Lec1-1 Note

$\in$: element of

$\subseteq$: subset of

$\supseteq$: contains

inclusive

$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$

$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$

$A = \{1,2,3\}$

$B = \{1,2,a,b\}$

$C = \{3,4\}$

$A \cup B = \{1,2,3,a,b\}$

$A \cap B = \{1,2\}$

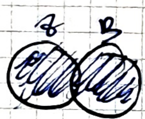
$B \cap C = \emptyset = \{\}$

* Think of it as minus

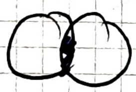
$A \setminus B = \{x \mid x \in A \text{ and } x \notin B\}$

= "complement / but not"

$A \cup B$



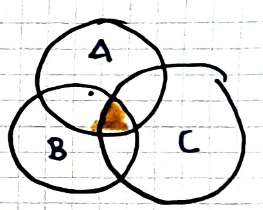
$A \cap B$



$A \setminus B$

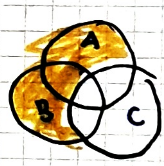
$B \setminus A$

$A \cap B \cap C$

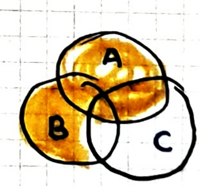


$(A \cup B) \setminus C$

$= A \setminus C \cup B \setminus C$



$A \cup (B \setminus C)$



exercise

Q. $(A \cup B \cup C) \setminus (A \cap B)$

$C \setminus (A \cap B)$

3) $(A \cup B) \setminus (A \cap B) \cup (C \setminus A \cap C \setminus B)$

$A \cup B$

(a

$(A \setminus B) \cup (B \setminus A) \cup (C \setminus A \setminus B)$

Lec Wk 1 - 2

Proof of $\sqrt{2}$ being irrational

m, n does not have common factor

Assume $\sqrt{2} = \frac{m}{n}$, $(m, n \in \mathbb{N}, n \neq 0)$

then $2 = \frac{m^2}{n^2}$

$2n^2 = m^2$

$\therefore$ m has a factor of 2

$\therefore$ m is an even number.

$\therefore$ m can be rewritten as

$m = \sqrt{2} \times \sqrt{2} \times q \quad (q \in \mathbb{N})$

$\sqrt{2} = \frac{\sqrt{2} \cdot \sqrt{2} \cdot q}{n}$

$\cancel{\sqrt{2}}n = \cancel{\sqrt{2}} \cdot \sqrt{2} \cdot q$

$n = \sqrt{2}q, \ q \in \mathbb{Z})$

$n^2 = 2q^2$

$\therefore$ n is also an even number.

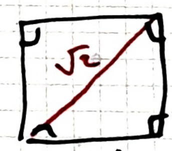
If m, n are both even integers they have common factor of 2.

$\therefore$ contradiction occurs

Therefore, $\sqrt{2}$ cannot be expressed in terms of $\frac{m}{n}$ wher $m, n \in \mathbb{N}$, $n \neq 0$, and m, n has no c.f.

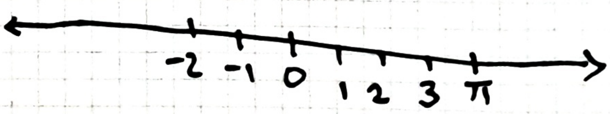
$\therefore \sqrt{2}$ is irrational

(unit square)



Real numbers

$\mathbb{N} \subset \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$



is ordered

for $a, b \in \mathbb{R}$ $\begin{cases} a > b & \text{or} \\ a < b & \text{or} \\ a = b \end{cases}$

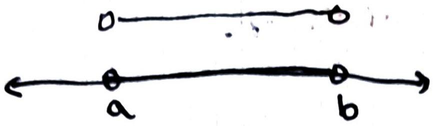
Absolute value (modulus)

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

Heart attack ♥

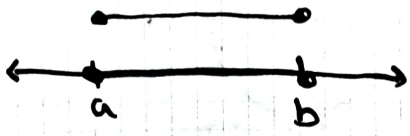
Interval Notation

$(a, b) =$



$= \{x \mid x \in \mathbb{R}, a < x < b\}$

$[a, b] =$



$= \{x \mid x \in \mathbb{R}, a \leq x \leq b\}$

$(1, \infty) =$

∞ x number but a state

modulus properties

1) $|zw| = |z| \times |w|$
2) $\left|\frac{z}{w}\right| = \frac{|z|}{|w|}$
3) $|z+w| \leq |z| + |w|$
4) $|z-w| \geq |z| - |w|$

Is $\mathbb{R}$ enough?

$x^2 + 1 = 0$ cannot solve over $\mathbb{R}$

* In comes complex number !

$i^2 + 1 = 0$

$i^2 = -1$

$i = \sqrt{-1}$

imaginary unit

$a + \beta i$

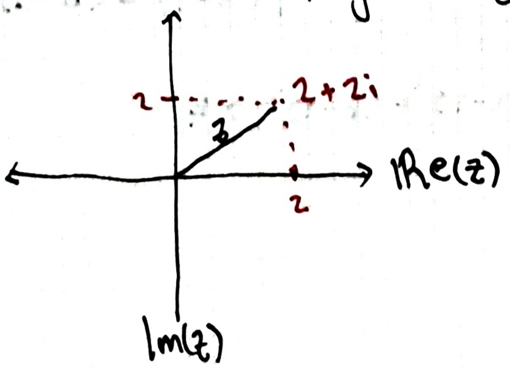
$\mathbb{C} = \{a + bi, a \in \mathbb{R}, b \in \mathbb{R}\}$

* Every polynomial with complex coefficients has a root in $\boxed{\mathbb{C}}$.

Addition, Substraction, Multiplication, Division of complex numbers.

Complex Conjugates

Argand Diagram



modulus or $\bar{z} = \sqrt{a^2 + b^2}$

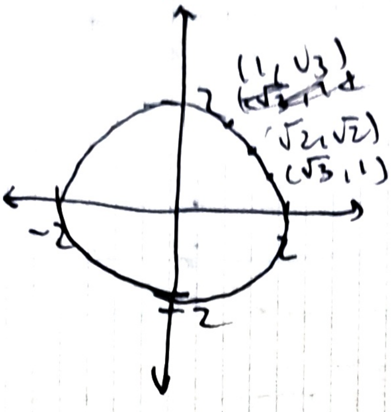
$\{z \in \mathbb{C} \mid |z| = 2\}$

$a + bi$ distance to origin $= 2$

$|z| = \sqrt{a^2 + b^2}$

$|z|^2 = a^2 + b^2 = 4$

equation of a circle



2. $\{z \in \mathbb{C} \mid |z-3| \leq 1\}$

$a+bi$

$|z-3|^2 = (a-3)^2 + b^2 \leq 1$

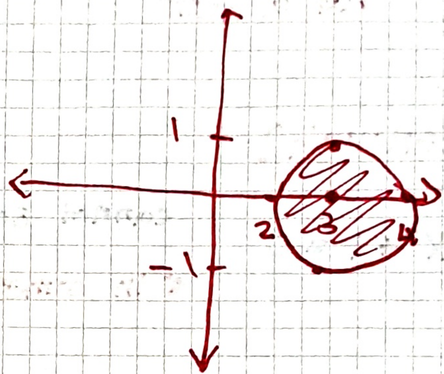
$|z-3|$

$a+bi$

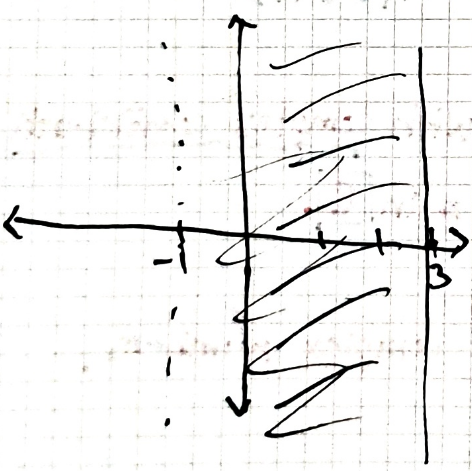
$|z| = \sqrt{(a-3)^2 + b^2}$

$|z| = \sqrt{a^2 - 6a + 9 + b^2}$

$|z|^2 = (a-3)^2 + b^2$



3. $\{z \in \mathbb{C} \mid -1 < Re(z) \leq 3\}$



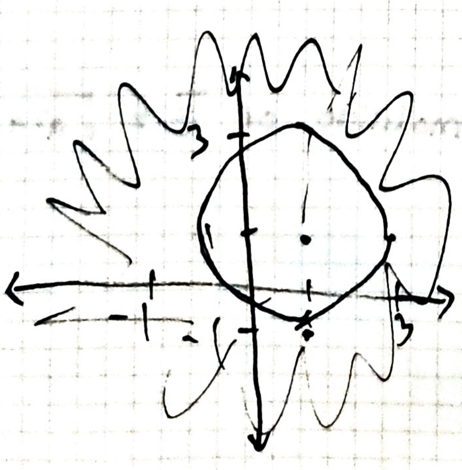
4) $\{z \in \mathbb{C} \mid |z - (1+i)| > 2\}$

$z = a + bi$

$\therefore$

$|z - (1+i)| = |(a-1) + (b-1)i|$

$= \sqrt{(a-1)^2 + (b-1)^2}$



$\therefore (a-1)^2 + (b-1)^2 > 4$