Project 1: Orientation estimation using smartphone sensors

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Analysis

In this report I will present my independent analysis of the questions related to home assignment X. I have discussed the solution with NAME1, NAME2 and NAME3 but I swear that the analysis written here are my own.

Task 1

Pros:

1. Simplicity: Using the gyroscope measurements as inputs can simplify the state space and reduce the complexity of the problem.

2. Accuracy: Gyroscopes in smartphones are typically quite accurate, which means their measurements can be reliably used as inputs to a model.

3. Bias-free: If the gyroscopes are not biased, their measurements do not require further processing or correction.

Cons:

1. Limited to rotation: Gyroscope measurements are only relevant for rotational motion, not linear motion. This approach would not be ideal in situations where linear motion is also of interest.

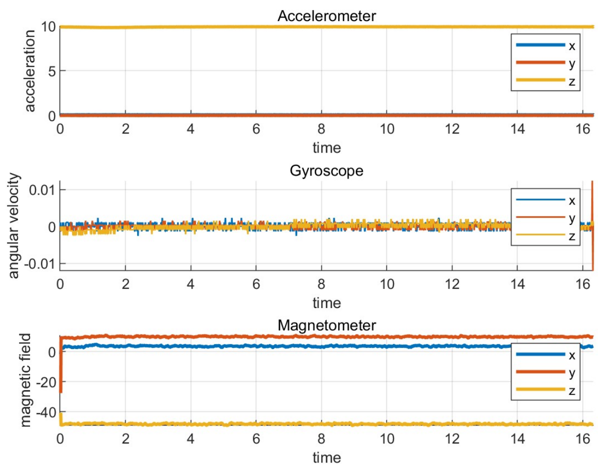
2. Assumption of Accuracy: If the gyroscope measurements are not as accurate as assumed, this could lead to inaccurate model predictions.

3. No bias estimation: If the gyroscope has any inherent bias, this modeling choice won't account for it, which could lead to errors over time.

A situation where it would not be a good choice to use gyroscope measurements as inputs might include cases where the gyroscope's accuracy is compromised, for example due to damage, drift, or a poorly calibrated device. Similarly, in environments with high levels of vibration or shock, the gyroscope readings might be noisy and unreliable.

Including angular velocities in the state vector would be a better choice when we need to account for system dynamics more accurately, especially for more complex systems. For instance, if the system involves controlling an aircraft, robot, or any other system where understanding the rate of change of the angular position is crucial for accurate control and navigation, including angular velocities in the state vector would be necessary. Furthermore, it would also be a more suitable approach when we want to estimate and correct for any possible bias in the gyroscope readings.

Task 2




```
acc_mean = 3×1
    0.0841
    0.0214
    9.8999

mag_mean = 3×1
    3.6143
    9.9278
  -48.0828

gyr_mean = 3×1
10^-4 ×
    0.2162
    0.1959
    0.0800
```

```
acc_cov = 3×3
10^-3 ×
    0.0363   -0.0030   -0.0475
   -0.0030    0.0370    0.0294
   -0.0475    0.0294    0.5257

mag_cov = 3×3
    0.1412    0.1231   -0.0001
    0.1231    0.9149   -0.1615
   -0.0001   -0.1615    0.1465

gyr_cov = 3×3
10^-6 ×
    0.5058    0.0391   -0.1011
    0.0391    0.4989    0.0474
   -0.1011    0.0474    0.7268
```

Task 3

Given the continuous-time model:

$$\dot{q}(t) = 0.5S(\omega_{k-1} + v_{k-1})q(t), \quad \text{for } t \in [t_{k-1}, t_k),$$

we want to derive a discrete-time model of the form:

$$q_k = F(\omega_{k-1})q_{k-1} + G(\hat{q}_{k-1})v_{k-1}.$$

Let's denote $\Delta t = t_k - t_{k-1}$. Assuming $\omega(t) = \omega_{k1}$ and $v(t) = v_{k1}$ are piecewise constant between the sampling times t_{k1} and t_k, integrating both sides of the continuous-time equation from t_{k-1} to t_k, and applying the approximation $\exp(A) \approx I + A$, we get:

$$q_k = q_{k-1} + 0.5\Delta t S(\omega_{k-1} + v_{k-1})q_{k-1} \approx (I + 0.5\Delta t S(\omega_{k-1} + v_{k-1}))q_{k-1}.$$

From this, identify the state transition model:

$$F(\omega_{k-1}) = I + 0.5\Delta t S(\omega_{k-1}).$$

For the process noise term, assuming it impacts the state linearly, we can choose:

$$G(\hat{q}_{k-1}) = 0.5\Delta t Sq(\hat{q}_{k-1}),$$

where Sq is the function defined in A.2.4. This choice is motivated by the Extended Kalman Filter (EKF) procedure, where the state transition function is linearized about the estimated state, in this case $\hat{q}_{k-1}$, rather than the actual state.

Therefore, the discrete-time model is:

$$q_k = F(\omega_{k-1})q_{k-1} + G(\hat{q}_{k-1})v_{k-1}.$$

Task 5

In the absence of measurements, the Extended Kalman Filter (EKF) can't estimate the absolute orientation of the phone. This is because gyroscopes only measure rotation rates, not absolute orientation. Starting from an initial estimate, the filter predicts future states by integrating these rotation rates. However, without any measurements to correct these predictions, the filter will simply propagate the initial estimate forward. This can be problematic as any initial errors will not be corrected, and the errors in the gyroscope measurements will accumulate over time, causing the estimated state to drift away from the true state.

Now, if start the filter with the phone on its side instead of face up on the desk, the filter's initial estimate of the phone's orientation will be incorrect. This initial estimate error will then propagate through the filter's predictions, resulting in an incorrect estimation of the phone's orientation throughout. This illustrates the importance of a good initial estimate in a prediction-only filter.

If shake the phone, the gyroscope will measure large rotation rates. The filter will then predict large changes in the phone's orientation. However, if the phone returns to its original orientation after the shake, the filter will not be able to recognize this because it lacks measurements to correct its predictions. As a result, the filter will incorrectly estimate that the phone's orientation has changed significantly.