Towards a Practical Cluster Analysis over Encrypted Data

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Joint work with

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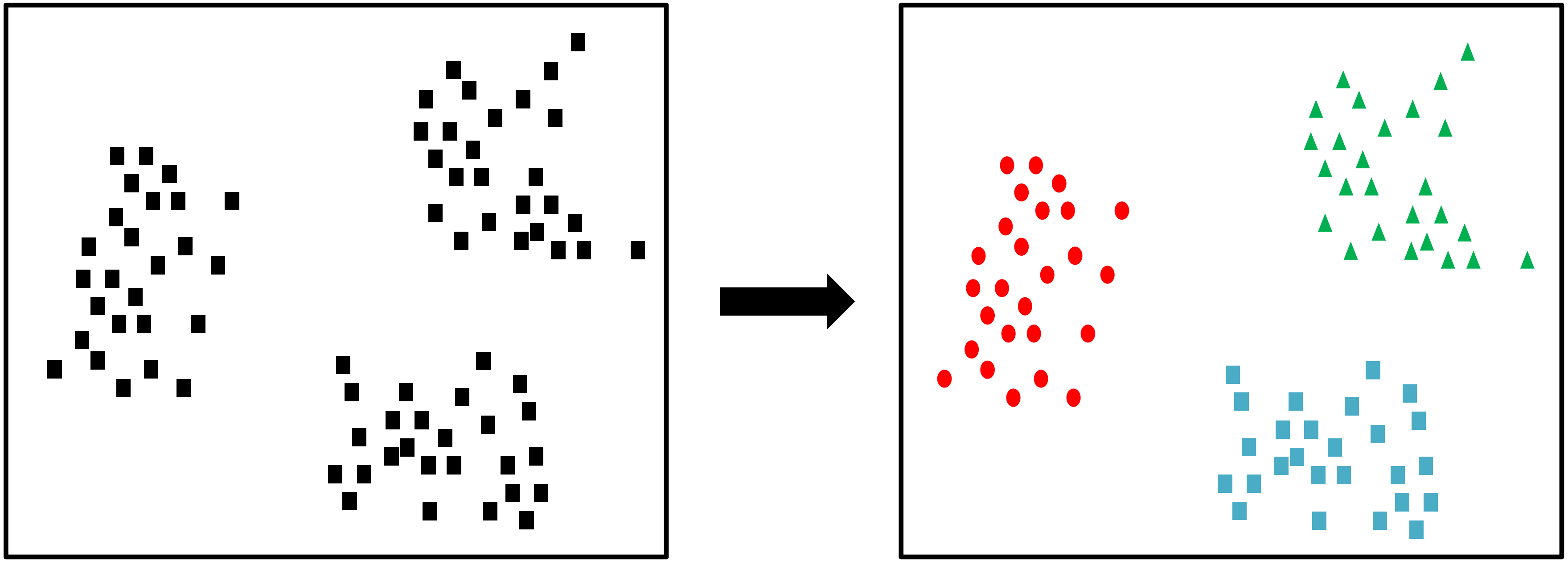
Summary of This Work

- The first privacy preserving non-interactive solution of mean-shift clustering algorithm based on homomorphic encryption
- Outstanding performance: Fast and Accurate
  - 99.99% accuracy on 262,144 data within only 82 min
  - 400 times faster than the previous work (SAC 18)

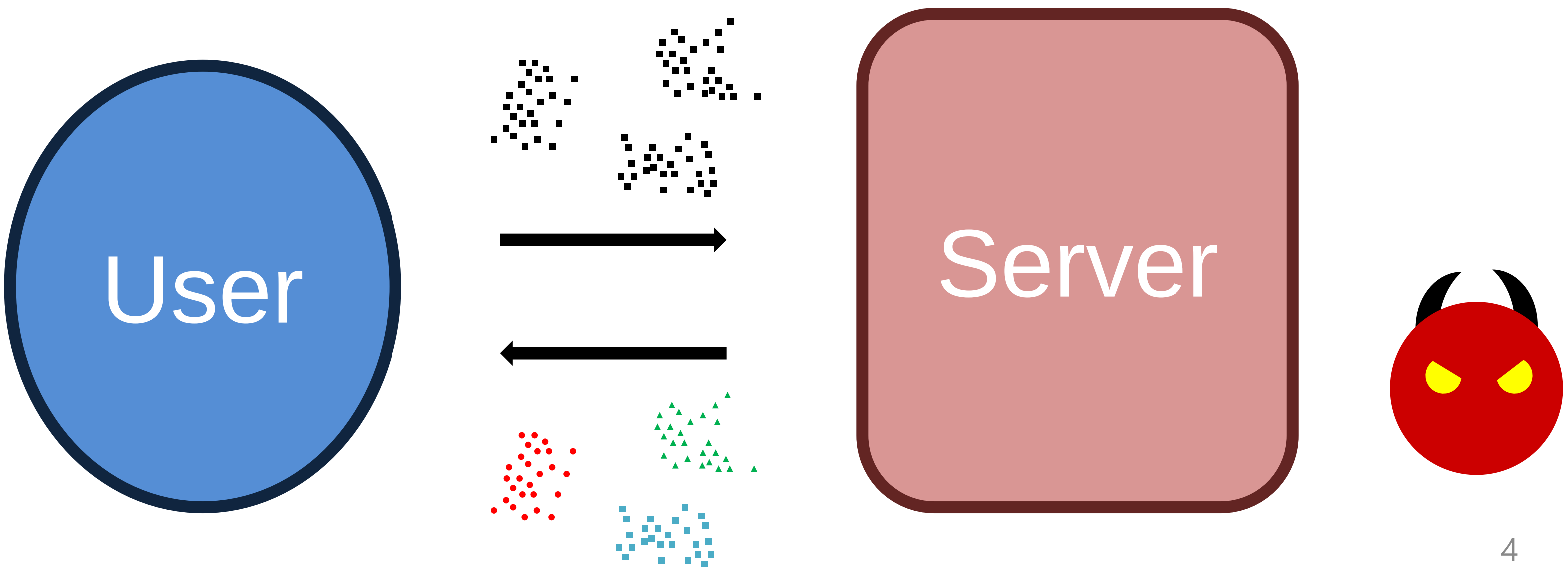
Data Clustering

- Grouping a set of given data into several subgroups
- Unsupervised machine learning task



Privacy Preserving Clustering

- Clustering is used in fields dealing with private information
  - Bioinformatics, finance, customer behavior analysis
- People do not want to delegate clustering of raw data to untrusted server



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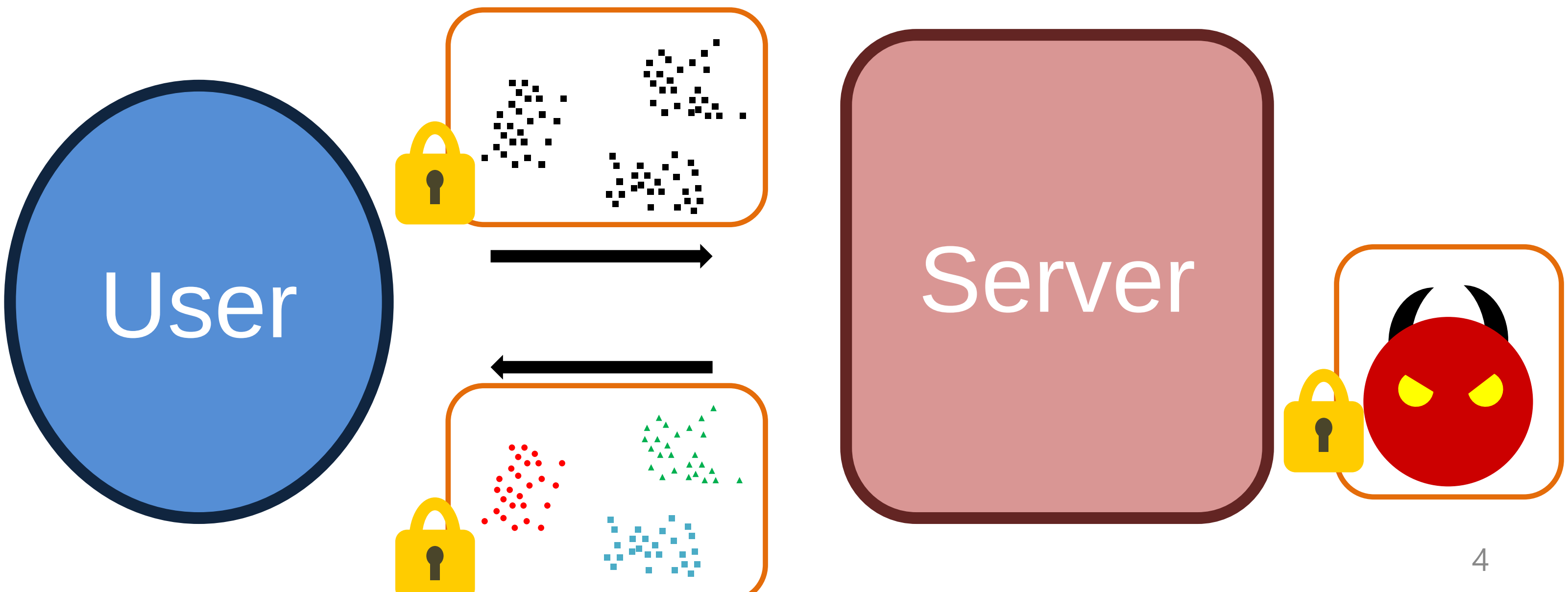
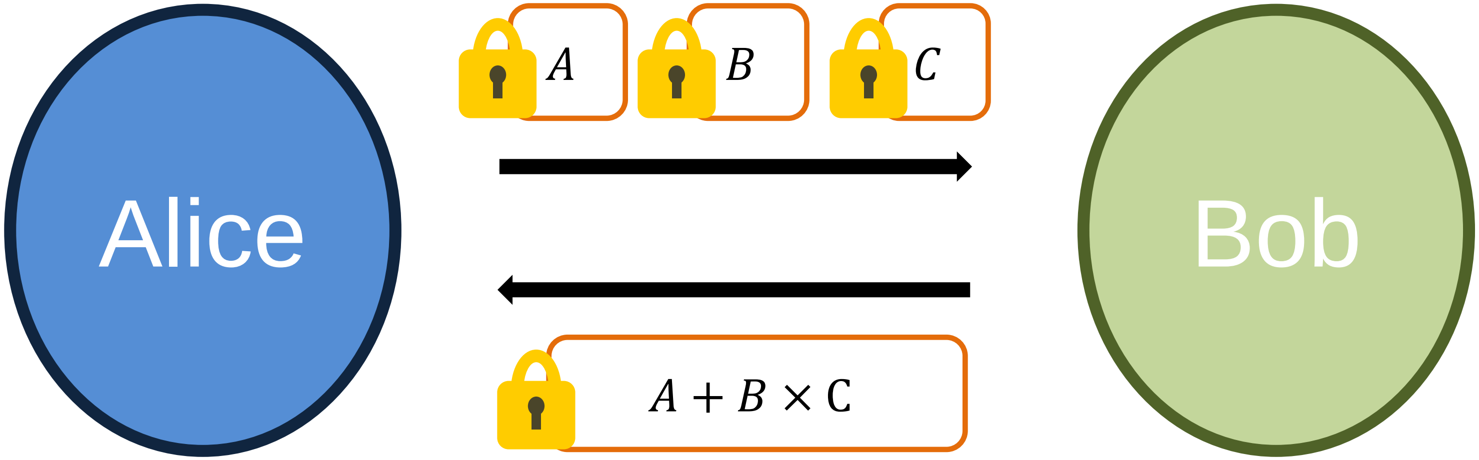


Homomorphic Encryption

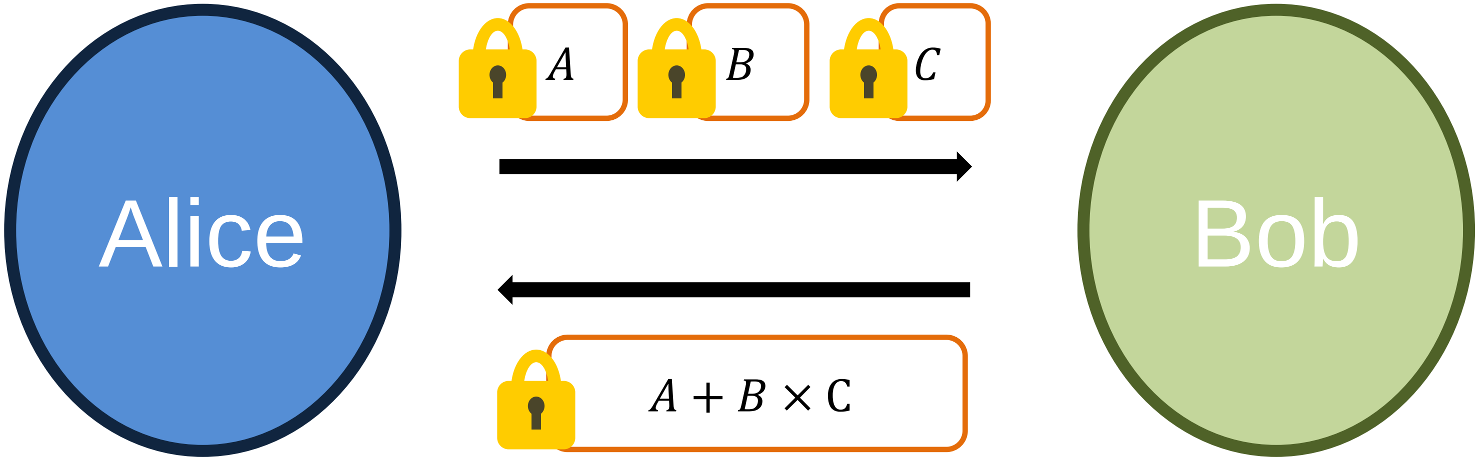
- Homomorphic encryption (HE) allows arithmetic (+, -, x) operations on ciphertexts without any decryption process



Homomorphic Encryption

+, -, x

- Homomorphic encryption (HE) allows arithmetic operations on ciphertexts without any decryption process
- Non-arithmetic operations (comparison, min, max) can be approximately computed
  - But expensive



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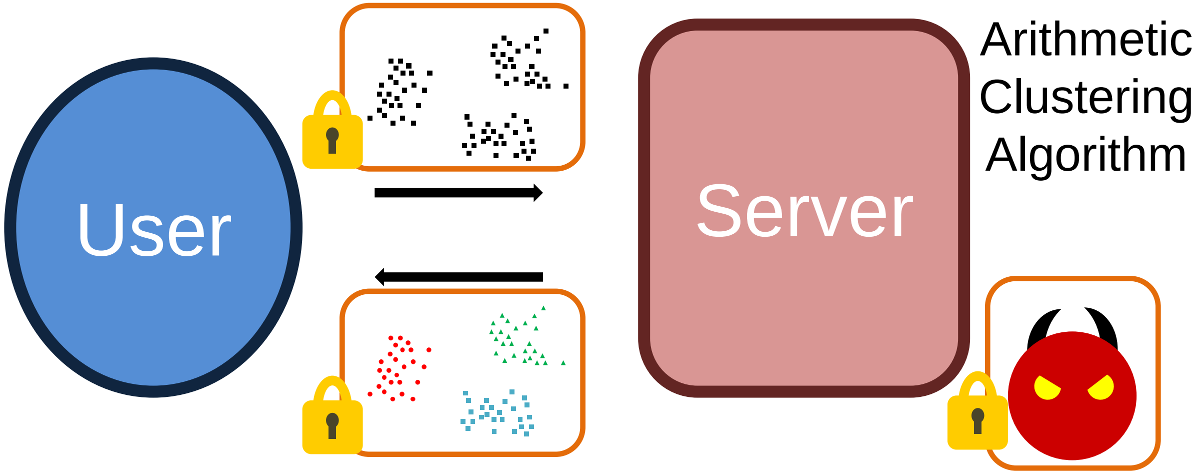

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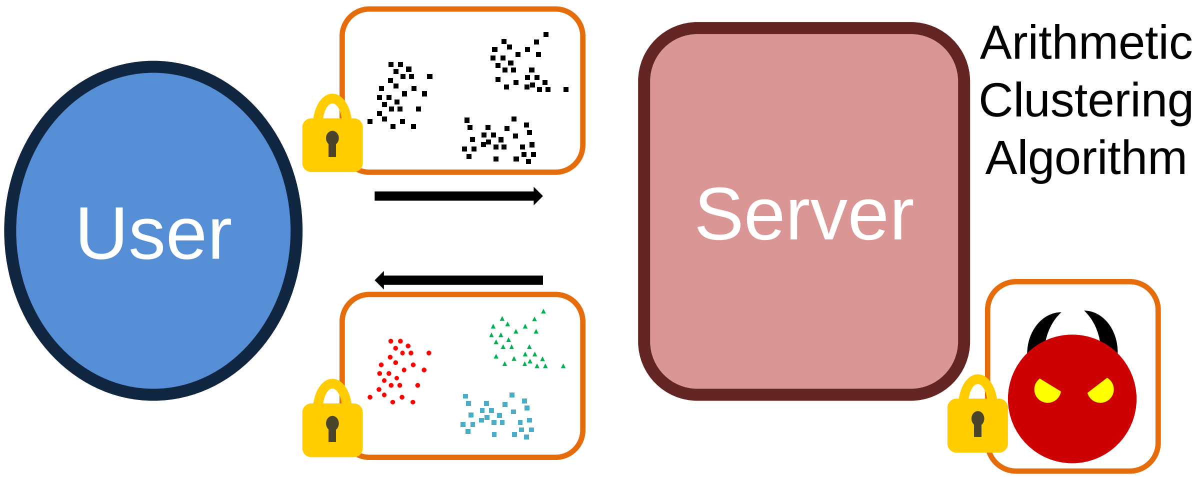

Two main issues:

1. Which clustering algorithm?
2. How to make it arithmetic?

K-means vs. Mean-shift

- K-means is faster
  - But uses <u>more pieces of information</u>

	K-means Clustering	Mean-shift Clustering
Complexity	$O(\#\text{clusters} \cdot \#\text{points} \cdot \#\text{iterations})$	$\mathbf{O(\#points^2 \cdot \#iterations)}$
Parameter	Number of Clusters	**None**
Shape of data	Should be convex	**None**
Comparison Operations	A number of comparison operations	**None**

K-means vs. Mean-shift

- K-means is faster
  - But uses more pieces of information
- Mean-shift clustering is more HE applicable
  - Non-parametric
  - No restriction on the shape of data
  - Does not use comparison operations

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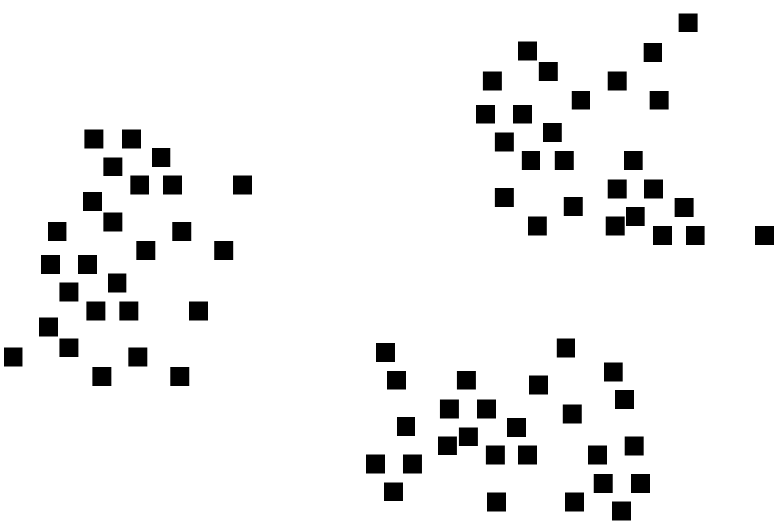
Mean-shift Clustering

- Clustering technique based on an estimated density map
  - Label each point by its closest local maximum (mode) of a Kernel Density Estimator (KDE)



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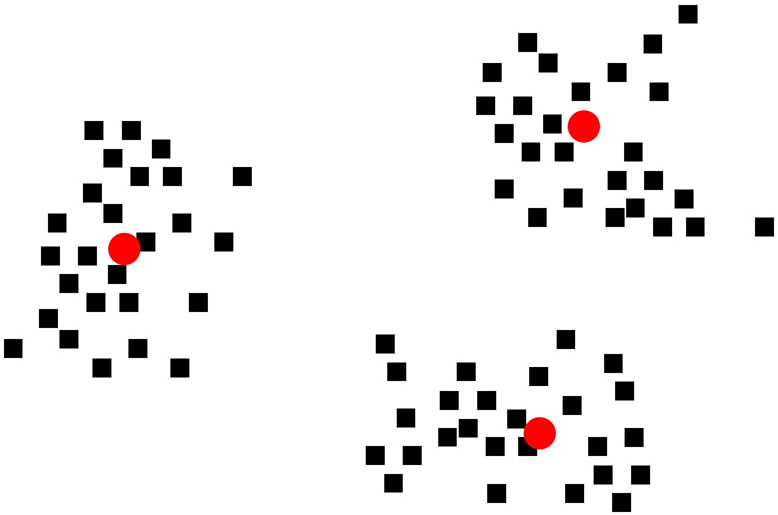

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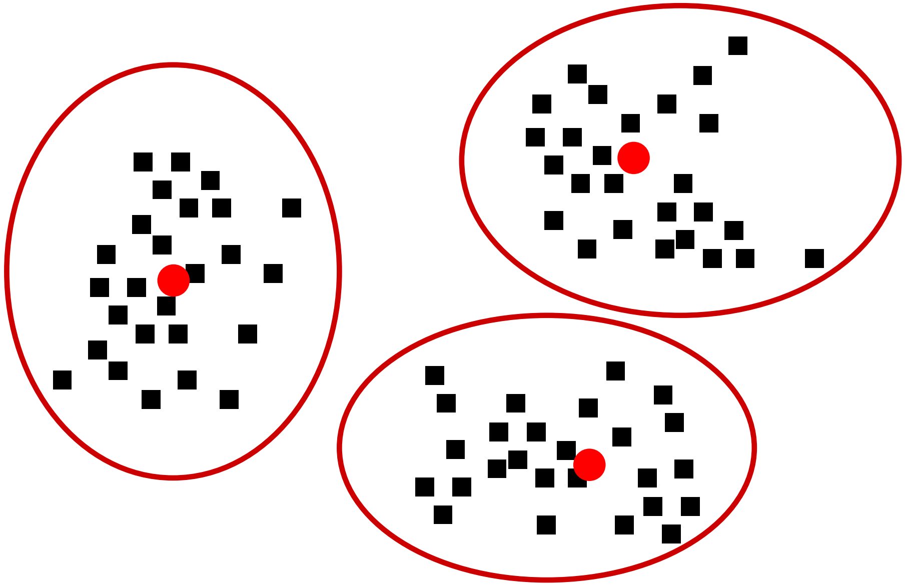

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profile k is a non-negative and decreasing function

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- **KDE map**
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- **Modes**
  – The local maxima of the KDE map

Mean-shift Clustering

- **Mean-shift process**

$$\boldsymbol{x} \leftarrow \boldsymbol{x} + \left(\sum_{i=1}^{p} \frac{k'(||\boldsymbol{x} - P_i||^2)}{\sum_{j=1}^{p} k'(||\boldsymbol{x} - P_j||^2)} \cdot P_i - \boldsymbol{x} \right)$$

  - Slightly moves each $\boldsymbol{x}$ to a denser point
  - Gradient descent method to seek modes

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- **Mean-shift process**
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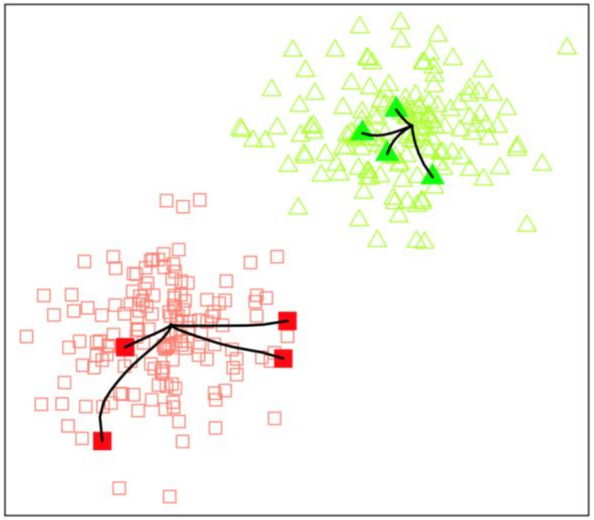
- **Mean-shift clustering**
  - Cluster each point by the mode it goes by mean-shift processes



Drawbacks of Mean-shift

1. Non-arithmetic kernel function

- Gaussian kernel function
  - $K_G(x, y) = c_{k_G} \cdot e^{-\frac{\|x-y\|^2}{\sigma^2}}$
  - Exponential function

2. Computationally expensive

- $O(\#\text{points}^2 \cdot \#\text{iterations})$

IDEA1: HE Friendly Kernel

- **New kernel function**

$$k(x) = (1-x)^{2^{\Gamma}+1}$$

1. Similar performance with usual kernels
    - Satisfies the necessary conditions of kernel functions
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    - Manage to group plaintexts of public datasets properly

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2. Arithmetic
3. Efficient
   - Requires log degree number of computations

IDEA2: Dust Sampling Method

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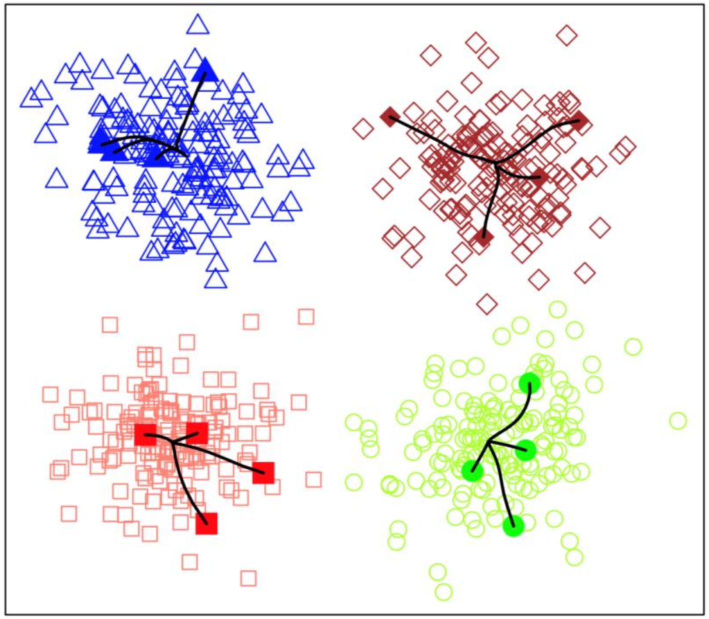
- Label each point by its closest mode
  - $O(\#\mathrm{dusts} \cdot \#\mathrm{points})$

IDEA2: Dust Sampling Method

	Original Mean-shift	Dust Sampling Method
Mean-shift	All points	**<u>Only sampled points</u>**
Structure	Find the modes and label the points at the same time	**Find the modes first, and label the points later**
Computational Complexity	$O(\#\text{points}^2)$	$\underline{\mathbf{O(\#dusts \cdot \#points)}}$

Our Modified Scheme

1. Sample *dusts* from given data



2. Apply mean-shift to dusts and find modes
   – Use HE friendly kernel

3. Label each points to its closest mode

Experimental Result

- **High accuracy on public datasets**
  - Covers various features of dataset: shape of data, number of data, number of attributes, and number of clusters
- **Fast and accurate performance on large scale dataset**

	Num of Data	Num of Attributes	Num of Clusters	Comp. Time	Quality Evaluation	
					Accuracy	Silh Coeff
Hepta	212	3	7	25 min	212/212	0.702 (0.702)
Tetra	400	3	4	36 min	400/400	0.504 (0.504)
Two Diamonds	800	2	2	38 min	792/800	0.478 (0.485)
Large Scale	262,144	4	4	82 min	262127 /262144	0.781 (0.781)

※ Use multi-threading (8 threads)

Experimental Result

- 400 times faster than the previous work (JA18) on Lsun public dataset

	JA18	Our work
Comp. Time	25.79 days	**83 min**
HE library	TFHE	**HEAAN**

※ Use a single thread

[JA18] Jäschke, A. and Armknecht, F., 2018, August. Unsupervised machine learning on encrypted data. In International Conference on Selected Areas in Cryptography (pp. 453-478). Springer, Cham.

Q&A

Thank you!