Week 3 Pre-tut / week 3 lec exercises (proofs)

Anti-Commutativity of Cross Product

$\underset{\sim}{a} = [a_1, a_2, a_3]$
$\underset{\sim}{b} = [b_1, b_2, b_3]$

$\underset{\sim}{a} \times \underset{\sim}{b} = [a_2b_3 - b_2a_3,\ a_3b_1 - b_3a_1,\ a_1b_2 - b_1a_2]$

$\underset{\sim}{b} \times \underset{\sim}{a} = [b_2a_3 - a_2b_3,\ \ldots\ ]$
$= [-(a_2b_3 - b_2a_3), -(\quad), -(\quad)]$

$\therefore \underset{\sim}{a} \times \underset{\sim}{b} = -(\underset{\sim}{b} \times \underset{\sim}{a})$

Distributivity of cross product

$\underset{\sim}{a} \times (\underset{\sim}{b} + \underset{\sim}{c})$

$\underset{\sim}{a} = [a_1, a_2, a_3]$
$\underset{\sim}{b} = [b_1, b_2, b_3]$
$\underset{\sim}{c} = [c_1, c_2, c_3]$
$d = [b_1+c_1, b_2+c_2, b_3+c_3]$

let $\underset{\sim}{d} = \underset{\sim}{b} + \underset{\sim}{c}$

$\underset{\sim}{a} \times \underset{\sim}{d} = [\underbrace{cp_1, cp_2, cp_3}]$

for any dimensional value for the result of $\underset{\sim}{a} \times \underset{\sim}{d}$ represented as

$= a_n d_m - d_n a_m \quad (n, m \in \{1,2,3\}\ a\ n \neq m)$

$= a_n(b_m + c_m) - (b_n + c_n)a_m$

$= a_nb_m + a_nc_m - b_na_m - c_na_m$

$= (a_nb_m - b_na_m) + (a_nc_m - c_na_m)$

$= \underset{\sim}{a} \times \underset{\sim}{b} + \underset{\sim}{a} \times \underset{\sim}{c}$

$\therefore \boxed{\underset{\sim}{a} \times (\underset{\sim}{b} + \underset{\sim}{c}) = \underset{\sim}{a} \times \underset{\sim}{b} + \underset{\sim}{a} \times \underset{\sim}{c}}$

Orthognality of Cross Product

$(\underset{\sim}{a} \times \underset{\sim}{b})$ is orthogonal to $\underset{\sim}{a}, \underset{\sim}{b}$
$(\underset{\sim}{b} \times \underset{\sim}{a})$

$$\underset{\sim}{a} \times \underset{\sim}{b} = [a_2b_3 - b_2a_3,\ a_3b_1 - b_3a_1,\ a_1b_2 - b_1a_2]$$

1 4

$a \cdot (\underset{\sim}{a} \times \underset{\sim}{b}) = 0$ $\therefore \underset{\sim}{b} \cdot (\underset{\sim}{b} \times \underset{\sim}{a}) = 0$

$(a_1a_2b_3 - a_1b_2a_3) + (a_2a_3b_1 - a_2b_3a_1) + (a_3a_1b_2 - a_3b_1a_2)$

$a_1a_2b_3 - a_1a_2b_3 + a_2a_3b_1 - a_2a_3b_1$
$+ a_1a_3b_2 - a_1a_3b_2 = \emptyset$

3 2

$b \cdot (\underset{\sim}{a} \times \underset{\sim}{b}) = 0$ $\therefore a \cdot (\underset{\sim}{b} \times \underset{\sim}{a}) = 0$

비슷하겠지 ㅋㅋ

Constant Multiplication

$c(\underset{\sim}{a} \times \underset{\sim}{b})$

$= \cancel{[c(a_2b_3 - b_2a_3),\ c(a_3b_1 - b_3a_1),\ c(a_1b_2 - b_1a_2)]}$

$= [a_2b_3c - b_2a_3c,\ a_3b_1c - b_3a_1c,\ a_1b_2c - b_1a_2c$

depending on how I group term

eg. $a_nc = A_n$
$b_nc = B_n$ $\Big) n \in \{1,2,3\}$

can be either
$[A_2b_3 - b_2A_3,\ A_3b_1 - b_3A_1,\ A_1b_2 - b_1A_2]$
$[a_2B_3 - B_2a_3,\ a_3B_1 - B_3a_1,\ a_1B_2 - B_1a_2]$

$= \left[\boxed{\begin{array}{l} \underset{\sim}{A} \times \underset{\sim}{b} = c\underset{\sim}{A} \times \underset{\sim}{b} \\ \underset{\sim}{a} \times \underset{\sim}{B} = \underset{\sim}{a} \times c\underset{\sim}{b} \end{array}} \right.$ $\therefore c(\underset{\sim}{a} \times \underset{\sim}{b}) = \left[\begin{array}{l} c(\underset{\sim}{a}) \times \underset{\sim}{b} \\ \text{or} \\ \underset{\sim}{a} \times c(\underset{\sim}{b}) \end{array} \right.$

Week 3 - pretut

$a = [2, -1, 2]$
$b = [1, 1, -1]$
$c = [3, 0, -4]$

i) $\underset{\sim}{a} \times \underset{\sim}{b} = [-1, 4, 3]$

ii) $90^\circ = \frac{\pi}{2}$

iii) $\|a \times b\| = \sqrt{1+16+9} = \sqrt{26}$

iv) $\sin\theta$

θ: angle between a & b

$\|a \times b\| = \underset{3}{\|a\|} \times \underset{\sqrt{3}}{\|b\|} \times \sin\theta$ 60+18

$\boxed{\sin\theta = \frac{\sqrt{26}}{3\times\sqrt{3}} = \frac{\sqrt{78}}{9}}$

2 | 78
3 | 39
  | 13

line ℓ (passes through (2, 3, 5)
direction of [1, 3, -1]

i) $\underset{\sim}{x} = [2, 3, 5] + t[1, 3, -1] \quad t \in R$

ii)
$$\left.\begin{aligned} x &= 2 + t \\ y &= 3 + 3t \\ z &= 5 - t \end{aligned}\right) \quad t \in R$$