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Study on the Cone Programming Reformulation of Active Noise Control Filter Design in the Frequency Domain

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ABSTRACT

In the practice of active noise control, the control filter can be designed in either the time domain or the frequency domain. Compared with the former method category, it is more convenient to use frequency-domain methods to apply constraints such as stability, robustness, disturbance enhancement, input limit of loudspeakers, etc. Better noise reduction performance can usually be achieved by frequency-domain design as well. However, the computational complexity of designing a filter in the frequency domain is usually significant, especially for multichannel systems with multiple constraints. This is one of the challenges of using frequency-domain design in practical applications. In this paper, the traditional optimization problem used in frequency-domain filter design was modified and reformulated to a cone programming problem, where the inequality constraints were reformulated as second-order cones and positive semidefinite cones. Because of its convex nature, the global minimum solution to the problem can always be found. Another advantage of this cone programming reformulation is that algorithms with high computational efficiency can be used. It was demonstrated that, compared with using the traditional sequential quadratic programming method, the calculation is more efficient if the filter design problem is reformulated to cone programming and solved by the primal-dual interior-point method.

1 INTRODUCTION

In active noise control (ANC), secondary sources, such as loudspeakers, are usually used to generate a sound field to reduce the original primary noise at desired locations. The commonly used ANC algorithms can be classified into different categories based on different criterions[1]. According to which domain the filter design formulation is based on, there are time-domain design methods and frequency-domain (or transform-domain) design methods[2]. Time-domain design methods are usually considered to be computationally more efficient than frequency-domain design methods. One important advantage for frequency-domain methods is the convenience of applying constraints such as stability, robustness, disturbance enhancement, input limit of secondary sources, etc. Also, better noise reduction performance can usually be achieved by using frequency-domain design methods. However, the computational complexity of designing a filter in the frequency domain is usually significant, especially for multichannel systems with multiple types of constraints[1], which makes it challenging to use frequency domain filter design methods in practical ANC applications.

To solve the ANC filter design problem in the frequency domain, Cheer and Elliott applied sequential quadratic programming (SQP) method to obtain the optimal solution and have demonstrated that this method can be used in practical ANC applications[3]. Similar approaches were discussed by Boyd et al[4], and Titterton and Olkin[5] as well. In previous studies, semidefinite programming (SDP) was also mentioned to be a possible alternative method for solving this problem[6,7], which has been applied to deal with robust design of controllers in ANC problems[8]. Recently, it was demonstrated that the mixed second-order cone programming (SCOP) and SDP problems can be solved efficiently by using primal-dual interior-point algorithm[9], which is considered as a powerful algorithm for large-scale nonlinear programming. And, it is often more efficient than active-set sequential quadratic programming on large problems[10]. Considering that, in ANC filter design problems, the involved filter coefficients form a large set of free variables, it is suggested in the current work that an alternative promising way to solve this filter design problem in the frequency domain is to reformulate original problem into a cone programming problem and solve it using the primal-dual interior-point method.

In this paper, the traditional optimization problem[3] for the frequency-domain ANC filter design was modified and reformulated. Firstly, this problem is relaxed to construct a convex optimization problem by replacing the stability constraint with an appropriate upper bound. A filter response amplitude constraint is also added to meet the practical purpose and resolve some numerical issues. Then this convex optimization problem is reformulated to a cone programming problem, where the inequality constraints were reformulated as second-order cones and positive semidefinite cones. Because of its convex nature, the global minimum solution to the problem can always be found[6]. Another advantage of this programming reformulation is that algorithms with high computational efficiency can be used, e.g., the primal-dual interior-point method. In the presented work, filter design based on the conventional formulation is solved by the sequential quadratic programming. And the cone programming formulation were solved by both sequential quadratic programming and the primal-dual interior-point method. The performance and the computational efficiency were compared in simulation with practical data. It was demonstrated that, compared with that using the traditional sequential quadratic programming method, the calculation is much more efficient if the filter design problem is reformulated to cone programming problem and the primal-dual interior-point algorithm is used to solve the optimization problem.

The rest of this paper is organized as follows. In section 2, the proposed method of modifying and reformulating the active noise control filter design problem in the frequency domain was explained. Simulation results and related analysis were presented in section 3. Conclusions of findings in the current work were drown in section 4.

2 THEORY OF THE CONE PROGRAMMING REFORMULATION OF ACTIVE NOISE CONTROL FILTER DESIGN PROBLEM

Cheer and Elliott[3] proposed a multichannel active noise control system designed in the frequency domain. They have also shown that the feedback and the feedforward control have similar model structures and the same filter design method can be applied to both control strategies[2,3]. For convenience, only the feedforward control system is discussed here in the presented work. The feedback control system design problem can be modified and reformulated using the same process.

2.1 Model structure and the original optimization problem

A multiple-input-multiple-output (MIMO) feedforward control system involving K microphones as reference sensors, M loudspeakers as secondary sources, L_e microphones as error sensors. The internal model control (IMC) structure can be used to cancel the acoustic feedback path[2,3,7]. The block diagram of this standard MIMO feedforward ANC system is shown in Figure 1. The G_e represents the transfer function of the acoustical responses of the secondary sources at the error sensor locations. d_e denotes the disturbance signal from the primary noise which is to be reduced by the ANC system. The controller is composed of two components. $\hat{G}_{s0}$ is a model of the acoustic feedback path G_{s0} (the acoustical responses of secondary sources at the reference sensor locations). In this work, the acoustic feedback path was assumed to be modelled perfectly, i.e., $\hat{G}_{s0} = G_{s0}$. The W_x is the transfer function of the ANC filter which is realized by a multi-channel finite impulse response (FIR) filter. The filter coefficients of W_x, denoted as w_F in the presented work, are variables that are calculated in the filter design problem. The optimal solution of these FIR filter coefficients, w_F^*, are obtained by solving an associated optimization problem, and are directly implemented in the real-time signal processing controller via time-domain filtering.

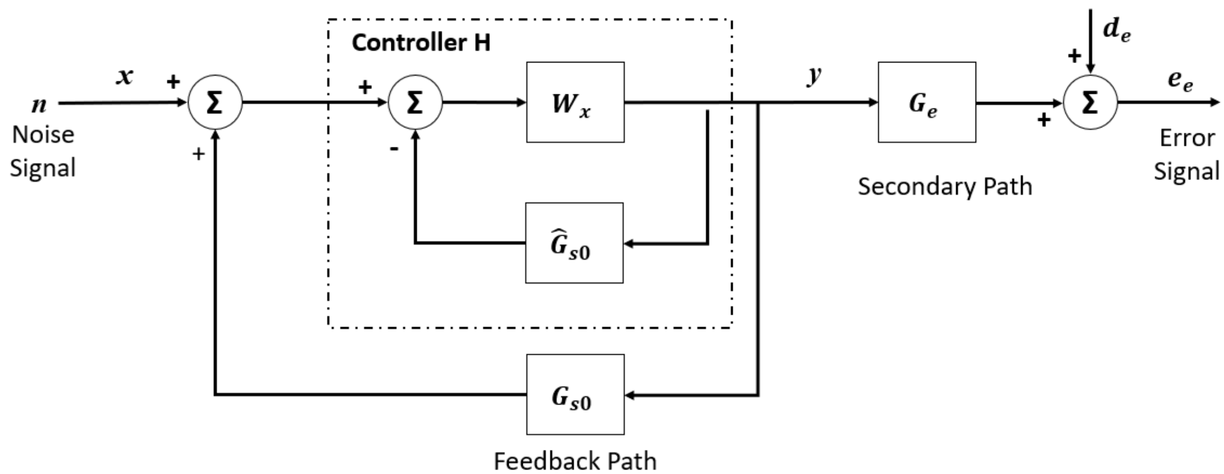

Figure 1: *Block diagram of the MIMO feedforward controllers using the internal model control structure.*

The use of FIR filter instead of infinite impulse response (IIR) filter is because the filter frequency response, W_x, is a direct linear transformation of the coefficients, w_F, and thus, the convexity can be preserved[6]. The frequency response of the ANC filter can be expressed as:

$$W_{x_{i,j}}(f) = F_z(f)\, w_{F_{i,j}} \ , \tag{1}$$

$$F_z(f) = \left[\, 1 \ , \ \mathrm{e}^{-j2\pi f \frac{1}{f_s}} \ , \ \mathrm{e}^{-j2\pi f \frac{2}{f_s}} \ , \dots , \ \mathrm{e}^{-j2\pi f \, \frac{N_t - 1}{f_s}} \,\right] , \tag{2}$$

where f represents the frequency; i and j represent the i-th output channel and the j-th input channel of the ANC filter. Each $w_{F_{i,j}}$ is a column vector including the coefficients of filter component associated with the input-output channel combination specified by i and j; f_s is the sampling frequency; N_t is the length of each FIR filter component.

The objective function $J_0(w_F)$ that is to be minimized is the total signal power of all error signals averaged among all desired frequencies. It was derived in Cheer and Elliott's work[3] :

$$J_0 = \frac{1}{N_f} \sum_{k=k_1}^{k_2} J(f_k) \ , \tag{3}$$

$$J(f_k) = tr\big[\, G_e(f_k) W_x(f_k) S_{xx}(f_k) W_x^{\mathrm{H}}(f_k) G_e^{\mathrm{H}}(f_k) \ + \ G_e(f_k) W_x(f_k) S_{xd_e}(f_k) \ + \ \dots \ S_{xd_e}^{\mathrm{H}}(f_k) W_x^{\mathrm{H}}(f_k) G_e^{\mathrm{H}}(f_k) \ + \ S_{d_e d_e}(f_k) \big] \ , \tag{4}$$

with

$$W_x(f_k) = \begin{bmatrix} W_{x_{1,1}}(f_k),\ W_{x_{1,2}}(f_k), \dots, W_{x_{1,N_r}}(f_k) \\ W_{x_{2,1}}(f_k), W_{x_{2,2}}(f_k), \dots, W_{x_{2,N_r}}(f_k) \\ \dots \\ W_{x_{N_{s},1}}(f_k), W_{x_{N_{s},2}}(f_k), \dots, W_{x_{N_s,N_r}}(f_k) \end{bmatrix},$$

where f_k denotes the k-th specified frequency component; H represents the complex conjugate transpose of a matrix; N_f is the total number of the specified frequency points; k_1 and k_2 represent the lower and upper bounds of the frequency range where noise attenuation is desired; S_{xx}, S_{xd_e}, and $S_{d_e d_e}$ are respectively the auto spectral density matrix of the reference signals measured at the reference microphones, the cross spectral density matrix between the reference signals and the disturbance signals measured at the error microphones, and the auto spectral density matrix of the disturbance signals.

A set of disturbance enhancement constraint is needed to make sure large enhancement is not produced outside of the desired attenuation frequency band[2,3]. A modified version of Cheer and Elliott's constraint[3] is used:

$$J(f_k)D_e(f_k) \le A_e \,, \tag{5}$$

$$D_e(f_k) = \frac{1}{tr\left(S_{d_e d_e}(f_k)\right)} \,, \tag{6}$$

where $D_e(f_k)$ is the reciprocal of total disturbance energy at k-th frequency; A_e is a constant value that specifies the maximum enhancement allowed in the error signals after the ANC system is activated.

Because of the existence of feedback cancellation path $\hat{G}_{s0}$ in the controller, stability issue needs to be considered. Cheer and Elliott[3] proposed a constraint that limits the controller's open-loop response trajectory associated with each eigenvalue of the system to be at the right hand side of the Nyquist point, i.e.

$$\min\left(\mathrm{Re}\left(\ \lambda\left(W_x(f_k)\hat{G}_{s0}(f_k)\right)\ \right)\ \right) > -1 \,, \tag{7}$$

where $\lambda()$ means the eigenvalues of a matrix; Re() means the real part of a complex number. Note that, compared with Cheer and Elliott's work[3], we modified it from $\hat{G}_{s0}W_x$ to $W_x\hat{G}_{s0}$. The non-zero eigenvalues for those two matrices are the same[11], but $W_x\hat{G}_{s0}$ is likely to have a smaller dimension, since the number of secondary sources M is usually smaller than the number of reference microphones K.

For robustness, the **M**-**Δ** structure[2,3] is used, and the small gain theory is applied, which then gives the resulting constraint as:

$$\max\left(\sigma\left(\ W_x(f_k)\hat{G}_{s0}(f_k)\right)\right) B(f_k) \le 1 \,, \tag{8}$$

where σ() means the singular values of a matrix; $B(f_k)$ is the upper bound of the output multiplicative plant uncertainty of $\hat{G}_{s0}$. For the same reason as explained in the stability constraint (Eq. (7)), $W_x\hat{G}_{s0}$ is used instead of $\hat{G}_{s0}W_x$.

In practice, there usually involves an upper limit on the amplitude of the input signal to the loudspeakers, especially at very low frequencies. So, a filter response amplitude constraint is also added to meet this practical purpose:

$$\left|W_{x_{i,j}}(f_k)\right| \leq C(f_k). \tag{9}$$

where i and j represent different rows and columns of the matrix $W_x(f)$; $C(f_k)$ is the upper limit of the amplitude of filter frequency response. Also, for the frequency range where the power of the disturbance signal is very small, the disturbance enhancement constraint (Eq. (5)) is likely to cause numerical issues when solving the optimization problem, since this makes $D_e(f_k)$ very large. At these frequencies, the response amplitude constraint, Eq. (9), is used to replace Eq. (5), which can still result in reasonable disturbance enhancement suppression but with better numerical stability.

The optimization problem for the ANC filter design is now constructed using equation (3) as the objective function, and using equations (5) (7) (8) (9) as constraints.

2.2 Modification and reformulation of the original optimization problem

The modification and reformulation have two parts: (1) modify the original optimization problem, constructed in section 2.1, to a general convex optimization problem; and (2) reformulate the convex optimization problem to a cone programming problem, which is a specific type of convex optimization problem.

2.2.1 Modify the original problem to a convex optimization problem

The objective function, Eq. (3), is essentially in a quadratic form. Some reformulation can be done to simplify it. Firstly, by means of the properties of trace operation, Frobenius inner product, and Kronecker product, the equation (4) can be reformulated as:

$$J(f_k) = w^{\mathrm{T}}\, A_J(f_k)\, w + 2\mathrm{Re}\left(b_J^{\mathrm{T}}(f_k)\, w\right) \;+\; c_J(f_k), \tag{10}$$

with

$$A_J(f_k) = \; F(f_k)^{\mathrm{H}}\left(G_e^{\mathrm{H}}(f_k)G_e(f_k) \otimes S_{xx}^{\mathrm{T}}(f_k)\right)F(f_k)\;,$$

$$b_J(f_k) = \; F(f_k)^{\mathrm{T}}\,\mathrm{vec}\left(S_{xd_e}(f_k)G_e(f_k)\right)\;,$$

$$c_J(f_k) = \; tr\left(S_{d_e d_e}(f_k)\right)\;,$$

$$w = \left[w_{F_{1,1}}^T,\; w_{F_{1,2}}^T, \ldots, w_{F_{1,N_r}}^T, w_{F_{2,1}}^T, w_{F_{2,2}}^T, \ldots, w_{F_{N_s,N_r}}^T\right]^T\;,$$

$$F(f_k) = \; I_{N_s N_r} \otimes F_z(f_k)\;,$$

where $I_{N_s N_r}$ is an identity matrix with a dimension of $N_s N_r \times N_s N_r$; T represents the transpose of a matrix; $\otimes$ represents the Kronecker product; vec() represents the vectorization operation for converting a matrix to a vector by stacking the columns[11].

Then, the equation (3) can be reformulated using equation (10):

$$J_0 = w^{\mathrm{T}}\left(\frac{1}{N_f}\sum_{k=k_1}^{k_2} A_J(f_k)\right)w + 2\mathrm{Re}\left(\frac{1}{N_f}\sum_{k=k_1}^{k_2} b_J^{\mathrm{T}}(f_k)\right)w \;+\; \frac{1}{N_f}\sum_{k=k_1}^{k_2} c_J(f_k), \tag{11}$$

where $A_J(f_k)$, $b_J^{\mathrm{T}}(f_k)$, and $c_J(f_k)$ are defined in Eq. (10). It is noted that Eq. (10) is a standard quadratic function where $G_e^{\mathrm{H}}(f_k)G_e(f_k)$ is always positive semidefinite; $S_{xx}^{\mathrm{T}}(f_k)$ is positive semidefinite according to its definition; and so, $G_e^{\mathrm{H}}(f_k)G_e(f_k) \otimes S_{xx}^{\mathrm{T}}(f_k)$ is positive semidefinite because of the property of Kronecker product[11]. This means that $A_J(f_k)$ is positive semidefinite, which makes the summation $\sum_{k=k_1}^{k_2} A_J(f_k)$ to be positive semidefinite. Thus, the function J_0 in Eq. (11) can be shown as a convex function since N_f is always positive [6].

For the disturbance enhancement, Eq. (5), it can be reformulated in a similar way by using Equations (10), which results in:

$$f_{1,k} = w^{\mathrm{T}} A_J(f_k)\, w + 2\mathrm{Re}\left(b_J^{\mathrm{T}}(f_k)\right) w + c_J(f_k) - \frac{A_e}{D_e(f_k)} \leq 0 \,. \tag{12}$$

It is a convex function with respect to w because $A_J(f_k)$ is positive semidefinite as we analyzed above.

For stability constraint, Eq. (7), it is equivalent to:

$$\max\left(\mathrm{Re}\left(\lambda\left(-W_x(f_k)\hat{G}_{s0}(f_k)\right)\right)\right) < 1 \,. \tag{13}$$

It is noted that the function on the left side is not convex. In the current work, this condition is relaxed to possess convexity by replacing it by its upper bound:

$$f_{2,k} = \max\left(\lambda\left(\frac{-W_x(f_k)\hat{G}_{s0}(f_k) + \left(-W_x(f_k)\hat{G}_{s0}(f_k)\right)^{\mathrm{H}}}{2}\right)\right) - (1-\epsilon_s) \leq 0 \,, \tag{14}$$

where ϵ_s is a small positive number added to ensure strict inequality. It is obvious that $f_{2,k}$ in Eq. (14) takes the largest eigenvalue of a Hermitian matrix and is guaranteed to be a convex function.

For robustness constraint, Eq. (8), it can be represented as:

$$f_{3,k} = \max\left(\sigma\left(W_x(f_k)\hat{G}_{s0}(f_k)\right)\right) B(f_k) - 1 \leq 0 \,, \tag{15}$$

in which $f_{3,k}$ is convex, because the largest singular value of a matrix is equivalent to a matrix norm.

For filter response amplitude constraint, Eq. (9), it is also convex because, by Eq. (1), it can be written as:

$$f_{4,k} = \|F_z(f_k)\, w_{F_{i,j}}\|_2 - C(f_k) \leq 0, \tag{16}$$

where the L^2 norm is a convex function.

Now, combining equations (11) (12) (14) (15) and (16), the ANC filter design can be formulated into a convex optimization problem:

$$\begin{array}{ll} \text{minimize} & J_0(w_F), \\ \text{subject to} & f_{l,k}(w_F) \leq 0 \quad l = 1,2,3,4, \text{for all frequency } f_k \,. \end{array} \tag{17}$$

2.2.2 Reformulate the convex optimization problem to a cone programming problem

Because the problem expressed in Eq. (17) is a convex optimization problem, the global minimum solution can always be found[6]. To solve this problem more efficiently, this problem can be reformulated into a cone programming problem, e.g., by using the CVX toolbox[12,13]. However, instead of the general reformulation algorithms as implemented in the toolboxes, it was found that the calculation time can be significantly further reduced, if the reformulation is done explicitly. Also, the physical meanings and some mathematical structures in the original problem can be used to make the resulting reformulated cone programming problem much simpler than that obtained by the direct use of a general toolbox.

The explicit reformulation process is presented in this section. The standard cone programming problem requires the objective function to be linear. In the objective function expressed in Eq. (11), the scaling factor in the front and the constant term can be removed without changing the solution of the problem. And, a new variable t_0 can be introduced to reformulate the objective function Eq. (11) to be:

$$\begin{aligned} &\text{minimize} && f_0 = t_0 + 2\mathrm{Re}\left(\sum_{k=k_1}^{k_2} b_J^{\mathrm{T}}(f_k)\right) w \\ &\text{subject to} && \|M_0\, w\|_2 \leq \sqrt{t_0\, \tilde{t}_0} \;\;, \quad \tilde{t}_0 = 1 \;\;, \end{aligned} \tag{18}$$

where the matrix M_0 is the matrix square root of $\sum_{k=k_1}^{k_2} A_J(f_k)$. The constraint is in the form of a rotated Lorentz cone, which can be reformulated as a second-order cone easily. Also, many solvers can directly accept the rotated Lorentz cone besides the second-order cone.

For disturbance enhancement constraint in Eq. (12), they can be reformulated in a similar way, which results in a number of linear equality constraints as well as inequality constraints in the forms of rotated Lorentz cones:

$$\begin{aligned} t_{1,k} \;+\; 2\mathrm{Re}\big(b_J^{\mathrm{T}}(f_k)\,\big)w + c_J(f_k) - \frac{A_e}{D_e(k)} = 0 \\ \|M_{1,k}\, w\|_2 \leq \sqrt{t_{1,k}\;\, \tilde{t}_{1,k}} \;\;, \quad \tilde{t}_{1,k} = 1 \;\;, \end{aligned} \tag{19}$$

where the matrix $M_{1,k}$ is the matrix square root of $A_J(f_k)$.

For stability constraints in Eq. (14), they can be reformulated into semidefinite cones:

$$W_x(f_k)\hat{G}_{s0}(f_k) + \left(W_x(f_k)\hat{G}_{s0}(f_k)\right)^{\mathrm{H}} + 2(1 - \epsilon_s) \succcurlyeq 0 \,, \tag{20}$$

where $\succcurlyeq 0$ means a matrix is positive semidefinite.

For robustness constraints in Eq. (15), they can also be reformulated into semidefinite cones:

$$\begin{bmatrix} \frac{1}{B(k)} I_{N_s} & W_x(k)\hat{G}_{s0}(k) \\ \left(W_x(k)\hat{G}_{s0}(k)\right)^H & \frac{1}{B(k)} I_{N_s} \end{bmatrix} \succcurlyeq 0 \,, \tag{21}$$

where I_{N_s} is an identity matrix with a dimension of $N_s \times N_s$.

For filter response amplitude constraints in Eq. (16), they can be reformulated into standard Lorentz cones:

$$\|F_z(f_k)\, w_{F_{i,j}}\|_2 \leq \; t_{3,k} \;\;, \quad t_{3,k} = C(f_k) \,. \tag{22}$$

The convex optimization problem presented in section 2.2.1 is now reformulated into a cone programming problem as described in equations (18) to (22). And, all the cones involved in this problem are self-dual. The CVX toolbox[12,13] can accept problems expressed in this form and then this optimization problem is solved by calling a cone programming solver such as SDPT3[14,15] or SeDuMi[16]. Those solvers can also be called directly if this cone programming problem is organized into the required format.

3 SIMULATION RESULTS

The computational efficiency of the two problem formulations, i.e., the original convex optimization formulation and the cone programming formulation, was investigated with different optimization algorithms applied to each formulation. The purpose of applying different algorithms to the same formulation is to study whether the efficiency difference is due to the difference in the formulations or due to the use of different algorithms. To compare the efficiency of solving the reformulated cone programming problem using primal-dual interior-point method and sequential quadratic programming method, simulations were conducted to compare the performance and the computation time required for different algorithms.

The simulation was performed based on reference signals, disturbance signals and frequency response functions (G_e and G_{s0}) that were measured in a environment. The ANC system includes two inputs and two outputs, i.e., $K = 2$, $M = 2$. Also, two error microphones were used, i.e., $L_e = 2$. The sampling frequency for the controller is $f_s = 8000$ Hz. Two choices of filter length, $N_t = 64$ and $N_t = 128$, were used in the simulations to demonstrate the computation time and ANC performance for different problem sizes. In the current work, the constraints are mainly applied on the lower and higher frequency range for specific numbers instead of the whole frequency range to simplify the problem because these constraints are usually found inactive in the middle frequency range in similar applications.

The following three combinations of formulations and algorithms are used in the simulations:

1. Use the CVX toolbox to further reorganize the cone programming problem formulated in section 2.2.2 and then call the solver SeDuMi. A reorganization was included instead of the direct call of the solver to solve the formulation described in section 2.2.2, because it usually results in a less chance to encounter numerical issues. The SeDuMi solver applies the primal-dual interior-point method to solve the cone programming problem. For convenience, this method is referred to as “cone programming + primal-dual interior-point” in this article.
2. The sequential quadratic programming algorithm (the MATLAB function “fmincon” was used with the algorithm option set to be “sqp”) to solve the original convex optimization problem described in section 2.2.1. For convenience, this method is referred to as “original formulation + sequential quadratic” in this article.
3. The sequential quadratic programming algorithm (the MATLAB function “fmincon” was used with the algorithm option set to be “sqp”) to solve the cone programming problem in section 2.2.2. It is noted that the “fmincon” function can only deal with real unknown variables, so, all the complex variables involved in the formulation were split into real and imaginary parts when using the “fmincon” function to solve the cone programming problem in section 2.2.2. For convenience, this method is referred to as “cone programming + sequential quadratic” in this article.

It is noted that there is not a case where the original formulation is solved by the primal-dual interior-point method. This is because the specific type of primal-dual interior-point method, which is implemented in the cone programming solvers such as SeDuMi, is for self-dual cone programming problems only.

The computation time for applying the above mentioned three methods to problems with two different sizes are summarized in Table 1. We can see that the use of method 1, “cone programming + primal-dual interior-point” has a significant advantage in computation efficiency. Comparing method 2, “original formulation + sequential quadratic”, and method 3, “cone programming + sequential quadratic”, we can see that when using the sequential quadratic programming, there is no significant difference in terms of the computation efficiency before and after the reformulation. These results suggest that the reformulation to cone programming without applying more efficient algorithms designed for cone programming problem specifically does not produce much improvement in computation efficiency, the main reason for the improvement is the use of more advanced cone programming algorithms, such as the type of self-dual primal-dual interior-point method that implemented in solver SeDuMi.

Table 1: Computation time for two problem sizes using different formulation-algorithm combinations.

FIR length	cone programming + primal-dual interior-point	original formulation + sequential quadratic	cone programming + sequential quadratic
64	8.0 s	1790.4 s	1943.2 s
128	28.4 s	7504.9 s	5980.7 s

Besides computation efficiency, the ANC performance of the three methods were compared as well. The noise reduction performance obtained by using different methods and different filter lengths is shown in Figure 2. The “Normalized SPL” is the dB scale (i.e., ten times the logarithm) of the ratio of the averaged sound pressure power spectral density function (PSD) at the error microphones to the PSD of the original disturbance signals averaged among the error microphones and then averaged over the whole frequency band. “ANC OFF” in Figure 2 is the original averaged disturbance PSD at error microphones. The noise reduction performance obtained by using different methods were found to be simpler, which is expected, since a reformulation of a problem and the use of different optimization algorithms should not change the nature of the problem. However, the performance resulted from method 3, “cone programming + sequential quadratic”, is slightly worse in the high frequency range. This is because the algorithm does not always converge to the optimal solution, instead, the iteration process stops when the maximum iteration times or function evaluation times is reached. On the other hand, the optimal solution can usually be found when using “cone programming + primal-dual interior-point” within the default maximum iteration times in the solver. Furthermore, a theoretical upper-bound of the number of iterations required to achieve convergence exists, $O\left(\sqrt{n}\log\left(\frac{1}{\epsilon}\right)\right)$, for the primal-dual interior-point method used here[16], which adds reliability when using this algorithm.

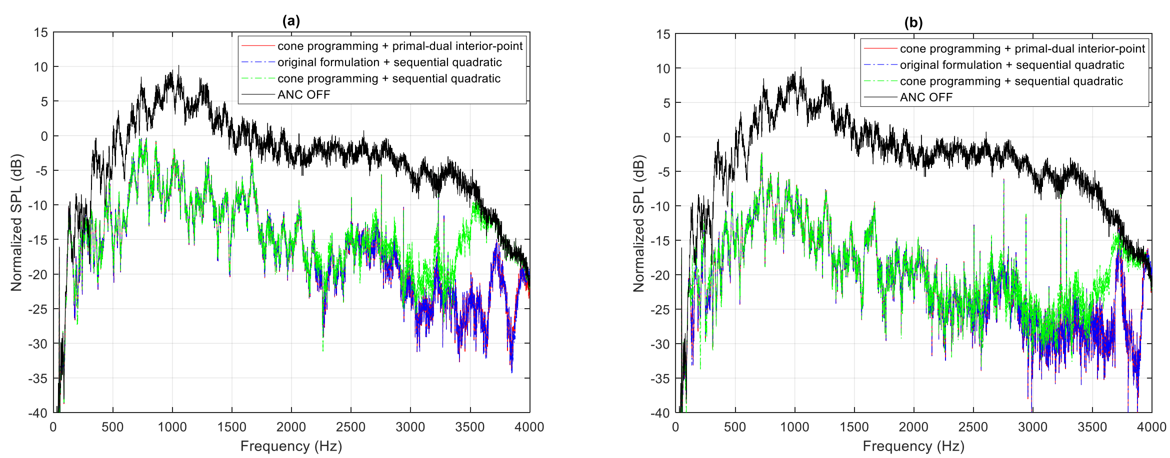

Figure 2: *(a) the attenuation performance for FIR length* $N_t = 64$. *(b) the attenuation performance for FIR length* $N_t = 128$

4 CONCLUDING COMMENTS

In this paper, the problem of active noise control filter design in frequency domain was modified and reformulated. The original problem was relaxed to a general convex optimization problem by replacing the stability constraint with its upper bound. Then this convex optimization problem is reformulated to a cone programming problem. The simulation results showed that the calculation using the primal-dual interior-point method for cone programming can be faster, compared with that using the commonly used sequential quadratic programming method.

For future work, further analysis can be done for this cone programming reformulation, for example, the specific reason why this can be faster than the original way. Also, some numerical issues will occur when a cone programming solver is used to solve the reformulated problem without the reorganization of the CVX toolbox, the reasons for this are not fully revealed yet. In many situations, the organization of cone programming problem is more numerically stable when using the CVX toolbox. But using CVX toolbox will require a relatively long time spent on reorganizing the formulations compared with calling the SeDuMi solver directly. Resolving these numerical issues can increase the reliability of the algorithm and reduce the computation time required. In the future, the physical meanings and some mathematical structures in the original problem can also be explored more to improve the efficiency of the general algorithms for this specific active noise control problem. If the efficiency of this algorithm can be further improved, it is possible to consider making this filter design method adaptive.

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