

$$\begin{aligned}
\text{Succ}_1((v_1, d_1, T_1, F_1)) &= \left\{ (v_2, d_2, T_2, F_2) \in S : \begin{array}{l} v_2 \in \text{Ord} \\ d_2 = d_1 - \text{Dist}(v_1, v_2) \wedge d_1 - \text{Dist}(v_1, v_2) \geq 0 \\ \exists i \in [k]: i \in T_1 \wedge T_2 = T_1 \setminus \{i\} \wedge F_2 = F_1 \cup \{i\} \\ \text{There exists a directed path } v_1 \rightarrow \dots \rightarrow v_2 \text{ on the map} \end{array} \right\} \\
\text{Succ}_2((v_1, d_1, T_1, F_1)) &= \left\{ (v_2, d_2, T_2, F_2) \in S : \begin{array}{l} v_2 \in \text{GasStations} \\ d_2 = d_{\text{refuel}} \wedge d_1 - \text{Dist}(v_1, v_2) \geq 0 \\ T_1 = T_2 \wedge F_1 = F_2 \\ \text{There exists a directed path } v_1 \rightarrow \dots \rightarrow v_2 \text{ on the map} \end{array} \right\} \\
\text{Succ}((v_1, d_1, T_1, F_1)) &= \text{Succ}_1((v_1, d_1, T_1, F_1)) \cup \text{Succ}_2((v_1, d_1, T_1, F_1))
\end{aligned}$$

$$\forall x_i \in x^t : \Pr(x_i) = \frac{\left(\frac{x_i}{\alpha}\right)^{-1/T}}{\sum_j \left(\frac{x_j}{\alpha}\right)^{-1/T}} = \frac{x_i^{-1/T} \cdot \alpha^{1/T}}{\sum_j \left(x_j^{-1/T} \cdot \alpha^{1/T}\right)} = \frac{x_i^{-1/T} \cdot \cancel{\alpha^{1/T}}}{\cancel{\alpha^{1/T}} \sum_j x_j^{-1/T}} = \frac{x_i^{-1/T}}{\sum_j x_j^{-1/T}}$$

First let us notice that the expression can be rewritten in the form

$$\forall x_i \in x : \Pr(x_i) = \frac{x_i^{-1/T}}{\sum_{j \in [N]} x_j^{-1/T}} = \dots = \frac{1}{1 + \sum_{i \neq j} \left(\frac{x_i}{x_j}\right)^{1/T}}$$

And by taking the limit $T \rightarrow 0$, we have two options:

1. if $x_i < x_j$ for every $i \neq j$ then $\sum_{i \neq j} \left(\frac{x_i}{x_j}\right)^{1/T} \xrightarrow{T \rightarrow 0} 0$ and $\lim_{T \rightarrow 0} \Pr(x_i) = \frac{1}{1+0} = 1$. This is the case where $x_i = \min_j \{x_j\}_{j=1}^N = \alpha$.
2. if there exists at least one j such that $x_i \geq x_j$ then we get:
 - (a) $\left(\frac{x_i}{x_j}\right)^{1/T} \xrightarrow{T \rightarrow 0} \infty$ if $x_i > x_j$, which leads to $\lim_{T \rightarrow 0} \Pr(x_i) = \infty$. This is the case where x_i is not the minimal element of $\{x_j\}_{j=1}^N$.
 - (b) $\left(\frac{x_i}{x_j}\right)^{1/T} = (1)^{1/T} = 1 \xrightarrow{T \rightarrow 0} 1$, if there is exactly one j that satisfies the equality $x_i = x_j$.
 - (c) If there are $\{x_k\}_{k \in K}$ where $K \subseteq N$ and $2 \leq |K| \leq |N|$ that satisfy $x_i = x_k$ for all $k \in K$, and all other coordinates satisfy $x_i < x_j$ (such that their summands vanish), we get $\lim_{T \rightarrow 0} \Pr(x_i) = \frac{1}{1+|K| \cdot 1 + (0 + \dots + 0)} = \frac{1}{1+K}$.

By using the equivalent expression of the probability function from question 21 and taking the limit $T \rightarrow \infty$ while assuming $x_j \neq 0$ for all j , we get

$$\lim_{T \rightarrow \infty} \Pr(x_i) = \lim_{T \rightarrow \infty} \frac{1}{1 + \sum_{i \neq j} \left(\frac{x_i}{x_j}\right)^{1/T}} = \frac{1}{1 + \underbrace{(1 + \dots + 1)}_{N-1 \text{ times}}} = \frac{1}{N}$$

and since in our case $N = 5$, we get that indeed the limit is 0.2, as can be seen in the plot.

$$h'(v) = \begin{cases} h(v) & \text{if } \text{Applicable}_h(v) \text{ is true} \\ 0 & \text{if } \text{Applicable}_h(v) \text{ is false} \wedge \text{isGoal}(v) \text{ is true} \\ \min_{u \in \text{Succ}(v)} (\text{cost}(v, u)) & \end{cases}$$