SSY191 Individual Home Assignment 1

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1 Problem 1

1.1 Prove

Given the transformation matrix ${}^{W}\xi_B$, I renote it as:

$$
\begin{aligned}
{}^{W}\xi_B &= \begin{bmatrix} R & d \\ \mathbf{0} & 1 \end{bmatrix} \\
R &= \begin{bmatrix} r11 & r12 & r13 \\ r21 & r22 & r23 \\ r31 & r32 & r33 \end{bmatrix} \\
d &= \begin{bmatrix} t1 \\ t2 \\ t3 \end{bmatrix}
\end{aligned} \tag{1}
$$

$R$is a rotation matrix and d is a translation vector.

Since R is an orthogonal matrix, the inverse of the rotation matrix R is its transpose,

$$
\begin{aligned}
R^{-1} &= R^T \\
RR^T &= R^T R = \boldsymbol{I}
\end{aligned} \tag{2}
$$

Define a matrix H^-1 as :

$$
H^{-1} = \begin{bmatrix} R^T & -R^T d \\ \mathbf{0} & 1 \end{bmatrix} \tag{3}
$$

And see the result of ${}^{W}\xi_B \times H^{-1}$:

$$
{}^{W}\xi_B \times H^{-1} = \begin{bmatrix} R & d \\ \mathbf{0} & 1 \end{bmatrix} \begin{bmatrix} R^T & -R^T d \\ \mathbf{0} & 1 \end{bmatrix} = \begin{bmatrix} RR^T & -RR^T d + d \\ \mathbf{0} & 1 \end{bmatrix} = \begin{bmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{bmatrix} \tag{4}
$$

which means:

$$
{}^{B}\xi_W = H^{-1} = \begin{bmatrix} R^T & -R^T d \\ \mathbf{0} & 1 \end{bmatrix} \tag{5}
$$

Q.E.D.

1.2 Calculate

I select my ${}^{B}\xi_W$ and p_W and calculate through Matlab

```
W_H_B = [ 0 0 1 1;
1 0 0 2;
0 1 0 3;
0 0 0 1 ];

p_W = [ 2; 2; 1; 1 ]

B_H_W = inv(W_H_B);
p_B = B_H_W * p_W
```

Output result:

```
p_W = ×41
 2
 2
 1
 1


 p_B = ×41
 0
-2
 1
 1
```

It show, the p point in W coordinate has position as $[2\ 2\ 1]$, and p in B coordinate has position as $[0\ \ -2\ 1]$.

2 Problem 2

$Ryxz = RotyRotxRotz =$

$$\begin{bmatrix} cos(\psi)cos(\theta) + sin(\phi)sin(\psi)sin(\theta) & cos(\psi)sin(\phi)sin(\theta) - cos(\theta)sin(\psi) & cos(\phi)sin(\theta) \\ cos(\phi)sin(\psi) & cos(\phi)cos(\psi) & -sin(\phi) \\ cos(\theta)sin(\phi)sin(\psi) - cos(\psi)sin(\theta) & sin(\psi)sin(\theta) + cos(\psi)cos(\theta)sin(\phi) & cos(\phi)cos(\theta) \end{bmatrix}$$

Extract the last column:

$$\begin{bmatrix} L1 = \cos(\phi)\sin(\theta) \\ L2 = -\sin(\phi) \\ L3 = \cos(\phi)\cos(\theta) \end{bmatrix}$$

Then we can get:

$$\begin{aligned} \theta &= \mathbf{atan2}(\mathbf{L1}, \mathbf{L3}) \\ \phi &= \mathbf{atan2}\big(-\mathbf{L2}, \sqrt{(\mathbf{L1} + \mathbf{L3}^2)}\big) \end{aligned} \tag{6}$$

3 Problem 3

In this derivation, I have converted the complex frequency domain directly to the Z-domain

Euler-backward discretization means:

$$s = \frac{z-1}{zT} \tag{7}$$

The original function is :

$$\begin{aligned} \theta &= G(s)\theta_a(s) + (1 - G(s))\theta_g(s) \\ \text{where } G(s) &= \frac{1}{\alpha s + 1} \end{aligned} \tag{8}$$

Replace all the s with equation 7, we get:

$$
\begin{aligned}
\theta(z) &= \frac{zT}{\alpha(z-1)+zT}\theta_{a(z)} + \frac{\alpha zT}{\alpha(z-1)+zT}Yg(z) \\
&= -\alpha\theta(z) + \alpha\theta(z) + zT\theta z = T\theta_a(z+1) + \alpha TYg(z+1) \\
&= \theta(z+1) = \frac{\alpha}{\alpha+T}(\theta(z) + TYg(z+1)) + \frac{T}{\alpha+T}\theta_{\alpha(z+1)}
\end{aligned}
\tag{9}
$$

Which can be expressed as:

$$
\begin{aligned}
&\theta(k) = \gamma(\theta(k-1) + hYg(k)) + (1-\gamma)\theta_a(k) \\
&\text{where } \gamma = \frac{\alpha}{\alpha+h} \text{and } h = T \text{ (T is sample interval)}
\end{aligned}
\tag{10}
$$

4 Problem 4

4.1 a

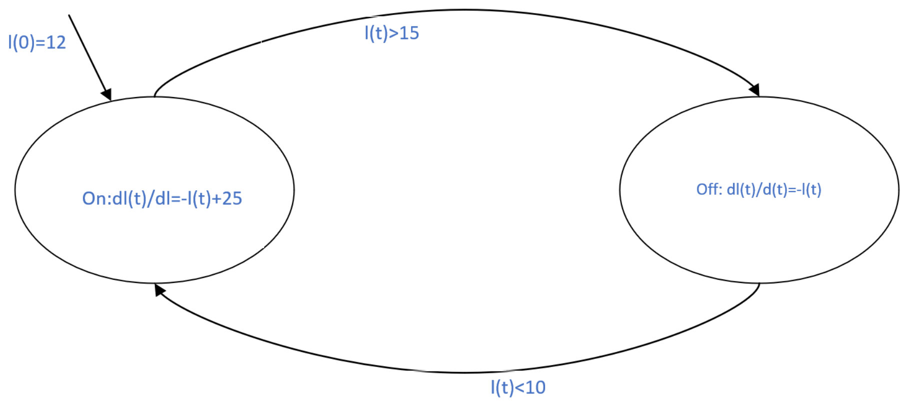


Figure 1: Automata

4.2 b

I used Simulink to do this modeling:

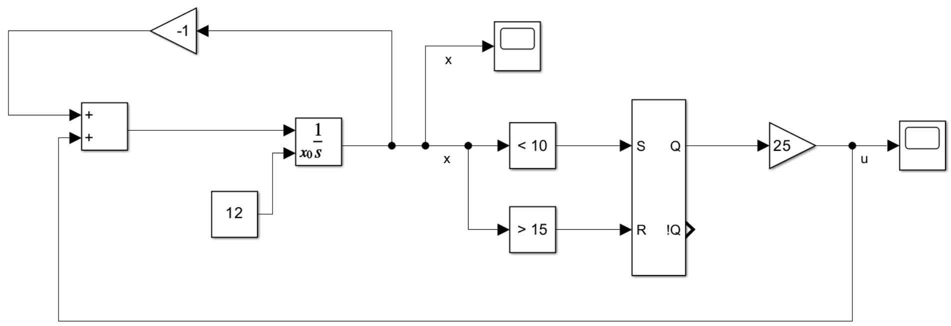
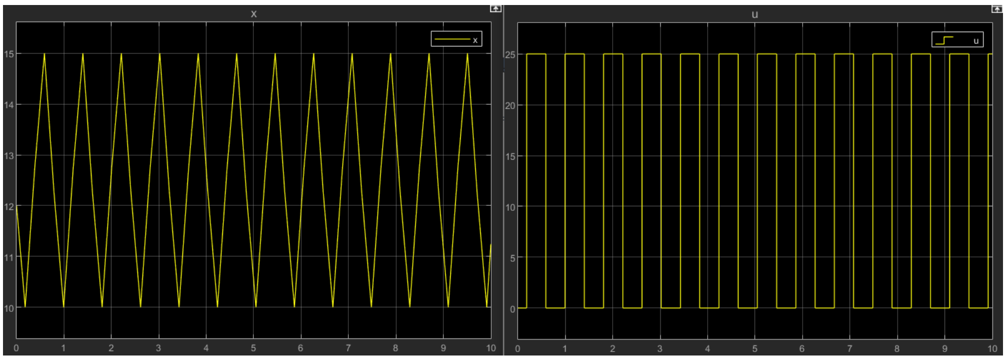
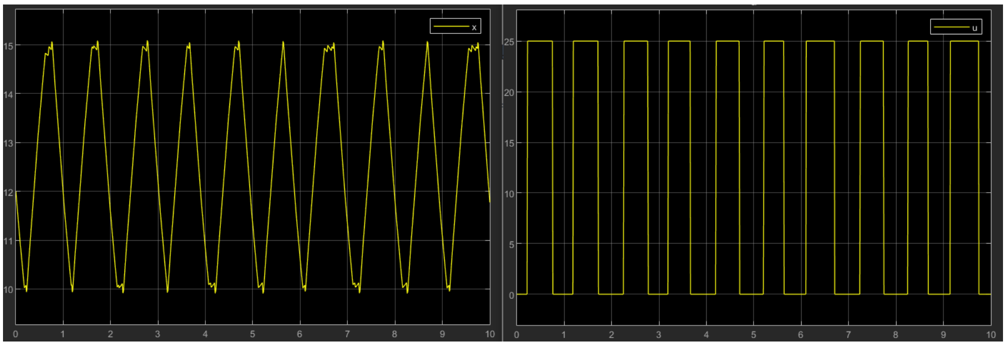


Figure 2: Model in Simulink

Here in figure 3 shows the difference between with and without zero-crossing detection enabled with variable step solver.



(a) With Zero Crossing



(b) Without Zero Crossing

Figure 3: Comparison of Zero Crossing on/off for variable step

As you can see, there is a 'shake' on the peak, but it is not easy to analyze and it can not match the lecture.

Now to change the solver to Fixstep:

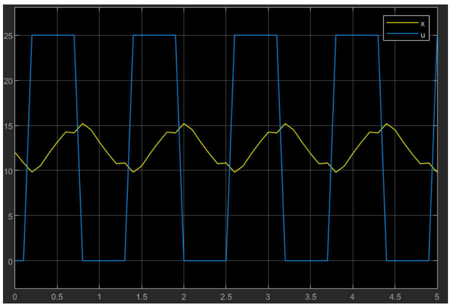


Figure 4: Off

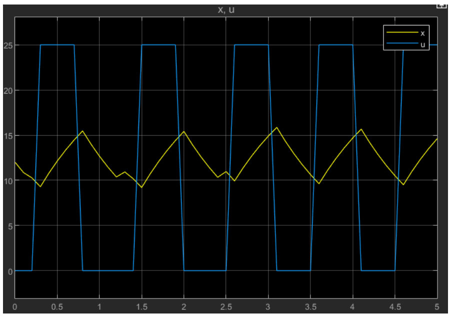


Figure 5: On

Figure 6: Comparison of Off and On in Fixstep

We can see it is obvious with the zero-crossing option enabled the state curve becomes smooth at the edge of the control signal u.

4.3 c

Zeno behavior occurs when the system appears to repeatedly approach a limit or event horizon without actually reaching it, but in our system, there is a gap between two event horizons so the Zeno behavior cannot approach.