NOISE-CON 2020

JAZZIN' UP NOISE CONTROL

Must Go On (line)!

A Virtual Event

Singular Vector Filtering Method for Disturbance Enhancement Mitigation in Active Noise Control Systems

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Introduction

Multichannel active noise control (ANC) systems:

- Better performance when large-size quiet zone is required
- Wide range of applications:

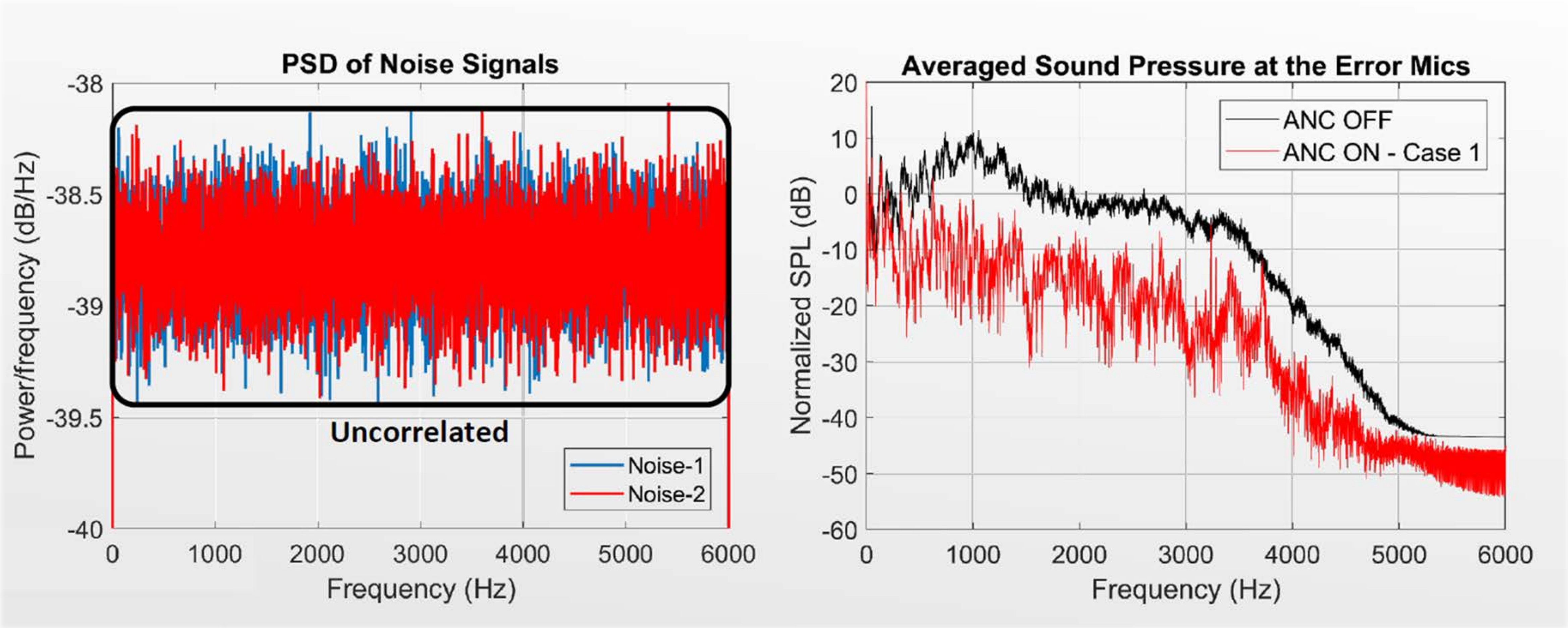
Interior of Vehicles

Range Hood

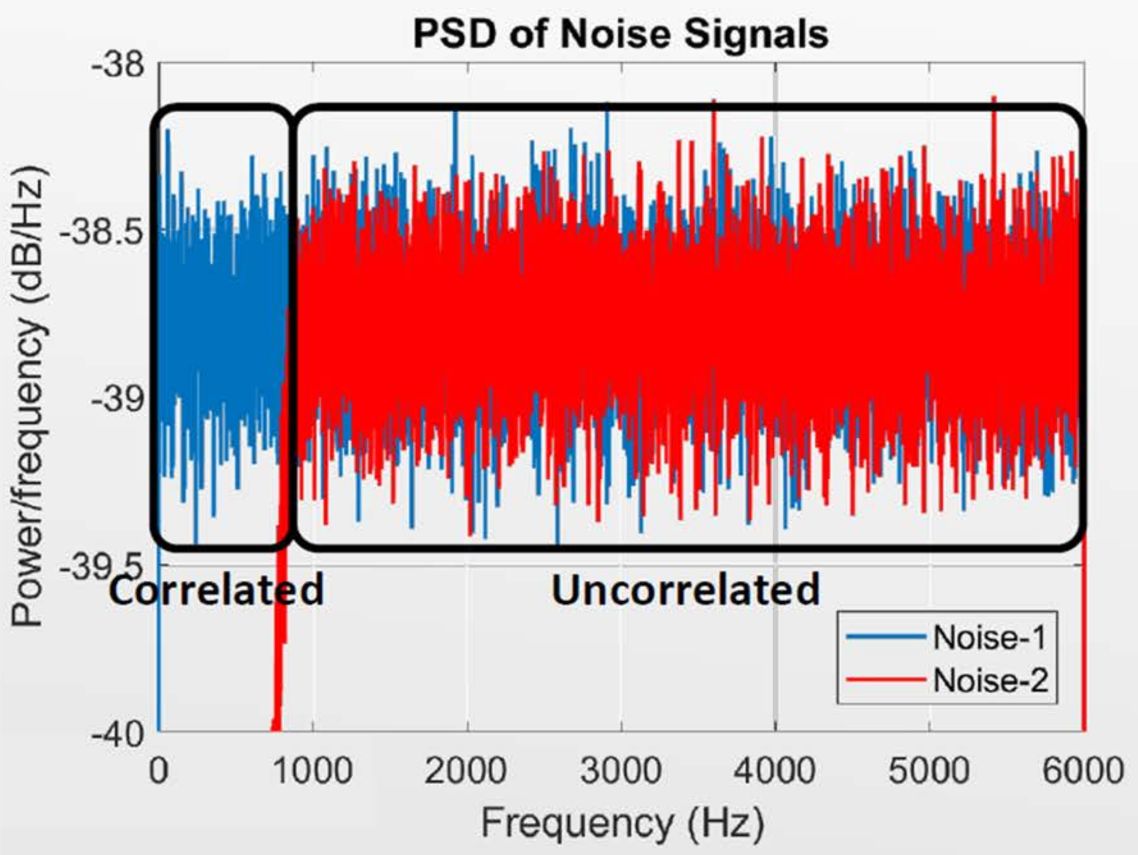
Infant Incubator

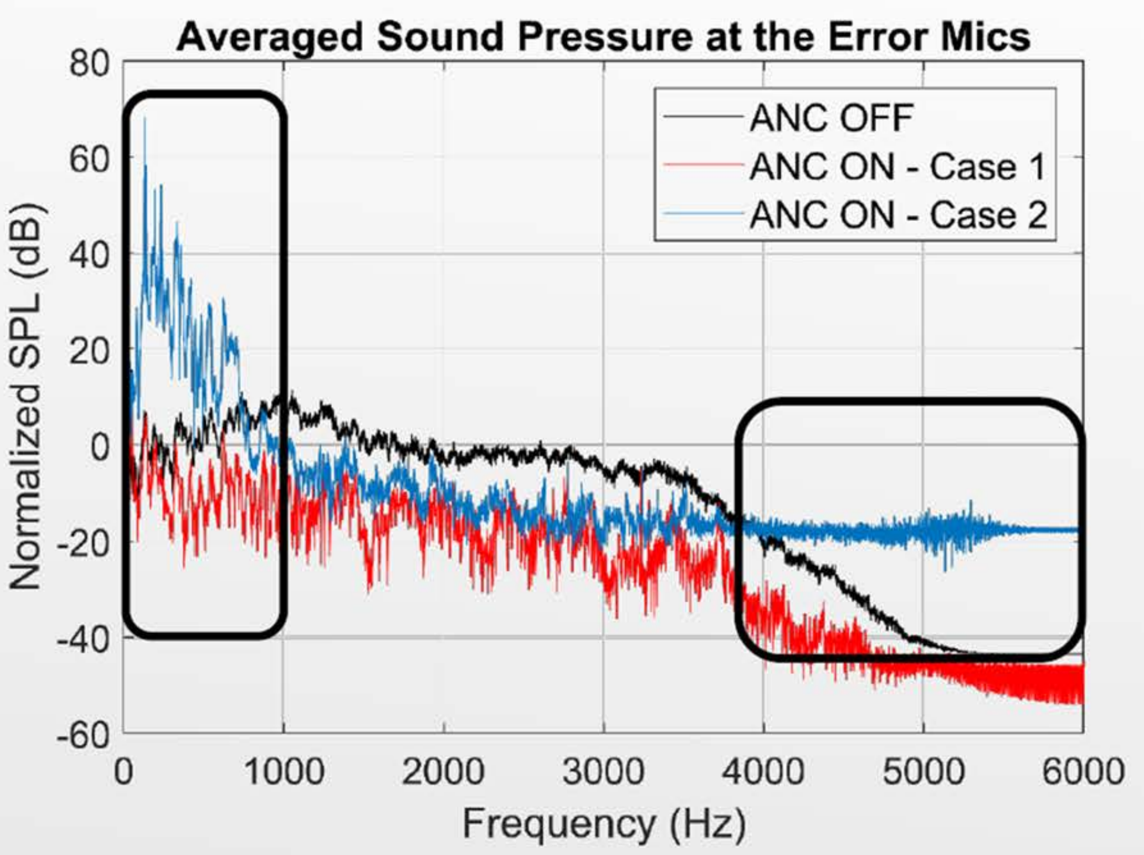
Air Conditioner

Introduction



Introduction





Introduction

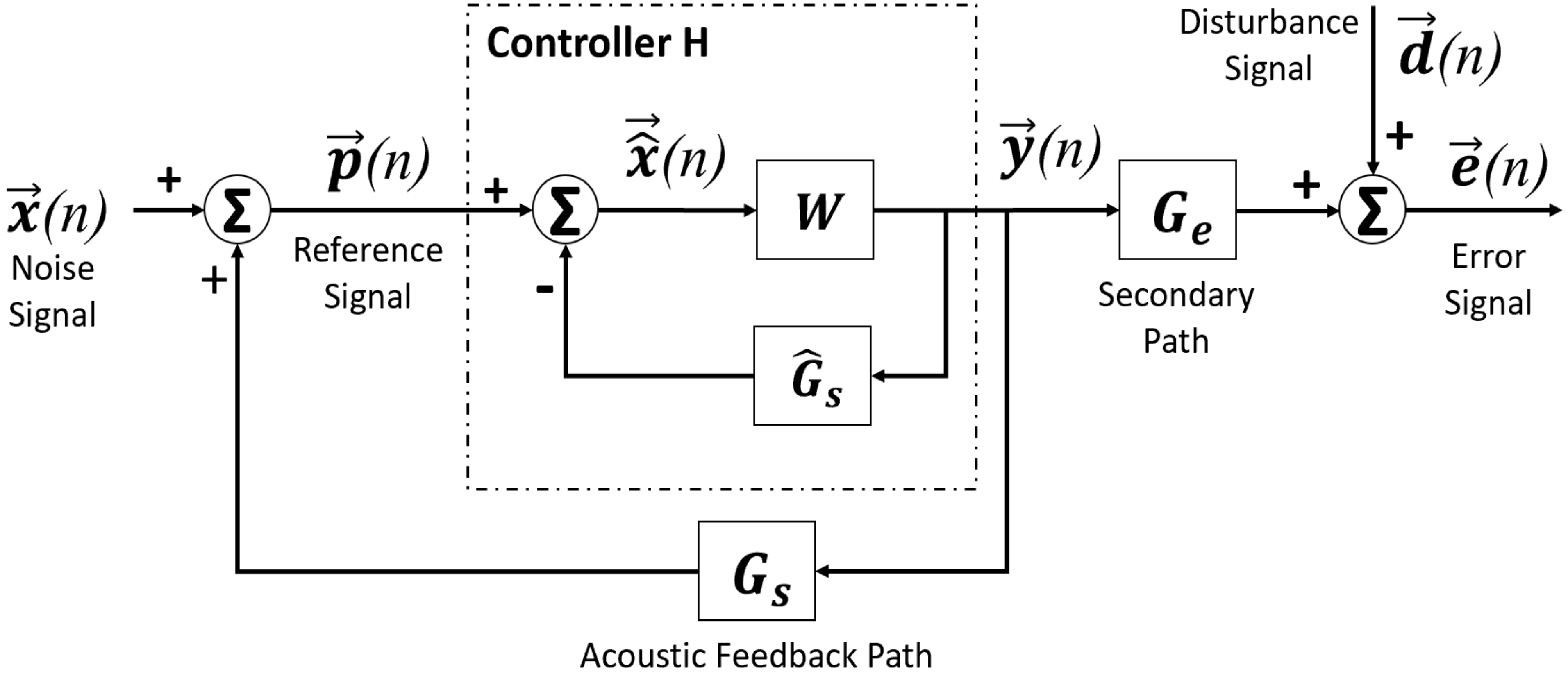
- When reference signals are correlated, the disturbance enhancement phenomenon is likely to occur, i.e., the resulting sound is enhanced instead of being reduced in those frequency bands.

- In previous work, a truncated singular value decomposition (SVD) method was applied to the auto-correlation matrix of filtered-reference signals to mitigate the disturbance enhancement.

- However, the removed singular values and singular vectors may contribute to the noise control performance in other frequency bands where the reference signals are not correlated. A direct truncation will sacrifice the noise control performance in those frequency bands.

- Thus, an improved method, singular vector filtering method, is proposed in the current work to reduce the impact on the noise control performance while still mitigate the disturbance enhancement.

Active Noise Control System



- Internal model control structure is used, and it is assumed that $\hat{G}_s = G_s$
- Design W such that $\vec{e}$ is minimized

Overview of Conventional Method

The cost function is:

$$\mathrm{E}[\vec{\boldsymbol{e}}^{\mathrm{T}}(n)\,\vec{\boldsymbol{e}}(n)] = \vec{\boldsymbol{w}}^{\mathrm{T}}\boldsymbol{A}\vec{\boldsymbol{w}} + 2\vec{\boldsymbol{w}}^{\mathrm{T}}\vec{\boldsymbol{b}} + c\ ,$$

where,

$$\boldsymbol{A} = \mathrm{E}[\boldsymbol{R}^{\mathrm{T}}(n)\,\boldsymbol{R}(n)],\ \vec{\boldsymbol{b}} = \mathrm{E}\left[\boldsymbol{R}^{\mathrm{T}}(n)\,\vec{\boldsymbol{d}}(n)\right],\ c = \mathrm{E}\left[\vec{\boldsymbol{d}}^{\mathrm{T}}(n)\,\vec{\boldsymbol{d}}(n)\right],$$

$$\vec{\boldsymbol{w}} = \left[w_{1,1,0}, w_{1,2,0}, \dots, w_{1,N_r,0}, \dots, w_{N_s,N_r,0}, \dots, w_{N_s,N_r,N_t}\right]^{\mathrm{T}},$$

$$\boldsymbol{R}(n) = \begin{bmatrix} \vec{\boldsymbol{r}}_1^{\mathrm{T}}(n) & \vec{\boldsymbol{r}}_1^{\mathrm{T}}(n-1) & \cdots & \vec{\boldsymbol{r}}_1^{\mathrm{T}}(n-N_t+1) \\ \vec{\boldsymbol{r}}_2^{\mathrm{T}}(n) & \vec{\boldsymbol{r}}_2^{\mathrm{T}}(n-1) & \cdots & \vec{\boldsymbol{r}}_2^{\mathrm{T}}(n-N_t+1) \\ \vdots & \vdots & \ddots & \vdots \\ \vec{\boldsymbol{r}}_{N_e}^{\mathrm{T}}(n) & \vec{\boldsymbol{r}}_{N_e}^{\mathrm{T}}(n-1) & \cdots & \vec{\boldsymbol{r}}_{N_e}^{\mathrm{T}}(n-N_t+1) \end{bmatrix},$$

$$\vec{\boldsymbol{r}}_q(n) = \left[r_{q,1,1}(n), r_{q,1,2}(n), \dots, r_{q,1,N_r}(n), r_{q,2,1}(n), \dots, r_{q,N_r,N_t}(n)\right]^{\mathrm{T}}, \qquad r_{q,m,l}(n) = \sum_{i=0}^{I-1} g_{q,m,j}\, x_l(n-j)$$

N_r, N_s, N_t are the number of reference sensors, control sources, and coefficients in each channel

$g_{q,m,j}(n)$ denotes the *j*-th coefficient of the FIR filter model of $\boldsymbol{G_e}$ that corresponds to the *m*-th input and the *q*-th output

Overview of Conventional Method

The optimal solution can be calculated by:

$$\vec{\boldsymbol{w}}_{opt} = -\boldsymbol{A}^{-1}\vec{\boldsymbol{b}}$$

When the filtered-reference signals $\vec{\boldsymbol{r}}_q(n)$ are correlated in some certain frequency bands:

- the auto-correlation matrix $\boldsymbol{A}$ may still be non-singular, thus $\boldsymbol{A}^{-1}$ exists.
- but the resulting sound level may be even higher than the original sound in those frequency bands, i.e., disturbance enhancement will occur in those frequency bands.

Overview of Truncation Method

The investigate the disturbance enhancement, Wang et al. applied singular value decomposition (SVD) to the auto-correlation matrix A:

$$\boldsymbol{A} = \boldsymbol{U}\boldsymbol{\Sigma}\boldsymbol{U}^{\mathrm{T}} = \begin{bmatrix} \vec{\boldsymbol{u}}_1 & \cdots & \vec{\boldsymbol{u}}_{N_r N_s N_t} \end{bmatrix} \begin{bmatrix} \boldsymbol{\sigma}_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \boldsymbol{\sigma}_{N_r N_s N_t} \end{bmatrix} \begin{bmatrix} \vec{\boldsymbol{u}}_1^{\mathrm{T}} \\ \vdots \\ \vec{\boldsymbol{u}}_{N_r N_s N_t}^{\mathrm{T}} \end{bmatrix}$$

Then, the optimal filter can be written as:

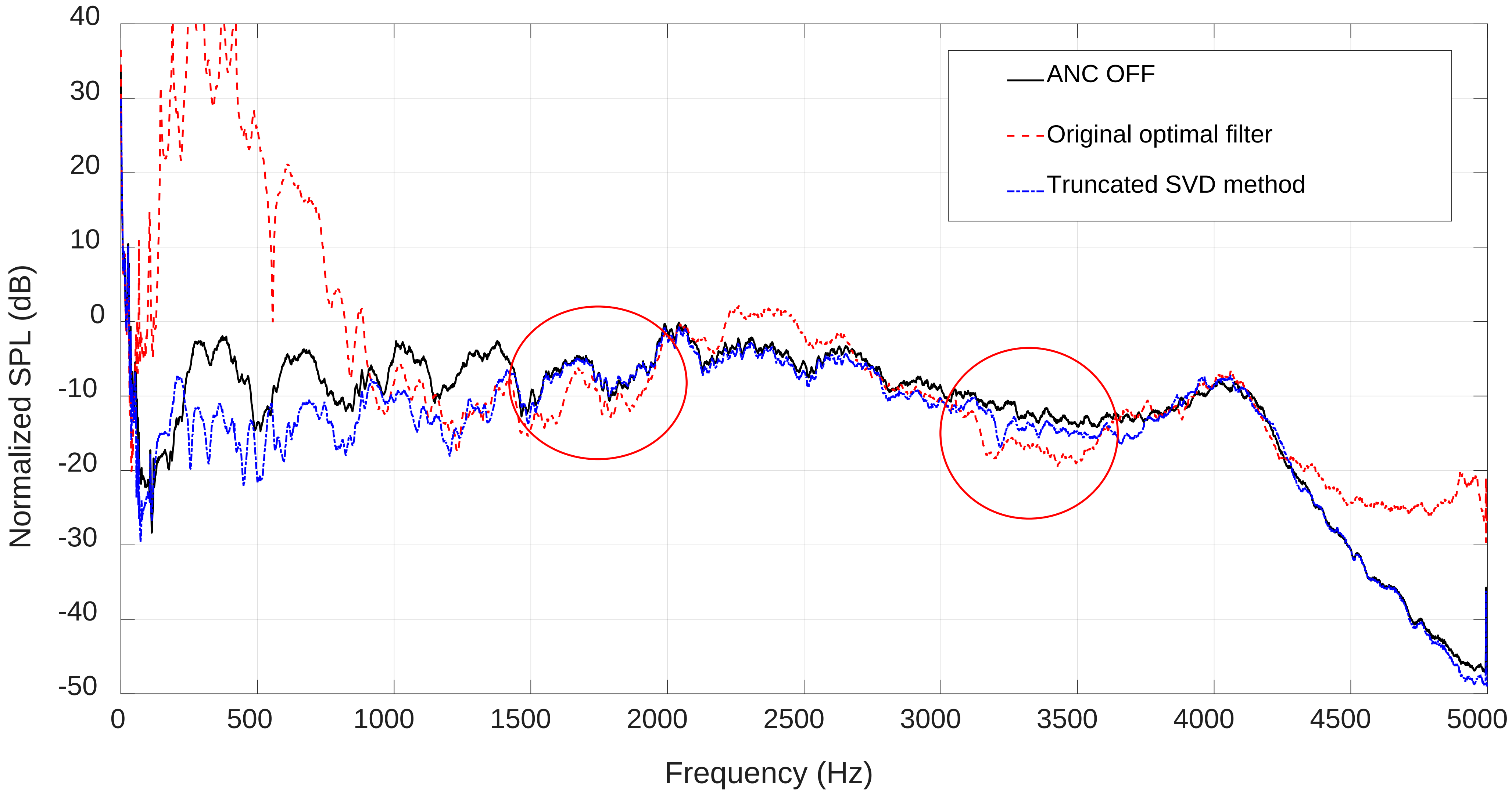
$$\vec{\boldsymbol{w}}_{opt} = -\sum_{k=1}^{N_r N_s N_t} \boldsymbol{\sigma}_k^{-1} \langle \vec{\boldsymbol{u}}_k, \vec{\boldsymbol{b}} \rangle \, \vec{\boldsymbol{u}}_k$$

Wang et al. demonstrated that when the correlation is strong among the reference signals, there will be some large value of $\boldsymbol{\sigma}_k^{-1} \langle \vec{\boldsymbol{u}}_k, \vec{\boldsymbol{b}} \rangle$, which should be truncated to mitigate the disturbance enhancement. The modified filter can be expressed as:

$$\vec{\boldsymbol{w}}_0 = -\sum_{k=1}^{l} \boldsymbol{\sigma}_k^{-1} \langle \vec{\boldsymbol{u}}_k, \vec{\boldsymbol{b}} \rangle \, \vec{\boldsymbol{u}}_k$$

where $l < N_r N_s N_t$ is an appropriate index.

Overview of Truncation Method



Singular Vector Filtering Method

The discarded singular values and vectors also contribute to the noise control performance at other frequency bands, thus a direct truncation will affect the overall noise control performance.

To reduce the impact on the noise control performance, singular vector filtering method is proposed in the current work.

First, the optimal filter can be treated as two groups:

$$\vec{\boldsymbol{w}}_{opt} = \vec{\boldsymbol{w}}_0 + \underbrace{\sum_{k=l+1}^{N_r N_s N_t}}_{\text{was truncated in truncation method}} \vec{\boldsymbol{w}}_k$$

$\vec{\boldsymbol{w}}_{opt}$ can be treated as a linear combination of different filters

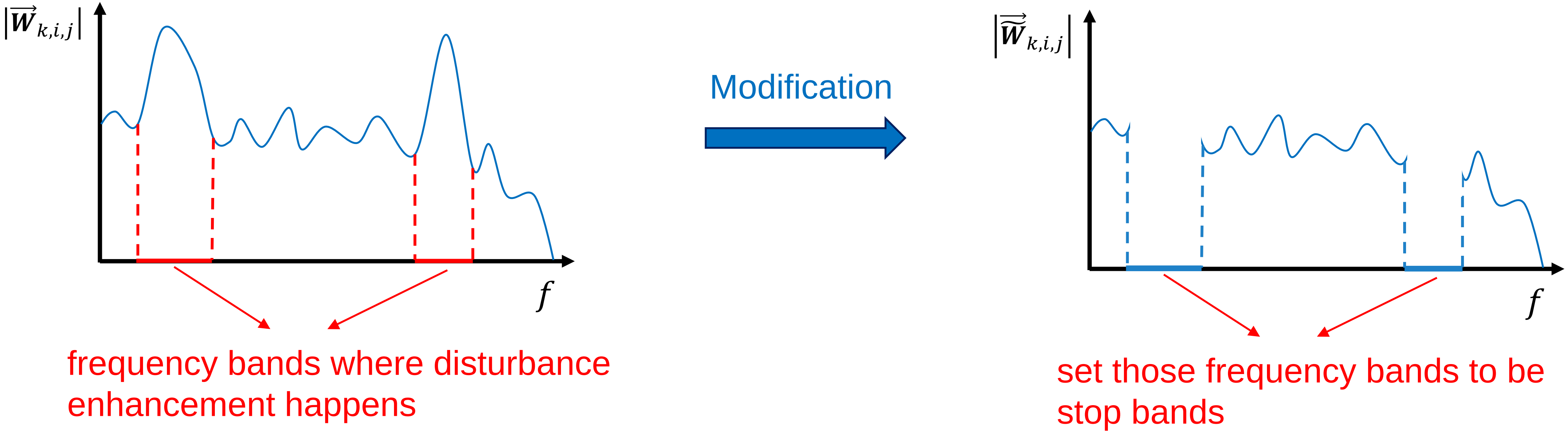
Each $\vec{\boldsymbol{w}}_k$ can be treated as a combination of filters from all channels.

$$\vec{\boldsymbol{w}}_k = -\boldsymbol{\sigma}_k^{-1} \langle \vec{\boldsymbol{u}}_k, \vec{\boldsymbol{b}} \rangle \, \vec{\boldsymbol{u}}_k = \left[w_{k,1,1,0}, w_{k,1,2,0}, \dots, w_{k,1,N_r,0}, \dots, w_{k,N_s,N_r,0}, \dots, w_{k,N_s,N_r,N_t} \right]^{\mathrm{T}}$$

Thus, it can be rearranged to filters $\vec{\boldsymbol{w}}_{k,i,j}$ (*i*-th output and *j*-th input) with length N_t.

Singular Vector Filtering Method

Let $\vec{\boldsymbol{W}}_{k,i,j}$ denotes the frequency response of $\vec{\boldsymbol{w}}_{k,i,j}$, $\overrightarrow{\widetilde{\boldsymbol{W}}}_{k,i,j}$ denotes the modified frequency response:



Then, a new set of filter $\vec{\boldsymbol{v}}_{k,i,j}$ is calculated to replace $\vec{\boldsymbol{w}}_{k,i,j}$ by:

$$\vec{\boldsymbol{v}}_{k,i,j} = \mathbf{arg}\min_{\vec{\boldsymbol{v}}_{k,i,j}} \left\| \boldsymbol{F}_z \vec{\boldsymbol{v}}_{k,i,j} - \overrightarrow{\widetilde{\boldsymbol{W}}}_{k,i,j} \right\|_2^2 ,$$

where $\boldsymbol{F}_z$ is the Fourier transform matrix

(Conventional band-stop filter is not used because it will introduce additional delay)

Singular Vector Filtering Method

Finally, the modified filter $\vec{\boldsymbol{w}}_{mod1}$ can be expressed as:

$$\vec{\boldsymbol{w}}_{mod1} = \vec{\boldsymbol{w}}_0 + \sum_{k=l+1}^{N_r N_s N_t} \vec{\boldsymbol{v}}_k \quad \Rightarrow \quad \text{Method 1}$$

Note that if $\vec{\boldsymbol{v}}_{k,i,j}$ is obtained separately for each *i*, *j* and *k*:
the optimization problem needs to be solved $N_r \times N_s \times (N_t - l)$ times.

To simplify the computation, $\sum_{k=l+1}^{N_r N_s N_t} \vec{\boldsymbol{w}}_k$ can be calculated first, then treat it as one filter and do the similar process and get $\vec{\boldsymbol{v}}_{sum}$. The optimal filter will be:

$$\vec{\boldsymbol{w}}_{mod2} = \vec{\boldsymbol{w}}_0 + \vec{\boldsymbol{v}}_{sum} \quad \Rightarrow \quad \text{Method 2}$$

Then the optimization problem needs to be solved only $N_r \times N_s$ times.

Results

Off-line Simulation based on experimental data.

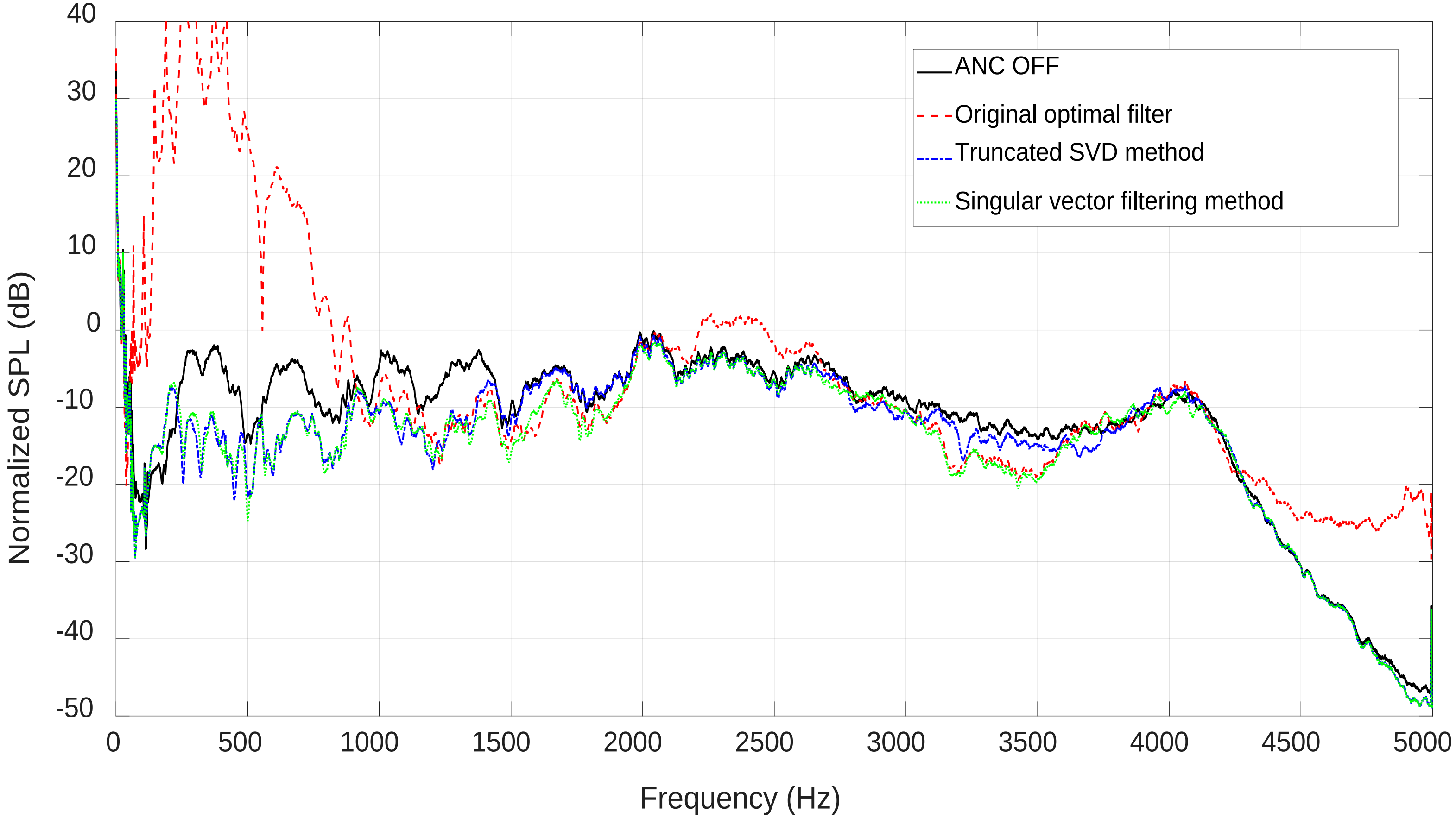
Experiment description:

- 2 reference microphones; 2 control loudspeakers; 4 error microphones
- sampling frequency is 10000 Hz
- Length of FIR filter for each channel is 128

  (i.e., a total of 512 singular values for matrix **A**)
- 2 loudspeakers playing white noise as noise source. To produce strongly correlated reference signals, input of one of the noise sources is further filtered through a high-pass filter with 1000 Hz cutoff frequency

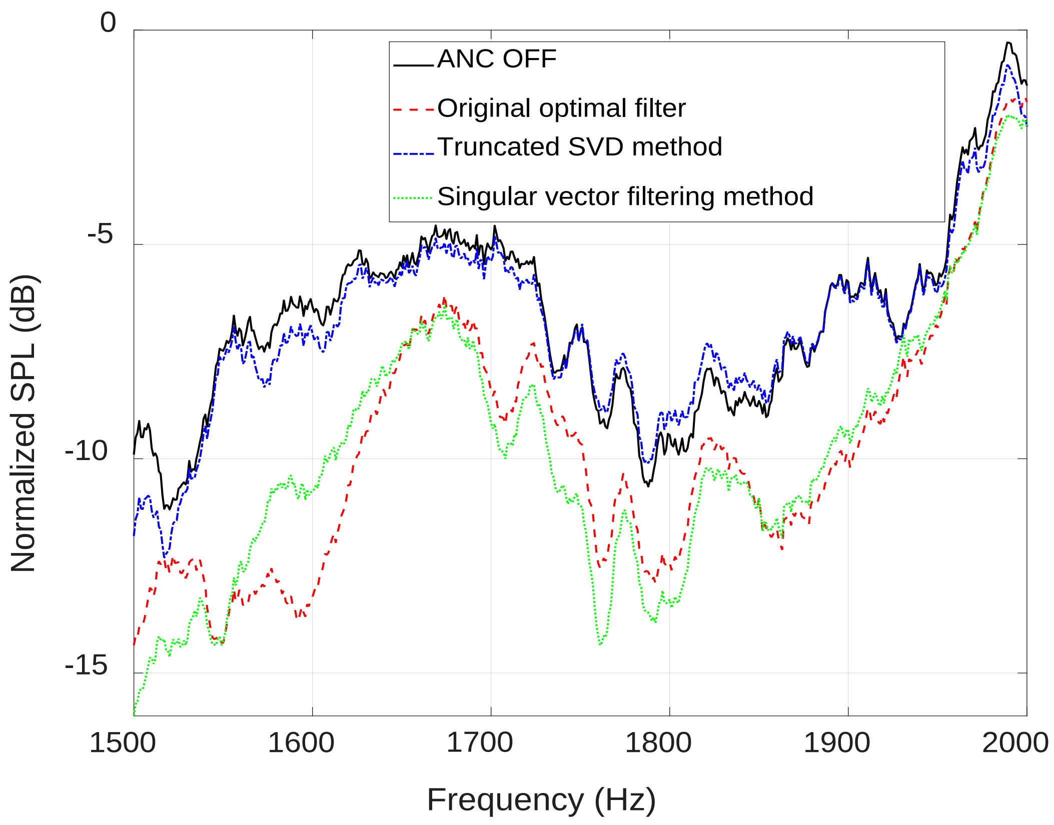
Results

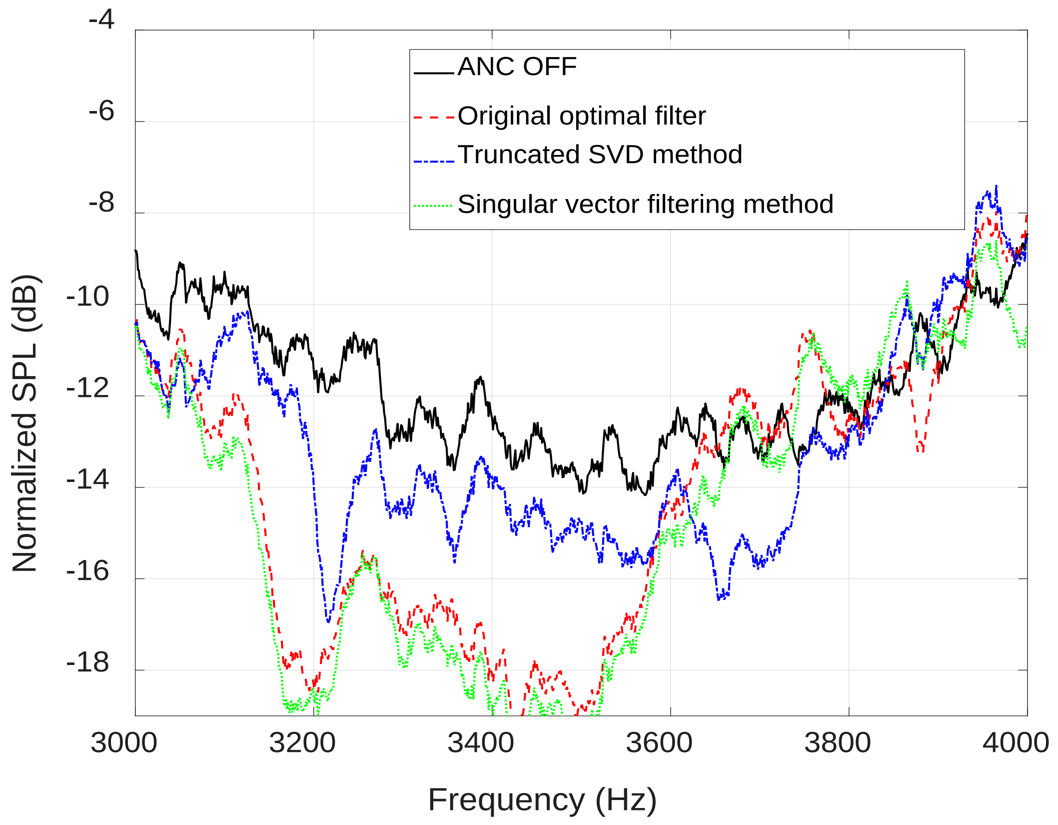
Comparison of averaged sound pressure at the error microphones for two different methods



The singular vector filtering method is using $\vec{\boldsymbol{w}}_{mod2}$ (method 2). Enhancement below 1000 Hz is due to the noise source setting, and enhancement between 2100 and 2700 Hz and above 4200 Hz is due to environment.

Results

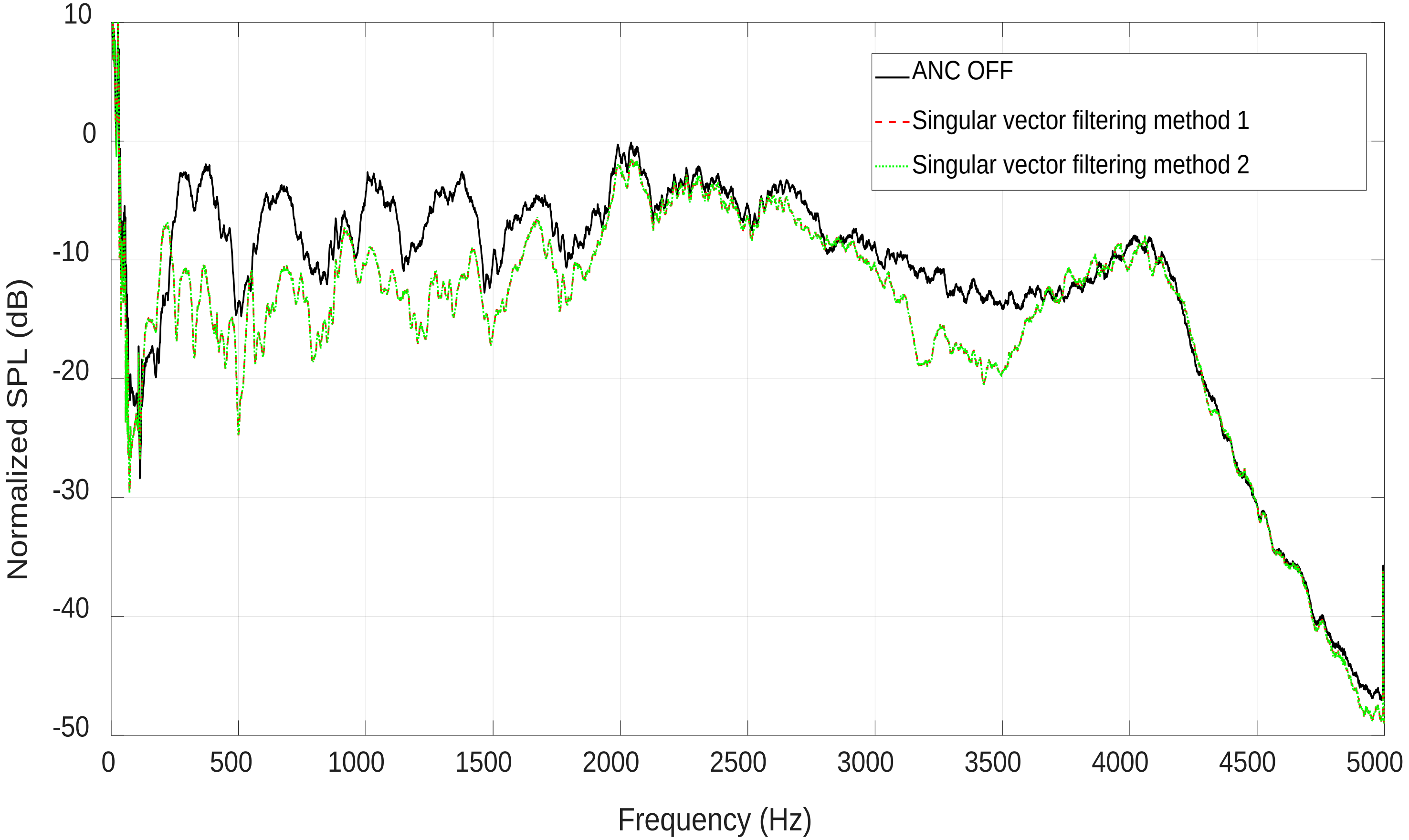




Compared with truncated SVD method, the singular vector filtering method has smaller impact on the noise control performance.

Results

Comparison of averaged sound pressure at the error microphones for two different singular vector filtering methods



This shows that modifying each $\vec{\boldsymbol{w}}_k$ separately (method 1) has similar performance compared with modifying $\sum_{k=l+1}^{N_r N_s N_t} \vec{\boldsymbol{w}}_k$ together (method 2).

Conclusion

- In the current work, a singular vector filtering method is proposed to mitigate the disturbance enhancement phenomenon caused by the correlation between reference signals.

- Compared with truncated singular value decomposition method, singular vector filtering method can effectively mitigate the disturbance enhancement and has a smaller impact on the noise control performance in other frequency bands.

- Compared with obtaining the filtered singular vectors separately, summing the singular vectors firstly then solving the optimization problem can have similar effect but with much lower computational complexity.

Thanks for listening!

Q&A