Week 2 Review

Linear combination

$v = \{ \cancel{c_1}\ v = c_1v_1 + c_2v_2 \cdots c_nv_n$ with $c_i \in \mathbb{R} \}$

v is linear comb of $v_1, v_2 \cdots v_n$ if

(some arbitary coefficient $\times$ $v_1 \cdots v_n$)'s sum equals to v.

↳ use system of equations to prove

- linear combination : ~~infinite sol~~ one sol
- not a linear combination : no sol
- $\vdots$ same line (v and v_1 and v_2)

ex) $\begin{bmatrix} 1 \\ 3 \end{bmatrix} = a\begin{bmatrix} 2 \\ 3 \end{bmatrix} + b\begin{bmatrix} -1 \\ -1 \end{bmatrix}$

$\begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 2a - b \\ 3a - b \end{bmatrix}$

$: \left[\begin{array}{cc|c} 3a & -b & 3 \\ 2a & -b & 1 \end{array}\right]$

$= \left[\begin{array}{cc|c} a & 0 & 2 \\ 2a & -b & 1 \end{array}\right]$

$= \left[\begin{array}{cc|c} a & 0 & 2 \\ 0 & -b & -3 \end{array}\right]$

$= \left[\begin{array}{cc|c} a & 0 & 2 \\ 0 & b & 3 \end{array}\right]$ $\boxed{\begin{array}{c} a = 2 \\ b = 3 \end{array}}$

$\therefore$ linear comb!

Dot Product

$v = [v_1, v_2, v_3]$

$w = [w_1, w_2, w_3]$

$v \cdot w = (v_1w_1) + (v_2w_2) + (v_3w_3)$

vector $\times$ vector $\to$ scalar

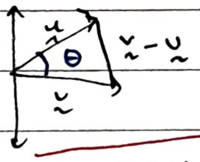
Identities

- $cu \cdot v = c(u \cdot v)$
- $a \cdot (b + c) = (a \cdot b) + (a \cdot c)$
- $u \cdot u \geq 0$ as $\sqrt{u \cdot u}$ is $|u|$ and $|u| \geq 0$

  $u \cdot u = \|u\|^2$ length of vectors over $\mathbb{R}^n$
- if $u \cdot v = 0$

  u and v are orthogonal..

$$\boxed{u \cdot v = \|u\|\|v\| \cos\theta}$$



- ~~[illegible]~~

  $\|c \times v\| = |c| \times \|v\|$

  ex) $\|-3v\| = 3\|v\|$

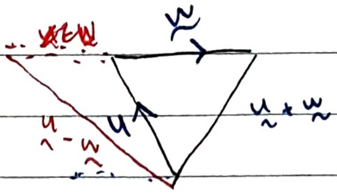
ref

$$\boxed{\operatorname{Proj}_{(u)}(v) = \frac{u \cdot v}{\|u\|^2}\, u}$$

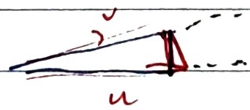
- (triangle inequality)

  $\|u + w\| \leq \|u\| + \|w\|$

  $\|u - w\| \geq \|u\| - \|w\|$

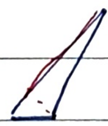


Projection



one side $\leq$ sum of other two sides

side 1 $\geq$ side 2 − side 3



$\frac{|v| \times \cos\theta \times |u|}{|u|} \times \frac{u}{|u|}$

$= \frac{u \cdot v}{\|u\|^2} \times v$

Week 2 Tut Review

Q(0)

$[2,3]$ is $\perp$ $[a+1, a-1]$

i) $[2,3] \cdot [a+1, a-1] = 0$

$2(a+1) + 3(a-1) = 0$

$2a + 2 + 3a - 3 = 0$

$5a - 1 = 0$

$\boxed{a = \frac{1}{5}}$

ii) $\|[3, \beta, 3\beta]\| = \|[12, 0, -5]\|$

$\|[3, \beta, 3\beta\| = 13$

$3^2 + \beta^2 + 9\beta^2 = 169$

$10\beta^2 = 160$

$\beta^2 = 16 \quad \boxed{\beta = \pm 4}$

Q(2) If $\underset{\sim}{a} \perp \underset{\sim}{b} \Rightarrow \|\underset{\sim}{a} + \underset{\sim}{b}\|^2 = \|\underset{\sim}{a}\|^2 + \|\underset{\sim}{b}\|^2$

$\boxed{\text{if } \underset{\sim}{a} \perp \underset{\sim}{b}}$

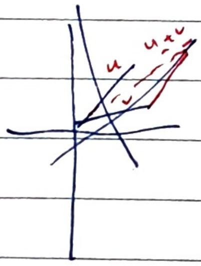
then $\underset{\sim}{a} \cdot \underset{\sim}{b} = 0$

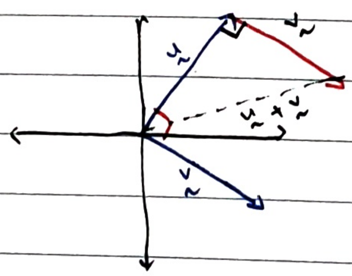
$\|\underset{\sim}{a} + \underset{\sim}{b}\|^2 = (a+b) \cdot (a+b)$

$= (a \cdot a) + 2(\cancelto{0}{a \cdot b}) + (b \cdot b)$

$= \|a\|^2 + \|b\|^2$

$\therefore$ If angle between $\underset{\sim}{a}, \underset{\sim}{b}$ is $90°$





pythagorean theorem