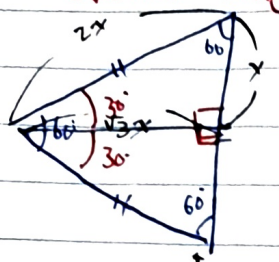
Khan Academy - Precalc ch.2 Trigonometry

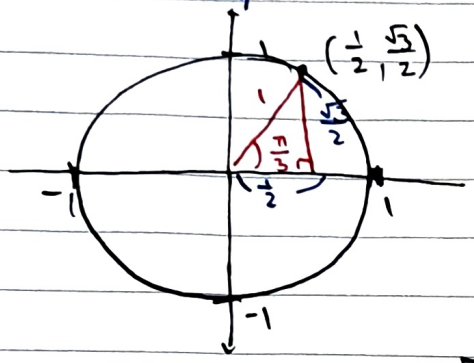
Special Angles

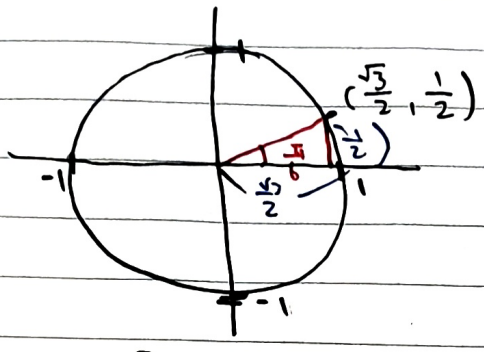


$\sin\frac{\pi}{6} = \frac{x}{2x} = \frac{1}{2}$ $\quad \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

$\cos\frac{\pi}{6} = \frac{\sqrt{3}x}{2x} = \frac{\sqrt{3}}{2}$ $\quad \cos\frac{\pi}{3} = \frac{1}{2}$

$\tan\frac{\pi}{6} = \frac{x}{\sqrt{3}x} = \frac{\sqrt{3}}{3}$ $\quad \tan\frac{\pi}{3} = \sqrt{3}$





$\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$ $\quad \sin\frac{\pi}{2} = 1$ $\quad \sin\emptyset = 0$

$\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$ $\quad \cos\frac{\pi}{2} = 0$ $\quad \cos\emptyset = 1$

$\tan\frac{\pi}{4} = 1$ $\quad \tan\frac{\pi}{2} = \text{undefined}$ $\quad \tan\emptyset = 0$

Symmetry (Sin & cos)

$\cos\theta = \cos(-\theta)$

$\sin\theta = -\sin(-\theta)$

$\cos(\pi - \theta) = -\cos\theta$ $\quad \cos(\pi + \theta) = -\cos\theta$

$\sin(\pi - \theta) = \sin\theta$ $\quad \sin(\pi + \theta) = -\sin\theta$

More about tangent

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\Delta x}{\Delta y} = \text{slope of line}$

$\tan(\pi + \theta) = \tan(\theta)$

$\left(= \frac{\sin(\pi+\theta)}{\cos(\pi+\theta)} = \frac{-\sin\theta}{-\cos\theta} = \frac{\sin\theta}{\cos\theta} = \tan\theta \right)$

$\tan(-\theta) = -\tan\theta$

$\left(\frac{\sin(-\theta)}{\cos(-\theta)} = \frac{-\sin\theta}{\cos\theta} = -\tan\theta \right)$

$\tan(\pi - \theta) = -\tan\theta$

$\frac{\sin(\pi-\theta)}{\cos(\pi-\theta)} = \frac{\sin\theta}{-\cos\theta} = -\tan\theta$

Based on $\sin(\frac{\pi}{2}-\theta)=\cos\theta$, $\cos(\frac{\pi}{2}-\theta)=\sin\theta$

More relationships

$\therefore \sin(\theta+\frac{\pi}{2})=\cos\theta$

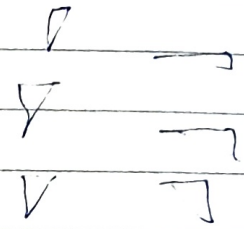
$\sin(\theta+\frac{\pi}{2})=\sin(\pi-\theta-\frac{\pi}{2})$

$=\sin(\frac{\pi}{2}-\theta)=\cos\theta$

$\therefore \cos(\theta+\frac{\pi}{2})=-\sin\theta$

$\cos(\theta+\frac{\pi}{2})=$

$-\cos(\pi-\theta-\frac{\pi}{2})=-\cos(\frac{\pi}{2}-\theta)=-\sin\theta$



Q $\tan 0.46=\frac{1}{2}$, which other θ can replace 0.46?

$\tan 0.46=\tan(\pi+0.46)$ opt 3

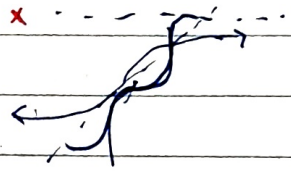
$\tan 0.46=\tan(0.46-\pi)$ λ in opt

$\tan 0.46=\tan(2\pi+0.46)$ opt 5

$\frac{\pi}{2}+\tan 0.46$ = perpendicular ✗

$\tan(\pi-0.46)=-\tan 0.46$ ✗

$\tan(2\pi-0.46)=\tan(-0.46)=-\tan 0.46$ ✗



Inverse of Trig functions

$\arcsin\frac{\sqrt{2}}{2}=?$

$?=\frac{\pi}{4}$ $(-\frac{\pi}{2}\le\theta\le\frac{\pi}{2})$

~~$\frac{\pi}{4}+2\pi n$, $\frac{3\pi}{4}+2\pi n$~~

$\sin^{-1}(-\frac{\sqrt{3}}{2})=-\frac{\pi}{3}$ $(-\frac{\pi}{2}\le\theta\le\frac{\pi}{2})$

arc sin range

~~$(0\le\theta\le\pi)$~~

~~arc cos range~~

~~$\tan^{-1}\theta=1$~~ $\tan^{-1}1=\theta$

$\tan 1$

$\tan\theta=1$

$\frac{\sin\theta}{\cos\theta}=1$

S

$\tan^{-1}(-)=\frac{3\pi}{4}$ or $-\frac{\pi}{4}$ $(-2\pi\le\theta\le2\pi)$

$=-\frac{\pi}{4}$ $(-\frac{\pi}{2}\le\theta\le\frac{\pi}{2})$

arctan range

$\cos^{-1}(-\frac{1}{2})=\theta$

$\cos\theta=-\frac{1}{2}$ $\cos\frac{\pi}{3}=\frac{1}{2}$

$\theta=\frac{2}{3}\pi$ $-\cos\frac{\pi}{3}=-\frac{1}{2}=\cos(\pi-\frac{\pi}{3})=\cos(\frac{2}{3}\pi)$

$\frac{2}{3}\pi$ $(0\le\theta\le\pi)$

$\cos^{-1}$ range

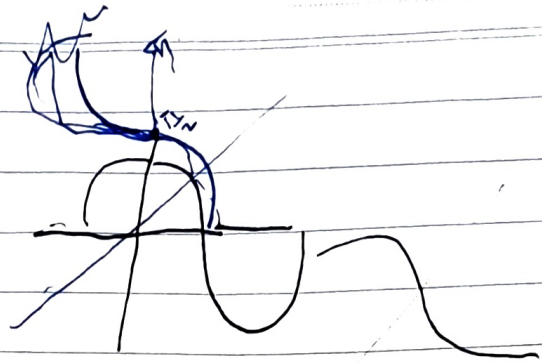
$\cos(\arccos A)=A$

$\arccos(\cos\theta)=\theta$ if $(0\le\theta\le\pi)$

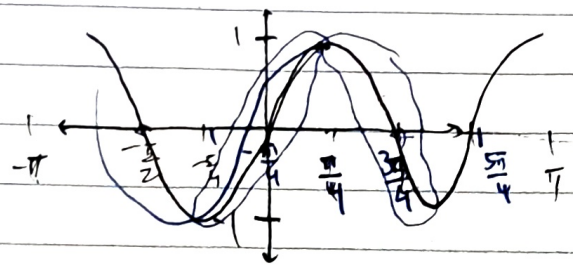
or

$\tan^{-1}$ range: $-1.57 \leq \theta \leq 1.57$

$\tan\theta = -1.4$.

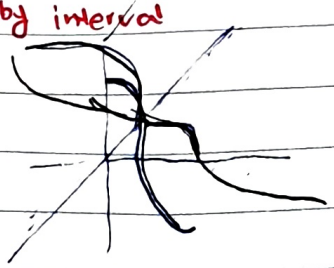


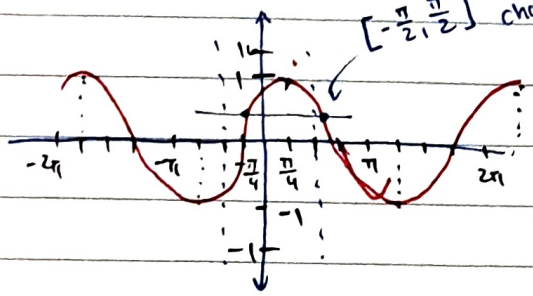
$\cos(x - \frac{\pi}{4})$



Domain can be restricted by interval

~~$[\frac{\pi}{2}, \frac{3\pi}{2}]$~~ ✗





$[-\frac{\pi}{2}, \frac{\pi}{2}]$ cannot be chosen as restricting interval

$[\frac{\pi}{4}, \frac{5\pi}{4}]$ or $[\frac{\pi}{4}, \frac{5\pi}{4})$ ← still invertible

$[-\frac{3\pi}{4}, \frac{\pi}{4}]$ ← but not recommended !!

or even $(\frac{\pi}{2}, \frac{3\pi}{2})$

$g(x) = \tan(x - \frac{3\pi}{2}) + 6$ $\quad x =$ ~~$\tan(A)$~~ $\quad \tan\theta = A$

$\tan^{-1} A = \theta$

$\frac{\pi}{2} \quad \frac{3\pi}{2} + \frac{\pi}{2} \quad \pi \sim 2\pi$

~~$\tan$~~ $\tan(\tan^{-1} A) = A$

$\tan^{-1}(\tan\theta) = \theta$

$-\frac{\pi}{2} \leq \theta \leq \frac{3\pi}{2}$

$\tan(x - \frac{3\pi}{2}) = g(x) - 6$

$x - \frac{3\pi}{2} = \tan^{-1}(g(x) - 6) + \frac{3\pi}{2}$

$g^{-1}(\tan(x - \frac{3\pi}{2}) + 6) = x$

$g(x) + 6 = \tan(x - \frac{3\pi}{2}) = A$

~~$\tan^{-1} A = x - \frac{3\pi}{2}$~~.

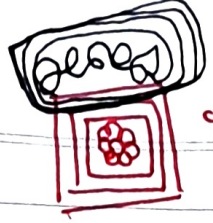
$g(x) = \frac{\tan(x - \frac{3\pi}{2}) + 6}{A}$

$g(A) = x$

~~$f(x) = x$~~ $\quad f(x) = \frac{1}{2}x$

$x = \frac{1}{2} f(x)$

$f^{-1}(x) = 2x$



$g(x) = \tan\left(x - \frac{3\pi}{2}\right) + 6$

$x = \tan\left(g^{-1}(x) - \frac{3\pi}{2}\right) + 6$

$x - 6 = \tan\left(g^{-1}(x) - \frac{3\pi}{2}\right)$

$\tan^{-1}(x-6) = g^{-1}(x) - \frac{3\pi}{2}$

$g^{-1}(x) = \tan^{-1}(x-6) + \frac{3\pi}{2}$

Domain = $(-\infty, \infty)$

Range = $(-\frac{\pi}{2}, \frac{\pi}{2})$

sin $(-\infty, \infty)$

$[-1, 1]$

$\sin^{-1}$ $D = [-1, 1]$

$R = [-\frac{\pi}{2}, \frac{\pi}{2}]$

$-\frac{\pi}{2} < x - 6 < \frac{\pi}{2}$

$-\frac{\pi}{2} + 6 < x < \frac{\pi}{2} + 6$

$D = [-1, 1]$

$R = [-\frac{\pi}{2}, \frac{\pi}{2}]$

~~$\tan\theta = x - 6$~~

$-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

any x even if slope is crazy high

$\theta < \frac{\pi}{2}$

Domain & Range of Inverse Trig Funcs

Sin $D = (-\infty, \infty)$ $R = [-1, 1]$

$\sin^{-1}$ $D = [-1, 1]$ $R = [-\frac{\pi}{2}, \frac{\pi}{2}]$

cos $D = (-\infty, \infty)$ $R = [-1, 1]$

$\cos^{-1}$ $D = [-1, 1]$ $R = [0, \pi]$

tan $D = \{x \mid x \in \mathbb{R}$ and $x \neq \frac{\pi}{2} + n\pi$ and $n \in \mathbb{Z}\}$

$R = (-\infty, \infty)$

$\tan^{-1}$ $D = (-\infty, \infty)$

$R = (-\frac{\pi}{2}, \frac{\pi}{2})$

Quiz

$\cos\frac{5\pi}{3} = \cos(-\frac{\pi}{3}) = \cos\frac{\pi}{3} = \frac{1}{2}$ $\sin\frac{5\pi}{3} = \sin^{-}\frac{\pi}{3} = -\sin\frac{\pi}{3} = -\frac{\sqrt{3}}{2}$