An adaptive constrained multi-channel ANC filter design approach using convex cone optimization

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- Methods
  - Review of traditional constrained ANC methods
  - Proposed online constrained optimization method
- Results
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Backgrounds

For wider applications:

Interior of Vehicles

Air Conditioner

Backgrounds

Three of the challenges when applying ANC to wider applications:

- Time-varying environment, especially for changing signal characteristics
  - **Require adaptive controllers**
- Controller should be stable and robust
  - **Require constraints on controllers**
- Larger quiet zone
  - **Require multi-channel systems**

- **Lower convergence rate** due to coupling in multi-channel systems
- **Complicated constraints** in multi-channel systems
- **Significant computational load**

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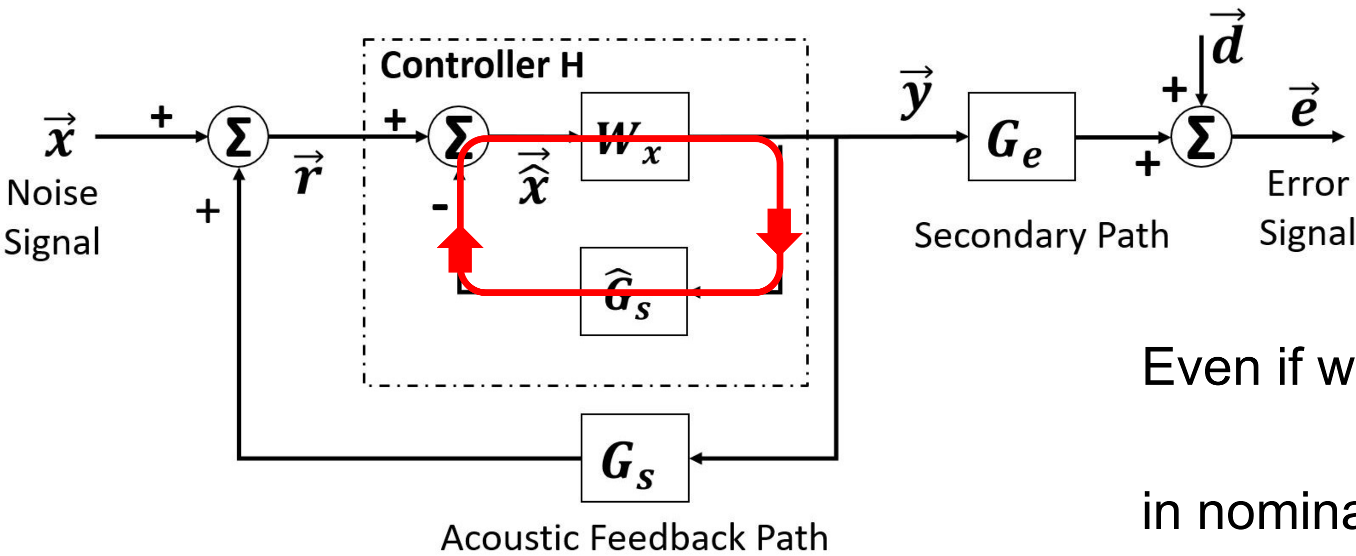
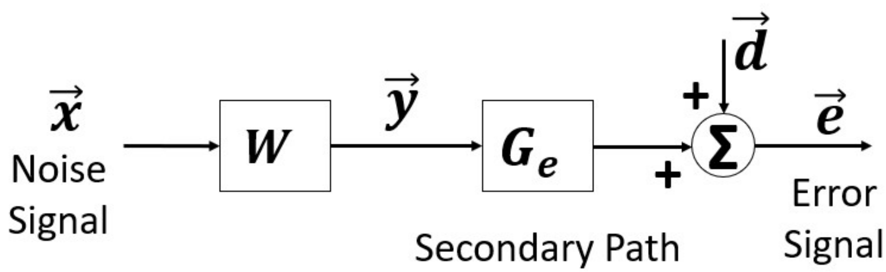
Methods – Basic block diagram



If $\hat{G}_{s_0} = G_{s_0}$



Even if we assume:

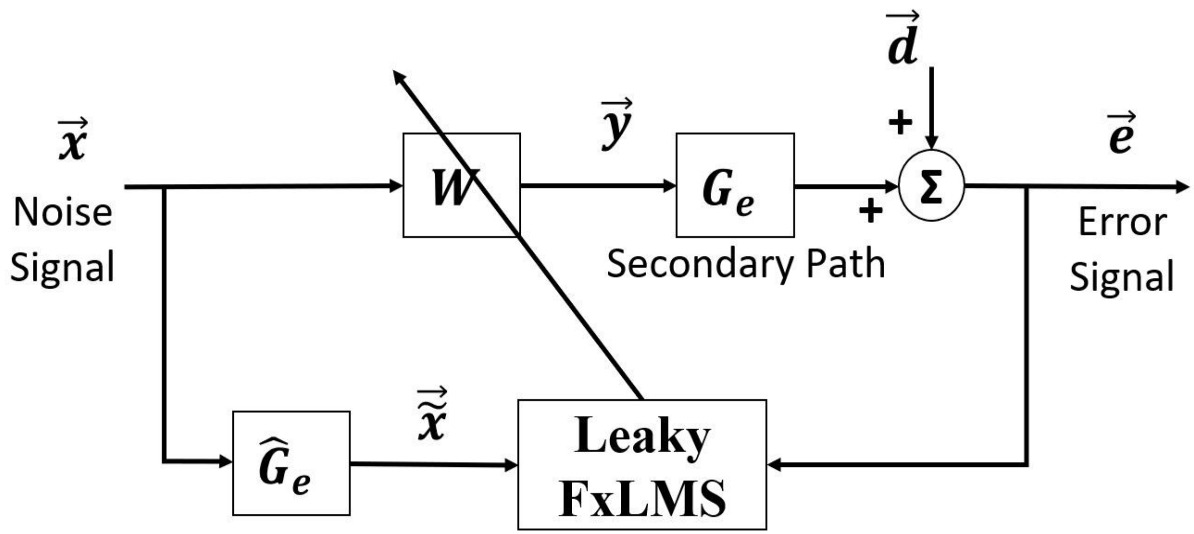
$$\hat{G}_{s_0} = G_{s_0}$$

in nominal operating condition.

The stability problem caused by the closed loop $W\hat{G}_s$ should still be considered.

Methods – Traditional Leaky FxLMS

Traditionally, the leaky FxLMS can be used



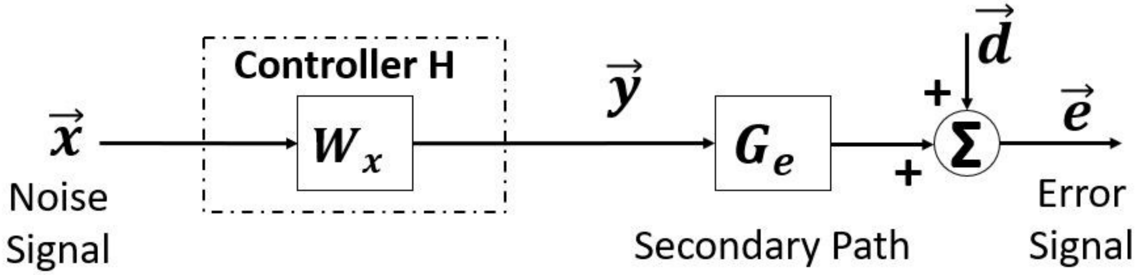
- When leakage factor $\boldsymbol{\beta}$ is large enough, most of the common constraints on controllers can be satisfied

- However, the designed controller can be **over-conservative** and sacrifices the ANC performance

$$w_{i,j,k}^{(n+1)} = w_{i,j,k}^{(n)} - \underbrace{\boldsymbol{\alpha}}_{\text{Step size}} \left(\sum_{l=1}^{N_e} \tilde{x}_{i,j,l}(n-k) e_l(n) + \underbrace{\boldsymbol{\beta}}_{\text{Leakage factor}} w_{i,j,k}(n) \right)$$

Methods – Traditional constrained optimization method

Alternatively, a constrained optimization problem can be formulated and solved to obtain the filter coefficients



- Good ANC performance for **non-adaptive** cases
- However, **significant computational load** prevents it from applying to adaptive controllers

Cost function:

$$\sum_{k=k_1}^{k_2} tr[E(f_k)E(f_k)^{\mathrm{H}}]$$

➡ **Total power of $\vec{e}$ cross all frequencies**

Stability constraints:

Acoustic feedback path

$$\min\left(\mathrm{Re}\left(\lambda\left(\boldsymbol{W}(f_k)\widehat{\boldsymbol{G}}_s(f_k)\right)\right)\right) > -1$$

➡ **Nyquist criterion, on the right of -1 point**

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Methods – Proposed constrained optimization method

Improvements were proposed by us on the constrained optimization method:

Zhuang and Liu, JASA 2021:

- Proposed a convex formulation from traditional constrained optimization problem for ANC filter design
- The **computational time can be reduced from the order of hours to seconds**

JASA ARTICLE

CrossMark

Constrained optimal filter design for multi-channel active noise control via convex optimization

Yongjie Zhuang[a] and Yangfan Liu[b]
Ray W. Herrick Laboratories, School of Mechanical Engineering, Purdue University, West Lafayette, Indiana 47907, USA

Zhuang and Liu, JASA 2022:

- A numerically stable formulation using dual form is proposed based on the previous convex formulation
- **Improves both the numerical efficiency and stability**

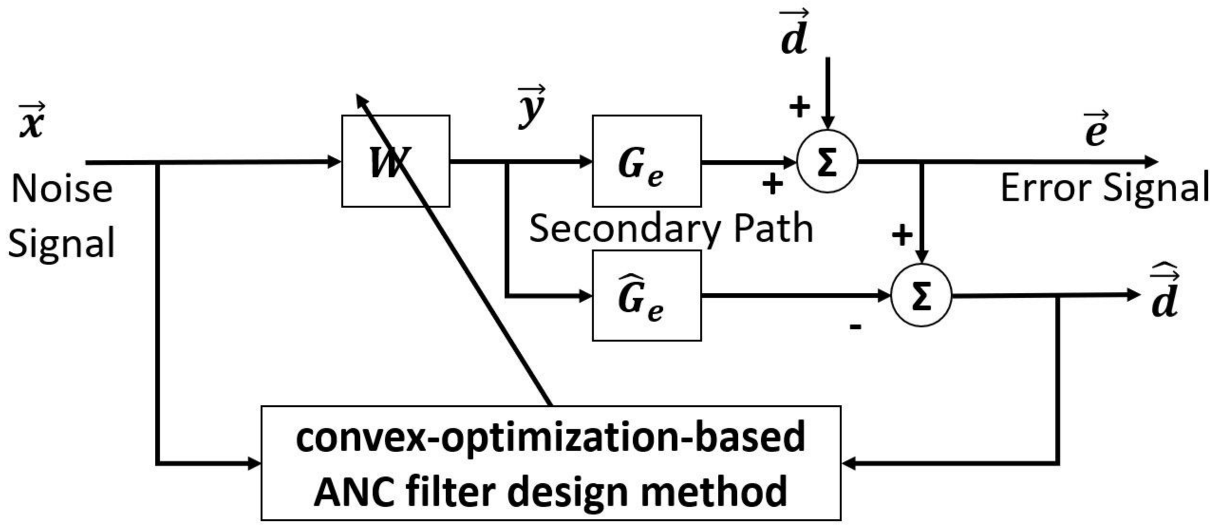
JASA ARTICLE

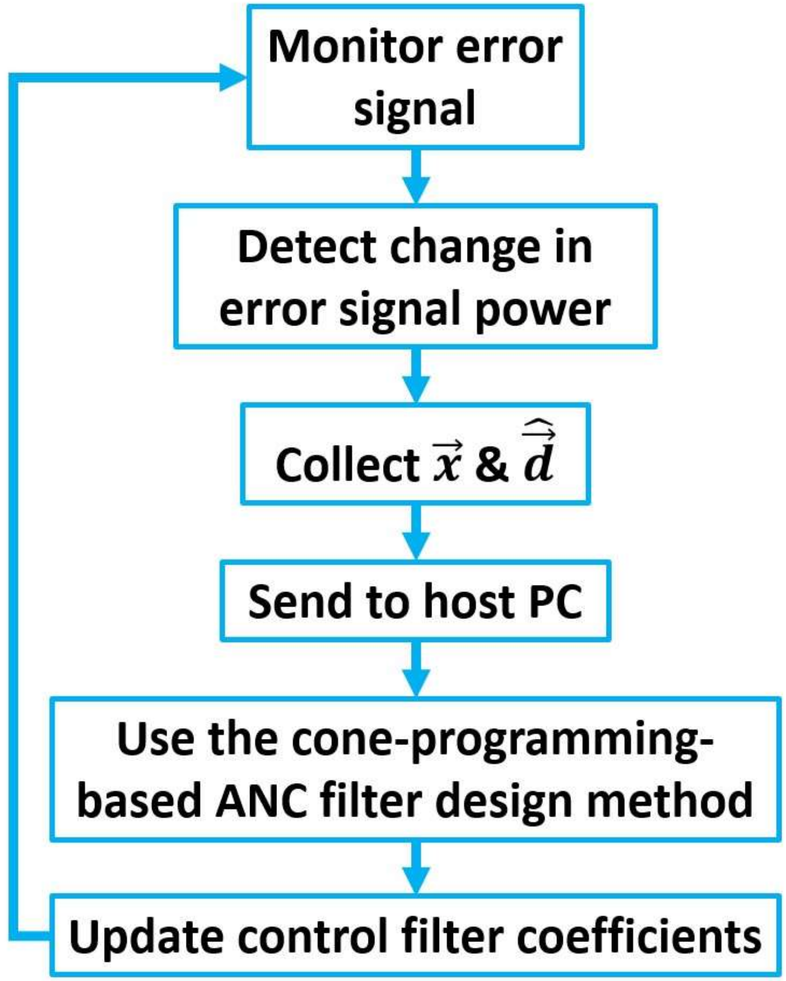
CrossMark

A numerically stable constrained optimal filter design method for multichannel active noise control using dual conic formulation

Yongjie Zhuang and Yangfan Liu[a]
Ray W. Herrick Laboratories, Mechanical Engineering, Purdue University, West Lafayette, Indiana 47907, USA

Methods – Proposed constrained optimization method





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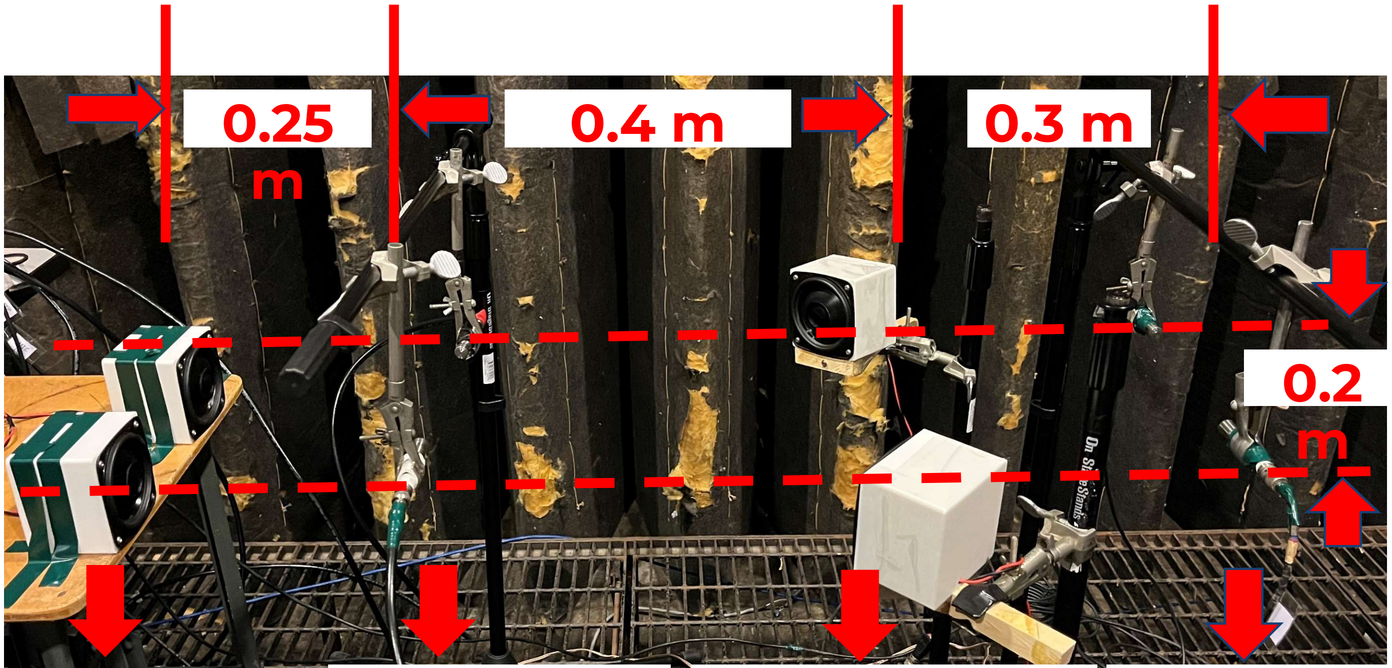
Results – Experimental setup



Two noise sources

Two reference microphones

Two secondary sources

Two error microphones

Sampling rate:

- DAQ : 9 kHz
- Controller : **3 kHz**

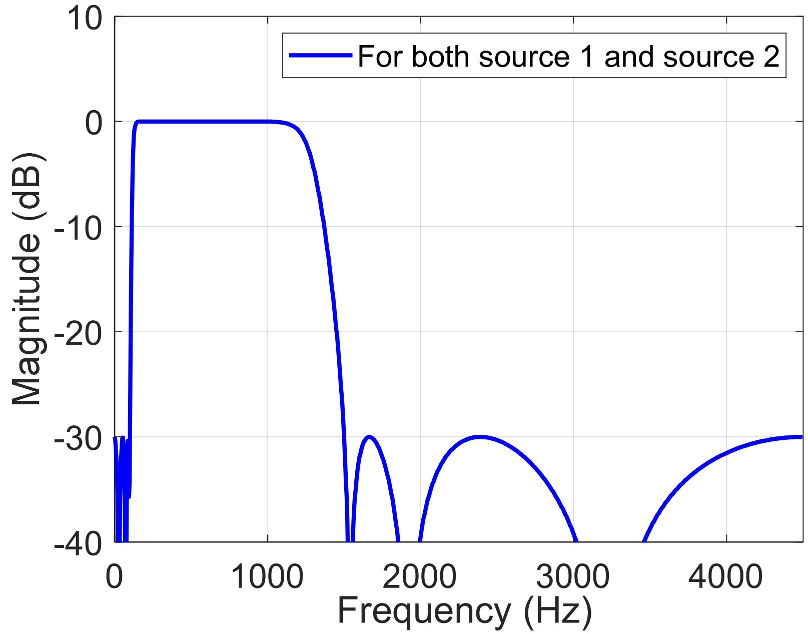
In each channel, filter length is **64** for:

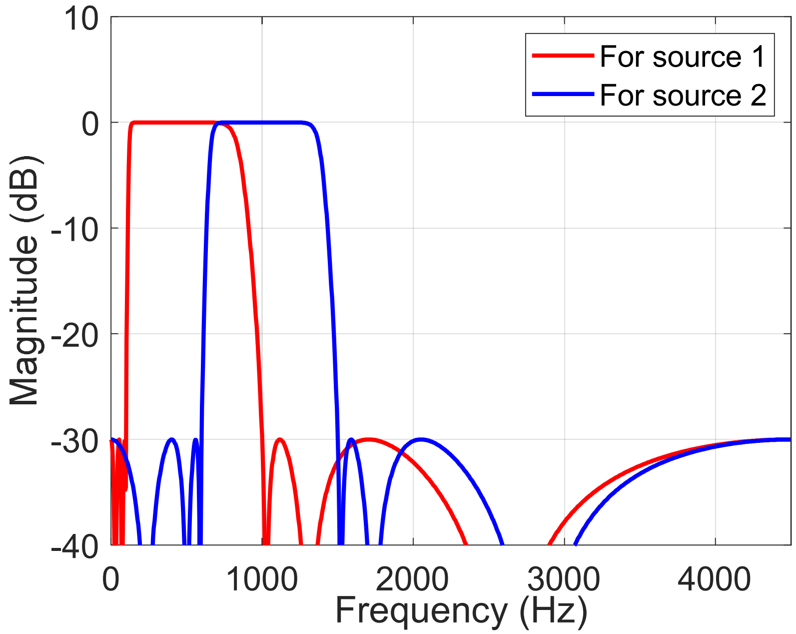
- ANC control filter
- Estimated secondary path
- Estimated acoustic feedback path

Results – Two types of sources – Full-band & Half-band cases

- Two independent white noises are generated digitally first
- Then they were filtered as the inputs for two noise sources

Magnitude of frequency responses of filters used for noise sources



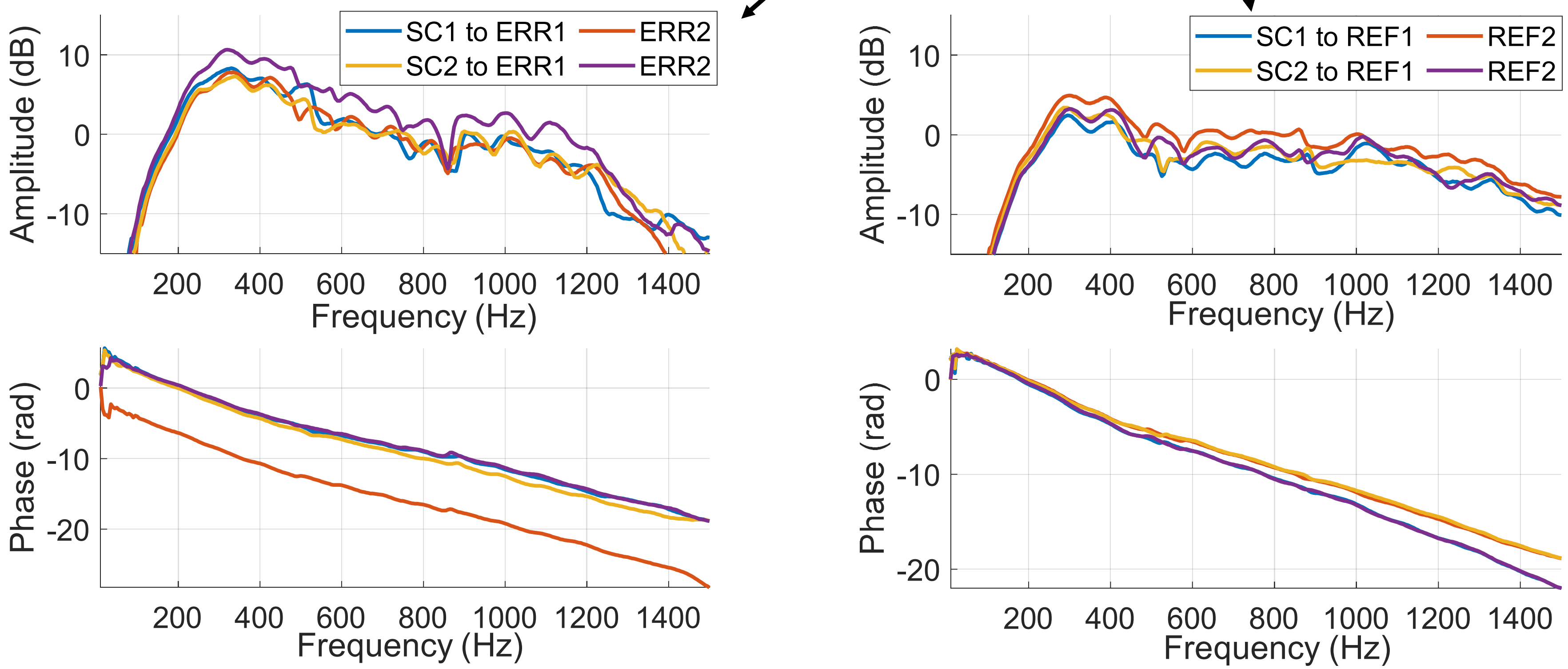


Full-band case:
both noise sources signal from 100 – 1450 Hz

Half-band case:
source 1: 100 – 950 Hz, **source 2**: 600 – 1450 Hz

Results – Measured transfer paths

Frequency responses of measured **secondary paths** and **acoustic feedback paths**



Acoustic feedback paths are **strong** compared with secondary paths

➡ **stability constraints are needed!**

Results – Choice of parameters

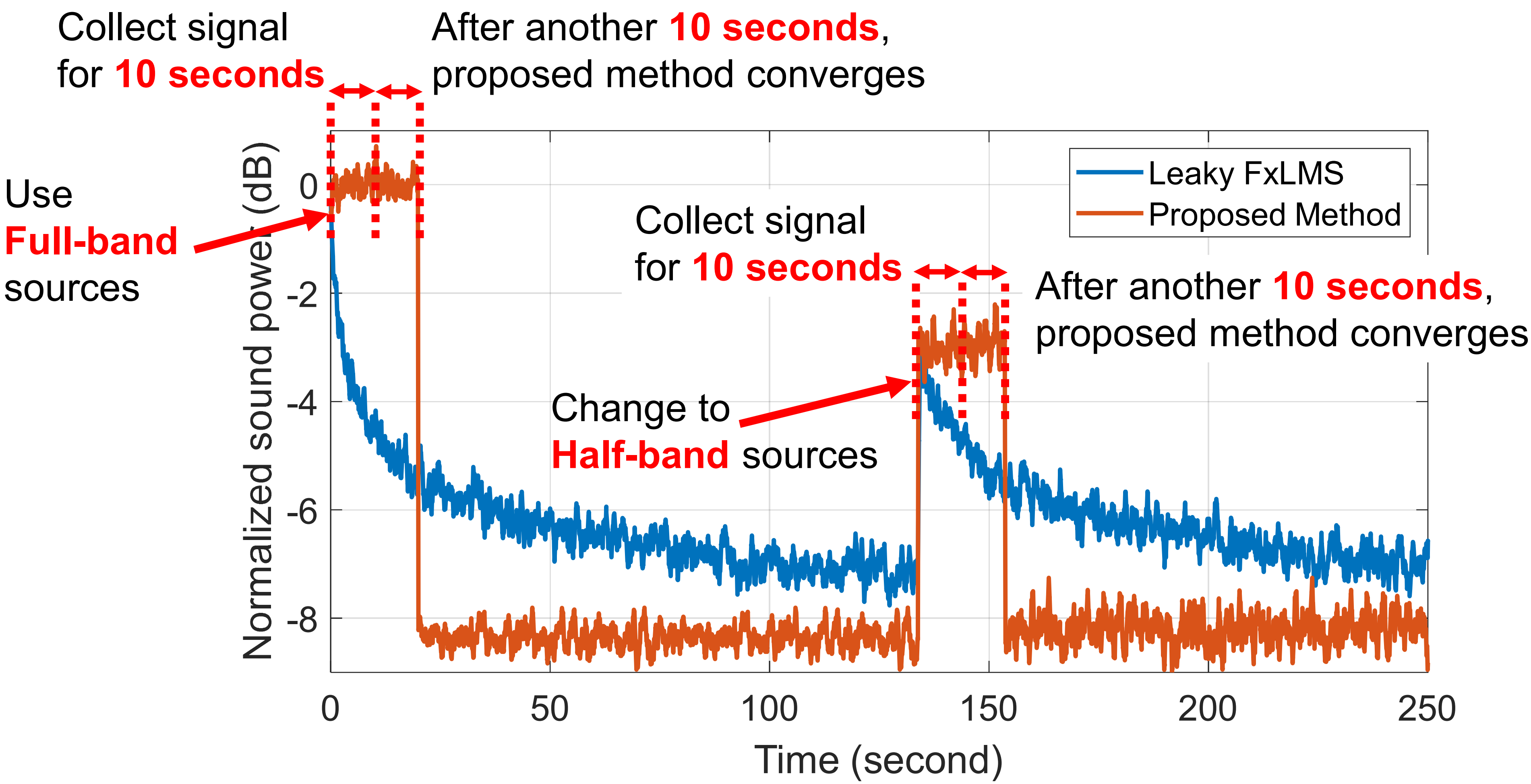
Parameters in leaky FxLMS

Parameters	Value	Reason
Leakage factor $\boldsymbol{\beta}$	1×10^{-5}	Tuned to get the smallest value that satisfy stability
Step length $\boldsymbol{\alpha}$	0.1	Tuned to get the largest value that satisfy convergence

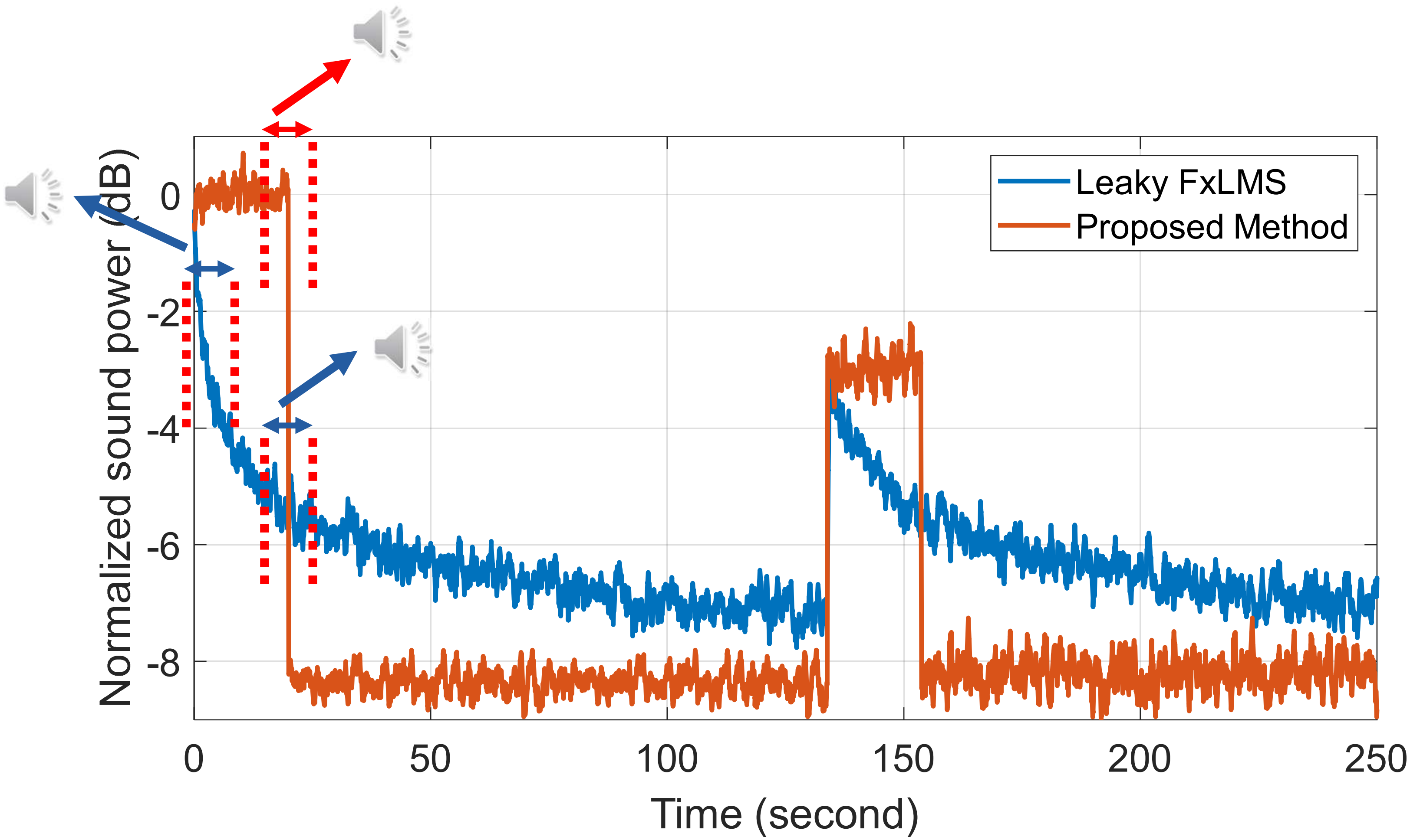
Parameters in proposed method

Parameters	Value	Reason
Data collection interval	10 seconds	To obtain enough data to compute signal spectrum
Filter coefficients updating interval	10 seconds	Time needed for the algorithm to converge
Frequency resolution	5 Hz	A fine resolution for better performance

Results – Total noise power in the time domain

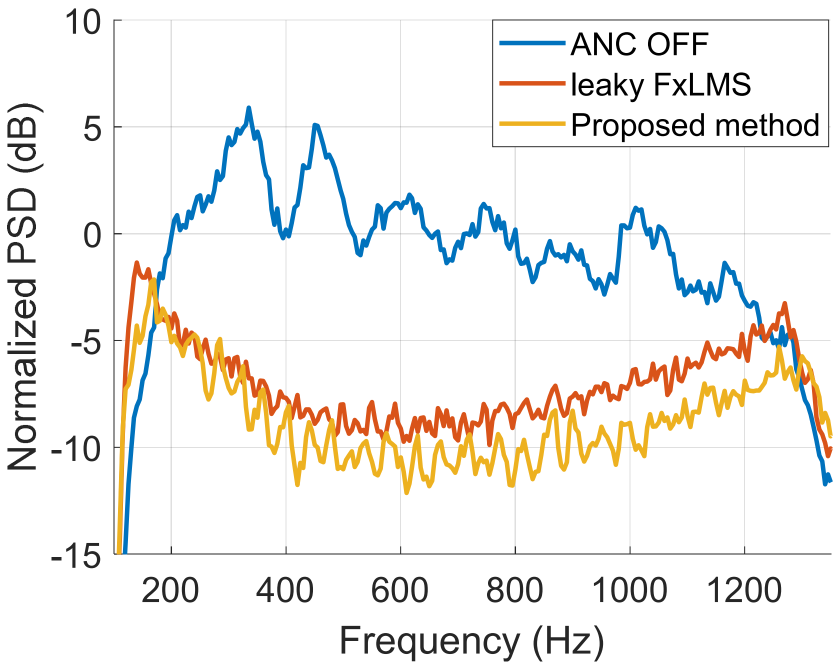


Results – Total noise power in the time domain

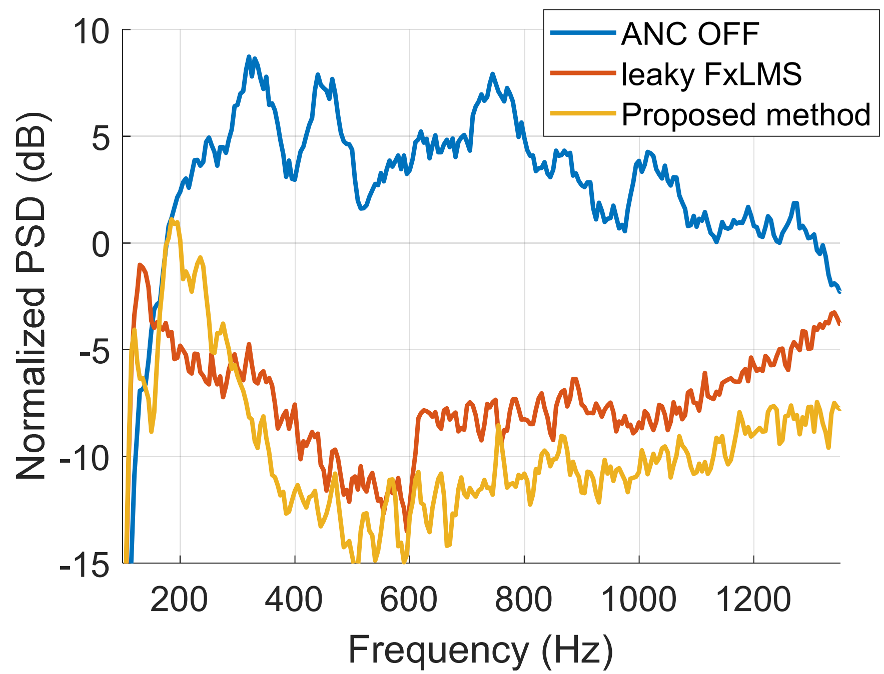


Results – Total noise power in the frequency domain

Full-band sources case



Half-band sources case



The proposed method has **better ANC performance** because it is less conservative in constraints.

Results – Analysis on computational time

Test 100 cases in host PC for 3-sigma limit (99.7%)

Process	CPU time (seconds)
Solving constrained optimization using proposed formulation	5.0 ± 1.8 seconds
Computing spectrum from collected data	0.19 ± 0.16

Maximum equivalent multiplications per sampling interval:

3 GHz × 5 s / 20 / (10 × 3000 Hz) = **25k**

- 3 GHz ⬇ **CPU clock speed**
- 5 s ⬇ **5 seconds in average**
- 20 ⬇ **20 cycles to do 64-bit multiplication once**
- (10 × 3000 Hz) ⬇ **Total number of sampling intervals**

The leaky FxLMS adaption part need 1536 multiplications per sampling interval, but it takes **12 times longer time** to converge, $1536 \times 12 \approx$ **18 k**

The total required computational power is not significantly different

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Summary

- A constrained adaptive ANC design method via online convex optimization is proposed

- Compared with traditional leaky FxLMS, the proposed method **converges faster** and has **better ANC performance**

- The proposed method can be suitable for cases where:
  - Signal characteristics change stage by stage, e.g., variable-speed HVAC systems, vehicles
  - Various products can share a host server, e.g., vehicles, smart home/office application

Questions?