# Warmstarting the Constrained Optimal Filter Design Problem for Active Noise Control Systems in Conic Formulation

Yongjie Zhuang (Presenter)
Zhuang Mo
Yangfan Liu

Ray W. Herrick Laboratories,
Purdue University

NOISE-CON 2022 June 13-15 Lexington, KY

# Applications

For many practical active noise control applications:

- **Multichannel systems :** for large-size quiet zone.

- **Multiple constraints** : robust stability, enhancement, filter output power.

Interior of Vehicles

Air Conditioner

Range Hood

Infant Incubator

# Background

One common approach for designing **constrained multichannel** controller: solve a **constrained optimization problem**

- ❑ Advantage: better noise control performance

- ❑ Challenge: **significant computational effort**
  (large channel number, filter order, number of the constraints)

# Background

NOISE-CON 2022 June 13-15 Lexington, KY

This work is a continuation of our previous work of convex & cone formulation:

- Zhuang and Liu, JASA 2021:

JASA ARTICLE

**Constrained optimal filter design for multi-channel active noise control via convex optimization**

Yongjie Zhuang[a] and Yangfan Liu[b]
*Ray W. Herrick Laboratories, School of Mechanical Engineering, Purdue University, West Lafayette, Indiana 47907, USA*

- Zhuang and Liu, InterNoise 2020:

Seoul inter-noise 2020 23-26 AUGUST

**Development and application of dual form conic formulation of multichannel active noise control filter design problem in frequency domain**

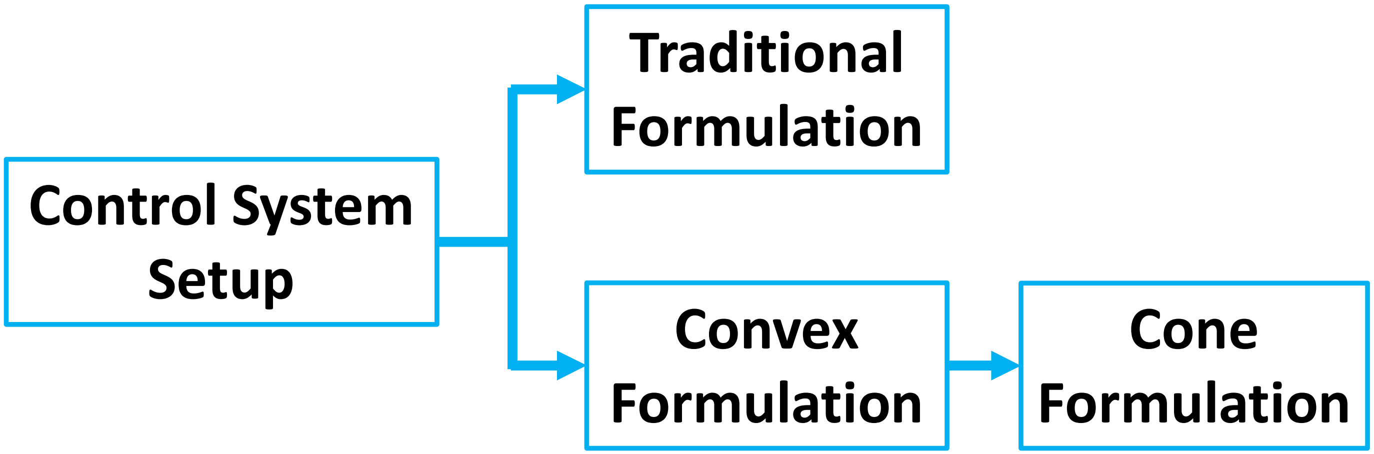
- Zhuang and Liu, NoiseCon 2019:

NEXT WAVE IN NOISE CONTROL ENGINEERING CATCH THE NOISE-CON 2019 AUG. 26-28 SAN DIEGO, CA

San Diego, CA
NOISE-CON 2019
2019 August 26-28

**Study on the Cone Programming Reformulation of Active Noise Control Filter Design in the Frequency Domain**

# Background




- **Proposed previously**
- **Computational time: hours → seconds**

Benefits of shorter computational time:

❑Reducing time and cost during product design circle

❑Make continuously design possible for time-varying environment.

# Motivation

For proposed formulation, **warmstarting** strategies are difficult.

❑**Cold start**:
choosing initial guess **without** information of approximate location of optimal solution.
e.g., use origin (0,0,…,0), or identity (1,1,…,1).

❑**Warm start**:
choosing initial guess **using** information of approximate location of optimal solution.
e.g., the optimal solution of a similar but different environmental setup

# Motivation

Why warmstarting strategies are important?

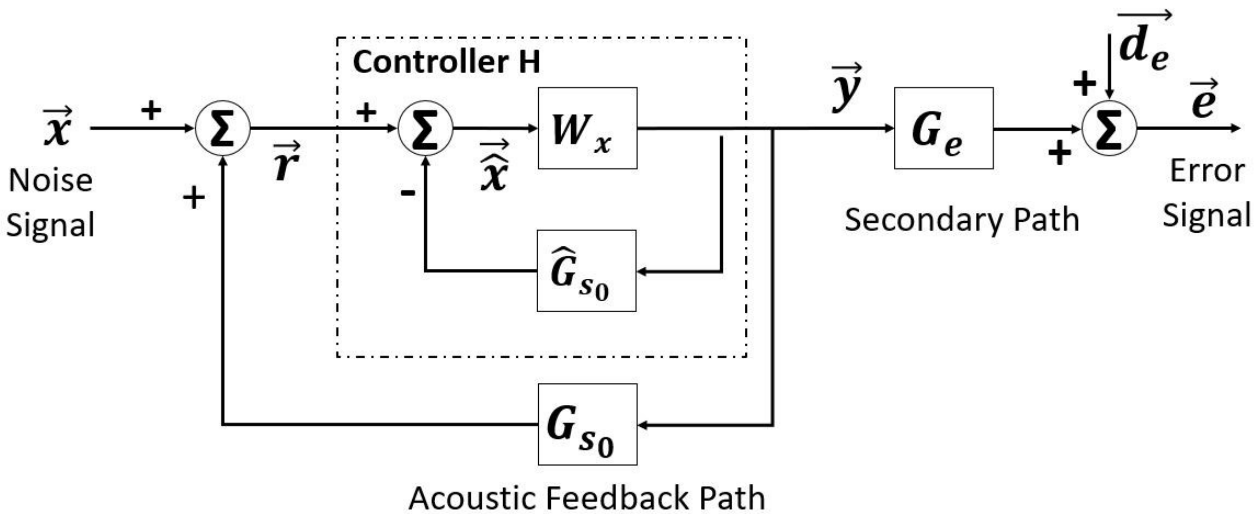
❑ **Commercial product design**:

- Current product model may be a variation of previous models
- Product differs from prototype by batch manufactural error

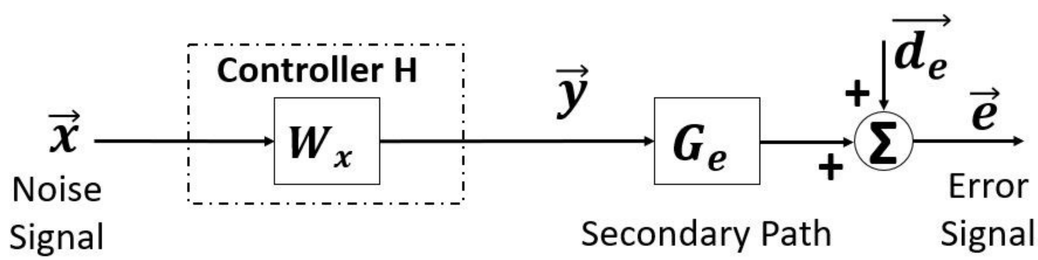
❑ **Time-varying applications**:

- the optimal filter coefficients of previous environment condition can be used as the initial guess when the condition changes.

# Review – Control diagram




If $\hat{G}_{s_0} = G_{s_0}$




- **Objective:**
  minimize the power of $\vec{e}$
- **Robust stability:**
  the feedback loop $W_x \hat{G}_{s_0}$
- **Output power:**
  Power of $W_x$ or $\vec{y}$
- **Disturbance enhancement:**
  $\vec{e}$ should not be amplified at certain frequency bands

# Review – Convex and cone formulation

**Convex formulation**

**Cost function:** Quadratic function

**Constraining total power of e**

**Constraints:**

Enhancement: Quadratic function

**Constraining normalized power of e**

Filter response: Quadratic function

**The magnitude of frequency response**

Stability: Max of eigenvalue

**Use Nyquist criterion**

Robustness: Max of singular value

**$\boldsymbol{M}$- $\boldsymbol{\Delta}$ structure and small gain theory**

**Cone formulation**

**Cost function:** **Linear**

**Constraints:**

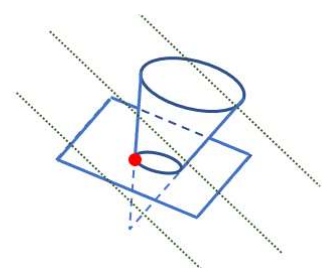
**Linear equalities or inequalities**

**Second-order cones:**

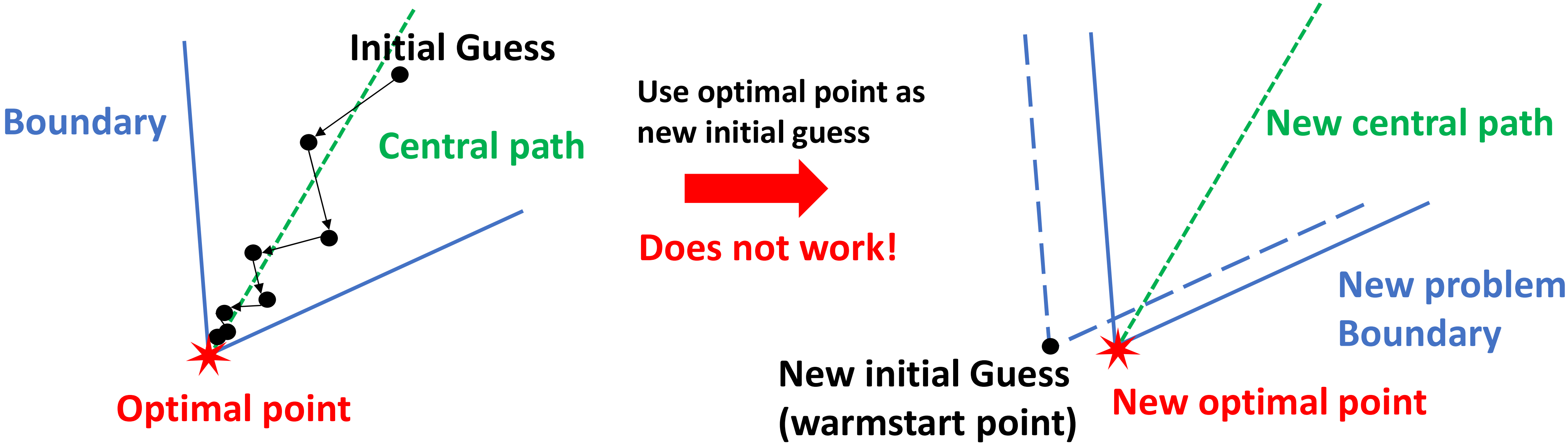
$$\left\{(y, \vec{\mathbf{x}}) \in \Re \times \Re^{n_i-1} : y \geq \|\vec{\mathbf{x}}\|_2\right\}$$

**Positive semidefinite cones:**

$$\left\{\mathrm{vec}(X) \in \Re^{n_i^2} : X \in \Re^{n_i \times n_i} \text{ is positive semidefinite}\right\}$$



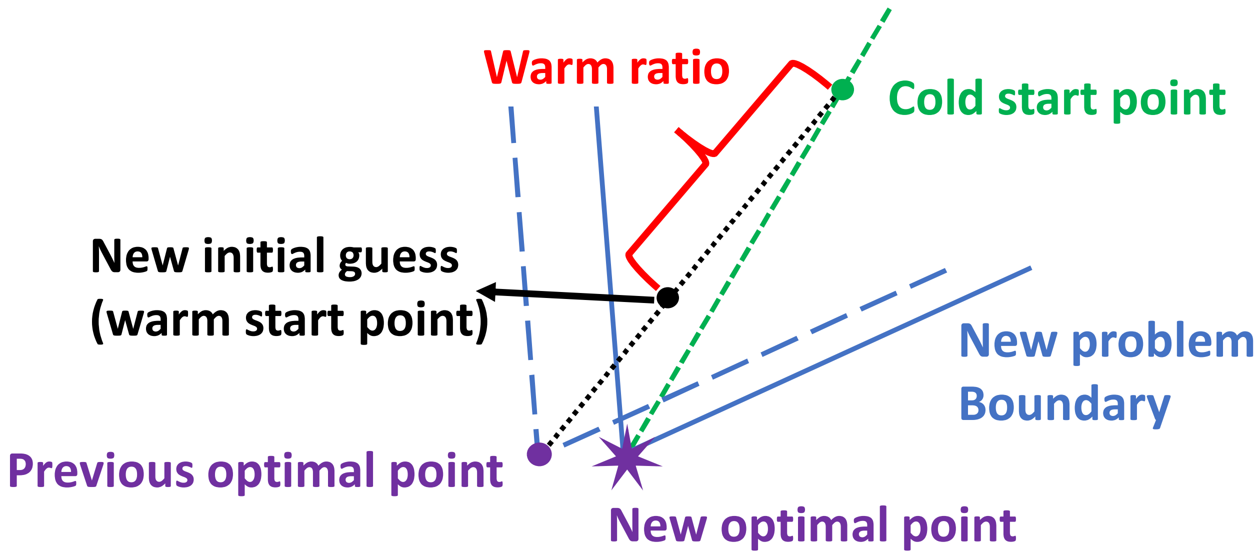
# Method – Warmstarting challenges




For cone programming algorithm, each iteration should:

- Inside the constraint boundaries
- Away from boundary as much as possible (follow the central path)

# Method – Warmstarting method




Proposed by
Anders Skajaa et al.
in 2013

Use convex combination of cold start point and previous optimal point:

- Guarantees a usable initial guess (close enough to cold start)
- Very little extra computational effort for warm start point

# Method – Convert PSD cones to SOCs

NOISE-CON 2022 June 13-15 Lexington, KY

**Convex formulation**

**Cost function:** Quadratic function

**Constraining total power of e**

**Constraints:**

Enhancement: Quadratic function

**Constraining normalized power of e**

Filter response: Quadratic function

**The magnitude of frequency response**

Stability: Max of eigenvalue

**Use Nyquist criterion**

Robustness: Max of singular value

**$\boldsymbol{M}$- $\boldsymbol{\Delta}$ structure and small gain theory**

- Need second-order cone (SOC) only
- The stability and robustness constraints can only be reformulated equivalently to positive semidefinite (PSD) cones
- Some relaxation must be done to convert them to second-order cones (SOCs)

# Method – Convert PSD cones to SOCs

NOISE-CON 2022 June 13-15 Lexington, KY

Stability: Max of eigenvalue

**Use Nyquist criterion**

Robustness: Max of singular value

**$\boldsymbol{M}$- $\boldsymbol{\Delta}$ structure and small gain theory**

Open Loop Response

$$\|\boldsymbol{W}_x(f_k)\widehat{\boldsymbol{G}}_{s0}(f_k)\|_2 \leq C(f_k)$$

**Method 1**: use max-norm properties:

$$\|M\|_{max} \leq \|M\|_2 \leq \sqrt{mn}\|M\|_2$$

PSD converts to SOCs:

$$\|\boldsymbol{W}_x(f_k)\|_{max} \leq \frac{C(f_k)}{\sqrt{N_r N_s}\|\widehat{\boldsymbol{G}}_{s0}(f_k)\|_2}$$

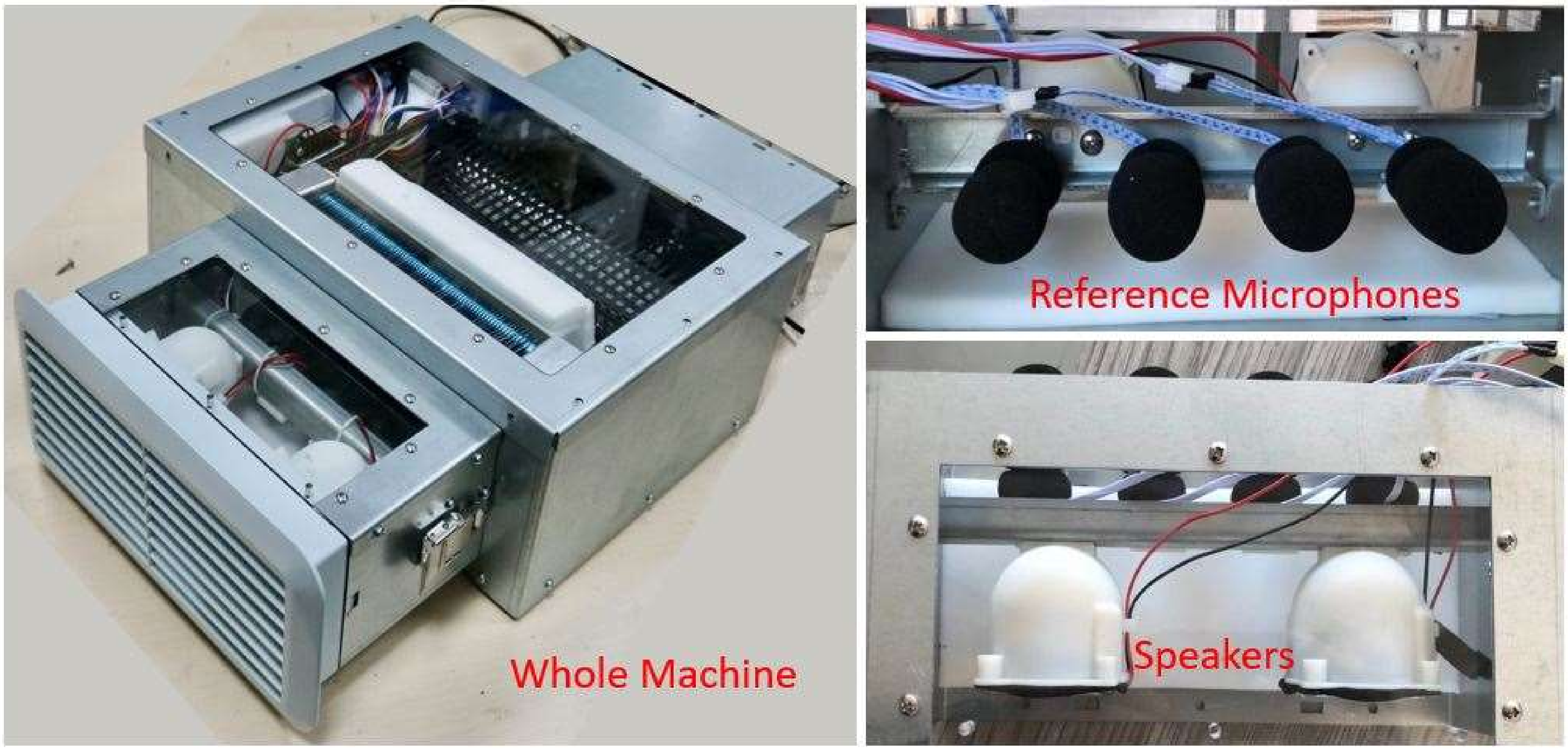
**Method 2**: use Frobenius norm properties:

$$\|M\|_2 \leq \|M\|_F$$

PSD converts to SOCs:

$$tr(\widehat{\boldsymbol{G}}_{s0}^{\mathrm{H}}(f_k)\boldsymbol{W}_x^{\mathrm{H}}(f_k)\boldsymbol{W}_x(f_k)\widehat{\boldsymbol{G}}_{s0}(f_k)) \leq C^2(f_k)$$

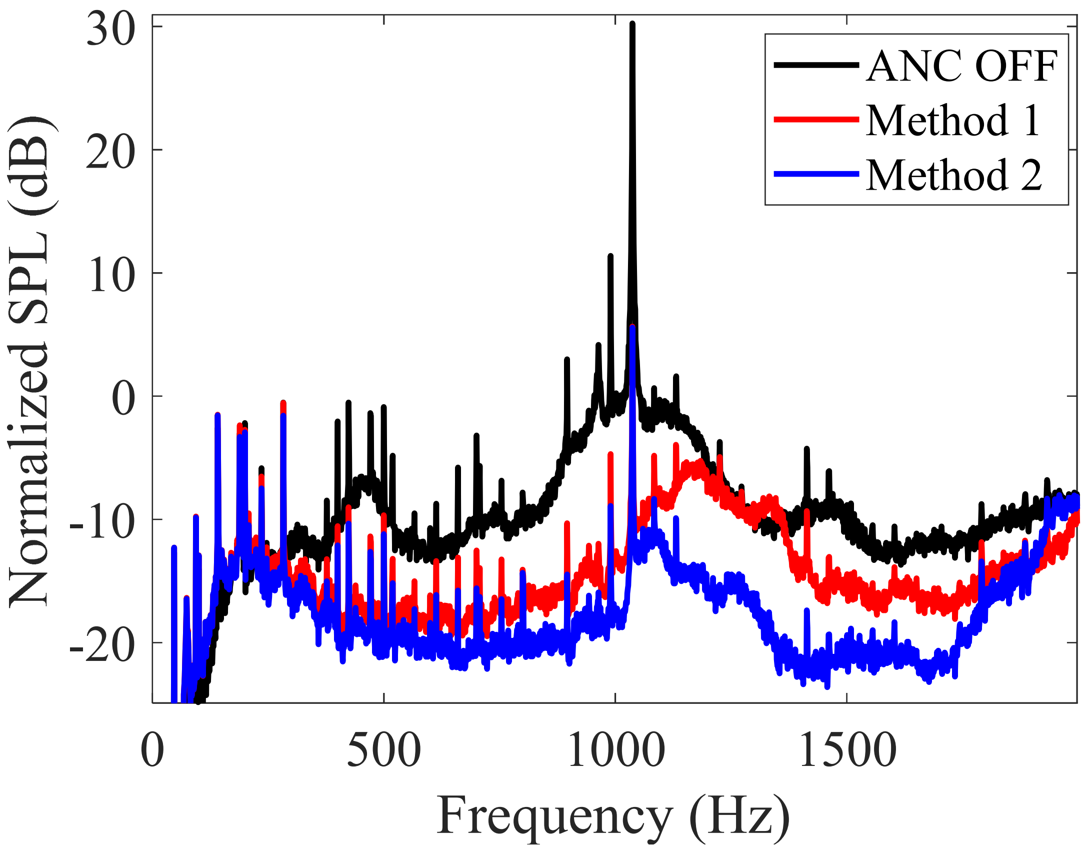
# Result – Experimental setup




A multi-channel active noise control system on a wind channel

# Result – Comparison of two methods

**Noise control performance**




**Method 1: use max-norm**

**Method 2: use Frobenius norm**

- Converting constraints will sacrifice performance
- Method 2 has better performance (less conservative)

# Result – Warmstarting performance



Auto spectral density function
of newly generated noise signal

$$\boldsymbol{E}_n = \begin{pmatrix} 1 & \cdots & 1 \\ \vdots & \ddots & \vdots \\ 1 & \cdots & 1 \end{pmatrix}$$

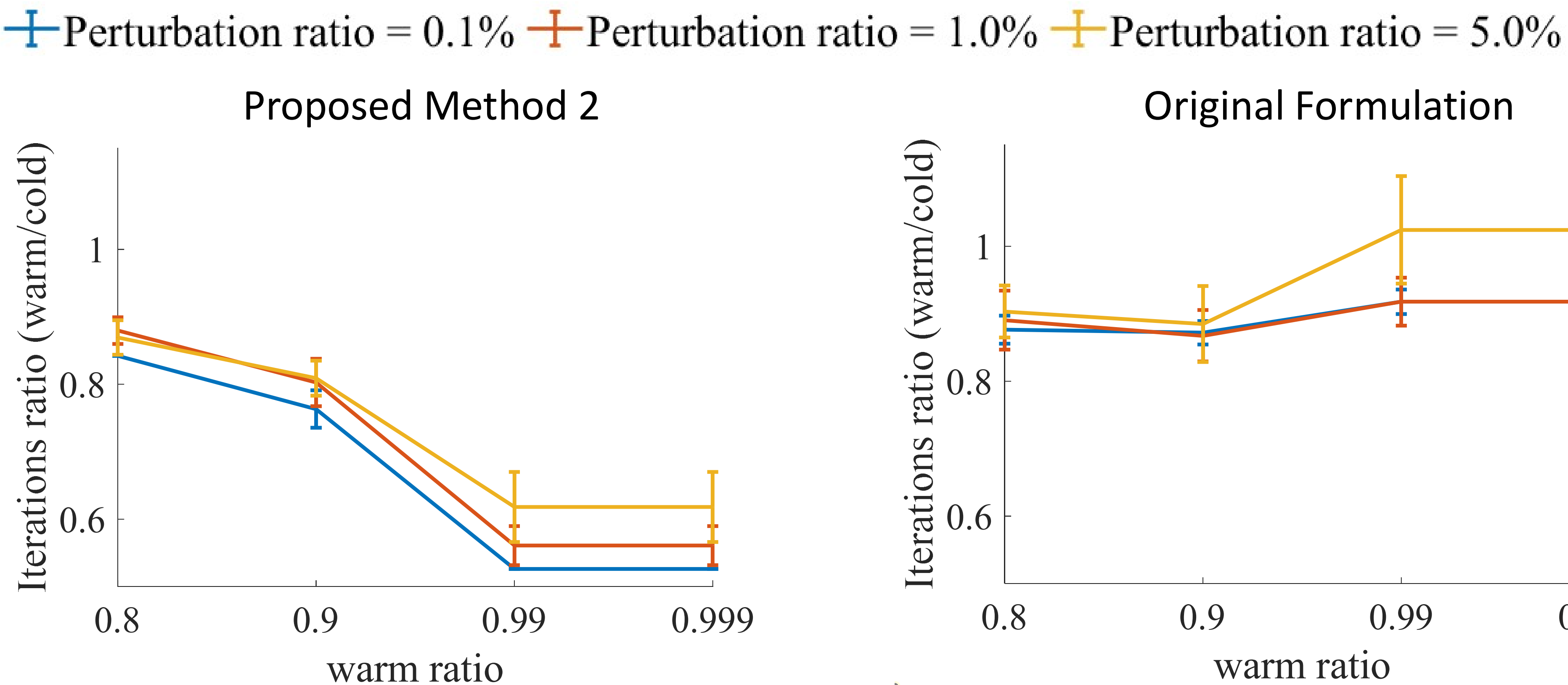
$$\boldsymbol{S}_{xx}^{new} \leq \boldsymbol{S}_{xx}(\boldsymbol{E}_n + \alpha \boldsymbol{P}_n)$$

Each element of $\boldsymbol{P}_n$ is generated by a standard Gaussian process

Measured auto spectral density function
-known optimal filter coefficients

Perturbation ratio
-represents the changes
of environmental setup

# Result – Warmstarting performance




Warm ratio: closer to 1, initial point closer to previous optimal solution
When warm ratio is higher than 0.999, it goes outside the constraints.

# Conclusion

- Two methods of converting the positive semidefinite cones into second order cones are proposed.

- After using the proposed formulation method 2, the iteration number can be **reduced up to 45%** when using the warmstarting strategy.

- For a relatively wide range of problem perturbation ratio (from 0.1% to 5%), the warmstarting method is **robust** when choosing the same warm ratio parameter.

# Thank you!