New Orleans, Louisiana
NOISE-CON 2020
2020 June 29 – July 1

Singular Vector Filtering Method for Disturbance Enhancement Mitigation in Active Noise Control Systems

Yongjie Zhuang
Yangfan Liu
Ray W. Herrick Laboratories
177 S, Russell Street
Purdue University
West Lafayette
IN 47907-2099, USA
yangfan@purdue.edu

Xuchen Wang
Academic building 1, 2/F
The Chinese University of Hong Kong
Sha Tin
Hong Kong SAR
999077, China

ABSTRACT
In multichannel active noise control systems, when reference signals are correlated, the disturbance enhancement phenomenon is likely to occur, i.e., the resulting sound is enhanced instead of being reduced in some frequency bands, if the filter is designed to minimize the total energy for all frequencies. In previous works, a truncated singular value decomposition method was applied to the system autocorrelation matrix to mitigate the disturbance enhancement due to the correlation of reference signals. Some small singular values and the associated singular vectors are removed, if they are responsible for unwanted disturbance enhancement in some frequency bands. However, some of these removed singular vectors may still contribute to noise control performance in other frequency bands, thus a direct truncation will degrade the noise control performance. In the present work, through an additional filtering process, the set of singular vectors that cause the disturbance enhancement are replaced by a set of new singular vectors whose frequency responses are attenuated in the frequency band where disturbance enhancement occurs, while the frequency responses in other frequency bands are unchanged. Compared with truncation, the proposed method can maintain the performance in the noise reduction bands, while mitigating the influence in disturbance enhancement bands.

1 INTRODUCTION
In active noise control (ANC), one or more control filters are designed to process the reference or error signals obtained by sensors such as microphones or accelerometers. The output signals of the control filters are used to drive secondary sources to produce an appropriate secondary sound field that can cancel or reduce the primary sound field at specified locations or regions. In the recent decades, because of the development of the computing devices, e.g., the Digital Signal Processor (DSP) and the Field-Programmable Gate Arrays (FPGA), ANC technologies were successfully applied to a wide range of applications, such as automobiles[1], headrests[2,3], and headphones[4,5].

One of the main focuses of ANC studies is the methods of designing appropriate control filters. The requirement for a control filter is to achieve good noise control performance and satisfy some practical criteria (stability, robustness, disturbance enhancement, etc.) at the same time. Among those criteria, the disturbance enhancement is particularly concerned in the current work.

It requires that the control filter should have satisfactory noise control performance at the desired frequency bands without significantly enhancing the noise level at other frequency bands. The design of the ANC filters can be accomplished either in the time domain, or in the frequency domain[6]. For multichannel ANC systems, if filters are designed by minimizing the energy of the resulting total sound, disturbance enhancement phenomenon occurs when two or more reference signals are correlated[7]. Different treatments for this disturbance enhancement phenomenon can be found in previous works, for example, the disturbance enhancement is constrained by adding enhancement constraints at each frequency band in the frequency domain to formulate an optimization problem[1,3,8]. Although this frequency-domain method is convenient in formulating the optimization problem, the computational complexity to solve the constrained optimization problem is significant, especially when the numbers of filter channels and coefficients are large. The enhancement bound can be also applied to the time-domain methods by introducing an adjustable penalty (or regularization) parameter into the formulation. However, this penalty will usually affect the entire frequency band and sacrifice the noise control performance significantly, especially for the frequency band where no enhancement phenomenon occurs[6,9].

Recently, Liu et al. demonstrated that the singular value decomposition (SVD) method can be used to extract independent sound field components[10]. Inspired by this work, Wang et al. proposed another time-domain treatment for disturbance enhancement where a truncated SVD method is applied[7]. SVD was firstly applied to the auto-correlation matrix of filtered-reference signals, and the singular values and singular vectors that contribute to the enhancement phenomenon were removed to mitigate the disturbance enhancement. Their results showed that this method can effectively mitigate the enhancement phenomenon resulted from strongly correlated reference signals. However, some of the removed singular values and singular vectors, although responsible for enhancement in certain frequency bands, may contribute to the noise control performance in other desired frequency bands. Thus, such a direct truncation will affect the noise control performance. To further reduce this effect, a filtering method is proposed in the present work. Instead of a direct truncation, a filtering process was introduced to replace the singular vectors contributing to the disturbance enhancement by a set of new singular vectors whose frequency responses are attenuated in the frequency band where disturbance enhancement occurs, while the frequency responses in other frequency bands are unchanged. The result showed that, compared with truncation method, the proposed method can reduce the impact on the noise control performance at desired frequency bands and, at the same time, mitigate the disturbance enhancement phenomenon.

The paper is organized as follows. In Section 2, the theory related to the proposed filtering method is introduced. In Section 3, the simulation results of this filtering method are presented and compared with the direct truncation method. Concluding comments are summarized in Section 4.

2 THEORY

In the current work, the proposed method is described in the context of a multichannel feedforward ANC system. For feedback ANC systems, it can be applied in a similar way. The only difference is that the error sensors would be also treated as reference sensors, and the acoustical feedback path is the same as the path from secondary sources to error sensors[1,6,11]. In this section, the typical algorithm for calculating the coefficients of control filters and the truncated singular value decomposition method are reviewed first, then the singular vector filtering method is described.

2.1 The block diagram and traditional algorithm for filter design

A multiple-input-multiple-output (MIMO) ANC system includes N_r reference sensors, N_s control sources, and N_e error sensors, The system block diagram of a typical MIMO ANC feedforward controller is shown in Figure 1, which is similar to the block diagram in Wang et al.'s work[7]. Elements in vector $\vec{\boldsymbol{p}}(n)$ are the reference signals measured by the reference sensors, when the ANC system is activated, it includes both the primary noise signals $\vec{\boldsymbol{x}}(n)$, representing the noise from the primary sources at the reference sensor locations, and the signals from secondary sources at the same reference sensor locations. $\vec{\boldsymbol{y}}(n)$ denotes the outputs of the controller, which are also the driving signals for secondary sources. $\vec{\boldsymbol{e}}(n)$ denotes the error signals measured at error sensor locations, which includes the disturbance signals $\vec{\boldsymbol{d}}(n)$ from the primary sources and the anti-noise signals from secondary sources. $\boldsymbol{G_s}$ is the acoustic feedback path from secondary sources to the reference sensors. To cancel influence of the acoustic feedback path, an internal model control (IMC) structure is used[1,3,11]. $\widehat{\boldsymbol{G}}_{\boldsymbol{s}}$ in the controller $\mathbf{H}$ is the estimate of the acoustic feedback path. In the current work, it is assumed that $\widehat{\boldsymbol{G}}_{\boldsymbol{s}} = \boldsymbol{G_s}$, so the estimate of noise signals $\vec{\hat{\boldsymbol{x}}}(n) = \vec{\boldsymbol{x}}(n)$. $\boldsymbol{G_e}$ is the secondary path from secondary sources to the error sensors. $\boldsymbol{W}$ denotes the $N_s \times N_r$ multichannel control filters. In the current work, each channel in the controller is a FIR filter with N_t coefficients.

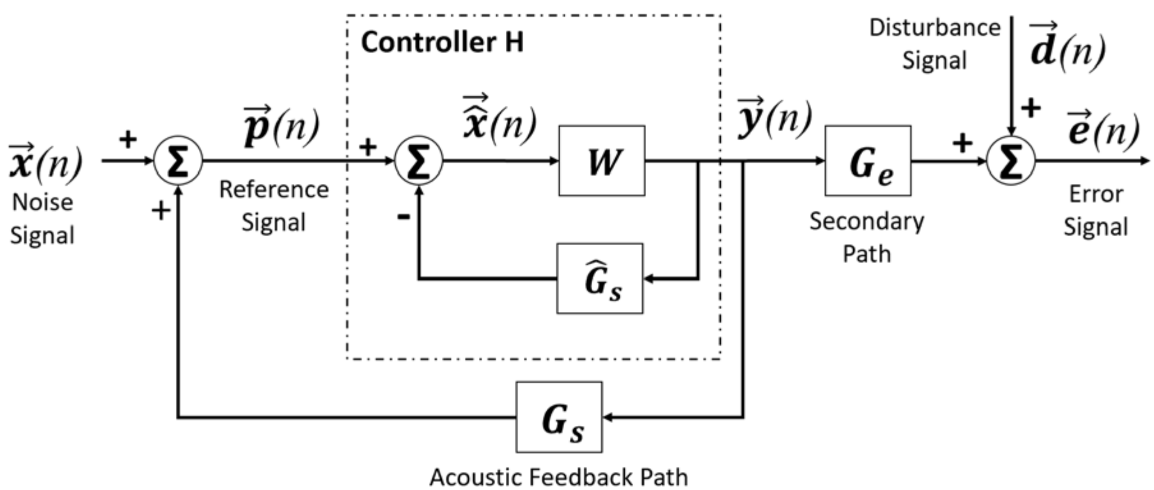

Figure 1: *Block diagram of the MIMO feedforward ANC controllers using the internal model control structure.*

The conventional method to calculate control parameters is to minimize the power of the error signals $\vec{\boldsymbol{e}}(n)$, when ANC system is activated. The cost function of this optimization problem can be expressed as[6,7]:

$$\mathrm{E}[\vec{\boldsymbol{e}}^{\mathrm{T}}(n)\,\vec{\boldsymbol{e}}(n)] = \vec{\boldsymbol{w}}^{\mathrm{T}}\boldsymbol{A}\vec{\boldsymbol{w}} + 2\vec{\boldsymbol{w}}^{\mathrm{T}}\vec{\boldsymbol{b}} + c, \tag{1}$$

where,

$$\boldsymbol{A} = \mathrm{E}[\boldsymbol{R}^{\mathrm{T}}(n)\,\boldsymbol{R}(n)], \qquad \vec{\boldsymbol{b}} = \mathrm{E}\big[\boldsymbol{R}^{\mathrm{T}}(n)\,\vec{\boldsymbol{d}}(n)\big], \qquad c = \mathrm{E}\big[\vec{\boldsymbol{d}}^{\mathrm{T}}(n)\,\vec{\boldsymbol{d}}(n)\big],$$

$$\vec{\boldsymbol{w}} = \big[w_{1,1,0}, w_{1,2,0}, \dots, w_{1,N_r,0}, \dots, w_{N_s,N_r,0}, \dots, w_{N_s,N_r,N_t}\big]^{\mathrm{T}},$$

$$\boldsymbol{R}(n) = \begin{bmatrix} \vec{\boldsymbol{r}}_1^{\mathrm{T}}(n) & \vec{\boldsymbol{r}}_1^{\mathrm{T}}(n-1) & \cdots & \vec{\boldsymbol{r}}_1^{\mathrm{T}}(n-N_t+1) \\ \vec{\boldsymbol{r}}_2^{\mathrm{T}}(n) & \vec{\boldsymbol{r}}_2^{\mathrm{T}}(n-1) & \cdots & \vec{\boldsymbol{r}}_2^{\mathrm{T}}(n-N_t+1) \\ \vdots & \vdots & \ddots & \vdots \\ \vec{\boldsymbol{r}}_{N_e}^{\mathrm{T}}(n) & \vec{\boldsymbol{r}}_{N_e}^{\mathrm{T}}(n-1) & \cdots & \vec{\boldsymbol{r}}_{N_e}^{\mathrm{T}}(n-N_t+1) \end{bmatrix},$$

$$\vec{\boldsymbol{r}}_q(n) = \big[r_{q,1,1}(n), r_{q,1,2}(n), \dots, r_{q,1,N_r}(n), r_{q,2,1}(n), \dots, r_{q,N_r,N_t}(n)\big]^{\mathrm{T}},$$

$$r_{q,m,l}(n) = \textstyle\sum_{i=0}^{I-1} g_{q,m,j} x_l(n-j),$$

where $g_{q,m,j}$ denotes the j-th coefficient of the FIR filter model of the component in $\boldsymbol{G_e}$ that corresponds to the m-th input and q-th output; $x_l(n)$ is the l-th element of $\vec{\boldsymbol{x}}(n)$. The optimal solution of $\vec{\boldsymbol{w}}$ is[6,7]:

$$\vec{\boldsymbol{w}}_{opt} = -\boldsymbol{A}^{-1}\vec{\boldsymbol{b}} \,. \tag{2}$$

When the filtered-reference signals $\vec{\boldsymbol{r}}_q(n)$ are correlated in some certain frequency bands, the matrix $\boldsymbol{A}$ may still be non-singular, thus $\vec{\boldsymbol{w}}_{opt}$ can be calculated by Equation (2), but the resulting sound level is even higher than the original sound in those frequency bands, i.e., disturbance enhancement occurs in those frequency bands.

2.2 Singular vector filtering method for mitigating disturbance enhancement

To investigate the disturbance enhancement phenomenon resulting from the mutual correlation of multichannel reference signals, Wang et al. applied SVD to the auto-correlation matrix A[7], which results in:

$$\boldsymbol{A} = \boldsymbol{U\Sigma U}^{\mathrm{T}} = \begin{bmatrix}\vec{\boldsymbol{u}}_1 & \cdots & \vec{\boldsymbol{u}}_{N_rN_sN_t}\end{bmatrix}\begin{bmatrix}\boldsymbol{\sigma}_1 & \cdots & 0\\ \vdots & \ddots & \vdots \\ 0 & \cdots & \boldsymbol{\sigma}_{N_rN_sN_t}\end{bmatrix}\begin{bmatrix}\vec{\boldsymbol{u}}_1^{\mathrm{T}}\\ \vdots \\ \vec{\boldsymbol{u}}_{N_rN_sN_t}^{\mathrm{T}}\end{bmatrix}, \tag{3}$$

$$\vec{\boldsymbol{w}}_{opt} = -\sum_{k=1}^{N_rN_sN_t} \boldsymbol{\sigma}_k^{-1}\langle\vec{\boldsymbol{u}}_k, \vec{\boldsymbol{b}}\rangle\,\vec{\boldsymbol{u}}_k \,. \tag{4}$$

The optimal filter $\vec{\boldsymbol{w}}_{opt}$ can be considered as a linear combination of the singular vectors, $\vec{\boldsymbol{u}}_k$, each of which represents a MIMO FIR filter with N_r outputs, N_s inputs, and N_t filter coefficients. When the correlation is strong among the reference signals, there will be some very large value of $\boldsymbol{\sigma}_k^{-1}\langle\vec{\boldsymbol{u}}_k, \vec{\boldsymbol{b}}\rangle$, which then cause the disturbance enhancement[7]. To mitigate this disturbance enhancement, Wang et al. truncated the singular values and the associated singular vectors, after some index l, to form a modified filter[7]:

$$\vec{\boldsymbol{w}}_0 = -\sum_{k=1}^{l} \boldsymbol{\sigma}_k^{-1}\langle\vec{\boldsymbol{u}}_k, \vec{\boldsymbol{b}}\rangle\,\vec{\boldsymbol{u}}_k \,. \tag{5}$$

Then $\vec{\boldsymbol{w}}_0$ will be used to replace $\vec{\boldsymbol{w}}_{opt}$ as the control filter. It was demonstrated that this method could mitigate the enhancement if l is chosen appropriately. However, the discarded singular values and vectors also contribute to the noise control performance at other frequency bands, thus such a direct truncation will negatively affect the overall noise control performance.

To reduce this impact on the noise control performance in desired frequency bands, a filtering method, instead of truncation, is proposed. Firstly, the original singular vector representation of the optimal filter, Equation (4), can be divided as two groups:

$$\vec{\boldsymbol{w}}_{opt} = \vec{\boldsymbol{w}}_0 + \sum_{k=l+1}^{N_rN_sN_t} \vec{\boldsymbol{w}}_k \,, \tag{6}$$

where $\vec{\boldsymbol{w}}_0$ is defined in Equation (5), which are the singular vectors with no enhancement contributions, and $\vec{\boldsymbol{w}}_k$ is the component in the subspace spanned by the singular vector contributing to enhancement in certain bands (i.e., the component that is removed in Wang, et al.'s work), which is expressed as:

$$\vec{\boldsymbol{w}}_k = -\boldsymbol{\sigma}_k^{-1}\langle\vec{\boldsymbol{u}}_k, \vec{\boldsymbol{b}}\rangle\,\vec{\boldsymbol{u}}_k = \left[w_{k,1,1,0}, w_{k,1,2,0}, \dots, w_{k,1,N_r,0}, \dots, w_{k,N_s,N_r,0}, \dots, w_{k,N_s,N_r,N_t}\right]^{\mathrm{T}}. \tag{7}$$

It is noted that $\vec{\boldsymbol{w}}_k$ can be rearranged to an $N_s \times N_r$ filters $\vec{\boldsymbol{w}}_{k,i,j}$ with a filter length of N_t for each input-output channel pair. The frequency response of $\vec{\boldsymbol{w}}_{k,i,j}$ can be evaluated appropriately at N_f frequency points:

$$\vec{\boldsymbol{W}}_{k,i,j} = \boldsymbol{F}_z\vec{\boldsymbol{w}}_{k,i,j} \,, \tag{8}$$

where,

$$\boldsymbol{F}_z = \begin{bmatrix} 1 & \mathrm{e}^{-j2\pi f_1 \frac{1}{f_s}} & \cdots & \mathrm{e}^{-j2\pi f_1 \frac{N_t-1}{f_s}} \\ 1 & \mathrm{e}^{-j2\pi f_2 \frac{1}{f_s}} & \cdots & \mathrm{e}^{-j2\pi f_2 \frac{N_t-1}{f_s}} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & \mathrm{e}^{-j2\pi f_{N_f} \frac{1}{f_s}} & \cdots & \mathrm{e}^{-j2\pi f_{N_f} \frac{N_t-1}{f_s}} \end{bmatrix},$$

f_k is the k-th frequency; i and j represent the indices of different input and output channels of the multichannel filter $\vec{\boldsymbol{w}}_k$; f_s is the sampling frequency. As mentioned earlier, although $\vec{\boldsymbol{w}}_k$ is responsible for the enhancement phenomenon in some frequency bands, it also contributes to active noise control performance in other bands, thus a direct removal of $\vec{\boldsymbol{w}}_k$, as in the work of Wang et al., will inevitably sacrifice the noise control performance. It is proposed in the current work that a suitably designed band-stop filter can be applied to each channel pair of $\vec{\boldsymbol{W}}_{k,i,j}$ to obtain a new filter, $\vec{\overline{\boldsymbol{W}}}_{k,i,j}$, such that the stopbands of the designed filter cover the frequency bands where disturbance enhancement phenomena occurs if the original $\vec{\boldsymbol{W}}_{k,i,j}$ were used. In this way, $\vec{\overline{\boldsymbol{W}}}_{k,i,j}$ can be treated as a filtered version of $\vec{\boldsymbol{W}}_{k,i,j}$, thus the proposed method is referred to as singular vector filtering method. It is important to note that the band-stop filter used in the current work differs from conventional band-stop filters in that the filter used in the current work should produce no change in either magnitude or phase in the pass-band, whereas, conventional band stop filters usually do not involve this zero phase requirement in the pass-band. Thus, instead of using conventional band-stop filter design methods, the FIR filter coefficients (denoted as $\vec{\boldsymbol{v}}_{k,i,j}$) representing the filtered filter $\vec{\overline{\boldsymbol{W}}}_{k,i,j}$ with the same length as $\vec{\boldsymbol{W}}_{k,i,j}$ are obtained by solving the following optimization problem:

$$\vec{\boldsymbol{v}}_{k,i,j} = \mathbf{arg}\min_{\vec{\boldsymbol{v}}_{k,i,j}} \left\| \boldsymbol{F}_z \vec{\boldsymbol{v}}_{k,i,j} - \vec{\overline{\boldsymbol{W}}}_{k,i,j} \right\|_2^2 . \tag{9}$$

Since the frequency response $\vec{\overline{\boldsymbol{W}}}_{k,i,j}$ is small in the frequency bands where disturbance enhancement occurs and is the same as $\vec{\boldsymbol{W}}_{k,i,j}$ in other frequency bands, the filter $\vec{\boldsymbol{v}}_{k,i,j}$ will not have enhancement problem while preserve the noise control performance at other frequency bands as much as possible. Then, $\vec{\boldsymbol{v}}_{k,i,j}$ can be rearranged to get:

$$\vec{\boldsymbol{v}}_k = \left[v_{k,1,1,0}, v_{k,1,2,0}, \ldots, v_{k,1,N_r,0}, \ldots, v_{k,N_s,N_r,0}, \ldots, v_{k,N_s,N_r,N_t}\right]^{\mathrm{T}}. \tag{10}$$

Finally, the modified filter by singular vector filtering method is:

$$\vec{\boldsymbol{w}}_{mod1} = \vec{\boldsymbol{w}}_0 + \sum_{k=l+1}^{N_r N_s N_t} \vec{\boldsymbol{v}}_k , \tag{11}$$

where $\vec{\boldsymbol{w}}_{mod1}$ can be used to replace $\vec{\boldsymbol{w}}_{opt}$ as control filters for the active noise control system.

However, if $\vec{\boldsymbol{v}}_{k,i,j}$ is to be obtained separately for each i, j pair and for each index k, the optimization problem specified in Equation (9) will need to be solved for $N_r \times N_s \times (N_t - l)$ times, which involves a significant calculation effort if $N_r \times N_s \times N_t$ is large and l is small. This process can be simplified if $\vec{\boldsymbol{v}}_{k,i,j}$ are obtained after summation over k. In usual ANC practices, the frequency bands where disturbance enhancement occurs are found to be similar for each $\vec{\boldsymbol{W}}_{k,i,j}$, this suggests that the same band-stop filter can be applied to all $\vec{\boldsymbol{W}}_{k,i,j}$. Thus, in order to simplify the proposed filtering process, $\vec{\boldsymbol{w}}_{k,i,j}$ can be added first, then calculated for its frequency response:

$$\vec{\boldsymbol{W}}_{sum,i,j} = \boldsymbol{F}_z\left(\sum_{k=l+1}^{N_r N_s N_t} \vec{\boldsymbol{w}}_{k,i,j}\right). \tag{12}$$

Then frequency responses of those band-stop filters can be multiplied to the frequency response $\vec{\boldsymbol{W}}_{sum,i,j}$ to obtain $\vec{\overline{\boldsymbol{W}}}_{sum,i,j}$ such that the stopbands cover all the frequency bands where disturbance enhancement occurs. Then the second term in Equation (11) can be calculated by a single step:

$$\vec{\boldsymbol{v}}_{sum,i,j} = \mathbf{arg}\min_{\vec{\boldsymbol{v}}_{sum,i,j}} \left\| \boldsymbol{F}_z \vec{\boldsymbol{v}}_{sum,i,j} - \vec{\overline{\boldsymbol{W}}}_{sum,i,j} \right\|_2^2 . \tag{13}$$

Using similar way in Equation (10) to rearrange $\vec{\boldsymbol{v}}_{sum,i,j}$ to get $\vec{\boldsymbol{v}}_{sum}$, then we have:

$$\vec{\boldsymbol{w}}_{mod2} = \vec{\boldsymbol{w}}_0 + \vec{\boldsymbol{v}}_{sum} , \tag{14}$$

where $\vec{\boldsymbol{w}}_{mod2}$ can be used to replace $\vec{\boldsymbol{w}}_{opt}$ as control filters. Using this method, i.e., filtering after summation, the optimization specified in Equation (13) only need to be solved for $N_r \times N_s$ times, which is much more computationally efficient than repetitively solving Equation (9) for different k indices. In the next section, results obtained by filtering after summation and by filtering separately will also be compared.

3 SIMULATION RESULT

In the present work, simulation results were based on experimental data obtained from a similar set up as that in Wang et al.'s work[7]. The ANC system used in the current simulation work consists of two reference microphones, four error microphones, two loudspeakers as secondary sources, and two loudspeakers as the primary noise sources. To produce strongly correlated reference signals, one of the two primary noise sources was playing a white noise, while the other noise source was playing a signal obtained by filtering another white noise uncorrelated with the first noise source through a high-pass filter with 1000 Hz cutoff frequency. Thus, the reference signals are strongly correlated below 1000 Hz. The sampling frequency was chosen to be 10000 Hz. The length of FIR filter for each channel is 128. Thus, there are 512 singular vectors in total after decomposing matrix $\boldsymbol{A}$.

Similar to Wang et al.'s work, the results presented in the current work are based on off-line simulation using experimentally measured data. The comparison of simulated sound pressure averaged among the error microphones for different controller design methods is shown in Figure 2. "ANC OFF" denotes the sound pressure power spectral density function (PSD) of original disturbance signals averaged among all the error microphones. The "Normalized SPL" is the dB scale (i.e., ten times the logarithm) of the ratio of error microphone averaged sound PSD when different controllers are in operation to the PSD firstly averaged over error microphones and then averaged over the whole frequency band when "ANC OFF". "Original optimal filter" denotes the result when ANC system is operating using $\vec{\boldsymbol{w}}_{opt}$ in Equation (2). "Truncated SVD method" denotes the result when ANC system is operating using $\vec{\boldsymbol{w}}_0$ in Equation (5). "Singular vector filtering method" denotes the result when ANC system is operating using $\vec{\boldsymbol{w}}_{mod2}$ in Equation (14).

From Figure 2 (a), by comparing "Original optimal filter" and "ANC OFF", we can see that the disturbance enhancement phenomenon occurs at frequencies below 1000 Hz (mainly due to the correlation of reference signals below 1000 Hz), from 2100 Hz to 2700 Hz, and above 4200 Hz. It is also noted that the original optimal filter can result in a reasonable ANC performance between 1000 and 2000 Hz. So, the stopbands of band-stop filters used in the current work are specified to cover those three frequency ranges. After trying different truncation values of l in Equation (5), it is finally chosen to be 256, since the singular vector with indices smaller than 256

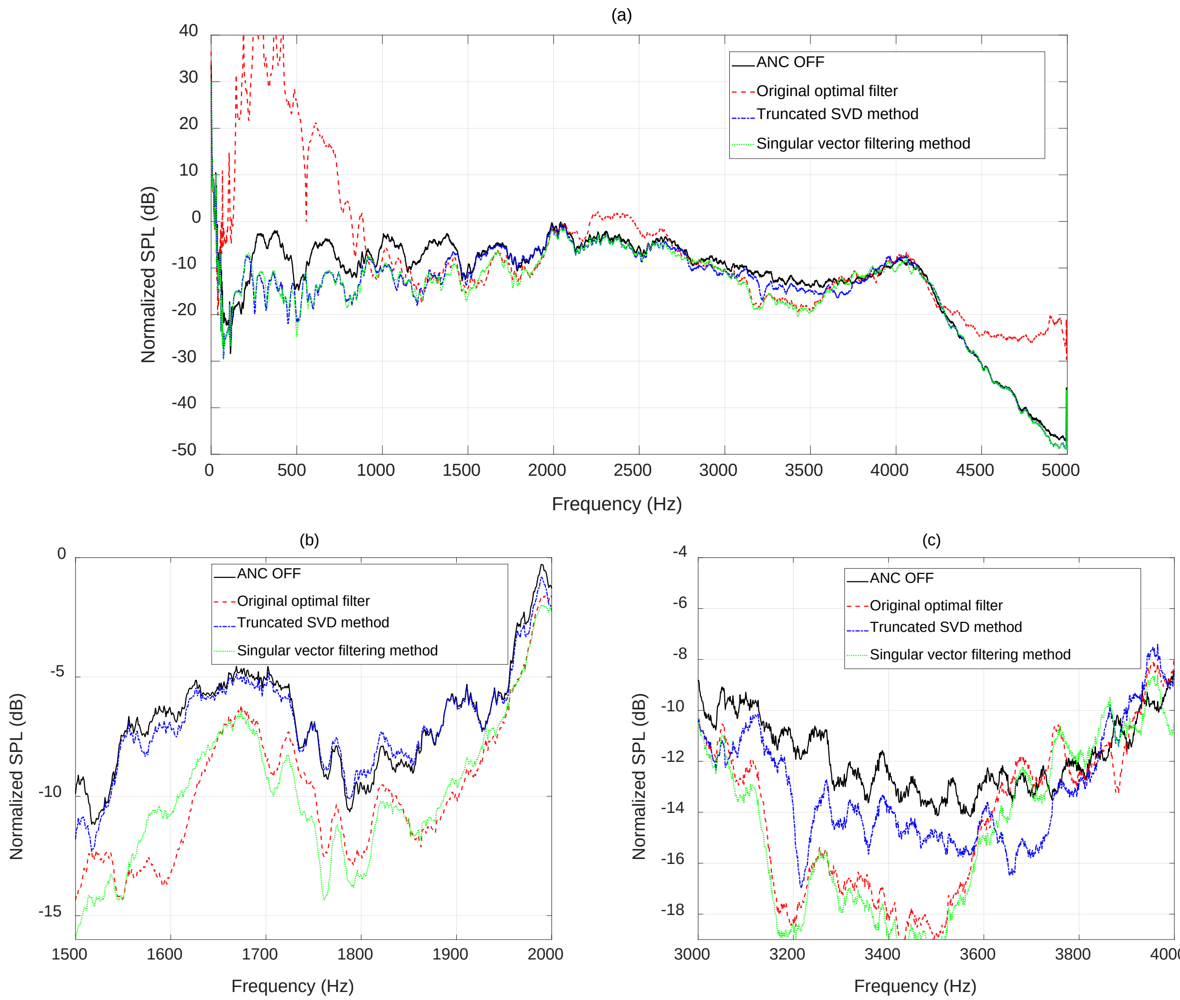

Figure 2: *(a) Comparison of averaged sound pressure at the error microphones for the whole frequency range. (b) Comparison of averaged sound pressure at the error microphones from 1500 Hz to 2000 Hz. (c) Comparison of averaged sound pressure at the error microphones from 3000 Hz to 4000 Hz.*

are shown to have no obvious enhancement contribution. From Figure 2 (a), it can be seen that both "Truncated SVD method" and "Singular vector filtering method" can mitigate the disturbance enhancement effectively. From Figure 2 (b) and (c), which are zoomed-in plots of Figure 2 (a), it can be seen that, compared with the truncated SVD method, the proposed singular vector filtering method can have a better noise control performance in the frequency bands where no enhancement occurs when the original optimal filter is used (in this example, 1500 Hz to 2000 Hz and 3000 Hz to 4000 Hz). In these frequency bands, the singular vector filtering method sacrifices noise control performance only slightly compared with the original optimal filter.

The performance of two different singular vector filtering methods mentioned in Section 2 are compared in Figure 3. "Singular vector filtering method 1" denotes the result when ANC system is operating using $\vec{\boldsymbol{w}}_{mod1}$ in Equation (11), i.e., filtering each filter separately. "Singular vector filtering method 2" denotes the result when ANC system is operating using $\vec{\boldsymbol{w}}_{mod2}$ in Equation (14), i.e., filtering after summation. It can be seen that the performance has no obvious difference between these two methods. However, the calculation of $\vec{\boldsymbol{w}}_{mod2}$ is much more computationally efficient, so using the "Singular vector filtering method 2" is preferable in practice.

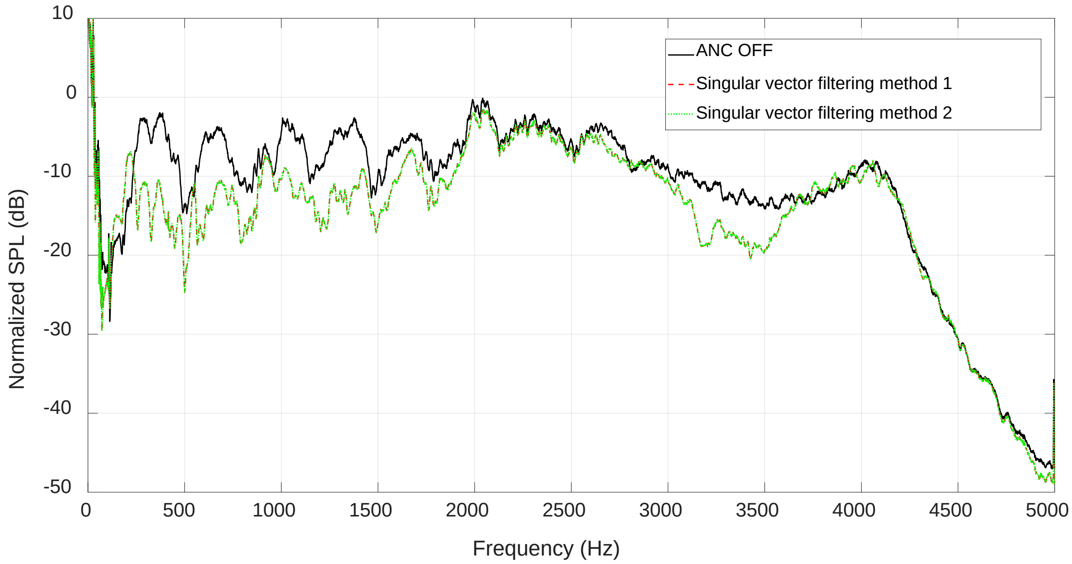

Figure 3: *Comparison of averaged sound pressure at the error microphones for two different singular vector filtering methods*

4 CONCLUDING COMMENTS

In the present work, a singular vector filtering method was proposed for ANC systems to mitigate the disturbance enhancement phenomenon due to the strong correlation among the reference signals. The proposed method firstly applies SVD to the auto-correlation matrix of filtered-reference signals. Then the singular vectors are divided into two groups. The group with larger singular values is unchanged, but the second group (the group with smaller singular values) is replaced by a new set of filters. The new set of filters are obtained by filtering the filters represented by the singular vectors in the second group through properly designed band-stop filters with the stopbands cover the frequency bands where disturbance enhancement occurs. In this way, the enhancement phenomenon can be mitigated. The band-stop filters were not designed or implemented using conventional methods, instead, the coefficients of the new set of filters are obtained by a least-square match to the frequency response of the singular vectors in the second group with the responses in the specified stop-bands changed to small magnitudes.

The simulation results showed that the proposed singular vector filtering method can mitigate the disturbance enhancement as effective as the truncated SVD method. Also, compared with truncated SVD method, the noise control performance can be better in other frequency ranges by using the singular vector filtering method. Also, two different singular vector filtering methods were compared, i.e., filtering each singular vector in the second group separately and filtering after summing up those singular vectors. The simulation results demonstrated that they produce the same noise control performance, if the disturbance occurs at similar frequency bands for each singular vector. Thus, filtering after summation is preferable because it is more computationally efficient than filtering separately.

So far, the proposed method is applied to non-adaptive filters. In the future, it may be converted into an adaptive algorithm by doing the calculation of eigenvalues and eigenvectors iteratively, and the minimization of Equation (13), i.e., a least-square estimation, iteratively.

5 ACKNOWLEDGEMENTS

The authors thank Beijing Ancsonic Technology Co. Ltd for providing financial support for the

present work.

6 REFERENCES

1. J. Cheer and S. J. Elliott, "Multichannel control systems for the attenuation of interior road noise in vehicles," *Mechanical Systems and Signal Processing,* vol. 60, pp. 753--769, 2015.
2. Y. Liu and J. Liu, "The Stochastic Domain Design of a Real-Time Controller for an Active Noise Control Headrest based on Finite Element Analysis," *INTER-NOISE and NOISE-CON Congress and Conference Proceedings,* vol. 255, pp. 488--499, 2017.
3. B. Rafaely and S. J. Elliott, "H2 / H_infty active control of sound in a headrest: design and implementation," *IEEE Transactions on control systems technology,* vol. 7, pp. 79-84, 1999.
4. S. M. Kuo, S. Mitra and W.-S. Gan, "Active noise control system for headphone applications," *IEEE Transactions on Control Systems Technology,* vol. 14, no. 2, pp. 331-335, 2006.
5. M. Guldenschuh and R. De Callafon, "Detection of secondary-path irregularities in active noise control headphones," *IEEE/ACM transactions on audio, speech, and language processing,* vol. 22, no. 7, pp. 1148-1157, 2014.
6. S. Elliott, "Multichannel Control of Stochastic Disturbances," *Signal Processing for Active Control,* Academic Press, 2001, pp. 233-270.
7. X. Wang, Y. Liu and J. S. Bolton, "Truncated Singular Value Decomposition Method for Mitigating Unwanted Enhancement in Active Noise Control Systems," *INTER-NOISE and NOISE-CON Congress and Conference Proceedings*, 2018.
8. Y. Zhuang and Y. Liu, "Study on the Cone Programming Reformulation of Active Noise Control Filter Design in the Frequency Domain," *INTER-NOISE and NOISE-CON Congress and Conference Proceedings*, 2019.
9. S. J. Elliott, C. C. Boucher and P. A. Nelson, "The behavior of a multiple channel active control system," *IEEE Transactions on signal processing*, vol. 40, no. 5, pp. 1041-1052, 1992.
10. Y. Liu, S. Wang and X. Wang, "A generalized spatial filtering method in broadband active noise control based on independent sound field component analysis," *INTER-NOISE and NOISE-CON Congress and Conference Proceedings*, 2017.
11. S. J. Elliott, "Design and Performance of Feedback Controllers," *Signal Processing for Active Control,* Academic Press, 2001, pp. 271-327.