme values (positive or negative) among these deviations may be computed using a variety of methods discussed in Chapter 2. These are reported as outliers.

Local Outliers

An observation in [523] is that all local deviations are not equally important from the perspective of outlier analysis. For example, consider the case in which sea-surface temperatures are measured at different spatial locations. In some spatial regions, the changes in temperatures may naturally show larger variations than others. Therefore, the same variation cannot be treated with equal importance in all regions. Specifically, the outlier scores in high-variance regions need to be suppressed. In such cases, it may be useful to quantify the changes around a data point in a local way. For example, instead of using the value of $f(o) - g(o)$ as discussed above, it is possible to use a normalized value of $\frac{f(o)-g(o)}{L(o)}$, where $L(o)$ represents a *spatially local* quantification of the deviations around o. For example, $L(o)$ could represent the standard deviations of the behavioral attribute values in the spatial neighbors of o.

In practice, a variety of different methods could be used in order to characterize the local deviations around the spatial object o. The work in [523] has also defined a deviation measure *SLOM* which is based on the LOF method for defining local spatial outliers. This approach is sensitive to the spatial heteroscedasticity of the data, in which the specific variance of the behavioral attribute in a particular spatial locality is carefully accounted for in constructing the outlier score.

11.2.1.2 Graph-Based Methods

In graph-based methods, spatial proximity is modeled with the use of links between nodes. Thus, nodes are associated with behavioral attributes, and strong variations in the behavioral attribute across neighboring nodes are recognized as outliers. Graph-based methods are particularly useful when the individual nodes are not associated with point-specific coordinates but may correspond to regions of arbitrary shape. In such cases, the links between nodes can be modeled on the basis of the neighborhood relationships between the different regions.

Graph-based methods define spatial relationships in a natural way, since semantic relationships can also be used to define neighborhoods. Typically, a spatial domain expert might construct the neighborhood graph. This allows for a more general view of neighborhoods. For example, two objects could be connected by an edge, if they are in the same *semantic* location such as a building, restaurant, or office. In many applications, the links may be weighted on the basis of the strength of the proximity relationship. For example, consider a disease-outbreak application in which the spatial objects correspond to county regions. In such a case, the weights of the links could correspond to the lengths of the boundaries between pairs of adjacent regions.

Let S be the set of neighbors of a given node o. Then, the concept of spatial continuity can be used to create a *predicted* value of the behavioral attribute based on those of its neighbors. The weight of the links between o and its neighbors can also be used in order to compute the predicted values as either the weighted mean or median on the behavioral attribute of the k nearest spatial neighbors. For a given spatial object o, with behavioral attribute value $f(o)$, let $o_1 \dots o_k$ be its k linked neighbors based on the relationship graph. Let the weight of the link (o, o_i) be $w(o, o_i)$. Then, the linkage-based weighted mean may be used to compute the predicted value $g(o)$ of the object o.

$$g(o) = \frac{\sum_{i=1}^{k} w(o, o_i) \cdot f(o_i)}{\sum_{i=1}^{k} w(o, o_i)} \tag{11.1}$$

Alternatively, the weighted median of the neighbor values may be used to compute $g(o)$. As the true value, $f(o)$, of the behavioral attribute is known, it can be used to model the deviations of the behavioral attributes from their predicted values. Specifically, the value of $f(o) - g(o)$ represents a deviation from the predicted values. Extreme value analysis can be used on these deviations in order to determine the spatial outliers as those objects for which the value of $f(o) - g(o)$ is unusually positive or negative. This process is identical to what was discussed before for the multidimensional case. As in all outlier analysis algorithms, a variety of extreme-value analysis methods of Chapter 2 can be used on these deviations in order to determine the outliers. The nodes with high values of the normalized deviations may be reported as outliers.

11.2.1.3 The Case of Multiple Behavioral Attributes

In many cases, multiple behavioral attributes may be associated with the contextual attributes. For example, in a meteorological application, both temperature and pressure values may be available with the spatial attributes. In these cases, the deviations may be computed on each behavioral-attribute, and then these values need to be combined into a single deviation value, which provides the final outlier score. For this purpose, any of the multivariate extreme value analysis methods in section 2.3 of Chapter 2 may be used. In particular, the work in [138] has proposed the use of the Mahalanobis distance-based method of Chapter 2 for extreme-value analysis.

11.2.2 Autoregressive Models

Spatial data shares a number of similarities with temporal data. Both types of data measure a behavioral attribute (e.g., temperature) with respect to a contextual attribute (e.g., space or time). In many scenarios, spatial data is available in the form of coordinates, and the values of the behavioral attribute may be available at *each possible spatial reference point in the grid*. Such data arises commonly in weather contour maps, images, MRI scans, and so on. In cases *in which the data is completely specified at most points in the grid*, it is possible to use autoregressive models in order to determine unusually large deviations in the data in an analogous way to the temporal scenario.

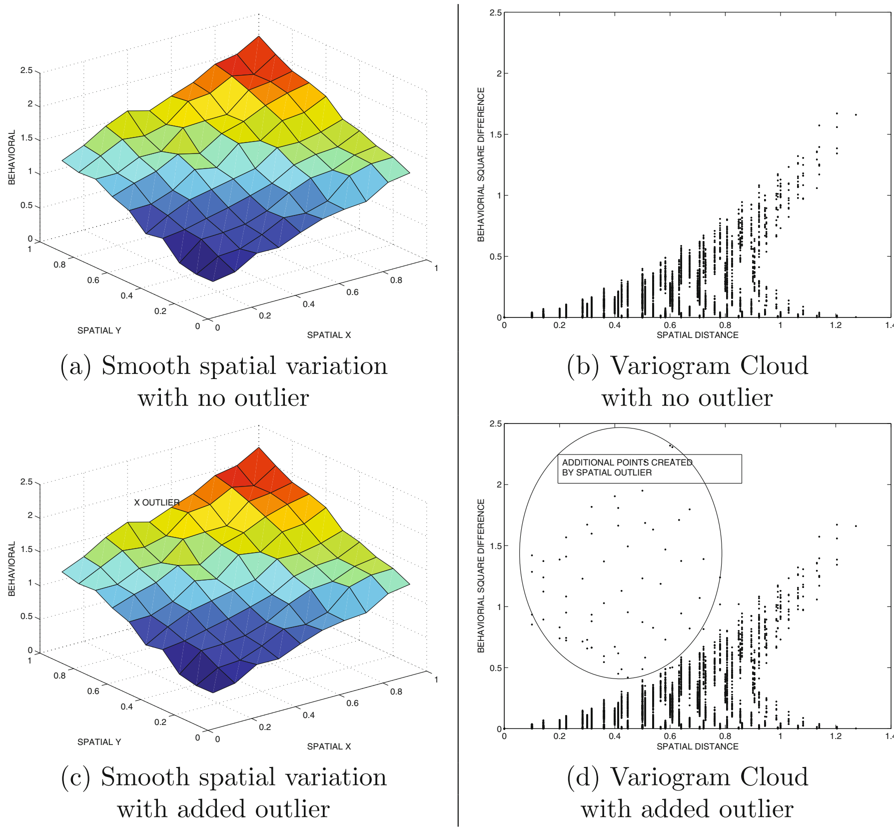
Let X_{t_1,t_2} be the value of the behavioral attribute at the spatial location (t_1, t_2). In the temporal autoregressive model, the predicted value of the behavioral attribute is based on a 1-dimensional window of *past* history of length p (see section 9.2.1 of Chapter 9). In the 2-dimensional spatial scenario, this can be generalized to a square window of size $(2 \cdot p + 1) \times (2 \cdot p + 1)$, with p coordinates in either direction. More generally, in the case of 3-dimensional spatial data, one can use a cube of size $(2 \cdot p+1) \times (2 \cdot p+1) \times (2 \cdot p+1)$. As in the case of the 1-dimensional autoregression for temporal data in section 9.2.1 of Chapter 9, a 2-dimensional model can be defined as follows.

$$X_{t_1,t_2} = \sum_{i=-p}^{p} \sum_{j=-p}^{p} a_{ij} \cdot X_{t_1-i,t_2-j} + c + \epsilon_{t_1,t_2}$$

The value of a_{00} is always set to 0, so that it is missing from the aforementioned summation. This is done in order to avoid the trivial solution in which a spatial value is used to predict itself. The values of a_{ij} need to be learned from the underlying training data. Thus, such an equation can be created for each value of (t_1, t_2). When the number of available spatial-coordinates is much larger than $(2 \cdot p + 1) \times (2 \cdot p + 1)$, this is an over-determined system of equations in which the least-squares error is minimized to determine the optimal values of the coefficients a_{ij}. The optimal values of the coefficients are computed with least-squares regression according to the approach discussed in section 3.2.1 of Chapter 3. Thus, the process of computing the regression coefficients is very similar to that in temporal data.

In the aforementioned system of equations, the value of c is a constant, and the value of ϵ_{t_1,t_2} represents the noise, or the *deviation* from the expected values. Large absolute values of this deviation represent the anomalies in the underlying data. These values are assumed to be independent and identically distributed random variables, which are drawn from a normal distribution. Thus, the extreme-value analysis methods of Chapter 2 can be used on these values to identify the abnormal locations.

The aforementioned discussion provides a generalization of Autoregressive (AR) models from temporal to spatial data for illustrative purposes. In practice, it is possible to generalize *all* the regression models (ARMA, ARIMA, PCA) to the spatial scenario, by using the appropriate slice of values from the spatial data. As in the temporal case, it is even possible to create multivariate spatial regression models, where multiple behavioral attributes are available. Typically such behavioral attributes may be correlated with one another (e.g., temperature and humidity), and it is desirable to determine unusually large local deviations with the help of multivariate correlations. Refer to section 9.2.2 of Chapter 9 for a discussion of multivariate time-series regression models. The spatial generalization to multiple behavioral attributes is similar to this case.



(a) Smooth spatial variation with no outlier

(b) Variogram Cloud with no outlier

(c) Smooth spatial variation with added outlier

(d) Variogram Cloud with added outlier

Figure 11.2: Effect of adding spatial outlier to variogram cloud

Although the autoregressive nature of spatial data is widely recognized, such models have rarely been used for anomaly detection in the research literature. This is partially a result of the high computational complexity of autoregressive models with an increasing number of coefficients. Such models also cannot easily handle spatial data which is incompletely specified by spatial location, region-based locations or semantic locations. Nevertheless, such models can be very useful in many scenarios such as image analysis or weather patterns in which large amounts of reasonably complete data are available for analysis. In such cases, the statistical robustness of these methods is likely to be higher than simpler neighborhood-based models.

11.2.3 Visualization with Variogram Clouds

A number of visualization techniques such as pocket plots and variogram clouds are used in order to visualize spatial outliers. The latter will be discussed detail here, because of their relative popularity. Since spatial outliers are based on *disagreement* in the continuity of the behavioral attribute *in relation* to the spatial attribute, a natural method to visualize this would be to create a scatter plot between the pairwise spatial distances and the pairwise behavioral attribute (square) deviation. The spatial distance is simply the Euclidean distance between a pair of points. The behavioral attribute deviation is defined as half the squared distance between the behavioral attribute values. A scatter plot is created between the spatial distances on the X-axis, and the behavioral squared deviations on the Y-axis for every pair of points in the data set. The idea is that smaller spatial distances correspond to smaller behavioral attribute variances and vice versa. In particular, large variations of the behavioral attribute for smaller spatial distances should be considered deviants. Such points on the variogram cloud can be traced back to the original data to determine pairs of points that are spatially close but behaviorally different.

In order to illustrate the impact of outliers on variogram clouds, a synthetic example will be used. First, the data set for the variogram clouds of Figure 11.2 will be described. In this case, a grid of 100 points on the spatial plane are used with coordinates drawn from $X, Y, = 0.1, 0.2 \ldots 1.0$. The value of the behavioral attribute Z is generated as follows:

$$Z = X + Y + \epsilon$$

Here, ϵ is a small amount of noise, which was randomly generated from the uniform distribution in $[0, 0.2]$. This spatial variation of the attribute is quite smooth, since the noise is small relative to the global variation in values of the behavioral attribute. The spatial profile of the generated data is illustrated in Figure 11.2(a), and the corresponding variogram cloud is illustrated in Figure 11.2(b). It is evident that low values of the spatial distance always correspond to low deviations of the behavioral attribute. While it is possible for high spatial deviations to be related to low behavioral deviations, the converse is not true.

Subsequently, a single outlier is added to the data by distorting the behavioral attribute of one of the spatial values in the grid of Figure 11.2(a). The corresponding outlier is shown in Figure 11.2(c), and is marked explicitly. Note that the spatial data sets in Figures 11.2(a) and 11.2(c) are virtually identical, with the only difference between them being the outlier created by a distorted behavioral attribute value. The corresponding variogram cloud is illustrated in Figure 11.2(d). It is evident that in this case, a new set of points have been added to the variogram cloud in which significant behavioral deviations exist even at low spatial distances. Multiple such deviant points are created corresponding to the different data points in the immediate spatial locality of the added outlier. Such points can easily be isolated visually and linked back to the original points in the data. Thus, this approach provides an easy and intuitive way of visually identifying the spatial outliers.

One challenge in creating a variogram cloud is the high computational complexity. Note that a single point exists in the variogram cloud for each pair of points in the original data. Therefore, the number of points in the variogram cloud scales quadratically with the number of points in the spatial data. This can make the approach rather slow, when the number of data points is large. In practice, it is difficult to create a variogram cloud for situations in which the data contains a few hundred thousand spatial data points. This can be a significant problem, since spatial data sets are often quite large in practice. Nevertheless, one can construct variogram clouds at rougher levels of granularity.

One observation about the variogram cloud is that it is not always necessary to represent *every pair* of points on the plot. Pairs of data points that are spatially very far away from one another add little insight about the outlier behavior. Therefore, each spatial dimension can be discretized into ranges, and this creates a 2-dimensional grid in the data. The pairwise relationships between all spatial points *within* this grid can be used in order to create the variogram cloud. This significantly reduces the computational complexity of creating the variogram cloud. For example, consider the case in which the original data set contains N points. Assume that these N points are discretized into a $t \times t$ grid with approximately[1]

[1]In practice, the different grid regions may contain a different number of data points because of spatial correlations. However, in many applications such as image data, pixels may be available for every spatial coordinate. Therefore, the division into grids will create a uniform division of the data points.

Figure 11.3: NASA satellite image of hurricane *Fran*: The anomalous shape is characteristic of a hurricane

N/t^2 data points in each. Then, the computational complexity of creating a variogram cloud for each grid is $O(N^2/t^4)$. Of course, since there are a total of t^2 grids, the aggregate computational complexity is $O(N^2/t^2) < O(N^2)$. This provides a speedup factor of $O(t^2)$. It is noteworthy that in this case, an optimistic scenario was assumed in which the data points were uniformly distributed into the grid structure. It can be shown theoretically that a speedup factor of at least t can be obtained with this approach. This is because the speed up achieved with a grid partitioning into $t \times t$ ranges will always be better than the discretization along only one dimension into t ranges with an equal number of data points. A significant speedup may be obtained even for modest values of t, without significant reduction in the quality of the visual discrimination between the outliers and the normal points.

11.2.4 Finding Abnormal Shapes in Spatial Data

The problem of finding unusual shapes in spatial data finds numerous applications such as image analysis. For example, the detection of unusual shapes from brain PET scans or MRI scans can help detect conditions such as tumors, Alzheimer, and sclerosis [448, 553], or can help identify anomalous conditions such as hurricanes from weather maps. For example, consider the satellite image illustrated in Figure 11.3. The anomalous shape in the image corresponds to hurricane *Fran*, which was a large destructive hurricane. This hurricane hit Cape Fear in North Carolina on September 1996, and it can easily be identified by its characteristic shape in the satellite image. However, such a shape may not appear in other similar satellite images on normal days, and is therefore an unusual event. Another example from the medical domain is illustrated in Figure 11.4, in which the PET scans from a normal person and an Alzheimer patient are presented. The colored regions correspond to the uptake of the radioactive tracer administered in a PET scan (behavioral attribute). It is evident that this behavioral attribute shows very different spatial behavior for normal

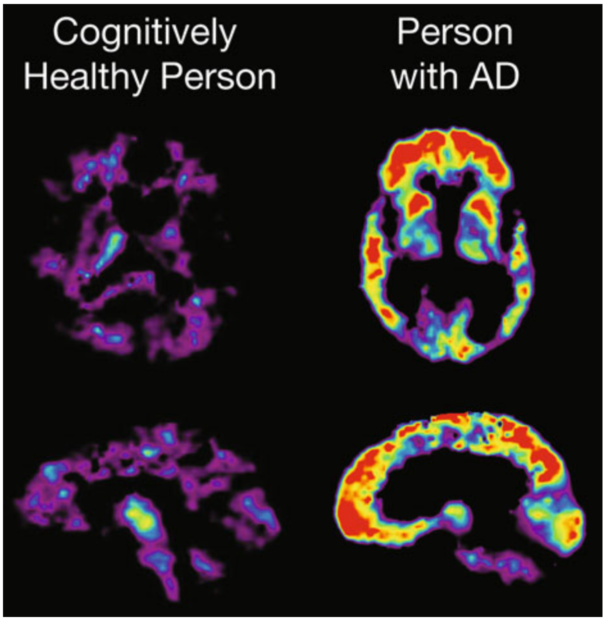


Figure 11.4: PET scans of brain for cognitively healthy person versus an Alzheimer patient: Image courtesy of the National Institute on Aging/National Institutes of Health

and diseased individuals.

Abnormal shapes in spatial data can be identified with both unsupervised and supervised methods. In general, supervised methods are always desirable because they allow the encoding the domain knowledge and training data in the prediction process. In the following, both unsupervised and supervised methods will be discussed. Many unsupervised and supervised methods use various feature transformation techniques such as contour extraction, multidimensional feature extraction, and multidimensional wavelet transformation. These different transformation techniques are relevant to different application domains, and should therefore be judiciously selected, depending on the problem at hand.

11.2.4.1 Contour Extraction Methods

In their simplest form, shapes can be modeled by the contours (or boundaries) of regions with particular ranges of behavioral attribute values in the data. For example, in the case of Figure 11.3, the boundaries of such regions can be extracted by direct analysis of sensor and satellite readings such as pressure, cloud cover, temperature, wind speed, and humidity or from the (already processed) color histogram of the corresponding image.

A key simplification for shape analysis is that the contours of an object can be converted into a time-series by using a feature-engineering trick. One possible way to achieve this is to use the distance from the centroid of the object to the boundary of the object, and compute a sequence of real numbers derived in a clockwise sweep of the boundary [602]. This yields a time series of real numbers, and is referred to as the *centroid distance signature*. This transformation can be used to map the problem of mining shapes to that of mining time-series, a domain which is much more easier to address from an analytical perspective. For example, consider the elliptical shape illustrated in Figure 11.5(a) with centroid denoted by X. Then, the time-series representing the distance from the centroid, by using 360 different equally spaced angular samples, is illustrated in Figure 11.5(b). In this case, the sample points are started at one of the major axes of the ellipse. Starting the sampling at a different place or rotating the shape (with the same angular starting point) causes a cyclic translation of the time-series. The resulting time-series may be normalized in different ways depending on the needs of the application at hand:

- If no normalization is performed, the outlier analysis approach is sensitive to the absolute size of the underlying object. This may be the case in many medical images such as MRI scans, in which all spatial objects are standardized in terms of the scale. Therefore, normalization is not necessary.

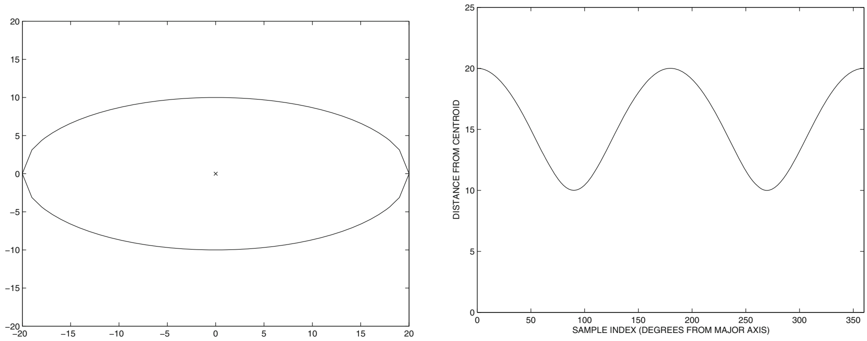
- If all time-series values are multiplicatively scaled down by the same factor to unit mean, then such an approach will allow the matching of shapes of different sizes, but will discriminate between different levels of relative variations in the shapes. For example, two ellipses with very different ratios of the major and minor axes will be discriminated well.

- If all time series are translated to zero mean and scaled to unit variance, such an approach will match shapes where *relative* local variations in the shape are similar, but the overall shape may be quite different. For example, such an approach will not discriminate very well between two ellipses with very different ratios of the major and minor axes, but will discriminate between two such shapes with different relative local deviations in the boundaries. Furthermore, noise effects in the contour will be differentially enhanced in shapes that are less elongated. For example, for two ellipses with similar noisy deviations at the boundaries, but different levels of elongation (major to minor axis ratio), the overall shape of the time-series will be similar, but the local noisy deviations in the extracted time series will be *differentially* suppressed in the elongated shape. This can sometimes provide a distorted picture from the perspective of shape analysis. A perfectly circular shape may show unstable and large noisy deviations in the extracted time-series because of image rasterization effects. The solution proposed in [565] is to treat circular shapes specially, although the unintended effects of such normalization may have unusually complex effects across a broader spectrum of shapes.

It is helpful to have a proper domain-specific understanding of the effect of a particular normalization, so that it may be selected in a domain-specific way.

The problem of shape analysis is further complicated by the effect that transformations such as rotations can have on the underlying data. For example, consider the images illustrated in Figure 11.6. All images correspond to the same object, but two of them are rotated with respect to the original shape, and the last is a mirror image of the original shape. It is clear that the rotation makes it more difficult to match the two images, if the time-series representation does not account for the rotation or the mirror image effects of the representation. Errors in matching the two shapes also lead to errors in outlier detection, especially when the outlier detection process uses a proximity-based method. It is important to note that all applications do not necessarily require the accounting of rotations. For example, in an MRI scan, where the correct orientation of the scan is known, such rotational transformations may not be needed.

An immediate observation is that *a rotation of the shape leads to a linear cyclic shifting of the time series generated by using the distances of the centroid of the shape to the contours*



(a) Elliptical shape (b) Distance from centroid

Figure 11.5: Conversion from shapes to time-series

Figure 11.6: Rotation and mirror-image effects on shape matching for outlier analysis

of the shape. For a time series of length n denoted by $a_1 a_2 \dots a_n$, a cyclic translation by i units leads to the time series $a_{i+1} a_{i+2} \dots a_n a_1 a_2 \dots a_i$. Then, the *rotation invariant Euclidean distance* $RIDist(T_1, T_2)$ between two time series $T_1 = a_1 \dots a_n$ and $T_2 = b_1 \dots b_n$ is given by the minimum distance between T_1 and all possible rotational translations of T_2 (or vice versa). Therefore, the following is true:

$$RIDist(T_1, T_2) = \min_{i=1}^{n} \sum_{j=1}^{n} (a_j - b_{1+(j+i) \bmod n})^2 \tag{11.2}$$

Note that the reversal of a time-series corresponds to the mirror-image of the underlying shape. Therefore, mirror images can also be addressed by using this approach.

The shape discords can then be determined by computing the series whose kth nearest neighbor distance to its closest neighbor is as large as possible. The top-r such shapes need to be found. As in all distance-based algorithms, a brute-force approach on a database with N shapes would require $O(N^2)$ distance computations, unless pruning methods are used.

The major difference between this problem and the unusual time-series shape discovery problem discussed in section 9.3 of Chapter 9 is that the rotational invariant distances are used instead of the Euclidean distances. Furthermore, the distances are computed on whole time-series instead of on subsequences. While it may be possible in theory to use the method of Chapter 9, by making some modifications to address rotational invariance, longer lengths of whole sequences (compared to subsequences), may cause greater challenges in pruning. For example, rotational variations can be addressed by explicitly incorporating rotational variations of the time-series into the database, just as subsequences of a time-series are incorporated into the database for subsequence discord discovery in section 9.3 of Chapter 9. Care needs to be taken in avoiding self-similarity from the same shape during the distance computations, just as self-similarity is avoided in time series discord discovery. Therefore, the techniques in section 9.3 of Chapter 9 can be used in theory in order to find discords. Of course, the addition of multiple rotational variations of the shapes to the database is likely to slow down the discovery process. It also leads to some redundancy in the representation, because all rotational variations of the same object will have the same outlier score.

The work in [565] uses a different pruning method based on LSH-approximations [281] of the symbolic aggregate approximations of the time-series. The overall organization of the approach is similar to the algorithm discussed in section 9.3 of Chapter 9. Both methods first sort the objects by approximate outlier tendency in order to perform the outlier search in an ordered way, which optimizes the pruning behavior. For each object, pruning is performed with approximate nearest-neighbor distances. However, the specific technique used for pruning is different in the two scenarios.

A nested loop approach is used to implement the method. The algorithm examines the candidate shapes iteratively in an outer loop, and progressively improves the estimate of each candidate's k-nearest neighbor distance in an inner loop. The inner loop essentially computes the distances of the other shapes to the candidate. At the end of the execution of a candidate-specific inner loop, the approach then either includes the candidate in the current set of top-n outlier score estimates, or discards the candidate at some point during the computation of its k-nearest neighbor in the inner loop. This is referred to as early inner loop termination. This inner loop can be terminated early, when the currently approximated k-nearest neighbor distance for that candidate shape is less than the score for the rth best outlier found so far. Clearly, such a shape cannot be an outlier. In order to obtain the best pruning results, the candidate shapes in the outer loop need to be heuristically ordered, so that the earliest shapes examined have the greatest tendency to be outliers. The pruning performance is also best, when the points are ordered in the inner loop, such that the k-nearest neighbors of the candidate shape are found early. It remains to explain how the heuristic orderings required for good pruning are achieved.

As in the case of time-series subsequences, each time series is mapped onto an LSH word with the use of Symbolic Aggregate Approximation. Assume that the resulting SAX words have length m. Locality sensitive hashing [281] randomly samples $s < m$ distinct positions in the SAX representation. Therefore, two SAX words that are more similar are more likely to map to the same string. This is also referred to as the *locality sensitivity property* of the LSH-hashing approach, and the similarity can be robustly quantified by examining the mapping behavior over multiple hash functions. However, this does not account for the rotational invariance of the matching process. In order to account for the possible rotations, a rotational invariant LSH function is defined. This function first picks $s < m$ position indices randomly, and then samples these s position indices from all possible m rotations of the SAX word. Clearly, similar shapes will lead to LSH-based collisions, even in the presence of rotations. The LSH-hashing process is repeated with multiple hash functions in order to provide greater robustness to the collision-based counts.

For each SAX word, a count is maintained of its number of LSH-based collisions. This provides approximate information about its outlier score. Shapes with smaller counts need to be processed first as candidates in the outlier loop, since they have greater likelihood of being outliers. Furthermore, shapes which collide with one another frequently in LSH-based hashing are more likely to be nearest neighbors. Therefore, shapes that have the largest number of collisions with the current outer loop candidate are examined first in the inner loop for distance computations. This provides the heuristic order of processing in the inner loop. The reader is referred to [565] for a detailed description of the algorithm.

11.2.4.2 Extracting Multidimensional Representations

Multidimensional methods are applicable in cases in which the behavioral attributes are specified along the entire grid of spatial coordinates. In this section, we assume that we are looking for abnormal grids of size $p \times p$ from the database of one or more spatial images. The first step is to extract all the grids of size $p \times p$. Subsequently, the p^2 behavioral values in the grid are converted into a p^2-dimensional record. Assume that N such points are extracted. This problem is therefore converted into that of discovering a point outlier in p^2-dimensional space. Any of the methods discussed in Chapters 1 through 6 can be used in this setting.

11.2.4.3 Multidimensional Wavelet Transformation

A second way of extracting a multidimensional representation is to use a wavelet transformation from a grid of values. Even though wavelets are inherently designed for the case in which there is a single contextual attribute, they can also be generalized to cases in which there is more than one contextual attribute (such as spatial data). In these cases, a Haar wavelet representation can be constructed by alternately dividing the spatial regions along the different axes and computing the differences. A detailed discussion of the multidimensional wavelet transformation is beyond the scope of this book, although a detailed discussion may be found in [33] (cf. section 16.2.2 of Chapter 16).

11.2.4.4 Supervised Shape Discovery

Spatial data is particularly common in many forms of image data such as weather maps, PET scans or MRI scans. For example, consider the case of MRI scans, where 3-dimensional images of the brain may be available for analysis. The anomalies in the data such as tumors and lesions may show up as characteristic regions in the data, which are rare but are nevertheless indicative of specific kinds of abnormalities. In such cases, previous examples of anomalous and normal scans may be available for the purposes of training. Although unsupervised anomaly detection can help outlier analysis up to a point, the use of supervision can increase the sophistication of the analysis by revealing specific *types* of abnormalities. In most applications, at least semi-supervision is used, where examples of normal spatial profiles are available for analysis. The collection of normal examples is typically not very difficult in most application-specific scenarios, since copious examples of normal instances are usually available.

For all forms of shape classification, the actual *representation* of the shape is the most important step. For example, the *centroid distance signature* discussed in this chapter [602] is one possible way of representing the shapes, but by no means the only one. A thorough review of shape representation techniques may be found in [602]. The shape to time-series transformation discussed in section 11.2.4 of this chapter can be used in order to transform the shape classification problem to the time-series classification problem. Any of a number of methods (such as subsequence-based k-nearest neighbor methods) can be used for time-series classification in this case. Furthermore, one can also use the multidimensional and wavelet transformations in the previous section, in combination with an off-the-shelf classifier.

Numerous methods for time-series classification may be found in the literature [408, 590]. These methods typically try to determine discriminative shapes of the series (or shapelets) which distinguish the normal and abnormal series. In the context of spatial data, such abnormal series are typically derived from abnormal shapes from a spatial perspective. In the *semi-supervised* case, the distances of the test series to examples of normal profiles can be used in order to create outlier scores for the underlying series. The only distinction from the available methods for time-series analysis is that care must be taken in order to account for different rotational variants of the shape in particular application-specific scenarios.

The problem of supervised classification of unusual shapes is also closely related to the problem of detecting and recognizing specific shapes in images. This problem has been studied extensively in the field of computer vision and image analysis. The problem of supervised shape recognition is an important area of research in its own right, and is beyond the scope of this book. The reader is referred to [69, 115, 375, 602] for a detailed description of such methods for image classification, analysis and change detection in the image domain. The major modification to these methods is the incorporation of rare class detection and cost-sensitive methods into these algorithms, using the methods of Chapter 7. Since many of the algorithms discussed in Chapter 7 are meta-algorithms, they can be used in conjunction with any of the classification techniques in the literature.

11.2.4.5 Anomalous Shape Change Detection

In spatial data such as weather data, PET scans, and MRI scans, unusual changes in the contours of the shapes may be used in order to predict anomalous events. For example, the formation of a hurricane or a tumor over multiple time stamps will show up as an unusual change in the shapes of the corresponding image representations of the weather data or the MRI scan. The determination of such changes is more complex than those of detecting unusual *point changes* in the data. However, the detection of unusual point changes can be a first step towards detecting regions of anomalous change in the data, by clustering the change points in the spatial data. Not all regions of change may necessarily correspond to anomalies. For example, increasing age may create certain characteristic change contours in an PET scan, which should be considered normal. In practice, this problem is not very different from finding unusual shapes in the original data, with the main difference being that the spatial contours of the behaviorally *changing* regions between two snapshots are extracted. The normally occurring changes in the data over time will usually be quite different from the anomalous changes. Therefore, the anomalous contours provide the unusually changing regions in the data.

This broader principle can also be used in conjunction with other outlier detection methods. In general, a differencing operation on two temporal snapshots of the data may be required as a pre-processing step before applying outlier analysis algorithms. Once the differencing step has been applied, the methods in the previous sections can be applied to the set of differenced features. A detailed description of many such change-analysis methods may be found in [115].

11.3 Spatiotemporal Outliers with Spatial and Temporal Context

Spatiotemporal data is very common in many real applications in which behavioral attribute values are continuously tracked at different spatial locations. For example, consider a chemical factory dumping chemicals in a river. In such cases, the concentrations of chemicals in the water cannot be described by using either only spatial or temporal contextual attributes. Thus, the contextual attributes need to contain *both* spatial and temporal components. Spatiotemporal data is extremely common in all forms of sensor data, in which behavioral attribute readings are continuously transmitted by sensors at different spatial locations. An example is provided in [569], where precipitation data from different spatial locations and times is aggregated. It is desirable to determine localized spatial regions that are also close together in time, whose precipitation values are significantly different from their "neighboring" values. So how should neighboring values be defined in the case of spatiotemporal data?

Virtually all the spatial methods discussed in earlier sections of this chapter can be generalized to spatiotemporal data, as long as the concept of neighborhood is properly defined in order to make it relevant for the spatiotemporal scenario:

- Spatial methods can be used on temporal snapshots of the data in order to determine the relevant outliers at different instants. However, such an approach is incomplete, because it fails to identify violations of temporal continuity.

- Some algorithms have been proposed in order to separately identify spatial outliers and temporal outliers, and then combining the results in order to provide the spatiotemporal outliers [89]. However, the decoupling of spatial and temporal aspects of the problem at an earlier stage is obviously a suboptimal solution.

- Spatiotemporal neighborhoods of data points may be used in order to determine predicted values. Thus, the only difference from purely spatial methods is that the expanded set of contextual attributes are used to define the neighborhoods for analysis and prediction. As in the previous case, deviations from the predicted values can be used to identify outliers. In some techniques such as neighborhood methods, the challenge is to combine the (contextual) distances along the spatial and temporal dimensions in a meaningful way. One simple way of achieving this would be to normalize the standard deviation across each of the contextual attributes to one unit before computation of distances. If desired, weights can be used to provide more importance to one or more of the contextual attributes.

The last of the aforementioned methods is the most general, because it can detect significant changes *both* across spatial and temporal attributes in an integrated and meaningful way. It is also important to note that spatial and temporal continuity may not be equally important, depending upon the underlying application. For example, in an application where precipitation level is the behavioral attribute [569], spatial continuity may be more important than temporal continuity. In such cases, appropriate scaling can be performed on the different dimensions to define neighborhoods in a way that provides greater importance to one or more contextual attributes.

11.4 Spatial Behavior with Temporal Context: Trajectories

A special case of spatiotemporal data is one in which the spatial attributes are behavioral and the temporal attribute is contextual. This data corresponds to trajectory data, which is encountered commonly in a variety of real-world settings. Such data can be treated as a form of bivariate temporal data by treating the X-coordinates and Y-coordinates of each object as the behavioral attributes and time as the only contextual attribute. This results in two related time series at the same instants. Thus, the methods for multivariate time-series data analysis can be applied effectively to such cases. Such analysis, when applied to single time-series, can identify sudden *changes* in trajectory directions and velocity. This can be very useful in detecting information about significant changes in cyclone or hurricane trajectories [117]. In other cases, a database of multiple trajectories may be available, and it is desirable to determine unusual shapes of trajectories. The temporal component is less important in this case, since the trajectories may have been created at different times. In such cases, it is possible to use subsequence analysis on these time-series in order to determine those trajectories which behave very differently from the remaining series by determining time-series of unusual shapes [360]. However, unlike the univariate scenario [360], spatial time-series are at least bivariate, and it is much harder to find unusual shapes in terms of the *combination behavior of the two time series*.

11.4.1 Real-Time Anomaly Detection

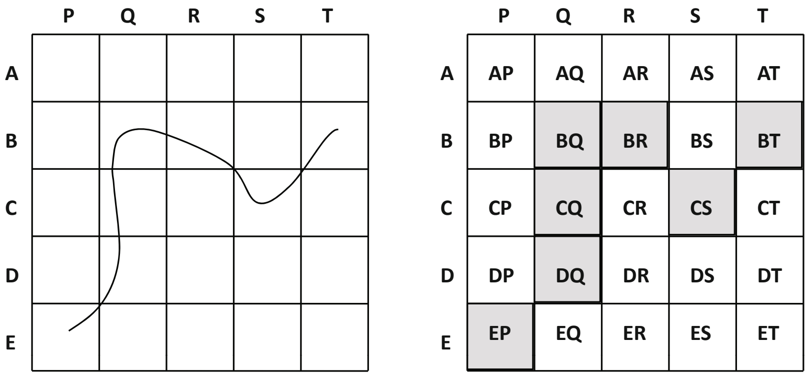
For the case of real-time change analysis, the prediction-based outlier detection methods discussed in section 9.2 of Chapter 9 can be applied separately on each of the X-coordinate and Y-coordinate time series. This results in a residual value along each of the two coordinates. If each of these residuals is modeled as a normal distribution, then the sum of the squares of the Z-values of these residuals is a χ^2 distribution with two degrees of freedom. This can provide an outlier score, along with a corresponding probability value, which can be derived from the χ^2-distribution. Since a trajectory is a bivariate or trivariate time-series, it is also possible to use multiple time-series regression models from section 9.2.2 of Chapter 9 in order to identify outliers.

11.4.2 Unusual Trajectory Shapes

Although real-time *change* analysis of such scenarios can be performed more effectively by using temporal modeling, *unusual shape* detection of trajectories can be best performed by abstracting out the temporal component, and performing the spatial analysis directly on the trajectories. In such cases, each spatial object has a shape, and the difference of this shape to its nearest neighbor trajectories are used in order to determine outliers. Since such trajectories may contain a large number of time-stamps, it may often be difficult to determine outliers on the entire sets of trajectories. In such cases, unusual subsequences of the trajectories may be used in order to identify outliers.

11.4.2.1 Segment-wise Partitioning Methods

Some specific methods such as TROAD have also been proposed in the literature [347] for unusual shape detection in trajectories. In particular, the partition-and-detect framework [347] first partitions the trajectories into a set of sub-trajectories. Note that this is



(a) Trajectory (b) Relevant grid regions

Figure 11.7: Grid-based discretization of trajectories

somewhat analogous to the concept of partitioning time series into subsequences (or finding outliers in subspaces of numerical data), since outliers cannot easily be determined on the full series (with high implicit dimensionality). The sub-trajectories are created with a two-level partitioning which is allowed to be coarse-grained at the higher levels, and fine-grained at the lower level. Subsequently, those sub-trajectories that are not similar to other ones in the data are reported as the outliers. The similarity is measured with the use of both distance-based and density-based methods. Note that the choice of the distance function is critical, and can regulate the nature of the outlier found. For example, a distance function which is sensitive to the *location* of the trajectory is likely to find an outlier based on location of the trajectories. On the other hand, a distance function which is sensitive to the angle between trajectory segments is likely to be sensitive to directions of movement. The precise definition of the distance function is application dependent, though a variety of such functions can be used in conjunction with the partitioned set of sub-trajectories.

The work in [347] defines a t-partition as a line segment from the trajectory. Intuitively, this can be considered analogous to comparison-unit schemes discussed in Chapter 10, which are used in the context of sequence data. A t-partition is said to be outlying using the variation[2] of the k-nearest neighbor distance definition, first proposed in [317]. Intuitively, a t-partition is considered an outlier, if a sufficient number of trajectories in the database are not close to it. The definition of closeness is based on measuring the portion of the trajectory, which is close to the t-partition. As in comparison-unit schemes for discrete sequences, the results from the different "units" (or partitions) are combined to declare a trajectory as an outlier, if a sufficient number of its partitions are also outlying. Furthermore, the locality sensitive density-based approach of [96] has also been generalized to this case, by creating a density-sensitive outlier score for the trajectories.

11.4.2.2 Tile-Based Transformations

A natural approach that is used to transform trajectory outlier detection to the case of sequence outlier detection is that of tile-based transformation [33]. The basic idea is to discretize the grid-regions of the space into a set of tiles, which are labeled. An examples

[2]That variation fixes the nearest neighbor distance, and computes the required value of k rather than the other way around.

of a trajectory together with its tile-based discretization is shown in Figure 11.7. Note that each tile in the grid has an X-label and a Y-label, and the combination of the two provides a unique label to the tile. For example, the upper-left tile has label AP and the lower-right tile has label ET. In this case, the relevant trajectory in Figure 11.7(a) can be converted into the shaded sequence of tiles in Figure 11.7(b). Therefore, the trajectory can now be represented by the following sequence:

EP, DQ, CQ, BQ, BR, CS, BT

For each trajectory in the database, one such sequence can be extracted. Subsequently, the sequence outlier detection methods discussed in Chapter 10 can be leveraged to discover outliers in such cases.

11.4.2.3 Similarity-Based Transformations

In many settings, similarity-based transformations can be very useful. The basic idea here is to treat a trajectory as a bivariate time-series. As discussed in Chapter 16 of [33], one can use multivariate similarity functions to compute similarities between pairs of trajectories. For example, multivariate forms of dynamic time-warping similarity functions are available for computing similarities between trajectories. For a database containing N trajectories, one can create an $N \times N$ similarity matrix. Note that distance functions can be converted into similarity functions with kernel transformations (like the Gaussian kernel in Chapter 3) where needed. Subsequently, the kernel trick can be applied as discussed in section 3.3.8 of Chapter 3. The basic idea is to extract a multidimensional embedding from the similarity matrix and then apply the Mahalanobis method (or any other distance-based method) in order to report outliers.

11.4.3 Supervised Outliers in Trajectories

In many cases, supervision may be available in the form of labels associated with trajectories. For example, consider a case where the trajectories of a large number of ships are available, and it is desirable to identify the suspicious ones based on their trajectory patterns. In some cases, previous examples of anomalous trajectories may be available. These can be used in order to detect significant anomalous patterns in the underlying data. This is a homogeneous attribute scenario, since the unusual shapes are based purely on the spatial and temporal attributes, rather than on a behavioral attribute.

The ROAM method [355] uses a discrete *symbolic* approximation of the trajectories, which converts the numerical coordinate sequence into a symbolic sequence based on the directions of movement and significant changes in this direction. For example, motifs could correspond to *right-turn*, *u-turn* or *loop*. Every movement pattern can be described as a sequence of these primitive movement patterns. The important motifs can be mined directly from the data by using a clustering approach. If desired, additional meta-attributes may be associated with the symbols corresponding to characteristics of the movement such as the speed. This is however different from the concept of behavioral attributes, since these attributes do not play the behavioral role in the learning process.

Once the discrete representation has been created, the sequences together with their labels can be fed to any sequence-based classifier, which identifies how different sequences are related to the class labels. While the ROAM method was applied in the context of supervised models, it is important to note that the feature transformation used in this work can also be used in the context of unsupervised scenarios.

Although such methods have not been explored extensively, it is possible to use the similarity-based and tile-based transformations of the previous sections. The similarity-based transformation can be used to transform the problem into the multidimensional domain, whereas the tile-based transformations can be used to transform the problem into the sequence mining domain. In both cases, one can leverage the large number of off-the-shelf classification tools in these settings. Furthermore, the similarity-based transformations need not be performed explicitly because one can directly use kernel SVMs in conjunction with the extracted similarity matrix.

11.5 Conclusions and Summary

The problem of spatial outlier detection arises in many domains such as demographic analysis, disease outbreaks, image analysis, and medical diagnostics. Spatial outlier detection shares significant resemblance with temporal outlier detection in terms of the effects of contextual attributes on the continuity of the behavioral attributes. Therefore, a number of methods in the temporal domain can be used for outlier detection in the spatial domain. Spatiotemporal outlier detection is even more complex and challenging, since it combines spatial and temporal characteristics effectively for outlier analysis. The spatial and temporal attributes can occur either as contextual or behavioral attributes. Depending on which attributes are contextual, the nature of the underlying data set is also quite different.

Spatial data can often be treated as an abstraction of image data, when the contextual attribute natural partitions the space into grid regions, and the values of the behavioral attribute(s) are known over all grid regions. In such cases, numerous methods for image analysis can be used for outlier detection. In fact, in many applications, such as MRI scans and weather maps, the data are *explicitly* represented as images. The analysis of such data involves the determination of unusual shapes from the distribution of the spatial attributes. Such analysis can be performed both in the unsupervised and supervised scenarios.

11.6 Bibliographic Survey

The problem of finding spatial outliers is different from that in multidimensional data because of the different types of attributes in spatial data. The simplest problem setting is that of discovering point outliers. In such cases, small changes in the behavioral attribute in a particular spatial proximity are used in order to identify outliers [2, 324, 376, 487, 488, 489, 490]. Spatial proximity can be defined either with the use of multidimensional distances or graph-based distances. Spatial distances are more relevant when the contextual attributes are expressed in terms of coordinates. On the other hand, when the contextual attributes correspond to spatial regions or semantic locations, graph-based methods are more relevant, because distances and proximity can be expressed as general functions across links. A random-walk approach to determine free-form spatial scan windows is discussed in [285]. The application of outlier detection to heterogeneous neighborhoods is discussed in [286]. The work in [573] introduces a spatial likelihood ratio test in order to determine local grid regions in which the variation of the behavioral attribute is different from the remaining data in a statistically significant way. Furthermore, such methods can also be used in the context of multiple behavioral attributes [138]. Spatial data also show local heterogeneity because of different levels of variance in different parts of the data. Therefore, a local method for spatial outlier detection was proposed in [523].

The standard autoregressive models for temporal data [467] can be extended to spatial data, in which the behavioral attribute values are completely specified over all the different reference values. This is often the case with many forms of image data. The problem of unusual shape detection in images is an important one from the perspective of outlier analysis. Some recent work [565] has been performed on finding unusual shapes in images in an efficient way. Supervised methods for shape detection and change analysis are also widely available in the literature [69, 115, 375, 602]. The work in [251] uses multivariate Gaussian Markov random fields in order to find unusual shapes in medical image data.

Spatial data is similar to temporal data in the context of the continuity shown by the behavioral attributes. Numerous methods for autoregressive modeling [467] can also be generalized to the case of spatial data. A significant amount of data in spatial domains also have a temporal component, when the attributes are tracked at multiple time-stamps. This requires methods for spatiotemporal outlier detection [141, 142]. An application of spatiotemporal outlier detection to precipitation data is discussed in [569]. A method for detecting flow anomalies in the context of sensors located upstream or downstream from one another is discussed in [304]. When the differences in the values of the sensors exceeds a given threshold, it is flagged as a spatiotemporal anomaly. A streaming method for explicitly quantifying the level of local change in a spatiotemporal data stream, which is discussed in section 9.4.2.1 of Chapter 9, was proposed in [19]. Methods for detecting anomalies in vegetation data with principal component analysis are discussed in [341].

The detection of outliers in trajectories can be modeled either spatially or temporally. Significant *changes* in trajectory directions are useful for many applications, such as hurricane tracking [117]. In such cases, the trajectory can be treated as bivariate temporal data, and change analysis can be applied to this representation. For this purpose, the prediction-based deviation detection techniques of the previous chapter can be helpful. The works in [102, 215] determine anomalies in moving object streams in real time, by examining patterns of evolution. On the other hand, the detection of anomalous trajectory *shapes* is a very different problem. The earliest methods for trajectory shape outlier detection were proposed in [319]. However, this method transforms the trajectories into point data by using a set of features describing meta-information about the trajectories. Unsupervised methods for trajectory outlier detection, which actually use the sequence information explicitly, were first investigated in [347, 409]. The work in [409] uses the discrete Fourier transform in order to represent the trajectories in terms of the leading coefficients, and find anomalies. In the second method [347], trajectories are divided into different line segments and anomalous patterns are identified in order to determine outliers. Supervised methods for anomaly detection in trajectory data may be found in [355]. These methods transform the data into discrete sequences, and a classifier is learned in order to relate the trajectories to the class labels. Another method proposed in [357] proposes methods for finding outliers in vehicle traffic data. However, these methods are not designed for determining outliers on individual objects, but are designed for finding anomalous traffic regions (or road segments) on the basis of aggregate spatial traffic characteristics.

11.7 Exercises

1. Construct the closed form solution to the AR regression model proposed in this chapter. Use the methods proposed in Chapter 3 for this purpose.

2. Construct PCA models for relating multiple behavioral attributes at the same spatial location and discovering points in the spatial grid that are anomalies. Use analogous

models to those discussed in Chapter 9 for this purpose.

3. Construct PCA models for relating multiple behavioral attribute values over spatially local slices of size $p \times p$. Use analogous spatial models to the time-series models proposed in Chapter 9 for this purpose.

4. What is the time complexity of the methods proposed in Exercises 2 and 3.

5. Create a generalization of the time-series shape detection algorithm discussed in section 9.3 of Chapter 9 [311] to the spatial shape detection scenario. Refer to the details in [311] for specific details of pruning based on Symbolic Aggregate Approximation.

6. Implement the algorithm developed in Exercise 5 using a C++ implementation. Test it over benchmark data sets discussed in [565].

7. Implement the algorithm discussed in this chapter for unusual shape detection. Refer to [565] for specific details of LSH-based pruning. Test it over benchmark data sets discussed in [565]. How does the speed compare to the algorithm developed in Exercise 6.