Date. /

1. $\frac{\partial f(x,y)}{\partial x} = a \qquad \frac{\partial f(x,y)}{\partial y} = b$

$\therefore \nabla f(x,y) = \begin{bmatrix} a \\ b \end{bmatrix}$

2. $\nabla f(x) = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ \vdots \\ \vdots \\ a_N \end{bmatrix}$

3. $z = A(x^2 - 2x_0x + x_0^2) + B(y^2 - 2y_0y + y_0^2) + C$

$= Ax^2 - 2x_0Ax + Ax_0^2 + By^2 - 2y_0B\cdot y + By_0^2 + C$

$\therefore f_x(x,y) = 2Ax - 2x_0A \qquad f_y(x,y) = 2By - 2y_0B$

4. $x^T = (3\ 1\ 4) \qquad y^T = \begin{pmatrix} 2 \\ 5 \\ 1 \end{pmatrix} \qquad x\cdot x = 9+1+16 = 26$

$x\cdot y^T = 6+5+4 = 15$

$x\cdot y = \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix}(2\ 5\ 1) = \begin{pmatrix} 6 & 15 & 3 \\ 2 & 5 & 1 \\ 8 & 20 & 4 \end{pmatrix}$

$y\cdot x = (2\ 5\ 1)\begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix} = 6+5+4 = 15$

$A\cdot x = \begin{pmatrix} 4 & 5 & 2 \\ 3 & 1 & 5 \\ 6 & 4 & 3 \end{pmatrix}\cdot\begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix} = \begin{pmatrix} 25 \\ 30 \\ 34 \end{pmatrix}$

$A\cdot B = \begin{pmatrix} 4 & 5 & 2 \\ 3 & 1 & 5 \\ 6 & 4 & 3 \end{pmatrix}\cdot\begin{pmatrix} 3 & 5 \\ 5 & 2 \\ 1 & 4 \end{pmatrix} = \begin{pmatrix} 39 & 38 \\ 19 & 37 \\ 41 & 50 \end{pmatrix}$

B.reshape(1,6) = [3, 5, 5, 2, 1, 4]

5.

Model : $y = mx+b$ Loss Function $= \sum_{i=1}^{N} (\hat{y}_i - M(\hat{x}_i, m, b))^2$

$$\frac{\partial L}{\partial m} = \frac{\partial \sum_{i=1}^{N} (\hat{y}_i - mx_i - b)^2}{\partial m} = \cancel{-2x_i \sum_{i=1}^{N} (y_i -}$$

$$= \frac{\partial \sum_{i=1}^{N} (\hat{y}_i^2 - 2y_i m x_i + 2y_i b + m^2 x_i^2 - 2mb x_i + b^2)}{\partial m}$$

$$= \boxed{-2 \sum_{i=1}^{N} x_i y_i + 2m \sum_{i=1}^{N} x_i^2 + 2b \cdot \sum_{i=1}^{N} x_i = 0} \quad ①$$

$$\frac{\partial L}{\partial b} = -2 \sum_{i=1}^{N} (y_i - mx_i - b) = -2 \sum_{i=1}^{N} y_i + 2m \sum_{i=1}^{N} x_i + 2nb = 0$$

$$\sum_{i=1}^{N} y_i = m \sum_{i=1}^{N} x_i + nb$$

$$\because ① : \sum_{i=1}^{N} x_i y_i = m \sum_{i=1}^{N} x_i^2 + b \sum_{i=1}^{N} x_i$$

$$\therefore m = \frac{\sum_{i=1}^{N} x_i y_i - b \sum_{i=1}^{N} x_i}{\sum_{i=1}^{N} x_i^2} \qquad \because b = \bar{y}_i - m\bar{x}_i$$

$$\therefore m = \frac{\sum (x_i y_i - \bar{y} x_i)}{\sum_{i=1}^{N} (x_i^2 - \bar{x} x_i)} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{N} (x_i - \bar{x})^2} = \frac{Cov(x, y)}{Var(x)}$$

$$\because y = mx+b \qquad \therefore b = \bar{y} - \frac{Cov(x, y)}{Var(x)} \cdot \bar{x}$$