Linear control system design SSY285

Assignment M3: Linear state estimation and control of DC motor with flywheel

Problem

Consider the DC motor with flywheel modeled and analyzed in the previous assignments. The starting point of this assignment is the discrete time state space representation obtained in subproblems d) and e) of assignment **M2**. The angle, ϕ_2, and angular velocity, ω_2, of the flywheel are both assumed to be measured in this assignment. In the following questions, assume that disturbance bounds correspond to a confidence interval of 99.7% of a normal distribution.

Questions

a) Suppose that (discrete time) white noise is added to both the external torque T_e and the applied motor voltage v_a, where the noise sequences are zero mean and uncorrelated. The voltage disturbances are mostly due to variations in the power supply unit, and are bounded by $\pm 0.3V$. The torque disturbance is estimated to be less than 10% of the maximum applied external torque value, which is assumed to be $T_e^{\max} = 1Nm$. Based on this information, propose a covariance matrix Q_w for the disturbance vector w. Which N matrix should be used in $x(k+1) = Ax(k) + Bu(k) + Nw(k)$?

b) Suppose also that measurement disturbances v_1, v_2 are added to the output. The disturbances are upper bounded by $0.02rad$ and $0.01rad/s$, respectively. Like above, they are assumed to be discrete time, zero mean uncorrelated white noises. Propose a covariance matrix Q_v for the measurement disturbance vector v.

c) Provided the cross spectrum between w and v is zero, compute a (discrete time) Kalman filter to estimate the "current" state $\hat{x}(k|k)$ of the system $x(k+1) = Ax(k) + Bu(k) + Nw(k), y(k) = Cx(k) + v(k)$. Find the the observer gain matrix, L. What is the covariance matrix of the state estimation error, P? What are the observer eigenvalues in this case?

d) Design a (discrete time) Linear Quadratic Gaussian controller and simulate the closed-loop answer to a step r_{ω_2}, jumping from an initial value of $10rads/s$ to $100rads/s$, for the discrete time and noise corrupted system above. Implement reference tracking for two cases: **(i)** using a reference gain and **(ii)** by integral action. Use the previously computed Kalman filter gain, answered in c) to reconstruct system states. For the LQ controller, choose the two appropriate dimensional weighting matrices Q_u and Q_x in order to achieve an "expensive" control. How do the results for case **(i)** and **(ii)** differ? Add an input disturbance of $1V$, and see how the results change.

Pre-approval of solution is mandatory before submission (by TA in tutorial session)