Assignment 1 - Group 22 - Mechanical

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1 (a)

$$\begin{aligned} J_1\dot{\omega}_1 &= K_T \cdot i_a - D_1(\phi_1 - \phi_2) \\ \dot{\omega}_1 &= \frac{1}{J_1}(K_T \cdot i_a - D_1(\phi_1 - \phi_2)) \end{aligned} \tag{1}$$

$$\begin{aligned} J_2\dot{\omega}_2 &= D_1(\phi_1 - \phi_2) - D_2(\phi_2 - \phi_3) \\ \dot{\omega}_2 &= \frac{1}{J_2}(D_1(\phi_1 - \phi_2) - D_2(\phi_2 - \phi_3)) \end{aligned} \tag{2}$$

$$\begin{aligned} v_a &= R \cdot i_a + L \cdot \dot{i}_a + K_E \cdot \omega_1 \\ \dot{i}_a &= \frac{1}{L}(v_a - R \cdot i_a - k_E \cdot \omega_1) \end{aligned} \tag{3}$$

$$\begin{aligned} \dot{\phi}_1 &= \omega_1 \\ \dot{\phi}_2 &= \omega_2 \\ \dot{\phi}_3 &= \omega_3 \end{aligned} \tag{4}$$

$$\begin{aligned} D_2(\phi_2 - \phi_3) &= T_e + B_f\omega_3 \\ \omega_3 &= \frac{1}{B_f}(D_2(\phi_2 - \phi_3) + T_e) \end{aligned} \tag{5}$$

and let

$$\begin{aligned} x &= [i_a \quad \phi_1 \quad \phi_2 \quad \phi_3 \quad \omega_1 \quad \omega_2]^{\mathrm{T}} \\ u &= [v_a \quad T_e] \end{aligned} \tag{6}$$

Finally, we put the equation (1-5) to describe the system, and the system is of 6 order.

Hence we have the state space model:

$$\dot{x} = \begin{bmatrix} -\dfrac{R}{L} & 0 & 0 & 0 & -\dfrac{K_E}{L} & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & \dfrac{D_2}{B_f} & -\dfrac{D_2}{B_f} & 0 & 0 \\ \dfrac{K_T}{J_1} & \dfrac{D_1}{J_1} & \dfrac{D_2}{J_1} & 0 & 0 & 0 \\ 0 & \dfrac{D_1}{J_2} & -\dfrac{(D_1+D_2)}{J_2} & \dfrac{D_2}{J_2} & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} i_a \\ \phi_1 \\ \phi_2 \\ \phi_3 \\ \omega_1 \\ \omega_2 \end{bmatrix} + \begin{bmatrix} \dfrac{1}{L} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & \dfrac{1}{B_f} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} v_a \\ T_e \end{bmatrix} \tag{7}$$

2 (b)

According to the question, we have

$$\begin{aligned} x &= [\phi_1 \quad \phi_2 \quad \phi_3 \quad \omega_1 \quad \omega_2]^{\mathrm{T}} \\ u &= [v_a \quad T_e] \end{aligned} \tag{8}$$

Since the $L \approx 0$, the rotor current i_a is no longer in state and written as:

$$i_a = \frac{v_a - K_E \cdot \omega_1}{R} \tag{9}$$

State-space model:

$$\dot{x} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & \dfrac{D_2}{B_f} & -\dfrac{D_2}{B_f} & 0 & 0 \\ -\dfrac{D_1}{J_1} & \dfrac{D_1}{J_1} & 0 & -\dfrac{K_T \cdot K_E}{J_1 \cdot R} & 0 \\ \dfrac{D_1}{J_2} & -\dfrac{(D_1+D_2)}{J_2} & \dfrac{D_2}{J_2} & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \omega_1 \\ \omega_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & \dfrac{1}{B_f} \\ \dfrac{K_T}{J_1 \cdot R} & 0 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} v_a \\ T_e \end{bmatrix} \tag{10}$$

3 (c)

3.1

$$y = \begin{bmatrix} \phi_2 \\ \omega_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \omega_1 \\ \omega_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} v_a \\ T_e \end{bmatrix} \tag{11}$$

3.2

$$y = \begin{bmatrix} i_a \\ \omega_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & -\dfrac{K_E}{R} & 0 \\ 0 & \dfrac{D_2}{B_f} & -\dfrac{D_2}{B_f} & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \omega_1 \\ \omega_2 \end{bmatrix} + \begin{bmatrix} \dfrac{1}{R} & 0 \\ 0 & \dfrac{1}{B_f} \end{bmatrix} \cdot \begin{bmatrix} v_a \\ T_e \end{bmatrix} \tag{12}$$

4 (d)

zero-pole cancellation: eigenvalues and poles are the same zero-pole cancellation may cause the poles of transition different from the system
With the parameters offered in the question, the A-matrix turns to be:

$$A = 1.0 \times 10^6 \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1000 & -1000 & 0 & 0 \\ -2000000 & 2000000 & 0 & -1000 & 0 \\ 500000 & -550000 & 50000 & 0 & 0 \end{bmatrix} \tag{13}$$

In Matlab, use 'pzmap' function to draw the zeros-poles map and get the zeros and poles:

4.1 case 1

$$Poles_{(1)} = \begin{bmatrix} -391.388379995380 + 1479.08290241348i \\ -391.388379995380 - 1479.08290241348i \\ -1.09288316514051e - 12 + 0.00000000000000i \\ -270.834675540027 + 0.00000000000000i \\ -946.388564469213 + 0.00000000000000i \end{bmatrix} \tag{14}$$

$$Zeros_{(1)} = \emptyset$$

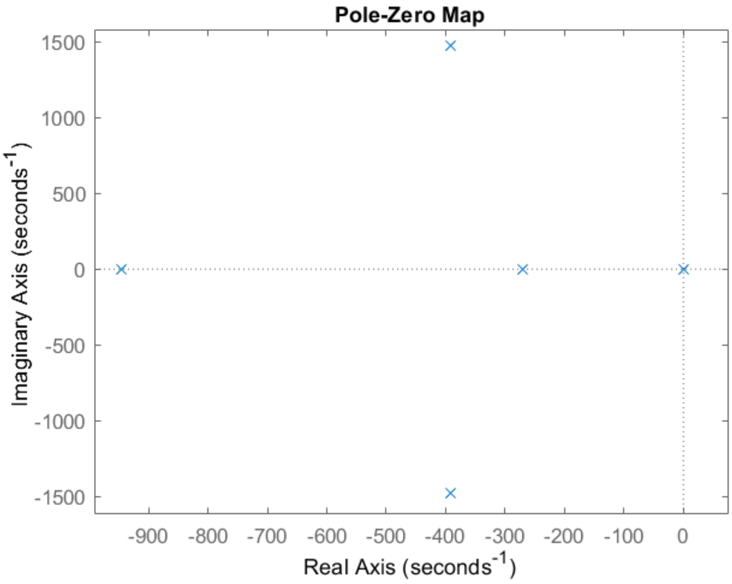

Figure 1: Poles and Zeros of case 1

In case 1, there are none zeros so pole-zero cancellation cannot happen.
From figure 1, we can see a pole is at (0,0) point and others on the left hand plane, so the system is margin stable.

4.2 case 2

$$Poles_{(2)} = \begin{bmatrix} -391.388379995380 + 1479.08290241348i \\ -391.388379995380 - 1479.08290241348i \\ -1.09288316514051e - 12 + 0.00000000000000i \\ -270.834675540027 + 0.00000000000000i \\ -946.388564469213 + 0.00000000000000i \end{bmatrix}$$
$$Zeros_{(1)} = \begin{bmatrix} 0.00000000000000 + 0.00000000000000i \\ -3.36398007510236e - 12 + 1584.34905645933i \\ -3.36398007510236e - 12 - 1584.34905645933i \\ -8.75526938901137e - 13 + 199.594757687765i \\ -8.75526938901137e - 13 - 199.594757687765i \end{bmatrix} \tag{15}$$

In case 2, we notice that pole-zero cancellation happens at (0,0) point.

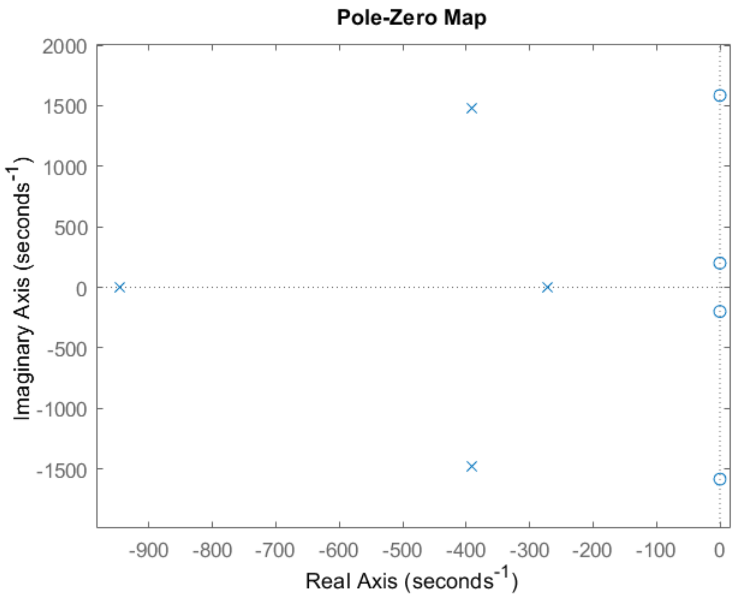

Figure 2: Poles and Zeros of case 2

From figure 2, we can see that there are 4 zeros on the axis y while other poles and zeros on the left hand plane, so the system is margin stable.

5 (e)

When the external torque is zero, the transfer function from the input, applied rotor voltage, to the output, defined in case (2) of question c) is given by

$$G(s) = C(sI - A)^{-1}B + D \tag{16}$$

```
From input to output...
     s^5 + 1000 s^4 + 2.55e06 s^3 + 2.5e09 s^2 + 1e11 s + 0.01988
 1:  -------------------------------------------------------------
     s^5 + 2000 s^4 + 3.55e06 s^3 + 3.05e09 s^2 + 6e11 s + 0.05601

                               5e12 s
 2:  -------------------------------------------------------------
     s^5 + 2000 s^4 + 3.55e06 s^3 + 3.05e09 s^2 + 6e11 s + 0.05601
```

Figure 3: transfer function

Compute the Zeros and Poles:

$$Poles_e = \begin{bmatrix} -391.388379995380 + 1479.08290241348i \\ -391.388379995380 - 1479.08290241348i \\ -1.09288316514051e - 12 + 0.00000000000000i \\ -270.834675540027 + 0.00000000000000i \\ -946.388564469213 + 0.00000000000000i \end{bmatrix}$$

$$Zeros_e = -1.75979850974953e - 14 \approx 0$$

The zero can be considered as numerically 0, then the pole-zero happens at (0,0) point.

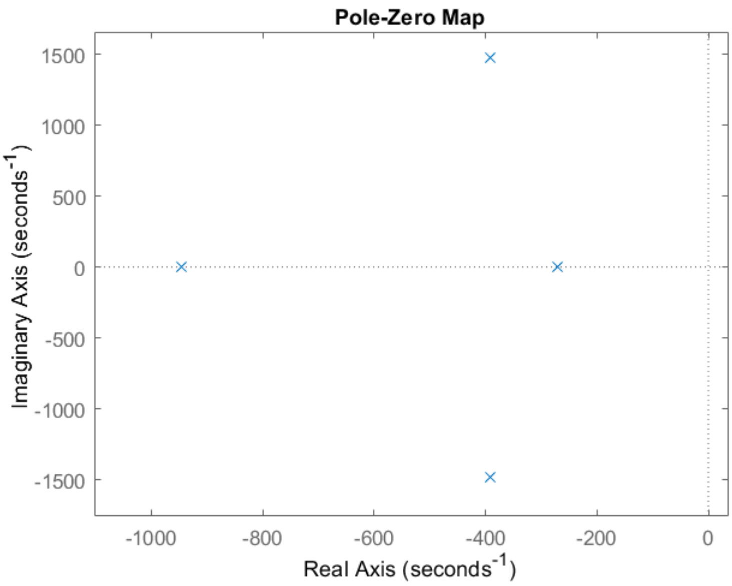

Figure 4: Poles and Zeros of question e after zero-pole cancellation

From the picture, we can that after pole-zero cancellation, all the poles are on the left hand plane, the system is stable and the system of minimum phase.

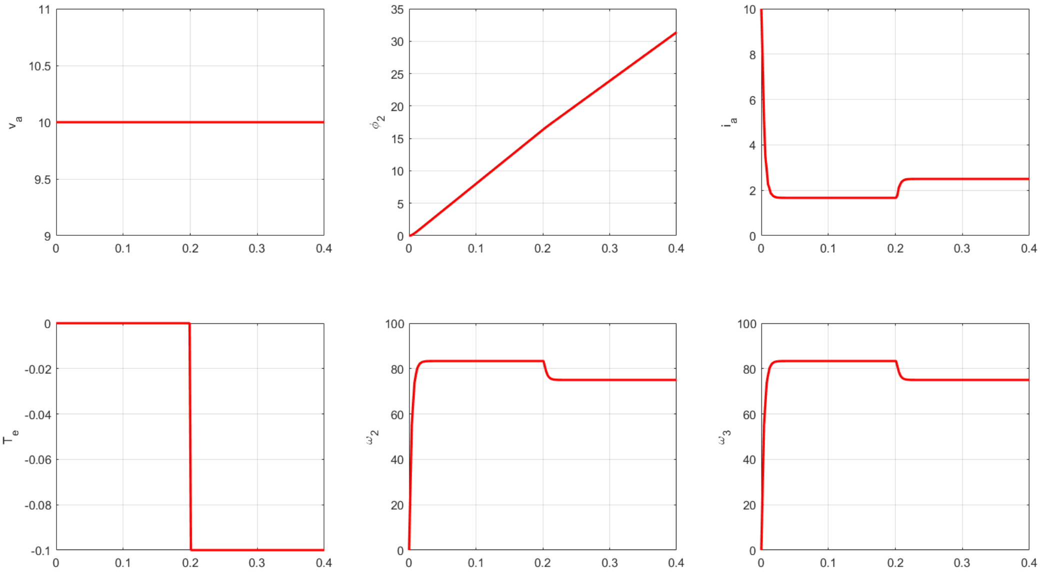

Figure 5: Outputs

In figure.5, we see i_a, ω_2 and ω_3 will eventually reach stable after implied the external torque. The $_2$ is unboundedly increasing since as the motor is working, the rotate angle is adding and accumulating.