Solution to analysis in Home Assignment 4

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1 Smoothing

1.1 a)

The filtered and smoothed trajectory can be seen in figure 1. One can clearly see that both lie on top of the true trajectory. The error covariance is bigger for the filter than the smoother. This is expected since the smoother has access to all measurements which naturally captures more information about the state. Hence the error covariance should be smaller in the smoothing case.

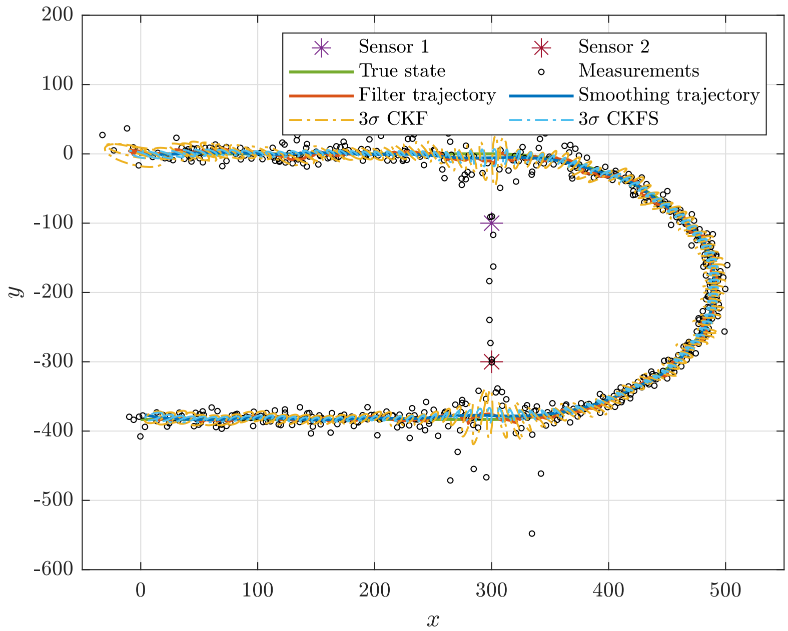


Figure 1: Filtered and smoothed trajectory along with true trajectory, measurements and the 3σ contour.

1.2 b)

Now we generate an outlier at $k = 300$. This outlier can be seen in figure 2 and 3. I we zoom in at the area we better see the impact of the outlier. One can clearly see that the filter reacts much more on the outlier than the smoother does. This can also be explained by the previous reasoning about the filter having access to more information. It is not reacting as much since the outlier only is a misleading measurement at one time instance. However, the filter does not know anything about the trajectory and has to react to this measurment.

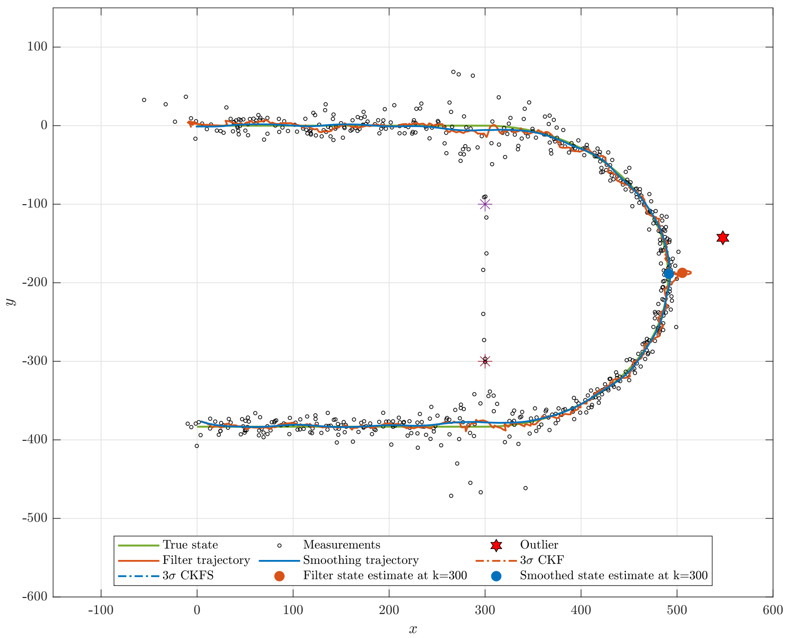


Figure 2: Outlier introduces at $k = 300$.

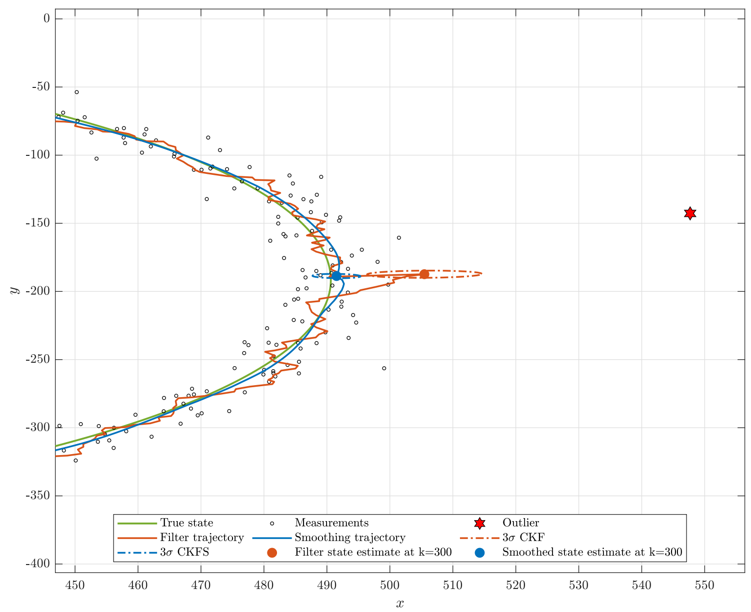


Figure 3: Zoomed in at the area where the outlier affect the two trajectories.

2 Particle filters for linear/Gaussian systems

2.1 a)

First we generate a linear trajectory of states and of measurements using the prior and variances stated in the description. Then we run our three filters. Plotting the true trajectory, measurements and visualizing the mean and covariance of the filter using Matlab function `errorbar` we get the result shown in figure 8.

We can clearly see how the PF without resampling deviates quite much from the KF and the PF with resampling.

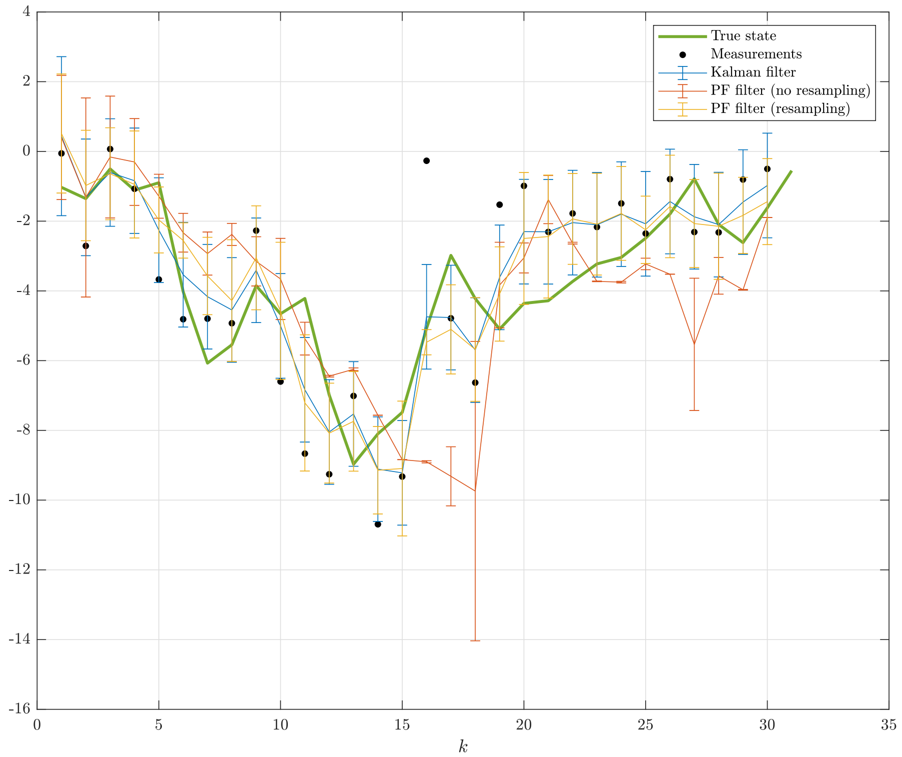


Figure 4: The three filter trajectories.

2.1.1 MSE

Calculating the MSE for different amount of particles we get the results show in table 1. In terms of MSE the PF with resampling is closer to performing as the KF than the PF without resampling. When we are 10000 particles the PF:s perform almost the same as the KF.

Table 1: MSE when changing the amount of partciles.

Particles	MSE_{KF}	MSE_{PF}	$MSE_{PF(resampling)}$
100	1.7333	3.7097	1.6201
1000	1.6168	1.4757	1.6683
10000	1.8355	1.7799	1.8292
100000	1.8612	1.7432	1.8653

2.1.2 Posterior densities

Comparing the posterior densities, at three different timesteps, of the PF:s with the KF yields figures 5, 6 and 7. Here we can see that the posterior for the KF and PF with resampling is closely related. Meanwhile the posterior density for the PF without resampling is clearly approximating with an error.

Note that these posterior densities was generated from another trajectory then the one in subtask a). However, the behaviour is similar, i.e in the beginning the three filters follows the same trajectory but as time goes on the no resample PF poorly approximates the true state trajectory. This behaviour is also depicted in figures 5, 6 and 7.

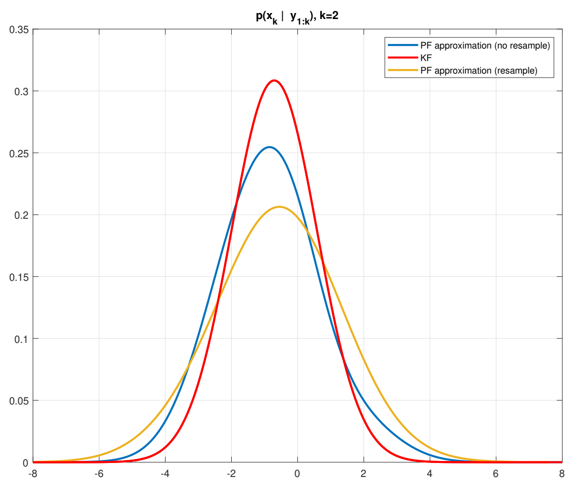


Figure 5: Posterior densities at time step $k = 2$.

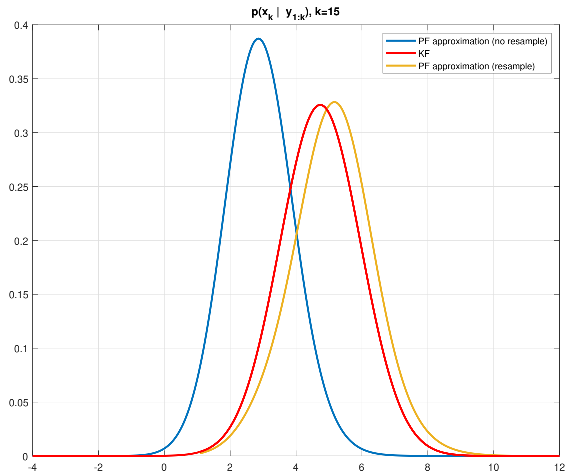


Figure 6: Posterior densities at time step $k = 15$.

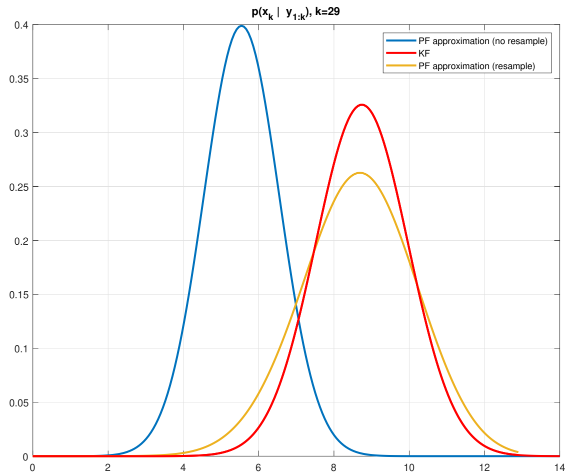


Figure 7: Posterior densities at time step $k = 29$.

2.2 b)

With an incorrect prior we see that the KF and PF with resampling recovers in a few time steps. PF without resampling does not handle the incorrect prior as good which can clearly be seen in figure 8. This is expected since a major consequence of not resampling is the bad state estimates that occurs.

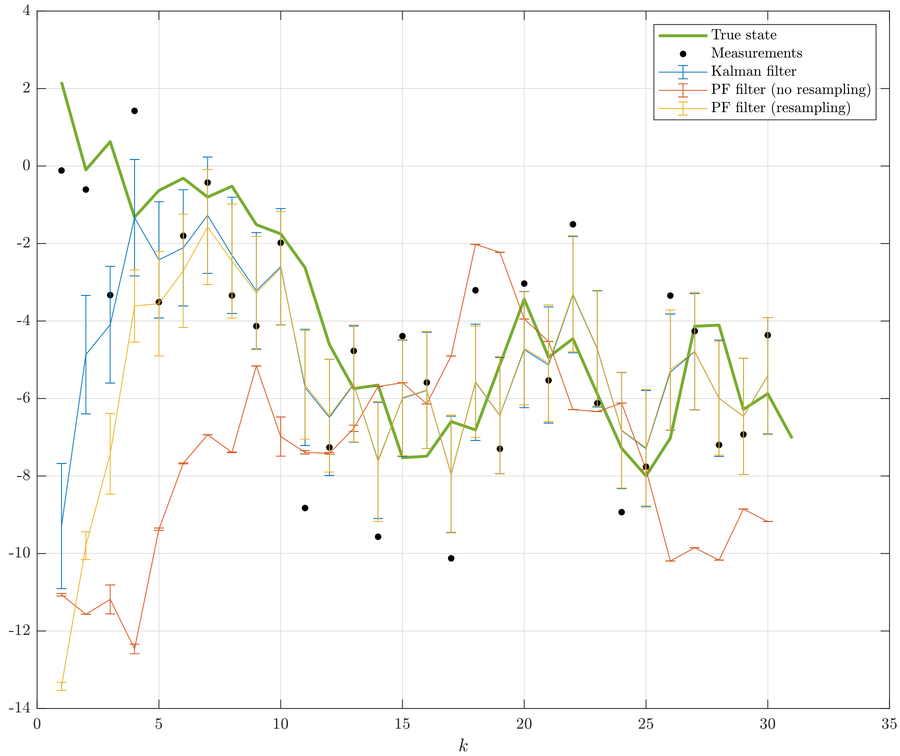


Figure 8: The three filter trajectories starting from an incorrect prior.

2.3 c) & d)

The particle trajectories for a PF with and without resampling can be seen in figure 9 and 10 when simulating for 100 particles and up to time $k = 100$.

For the PF without resampling, figure 9, we can see that the particles are scattered and does not describe the true state trajectory especially well. This will affect the filter in a negative way since we have particles values inaccurately describing the approximated posterior. On the other hand, the behaviour of the PF trajectories with resampling, figure 10, is quite the opposite. Here we can see that the particles are centered around the true state trajectory, i.e better approximates the posterior, which is desirable.

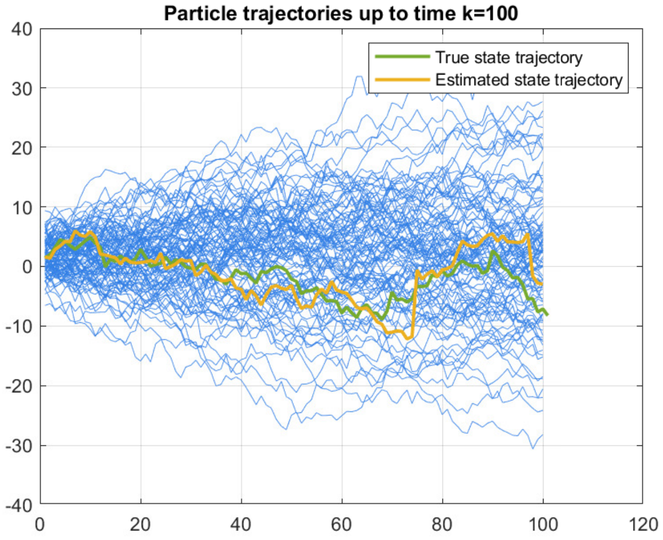


Figure 9: 100 particle, without resampling.

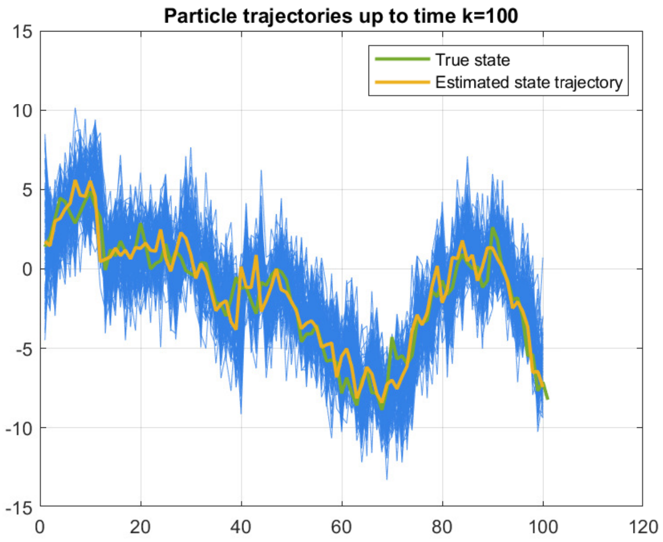
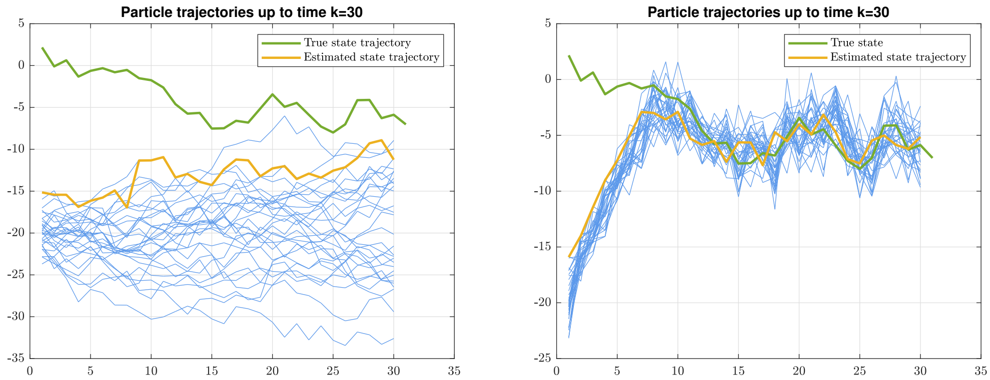


Figure 10: 100 particles, with resampling.

Doing another simulation but with less particles and a new state trajectory we get following figures. They behave similarly as described above.



(a) Without resampling.

(b) With resampling.

Figure 11: Simulating with 30 particles.