Universal Partitioning of a Large Array for Communications in Environments with Limited Spatial Coherence

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***Abstract*—Large aperture arrays improve communication performance with higher gain, but are susceptible to limited spatial coherence of signals, especially in shallow water acoustic environments. An acoustic communication receiver's ability to combine the signal wavefront coherently degrades when there are losses across the array in either phase coherence or the ability to track phase variation. To mitigate such coherence issues, the array can be separated into smaller segments of sensors known as subapertures. The phase variations can be tracked and mitigated on each subaperture. Subsequently, each subaperture can be coherently combined to realize the full array gain, through this two-stage process. However, subaperture size selection remains an open problem, especially in dynamic, uncertain environments like those experienced in underwater acoustic communications. In this paper, a universal algorithm is proposed for realizing the performance of the best possible partitioning among a collection of partitions of the receiver into subapertures, such that the overall receiver performance is as good as if the best partitioning were known a priori. This work builds on previous work in universal adaptive filtering and beamforming.**

***Index Terms*—Universal, subaperture, underwater acoustic communication, phase-locked loop, phase coherence, bit error ratio, mixture of experts**

I. Introduction

Underwater acoustic (UWA) communication applications often exhibit low signal-noise-ratios (SNRs) and dynamic time-varying channel characteristics [1], especially when the source-receiver distance is large, the signal exhibits substantial multipath (such as in shallow water), and the relative signal bandwidth is high [2]. Multiple receivers are often placed in an array to provide SNR gain and improve the performance of the communication algorithm [3], [4].

A commonly-used architecture for this multichannel communication framework employs a multiple-input decision feedback equalizer (DFE), in which a feedforward (FF) filterbank is applied to the full receiver array, which is then phase-compensated using a phase-locked loop (PLL) [5] to form an estimate of the transmitted symbols. These estimates are then quantized to the nearest symbol in the transmission alphabet, and linear combinations of previously decided symbols are used to improve the phase-compensated, feedforward filter outputs. This framework is susceptible to limited spatial phase-coherence of received signals, especially in shallow water acoustic environments. The phase-coherence tends to degrade across larger arrays, due to calibration errors, sensor position variation, sound speed variations, and variations in the acoustic path from the source, for example. An acoustic communication receiver's ability to combine the signal wavefront coherently degrades when there are losses across the array in either phase coherence or the ability to track phase variation. Thus, another commonly used multichannel framework is to implement a phase-locked loop (PLL) for each sensor output before or after applying the feedforward filterbank [1]. The disadvantage of this framework is that the PLL cannot benefit from the SNR gain when using multiple sensors.

An improvement on the abovementioned frameworks is to partition the receiver array into smaller segments of sensors known as subapertures or subarrays. The phase variations can be tracked and mitigated on each subaperture using a dedicated PLL. Subsequently, each subaperture can be coherently combined to realize the full array gain, through this two-stage process. However, selecting sub-aperture size remains an open problem, especially in dynamic, uncertain environments like the UWA environment. In this paper, a universal algorithm is proposed for realizing the performance of the best possible partitioning of the receiver into coherent subapertures, such that the overall receiver performance is as good as if the best partitioning were known a priori. This work builds on previous work in universal adaptive filtering [6] and beamforming [7]. The basic idea of such universal approaches is to permit the data to blend the outputs of a variety of different candidate algorithms or parameter choices, based on their individual performance. In doing so, the underlying mathematical framework enables performance guarantees to be derived, and explicitly bounded, for a broad class of problems. This approach has also been successfully applied to various applications including active noise control [8], power spectral density estimation [9], portfolio management [10], and a variety of prediction problems, including nonlinear [11] and time-varying predictor structures [12].

II. Methodology

A. Review of Traditional Frameworks

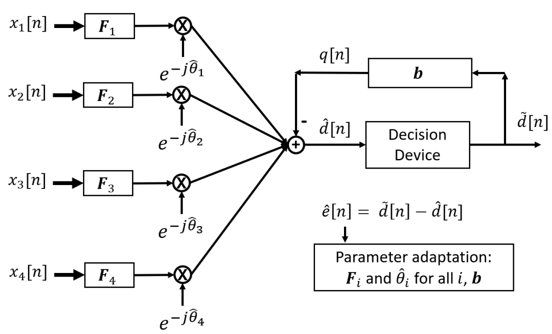
In this article, the transmitted signal is represented in its equivalent complex baseband form as [5]

$$u(t) = \sum_n d[n] g(t - nT), \tag{1}$$

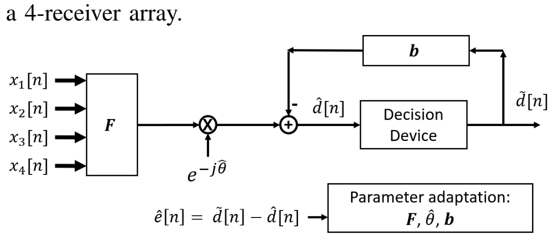
where $d[n]$ are the transmitted data symbols, $g(t)$ is the transmitter equivalent complex baseband pulse, and T is the signaling interval. Suppose there are K receivers placed at different locations, the received signal at kth receiver after appropriate pre-processing such as alignment in time, discretization, and shifting to baseband, can be expressed as

$$x_k[n] = \sum_i d[i] h_k[n - iT_s] e^{j\theta_k[n]} + \nu_k[n], \tag{2}$$

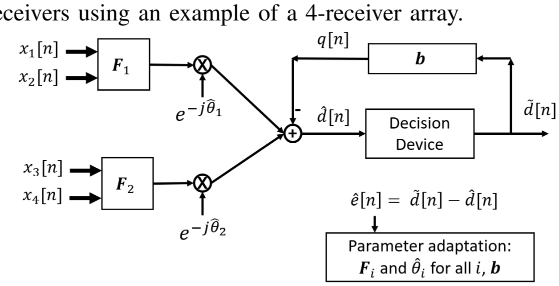
where $h_k[n]$ is the equivalent baseband impulse response of the UWA channel associated with kth receiver, $\theta_k[n]$ is a path-dependent phase variation, and $\nu_k[n]$ is noise which is assumed to be a Gaussian white noise in this article.



(a) Applying PLL to each receiver separately using an example of a 4-receiver array.



(b) Applying PLL to the linearly combined output of all receivers using an example of a 4-receiver array.



(c) Partitioning the receiver array into two subarrays and applying PLL to the linearly combined outputs of each subarray using an example of a 4-receiver array.

Fig. 1: Some examples of using different array partitions in the multichannel adaptive UWA communication.

In general, the received signal $x_k[n]$ should be filtered through FF filters, and then a PLL can be applied to compensate for the phase variation. A FB filter can be used to cancel the intersymbol interference (ISI). More details of this framework are found in Reference [5] for the single-channel example. Some common examples of partitioning arrays for multichannel adaptive UWA communication algorithms are shown in Fig. 1 using the case where the number of receivers $K = 4$. In Fig. 1(a), the 4-sensor array is segmented into 4 subarrays with one receiver in each subarray (we denote it as "1+1+1+1"). Figure 1(b) uses the whole array without partition. Figure 1(c) partitions the array into 2 sub-arrays with 2 receivers in each sub-array. The selection of the optimal array partition depends on the environment, especially the phase variation discrepancy between different sensors, which is usually unknown. Thus, selecting an appropriate array partition remains a challenging problem in practical UWA applications.

B. Proposed Universal Framework

To solve the above-mentioned array partition challenge, we propose a universal multichannel adaptive UWA communication algorithm shown in Fig. 2 using an example of a 4-receiver array, which incorporates the concept of a "mixture of experts". Without loss of generality, suppose that 2 candidate array partitions, "2+2" and "3+1", are considered. It is obvious that this structure can be extended to any number of elements and to any number of candidate array partitions.

An input vector $\mathbf{x}_k$ for the kth receiver can be formed

$$\mathbf{x}_k[n] = \big[x_k[n], \quad x_k[n-1], \quad ..., \quad x_k[n - N_F + 1]\big]^{\mathrm{T}}, \tag{3}$$

where N_F is the tap length for FF filters. Let the vector $\mathbf{a}_{m,k}$ be the FF filter coefficients associated with the mth candidate partition and kth receiver, we have the FF filter output of lth subarray in mth candidate array partition $y_{m,l}[n]$

$$y_{m,l}[n] = \sum_{k \in \mathbf{S}_{m,l}} \mathbf{a}^{\mathrm{H}}_{m,k} \mathbf{x}_{\mathbf{k}}[n], \tag{4}$$

where $\mathbf{S}_{m,l}$ is the set of receiver indices in lth subarray for the mth candidate array partition. The carrier phase update can then be performed

$$\begin{aligned} p_{m,l}[n] &= y_{m,l}[n] e^{-j\hat{\theta}_{m,l}} \\ p_m &= \sum_l p_{m,l} \end{aligned} \tag{5}$$

For each candidate array partition m, an FB filter can be used to cancel the ISI in the summed FF filter output after phase compensation. Let

$$\mathbf{d}_m[n-1] = \big[\tilde{d}_m[n-1], \quad \tilde{d}_m[n-2], \quad ..., \quad \tilde{d}_m[n - N_B]\big]^{\mathrm{T}}, \tag{6}$$

where N_B is the tap length for the FB filter. Then an estimate of the ISI for the mth candidate array partition is

$$q_m[n] = \mathbf{b}^{\mathrm{H}}_m \mathbf{d}_m[n-1]. \tag{7}$$

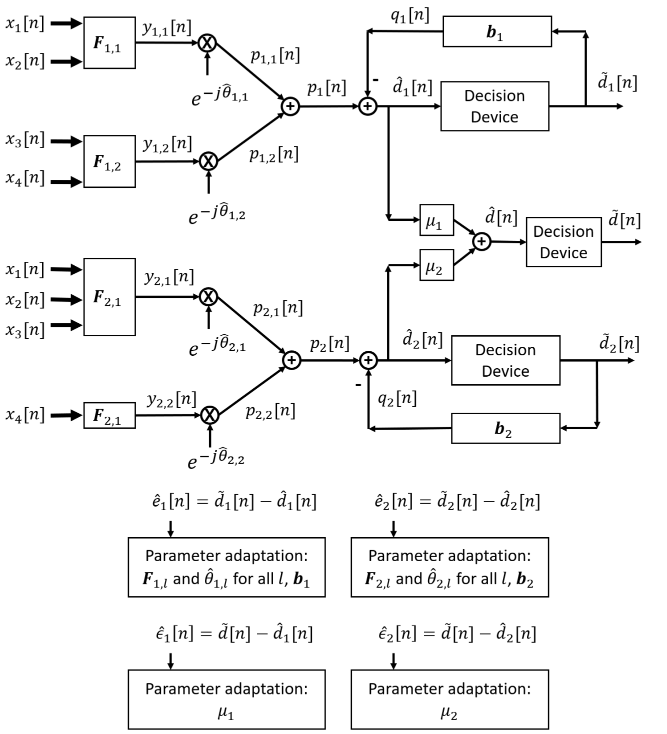


Fig. 2: A demonstration of the proposed universal adaptive multichannel UWA communication algorithms using an example of a 4-receiver array and a selection of two candidate array partitions ("2+2" and "3+1").

Then, the estimate of the data symbol before the decision device for the mth array partition is $\hat{d}_m = p_m[n] - q_m[n]$.

Using a derivation that minimizes the estimated mean square error (MSE) in Reference [5], we can derive the adaptation rule for the parameters. The FF filter, FB filter, and PLL parameters are updated using the locally estimated error for mth candidate partition $\hat{e}_m[n] = \tilde{d}_m - \hat{d}_m$. In the training stage, when $d[n]$ is known, we can replace $\tilde{d}_m$ with the known sequence $d[n]$. For the FF filter parameters of kth receiver partitioned to the lth subarray in the mth partition:

$$\mathbf{a}_{m,k}^{(n+1)} = \mathbf{a}_{m,k}^{(n)} + \alpha \mathbf{x}_k[n] \hat{e}_m[n]^* e^{-j\hat{\theta}_l} \tag{8}$$

where α is the step size. The phase estimate for the lth subarray in the mth partition is [5]

$$\hat{\theta}_{m,l}^{(n+1)} = \hat{\theta}_{m,l}^{(n)} + K_{f_1} \Phi_{m,l}^{(n)} + K_{f_2} \sum_{i=0}^{n} \Phi_{m,l}^{(i)}, \tag{9}$$

where $\Phi_{m,l}^{(n)} = \mathrm{Im}\{p_m[n]\hat{e}_m[n]^*\}$, K_{f_1} and K_{f_2} are the proportional and integral tracking constants [5]. The adaptation of the FB filter coefficients for the mth candidate array partition is

$$\mathbf{b}_m^{(n+1)} = \mathbf{b}_m^{(n)} - \beta \mathbf{d}_m[n-1] \hat{e}_m[n]^*, \tag{10}$$

where β is the step size. Each array partition will give an estimate $\hat{d}_m$ before the decision device. The essence of the proposed universal algorithm is to blend them based on their individual performance. First, we can define global estimated errors $\hat{\epsilon}_m[n] = \tilde{d}[n] - \hat{d}_m$. Similarly, we can replace $\tilde{d}[n]$ with $d[n]$ in the training stage. The cumulative global errors for each candidate partition are

$$E_m[n] = \sum_{i=0}^{n} |\hat{\epsilon}_m[i]|^2. \tag{11}$$

The mixture weights μ_m for the mth partition can be computed using [6]

$$\mu_m[n] = \frac{e^{-\frac{1}{2c} E_m[n-1]}}{\sum_{r=1}^{M} e^{-\frac{1}{2c} E_r[n-1]}}, \tag{12}$$

where M is the total number of candidate array partitions, and c is a hyperparameter that controls the learning rate of universal algorithms. The mixture weights will allow the array partition that performs better in the past to be dominant in the future blended estimate.

III. Results

To investigate the proposed universal algorithm, UWA channel impulse responses and the phase variation extracted from the SPACE'08 dataset are used. $K = 12$ receivers are selected from a 32-element cross array. Quadrature Phase Shift Keying (QPSK) is used for modulation. Two samples per symbol interval are used. One hundred Monte Carlo trials are simulated and averaged to reduce the variance in the simulation results. In each trial, 3k samples were sent for training followed by 27k data samples. The baseband sampling rate is 13020 Hz for the FF filters. The equivalent baseband impulse responses for all receivers are truncated at 160 taps. The magnitude of the UWA baseband impulse responses between the transmitter source and four typical receivers (receivers 1, 4, 7, and 10) are shown in Fig. 3. The impulse responses are assumed to be fixed for the whole training and data transmission stage. The phase variation, on the other hand, is time-varying. The phase fluctuations θ extracted from the SPACE'08 dataset at those example receivers are shown in Fig. 4 and used in the simulation. Gaussian white noise is added to achieve the desired SNR.

In the universal communication algorithm, each FF filter has tap length $N_F = 20$ and each FB filter has tap length $N_B = 80$. The step length α for FF filters is 0.02 in the training stage and 0.005 for the data transmission stage. The step length β for the FB filters is 0.001 for the training stage and 0.0002 for the data transmission stage. In the PLL, $K_{f_1} = 0.005$ and $K_{f_2} = K_{f_1}/10$ for both the training stage and data transmission stage. The learning rate c in the mixture weighting computation is 50.

A set of $M = 6$ candidate array partitions are considered: uniformly partition the 12 receivers into 12, 6, 4, 3, 2, and 1 subarrays respectively (denoted as "cand0", "cand1", ..., "cand5"). Apparently, "cand0" is a similar partition in Fig. 1(a) that uses only 1 receiver in each subarray and "cand5" a similar partition in Fig. 1(b) that uses the whole array. The MSE of using all 6 candidate array partitions and the proposed universal algorithm during the data transmission stage when

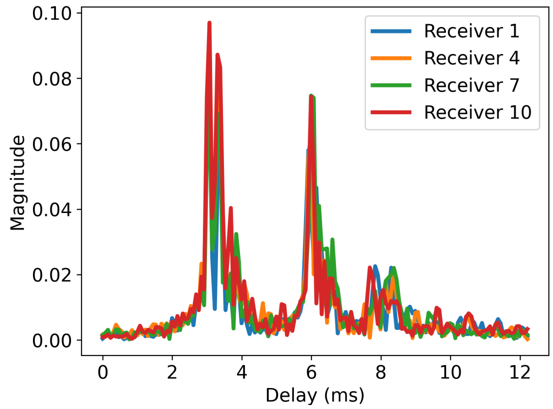


Fig. 3: Magnitude of UWA baseband impulse responses between the transmitter source and some example receivers.

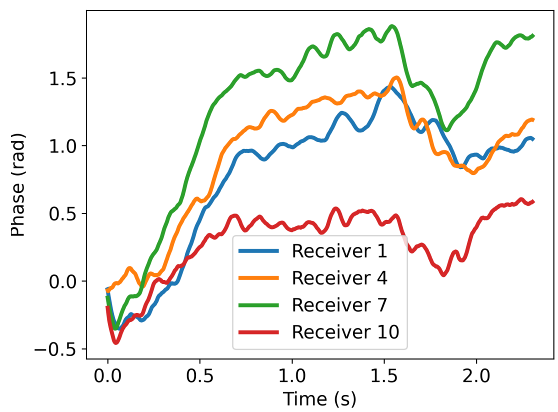


Fig. 4: Phase fluctuation in time at some example receivers.

averaged single receiver SNR = 6 dB is shown in Fig. 5. The figure clearly demonstrates that using all 12 receivers in an FF filter and using one PLL ("cand5") cannot achieve a satisfactory MSE. This also matches with the observation from Fig. 4 that different receivers have different phase variation patterns. The proposed universal algorithm can indeed converge to the best candidate when measured by the MSE. The universal algorithm can even provide a performance improvement at the early stage (less than 10k samples) by effectively blending different candidate array partitions.

The bit error ratio (BER) of using all 6 candidate array partitions and the proposed universal algorithm during the data transmission stage at SNR range from 0 dB to 6 dB is shown in Fig. 6. The proposed universal algorithm achieves a lower BER compared with each of the different candidate array partitions.

IV. Conclusion

Selecting an optimal array partition for subaperture processing in a spatial phase coherence-limited environment is challenging. Instead of pre-selecting an array partition, we proposed to use a universal algorithm that blends outputs of

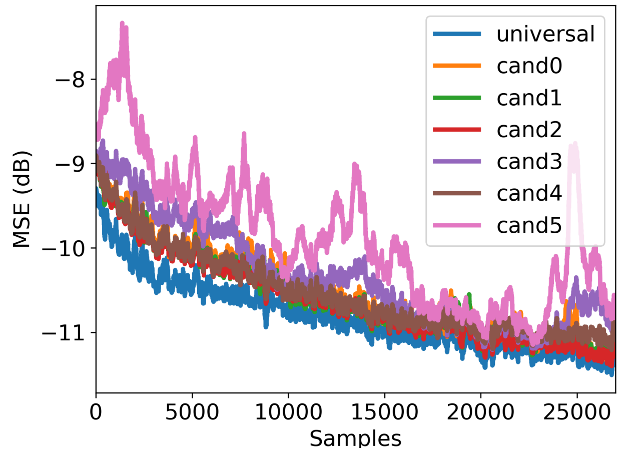


Fig. 5: The MSE of using all 6 candidate array partitions and the proposed universal algorithm during the data transmission stage when averaged single receiver SNR = 6 dB.

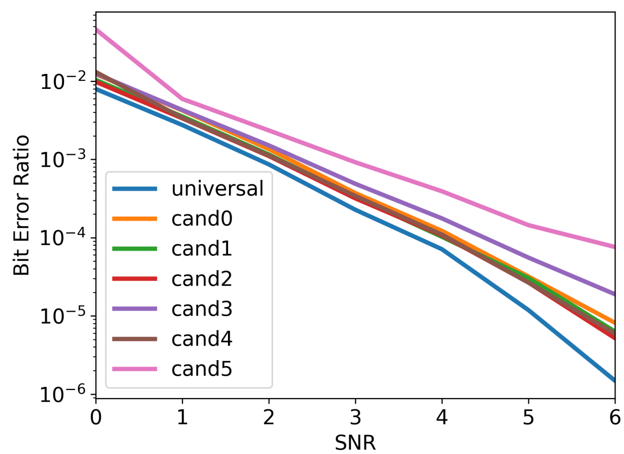


Fig. 6: The bit error ratio of using all 6 candidate array partitions and the proposed universal algorithm during the data transmission stage.

all candidate array partitions based on their past performance in real time. The algorithm structure and parameter adaptation rules are given. Simulation results based on the SPACE'08 dataset are used to test the proposed universal algorithm. The results demonstrate that the universal algorithm can effectively blend the outputs of different array partitions and achieve better MSE and BER.

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REFERENCES

[1] A. C. Singer, J. K. Nelson, and S. S. Kozat, "Signal processing for underwater acoustic communications," *IEEE Communications Magazine*, vol. 47, no. 1, pp. 90–96, 2009.

[2] M. Stojanovic and J. Preisig, "Underwater acoustic communication channels: Propagation models and statistical characterization," *IEEE communications magazine*, vol. 47, no. 1, pp. 84–89, 2009.

[3] A. Song, M. Badiey, and V. K. McDonald, "Multichannel combining and equalization for underwater acoustic mimo channels," in *OCEANS 2008*. IEEE, 2008, pp. 1–6.

[4] M. Stojanovic and Z. Zvonar, "Multichannel processing of broad-band multiuser communication signals in shallow water acoustic channels," *IEEE Journal of Oceanic Engineering*, vol. 21, no. 2, pp. 156–166, 1996.

[5] M. Stojanovic, J. A. Catipovic, and J. G. Proakis, "Phase-coherent digital communications for underwater acoustic channels," *IEEE journal of oceanic engineering*, vol. 19, no. 1, pp. 100–111, 1994.

[6] A. C. Singer and M. Feder, "Universal linear prediction by model order weighting," *IEEE Transactions on Signal Processing*, vol. 47, no. 10, pp. 2685–2699, 1999.

[7] J. R. Buck and A. C. Singer, "A performance-weighted blended dominant mode rejection beamformer," in *2018 IEEE 10th Sensor Array and Multichannel Signal Processing Workshop (SAM)*. IEEE, 2018, pp. 124–128.

[8] Y. Zhuang, M. Mittal, R. Fernando, R. M. Corey, and A. C. Singer, "Active noise control without tap length selection: a model order weighting method," in *INTER-NOISE and NOISE-CON Congress and Conference Proceedings*, vol. 269, no. 2. Institute of Noise Control Engineering, 2024, pp. 902–913.

[9] D. C. Anchieta and J. R. Buck, "Robust power spectral density estimation via a performance-weighted blend of order statistics," in *Proceedings of Meetings on Acoustics*, vol. 52, no. 1. AIP Publishing, 2023.

[10] S. S. Kozat, A. C. Singer, and A. J. Bean, "Universal portfolios via context trees," in *2008 IEEE International Conference on Acoustics, Speech and Signal Processing*. IEEE, 2008, pp. 2093–2096.

[11] S. S. Kozat, A. C. Singer, and G. C. Zeitler, "Universal piecewise linear prediction via context trees," *IEEE Transactions on Signal Processing*, vol. 55, no. 7, pp. 3730–3745, 2007.

[12] S. S. Kozat and A. C. Singer, "Universal switching linear least squares prediction," *IEEE Transactions on Signal Processing*, vol. 56, no. 1, pp. 189–204, 2007.