Solution to analysis in Home Assignment 4

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1 Part 2

1.1 How to design LQI controller

A stationary LQR can be used to find the feedback control gain K, which minimizes thefollowing quadratic cost, with respect to a control input u(t):

$$J = \frac{1}{2}\int_0^{\infty} \left(x(t)^T Q_x x(t) + u(t)^T Q_u u(t)\right) dt \tag{1}$$

where Q_x and Q_u are weighting matrices, telling the controller about the priorities of tunning states and control signals.

The original LQR method can be extended to achieve reference tracking. To do this, the system needs to be augmented with integral states z_i, which allow the computation of the cumulative tracking error $\int (y - r)dt$:

$$\begin{bmatrix} \dot{x}(t) \\ \dot{z}(t) \end{bmatrix} = \underbrace{\begin{bmatrix} A_\delta & 0 \\ C_I & 0 \end{bmatrix}}_{A_{aug}} \begin{bmatrix} x(t) \\ z(t) \end{bmatrix} + \underbrace{\begin{bmatrix} B_\delta \\ 0 \end{bmatrix}}_{B_{aug}} u(t) + \begin{bmatrix} K_r \\ -I \end{bmatrix} r(t) \tag{2}$$

$$u(t) = -K \begin{bmatrix} x(t) \\ z(t) \end{bmatrix} = -[K_P K_I] \begin{bmatrix} x(t) \\ z(t) \end{bmatrix} \tag{3}$$

where C_I selects the system states needed to compute the new integral states, and K can be calculated by Matlab function `lqr`.

1.2 Application and verify

For the assignment 1 part 2, the system does not have other input singals expect a singal reference signal **r**.

The augmented system is :

$$
\begin{aligned}
\begin{bmatrix} \dot{x} \\ \dot{x}_i \end{bmatrix} &= \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix} \begin{bmatrix} x \\ x_i \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} r \\
\mathbf{A_{aug}} &= \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix} \\
\mathbf{B_{aug}} &= \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}
\end{aligned}
\tag{4}
$$

where $\mathbf{C}$ is selected as $\mathbf{C}(\mathbf{1},:)$, since $\mathbf{y} = \mathbf{Cx}$ and $y_1 = \alpha$.

Select Q, R after tunning:

```
Q = diag([0 100 0 0 0 1]);
R = diag([1 1 1]);
```

And use the build- in function to get the K matrix:

```
[K,  , ] = lqr(A_aug, B_aug, Q, R)
```

Now rebuild the new closed loop feedback system as :

```
Ae = [A_lqr-B_lqr*K]; % 6x6
Be = [0 0 0 0 0 1]'; % 6x1
Ce = [C zeros(3, 1)]; % 3x6
De = 0; % 1x1
sys_e = ss(Ae, Be, Ce, De);
```

The respond of a step signal shows below:

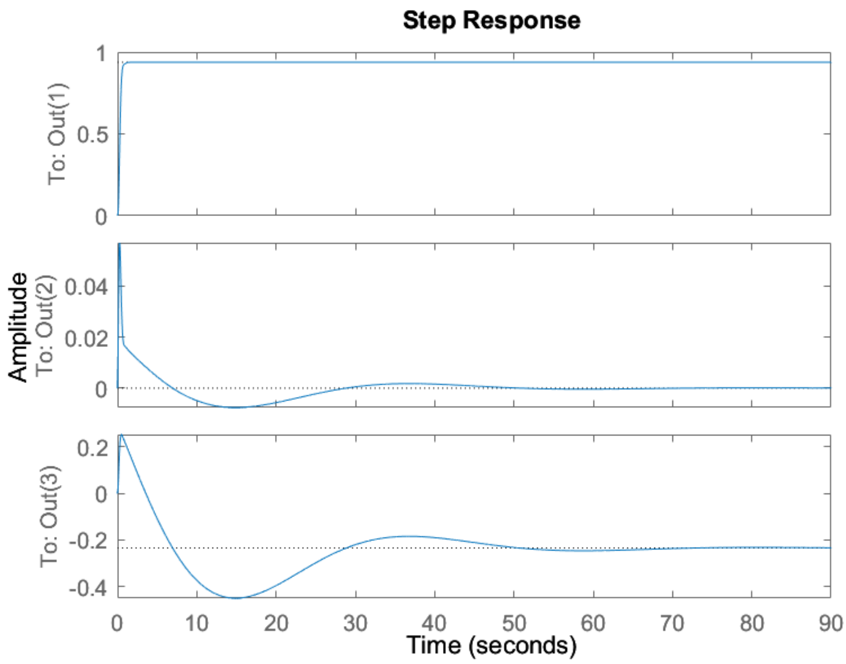

Figure 1: Step respond

From the picture **??**, it is clear we achieved a reference tracking for the state α.

$$\begin{bmatrix} y\Delta \\ z_e \\ z_p \\ z_u \\ v_r \\ v_{\tilde{y}} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & G_a \\ -W_e G_n W_m & -W_e G_n W_d & W_e W_{r\alpha} & 0 & -W_e G_n G_a \\ W_p G_n W_m & W_p G_n W_d & 0 & 0 & W_p G_n G_a \\ W_u W_m & W_u W_d & 0 & 0 & W_u W_a \\ 0 & 0 & I & 0 & 0 \\ G_n W_m & G_n W_d & 0 & W_n & G_n G_a \end{bmatrix} = \begin{bmatrix} u\Delta \\ d \\ r \\ n \\ u \end{bmatrix} \tag{5}$$