Wk 1 - tutorial (Calculus)

1. $X = \{n \in \mathbb{Z} \mid n^2 \leq 5\}$

a) $X = \{-2, -1, 0, 1, 2\}$

b) True, False, False, False

2. $z = 2 - i$, $w = -4 + 3i$

$z + w = -2 + 2i$

$z - w = 6 - 4i$

$|z| = \sqrt{4 + 1} = \sqrt{5}$

$\overline{w} = -4 - 3i$

$2w = -8 + 4i + 6i - 3i^2$

$= -8 + 3 + 10i$

$= -5 + 10i$

$\frac{z}{w} = \frac{2 - i}{-4 + 3i} \frac{(-4 - 3i)}{(-4 - 3i)} = \frac{-8 + 4i - 6i - 3}{16 + 9}$

$= \frac{-11 - 2i}{25}$

$= -\frac{11}{25} - \frac{2}{25}i$

4. a. $\frac{1+i}{1-i} \frac{(1+i)}{(1+i)} = \frac{1 + 2i + i^2}{2} = i$

b. $(2 + 3i)(5 - 6i)$

$= 10 + 15i - 12i - 18i^2$

$= 10 + 3i + 18 = 28 + 3i$

c. $\frac{(1 - i) - 3i^2}{(i)(1 - i)} = \frac{1 - i + 3}{i - i^2} = \frac{(4 - i)(1 - i)}{(1 + i)(1 - i)} = \frac{4 - i - 4i - 1}{2} =$

$\frac{3 - 5i}{2} = \frac{3}{2} - \frac{5}{2}i$

d. $i^{123} - 4i^9 - 4i$

$= -i - 4i - 4i = -9i$

$i^{123} = i^{120} \times i^3 = i^3 = -i$

$i^9 = i$

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a) $3z^2 - 4z + 4 = 0$

$\sqrt{16 - 4(3)(4)} = \sqrt{16 - 48}$

$= \sqrt{-32} = \sqrt{32}\,i = 4\sqrt{2}\,i$

$\frac{4 \pm 4\sqrt{2}\,i}{6} = \frac{2 \pm 2\sqrt{2}\,i}{3}$ $z = \boxed{\frac{2}{3} + \frac{2\sqrt{2}}{3}i,\ \frac{2}{3} - \frac{2\sqrt{2}}{3}i}$

b) $z^4 = 1 \quad A = z^2$

$A^2 = 1$

$A = \pm 1$ $\begin{cases} z^2 = 1, & z = \boxed{1 \text{ or } -1} \\ z^2 = -1, & z = \boxed{i \text{ or } -i} \end{cases}$

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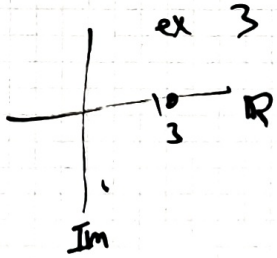
a) Square of imaginary num is always real.

$bi,\ b \in R$

$(bi)^2 = b^2 i^2 = -b^2$ ∴ True

b) False because the magnitudes (modulus) of complex numbers can indeed be ordered as they are positive real numbers.

c) False, for $x \in R$ y-coordinate $= 0$ x-axis



d) False $A = \frac{a}{b + ci} \quad (a, b, c \in R)$

assume search for counterexample

i.e. A is real

$A(b + ci) = a$ if $c = 0$

for any $a, b \in R$ A is also real.

✱ During Tutorial

#3 - should be easy-peasy

#6 - ask tutor

#8 - easy, work with peers

#7 - go over detailed solutions.