

```

close all
clear
clc

%---test codes-----
disp("Test codes for question (B): ");
Q=[sqrt(0.2),sqrt(0.3),sqrt(0.4),sqrt(0.1)];
Rotation=Quat_to_Rot(Q);
disp("Rotation Matrix R: ");
disp(Rotation);
Quat=Rot_to_Quat(Rotation);
disp("Input Quaternion for Comparison: ");
disp(Q);
disp("Quaternion: ");
disp(Quat);
%---test ends-----

%-----question (c)-----
disp("*****");
disp("Question (C) section: ")
% first of all, generate 5 sets of random rpy angles and convert them into
% SO(3) Rotation Group
numOfRandom = 5;
% declare the equality.
equality=false;
for i=1:numOfRandom
    % generate random set of rpy vector
    rpy=2*pi*rand(1,3);
    % convert it into Rotation Matrix
    R=RPY_to_Rot(rpy);
    % Convert random Rotation Matrix into Quaternions using functions below
    Quaternion=Rot_to_Quat(R);
    % Convert the Quaternions back to Rotation Matrix using functions below
    R_c=Quat_to_Rot(Quaternion);
    % Compute the difference between the input and output Rotation Matrix
    diff=R_c-R;
    % display the results
    disp("-----");
    disp("The random set " + i + ": ");
    disp("    The input Rotation Matrix: ");
    disp(R);
    disp("    The output Rotation Matrix: ");
    disp(R_c);
    disp("    The difference is: ");
    disp(diff);
    if max(diff)>1E-10
        disp("The codes fail and the random generation of matrix stops!");
        equality=false;
        break;
    else
        disp("Equality for set " + i + " holds!" + newline);
        equality=true;
    end
end

% check the overall equality and make conclusion
disp("-----");
disp(newline+"Conclusion:");
switch equality
    case false
        disp("    The codes fail!");
    otherwise
        disp("    The codes are verified to be correct!");
end

```

end

%-----question (a)-----

function R = Quat_to_Rot(Q)

% Assign the elements in Quaternions for convenience

q0 = Q(1);

q1 = Q(2);

q2 = Q(3);

q3 = Q(4);

% Compute the Rotation Matrix Elements using Elements of Quaternions above

r11 = q0^2+q1^2-q2^2-q3^2;

r12 = 2*(q1*q2-q0*q3);

r13 = 2*(q1*q3+q0*q2);

r21 = 2*(q1*q2+q0*q3);

r22 = q0^2+q2^2-q1^2-q3^2;

r23 = 2*(q2*q3-q0*q1);

r31 = 2*(q1*q3-q0*q2);

r32 = 2*(q2*q3+q0*q1);

r33 = q0^2+q3^2-q1^2-q2^2;

% Assign the elements into the Rotation Matrix

R = [r11,r12,r13;r21,r22,r23;r31,r32,r33];

end

%-----question (a) ends-----

%-----question (b)-----

function Q = Rot_to_Quat(R)

% Assign the elements in Rotation matrix for convenience

r11 = R(1,1);

r12 = R(1,2);

r13 = R(1,3);

r21 = R(2,1);

r22 = R(2,2);

r23 = R(2,3);

r31 = R(3,1);

r32 = R(3,2);

r33 = R(3,3);

% declare column vector b expressed in Equation (2.23) in lecture notes

b = [r11;r22;r33;1];

% declare the inverse of a matrix that represents the coefficients

% expressed in equation (2.21) in lecture notes. The inverse of that matrix

% is just 0.25 times the transpose of that matrix itself!

invA = 0.25*[1 1 1 1;1 -1 -1 1;-1 1 -1 1;-1 -1 1 1];

% compute the Qsquare term

Qsquare = invA*b;

% find the index of the maximum value in the vector, if there are multiple

% maximum values (less likely), then return the first one.

maxIndex=find(Qsquare==max(Qsquare),1,'first');

qknown=sqrt(max(Qsquare));

% determine which solution will be used.

switch maxIndex

% solution 1 will be used

case 1

q0=qknown;

q1=(r32-r23)/(4*q0);

q2=(r13-r31)/(4*q0);

q3=(r21-r12)/(4*q0);

% solution 2 will be used

case 2

q1=qknown;

q0=(r32-r23)/(4*q1);

q2=(r12+r21)/(4*q1);

```

        q3=(r13+r31)/(4*q1);
% solution 3 will be used
case 3
    q2=qknown;
    q0=(r13-r31)/(4*q2);
    q1=(r12+r21)/(4*q2);
    q3=(r23+r32)/(4*q2);
otherwise
    q3=qknown;
    q0=(r21-r12)/(4*q3);
    q1=(r13+r31)/(4*q3);
    q2=(r23+r32)/(4*q3);
end

Q=[q0,q1,q2,q3];

end

%-----question (b) ends-----

%-----question (c)-----
function R = RPY_to_Rot(RPY_angles)
% assign the RPY angles for convenience
% Roll angle
gamma = RPY_angles(1);
% Pitch angle
beta = RPY_angles(2);
% Yaw angle
alpha = RPY_angles(3);
% Compute each elementary rotation matrices for RPY.
% Roll Matrix
R1 = [1,0,0;0,cos(gamma),-sin(gamma);0,sin(gamma),cos(gamma)];
% Pitch Matrix
R2 = [cos(beta),0,sin(beta);0,1,0;-sin(beta),0,cos(beta)];
% Yaw Matrix
R3 = [cos(alpha),-sin(alpha),0;sin(alpha),cos(alpha),0;0,0,1];
% Total Rotation Matrix
R = R3*R2*R1;
end
%-----question (c) ends-----

```

Test codes for question (B):

Rotation Matrix R:

```

-0.0000    0.4100    0.9121
 0.9757    0.2000   -0.0899
-0.2193    0.8899   -0.4000

```

Input Quaternion for Comparison:

```

0.4472    0.5477    0.6325    0.3162

```

Quaternion:

```

0.4472    0.5477    0.6325    0.3162

```

Question (C) section:

The random set 1:

The input Rotation Matrix:

```

0.1146    0.0145    0.9933
 0.2143    0.9760   -0.0389
-0.9700    0.2173    0.1088

```

The output Rotation Matrix:

0.1146	0.0145	0.9933
0.2143	0.9760	-0.0389
-0.9700	0.2173	0.1088

The difference is:

1.0e-15 *

-0.1388	0.0069	0.1110
0	0.1110	0.0069
0	0	-0.2220

Equality for set 1 holds!

The random set 2:

The input Rotation Matrix:

-0.0274	0.3495	-0.9365
0.9097	-0.3796	-0.1683
-0.4143	-0.8566	-0.3075

The output Rotation Matrix:

-0.0274	0.3495	-0.9365
0.9097	-0.3796	-0.1683
-0.4143	-0.8566	-0.3075

The difference is:

1.0e-15 *

-0.0902	-0.0555	0
0	-0.0555	-0.1110
0	-0.1110	0.1110

Equality for set 2 holds!

The random set 3:

The input Rotation Matrix:

-0.2759	-0.3450	-0.8971
0.4093	0.8023	-0.4344
0.8697	-0.4871	-0.0802

The output Rotation Matrix:

-0.2759	-0.3450	-0.8971
0.4093	0.8023	-0.4344
0.8697	-0.4871	-0.0802

The difference is:

1.0e-15 *

0.1665	-0.0555	0
0.1110	0	-0.0555
0	0	0.1804

Equality for set 3 holds!

The random set 4:

The input Rotation Matrix:

-0.0862	-0.1309	-0.9876
-0.1700	0.9787	-0.1149
0.9817	0.1580	-0.1066

The output Rotation Matrix:

-0.0862	-0.1309	-0.9876
---------	---------	---------

-0.1700	0.9787	-0.1149
0.9817	0.1580	-0.1066

The difference is:
1.0e-15 *

-0.2082	-0.0555	0.1110
0.0555	-0.1110	0.0555
0	-0.0555	-0.1388

Equality for set 4 holds!

The random set 5:

The input Rotation Matrix:

0.6066	-0.6332	0.4807
-0.5229	-0.7732	-0.3586
0.5988	-0.0338	-0.8002

The output Rotation Matrix:

0.6066	-0.6332	0.4807
-0.5229	-0.7732	-0.3586
0.5988	-0.0338	-0.8002

The difference is:
1.0e-15 *

-0.1110	0	-0.0555
0	0.3331	-0.0555
-0.1110	-0.0416	0

Equality for set 5 holds!

Conclusion:

The codes are verified to be correct!

.....
Published with MATLAB® R2018b