Assignment 1 - Group 22 - Mechanical

Peilin Lv, Jing Zhang, Yongzhao Chen

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1 (a)

$$\begin{aligned} J_1\dot{\omega}_1 &= K_T \cdot i_a - D_1(\phi_1 - \phi_2) \\ \dot{\omega}_1 &= \frac{1}{J_1}(K_T \cdot i_a - D_1(\phi_1 - \phi_2)) \end{aligned} \tag{1}$$

$$\begin{aligned} J_2\dot{\omega}_2 &= D_1(\phi_1 - \phi_2) - D_2(\phi_2 - \phi_3) \\ \dot{\omega}_2 &= \frac{1}{J_2}(D_1(\phi_1 - \phi_2) - D_2(\phi_2 - \phi_3)) \end{aligned} \tag{2}$$

$$\begin{aligned} v_a &= R \cdot i_a + L \cdot \dot{i}_a + K_E \cdot \omega_1 \\ \dot{i}_a &= \frac{1}{L}(v_a - R \cdot i_a - k_E \cdot \omega_1) \end{aligned} \tag{3}$$

$$\begin{aligned} \dot{\phi}_1 &= \omega_1 \\ \dot{\phi}_2 &= \omega_2 \\ \dot{\phi}_3 &= \omega_3 \end{aligned} \tag{4}$$

$$\begin{aligned} D_2(\phi_2 - \phi_3) &= T_e + B_f\omega_3 \\ \omega_3 &= \frac{1}{B_f}(D_2(\phi_2 - \phi_3) + T_e) \end{aligned} \tag{5}$$

and let

$$\begin{aligned} &x = [i_a \quad \phi_1 \quad \phi_2 \quad \phi_3 \quad \omega_1 \quad \omega_2]^{\mathrm{T}} \\ &u = [v_a \quad T_e] \end{aligned} \tag{6}$$

Finally, we put the equation (1-5) to describe the system, and the system is of 6 order.

Hence we have the state space model:

$$\dot{x} = \begin{bmatrix} -\dfrac{R}{L} & 0 & 0 & 0 & -\dfrac{K_E}{L} & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & \dfrac{D_2}{B_f} & -\dfrac{D_2}{B_f} & 0 & 0 \\ \dfrac{K_T}{J_1} & \dfrac{D_1}{J_1} & \dfrac{D_2}{J_1} & 0 & 0 & 0 \\ 0 & \dfrac{D_1}{J_2} & -\dfrac{(D_1+D_2)}{J_2} & \dfrac{D_2}{J_2} & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} i_a \\ \phi_1 \\ \phi_2 \\ \phi_3 \\ \omega_1 \\ \omega_2 \end{bmatrix} + \begin{bmatrix} \dfrac{1}{L} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & \dfrac{1}{B_f} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} v_a \\ T_e \end{bmatrix} \tag{7}$$

2 (b)

According to the question, we have

$$\begin{aligned} x &= [\phi_1 \quad \phi_2 \quad \phi_3 \quad \omega_1 \quad \omega_2]^{\mathrm{T}} \\ u &= [v_a \quad T_e] \end{aligned} \tag{8}$$

Since the $L \approx 0$, the rotor current i_a is no longer in state and written as:

$$i_a = \frac{v_a - K_E \cdot \omega_1}{R} \tag{9}$$

State-space model:

$$\dot{x} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & \dfrac{D_2}{B_f} & -\dfrac{D_2}{B_f} & 0 & 0 \\ -\dfrac{D_1}{J_1} & \dfrac{D_1}{J_1} & 0 & -\dfrac{K_T \cdot K_E}{J_1 \cdot R} & 0 \\ \dfrac{D_1}{J_2} & -\dfrac{(D_1+D_2)}{J_2} & \dfrac{D_2}{J_2} & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \omega_1 \\ \omega_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & \dfrac{1}{B_f} \\ \dfrac{K_T}{J_1 \cdot R} & 0 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} v_a \\ T_e \end{bmatrix} \tag{10}$$

3 (c)

3.1

$$y = \begin{bmatrix} \phi_2 \\ \omega_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \omega_1 \\ \omega_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} v_a \\ T_e \end{bmatrix} \tag{11}$$

3.2

$$y = \begin{bmatrix} i_a \\ \omega_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & -\dfrac{K_E}{R} & 0 \\ 0 & \dfrac{D_2}{B_f} & -\dfrac{D_2}{B_f} & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \omega_1 \\ \omega_2 \end{bmatrix} + \begin{bmatrix} \dfrac{1}{R} & 0 \\ 0 & \dfrac{1}{B_f} \end{bmatrix} \cdot \begin{bmatrix} v_a \\ T_e \end{bmatrix} \tag{12}$$

4 (d)

the A-matrix is

$$A = 1.0 \times 10^6 \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1000 & -1000 & 0 & 0 \\ -2000000 & 2000000 & 0 & -1000 & 0 \\ 500000 & -550000 & 50000 & 0 & 0 \end{bmatrix} \tag{13}$$

through matlab, the eigenvalues of the A-matrix are:

```
eigenvalue = 5×1 complex
10^3 ×
    -0.3914 + 1.4791i
    -0.3914 - 1.4791i
    -0.0000 + 0.0000i
    -0.2708 + 0.0000i
    -0.9464 + 0.0000i
```

Figure 1: the eigenvalues of the A-matrix

Since in case 1 and 2, poles are the same as above in Figure 1, We notice that all the eigenvalues of A-matrix are less or equal to zero, so this system is stable. Additionally, since it is a LTI system, it is input-output stable.

5 (e)

When the external torque is zero, the transfer function from the input, applied rotor voltage, to the output, defined in case (2) of subproblem c) is given by

$$G(s) = C(sI - A)^{-1}B + D \tag{14}$$

```
From input to output...
     s^5 + 1000 s^4 + 2.55e06 s^3 + 2.5e09 s^2 + 1e11 s + 0.01988
 1:  -------------------------------------------------------------
     s^5 + 2000 s^4 + 3.55e06 s^3 + 3.05e09 s^2 + 6e11 s + 0.05601

                                5e12 s
 2:  -------------------------------------------------------------
     s^5 + 2000 s^4 + 3.55e06 s^3 + 3.05e09 s^2 + 6e11 s + 0.05601
```

Figure 2: transfer function

Compute the Zeros and Poles:

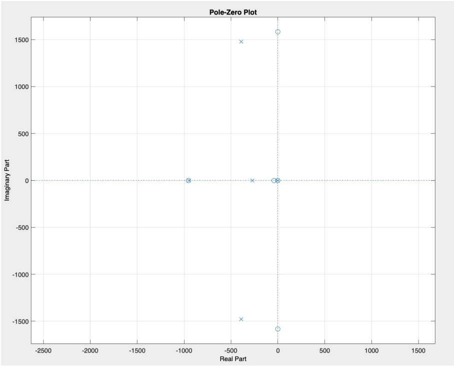
```
z3 = 5×1 complex
10^3 ×
    -0.0015 + 1.5834i
    -0.0015 - 1.5834i
    -0.9553 + 0.0000i
    -0.0418 + 0.0000i
    -0.0000 + 0.0000i

p3 = 5×1 complex
10^3 ×
    -0.3914 + 1.4791i
    -0.3914 - 1.4791i
    -0.9464 + 0.0000i
    -0.2708 + 0.0000i
    -0.0000 + 0.0000i
```

Figure 3: Poles and Zeros of Transfer function 1

```
z3 = 0
p3 = 5×1 complex
10^3 ×
    -0.3914 + 1.4791i
    -0.3914 - 1.4791i
    -0.9464 + 0.0000i
    -0.2708 + 0.0000i
    -0.0000 + 0.0000i
```

Figure 4: Poles and Zeros of Transfer function 2

From the lists above, there are no poles or zeros on the right complex plane. Then we can draw the conclusion that both transfer functions are stable. Also, the poles and zeros can be depicted in the complex plane:



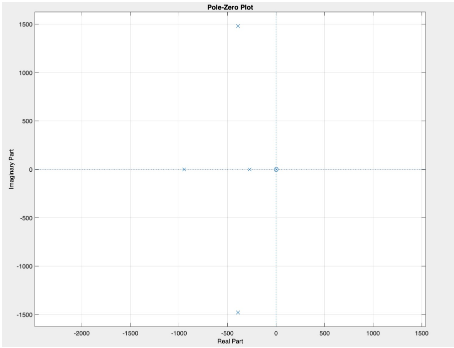


Figure 5: zeros and poles of the first transfer function

Figure 6: zeros and poles of the second transfer function

Both the poles and zeros are in the left-hand plane, which means the two systems are of minimum phase.

5.1 f

the outputs of both cases in subproblem c) can be plotted as:

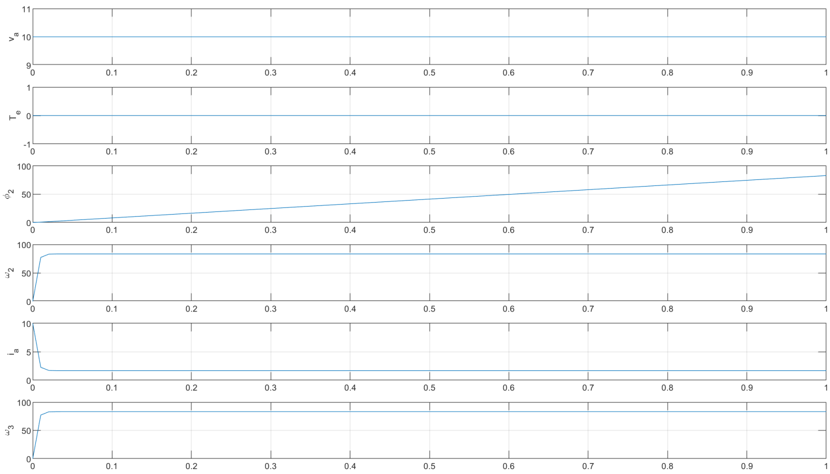


Figure 7: find steady time

From figure 7 we know that the system reaches a steady state before 0.1s, so it is OK to add torque at this time point.

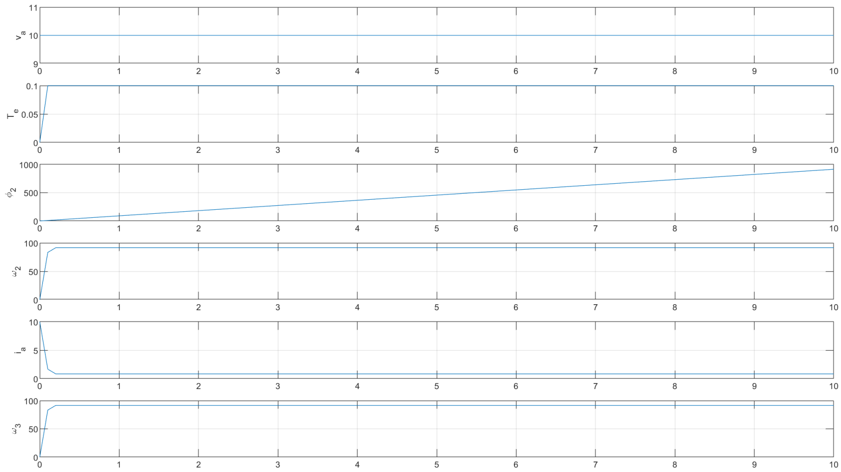


Figure 8: simulation results

In figure 8, we extend the duration to 10 seconds, it is reasonable that the angles are increasing unboundedly because angles are constantly accumulative as long as the motor works. i_a, ω_2 and ω_3 become stable when $t > 0.1$. At the time $t = 0.1$, the external torque implied and makes i_a, ω_2, and ω_3 a small change, then becomes stable again.