wk 4 pre-tut

Q1) e_1, e_2 : standard vectors in $\mathbb{R}^2$

$[1,0]$ $[0,1]$

Q span(e_1, e_2) ?

span(e_1, e_2) + $s[1,0] + t[0,1]$ $s, t \in \mathbb{R}$

$= [s,t]$ $s,t \in \mathbb{R}$

$= \mathbb{R}^2$

Q2)

i) $\underset{\sim}{u} = [2,-1,3]$, $\underset{\sim}{v} = [-1,2,3]$

$c_1[2,-1,3] + c_2[-1,2,3] = [0,0,0]$

c_1, c_2 ~~one of th~~ can't be both $\emptyset$.

$2c_1 - c_2 = 0$

$-c_1 + 2c_2 = 0$

$3c_1 + 3c_2 = 0$

$$\left[\begin{array}{cc|c} 2 & -1 & 0 \\ -1 & 2 & 0 \\ \cancel{3} & \cancel{3} & 0 \end{array}\right] = \left[\begin{array}{cc|c} 1 & 1 & 0 \\ -1 & 2 & 0 \\ 2 & -1 & 0 \end{array}\right]$$

~~$c_1 + c_2 = 0$ ∴ $c_1 = -c_2$~~ $= \left[\begin{array}{cc|c} 1 & 1 & 0 \\ 1 & -2 & 0 \\ 2 & -1 & 0 \end{array}\right] = \left[\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 3 & 0 \\ 2 & -1 & 0 \end{array}\right]$

$c_2 = 0, c_1 = 0$

c_1, c_2 are both zero

∴ $\underset{\sim}{u}, \underset{\sim}{v}$ are linearly independent

or assume $\underset{\sim}{u} \parallel \underset{\sim}{v}$

then: $[2,-1,3] = k[-1,2,3]$ for some k

$2 = -k$

$-1 = 2k$

$3 = 3k$

$k = 1$

$2 = -1$ contradiction

∴ $\underset{\sim}{u}, \underset{\sim}{v}$ are not parallel

also, $\underset{\sim}{u}, \underset{\sim}{v}$ are non-zero

∴ $\underset{\sim}{u}, \underset{\sim}{v}$ are independent

$|\underset{\sim}{u} \times \underset{\sim}{v}| \neq 0$

$-1 \quad 3 \quad 2 \quad -1$

$2 \quad 3 \quad -1 \quad 2$

~~$[-3-6,$~~

ii) $\underset{\sim}{u} = [1,1]$, $\underset{\sim}{v} = [-1,0]$, $\underset{\sim}{w} = [3,5]$

$5\underset{\sim}{u} + 2\underset{\sim}{v} = [5,5] + [-2,0] = [3,5] = w$

$\therefore \underbrace{\underset{\sim}{w} = 5\underset{\sim}{u} + 2\underset{\sim}{v}}_{\text{dependence relation}}$ ∴ $\underset{\sim}{u}, \underset{\sim}{v}, \underset{\sim}{w}$ are linearly dependent.

Q3) Find normal, general form

$P = (2,3,5)$

$\underset{\sim}{n} = [1,3,-1]$

$[1,3,-1][x-2, y-3, z-5] = 0$

$(x-2) + 3(y-3) - (z-5) = 0$

$x - 2 + 3y - 9 - z + 5 = 0$

$g.f = x + 3y - z = 6$

$n.f = [1,3,-1]\underset{\sim}{x} = 6$

Q4)

$P = (1,2,3)$

$Q = (-1,-2,-3)$

$R = (4,-4,4)$

plane contain P, Q, R.

$\vec{PQ} = [-2,-4,-6]$

$\vec{PR} = [3,-6,1]$

$v.f = \underset{\sim}{x} = (1,2,3) + s[-2,-4,-6] + t[3,-6,1]$

$= \cancel{(1,2,3) + s[1,2,3] + t[3,-6,1]}$

$\vec{PQ} \times \vec{PR} =$

$$\begin{array}{cccc} -4 & -6 & -2 & -4 \\ -6 & 1 & 3 & -6 \end{array}$$

$= -2[1,2,3] \times [3,-6,1]$

$(-4-36) + (-18+2) + (12+12)$

$\cancel{(-40) + (-16) + (24)}$

$\cancel{= -40 + 8 = -32}$

$[-40,-16,24]$

$$\begin{array}{cccc} 2 & 3 & 1 & 2 \\ -6 & 1 & 3 & -6 \end{array}$$

$(2+18) + (9-1) + (-6-6)$

$\cancel{20 + 8 + 12}$ $-2[20,8,-12]$

$\cancel{= 16 \times -2}$ $= [-40,-16,24]$

$\cancel{= \boxed{-32}}$

$$\begin{array}{cccc} 2 & 3 & 1 & 2 \\ -6 & 1 & 3 & -6 \end{array}$$

$[20, 8, -12]$

$= 4[5,2,-3]$

$n.f = [-40,-16,24] \cdot \underset{\sim}{x} = 0$

$5x + 2y - 3z = 0$

Q5

i)

$$\left[\begin{array}{cc|c} 1 & 1 & 6 \\ 2 & -3 & 2 \end{array}\right] = \left[\begin{array}{cc|c} 1 & 1 & 6 \\ 0 & -5 & -10 \end{array}\right] = \left[\begin{array}{cc|c} 1 & 1 & 6 \\ 0 & 1 & 2 \end{array}\right]$$

$$= \left[\begin{array}{cc|c} 1 & 0 & 4 \\ 0 & 1 & 2 \end{array}\right] \quad x = 4,\ y = 2$$

ii)

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 1 & -1 & 1 & 0 \\ 5 & 2 & 0 & 0 \end{array}\right] \leftarrow \text{homogeneous}$$

system of linear expressions are homogeneous

all zeroes

Week 4 tutorial

Q1 Find general form, normal form of the equation for plane containing point P, ~~and~~ normal vector $= \underset{\sim}{n}$.

i) $P = (0,0,0)$, $\underset{\sim}{n} = [3,1,4]$

$\underset{\sim}{n} \times \underset{\sim}{x} = 0$

general form : $3x + y + 4z = 0$

normal ″ $[3,1,4] \cdot \underset{\sim}{x} = 0$

ii) $P = (7,5,-3)$, $\underset{\sim}{n} = [2,0,1]$

$[2,0,1] \cdot [x-7, y-5, z+3] = 0$

$2(x-7) + 1(z+3) = 0$

$2x - 14 + z + 3 = 0$

g.f : $2x + z = 11$

n.f : $[2,0,1] \cdot \underset{\sim}{x} = 11$

iv) $P = (-6,5,6)$ $\underset{\sim}{n} = [0,1,0]$

$[0,1,0] \cdot [x+6, y-5, z-6] = 0$

$(y-5) = 0$

g.f $=$ $\boxed{y = 5}$

n.f $=$ $\boxed{[0,1,0] \cdot \underset{\sim}{x} = 5}$

Q2) Test if $\underset{\sim}{u}, \underset{\sim}{v}$ are linearly independent.
If so, find n.f and p.eq^n.

↳ plane : independent

↳ line : dependent

ii) $P = (5,0,2)$

$\underset{\sim}{u} = [3,0,0]$

$\underset{\sim}{v} = [1,0,3]$

if $\underset{\sim}{u} \parallel \underset{\sim}{v}$ then, $[3,0,0] = k[1,0,3]$ for some $k \in \mathbb{R}$

$3 = k$ → $k = -3$

$0 = 3k$

$0 = -9$ ∴ k doesn't exist

∴ $\underset{\sim}{u}, \underset{\sim}{v}$ are linearly independent.

v.f $= (5,0,2) + s\underset{\sim}{u} + t\underset{\sim}{v}$

$x = 5 + 3s + t$ $z = 2 + 3t$

$y = 0$

iv) $P = (-5,3,6)$ $\underset{\sim}{u} = [1,-2,3]$, $\underset{\sim}{v} = [-3,6,-9]$

$\underset{\sim}{u} = -\frac{1}{3}\underset{\sim}{v}$ $\therefore \underset{\sim}{u} \parallel \underset{\sim}{v}$

$\therefore \underset{\sim}{u}, \underset{\sim}{v}$ are linearly dependent

$\underset{\sim}{x} = (-5,3,6) + t[1,-2,3]$

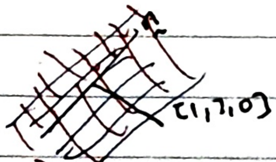
$$\left.\begin{aligned} x &= -5 + t \\ y &= 3 - 2t \\ z &= 6 - 3t \end{aligned}\right\} t \in \mathbb{R}$$

Q3) $P = (2,0,0)$
$Q = (0,1,3)$ $\overrightarrow{PQ} = [-2,1,3]$
$R = (+1,-3,5)$ $\overrightarrow{PR} = [-1,-3,5]$

v.f $= (2,0,0)$ $\overrightarrow{PQ} \times \overrightarrow{PR} = [14, 7, 7]$
$+ [-2,1,3]s + [-1,-3,5]t \mid \overrightarrow{PQ} \times \overrightarrow{PR}) \neq 0$ $\therefore$ linearly independent

n.f $= [2,1,1] \cdot [x-2, y, z] = 0$
$2(x-2) + y + z = 0$
$2x - 4 + y + z = 0$
g.f $= 2x + y + z = \cancel{0}\ 4$
n.f $= [2,1,1] \cdot \underset{\sim}{x} = 4$

Q4) $P = 2x - y + 3z = 5$ – slope $\perp \underset{\sim}{n}$



$\underset{\sim}{n} = [2,-1,3]$

i) $x + 2y = 4$ $\underset{\sim}{n} \cdot [1,2,0] = 2 + (-2) + 0 = 0$
$x + 2y + 0z = 4$ $[1,2,0]$
$\llcorner$ slope $\perp [1,2,0]$ $\underset{\sim}{n} \perp [1,2,0]$
$\therefore$ Slpo

perpendicular

ii) $2x - y + 3z = 0$

parallel

$[2,-1,3]$

iii) $4x - 2y + 6z = 1$

$[2,-1,3]$

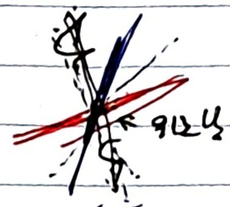
parallel

iv) $[3,1,-1] \cdot \; = (6) + (-1) + (-3) = 2$

neither

Q4) $(x+y+z=2)$, $(x-y+3z=0)$

intersect in a line l.



1

Line intersecting 2 places

$$x + y + z = 2$$
$$x - y + 3z = 0$$
$$2x + 4z = 2$$
$$x + \cancel{2}z = 1$$

$$x + y + z = 2$$
$$-x + y - 3z = 0$$
$$2y - 2z = 2$$
$$y - \cancel{z} = 1$$

is perpendicular to ... plane

i) $P = (1,1,0)$

$d = [1,1,1] \times [1,-1,3]$

$= [4,-2,-2] = 2[2,-1,-1]$

$(1,1,0) + t[2,-1,-1]$

$x = 1 + 2t$

$y = 1 - t$

$z = -t$

Q6) $\underset{\sim}{u} = [1,2,1]$

$\underset{\sim}{v} = [5,2,-4]$

$\underset{\sim}{w} = [-3,2,6]$

i) is w in span of $\underset{\sim}{u}, \underset{\sim}{v}$ 이걸 좀 갈래!?

$\text{span}(\underset{\sim}{u}, \underset{\sim}{v}) = a[1,2,1] + b[5,2,-4] \quad a, b \in \mathbb{R}$

$a + 5b = -3$

$2a + 2b = 2$

$a - 4b = 6$

$a + b = 1$

$a = -b$

$a - 4(-a) = 6$

$a + 4a = 6$

$5a = 6 \quad a = \frac{6}{5} \quad b = -\frac{6}{5}$

$\frac{6}{5} - \frac{30}{5} = -\frac{24}{5}$

$\frac{6}{5} + 5\left(-\frac{6}{5}\right) = -3$

$\frac{6}{5} - 6 = -3$

$\frac{6}{5} = 3$ no solution

$a + b = 1$

$\boxed{a = 1 - b}$ $\boxed{b = 1 - a}$

$a - 4(1-a) = 6$

$a - 4 + 4a = 6$

$\therefore$ No $5a = 10$

$\boxed{a = 2, \; b = -1}$ $\therefore$ Yes w is in span(u, v)

ii) for $\underset{\sim}{w} \parallel \underset{\sim}{x}$ where $\underset{\sim}{x} = [1,3,-2] + s\underset{\sim}{u} + t\underset{\sim}{v}$ to be true,

if $\underset{\sim}{w} \perp \underset{\sim}{u} \times \underset{\sim}{v}$

$\underset{\sim}{u} \times \underset{\sim}{v} =$ 2 1 1 2 / 2 -4 5 2 $\quad [-10, 9, -8]$

$[-3,2,6] \cdot [-10,9,-8]$

$= 30 + 18 - 48 = 0 \quad \therefore \underset{\sim}{w} \parallel \underset{\sim}{x}$

YES also w lies on $\underset{\sim}{x}$

Q7) V.f. for equation of line

$P = (1,4,2)$

perpendicular to $[2,-1,5]$

$[a, b, c] \cdot [2,-1,5]$

$2a - b + 5c = 0$

$[1,2,0]$

or $[0,5,1]$

$V.f = (1,4,2) + [1,2,0]t$

or $(1,4,2) + [0,5,1]t$

perpendicular to $2x - y + 5z = 1$

$P \quad [2,-1,5]\,\underset{\sim}{x} = 1$

t slope of l l

$l \perp P$

$V.f = (1,4,2) + t[2,-1,5] \quad t \in \mathbb{R}$

8) $P = (0,2,3)$

parallel to $4x + y - 3z = 2$

normal form of P in $\mathbb{R}^3$

$P = 4x + y - 3z = d$

$d = 0 + 2 - 3(3) = -7$

$= [4,1,-3] \cdot \underset{\sim}{x} = -7$

Q9) $\underset{\sim}{u} = [1, 0, -1]$
$\underset{\sim}{v} = [0, 1, 0]$
$\underset{\sim}{w} = [2, 0, 2]$

let $c_1 \underset{\sim}{u} + c_2 \underset{\sim}{v} + c_3 \underset{\sim}{w} = \underset{\sim}{0}$

$0 = c_1 + 2c_3$
$0 = c_2$
$0 = -c_1 + 2c_3$

$c_2 = 0$

$$\begin{array}{r} c_1 + 2c_3 = 0 \\ -c_1 + 2c_3 = 0 \\ \hline 4c_3 = 0 \end{array}$$

$\therefore c_3 = 0$

$0 = c_1 + 0$

$\therefore c_1 = 0$

$$\boxed{\begin{array}{c} c_1 = 0 \\ c_2 = 0 \\ c_3 = 0 \end{array}}$$

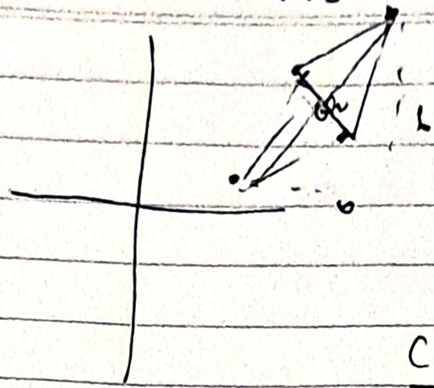
$\therefore$ Linearly Independent

(not parallel)

Week 4 Quiz

$\sqrt{20}$

$[-3, 4, 0]$ $-2, 4$



$\vec{AB} = [4, 1, 0]$

$\vec{AC} = [2, 5, 0]$

$\vec{BD} = [2, 5, 0]$

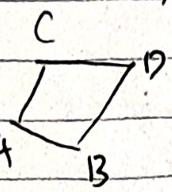
$\frac{\sqrt{20}}{2} = \sqrt{5}$

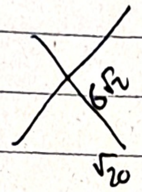
$6\sqrt{2} \times$

$6, 6$

$AC \parallel BD$

$6\sqrt{2} \times \sqrt{5} =$





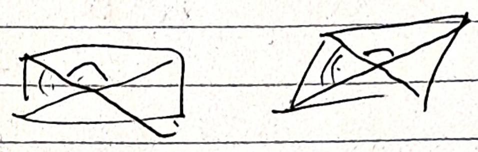
$\| \vec{AB} \times \vec{AC} \| =$

$$\begin{matrix} 1 & 0 & 4 & 1 \\ 5 & 0 & 2 & 5 \end{matrix}$$

$= 18$

$\boxed{= [0, 0, 18]}$

$6\sqrt{2}$



$[1, -3] \cdot [x, y] = 1$

$\hat{n} = [1, -3]$

$[1, -3] \cdot [x-2, y-9] = 0$

$1(x-2) - 3(y-9) = 0$

$x - 2 - 3y + 27 = 0$

$(5, 5, 3) + t[-2, 3, 5]$

$x - 3y + 25 = 0$

$x - 3y = -25$

$$\begin{matrix} \underset{\sim}{u}: & 3 & -2 & 4 & 3 \\ \underset{\sim}{v}: & -2 & 1 & -4 & -2 \end{matrix}$$

$= [3-4, 8-4, -8-12]$

$\underset{\sim}{u} \times \underset{\sim}{v} = [-1, 4, -20]$

$\underset{\sim}{v} \times \underset{\sim}{u} = [1, -4, 20]$

A

$\|\underset{\sim}{v} \times \underset{\sim}{w}\| = \|\underset{\sim}{v}\| \times \|\underset{\sim}{w}\| \times \sin\theta$

$\cos\theta = \dfrac{\underset{\sim}{u} \cdot \underset{\sim}{v}}{\|\underset{\sim}{u}\| \times \|\underset{\sim}{v}\|}$ A

$\|\underset{\sim}{v} \times \underset{\sim}{w}\| = \sqrt{a+1+4} = \sqrt{14}$

$\sqrt{14} = A\sin\theta$

$\cos\theta = \dfrac{3}{A}$

$A\cos\theta = 3$

$(A\sin\theta)^2 + (A\cos\theta)^2 = 14+9$

$A^2\sin^2\theta + A^2\cos\theta = 23$

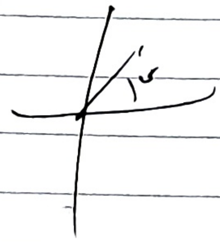
$A^2 = 23$

$A = \cancel{\pm\sqrt{23}}$ $\sqrt{23}$

$\sin\theta = \dfrac{\sqrt{14}}{\sqrt{23}}$

~~sin~~ $\theta = \sin^{-1}\left(\sqrt{\frac{14}{23}}\right)$

$\tan\left(\sin^{-1}\left(\frac{\sqrt{14}}{\sqrt{23}}\right)\right)$



$[-5, -25]$

$= 5[1, 5]$

$[3, \cancel{18}^{15}]$

$= [1, 5]$

$\vec{AB} = [-1, 0, 8]$

$\vec{AC} = [-2, 5, 3]$

$\begin{matrix} 0 & 8 & -1 & 0 \\ 5 & 3 & -2 & 5 \end{matrix}$

$= [-40, -13, -5]$

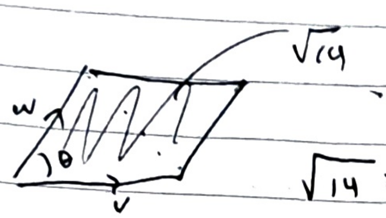
$[\begin{matrix} 3 & -2 & 4 & 3 \\ -2 & 1 & -4 & -2 \end{matrix}$

$[3-4, 8-4, -8-\cancel{4}=12)]$

$= [-1, 4, 4]$

16 $\underset{\sim}{v} \times \underset{\sim}{w} = [-3, 1, 2]$

$\| \underset{\sim}{v} \times \underset{\sim}{w} \| = \sqrt{9+1+4} = \sqrt{14}$



$\sqrt{14} = |\underset{\sim}{u}| \, |\underset{\sim}{v}| \sin\theta$

$\underset{\sim}{v} \cdot \underset{\sim}{w} = 3$

$\dfrac{\underset{\sim}{v} \cdot \underset{\sim}{w} \; 3}{|\underset{\sim}{v}| \times |\underset{\sim}{w}|} = \cos\theta$

$\sqrt{14} = A \sin\theta$

$3 = A\cos\theta$

$A^2 \sin^2\theta + A^2 \cos^2\theta = 14 + 9$

$A^2 = 23$

$\boxed{A = \sqrt{23}}$

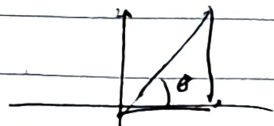
$\sin\theta = \dfrac{\sqrt{14}}{A}$

$\cos\theta = \dfrac{3}{A}$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{\sqrt{14}}{3}$

$\sin\theta = \dfrac{\sqrt{14}}{\sqrt{23}}$

$\cos\theta = \dfrac{3}{\sqrt{23}}$



$\dfrac{\dfrac{\sqrt{14}}{\cancel{\sqrt{23}}}}{\dfrac{3}{\cancel{\sqrt{23}}}} = \dfrac{\sqrt{14}}{3}$

$\sqrt{\dfrac{14}{23}}$