Wk 0 Tutorial

1a. set of positive odd integers or odd ~~whole~~ natural numbers

1b. set of even integers

1c. set of positive even integers. or even natural numbers

2a. ~~example~~

$\cancel{x} \in \{1, 10, 100\}$

2b. $\{n \mid n > 5, n \in \mathbb{Z}\}$ or $\{6, 7, 8, 9 \cdots\}$

2c. $\{n \mid n < 5, n \in \mathbb{N}\}$ or $\{1, 2, 3, 4\}$

2d. $\varnothing$ $\cancel{\{\varnothing\}}$? $\{\}$

3a. ~~yes~~ no

3b. yes

3c. $\{x, y, z\}$

3d. $\{x, y\}$

3e. $\{(x,x), (x,y), (y,x), (y,y), (z,x), (z,y)\}$

3f. $\{\varnothing, \{x\}, \{y\}, \{x,y\}\}$

4. Use Induction to show

$$\sum_{i=0}^{n} 2^{i} = 2^{i+1} - 1$$

$$2 \times 2^{i} - 1$$

$n = 0, \quad 2^0 = 1$

$n = 1, \quad 2^1 = 2$

$n = 2, \quad 2^2 = 4$

3   8

4   16

5   ⋮

$n=0, \ \Sigma 2i = 1 = 2^1 - 1 =$

$n=1, \ \Sigma 2^1 = 1 + 2 = 3 = 2^2 - 1$

$n=2 \ \ \Sigma 2^2 = 1 + 2 + 4 = 7$

$2^3 - 1 = 8 - 1 = 7$

$n = x$

$\Sigma 2^x = 2^0 + 2^1 + 2^2 + \cdots 2^x$

$2^{x+1} - 1 = 2 \cdot 2^x - 1$

$2^0 + 2^1 + 2^2 + \cdots 2^x = 2^x + 2^x - 1$

$2^0 + 2^1 + \cdots 2^{x-1} = 2^x - 1$

$\dfrac{\cancel{1+}2^0 + 2^1 + 2^2 + \cdots + 2^x}{2} = \dfrac{2 \cdot 2^x \cancel{-1}}{2}$

$1 + 1 + 2^1 + \cdots 2^{x-1} = 2^x$

$$P(n) = \sum_{i=1}^{n} i = \frac{n(n+1)}{2} = \sum_{i=1}^{n-1} i + n = P(n-1) + n$$

$P(1) = 1$

~~Assume $P(k) = \frac{k(k+1)}{2}$ is true.~~

$P(2) = 3$

$$P(n) = \sum_{i=1}^{n} i = \sum_{i=1}^{n-1} + n = P(n-1) + n$$

$P(1) = 1$

$P(2) = 3$

Assume $\sum_{i=1}^{n} i = \frac{n(n+1)}{2} = \frac{n^2+n}{2}$

$$P(n+1) = \frac{(n+1)(n+2)}{2} = \frac{n^2+3n+2}{2}$$

$$P(n+1) = \frac{2n+2}{2} = n+1$$

$$= P(n) + n$$

4) Base case:

n = 0,

$$\sum_{i=0}^{0} 2^i = 1$$

$$2^1 - 1 = 1$$

$\therefore$ base case is true

Induction step:

$$\sum_{i=0}^{n-1} 2^{i+1} = 2^i - 1$$

Assume n = k

$$\sum_{i=0}^{k} 2^k = 2^{k+1} - 1$$

Show n = k+1 is true

$$\sum_{i=0}^{k+1} 2^{k+1} = 2^{(k+1)+1} - 1$$

$$\sum_{i=0}^{k} 2^k + 2^{k+1} = 2^{k+2} - 1$$

$$(2^{k+1} - 1) + 2^{k+1} = 2 \cdot 2^{k+1} - 1$$

$$2^{k+1} + 2^{k+1} - 1 = 2^{k+1} + 2^{k+1} - 1$$

$0 = 0$ ~~both~~ two sides are identical

$F(1) = 1$

$F(2) = 2$

$F(n) = F(n-1) + F(n-2) \quad , n > 2$

Base case:

$F(1) < 2^1 \qquad 1 < 2 \qquad \therefore$ true

$F(2) < 2^2 \qquad 2 < 4 \qquad \therefore$ true

Induction step:

Assume $F(n-1) < 2^{n-1}$

$F(n-2) < 2^{n-2}$

$2^{n-1} + 2^{n-1}$

$2 \cdot 2^{n-1}$

Show $F(n) < 2^n$

$F(n-1) + F(n-2) < 2^{n-1} + 2^{n-1}$

$F(n-1) + F(n-2) < 2^{n-1} + 2^{n-2} + 2^{n-2}$

$0 < (2^{n-1} - F(n-1)) + (2^{n-2} - F(n-2)) + 2^{n-2}$

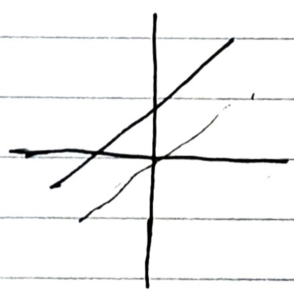
Because $2^{n-1} - F(n-1) > 0$, ~~and~~

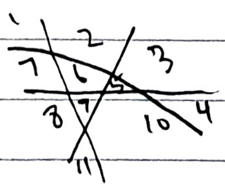
$2^{n-2} - F(n-2) > 0$, ~~and~~

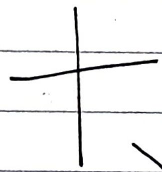
$2^{n-2} > 0$, [illegible] holds true ~~for any real number~~ when $n > 2$, and

addition of ~~no~~ positive real numbers are bounded to be positive,

$\rightarrow F(n) < 2^n$ is true







0	1
1	2
2	4
3	7
4	11