Toyler expansion approximation examples

$$f(x+\delta) \approx f(x) + \nabla f(x)^T \delta + \frac{1}{2}\delta^T \nabla^2 f(x)\delta + O(\|\delta\|^3)$$

$$f(\delta) \approx f(x_0) + \nabla f(x_0)^T(\delta - x_0) + \frac{1}{2}(\delta - x_0)^T \nabla^2 f(x)(\delta - x_0) + O\left(\|(\delta - x_0)\|^3\right)$$

One-dimension approximation based on gradient and hessian.

```
% define function
f = @ (x) sqrt(x);
% define gradient
g = @ (x) 1/2.*x.^(-1/2);
% define hessian
h = @ (x) -1/4.*x.^(-3/2);
% define first and second order approximation
appro1 = @ (x0, delta) f(x0) + g(x0)' * delta;
appro2 = @ (x0, delta) f(x0) + g(x0)' * delta + 1/2 .* delta' * h(x0) * delta;

% define x0 and delta
x0 = 4;
delta = 0.1;
actual_value = f(x0+delta)
```

```
actual_value = 2.0248
```

```
approximate_value_1st = appro1(x0, delta)
```

```
approximate_value_1st = 2.0250
```

```
approximate_value_2nd = appro2(x0, delta)
```

```
approximate_value_2nd = 2.0248
```

Two-dimension approximation based on gradient and hessian.

```
f = @(x) x(1).^2 - 2.*x(2).*cos(x(1)) + 9;
g1 = @(x) 2.*x(1) + 2.*x(2).*sin(x(1));
g2 = @(x) 2.*sin(x(1));
g = @(x) [g1(x); g2(x)];
h11 = @(x) 2+2.*x(2).*cos(x(1));
h12 = @(x) 2.*sin(x(1));
h21 = h12;
h22 = @(x) 0;

h = @(x) [h11(x), h12(x); h21(x), h22(x)];

x0 = [1;1];
delta = [0.1;0.1];
appro1 = @ (x0, delta) f(x0) + g(x0)' * delta;
appro2 = @ (x0, delta) f(x0) + g(x0)' * delta + 1/2 .* delta' * h(x0) * delta;

actual_value = f(x0+delta)
```

```
actual_value = 9.2121
```

```
approximate_value_1st = appro1(x0, delta)
```

```
approximate_value_1st = 9.4560
```

```
approximate_value_2nd = appro2(x0, delta)
```

```
approximate_value_2nd = 9.4882
```