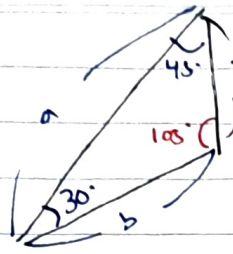
Khan - Law of Sine,



〈 For any triangle 〉

$$\frac{\sin\theta}{\text{len(opposite)}} = \sin$$

$$\frac{\sin 30^\circ}{2} = \frac{\sin 105^\circ}{a} = \frac{\sin 45^\circ}{b}$$

$$\frac{\frac{1}{2}}{2} = \frac{\sin 45^\circ}{b} \qquad \frac{1}{4} = \frac{\sin 105^\circ}{a}$$

$$4\sin 45^\circ = b \qquad 4\sin 105^\circ = a$$

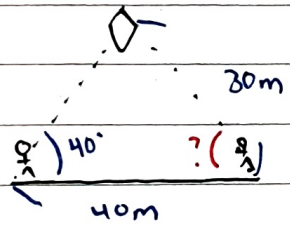
$$b = 4 \cdot \frac{\sqrt{2}}{2} = 2\sqrt{2} \qquad a = 4\sin 75^\circ$$

$$\approx 3.86$$

$a = 3.86$

$b = 2\sqrt{2}$

ex)



$$\frac{\sin 40^\circ}{30m} = \frac{\sin(140 - x)}{40} = \frac{\sin x}{?}$$

$$40 \times \sin 40^\circ = 30A$$

$$A = \frac{4}{3}\sin 40^\circ \qquad A \approx 0.857$$

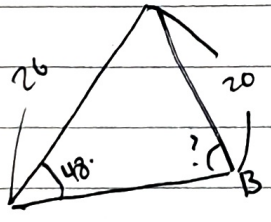
$$140 - x = \sin^{-1}A$$

$$-180 + x = -\sin^{-1}(0.857)$$

$$x = \sin^{-1}(-0.857) + 180^\circ$$

$$\approx -59^\circ + 180^\circ \approx 121^\circ$$

? = 81°

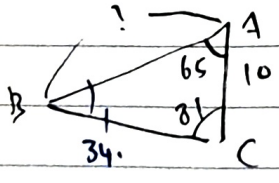


$$\frac{\sin 48^\circ}{20} = \frac{\sin x}{26}$$

$$\sin x = \frac{26 \cdot \sin 48^\circ}{20} = \frac{13}{10} \cdot \sin 48^\circ$$

$$x = \sin^{-1}\left(\frac{13}{10} \cdot \sin 48^\circ\right)$$

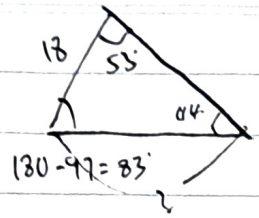
$$x \approx 75.04^\circ$$



$$\frac{\sin 34^\circ}{10} = \frac{\sin 81^\circ}{x}$$

$$x = \frac{\sin 81^\circ \times 10}{\sin 34^\circ}$$

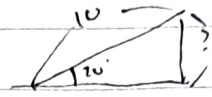
$$x \approx 17.663$$



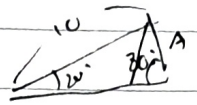
$$\frac{\sin 53^\circ}{x} = \frac{\sin 44^\circ}{18}$$

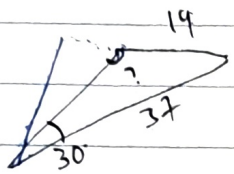
$$x = \frac{\sin 53^\circ}{\sin 44^\circ} \times 18$$

$$x \approx 20.7$$



$\sin 20^\circ \times 10$





$$\frac{\sin x^\circ}{37} = \frac{\sin 30^\circ}{19}$$

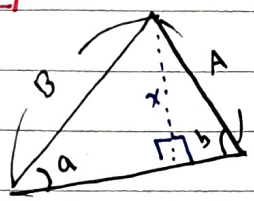
$$\sin x^\circ = \frac{\frac{1}{2}}{19} \cdot 37$$

$$= \frac{1}{38} \cdot 37 = \frac{37}{38}$$

$$x^\circ = -\sin^{-1}\left(\frac{37}{38}\right) + 180^\circ$$

$$\approx 103.2^\circ$$

Proof



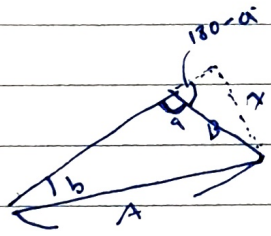
(Arbitary △)

$$x = \sin b^\circ \times A$$

$$x = \sin a^\circ \times B$$

$$\sin b^\circ A = \sin a^\circ B$$

$$\frac{\sin b^\circ}{B} = \frac{\sin a^\circ}{A}$$



$$x = \sin b^\circ A$$

$$x = \sin(180 - a)^\circ B$$

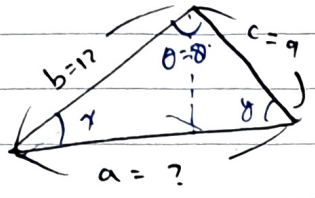
$$= \sin a^\circ B$$

$$\frac{\sin a^\circ}{A} = \frac{\sin b^\circ}{B}$$

Law of Cosine

5 3

$25+9$
$-30(-1)$
$= 34 + 30\sqrt{4}$ $\quad 25+9+30$ $\quad = 34\sqrt{4}$
$= 8.$



$x = 180 - \theta - y$

$x = 93 - y$

or

$y = 93 - x$

$a = \cos x^\circ (12) + \cos(93 - x^\circ)(9)$???

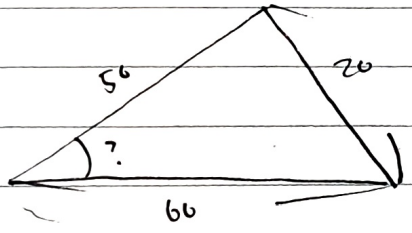
$a^2 = b^2 + c^2 - 2bc\cos\theta$ ← between b & c (opposite of angle of interest)

$= 144 + 81 - 2(12)(9)\cos\theta$

$a^2 = 225 - 216\cos\theta$

$a = \sqrt{225 - 216\cos 83^\circ}$

≈ 14.6



$20^2 = 50^2 + 60^2 - 2(50)(60)\times\cos\theta$

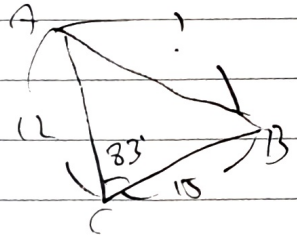
$400 = 2500 + 3600 - 6000\cos\theta$

$6000\cos\theta = 5700$

$\cos\theta = \frac{57}{60} = \frac{19}{20}$

$\theta = \cos^{-1}\left(\frac{19}{20}\right)$

$\approx 18.2^\circ$



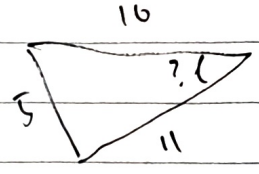
$x^2 = 12^2 + 15^2 - 2(12)(15)\cos 83^\circ$

$x^2 = 144 + 225 - 2(180)\cos 83^\circ$

$x^2 = 369 - 360\cos 83^\circ$

$x = \sqrt{369 - 360\cos 83^\circ}$

$= 18.03$

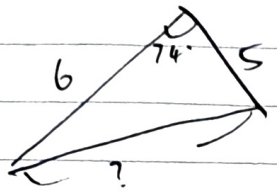


$5^2 = 10^2 + 11^2 - 2(10)(11)\cos\theta$

$2(10)(11)\cos\theta = 10^2 + 11^2 - 5^2$

$\cos\theta = \frac{10^2 + 11^2 - 5^2}{2(10)(11)}$

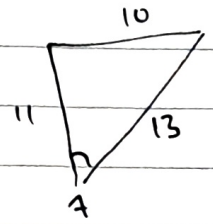
$\theta = 27^\circ$



$x^2 = 6^2 + 5^2 - 2(6)(5)\cos 74°$

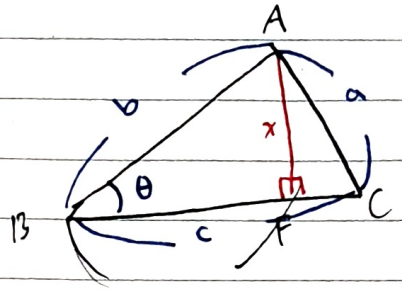
$x = \sqrt{6^2+5^2 - 60\cos 74°}$

≈ 6.668



$\cancel{2(11)(13)}\cos\theta = \dfrac{11^2+13^2-10^2}{2(11)(13)}$

$\theta \approx 48.4°$



$\overline{BF} = b\cdot\cos\theta$

$\overline{CF} = (c - b\cos\theta)$

$\overline{AF} = \sin\theta \times b$

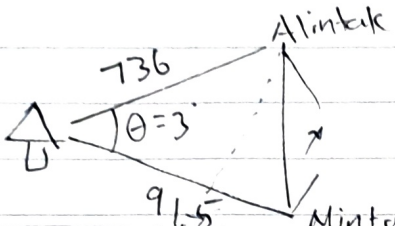
$a^2 = (c - b\cos\theta)^2 + (b\sin\theta)^2$

$= c^2 - 2cb\cos\theta + b^2\cos^2\theta + b^2\sin^2\theta$

$= c^2 - 2bc(\cos\theta) + b^2(\cos^2\theta + \sin^2\theta)$

$a^2 = b^2 + c^2 - 2bc(\cos\theta)$

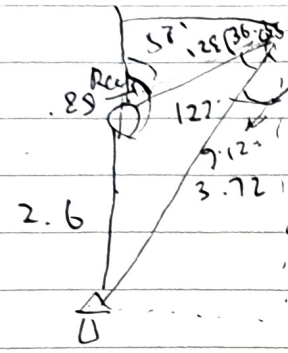
Khan - Law of Sines & Cosines Word Problems



$x^2 = 736^2 + 915^2 - 2(736)(915)\cos\theta$

$x^2 \approx 33886.35$

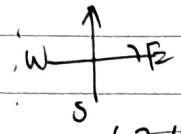
$x \approx 184.08$ light years



$\frac{\sin 122°}{3.72} = \frac{\sin x°}{2.6}$

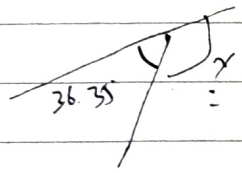
$\sin x° = \sin 122° \cdot \frac{2.6}{3.72}$

$x° = 36.35°$



~~68.4° south of east~~

or 21.6° east of north



$\frac{\sin 28°}{80} = \frac{\sin 83}{x}$

$\frac{\sin 83}{\sin 28°} 80$

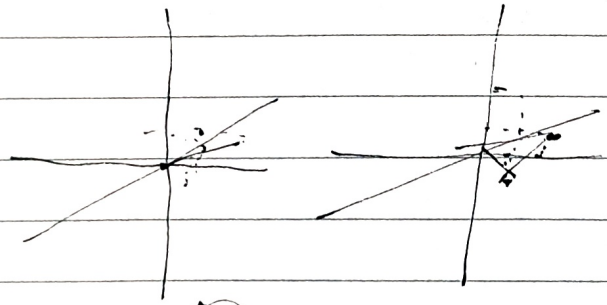
$A = (1,0,1)$ $B = (1,2,-1)$

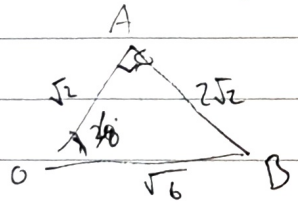
$O = (0,0,0)$ (AI Preview)

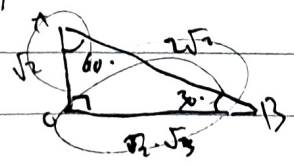
$\overline{OA} = \sqrt{2}$

$\overline{OB} = \sqrt{6}$

$\overline{AB} = 2\sqrt{2}$







$\sqrt{2} \cdot \sqrt{6} = \frac{1}{2}\sqrt{12}$.

$= \frac{1}{2} 2\sqrt{3}$.

$= \boxed{\sqrt{3}}$

$\overrightarrow{OA} = [1,0,1]$ $\overrightarrow{OB} = [1,2,-1]$

$\cos\theta = \frac{\overrightarrow{OA} \cdot \overrightarrow{OB}}{|\overrightarrow{OA}||\overrightarrow{OB}|} = \frac{(1+0-1)}{|OA||OB|} = 0$

$\theta = 90°$

(wk 2)

LH - Cartesian Components of vectors

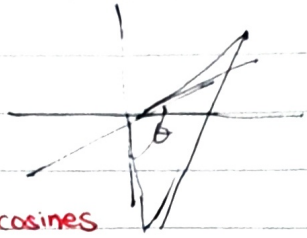
10 (1) & 2)

(a) $v = 2\hat{i} + 4\hat{j} + 3\hat{k}$

$\|v\| = \sqrt{2^2+4^2+3^2} = \sqrt{4+16+9} = \sqrt{29}$

$\cos\theta = \frac{2}{\sqrt{29}}, \frac{4}{\sqrt{29}}, \frac{3}{\sqrt{29}}$

$\theta = 68.2°, 56.1°, 42.0°$



direction cosines

sum of three direction vectors = 1

(b) $5\hat{i} - 2\hat{j} + \hat{k}$

length $= \sqrt{25+4+1} = \sqrt{30}$

$\cos\alpha = \frac{5}{\sqrt{30}}$ $\alpha = 24.1°$

$\frac{-2}{\sqrt{30}}$ $\beta = 111.4°$

$\frac{1}{\sqrt{30}}$ $\gamma = 79.5°$

Q 3 & 4)

(c) $A = (9,3,-2)$ $B = (1,3,4)$

$\vec{AB} = [-8, 0, 6]$

$= -8\hat{i} + 6\hat{k}$

unit vector $= \left[-\frac{4}{5}, 0, \frac{3}{5}\right] = -\frac{4}{5}\hat{i} + \frac{3}{5}\hat{j}$

10 ~~log n~~ = 10

10 ~~$\left[-\frac{4}{5}\hat{i} + \frac{3}{5}\hat{j}\right]$~~