

SSY191 Individual Home Assignment 1

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1 Problem 1

1.1 Prove

Given the transformation matrix ${}^W\xi_B$, I renotate it as:

$$\begin{aligned} {}^W\xi_B &= \begin{bmatrix} R & d \\ \mathbf{0} & 1 \end{bmatrix} \\ R &= \begin{bmatrix} r11 & r12 & r13 \\ r21 & r22 & r23 \\ r31 & r32 & r33 \end{bmatrix} \\ d &= \begin{bmatrix} t1 \\ t2 \\ t3 \end{bmatrix} \end{aligned} \tag{1}$$

R is a rotation matrix and d is a translation vector.

Since R is an orthogonal matrix, the inverse of the rotation matrix R is its transpose,

$$\begin{aligned} R^{-1} &= R^T \\ RR^T &= R^T R = \mathbf{I} \end{aligned} \tag{2}$$

Define a matrix H^{-1} as :

$$H^{-1} = \begin{bmatrix} R^T & -R^T d \\ \mathbf{0} & 1 \end{bmatrix} \tag{3}$$

And see the result of ${}^W\xi_B \times H^{-1}$:

$${}^W\xi_B \times H^{-1} = \begin{bmatrix} R & d \\ \mathbf{0} & 1 \end{bmatrix} \begin{bmatrix} R^T & -R^T d \\ \mathbf{0} & 1 \end{bmatrix} = \begin{bmatrix} RR^T & -RR^T d + d \\ \mathbf{0} & 1 \end{bmatrix} = \begin{bmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & 1 \end{bmatrix} \tag{4}$$

which means:

$${}^B\xi_W = H^{-1} = \begin{bmatrix} R^T & -R^T d \\ \mathbf{0} & 1 \end{bmatrix} \tag{5}$$

Q.E.D.

1.2 Calculate

I select my ${}^B\xi_W$ and p_W and calculate through Matlab

```
1   W_H_B = [ 0 0 1 1;  
2   1 0 0 2;  
3   0 1 0 3;  
4   0 0 0 1 ];  
5  
6   p_W = [ 2; 2; 1; 1 ]  
7  
8   B_H_W = inv(W_H_B);  
9   p_B = B_H_W * p_W
```

Output result:

```
1   p_W = ×41  
2   2  
3   2  
4   1  
5   1  
6  
7  
8   p_B = ×41  
9   0  
10  -2  
11  1  
12  1
```

It show, the p point in W coordinate has position as $[2 \ 2 \ 1]$, and p in B coordinate has position as $[0 \ -2 \ 1]$.

2 Problem 2

$$R_{yxz} = Rot_y Rot_x Rot_z =$$

$$\begin{bmatrix} \cos(\psi)\cos(\theta) + \sin(\phi)\sin(\psi)\sin(\theta) & \cos(\psi)\sin(\phi)\sin(\theta) - \cos(\theta)\sin(\psi) & \cos(\phi)\sin(\theta) \\ \cos(\phi)\sin(\psi) & \cos(\phi)\cos(\psi) & -\sin(\phi) \\ \cos(\theta)\sin(\phi)\sin(\psi) - \cos(\psi)\sin(\theta) & \sin(\psi)\sin(\theta) + \cos(\psi)\cos(\theta)\sin(\phi) & \cos(\phi)\cos(\theta) \end{bmatrix}$$

Extract the last column:

$$\begin{bmatrix} L1 = \cos(\phi) \sin(\theta) \\ L2 = -\sin(\phi) \\ L3 = \cos(\phi) \cos(\theta) \end{bmatrix}$$

Then we can get:

$$\begin{aligned} \theta &= \text{atan2}(\mathbf{L1}, \mathbf{L3}) \\ \phi &= \text{atan2}(-\mathbf{L2}, \sqrt{(\mathbf{L1} + \mathbf{L3}^2)}) \end{aligned} \tag{6}$$

3 Problem 3

In this derivation, I have converted the complex frequency domain directly to the Z-domain

Euler-backward discretization means:

$$s = \frac{z - 1}{zT} \tag{7}$$

The original function is :

$$\begin{aligned} \theta &= G(s)\theta_a(s) + (1 - G(s))\theta_g(s) \\ \text{where } G(s) &= \frac{1}{\alpha s + 1} \end{aligned} \tag{8}$$

Replace all the s with equation 7, we get:

$$\begin{aligned}
\theta(z) &= \frac{zT}{\alpha(z-1) + zT} \theta_{a(z)} + \frac{\alpha zT}{\alpha(z-1) + zT} Yg(z) \\
&= -\alpha\theta(z) + \alpha\theta(z) + zT\theta z = T\theta_a(z+1) + \alpha TYg(z+1) \\
&= \theta(z+1) = \frac{\alpha}{\alpha+T}(\theta(z) + TYg(z+1)) + \frac{T}{\alpha+T}\theta_{a(z+1)}
\end{aligned} \tag{9}$$

Which can be expressed as:

$$\begin{aligned}
\theta(k) &= \gamma(\theta(k-1) + hYg(k)) + (1-\gamma)\theta_a(k) \\
\text{where } \gamma &= \frac{\alpha}{\alpha+h} \text{ and } h = T \text{ (T is sample interval)}
\end{aligned} \tag{10}$$

4 Problem 4

4.1 a

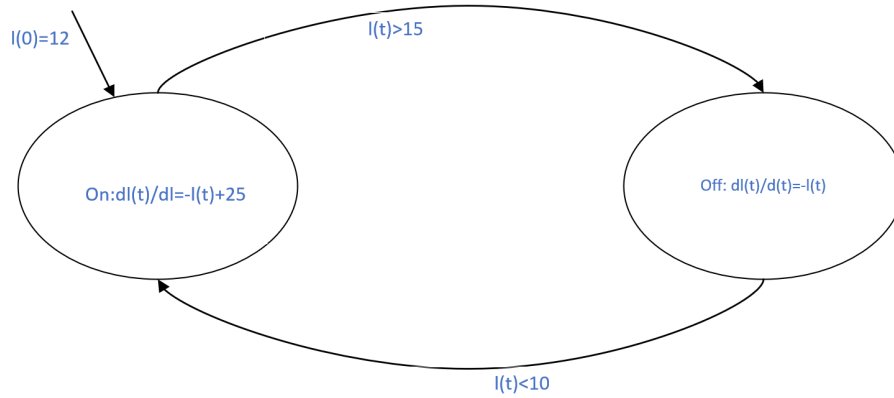


Figure 1: Automata

4.2 b

I used Simulink to do this modeling:

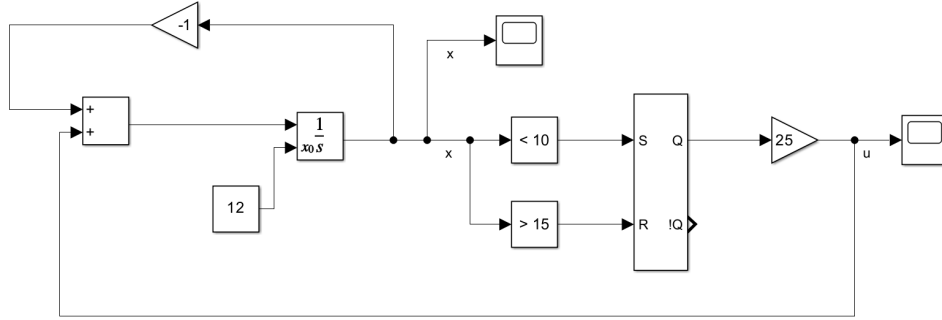
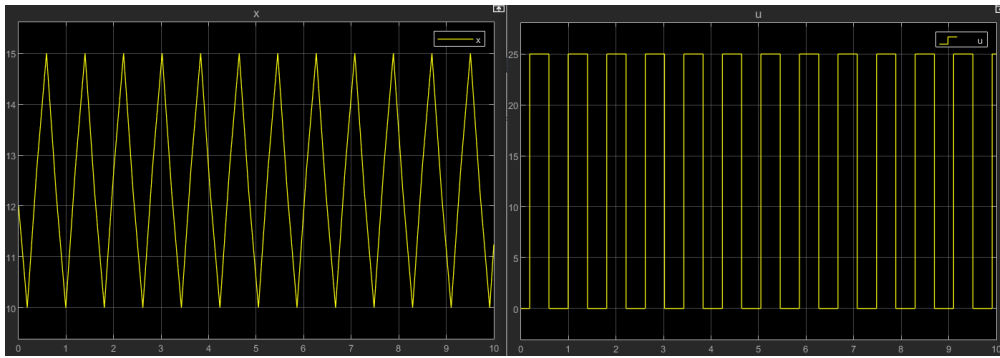
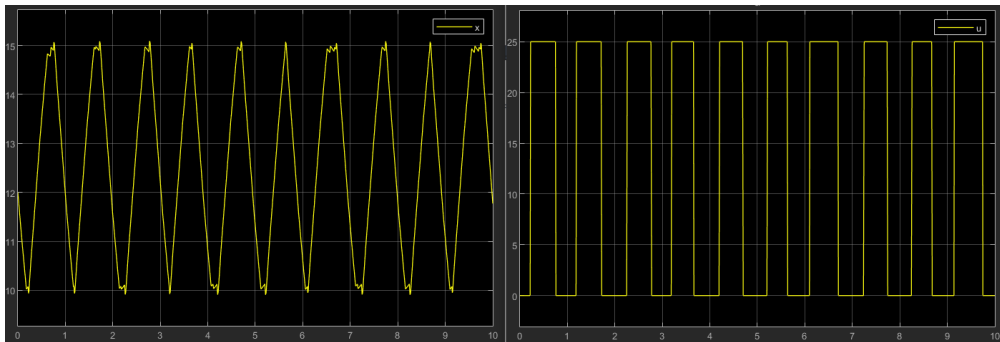


Figure 2: Model in Simulink

Here in figure 3 shows the difference between with and without zero-crossing detection enabled with variable step solver.



(a) With Zero Crossing



(b) Without Zero Crossing

Figure 3: Comparison of Zero Crossing on/off for variable step

As you can see, there is a 'shake' on the peak, but it is not easy to analyze

and it can not match the lecture.

Now to change the solver to Fixstep:

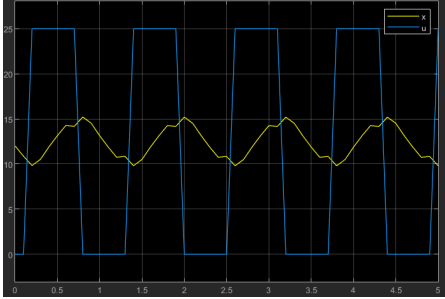


Figure 4: Off

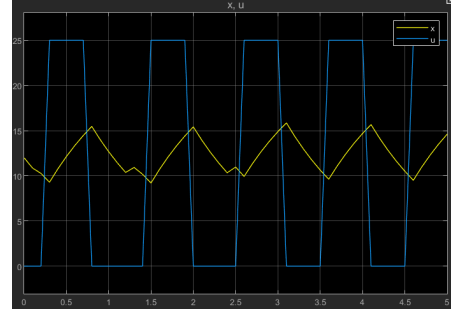


Figure 5: On

Figure 6: Comparison of Off and On in Fixstep

We can see it is obvious with the zero-crossing option enabled the state curve becomes smooth at the edge of the control signal u .

4.3 c

Zeno behavior occurs when the system appears to repeatedly approach a limit or event horizon without actually reaching it, but in our system, there is a gap between two event horizons so the Zeno behavior cannot approach.