

# Wk1 Exercise Sheet

#1

$$A = \{1, 2, 3, 4\} \quad B = \{3, 4, 5, 6\}$$

a)  $A \cup B = \{1, 2, 3, 4, 5, 6\}$

b)  $A \cap B = \{3, 4\}$

c)  $A \setminus B = \{1, 2\}$

d)  $B \setminus A = \{5, 6\}$

e)  $(A \setminus B) \cup (B \setminus A) = \{1, 2, 5, 6\}$

#2

$$z = 1 - 2i, \quad w = 3 + i$$

*complex imaginary num*

a)  $z + w = (1 - 2i) + (3 + i) = 4 - i$

b)  $z - w = (1 - 2i) - (3 + i) = -3 - 3i$

c)  $|z| = \left\{ \begin{array}{l} 1 - 2i, i < \frac{1}{2} \\ -1 + 2i, i \geq \frac{1}{2} \end{array} \right\}$

$$\sqrt{1+4} = \sqrt{5}$$

$$\sqrt{9+1} = \sqrt{10}$$

d)  $|w| = \left\{ \begin{array}{l} -i - 3, i < 3 \\ 3 + i, i \geq 3 \end{array} \right\}$

$$z = a + bi$$

$$\bar{z} = a - bi$$

$$w = c + di$$

$$\bar{w} = c - di$$

e)  $\bar{z} = 1 + 2i$  *complement conjugate*

$$(a+c) + (b+d)i$$

$$= (a+c) - (b+d)i$$

f)  $\bar{w} = 3 - i$

g)  $zw = (1 - 2i)(3 + i) = 3 - 6i + i - 2i^2 = 3 - 5i + 2 = 5 - 5i$

h)  $\frac{1 - 2i}{3 + i} = \frac{(1 - 2i)(3 - i)}{(3 + i)(3 - i)} = \frac{3 - 6i - i + 2i^2}{9 + 1} = \frac{-2 - 7i}{10} = \frac{-2 - 7i}{10}$

i)  $\frac{3 + i}{1 - 2i} = \frac{(3 + i)(1 + 2i)}{(1 - 2i)(1 + 2i)} = \frac{3 + 7i - 2}{1 + 4} = \frac{1 + 7i}{5} = \frac{1}{5} + \frac{7}{5}i$

$$= \frac{3 + 7i - 2}{1 + 4} = \frac{1 + 7i}{5} = \frac{1}{5} + \frac{7}{5}i$$



#3

$$(a) (2+3i) + (5-6i) \\ = 7-3i$$

$$(b) (2+3i) - (5-6i) \\ = -3+9i$$

$$(c) (1+i)(1-i) \\ = 1^2 - i^2 \\ = 1 - (-1) = 2$$

$$(d) \frac{(1-i)(1-i)}{(1+i)(1-i)} = \frac{1-2i+i^2}{2} = -i$$

#4  $Y = \{n \in \mathbb{N} \mid 5 \leq n^3 \leq 100\}$ 1, 2, 27, 64 | 125

$$(a) Y = \{2, 3, 4\}$$

$$(b) Y \subseteq \mathbb{N} \text{ true}$$

$$Y \subseteq \mathbb{Z} \text{ true}$$

$$5 \in Y \text{ false}$$

$$100 \notin Y \text{ true}$$

#5  $z = -3, w = 2i, v = 4-3i$ 

$$(a) z + w - v \\ = (-3) - (2i) - (4-3i) \\ = -3 - 2i - 4 + 3i \\ = -7 + i$$

$$(b) |z| = 3$$

$$(c) \bar{z} = -3$$

$$(d) \frac{z-w}{v} = \frac{-3-2i(4+3i)}{4-3i(4+3i)} \\ = \frac{-12-8i-9i+6}{16+9} = \frac{-6-17i}{25} \\ = -\frac{6+17i}{25}$$

$$(f) |wv| = |-3(4-3i)| \\ = |-12+9i| \\ = \sqrt{144+81} = \sqrt{225} \\ = 15$$

$$(e) \bar{v} - \frac{z}{w} = (4+3i) + \frac{3}{2i} \\ = \frac{(4+3i)(2i)+3}{2i} \\ = \frac{8i-6+3}{2i} = \frac{-3+8i}{2i}(-2i) \\ = \frac{6i+16}{4} = 4 + \frac{3}{2}i$$

$$(g) \frac{z}{2w+3v} = \frac{-3}{4i+12(4i)} \\ = -\frac{3}{4i+48i} = -\frac{3}{52i} \\ = \frac{-3(i-3)}{4(i+3)(i-3)} = \frac{9-3i}{4(-10)} = -\frac{9-3i}{40}$$

Q How can a programmer produce more reliable code by knowing the systems better?

59)

$$\frac{z}{2w + 3v} = \frac{-3}{4i + 12 - 9i} = \frac{-3}{(12 - 5i)(12 + 5i)}$$

$$= \frac{(-36 - 15i)}{(144 + 25)} = \frac{-(36 + 15i)}{169} = -\frac{36}{169} - \frac{15i}{169}$$

$$\frac{-36 - 15i}{169}$$

#60

$$7i^{181} = 7i^{180+1} = 7 \times i^{4(45)} \times i$$

$$= 7 \times i^4 \times i^{45} \times i$$

$$= 7 \times (i^4)^{45} \times i$$

$$= 7 \times 1^{45} \times i = 7 \times 1 \times i = 7i$$

#7

a)  $z^2 + 3z + 2 = 0$

$$(z+2)(z+1) = 0$$

$$\underline{z = -2 \text{ or } -1}$$

b)  $z^2 + z + 1 = 0$

$$z = \frac{-1 \pm \sqrt{1 - 4(1)(1)}}{2}$$

$$= \frac{-1 \pm \sqrt{-3}}{2} = \frac{-1 \pm \sqrt{3} \cdot \sqrt{-1}}{2} = \frac{-1 \pm \sqrt{3}i}{2}$$

c) ~~2030~~

$$-z^2 + 2z - 10 = 0$$

$$\frac{-2 \pm \sqrt{4 - 4(10)}}{-2} = \frac{-2 \pm \sqrt{-36}}{-2}$$

$$= \frac{-2 \pm 6i}{-2}$$

$$= 1 \pm 3i$$

$$\frac{-1 \pm \sqrt{3}i}{2}$$

d)  $x^4 = 16$

$$A = x^2$$

$$A^2 = 16$$

$$A = \pm 4$$

$$x^2 = 4$$

$$x = \pm 2$$

$$x^2 = -4$$

$$x = \pm 2i$$