Wk 2 - preTut

1) $a = [2,-1,2]$ $\quad b = [1,1,-1]$ $\quad c = [3,0,-4]$ 1)

a) $a \cdot b = (2) + (-1) + (-2) = -1$

b) $c \cdot c = 9 + 16 = 25$

c) $a \cdot (b + c) = -1 + (a \cdot c) = -1 + (6-8) = -3$

d) $\|b\| = \sqrt{3}$

e) $\|-2b\| = \|[-2,-2,2]\|$

$= \sqrt{12} = 2\sqrt{3}$

f) $c = 5\boxed{\left[\frac{3}{5}, 0, -\frac{4}{5}\right]}$

2) $v = [2,-1,1]$ $\quad w = [1,-2,-1]$

$\cos\theta = \dfrac{v \cdot w}{\|v\| \cdot \|w\|}$ $\quad v \cdot w = (2) + (2) + (-1) = 3$

$\|v\| = \sqrt{6}$

$\cos\theta = \dfrac{3}{2\sqrt{6}} = \cancel{\dfrac{3\sqrt{6}}{12}} = \dfrac{\sqrt{6}}{4}$ $\quad \|w\| = \sqrt{6}$ $\quad \dfrac{3}{2\sqrt{6}} =$

$\cancel{\cos\theta = \dfrac{\sqrt{2}\sqrt{3}}{4} = \dfrac{\sqrt{2}}{2}\dfrac{\sqrt{3}}{2}} = \dfrac{3\sqrt{6}}{12} = \dfrac{\sqrt{6}}{4}$

곱하기는 못하면 어떡해!!

$\cos\theta = \dfrac{3}{\sqrt{6}\cdot\sqrt{6}} = \dfrac{3}{6} = \dfrac{1}{2}$

$\theta = \dfrac{\pi}{3}$

b) $\mathrm{proj}_v w$ $\quad \dfrac{3}{6}[1,-2,-1] = \left[\frac{1}{2}, -1, -\frac{1}{2}\right]$

Wk 2 - tutorial

1) $A = (2,1)$ $B = (3,-1)$ $C = (0,5)$ $D = (-2,-2)$

$\vec{AB} = [1, -2]$ $\vec{AB} \cdot \vec{CD} = (-2) + (14) = \boxed{12}$

$\vec{CD} = [-2, -7]$

2) $\underset{\sim}{a} = [3,1]$ $\underset{\sim}{b} = \left[\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right]$ $c = [-1, 2]$

a) $a \cdot b = \frac{3}{\sqrt{2}} + \left(-\frac{1}{\sqrt{2}}\right) = \frac{2}{\sqrt{2}} = \boxed{\sqrt{2}}$

$a \cdot c = (-3) + (2) = \boxed{-1}$

b) $\|\underset{\sim}{a}\| = \sqrt{10}$

$\|b\| = 1$

$\|c\| = \sqrt{5}$

c) $\underset{\sim}{a} = \sqrt{10}\left(\frac{3}{\sqrt{10}}, \frac{1}{\sqrt{10}}\right)$ — unit vector $c = \sqrt{5}\left[-\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right]$

$\underset{\sim}{b} = \left[\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right] = \boxed{b}$ * 그냥 b라고 써도 되었을 듯

d) proj of a onto b

$\frac{a \cdot b}{\|b\|^2} \cancel{a} = \frac{\sqrt{2}}{1\cancel{\sqrt{10}}} \left[\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right] \times \cancel{[3,1]} = \frac{\cancel{\sqrt{20}}\ 2\sqrt{5}}{10}[3,1]$

$\cancel{= \frac{\sqrt{5}}{5}[3,1] = \left[\frac{3\sqrt{5}}{5}, \frac{\sqrt{5}}{5}\right]}$

$\sqrt{2}\left[\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right] = [1, -1]$

e) $\frac{-1}{\sqrt{10}}$ [$\frac{c \cdot a}{\|a\|^2}\tilde{a} = \frac{-1}{10}[3,1] = \cancel{\left[\frac{-3\sqrt{5}}{10}, \frac{\sqrt{5}}{10}\right]}$

$= \left[-\frac{3}{10}, -\frac{1}{10}\right]$

3) $u = [1,2,2]$ $v = [-4,4,1]$

i) $u \cdot v = (-4) + (8) + (2) = 6$

ii) $\dfrac{u \cdot v}{\|u\| \cdot \|v\|} = \dfrac{6}{3 \cdot \sqrt{33}} = \dfrac{2}{\sqrt{33}}$

iii) proj of u to v $\dfrac{6}{\sqrt{33}}[-4,4,1]$

proj(u) $= \left[\dfrac{-24}{\sqrt{33}}, \dfrac{24}{\sqrt{33}}, \dfrac{6}{\sqrt{33}}\right]$

4) $v = [2,-6,9,0]$ $w = [4,0,2,-4]$

i) $\underset{\sim}{v} \cdot \underset{\sim}{w} = (8) + 0 + (18) + (0)$
$= \boxed{26}$

ii) $|\underset{\sim}{v}| = \sqrt{4+36+81} = \sqrt{121} = 11$

unit vector in direction of $\underset{\sim}{v}$ = $\boxed{\left[\frac{2}{11}, -\frac{6}{11}, \frac{9}{11}, 0\right]}$

$|\underset{\sim}{w}| = \sqrt{16+4+16} = \sqrt{36} = 6$

unit vector in direction of $\underset{\sim}{w}$ = $\boxed{\left[\frac{2}{3}, 0, \frac{1}{3}, -\frac{2}{3}\right]}$

iii) $\|\underset{\sim}{v} + \underset{\sim}{w}\|$

$= \|[6,-6,11,-4]\|$

$= \sqrt{36+36+121+16}$

$= \sqrt{72+137}$

$= \boxed{\sqrt{209}}$

$= \sqrt{11 \times 19}$ can't be further simplified

$110 + 99$
$= 11 \times 19$

5) $u \cdot (v \cdot w)$ → $v \cdot w$ results in scalar

dot product can only be done between two vectors, not between vector and a scalar. we can't take dot product of vect & scalar.

$c \cdot (v+u)$

$(u \cdot v) + w$: scalar can't be added to vector.

vw : multiplication between vectors have not been defined. ✓

6) Assume $\|a\| = \|b\|$

then, $\|a\|^2 = \|b\|^2$ (both sides are positive ~~so~~ due to $\|A\|$ and $\|b\|$ being length

then, $(a \cdot a) = (b \cdot b)$

Assume $v = a + b$ $\quad w = a - b$

$$\cos(\theta) = \frac{(v \cdot w)}{\|v\|\,\|w\|}$$

angle between v, w

If $v \cdot w = 0$, $\theta = 90°$

$$\begin{aligned}
v \cdot w &= (a+b)\cdot(a-b) \\
&= (a)\cdot(a-b) + (b)\cdot(a-b) && \text{distributivity} \\
&= (a\cdot a) - (a \cdot b) + (b \cdot a) - (b - b) && \text{distributivity} \\
&= (a \cdot a) - (b \cdot b) - \cancel{(a \cdot b) + (a \cdot b)} && \text{commutativity} \\
&= (a \cdot a) - (b \cdot b) \\
&= A - A \quad (A = (a \cdot a) = (b \cdot b)) \\
&= 0
\end{aligned}$$

$\therefore \theta = 90°$

$\therefore (a+b)$ and $(a-b)$ are ~~orthognal~~ orthogonal

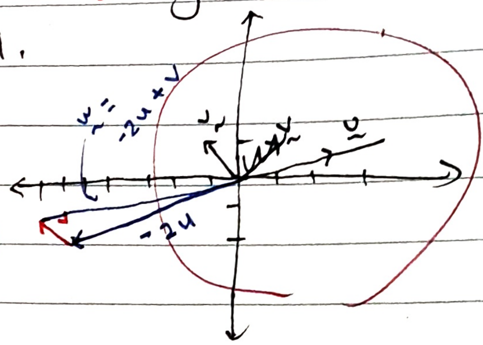
~~angle betw~~ when $\|a\| = \|b\|$.

7) $u = [3, 1]$

$v = [-1, 1]$

$w = [-7, -1]$

How about using c_1, c_2 instead of a, b



w is linear combination of u and v if:

$au + bv = w$ ~~applies~~ holds true for any a, b where $a, b \in$

$au + bv = w$ holds true when:

$[3a, a] + [-b, b] = [-7, -1]$

$3a - b = -7$

$a + b = -1$

$4a = -8$

~~$a = -2, b =$~~

$\boxed{a = -2,\ b = 1}$

$\therefore w$ is linear comb of u, v

$(w = -2u + v)$

Wk 2 tut Cont.

8) $\underset{\sim}{v} = \overrightarrow{PQ}$ where $P = (-3,2,0)$ $Q = (4,-2,3)$

$2\overline{)74}$
$\quad 37$

$\underset{\sim}{v} = Q - P = \boxed{[7,-4,3]}$

$\|\underset{\sim}{v}\| = \sqrt{49+16+9} = \sqrt{49+25} = \boxed{\sqrt{74}}$

θ to positive x-axis $[1,0,0]$

$\cos\theta = \cancel{\frac{7}{1}[1,0,0]} = \frac{7}{\sqrt{74}\times 1} = \frac{7\sqrt{74}}{74}$

$\theta = 35.54° \approx \boxed{36°}$

θ to positive y-axis $[0,1,0]$

$\cos\theta = \frac{\cancel{4}\,-4}{\sqrt{74}\times 1}$ $\quad \cancel{\boxed{\theta = 36°}}$ $\quad \theta = 112°$

θ to positive z-axis $[0,0,1]$ $\quad \cancel{\boxed{\theta = 36°}}$

$\cos\theta = \frac{3}{\sqrt{74}}$ $\quad \theta = 70°$

9) in $\square ABCD$ (parallelogram) $\quad A = (1,2,-3)$

$\overline{AB} \parallel \overline{CD}$ $\quad B = (-2,1,1)$

i) $\overrightarrow{AB} = -\overrightarrow{CD} = \overrightarrow{DC}$ $\quad C = (0,2,1)$

$[-3,-1,4] = +[0-D_x, 2-D_y, 1-D_z]$

where $D = (D_1, D_2, D_3)$

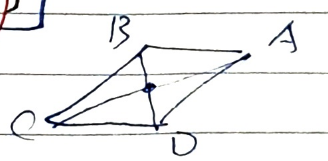
$-3 = -D_x \quad \therefore D_x = 3$

$-1 = 2 - D_y \quad \therefore D_y = +3$

$4 = 1 - D_z \quad \therefore D_z = -3$

$\therefore \cancel{D = [3,3,-3]}$

$\boxed{D = (3,3,-3)}$



ii) P = midpoint of $\overrightarrow{AC} = A + \frac{1}{2}\overrightarrow{AC}$

$\overrightarrow{AC} = [-1, 0, 4]$

$P = (1,2,-3) + [-\frac{1}{2}, 0, 2] = \boxed{(\frac{1}{2}, 2, -1)}$

iii) $\overrightarrow{BD} = [\frac{5}{2}, 1, -2]$

$\overrightarrow{PD} = [-\frac{5}{2}, -1, 2]$

Let Q be midpoint of $\overrightarrow{BD}$

$Q = B + \frac{1}{2}\overrightarrow{BD} = (-2,1,1) + [\frac{5}{2}, 1, -2]$

$= (\frac{1}{2}, 2, -1)$

$P = Q$

$\therefore$ midpoints of $\overline{AC}$ & $\overline{BD}$ are same

$\therefore$ $\overline{AC}$ & $\overline{BD}$ intersects

10) $\underset{\sim}{[2,3]}$ is orthogonal to $\underset{\sim}{[a+1, a-1]}$

$v \cdot \underset{\sim}{w}$ $2(a+1) + 3(a-1)$

i) $= 2a + 2 + 3a - 3 = 5a - 1$

$5a - 1 = 0$

$\boxed{a = \frac{1}{5}}$

ii) $\underset{\sim}{[3, \beta, 3\beta]}$ $\underset{\sim}{w} = [12, 0, -5]$

$\|\underset{\sim}{w}\| = 13$

$\|\underset{\sim}{v}\| = \sqrt{9 + \beta^2 + 9\beta^2}$

$= \sqrt{9 + 10\beta^2}$

$9 + 10\beta^2 = 169$

$10\beta^2 = 160$

$\beta^2 = 16$ $\boxed{\beta = \pm 4}$

12) LHS $\|a + b\|^2$ RHS $\|a\|^2 + \|b\|^2$

$=$

$(a+b) \cdot (a+b)$

$= (a \cdot a) + 2(a \cdot b) + (b \cdot b)$

$= \|a\|^2 + 2(a \cdot b) + \|b\|^2$

Put LHS a RHS together

$\cancel{\|a\|^2} + 2(a \cdot b) + \cancel{\|b\|^2} = \cancel{\|a\|^2} + \cancel{\|b\|^2}$

$(a \cdot b) = 0$

$\|\underset{\sim}{a}\| \times \|b\| \times \cos\theta = 0$

where θ is angle between $\underset{\sim}{a}$ and $\underset{\sim}{b}$

$\cos\theta = 0$

$\theta = 90°$

13) $U \cdot V = U \cdot w \Rightarrow \underset{\sim}{v} = \underset{\sim}{w}$?

let $u = [1, 1]$

$\underset{\sim}{v} = [2, 0]$ $\underset{\sim}{u} \cdot \underset{\sim}{v} = 2$

$\underset{\sim}{w} = [0, 2]$ $\underset{\sim}{u} \cdot \underset{\sim}{w} = 2$ $\therefore \underset{\sim}{u} \cdot \underset{\sim}{v} = \underset{\sim}{u} \cdot \underset{\sim}{w}$

Yet, $\underset{\sim}{v} \neq \underset{\sim}{w}$

Wk $\frac{1}{\cancel{2}}$ Quiz (in wk 2)

(1)

$u = [3, 7, 9]$

$v = [-9, 4, -1]$

$w = [-3, 8, 7]$

$3u + 9v + 3w = [9, 21, 27] + [-81, 36, -9] + [-9, 24, 21]$

$= [-81, 81, 39]$

(2)

$x = [-3, -5, 1]$ $y = [9, 9, -9]$

$x + y = [6, 4, -8]$

$3x = [-9, -15, 3]$

$x - y = [-12, -14, 10]$

$3x + 10y = [-9 + 90, -15 + 90, 3 + (-90)]$

$= [81, 75, -87]$

(4)

$c_1 + \tilde{u} c_2$ is undefined

Scalar + vector is NOT defined

$a + b$ $a - b$

$$\frac{a^2 - b^2}{\sqrt{a^2+b^2} \times \sqrt{a^2+b^2}} = \frac{a^2 - b^2}{a^2 + b^2}$$