COMP2123

wk 0 Review

Induction

① Given/or formulate ⟨Induction Hypothesis⟩

② Prove ~~base case~~ Induction hypothesis is true for base case

③ Induction step.

- assume $n = k$ also arbitary ~~n~~ that satisfies I.H.
- show $n = k+1$ satisfies Induction hypothesis.

Proof by Induction ex1. $\boxed{\text{tut 0-4}}$

Induction hypothesis: $\sum_{i=0}^{n} 2^i = 2^{n+1} - 1$

1) Assume that hypothesis is true for $n = k$, $(k \in \mathbb{N})$ arbitary natural number

$$\sum_{i=0}^{k} 2^i = 2^{k+1} - 1$$

2) We can prove that hypothesis is also true for $n = \cancel{k+1}$

$$\sum_{i=0}^{k+1} 2^i = 2^{(k+1)+1} - 1$$

$$\sum_{i=0}^{k} 2^i + \cancel{2^{(k+1)}} = 2^1 \times (2^{k+1}) - 1$$

$$= \cancel{(2^{k+1})} + (2^{k+1}) - 1$$

$$\therefore \sum_{i=0}^{k} 2^i = 2^{k+1} - 1$$

3) Since hypothesis is true for base case $n = 0$,

$$\sum_{i=0}^{0} 2^i = 2^{0+1} - 1$$

$$2^0 = 2^1 - 1$$

$$1 = 1$$

it is also true for $n = \{1, 2, 3, 4 \cdots\}$.

~~for other~~

$\therefore \sum_{i=0}^{n} 2^i = 2^{n+1} - 1$ is true for n $(n \in \mathbb{N})$

Wk 0 Induction exercises

Tut 0 - 5

fibonacci sequence:

$F(1) = 1$

$F(2) = 2$

$F(n) = F(n-2) + F(n-1)$ for $n > 2$

Inductive hypothesis: $F(n) < 2^n$

Assume that hypothesis is satisfied when $n = k$, $n = k+1$ $(k \in \mathbb{N}, k \geq 1)$

$F(k) < 2^k$

$F(k+1) < 2^{k+1}$

It can be proved that hypothesis is also satisfied for $n = k+2$

$F(k+2) < 2^{k+2}$?

LHS	RHS
$F(k+2)$	2^{k+2}
$= F(k) + F(k+1)$, by definition	$= 2 \times (2^{k+1})$
	$= (2^{k+1}) + (2^{k+1})$
	$= 2(2^k) + (2^{k+1})$
	$= (2^k + 2^k) + (2^{k+1})$

$F(k) + F(k+1) < 2^k + 2^k + 2^{k+1}$

$0 < (2^k - F(k)) + (2^{k+1} - F(k+1)) + 2^k$

$0 < A + B + 2^k$, $(A, B \in \mathbb{N}$ and $A, B > 0)$

$0 < C$ $(C \in \mathbb{N}, C > 0)$

= true

Since hypothesis is true for $n = 1$, $n = 2$

$F(1) = 1 < 2^1 = 2$

$F(2) = 3 < 2^2$

hypothesis is true for all n. $(n \geq 1, n \in \mathbb{N})$

tut 0-6

$R(n)$ = # of regions on plane divided after n^{th} line intersection.

- lines are not parallel
- ~~[illegible]~~ 3 lines do not insect at one point.

∴ at most 2 lines intersect at one time

∴ There are no line that never meets.

Hence, for n^{th} line, the line intersects with ~~other~~ every other line on $(n-1)$ points.

when ~~[illegible]~~ $n \geq 2$,

~~[illegible]~~ n^{th} line cuts through $1+(n-2)+1$ ~~regions~~ segments ∴ hence creating $1+(n-2)+1$ many new regions

1 ~~region~~ segmen : ~~region~~ segment between edge & closest point

1 ~~region~~ seg : ~~region~~ segment betwee opposite edge & closest point.

$(n-2)$ ~~region~~ seg : segments between $(n-1)$ points

∴ n^{th} line cuts through $1+(n-2)+1 = n$ regions.