Week 4 Review

Lines in $\mathbb{R}^3$

General form

↳ $ax+by+cz=d$

Normal form

$\underset{\sim}{n} \cdot \underset{\sim}{x} = d$ — (x,y,z)

Span$(v_1, v_2 \cdots v_n)$: set of linear combinations of $v_1, v_2 \cdots v_n$

- In $\mathbb{R}^2$, if v_1, v_2 are linearly independent
  → $\text{span}(v_1, v_2) \in \mathbb{R}^2$ = not parallel
- In $\mathbb{R}^3$, 2 non-zero, non-parallel vectors
  spans a plane thru the origin. ← if parallel, $\text{span}(v_1, v_2)$ = line

Linear Independence

2 vectors : $\boxed{\text{not parallel}}$, non-zero

$\boxed{\underset{\sim}{u}, \underset{\sim}{v} \quad \text{if } \|\underset{\sim}{u} \times \underset{\sim}{v}\| = 0,\ \underset{\sim}{u} \parallel \underset{\sim}{v}}$

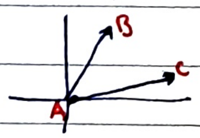
3+ vectors : not parallel (one vector can't be expressed in ~~...~~ comb of other vectors)

Vector form

$\underset{\sim}{x} = p + s\underset{\sim}{u} + t\underset{\sim}{v} \qquad s,t \in \mathbb{R}$

two direction vectors : $\vec{AB}, \vec{AC}$

eg.



$P = A = (0,0,0)$

normal vector : $\underset{\sim}{u} \times \underset{\sim}{v}$

System of equations

$$\begin{aligned} x + 2y + 3z &= 6 \\ -2x \quad\quad + 5z &= 2 \\ 6y - z &= -5 \end{aligned} \Rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ -2 & 0 & 5 & 2 \\ 0 & 6 & -1 & -5 \end{array}\right]$$

If constant term are all zero, then the systems are $\boxed{\text{homogeneous}}$

"Augmented Matrix"

Line intersecting two planes

- Line is perpendicular to both planes