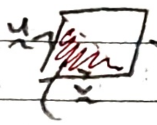
Assignment 1 (Q1-a, 1-b)

Ques 1 $\quad \underset{\sim}{u} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \qquad \underset{\sim}{v} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$

(a) Area of the triangle inscribed by $\underset{\sim}{u}$ and $\underset{\sim}{v}$.

Answer: $\sqrt{3}$

Area of parallelogram with side $\underset{\sim}{u}$ and $\underset{\sim}{v}$ is



's Area $= \|\underset{\sim}{u} \times \underset{\sim}{v}\|$ $\qquad$ * $\underset{\sim}{v}, \underset{\sim}{u}$ must be in $\mathbb{R}^3$

And, Area inscribed by $\underset{\sim}{u}$ and $\underset{\sim}{v}$ is half of $\|\underset{\sim}{u} \times \underset{\sim}{v}\|$

Area of $\triangle$ $= \frac{1}{2}\|\underset{\sim}{u} \times \underset{\sim}{v}\| = \frac{1}{2}\|[-2,2,2]\|$

* $\underset{\sim}{u} \times \underset{\sim}{v} = [-2,2,2]$

$$\begin{matrix} 0 & 1 & 1 & 0 \\ 2 & -1 & 1 & 2 \end{matrix}$$

$= \frac{1}{2}\sqrt{(-2)^2 + 2^2 + 2^2}$

$= \frac{1}{2}\sqrt{12} = \frac{1}{2} \cdot 2\sqrt{3} = \boxed{\sqrt{3}}$

(b) Equation of line ℓ in $\mathbb{R}^3$ where (in vector form)

- ℓ passes through $(3,-2,3)$
- ℓ is perpendicular to $\underset{\sim}{u}$ and $\underset{\sim}{v}$ $\qquad \underset{\sim}{p} = [3,-2,3]$

From the fact that ℓ is perpendicular to $\underset{\sim}{u}$ and $\underset{\sim}{v}$, we know that direction vector of ℓ is either

$\underset{\sim}{d} = \underset{\sim}{u} \times \underset{\sim}{v} = [-2,2,2]$ or ($\underset{\sim}{d} = \underset{\sim}{v} \times \underset{\sim}{u} = [2,-2,-2]$ but I will use the former)

vector form: $\underset{\sim}{p} + t\underset{\sim}{d} \quad t \in \mathbb{R}$

Answer

$$\boxed{\ell: \underset{\sim}{x} = [3,-2,3] + t[-2,2,2] \quad t \in \mathbb{R}}$$

Assignment 1 (Q1-c)

Ques 1 $\quad \underset{\sim}{u} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \quad \underset{\sim}{v} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$

(c) From pervious parts (a), (b) we know that

$\underset{\sim}{u} \times \underset{\sim}{v} = [-2, 2, 2] = 2[-1, 1, 1]$

$\underset{\sim}{x} = s\underset{\sim}{v} + \frac{t}{\sqrt{2}} \underset{\sim}{u} \times \underset{\sim}{v}$

$\underset{\sim}{x} = s[1, 2, -1] + \frac{t}{\sqrt{2}} \times 2[-1, 1, 1]$ ∘∘∘ $c\underset{\sim}{a} \times \underset{\sim}{b} = c(\underset{\sim}{a} \times \underset{\sim}{b})$

$\underset{\sim}{x} = s[1, 2, -1] + \sqrt{2}t[-1, 1, 1]$

For $\|\underset{\sim}{x}\| \leq \sqrt{6}$ to be true, applicable because both sides are positive

$\underline{\|\underset{\sim}{x}\|^2 \leq 6 \text{ must be true}}$

$\underset{\sim}{x} = [s, 2s, -s] + [-\sqrt{2}t, \sqrt{2}t, \sqrt{2}t]$

let $n = \sqrt{2}t$

$\underset{\sim}{x} = [s-n, 2s+n, -s+n]$

$\|\underset{\sim}{x}\|^2 = (\underset{\sim}{x} \cdot \underset{\sim}{x}) = (s-n)^2 + (2s+n)^2 + (-s+n)^2$

$= s^2 - 2sn + n^2 + 4s^2 + 4sn + n^2 + s^2 - 2sn + n^2$

$= 6s^2 + 3n^2$

$= 6s^2 + 3(\sqrt{2}t)^2$

$= 6s^2 + 6t^2$

$\|\underset{\sim}{x}\|^2 = 6s^2 + 6t^2 \leq 6$

$6(s^2 + t^2) \leq 6$

answer $\rightarrow$ $\boxed{s^2 + t^2 \leq 1 \quad (s, t \in \mathbb{R})}$

Assignment 1 (Q2-a, 2-b)

Ques 2

$$\underset{\sim}{w} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \qquad \underset{\sim}{z} = \begin{bmatrix} 3t-2 \\ t+3 \\ 2t-3 \end{bmatrix}$$

(a) For $\underset{\sim}{w}$ and $\underset{\sim}{z}$ to be orthogonal, it must satisfy that

answer: $t = 2$

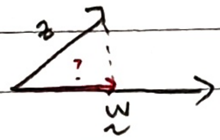
$$\underset{\sim}{w} \cdot \underset{\sim}{z} = 0$$

$$\underset{\sim}{w} \cdot \underset{\sim}{z}$$
$$= [1,-1,1] \cdot [3t-2,\ t+3,\ 2t-3]$$
$$= (3t-2) - (t+3) + (2t-3)$$
$$= 3t - 2 - t - 3 + 2t - 3$$
$$= 4t - 8$$

$4t-8$ must be $\emptyset$.

$$\therefore \boxed{t = 2}$$

(b) Projection of $\underset{\sim}{z}$ onto $\underset{\sim}{w}$



$$\underset{\underset{\sim}{w}}{\text{Proj}}(\underset{\sim}{z}) = \frac{(\underset{\sim}{w} \cdot \underset{\sim}{z})}{\|\underset{\sim}{w}\|^2} \times \underset{\sim}{w}$$

$\underset{\sim}{w} \cdot \underset{\sim}{z} = 4t - 8$ (from part a)

$$\|\underset{\sim}{w}\|^2 = (\underset{\sim}{w} \cdot \underset{\sim}{w}) = (1)^2 + (-1)^2 + (1)^2 = 3$$

$$\therefore \underset{\underset{\sim}{w}}{\text{Proj}}(\underset{\sim}{z}) = \frac{4(t-2)}{3}[1,-1,1]$$

answer

$$= \boxed{\frac{4}{3}(t-2)[1,-1,1]}$$

or

$$= \boxed{\left[\frac{4t-8}{3},\ \frac{8-4t}{3},\ \frac{4t-8}{3}\right]}$$ (expanded)

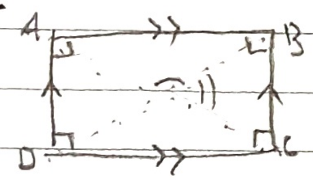
Assignment 2 (Q2-3)

Ques 2

(c) Find p in $\mathbb{R}^3$ such that

$(0,0,0)$, $(1,-1,1)$, $(4,5,1)$, and P forms a rectangle.

For a shape to be a rectangle:



- Adjacent sides need to be orthogonal
- Be parallel with opposite side
- diagnals aren't orthogonal unless it's square

let $A=(0,0,0)$, $B=(1,-1,1)$, $C=(4,5,1)$

$\vec{AB} = [1,-1,1]$

$\vec{AC} = [4,5,1]$

$\vec{BC} = [3,6,0]$

$\square ABCP$ is not a square because ($\sqrt{3}$, $\sqrt{45}$, $\sqrt{42}$)

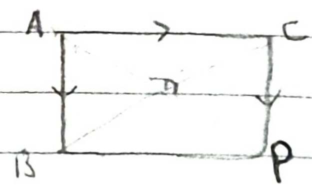
$\|\vec{AB}\| \neq \|\vec{BC}\|$ and $\|\vec{AB}\| \neq \|\vec{AC}\|$

↳ $\vec{AB}$ and ($\vec{BC}$ or $\vec{AC}$) has to be adjacent side but neither has equal length as $\vec{AB}$

$\therefore$ Only adjacent sides (not diagnals) can be orthogonal

$\vec{AB}$ and $\vec{BC}$ are not adjacent

because $\vec{AB} \cdot \vec{BC} = (3) + (-6) + 0 \neq 0$



$\therefore \vec{AB}$ is adjacent with $\vec{AC}$ and $\vec{BP}$

$\therefore \vec{AP} = \underset{\sim}{P} - \underset{\sim}{A} = \vec{AC} + \vec{CP}$

$= \vec{AC} + \vec{AB} = [5,4,2]$

$\underset{\sim}{P} - \underset{\sim}{A} = \underset{\sim}{P} - \underset{\sim}{0} = [5,4,2]$

$\therefore \underset{\sim}{P} = [5,4,2]$

Answer → $\boxed{\therefore P = (5,4,2)}$