A Sub-band Filter Design Approach for Sound Field Reproduction

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Content

- Introduction
- Methodology
- Results
- Conclusions

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Statement of research problem

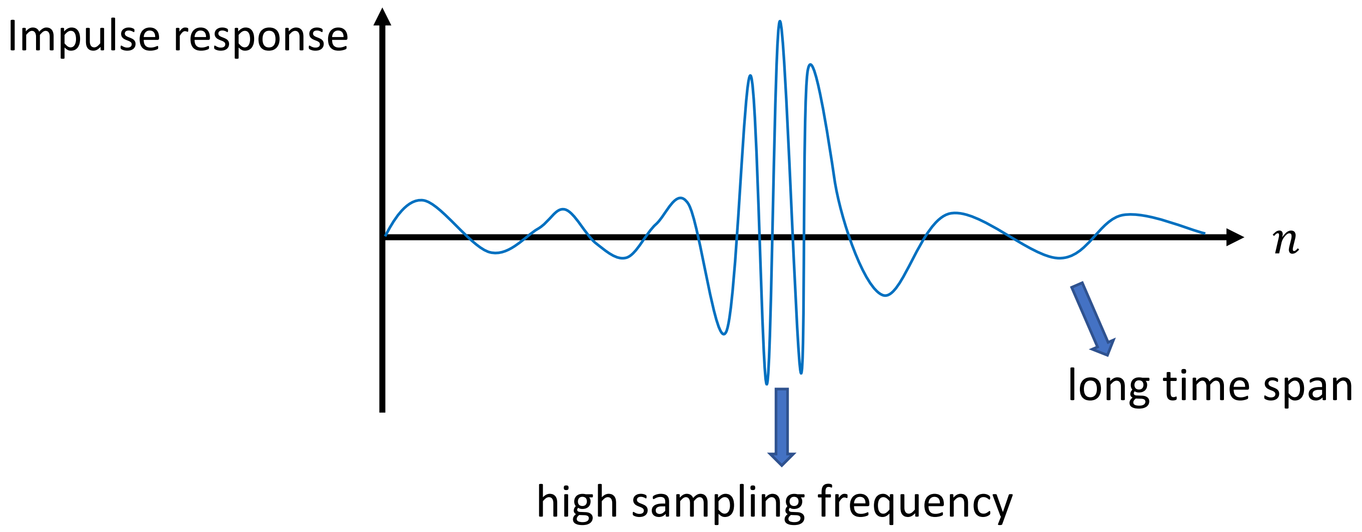
- ❑ Sound field reproduction uses loudspeakers to produce desired sound at locations.

- ❑ When designing filter for sound that spans a wide frequency range:

Low frequency band → longer time span

High frequency band → higher sampling frequency

→ Large number of filter coefficients



Statement of research problem

An approach is proposed to design filter in a sub-band form:

- Design all sub-band filters directly in one optimization problem:

  The **transition region** between two sub-band filters can be designed conveniently

- The **computational load** can be reduced even if sub-band filters structure is not required

Content

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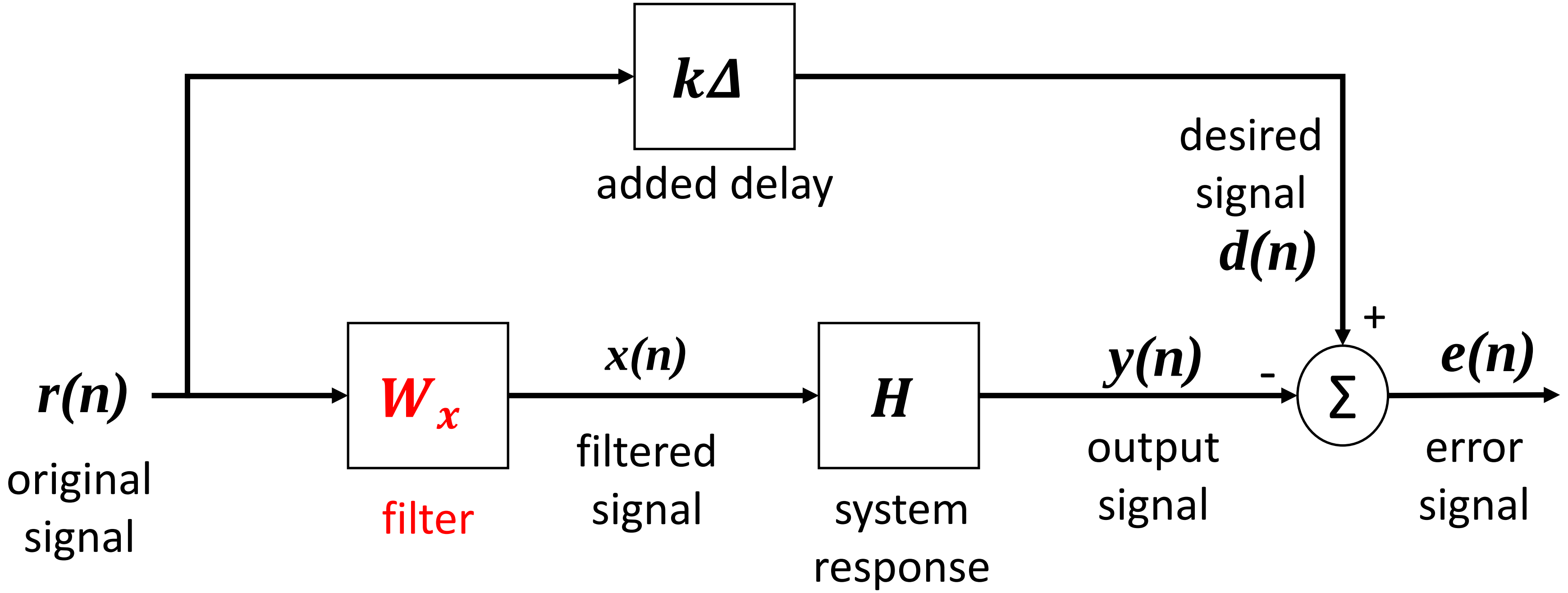
- Methodology
- Results
- Conclusions

Designing filter directly



❑ **Example**:

Use loudspeaker to produce desired sound at certain locations

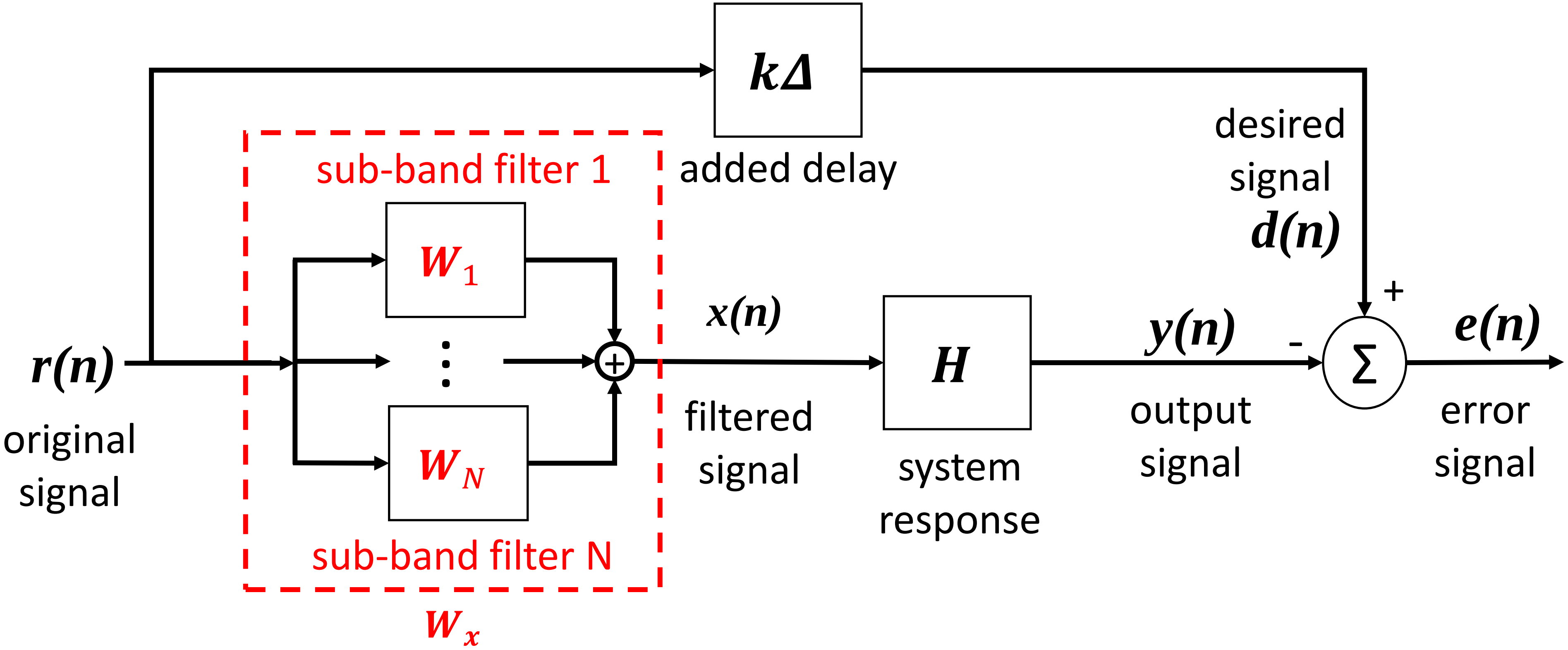
❑ **Cost function**:

Minimizing the power of error signal $\boldsymbol{e}$

❑ **Constraints:**

Filter response $\boldsymbol{W}_x$(f)

Designing filter when sub-band technique is used



- **Example**:

  Use loudspeaker to produce desired sound at certain locations

- **Cost function**:

  Minimizing the power of error signal e

- **Constraints:**

  Filter response W_i(f)

Expressing sub-band filters as one equivalent filter

Conventional method (one single filter)

frequency response of designed filter at frequency f_k :

$$W_x(f_k) = F(f_k, f_s, N_t)\,\vec{w}_x \ , \qquad F(f_k, f_s, N_t) = \begin{bmatrix} 1 & e^{-\frac{j2\pi f_k}{f_s}} & \cdots & e^{-\frac{j2\pi f_k (N_t - 1)}{f_s}} \end{bmatrix}$$

f_s is the sampling frequency,
N_t is the number of filter coefficients,
$\vec{w}_x$ is the filter coefficients

Sub-band structure

frequency response of designed filter at frequency f_k:

$$\sum_{i=1}^{N} W_i(f_k) = \sum_{i=1}^{N} \begin{bmatrix} 1 & e^{-\frac{j2\pi f_k}{f_{s_i}}} & \cdots & e^{-\frac{j2\pi f_k (N_t - 1)}{f_{s_i}}} \end{bmatrix} \vec{w}_i$$

Expressing sub-band filters as one equivalent filter

So designing sub-band filters can be treated as:
designing one filter $\vec{\tilde{w}}_x$ with modified Fourier matrix $\tilde{F}(f_k)$

$$\tilde{W}_x(f_k) = \sum_{i=1}^{N} W_i(f_k) = \sum_{i=1}^{N} \begin{bmatrix} 1 & e^{-\frac{j2\pi f_k}{f_{s_i}}} & \cdots & e^{-\frac{j2\pi f_k (N_t-1)}{f_{s_i}}} \end{bmatrix} \vec{w}_i = \tilde{F}(f_k)\, \vec{\tilde{w}}_x \,,$$

$$\tilde{F}(f_k) = \left[F\left(f_k, f_{s_1}, N_{t_1}\right) \quad \cdots \quad F\left(f_k, f_{s_N}, N_{t_N}\right) \right],$$

$$\vec{\tilde{w}}_x = \begin{bmatrix} \vec{w}_1 \\ \vdots \\ \vec{w}_N \end{bmatrix}$$

So all the sub-band filters can be designed in one optimization problem if designed in the frequency domain.
The transition region can be designed more conveniently.

Overview of proposed design process

Conventional method

General Nonlinear algorithm (e.g., sequential quadratic programming)

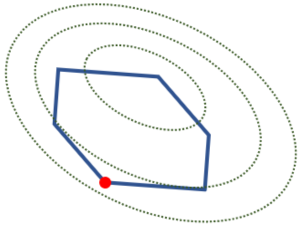
Proposed method

Design Problem → **Convex Optimization** → **Cone Programming** → **Primal-dual Interior-point**

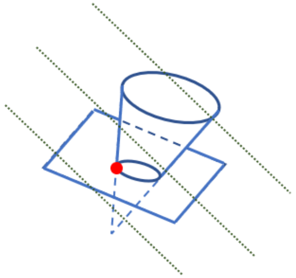
Cost function:
Power of ***e***

Constraints:
Filter response

Global minimum usually guaranteed



Possible to apply efficient algorithms



- Efficient
- Upper-bound for iteration times

Problem formulation

Design Problem Expressed in Convex Problem

Cost function:

Total power of e:

$$\sum_{k=k_1}^{k_2} |E(f_k)|^2, \quad \Longrightarrow \quad \vec{\widetilde{w}}_x^{\,\mathrm{T}} \left(\sum_{k=k_1}^{k_2} A_J(f_k)\right) \vec{\widetilde{w}}_x + 2\mathrm{Re}\left(\sum_{k=k_1}^{k_2} b_J^{\mathrm{T}}(f_k)\right) \vec{\widetilde{w}}_x + \sum_{k=k_1}^{k_2} c_J(f_k)$$

Convex ✓

- Quadratic
- $A_J(f_k)$ p.s.d

Constraints:

Filter response:

The magnitude of frequency response:

$$|W_i(f_k)| \le C_i(f_k) \quad \Longrightarrow \quad \|F(f_k, f_{s_1}, N_{t_1})\,\vec{w}_i\|_2 - C_i(f_k) \le 0 \quad \Longrightarrow$$

Vector norm **Convex** ✓

Cone Programming Reformulation

Convex Problem

Cost function:

$$\vec{\tilde{w}}_x^{\mathrm{T}} \left(\sum_{k=k_1}^{k_2} A_J(f_k)\right) \vec{\tilde{w}}_x + 2\mathrm{Re}\left(\sum_{k=k_1}^{k_2} b_J^{\mathrm{T}}(f_k)\right) \vec{\tilde{w}}_x + \sum_{k=k_1}^{k_2} c_J(f_k)$$

Constraints:

$$\|F(f_k, f_{s_1}, N_{t_1})\,\vec{w}_i\|_2 - C_i(f_k) \le 0$$

?

Standard Cone Programming

Cost function: $c^{\mathrm{T}}x$

Constraints: $x \in K_i, \quad i = 1, 2, 3 \ \ldots$

$$Ax = b$$

c **to be a constant vector**

K_i **to be a convex cone**

A, b **to be a constant matrix and vector**

Cone Programming Reformulation

Convex Problem ➡ **Cone Programming**

- **Reformulate quadratic cost function**

Cost function: $x^{\mathrm{T}} A\, x + b^{\mathrm{T}} x \; + c$

$\Updownarrow$

Cost function: $t_0 + b^{\mathrm{T}} x$ ➡ Linear cost function

Constraints: $\|\sqrt{A}\, x\|_2 \leq \sqrt{t_0\, \tilde{t}_0}$ ➡ Rotated second-order cone

$\tilde{t}_0 = 1$ ➡ Linear constraint

- **The vector norm constraint**

Constraints: $\|x\|_2 - c \leq 0$

$\Updownarrow$

Constraints: $\|x\|_2 \leq t$ ➡ Second-order cone

$t = c$ ➡ Linear constraint

Cone Programming Reformulation

Convex Problem

Cost function:

$$\vec{\tilde{w}}_x^{\ \mathrm{T}} \left(\sum_{k=k_1}^{k_2} A_J(f_k)\right) \vec{\tilde{w}}_x + 2\mathrm{Re}\left(\sum_{k=k_1}^{k_2} b_J^{\mathrm{T}}(f_k)\right) \vec{\tilde{w}}_x + \sum_{k=k_1}^{k_2} c_J(f_k)$$

Constraints:

$$\|F\left(f_k, f_{s_1}, N_{t_1}\right) \vec{w}_i\|_2 - C_i(f_k) \leq 0$$

Cone Programming

Cost function:

$$t_0 + 2\mathrm{Re}\left(\sum_{k=k_1}^{k_2} b_J^{\mathrm{T}}(f_k)\right) \vec{\tilde{w}}_x$$

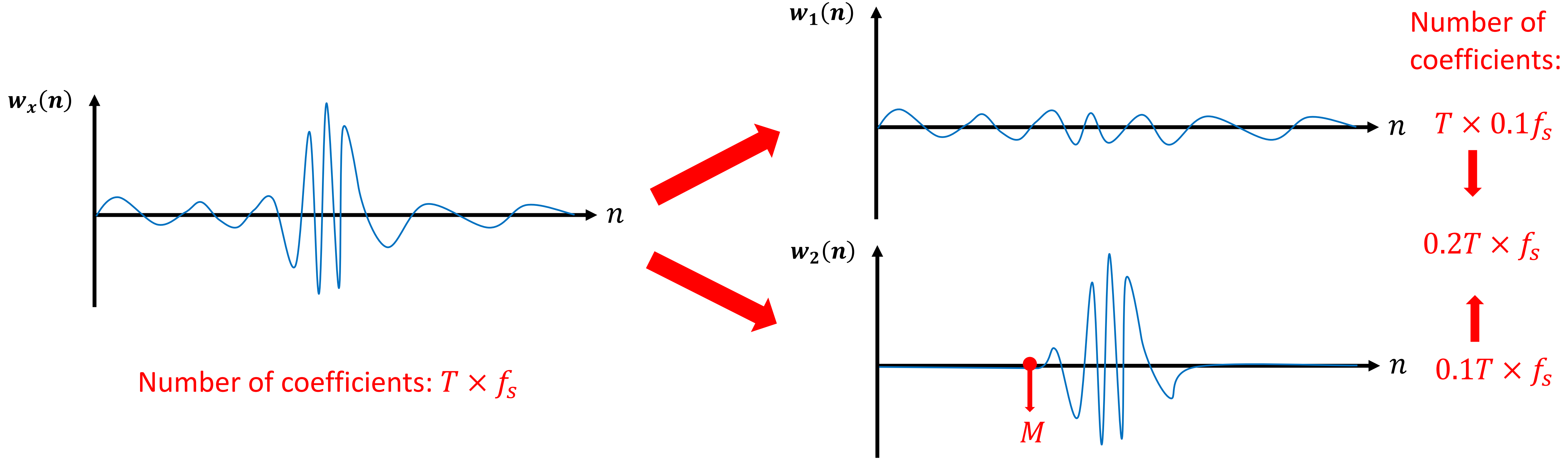
Constraints:

$$\|F\left(f_k, f_{s_1}, N_{t_1}\right) \vec{w}_i\|_2 \leq \ t_{3,k} \ ,$$

$$t_{3,k} = C(f_k)$$

A reduced order technique

Sometimes, the designed filter has high frequency response concentrated in small time span:



In this case, $\vec{w}_i$ (with higher sampling frequency) can be chosen to start with $t = M\Delta$, where $M > 0$, then we have:

$$F_r(f_k, f_s, N_t) = \left[e^{-\frac{j2\pi f_k M}{f_s}} \quad e^{-\frac{j2\pi f_k (M+1)}{f_s}} \quad \cdots \quad e^{-\frac{j2\pi f_k (N_t-1)}{f_s}} \right]$$

Content

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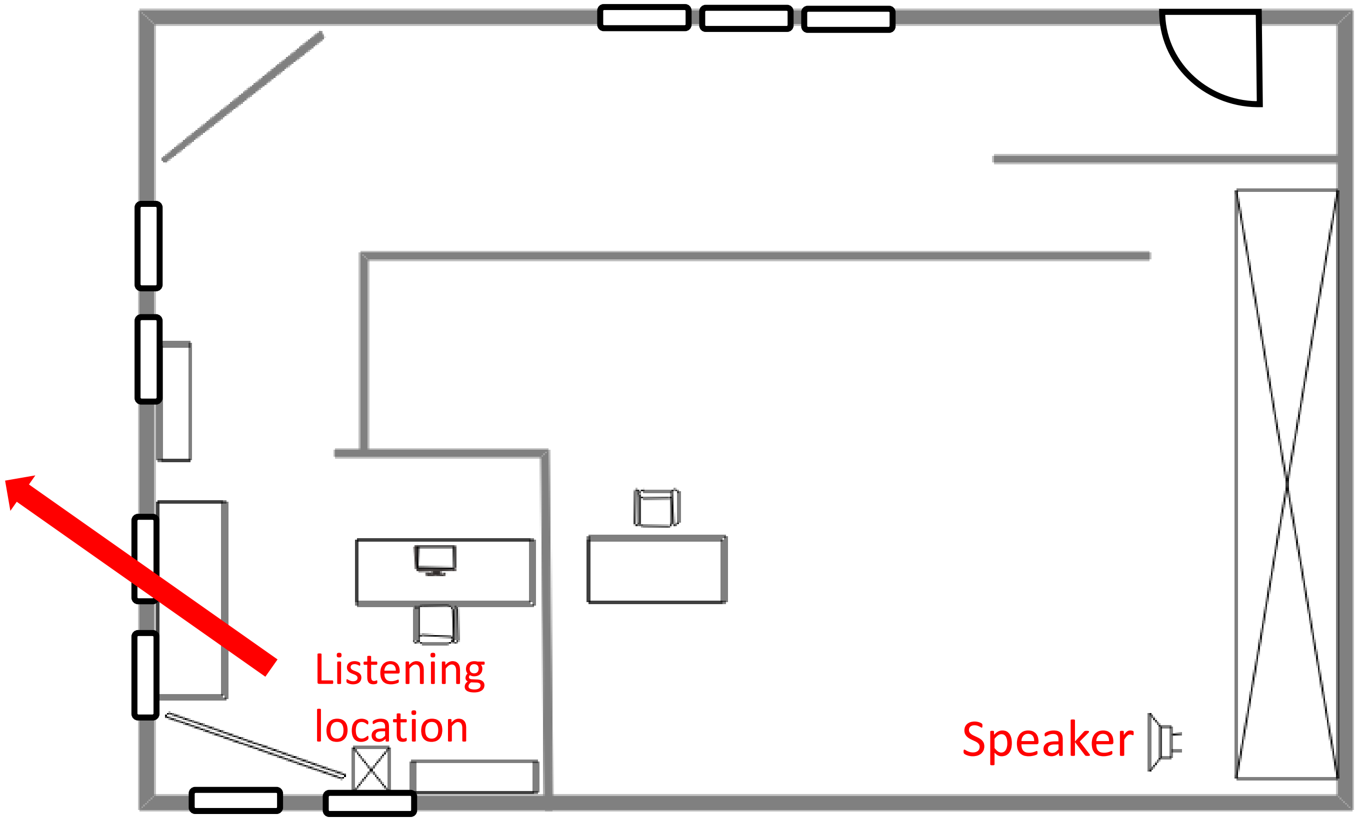
Experimental setup

- An experimental setup for psychoacoustic listening test
- Speaker should produce desired sound at listening location



Experimental setup

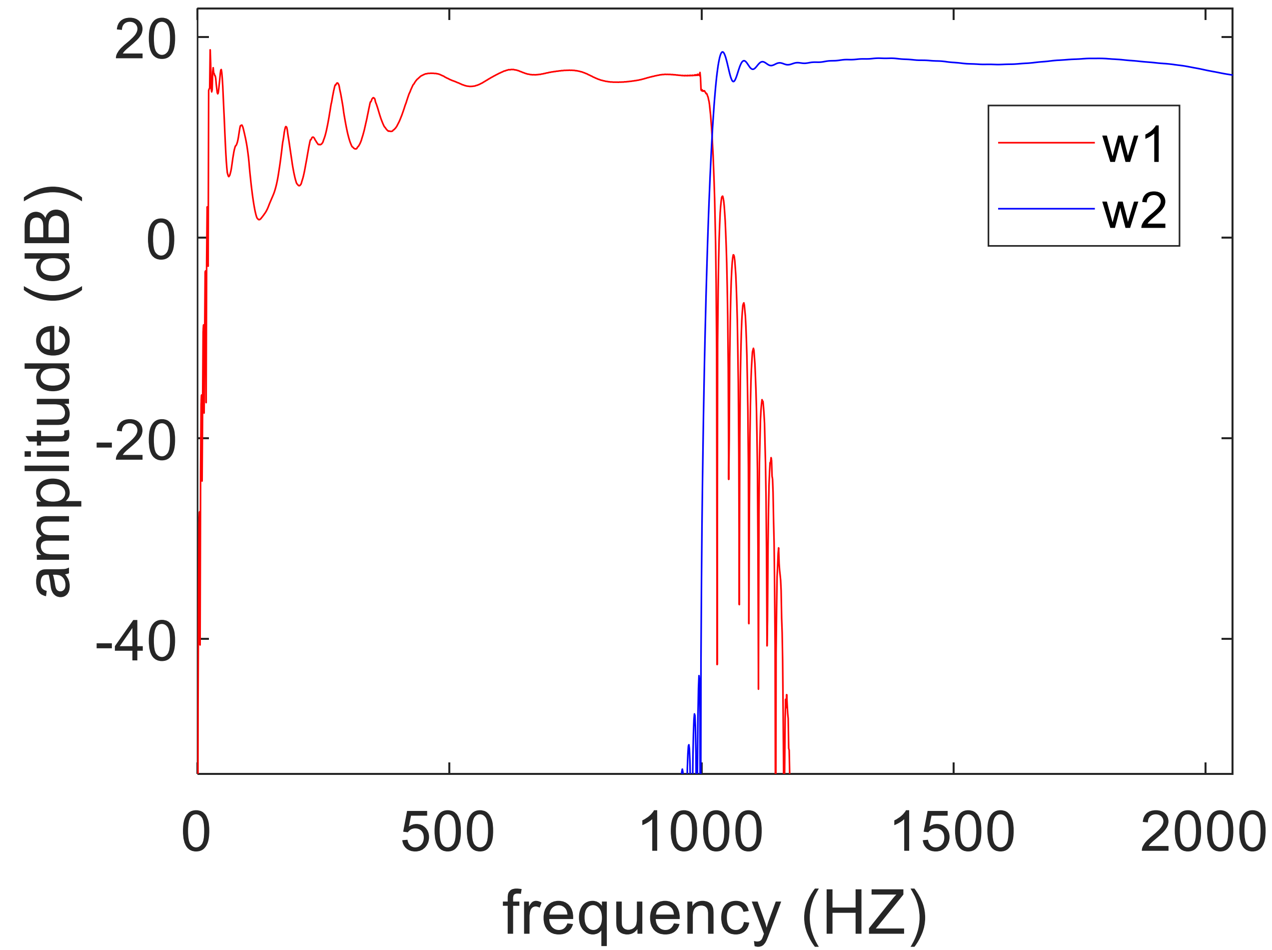
- Required sampling frequency: **48 kHz (Δ =20.83 us)**
- Desired delay: **19200 Δ**
- Two sub-band filters:

	Sampling frequency	**Filter coefficients**	**Starting time**
Filter 1	2.4 kHz	1920	0
Filter 2	48 kHz	3000	17700 **Δ**

- SeDuMi is used to solve the reformulated cone programming problem

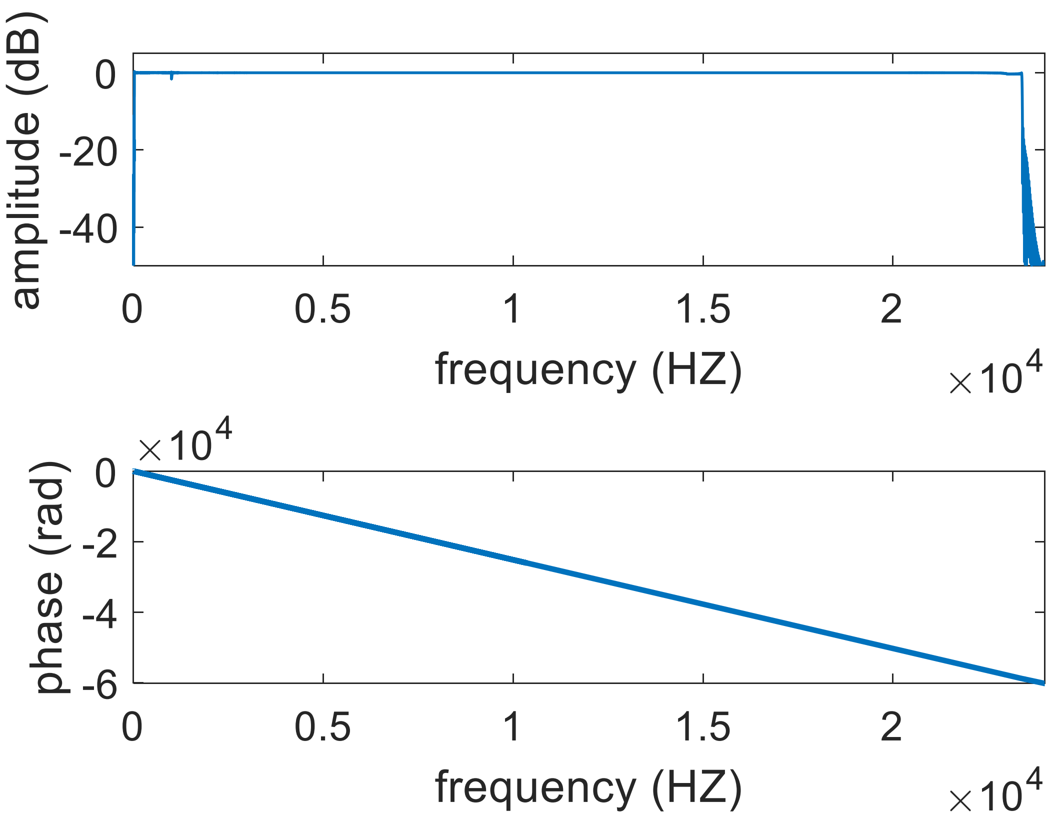
Result

The frequency response of both filter around 1200 Hz

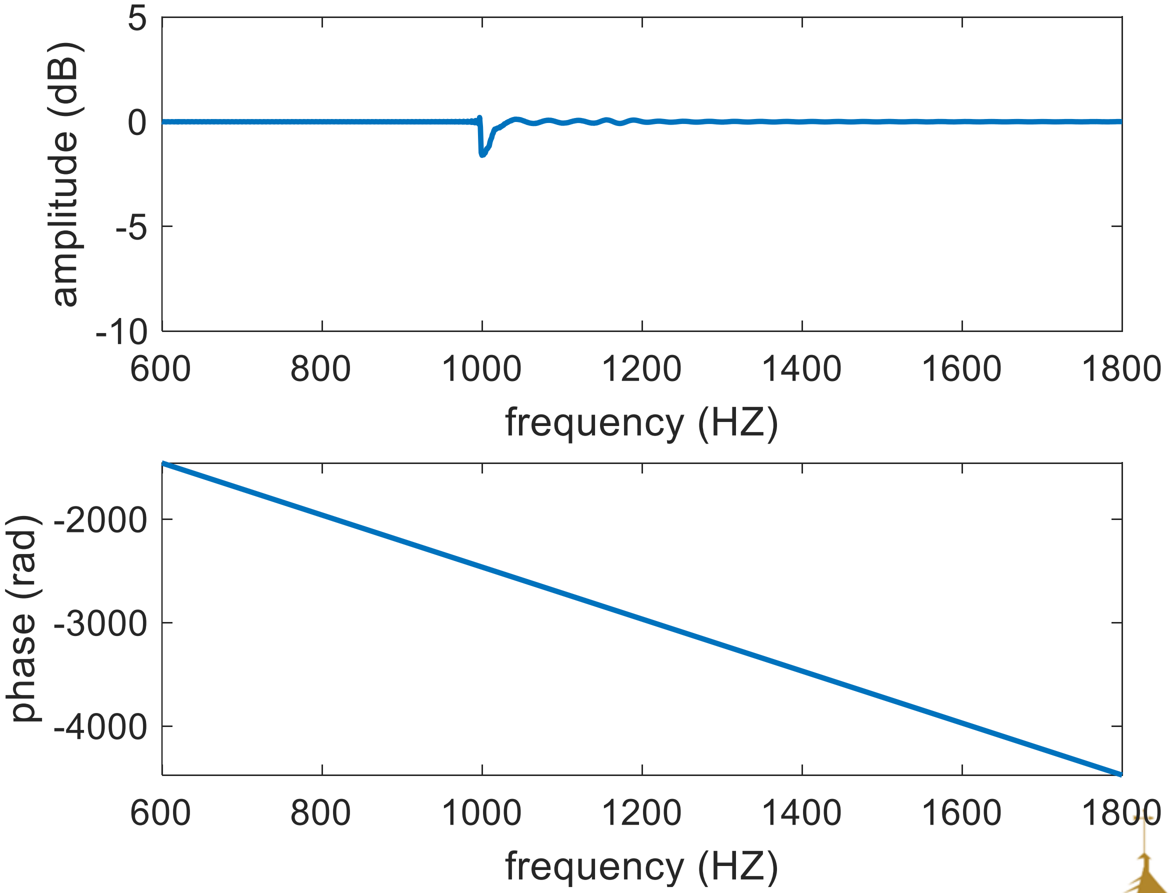


Result

The frequency response of $\mathrm{H(f)}\widetilde{W}_x(f)$



$\mathrm{H(f)}\widetilde{W}_x(f)$ around 1200 Hz



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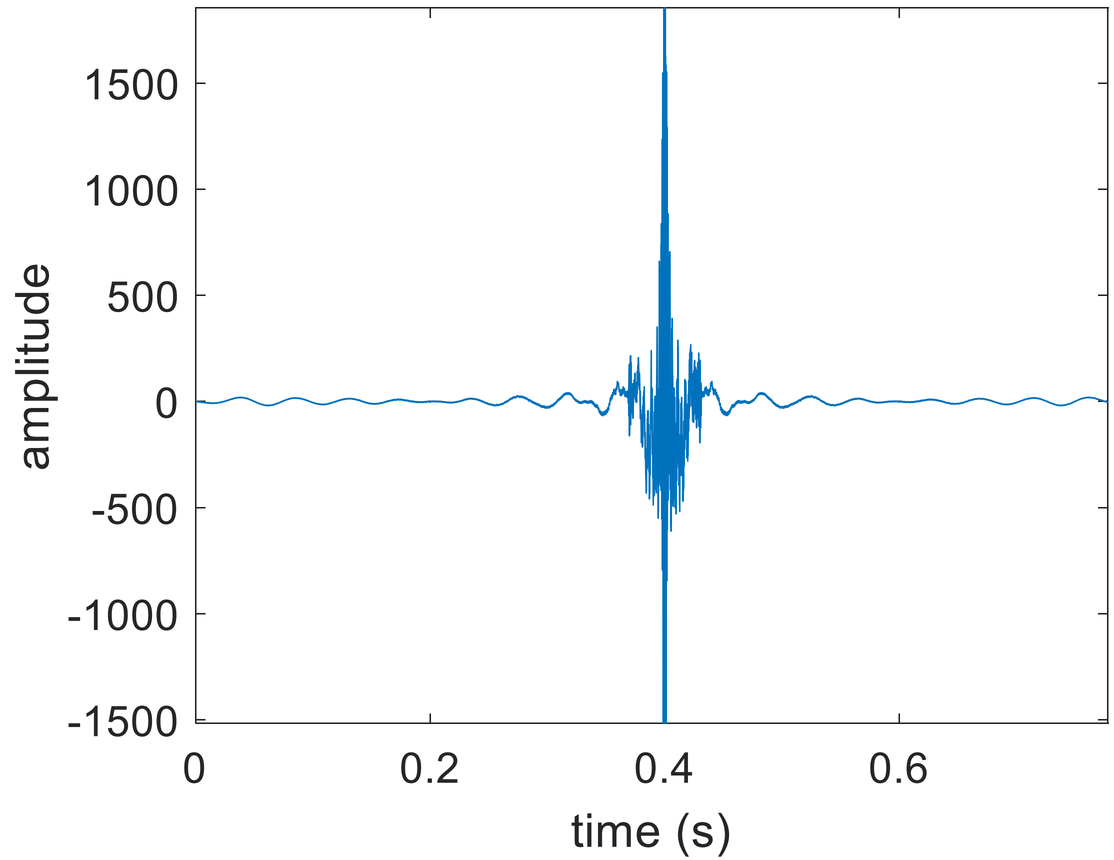
Result

Combining two sub-band filters together in time domain

The combination is done by:

- Upsampling the sub-band filter 1 with lower sampling frequency
- Adds the upsampled filter 1 with filter 2



The designed filter coefficients are 1920+3000 = 4920, which is much smaller than $48000 \times \frac{1920}{2400} = 38400$

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- Introduction
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Conclusions

- The proposed method can design sub-band filters for sound field reconstruction in one optimization problem, so designing transition region is more convenient.

- The optimization problem can be reformulated to a convex problem, then further reformulated to a cone programming problem. These guarantees the global optimal solution can be found in an efficient way.

- A reduced-order technique can be used to reduce the variables in filter design problem if different frequency bands of required filter have impulse response concentrated in different time intervals.

Q&A

References

- Zhuang, Yongjie, and Yangfan Liu. "Study on the cone programming reformulation of active noise control filter design in the frequency domain." *INTER-NOISE and NOISE-CON Congress and Conference Proceedings*. Vol. 260. No. 1. Institute of Noise Control Engineering, 2019.
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