Bayesian Inference and Neural Estimation of Acoustic Wave Propagation

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Introduction

- Acoustic wave analysis is vital for robotics, virtual reality, and architectural acoustics.
- Focus of this work: Characterizing the room impulse response (RIR) for localization.
- Challenges: Traditional methods are computationally intensive and struggle with noise and uncertainty.
- Proposed solution: Integrate physics-based and data-driven methods.

Proposed Framework

- Combines three methods for spectral acoustic analysis:
  - Bayesian inference for uncertainty quantification.
  - Neural-physical model with forward-backward physical losses.
  - Non-linear least squares as a benchmark.

- Goal: Estimate wave propagation coefficients $\gamma(\omega) = \alpha(\omega) + i\kappa(\omega)$:
  - $\gamma(\omega)$: Complex-valued coefficient governing wave behavior.
  - $\alpha(\omega)$: Attenuation coefficient, measures energy loss.
  - $\kappa(\omega)$: Wave number, indicates spatial frequency.

Bayesian Inference Approach

- Based on the frequency domain wave equation:

$$\frac{\partial^2}{\partial x^2}\tilde{P}(x,\omega) + \frac{\omega^2}{c^2}\tilde{P}(x,\omega) = 0$$

- Solution for one-direction wave:
  $\tilde{P}(x,\omega) = \tilde{P}(x_0,\omega)e^{-\gamma(\omega)(x-x_0)}$.
- Likelihood model:

$$\begin{bmatrix} \tilde{P}^m(x_2,\omega) - \tilde{P}^m(x_1,\omega)e^{-\gamma(\omega)(x_2-x_1)} \\ \tilde{P}^m(x_1,\omega) - \tilde{P}^m(x_2,\omega)e^{\gamma(\omega)(x_2-x_1)} \end{bmatrix} \sim \mathcal{N}(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma^2 \\ \sigma^2 \end{bmatrix})$$

- Uses NUTS sampler for posterior inference of $\alpha(\omega)$ and $\kappa(\omega)$.

Neural-Physical Model

- Neural network predicts α and κ, integrated with physical constraints.
- Forward-backward loss design ensures physical consistency.

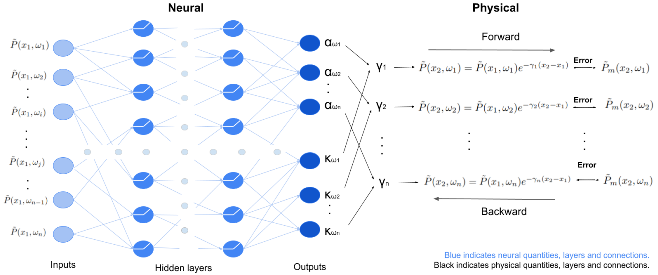


Figure: Neural-physical model architecture.

Non-Linear Least Squares Estimation

- Serves as a benchmark for estimating $\gamma(\omega)$.
- Minimizes the difference between measured and predicted wave profiles.
- Solution using log transform:

$$\hat{\Gamma} = -(\Delta X)^{-1} \Delta \log \tilde{P}$$

  where:

  $\hat{\Gamma}$: Estimated wave propagation coefficients matrix (see paper).
  ΔX: Diagonal matrix of travel distances $(x_2 - x_1)$.
  $\Delta \log \tilde{P}$: Log difference of measured wave profiles.

- Provides interval estimates under noise conditions.

RIR Estimation

- RIR defined in time domain:

$$P(x_1, t) \circledast RIR(x_2 - x_1, t) = P(x_2, t)$$

- In frequency domain:

$$\tilde{RIR}(x_2 - x_1, \omega) = \tilde{P}(x_2, \omega)./\tilde{P}(x_1, \omega)$$

- Reflects medium properties (e.g. air density, humidity, etc).
- Used for relocalization tasks in robotics.

Relocalization Application

- Estimates distance Δx using: $\Delta x = -\frac{1}{\gamma} \ln \frac{\tilde{P}(x_2,\omega)}{\tilde{P}(x_1,\omega)}$.
- Localization via intersection of multiple speaker signals.

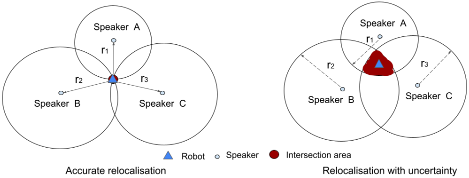
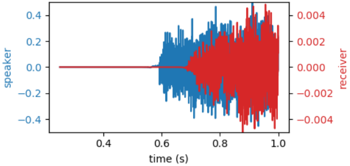


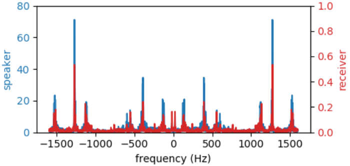
Figure: Relocalization using acoustic signals.

Experimental Setup

- Simulated using *SoundSpaces 2.0* with *Matterport3D* dataset.
- Setup: 1 speaker, 9 receivers, 1-second signals at 16kHz.
- Training: 1 pair for Bayesian, 8 pairs for neural model.



(a) Time domain.

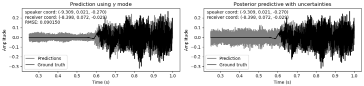


(b) Frequency domain.

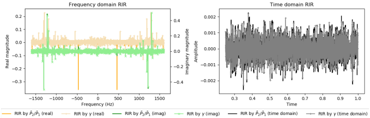
Figure: Simulated speaker-receiver signal pair.

Experimental Results

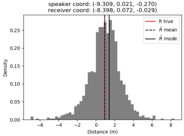
- Bayesian inference outperforms neural and least squares methods in terms of distance estimation accuracy.
- MCMC sampling shows successful convergence for α, κ.



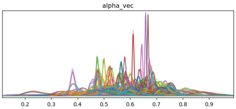
(a) Wave profile prediction.



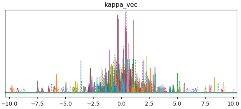
(b) RIR estimation.



(c) Distance estimation.



(d) α



(e) κ

Figure: (a) Test wave profile prediction, (b) RIR estimation, (c) Distance estimation for test receiver 2, (d-e) MCMC sample trajectories.

Comparison of Methods

	Test receiver 1			Test receiver 2		
Statistics	Mean	Mode	Std	Mean	Mode	Std
Bayesian MAP	**7.319**	**7.407**	**0.466**	**0.936**	1.425	1.715
Bayesian mean	2.749	2.896	1.311	0.416	0.487	**0.855**
Neural (MLP)	6.290	7.883	3.955	0.986	1.509	2.395
Neural (Autoencoder)	6.308	7.623	3.880	0.961	1.869	2.375
Least squares	5.458	4.998	2.968	0.854	**1.318**	2.016
Ground truth	**7.407**	-	-	**0.943**	-	-

Table: Comparison of distance estimation methods.

Conclusions

- Integrated framework enhances accuracy and robustness in acoustic analysis.
- Bayesian inference excels in uncertainty quantification and accuracy.
- Applicable to robotics, VR, and acoustics engineering.
- Future work: Explore complex wave phenomena and scalable Bayesian methods.