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Warmstarting the constrained optimal filter design problem for active noise control systems in conic formulation

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ABSTRACT

In practical active noise control (ANC) systems, constraints such as stability, need to be considered in the controller design process. The optimal control filters can be obtained by solving a constrained optimization problem, which requires a significant computational effort. Recently, a convex formulation in conic form was proposed for ANC applications which was shown to result in a computational time reduction by several orders. It is desirable to further improve its efficiency so that the optimal filter design process can be continuously repeated to achieve adaptive control for slow varying operating conditions. One potential way is to introduce a warmstart technique where the filter solution of a similar system or environment is used as the starting point of the optimization algorithm. However, the conic formulation should be solved by the interior-point method which, in general, is challenging for applying warm start techniques. In the current work, modifications are proposed to the original ANC filter design formulation so that the warmstart techniques can be applicable. The performance of warmstarting technique is investigated. Results show that an appropriate choice of warmstart strategy can significantly reduce the number of iterations required for solving the proposed conic formulation of ANC filter design problem.

1. INTRODUCTION

In practical active noise control (ANC) applications, various types of constraints may be needed. For example, robust stability constraints, filter response constraints, and disturbance enhancement

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constraints [1]. When one constraint is required, one commonly used constrained ANC filter design method is to introduce a regularization term in the filter coefficient optimization process [1], which can be extended to leaky LMS method [2, 3]. However, when multiple constraints are required, filter coefficients obtained by solving a constrained optimization method formulated using H_2/H_∞ method can achieve better noise control performance [4].

An initial point is usually required due to the iterative nature of solving constrained optimal ANC filter. So far, cold start approach is usually used, i.e., no a priori information of the filter design problem is considered while choosing the initial point. For example, starting from origin when using leaky LMS, or starting from points well centered at feasible set when using constrained optimization method [4–6]. In many practical ANC applications, a reasonable guess of the optimal filter coefficients may be known. For example, in commercial product design, some product model is a variation of previous model, or a specific product is an variation to an prototype product due to batch manufacture process error. In these cases, if the optimal filter coefficients of one product are known, it can be used as a initial guess of the optimal filter coefficients of other products. If the initial point is chosen according to some priori information, it is called the warmstarting method.

Recently, Zhuang and Liu proposed a convex formulation and cone programming reformulation of ANC filter design problem [4, 7, 8] and demonstrated that the computational time can be reduced significantly from the order of hours to seconds [4, 7]. However, that method used the interior point methods (IPM) where warmstarting strategies are difficult to apply because the priori information usually leads to an initial point that is close to feasible set boundaries or even an infeasible point and causes numerical problems [6]. This limits their proposed method on the applications where approximate value of optimal filter coefficients are known. On the other hand, when the constrained optimization method is applied on time-varying applications, the optimization problem associated with current adaptation iteration can be treated as a perturbed problem with respect to that of a previous adaptation iteration. The optimal solution between two adaptation iterations should be similar to each other such that the previous optimal solution can be used to obtain a warm-start initial point for the current iteration.

In previous studies, the warmstarting strategies for primal-dual IPM have been investigated. Xia [9] proposed a warmstart strategy based on semismooth Newton's method, which requires the complementary conditions to be transformed into equations. In Gondzio and Grothey's work [10], a method leading the iterations back to feasible region was proposed, which relies on sensitivity analysis on the Newton step. Skajaa et al. [11, 12] proposed a concise strategy with convex combination of the cold start point of current problem and the optimal solution of a similar problem.

In this article, the convex formulation and cone programming reformulation for ANC filter design problem proposed by Zhuang and Liu [4, 7, 8] is firstly adopted. The warmstarting strategy for homogeneous and self-dual IPM proposed by Skajaa et al. [12] is combined with the cold start initial point used in SDPT3 [13] as the warm-starting method for conic form ANC optimization problem. The strategy proposed by Skajaa [12] is mainly used for linear programming (LP) and second-order cone (SOC) programming, but the conic form ANC problem proposed by Zhuang and Liu [4, 7, 8] contains positive semidefinite (PSD) cones reformulated from the stability and robustness constraints. Thus, it is also proposed to reformulate the stability and robustness constraints as SOC. The result demonstrated that compared with solving ANC optimization problem with mixed SOC and PSD cones, the warm-starting strategy has a better performance when solving the ANC optimization problem with only SOC. The warmstarting performance of using different warmstarting ratio on problems with different perturbation is also investigated.

2. THEORY

In this section, the multiple-input-multiple-output (MIMO) ANC control system, the H_2/H_∞ framework formulation for constrained optimal filter design, and the conic formulation for improving computational efficiency are reviewed. The formulation in this article is based on a feedforward control structure. The feedback control structure can be formulated in a similar way if an internal model control structure (IMC) is used [14].

2.1. ANC System Diagram and Notation

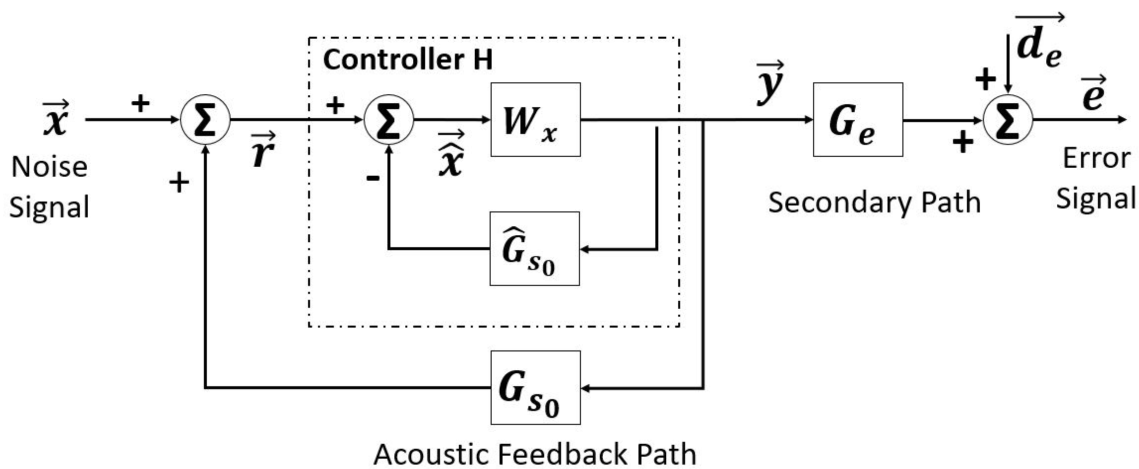

Figure 1: Block diagram of the MIMO feedforward controllers with the acoustic feedback path and internal model control structure. [4]

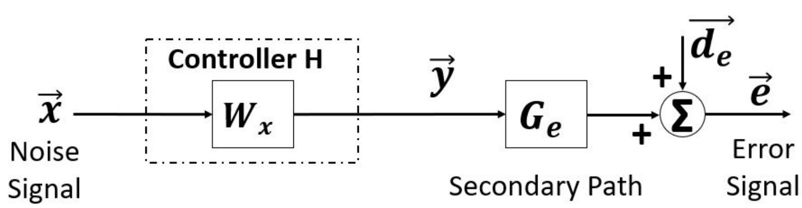

Figure 2: Block diagram of the MIMO feedforward controllers in a standard feedforward form when assuming internal model control structure perfectly cancels the acoustic feedback path effect. [4]

The MIMO ANC feedforward control diagram is shown in Fig. 1. There are N_r microphones, N_s loudspeakers, and N_e microphones used as reference sensor, secondary sources, and error sensors respectively. $\vec{\mathbf{x}}$ denotes the primary sound field signals at reference sensor locations. $\vec{\mathbf{r}}$ denotes the signal obtained from the reference microphones when the ANC system is activated. $\vec{\mathbf{y}}$ is the output of the ANC controller. $\vec{\mathbf{d}}_e$ is the disturbance signal. $\vec{\mathbf{e}}$ denotes the error signal, which is attenuated by the ANC system. $\mathbf{G}_{s0}$ is the acoustic feedback path matrix that represents the acoustic responses of secondary sources at the reference sensors. To estimate the primary noise signals, $\vec{\mathbf{x}}$, from the reference signals $\vec{\mathbf{r}}$, an internal model cancellation (IMC) structure is used to cancel the acoustic feedback path effect [1, 14, 15]. When $\hat{\mathbf{G}}_{s_0}$, a model of the physical feedback path $\mathbf{G}_{s_0}$, is assumed to be a perfect model, i.e., $\hat{\mathbf{G}}_{s_0} = \mathbf{G}_{s_0}$, this system is simplified to a standard feedforward system in Fig. 2. $\mathbf{G}_e$ represents the acoustical responses matrix of the secondary sources at the error sensor positions. $\mathbf{W}_x$ is the frequency response matrix of the multi-channel ANC FIR filters, $\mathbf{w}_F$, which need to be designed.

2.2. Review of Convex and Cone Formulation for ANC Design Problem

In practice, sometimes constraints need to be satisfied while designing the control filter $\mathbf{w}_F$ [14]. For example, robust stability constraints due to the acoustic feedback path and IMC structure, disturbance enhancement constraint, and filter response magnitude constraint. One of the commonly used design approach is to solve a constrained optimization problem [14, 15] which requires significant computational power. To reduce the computational load and improve numerical stability of this type of constrained optimization method, Zhuang and Liu [4, 7, 8] proposed a convex and cone formulation method. The formulation is briefly reviewed in this section, more details can be referred to reference [4, 7, 8].

The multi-channel ANC filter design can be formulated as a convex optimization problem using the H_2/H_∞ framework with some relaxation and reformulation [4]:

$$
\begin{aligned}
\min_{\mathbf{w}_F}. \quad & \vec{\mathbf{w}}^{\mathrm{T}} \left(\textstyle\sum_{k=1}^{N_f} \mathbf{A}_J(f_k) \right) \vec{\mathbf{w}} + \textstyle\sum_{k=1}^{N_f} \vec{\mathbf{b}}_J^{\mathrm{T}}(f_k)\, \vec{\mathbf{w}} && \text{(1a)} \\
\text{s.t.} \quad & \vec{\mathbf{w}}^{\mathrm{T}} \mathbf{A}_J(f_k) \vec{\mathbf{w}} + \vec{\mathbf{b}}_J^{\mathrm{T}}(f_k)\, \vec{\mathbf{w}} + \tilde{c}_J(f_k) \le 0, && \text{(1b)} \\
& \|\vec{\mathbf{F}}_z^{\mathrm{T}}(f_k) \vec{\mathbf{w}}_{F_{i,j}}\|_2 - C_{i,j}(f_k) \le 0, && \text{(1c)} \\
& \max \left(\lambda \left(\tfrac{\mathbf{A}_s + \mathbf{A}_s^{\mathrm{H}}}{2} \right) \right) - (1 - \epsilon_s) \le 0, && \text{(1d)} \\
& \max \left(\sigma \left(\mathbf{A}_s \right) \right) B(f_k) - 1 \le 0, && \text{(1e)}
\end{aligned}
$$

for all f_k, i, and j. The objective function is the total power of error signals across all desired frequencies, which can be further reformulated as second-order cone (SOC) in the cone programming formulation [7]. The constraints, Eq. (1b) is introduced to prevent large enhancement at some frequency band. Equation. (1c)[U+FF0C] is used to constrain the amplitude of the control filter response for each channel. Both Eq. (1b) and Eq. (1c) can be further reformulated equivalently to SOC in conic formulation [7]. Equation (1d) and (1e) are applied to ensure the stability and robustness of controller $\mathbf{H}$ respectively, where

$$
\mathbf{A}_s = -\mathbf{W}_x(f_k) \hat{\mathbf{G}}_{s_0}(f_k),
$$

$\lambda()$ and $\sigma()$ denote the eigenvalues and singular values of a matrix, and $B(f_k)$ is the upper bound on the output multiplicative plant uncertainty at frequency f_k. Both Eq. (1d) and Eq. (1e) can be reformulated equivalently to positive-semidefinite (PSD) cones [7]. Thus, Eq. (1) can be further reformulated to cone programming formulation [7, 8, 16]:

$$
\begin{aligned}
\min. \quad & \vec{\mathbf{c}}^{\mathrm{T}} \vec{\mathbf{x}} \\
\text{s.t.} \quad & \mathbf{A} \vec{\mathbf{x}} = \vec{\mathbf{b}} \\
& \vec{\mathbf{x}} \in K,
\end{aligned} \tag{2}
$$

where K is a Cartesian product of various SOC and PSD cones. More details on this conic formulation can be refereed to Zhuang and Liu's work [7, 8, 16].

2.3. Review of Warmstarting Strategy

The dual problem of Eq. (2) is:

$$\begin{aligned} \max. \quad & \vec{\mathbf{b}}^{\mathrm{T}}\vec{\mathbf{y}} \\ \text{s.t.} \quad & \mathbf{A}^{\mathrm{T}}\vec{\mathbf{y}} + \vec{\mathbf{s}} = \vec{\mathbf{c}} \\ & \vec{\mathbf{s}} \in K, \end{aligned} \tag{3}$$

where $\vec{\mathbf{y}}$ and $\vec{\mathbf{s}}$ are dual variables. To solve the optimization problem Eq. (2), a homogeneous and self-dual model of Eq. (2) and Eq. (3) is formulated as [12]

$$\begin{aligned} \min. \quad & \theta\mu(z^0) \\ \text{s.t.} \quad & \mathbf{A}\vec{\mathbf{x}} - \vec{\mathbf{b}}\tau = \theta r_p(z^0) \\ & -\mathbf{A}^{\mathrm{T}}\vec{\mathbf{y}} - \vec{\mathbf{s}} + \vec{\mathbf{c}}\tau = \theta r_d(z^0) \\ & \vec{\mathbf{b}}^{\mathrm{T}}\vec{\mathbf{y}} - \vec{\mathbf{c}}^{\mathrm{T}}\vec{\mathbf{x}} - \kappa = \theta r_g(z^0) \\ & r_p(z^0)^{\mathrm{T}}\vec{\mathbf{y}} - r_d(z^0)^{\mathrm{T}}\vec{\mathbf{x}} + r_g(z^0)\tau = \mu(z^0) \\ & (\vec{\mathbf{x}}, \tau) \geq 0, (\vec{\mathbf{s}}, \kappa) \geq 0, (\vec{\mathbf{y}}, \theta) \text{ free}, \end{aligned} \tag{4}$$

where $z = (\vec{\mathbf{x}}, \tau, \vec{\mathbf{y}}, \vec{\mathbf{s}}, \kappa)$ are the variables (τ and κ are two additional variables to make the formulation self-dual), $z^0 = (\vec{\mathbf{x}}^0, \tau^0, \vec{\mathbf{y}}^0, \vec{\mathbf{s}}^0, \kappa^0)$ are the initial points, and

$$\begin{aligned} r_p(z) &= \mathbf{A}\vec{\mathbf{x}} - \vec{\mathbf{b}}\tau \\ r_d(z) &= -\mathbf{A}^{\mathrm{T}}\vec{\mathbf{y}} - \vec{\mathbf{s}} + \vec{\mathbf{c}}\tau \\ r_g(z) &= \vec{\mathbf{b}}^{\mathrm{T}}\vec{\mathbf{y}} - \vec{\mathbf{c}}^{\mathrm{T}}\vec{\mathbf{x}} - \kappa \\ \mu(z) &= (\vec{\mathbf{x}}^{\mathrm{T}}\vec{\mathbf{s}} + \tau\kappa)/(\nu + 1), \end{aligned}$$

We can denote the commonly used cold start point as z^c. In this article, $z^c = (\vec{\mathbf{x}}^c, \tau^c, \vec{\mathbf{y}}^c, \vec{\mathbf{s}}^c, \kappa^c)$ is chosen using the same strategy in SDPT3 [13] which is a point well centered in the feasible set. Then the warmstarting point proposed is [12]:

$$\begin{aligned} \vec{\mathbf{x}}^0 &= g\vec{\mathbf{x}}^* + (1-g)\vec{\mathbf{x}}^c \\ \vec{\mathbf{s}}^0 &= g\vec{\mathbf{s}}^* + (1-g)\vec{\mathbf{s}}^c \\ \vec{\mathbf{y}}^0 &= g\vec{\mathbf{y}}^* \\ \tau^0 &= \tau^c \\ \kappa^0 &= \vec{\mathbf{x}}^{0\mathrm{T}}\vec{\mathbf{s}}^0/n, \end{aligned} \tag{5}$$

where $\vec{\mathbf{x}}^*$, $\vec{\mathbf{s}}^*$, and $\vec{\mathbf{y}}^*$ are known optimal solutions (i.e., the priori information) of a similar (or perturbed) optimization problem. $0 \leq g \leq 1$ is the warm ratio representing the relative location of the warmstarting point between the optimal solution of a similar optimization problem and the cold start point of the current optimization problem.

2.4. Proposed Constraint Modification

The warmstarting methods in previous studies are usually applied to cone programming problem with SOC only [12]. In the ANC filter design problem, PSD cones exist because of the robust stability constraint which is one of the most essential constraints. In this section, two different relaxations are proposed to reformulate the PSD cones to SOC.

First, robust stability constraints Eq. (1d) and Eq. (1e) are satisfied if the following holds [4]:

$$||\mathbf{W}_x(f_k)\hat{\mathbf{G}}_{s0}(f_k)||_2 \leq \min\{1-\epsilon_s, 1/B^2\}. \tag{6}$$

A relatively very conservative relaxation is to consider that max-norm of an arbitrary $m \times n$ matrix $\mathbf{M}$ satisfies:

$$||\mathbf{M}||_{\max} \leq ||\mathbf{M}||_2 \leq \sqrt{mn}||\mathbf{M}||_{\max}. \tag{7}$$

Thus, Eq. (6) can be relaxed as:

$$\sqrt{N_r N_s}||\mathbf{W}_x(f_k)||_{\max}||\hat{\mathbf{G}}_{s0}(f_k)||_2 \leq \min\{1-\epsilon_s, 1/B^2\}. \tag{8}$$

So the following constraint (Eq. (9)), which can be formulated equivalently to a SOC, can be used to replace Eq. (1d) and Eq. (1e):

$$||\mathbf{W}_x(f_k)||_{\max} \leq \frac{\min\{1-\epsilon_s, 1/B^2\}}{\sqrt{N_r N_s}||\hat{\mathbf{G}}_{s0}(f_k)||_2}. \tag{9}$$

Alternatively, a less conservative relaxation is to consider the Frobenius norm properties:

$$||\mathbf{W}_x(f_k)\hat{\mathbf{G}}_{s0}(f_k)||_2 \leq ||\mathbf{W}_x(f_k)\hat{\mathbf{G}}_{s0}(f_k)||_F, \tag{10}$$

where,

$$||\mathbf{W}_x(f_k)\hat{\mathbf{G}}_{s0}(f_k)||_F = \sqrt{\mathrm{tr}\left(\hat{\mathbf{G}}_{s0}(f_k)^{\mathrm{H}}\mathbf{W}_x(f_k)^{\mathrm{H}}\mathbf{W}_x(f_k)\hat{\mathbf{G}}_{s0}(f_k)\right)}. \tag{11}$$

Thus, the following constraint (Eq. (12)), which can also be formulated equivalently to a SOC, can be used to replace Eq. (1d) and Eq. (1e) as well:

$$\mathrm{tr}\left(\hat{\mathbf{G}}_{s0}(f_k)^{\mathrm{H}}\mathbf{W}_x(f_k)^{\mathrm{H}}\mathbf{W}_x(f_k)\hat{\mathbf{G}}_{s0}(f_k)\right) \leq \min\{1-\epsilon_s, 1/B^2\}. \tag{12}$$

The ANC performance results using these two proposed relaxation methods, Eq. (9) and Eq. (12), will be compared in the next section.

3. RESULTS

An ANC system installed on the wind channel of a central air handling system was used as the experimental setup. The experimental setup and its dimension is shown in Fig. 3 and Fig. 4. There were two speakers (served as secondary sources), four reference microphones and four error microphones in this system. The experimental setup is the same as in the reference [4].

3.1. Comparison of Different Proposed Relaxation Methods

The noise control performance when using different proposed relaxation methods were compared: (1) using the original PSD cones (Eq. (1d)and Eq. (1e)); (2) using Eq. (9) which is referred to as method 1 in this section; and (3) using Eq. (12) which is referred to as method 2 in this section. The noise control performance comparison is shown in Fig. 5, the comparison of the value of $\max\left(\lambda\left(\frac{\mathbf{A}_s+\mathbf{A}_s^{\mathrm{H}}}{2}\right)\right)$ is shown in Fig. 6. The ϵ_s is set to be 0.2, thus the value of $\max\left(\lambda\left(\frac{\mathbf{A}_s+\mathbf{A}_s^{\mathrm{H}}}{2}\right)\right)$ should be less than 0.8 to be considered as stable. It can be observed from Fig. 5 that, compared with the proposed relaxation method 1, the proposed relaxation method 2 has a much better noise control

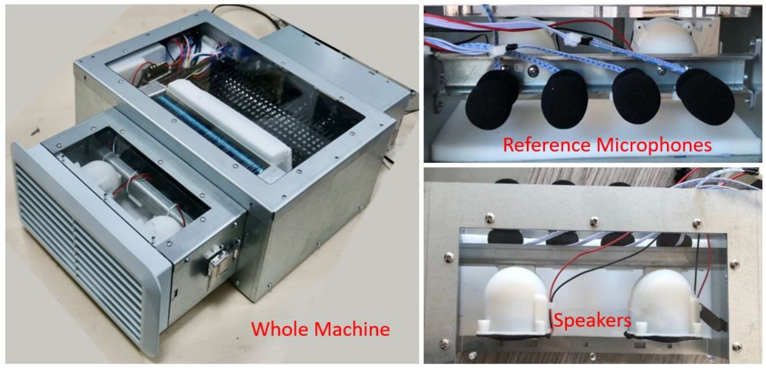

Figure 3: The experimental setup. [4]

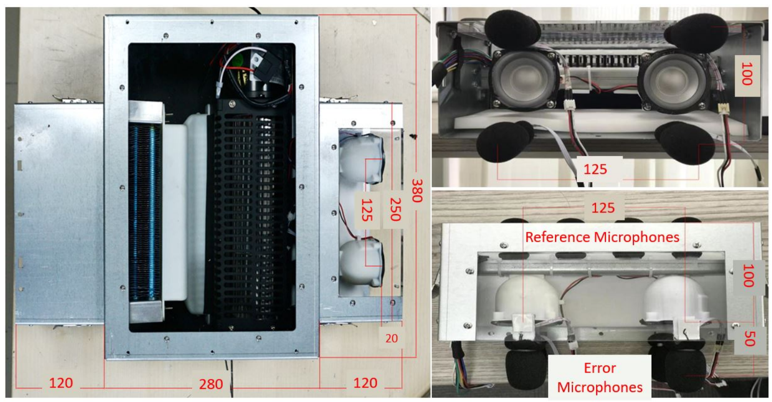

Figure 4: Dimension of the experimental setup. [4]

performance. This result can be confirm from Fig. 6. All three methods have stable filter design result because the value of all three methods are less than 0.8. Compared with original PSD cones which is very close to 0.8, the proposed relaxation method 2 is closer to 0.8 and the proposed relaxation method 1 is much less than 0.8. This demonstrated that proposed relaxation method 1 is a more conservative relaxation compared with method 2. Therefore, in the investigation of warmstarting method in the next result section, method 2 will be used for SOC only case.

3.2. Investigation of Warmstarting Method Performance

To investigate the warmstarting method performance, a number of perturbed problems were generated from the measured reference signal:

$$\mathbf{S}_{xx}^{new} = \mathbf{S}_{xx}(\mathbf{E}_n + \alpha\mathbf{P}_n). \tag{13}$$

where $\mathbf{S}_{xx}^{new}$ denotes the cross spectral density matrix of reference signal in the perturbed problems; $\mathbf{S}_{xx}$ denotes the cross spectral density matrix of measured reference signal (the optimal solution will be used as $\vec{\mathbf{x}}^{*}$, $\vec{\mathbf{s}}^{*}$, and $\vec{\mathbf{y}}^{*}$). $\mathbf{E}_n$ denotes a $n \times n$ matrix with all elements being 1. α denotes the perturbation ratio as a measure of how much the problem is changed. $\mathbf{P}_n$ is a $n \times n$ matrix with each element generated from a standard Gaussian process.

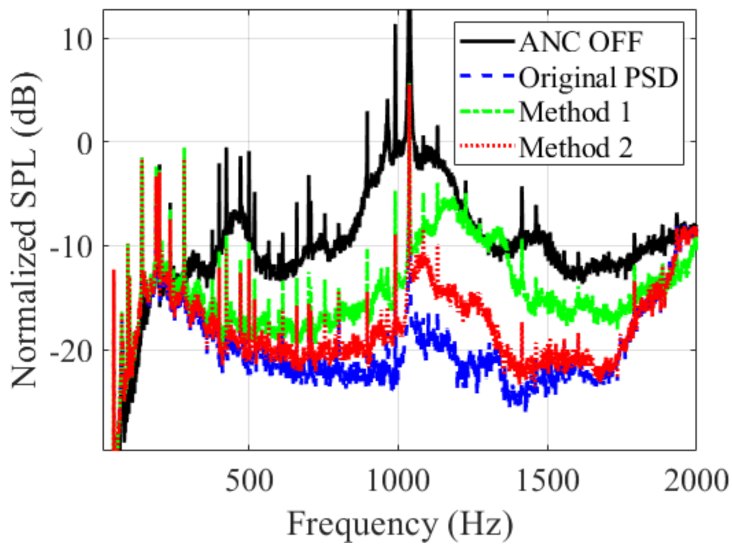

Figure 5: The comparison of noise control performance between the originally used PSD cones robust stability constraints, relaxed SOC robust stability constraints using method 1, and relaxed SOC robust stability constraints using method 2.

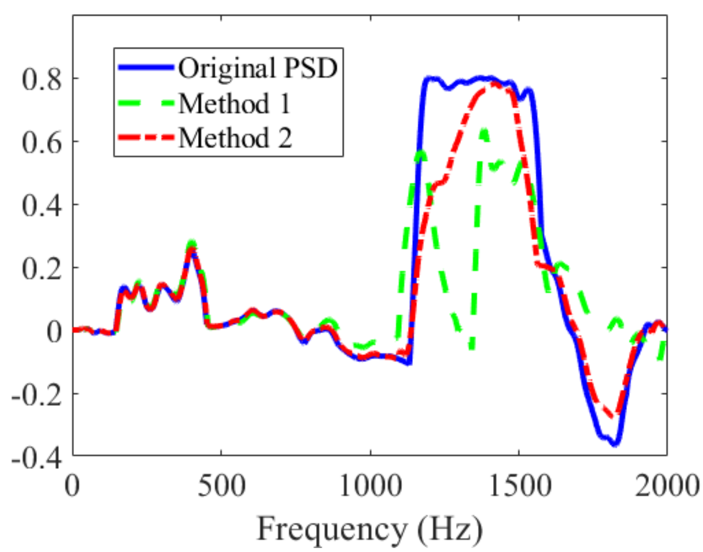

Figure 6: The comparison of the value of $\max\left(\lambda\left(\frac{\mathbf{A}_s+\mathbf{A}_s^{\mathrm{H}}}{2}\right)\right)$ when using the originally used PSD cones robust stability constraints, relaxed SOC robust stability constraints using method 1, and relaxed SOC robust stability constraints using method 2. The ϵ_s is set to be 0.2 and thus the value should be less than 0.8 to be considered as stable.

Two different ANC design problems were investigated: (1) the original convex formulation Eq. (1), and (2) the problem using proposed relaxation method 2 (i.e., Eq. (1d) and Eq. (1e) are replaced by Eq. (12)). For each design problem and perturbation ratio combination, 10 different perturbed problems are generated randomly and the numbers of iterations needed to solve the filter design optimization problems are compared. It is noted that the reason the number of iterations is used instead of the solving time is that the computation time for each iteration in similar problems are usually the same [17].

Fig. 7 shows the mean and standard deviation of iteration ratio (i.e., the ratio of the number of iterations using warmstarting method to the number of iterations using cold start method) for SOC cases (proposed relaxation method 2) when using different warm ratio g in Eq. (5) and perturbation ratio α in Eq. (13). Fig. 7 demonstrated the warmstarting method can effectively reduce the required number of iterations. It is shown (and is expected) that the smaller the perturbation ratio is (i.e., the closer the new problem is to the original problem), the warmstarting method performs better. When the perturbation ratio is about 0.1%, warmstarting ratio higher than 0.99 can achieve almost 50% of reduction in iterations, which significantly reduce the required computational effort of designing ANC filter. Even At relatively high perturbation ratio (5%), the warmstarting method can still reduce about 40% iterations. The standard deviation of iterations ratio is not high which means that warmstarting method is relatively reliable for different problems with similar environment settings.

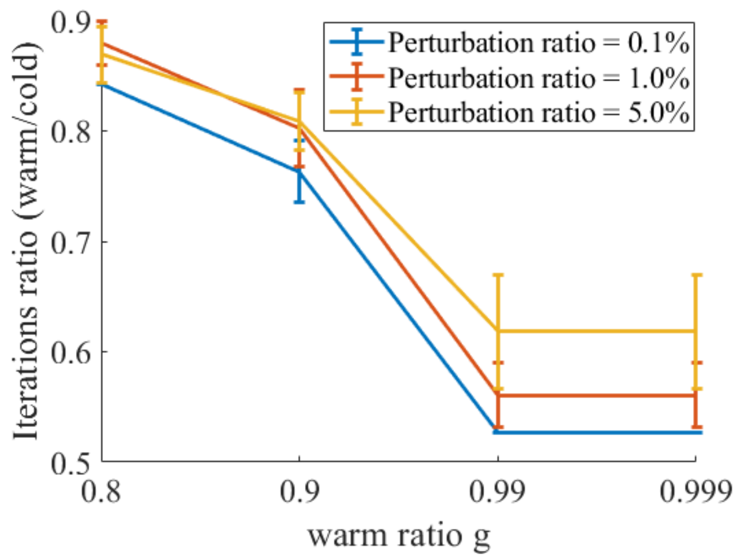

Figure 7: The mean and standard deviation of iteration ratio (i.e., iterations using warmstarting method / iterations using cold start method) when using different warm ratio g and perturbation ratio α. Ten cases with SOC only (using proposed relaxation method 2) are randomly generated and tested for each combination of warm ratio and perturbation ratio.

Fig. 8 shows the mean and standard deviation of iteration ratio for mixed SOC and PSD cones cases (the original convex formulation Eq. (1)) when using different warm ratio g in Eq. (5) and perturbation ratio α in Eq. (13). Compared with Fig. 7, the Fig. 8 shows that if the proposed relaxation method is not applied to convert PSD cones into SOC, the warmstarting method cannot have satisfactory result (the iterations ratios are around 1).

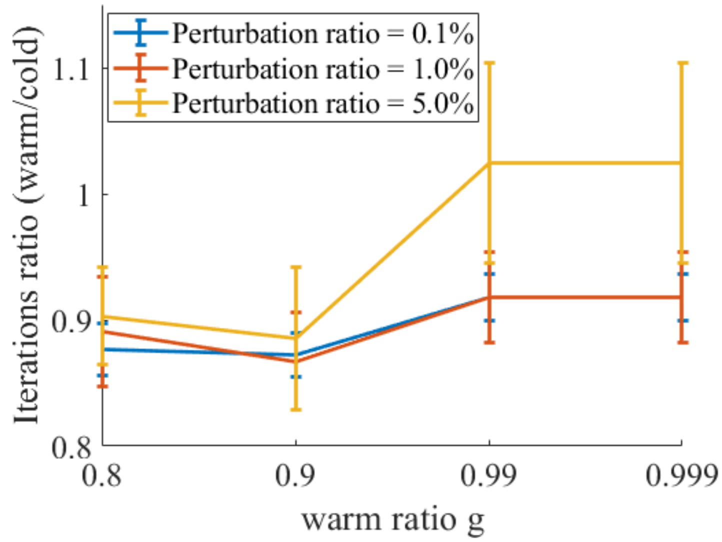

Figure 8: The mean and standard deviation of iteration ratio (i.e., iterations using warmstarting method / iterations using cold start method) when using different warm ratio g and perturbation ratio α. Ten cases with mixed SOC and PSD cones (using original convex formulation) are randomly generated and tested for each combination of warm ratio and perturbation ratio.

4. CONCLUSIONS

In this article, the warmstarting strategies for constrained ANC optimal filter design problem is investigated. First, two relaxation methods are proposed to convert original ANC design problem (i.e., mixed PSD and SOC problem) to a SOC-only problem. Results show that the relaxation method 2 (i.e., the relaxation of stability and robustness constraints using Frobenius norm properties) has a better noise control performance compared with relaxation method 1 (i.e., the relaxation of stability and robustness constraints using max-norm properties). Then the warmstarting performance is compared for different problem types, warm ratios, and perturbation ratios. It is demonstrated that it is important to apply proposed relaxation methods to the original ANC problem if the warmstarting method is to be applied. After the relaxation, warmstarting method can significantly reduce the required number of iterations in solving the filter design optimization problem. Practically, this is helpful for commercial product design that involves designing ANC filter in similar but different environmental setup.

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