MATH1021 A1

SID: 510603294

Q1

$A = \{1,2,3,4,5,6,7\}$

$B = \{1,3,5,8,9\}$

$C = \{1,4,6,8\}$

$D = \{3,5,7,8\}$

(a) $(A \cap B) \setminus C$

$A \cap B = \{1,3,5\}$

$\{1,3,5\} \setminus C = \boxed{\{1\}}$

$\cancel{\boxed{\{3\}}}$ $\boxed{\{1\}}$

(b) $(A \setminus D) \cup (D \setminus A)$

$= \{1,2,4,6\} \cup \{8\}$

$= \boxed{\{1,2,4,6,8\}}$

$(A \cup D) \setminus (A \cap D)$

$\{1,2,3,4,5,6,7,8\} \setminus \{3,5,7\}$

$= \boxed{\{1,2,4,6,8\}}$

Q2

$z =$

(a) $z = (1+i)(-2+3i)$

$= -2 - 2i + 3i + 3i^2$

$= -2 - 3 + i$

$= \boxed{-5+i}$

(b) $z = \left(\frac{1+i}{1-i}\right)^{2023} = A^{2023} \quad \left(A = \frac{1+i}{1-i}\right)$

$A = \frac{1+i}{1-i} \times \frac{1+i}{1+i} = \frac{1-1+2i}{2} = \frac{2i}{2} = i$

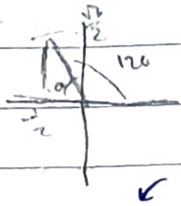
$\therefore z = i^{2023} = i^{2000} \times i^{20} \times i^{3}$

$= 1 \times 1 \times i^3 = i^3 = \boxed{-i}$

Q3 $z = -3 + 3\sqrt{3}\,i$

(a) $|z| = \sqrt{(-3)^2 + (3\sqrt{3})^2} = \sqrt{9+27} = \sqrt{36} = 6$

$z = 6\left(-\frac{1}{2} + \frac{\sqrt{3}}{2} i\right)$



$\text{Arg}(z) = \frac{2\pi}{3}$

$\boxed{z = 6\left(\cos\left(\tfrac{2}{3}\pi\right) + i\sin\left(\tfrac{2}{3}\pi\right)\right)}$ ← standard polar form

(b) $\boxed{z = 6e^{i\left(\frac{2}{3}\pi\right)}}$ ← polar exponential form

Q4 $z^5 = -2$

magnitude = 2
Arg(−2) = π

let $z = r(\cos(\theta) + i\sin(\theta))$

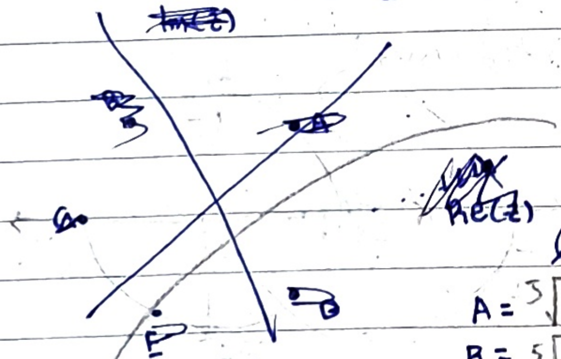
$z^5 = r^5(\cos(5\theta) + i\sin(5\theta))$

$r^5 = |-2| = 2 \quad \therefore \quad r = 2^{\frac{1}{5}} = \sqrt[5]{2}$

$\cos 5\theta = -1 \quad (-\pi < \theta \leq \pi)$

$5\theta = \pi + 2\pi k, \quad k \in \mathbb{Z}$

$\therefore \theta \in \left\{\frac{1}{5}\pi, \frac{3}{5}\pi, \pi, -\frac{1}{5}\pi, -\frac{3}{5}\pi\right\}$



{A, B, C, D, E} ← [5^{th} roots of −2 plotted on the Argand plane]

labels

$A = \sqrt[5]{2}\left(\cos\left(\frac{\pi}{5}\right) + i\sin\left(\frac{\pi}{5}\right)\right)$

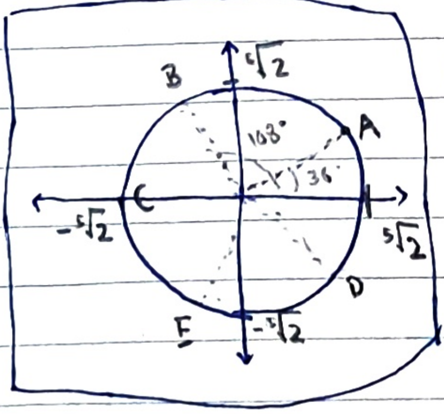
$B = \sqrt[5]{2}\left(\cos\left(\frac{3}{5}\pi\right) + i\sin\left(\frac{3}{5}\pi\right)\right)$

$C = \sqrt[5]{2}\left(\cos(\pi) + i\sin(\pi)\right)$

$D = \sqrt[5]{2}\left(\cos\left(-\frac{\pi}{5}\right) + i\sin\left(-\frac{\pi}{5}\right)\right)$

$E = \sqrt[5]{2}\left(\cos\left(-\frac{3}{5}\pi\right) + i\sin\left(-\frac{3}{5}\pi\right)\right)$

Plot



Q5

$5z^2 - 6z + 5 = 0$

* use quadratic formula

(a)

$$\frac{6 \pm \sqrt{36-4(5)(5)}}{10} = \frac{6 \pm \sqrt{-64}}{10}$$

$$= \frac{6 \pm 8i}{10} = \frac{6}{10} \pm \frac{8}{10}i = \frac{3}{5} \pm \frac{4}{5}i$$

$$\boxed{z = \frac{3}{5} \pm \frac{4}{5}i}$$

$|-3| + |2| \leq |0|$

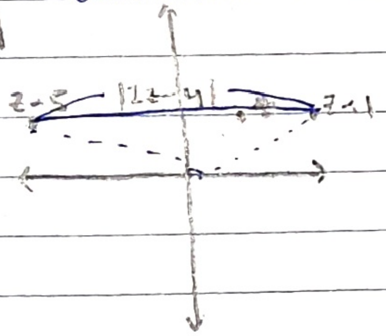
(b) $|z-5| + |z+1| \leq |z-2|$

By triangle inequality theorem,

$|(z-5)+(z+1)| \leq |z-5| + |z+1|$

$|2z-4| \leq |z-5| + |z+1|$

(ex illustration)



$\therefore |2z-4| \leq |z-2|$

let $z = x + yi$

$|2(x+yi) - 4| \leq |x+yi-2|$

$|(2x-4)+2yi| \leq |(x-2)+yi|$ Both sides are positive

$\sqrt{(2x-4)^2+(2y)^2} \leq \sqrt{(x-2)^2+y^2}$

Square both sides

$(2x-4)^2 + 4y^2 \leq (x-2)^2 + y^2$

$2^2(x-2)^2 + 3y^2 \leq (x-2)^2 \ldots$

$4(x-2)^2 + 3y^2 \leq (x-2)^2$

$3(x-2)^2 + 3y^2 \leq 0$

$(x-2)^2 + y^2 \leq 0$

$(x = 2, y = 0)$

Check the (x,y) that satisfies $|2z-4| \leq |z-2|$ also satisfies

$|z-5| + |z+1| \leq |z-2|$

$|2-5| + |2+1| \leq 0$

$6 \leq 0$

↳ No Solution

$\boxed{\text{No Solution}}$ ← set of z that satisfies the given inequality