# Week 3 - Tut

1) $a = [2,-1,0]$ $\quad b = [1,1,1]$ $\quad c = [-2,0,1]$

$a \times b =$ -1 0 2 -1 / 1 1 1 1

$\rightarrow [-1,-2,3]$

~~$b \times c =$~~ 1 1 1 1 / 0 1 -2 0

$\rightarrow [1,-3,2]$

~~$c \times a =$~~ $\quad [1,2,2]$ $\qquad \underset{\sim}{a} \times \underset{\sim}{c} = [-1,-2,-2]$

0 1 -2 0 / -1 0 2 -1

i) $b \times a = [1,2,-3]$

ii) $\cancel{\underset{\sim}{a} \times (\underset{\sim}{a} + \underset{\sim}{c})} = \underset{\sim}{a} \times \underset{\sim}{a} + \underset{\sim}{a} \times c$

$= \underset{\sim}{0} + \underset{\sim}{a} \times \underset{\sim}{c} = -\underset{\sim}{c} \times a = [-1,-2,-2]$

iii) $(\underset{\sim}{a} \times \underset{\sim}{a}) \times \underset{\sim}{c}$

$= \underset{\sim}{0} \times \underset{\sim}{c} = \underset{\sim}{0}$

iv) $\underset{\sim}{a} \times (\underset{\sim}{b} - 2\underset{\sim}{c})$ $\qquad$ ~~$b - 2c =$~~

$= \underset{\sim}{a} \times (\cancel{[1-(-4)),}$ $\qquad$ ~~$[1-(-4), 1-0, 1-2]$~~

$= \underset{\sim}{a} \times \underset{\sim}{b} - 2(\underset{\sim}{a} \times \underset{\sim}{c})$ $\qquad$ ~~$= [5, 1, -1]$~~

부호 잘못 적음

$= [-1,-2,+3] - 2[-1,-2,-2]$

$= [-1,-2,+3] + [2,4,4] = [1,2,7]$

v) $\|\underset{\sim}{a} \times \underset{\sim}{b}\| = \|a\| \|b\| \sin\theta$

$\sqrt{1+4+9} = \sqrt{4+1} \times \sqrt{3} \times \sin\theta$

$\sqrt{14} = \sqrt{5} \times \sqrt{3} \times \sin\theta$

$\sin\theta = \dfrac{\sqrt{14}}{\sqrt{15}} = \sqrt{\dfrac{14}{15}}$

vi) ~~$\sqrt{14}$~~ $\quad \|a \times c\| = \sqrt{1+4+4} = 3$

2) $\underset{\sim}{v} \times \underset{\sim}{w} =$

$$\begin{array}{cccc} 2 & -7 & 1 & 2 \\ 4 & 1 & 5 & 1 \end{array} = [2+7, -35-1, 1-10]$$

$$= [9, -36, -9]$$

$$= 9\,[1, -4, -1]$$

$$= \cancel{9 \times \sqrt{18}} \left[\frac{1}{\sqrt{18}}, -\frac{16}{\sqrt{18}}, -\frac{1}{\sqrt{18}}\right]$$

$$\left[\frac{-1}{\sqrt{18}}, +\frac{16}{\sqrt{18}}, +\frac{1}{\sqrt{18}}\right] = \frac{1}{\sqrt{18}}[-1, 16, 1] = \frac{1}{3\sqrt{2}}\frac{\sqrt{2}}{\sqrt{2}} \quad \frac{\sqrt{2}}{6}[-1, 16, 1]$$

$$\propto$$

$$\left[\frac{1}{\sqrt{18}}, -\frac{16}{\sqrt{18}}, \frac{-1}{\sqrt{18}}\right] \qquad \frac{\sqrt{2}}{6}[1, -16, -1]$$

답지는 뭐여서 이렇게 하네?

3) $P = (8, 4, -1)$ $\qquad Q = (6, 3, -4)$

$R = (7, 5, -5)$

$\overrightarrow{QP} \times \overrightarrow{QR}$ ? $\qquad \triangle PQR$'s Area ?

$\underset{\sim}{u} = \overrightarrow{QP} = [2, 1, 3]$

$\underset{\sim}{v} = \overrightarrow{QR} = [1, 2, -1]$

$$\times = \begin{array}{cccc} 1 & 3 & 2 & 1 \\ 2 & -1 & 1 & 2 \end{array}$$

$$\underset{\sim}{u} \times \underset{\sim}{v} = [-7, 5, 3]$$

$$\frac{1}{2}\|\underset{\sim}{u} \times \underset{\sim}{v}\| = \frac{1}{2}\sqrt{49+25+9} = \frac{\sqrt{83}}{2}$$

$$= \frac{1}{2}\sqrt{83}$$

4) $\underset{\sim}{a}, \underset{\sim}{b} \in \mathbb{R}^3$ $\quad \|a\| = 7 \quad \|b\| = 4 \quad a \cdot b = -21$

$$\|a \times b\|^2 = \|a\|^2 \, \|b\|^2 - (\underset{\sim}{a} \cdot \underset{\sim}{b})^2$$
$$= 49 \times 16 - (21)^2$$
$$= 343$$
$$\cancel{\therefore \sqrt{343} \approx 18.5}$$
$$\sqrt{343}$$

$a \cdot b = \|a\| \, \|b\| \cos\theta$

$-21 = 28 \cos\theta$

$\cos\theta = -\frac{3}{4}$

$\|a \times b\| = 7 \times 4 \times \sqrt{\left(1 - \frac{9}{16}\right)}$

$= 28 \times \sqrt{\frac{7}{16}} =$

$28 \times \frac{\sqrt{7}}{4} = 7\sqrt{7}$

I like the approach better!

5) passes thru P, direction = [2, 1]

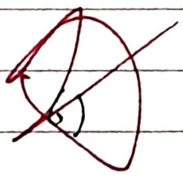
1) $P = (-6, 5)$ $\qquad -x + 2y = c$

$[-1 \;\; 2][x, y] = \underset{\sim}{n} \cdot \underset{\sim}{p}$ $\qquad c = 6 + 10 = 16$

$= 6 + 10 = 16$

general form: $-x + 2y - 16 = 0$

parametric " : $[-1, 2] \cdot [x, y] = 16$



Wk 3 - tut cont.

5) P = point on $\ell$ $\quad$ $\underset{\sim}{v}$ = direction vector

i) $P = (-6, 5)$ $\quad$ $\underset{\sim}{v} = [2, 1]$

$$\underset{\sim}{x} = [-6, 5] + t[2, 1]$$

$$\left.\begin{array}{l} x = -6 + 2t \\ y = 5 + t \end{array}\right\} t \in \mathbb{R}$$

ii) $P = (1, 0)$ $\quad$ $\underset{\sim}{v} = [2, 2]$

$$\underset{\sim}{x} = [1, 0] + t[2, 2]$$

$$\left.\begin{array}{l} x = 1 + 2t \\ y = 2t \end{array}\right\} t \in \mathbb{R}$$

iii) $P = (0, 1, -1)$ $\quad$ $\underset{\sim}{v} = [1, 2, 0]$

$$\underset{\sim}{x} = [0, 1, -1] + t[1, 2, 0]$$

$$\left.\begin{array}{l} x = t \\ y = 1 + 2t \\ z = -1 \end{array}\right\} t \in \mathbb{R}$$

6) line $\ell$ passes through Q and P

i) $P = (3, 1)$ $\quad$ $Q = (5, 4)$

$\underset{\sim}{v} = [2, 3]$

v.f. $\quad \underset{\sim}{x} = [3, 1] + t[2, 3]$

p.eq $\quad \left.\begin{array}{l} x = 3 + 2t \\ y = 1 + 3t \end{array}\right\} t \in \mathbb{R}$

[ ]·[ ] = c form 말고

이 form로 써야지!

g.e $\quad [-3, 2] \cdot [x, y] = -7$ $\quad$ $\cancel{-3x + 2y = -7}$ $\quad$ OR $\quad 3x - 2y = 7$

n.f $\quad [-3, 2] \cdot [x - 3, y - 1] = 0$ $\quad$ &

or

$\cancel{[3, -2] \cdot [x - 3, y - 1] = 0}$

$[3, -2] \cdot ([x, y] - [3, 1]) = \emptyset$

ii) $P = (-4, 3, 5)$ $\quad Q = (-2, 4, 1)$

$\underset{\sim}{v} = [2, 1, -4]$ $\qquad [0, 5, -\frac{3}{2}]$

v.f. $\underset{\sim}{x} = [-4, 3, 5] + t[2, 1, -4]$

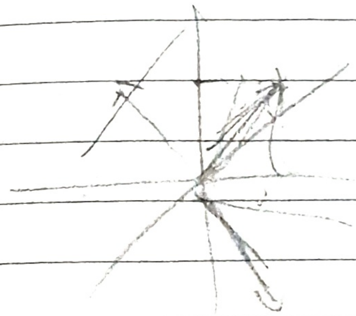
p. eqs
$$\left.\begin{aligned} x &= -4 + 2t \\ y &= 3 + t \\ z &= 5 - 4t \end{aligned}\right\} t \in \mathbb{R}$$

Q lines represented are multiples

<2: slider>

$P \times Q = [-17, -6, -10]$ $\qquad \frac{17}{4}$

$Q \times P = [17, 6, 10]$ $\qquad 110 + 28$



g.f

$[-1, 2, 0][x, y, z] = 10$

$-x + 2y = 10$ $\quad z$ is free

Q How do I find perpendicular slope to 3D line in?

$3x + 2(y+z) = -12 + \cancel{6t} + 6 + \cancel{2t} + 10 - \cancel{8t}$

$= \cancel{-6} + \cancel{10}\ 4$

모르겠당 ㅎ

$3x + 2y + 2z = 4$

$[2, 1, -4] \cdot [3, 2, 2]$

$= 6 + 2 - 8 = 0$ ?

$4(x + y) + 3z = 4(-1 + 3t) + 3(5 - 4t)$

$= -4 + \cancel{12t} + 15 - \cancel{12t}$

$= 11$

$4x + 4y + 3z = 11$

?

7) To show two lines aren't skewed
we can either show that
· lines are parallel
or
· lines intersect

"완벽한 풀이였다? ㅋㅋㅋ"

$l_1: x_1 = [1,1,1] + t[3,-1,4]$
$l_2: x_2 = [6,-6,1] + s[-7,5,-6]$ $\Big\} t, s \in \mathbb{R}$

$1 + 3t = 6 - 7s$
$1 - t = -6 + 5s$
$1 + 4t = 1 - 6s$

~~$2 + 2t = -2s$~~
~~$4 + 4t = -s$~~ $-4s$
$1 + 4t = 1 - 6s$

$3 =$

-8

$t = -3, \ s = 2$

$4 + 4t = -4s$
$- \ 1 + 4t = 1 - 6s$
$3 = 2s + 1$
$s = 2, \ t = -3$

two lines intersect when
$t = -3 \quad s = 2$

$x_1 = [1,1,1] + -3[3,-1,4]$
$= [1,1,1] + [-9,3,-12] = [-8,4,-11]$

$x_2 = [6,-6,1] + 2[-7,5,-6]$
$= [6,-6,1] + [-14,10,-12] = [-8,4,-11]$

$\therefore x_1 = x_2$
when
$t = -3, s = 2$

$\therefore$ two lines aren't skewed