Wk 2 Tutorial

Find the modulos & Principal Argument

①

(a) $z = i$ $\quad |z| = 1 \quad Arg(z) = \frac{\pi}{2}$

(b) $z = 1 + i$

$|z| = \sqrt{2}, \quad Arg(z) = \frac{\pi}{4}$

(c) $z = 1 - i$

$|z| = \sqrt{2}, \quad Arg(z) = -\frac{\pi}{4}$

(d) $z = -6i$

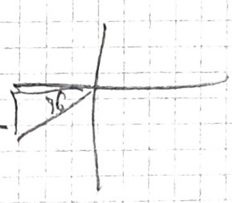
$|z| = 6 \quad Arg(z) = \tan^{-1}\left(\frac{-6}{0}\right) = -\frac{\pi}{2}$

(e) $z = 1 + \sqrt{3}i$

$|z| = 2, \quad Arg(z) = \frac{\pi}{3}$

(f) $z = -\sqrt{3} - i$

$|z| = 2, \quad Arg(z) = -\frac{5\pi}{6}$



2) Standard Polar form

(a) $i = (\cos\frac{\pi}{2} + i\sin\frac{\pi}{2})$

(b) $1+i = \sqrt{2}(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4})$

(c) $1-i = \sqrt{2}(\cos(-\frac{\pi}{4}) + i\sin(-\frac{\pi}{4}))$

③ $z = \sqrt{3} + i = 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

$w = 1 + \sqrt{3}i = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

$\underline{z \times w} = (\sqrt{3}+i)(1+\sqrt{3}i)$

$= \sqrt{3} + i + 3i - \sqrt{3}$

$= 4i$

$= 4\left(\cos\left(\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{2}\right)\right)$

$\cos\frac{\pi}{6} \times \cos\frac{\pi}{3} =$

Review Trigonometry Angle Addition Properties

$z/w = \frac{\sqrt{3}+i}{1+\sqrt{3}i} \times \frac{1-\sqrt{3}i}{1-\sqrt{3}i}$

$= \frac{\sqrt{3}+i-3i+\sqrt{3}}{1+3} = \frac{2\sqrt{3}-2i}{4} = \frac{\sqrt{3}}{2} - \frac{1}{2}i$

$= \cos\left(-\frac{\pi}{6}\right) + i\sin\left(-\frac{\pi}{6}\right)$

$\frac{1}{z} = \frac{1}{\sqrt{3}+i} \times \frac{(\sqrt{3} \times -i)}{(\sqrt{3}-i)}$

$= \frac{\sqrt{3}-i}{3+1} = \frac{\sqrt{3}}{4} - \frac{i}{4}$

mod: $\sqrt{\frac{3}{16} + \frac{1}{16}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$

$\frac{1}{2}\left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right)$

$= \frac{1}{2}\left[\cos\left(-\frac{\pi}{6}\right) + i\sin\left(-\frac{\pi}{6}\right)\right]$

④ $(1-i)^{24}$

$z = 1-i$

$|z| = \sqrt{2}$ $Arg(z) = -\frac{\pi}{4}$

$z = \sqrt{2}\, e^{i(-\frac{\pi}{4})}$

$z^{24} = \sqrt{2}^{24} e^{i(\frac{-24\pi}{4})}$

$= 2^{12} e^{i \cancel{6\pi}\, 0} = \boxed{2^{12}} = \boxed{4096}$

$z = (3\sqrt{3} + 3i)^3$

$|z| = \sqrt{27+9} = \sqrt{36} = 6$

$z = 6\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)$

$Arg(z) = \frac{\pi}{6}$

$z = 6 \times e^{i\frac{\pi}{6}}$

$z^3 = 6^3 \times e^{i\frac{3\pi}{6}\,\frac{\pi}{2}}$

$= 6^3 \times (\cancel{\cos\frac{\pi}{2}} + \cancel{i\sin\frac{\pi}{2}})$ 0 1

$= 6^3 \times i = \boxed{216i}$ $\frac{3}{12}$ $\frac{5}{-12}$

Division 에서 polar exp 로 있기 없고 사용하라!

⑤

e) $\frac{\sqrt{3}-i}{\sqrt{3}+i} \times \frac{(\sqrt{3}+i)}{(\sqrt{3}+i)}$

$= \frac{\sqrt{3}-\sqrt{3}i+i+1}{3+1} = \frac{(1+\sqrt{3})+(1-\sqrt{3})i}{4}$

$|z| = \sqrt{\frac{4}{16}}$

$= \frac{1}{2}$

$\alpha^2 + 2\alpha\beta + \beta^2$

$= (1+2\sqrt{3}+3) + 2(1-3) + (1-2\sqrt{3}+3)$

$= (4+\cancel{2\sqrt{3}}) + (-4) + (4-\cancel{2\sqrt{3}})$

$= 4$

~~$1-i = \sqrt{2}(\cos -\frac{\pi}{4} + i\sin -\frac{\pi}{4})$~~

~~$\sqrt{3}-i = 2(\cos -\frac{\pi}{6} + i\sin -\frac{\pi}{6})$~~

~~$\frac{1}{\sqrt{2}}\left(\cos -\frac{5\pi}{12} + i\sin -\frac{5\pi}{12}\right)$~~

$\frac{\sqrt{2}\, e^{i(-\frac{\pi}{4})}}{2\, e^{i(-\frac{\pi}{6})}}$

$2\left[\frac{1+\sqrt{3}}{2} + i\frac{1+\sqrt{3}}{2}\right]$???

$\frac{1}{2} + \frac{\sqrt{3}}{2}$ $60°$

$-\frac{1}{2} + -\frac{\sqrt{3}}{2}$ $-60°$

$\frac{\sqrt{3}}{2} - \frac{1}{2}$ $-30°$

~~+30~~

$= \frac{1}{\sqrt{2}} e^{i -\frac{\pi}{4} + i -\frac{\pi}{6}}$

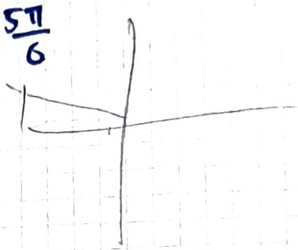
$= \frac{1}{\sqrt{2}} e^{i\frac{5\pi}{6}}$

(f) $(\sqrt{3}-i)^{19}$

$\sqrt{3}-i = 2\left[\cos\frac{-\pi}{6} + i\sin\frac{-\pi}{6}\right]$ Polar exp form 으로 나타내야지!!!

$= 2e^{i(-\frac{\pi}{6})}$

$z^{19} = 2^{19}e^{i(-\frac{\cancel{19}\,17}{6}\pi)}$ $\frac{5}{6}\pi$ $= 2^{19}e^{i\frac{5\pi}{6}}$



$= 2^{19}\left(\cos\frac{5}{6}\pi + i\sin\frac{5\pi}{6}\right)$

$= 2^{19}\left(-\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)$

(6) $z^5 = 1$ $\quad 1 = e^{i0}$

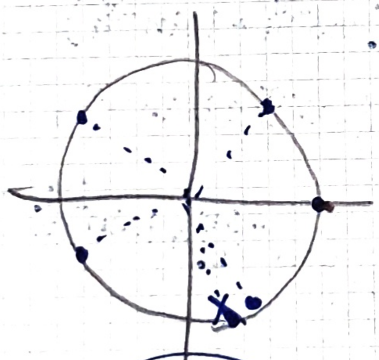
$z^5 = 1^{\frac{1}{5}}e^{i\left(\frac{0+2\pi k}{n}\right)}$

↳ $0, \frac{2\pi}{5}, \frac{4}{5}\pi, \frac{6}{5}\pi, \frac{8}{5}\pi$

$= z = e^{i\theta}, \theta \in \{0, \frac{2\pi}{5}, \frac{4\pi}{5}, \cancel{\frac{6}{5}\pi, \frac{8}{5}\pi}\}$

not p.18

$-\frac{4\pi}{5}, -\frac{2\pi}{5}$



$\frac{-\frac{\pi}{3}+2\pi k}{3}$

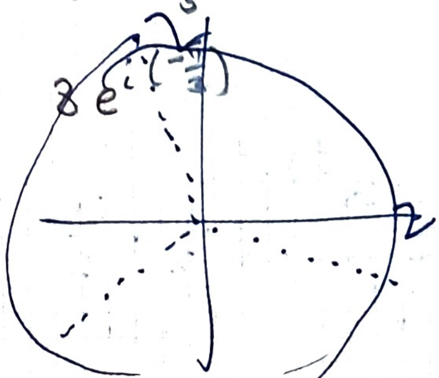
(6) $z^3 = \underline{4-4\sqrt{3}i}$ $\quad |w| = \sqrt{16+16\times3} = \sqrt{64} = 8$

w $\quad w = 8\left(\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)$

$\frac{\frac{\pi}{3}+6\pi k}{3}$ $\quad \mathrm{Arg}(w) = -\frac{\pi}{3}$

$z = 8^{\frac{1}{3}}e^{i\left(\frac{\frac{\pi}{3}+2\pi k}{n}\right)}$ $\quad w = 8e^{i(-\frac{\pi}{3})}$

↳ $\frac{\pi}{9}, \frac{\cancel{9}\,7\pi}{9}, \frac{\cancel{12}\,11}{9}, \cancel{\frac{19\pi}{9}}$



$= 2e$ $\quad -\frac{7\pi}{9}, -\frac{\pi}{9}, \frac{5\pi}{9}$

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$z^3 + i = 0$

$z^3 = -i$ (w)

$|w| = 1$

$\mathrm{Arg}(w) = -\frac{\pi}{2}$

$w = e^{i(-\frac{\pi}{2})}$

$-\frac{1}{2} - \frac{4}{2}$

$$z = 1^{\frac{1}{3}} e^{i\left(\frac{-\frac{\pi}{2} + 2\pi k}{3}\right)}$$

$\to -\frac{5\pi}{6}, -\frac{\pi}{6}, \frac{1}{2}\pi$

$= 1($

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~~3th roots of z~~

3th roots of $-i$

or

$z = \cos(-\frac{5}{6}\pi) + i\sin(-\frac{5}{6}\pi)$,

$\cos(-\frac{1}{6}\pi) + i\sin(-\frac{1}{6}\pi)$,

$\cos(\frac{1}{2}\pi) + i\sin(\frac{1}{2}\pi)$

$\cos\theta + i\sin\theta$

where

$\theta \in \{\frac{\pi}{2}, -\frac{\pi}{2}, 0, \frac{2\pi}{3}, -\frac{2\pi}{3}\}$

(b) $z^5 + z^3 - z^2 - 1 = 0 = p(z)$

$\boxed{z = i}$

$i^5 + i^3 - i^2 - 1 = 0$

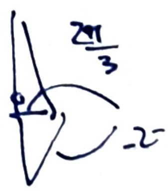
$= i - i + 1 - 1 = 0$

$i, \boxed{z = -i}$

$$\begin{array}{r} z^3 - 1 \\ z^2 + 1 \overline{)\, z^5 + z^3 - z^2 - 1} \\ \underline{-z^5 - z^3} \\ -z^2 - 1 \\ \underline{+z^2 + 1} \\ 0 \end{array}$$

then $p(z) = (z - i)(z + i)\,q(z)$

$= (z^2 + 1)\,q(z)$



$z^2 + z + 1 = 0$

~~$z(z+1) + 1 = 0$~~

$z = \frac{-1 \pm \sqrt{1 - 4}}{2} = \frac{-1 \pm \sqrt{3}i}{2}$

$q(z) = z^3 - 1$

$= (z - 1)(z^2 + z + 1)$

$\boxed{z = 1}$ $\boxed{z = -\frac{1}{2} + \frac{\sqrt{3}}{2}i}$ $\boxed{z = -\frac{1}{2} - \frac{\sqrt{3}}{2}i}$

(8) ~~Find~~ $e^z = \underset{r}{e^x}\, \underset{\theta}{e^{iy}}$ $z = x + yi$

$\cancel{z = 2i}$

$|z| = 2$

$Arg(z) = \frac{\pi}{2}$

$\cancel{e^z = 2 \times e^{i\frac{\pi}{2}}}$

$\cancel{= 2\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)}$

$= \boxed{\cancel{2i}}$

$e^z = e^x e^{iy}$

$e^z = e$

$z = 2i = 0 + 2i$

$e^0 \times e^{i(2i)}$

$= e^{-2} =$

$e^0 \times e^{2i} = \cos 2 + i\sin 2 =$

$= \cancel{1 + 0.035i}$ $-0.42 + 0.91i$

$z = 4 + \pi i$

$\cancel{|z| = \sqrt{16+\pi^2}}$

$\cancel{\sqrt{16+\pi^2}\left(\cos\frac{1}{\theta} + i\sin\frac{1}{\theta}\right)}$

$\sqrt{16+\pi^2}\; e^{i}$

$\cancel{Arg(z) = \tan^{-1}\left(\frac{\pi}{4}\right)}$

$\cancel{= 1}$

e^z when $z = \underset{x}{4} + \underset{y}{\pi} i$

$e^x e^{yi} = e^4 e^{\pi i}$ $\cos(\pi) + i\sin(\pi)$
-1 $\qquad 0$

$= e^4 \times -1$

$= -e^4$

a) $e^{z} = \sqrt{3} - i$

$\cancel{e^{x} e^{yi}} = 2\left[\cos -\frac{\pi}{6} + i\sin -\frac{\pi}{6}\right]$

$e^{x+yi} = 2e^{i(-\frac{\pi}{6})}$

$e^{x} + e^{yi} = e^{\ln(2)} e^{-\frac{\pi}{6}i}$

$x = \ln(2)$

$y = -\frac{\pi}{6}$

$$\boxed{z = \ln(2) + i\left(-\frac{\pi}{6} + 2\pi k\right),\ k \in \mathbb{Z}}$$