Crimes Imputation

using bayesian regression



2조

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Project Goal



CONTENTS

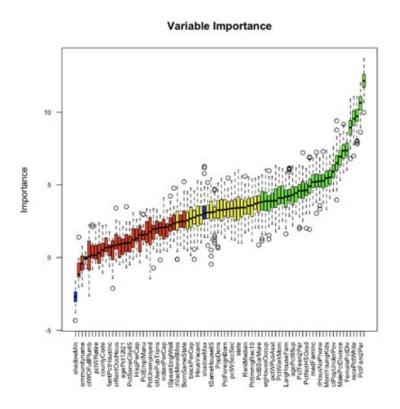
- **1** EDA Summary
- Model Selection
 - Bayesian Regression
 - Other Methods; LASSO SPCA
- 3 Conclusion
 - Model Comparison
 - Suggestions for Improvement Mixture Distribution;
 K-means Clustering



EDA

Variable Selection – Boruta (Random Forest를 기반으로)

```
# Do a tentative rough fix
roughFixMod <- TentativeRoughFix(boruta_output)
boruta_signif <- getSelectedAttributes(roughFixMod)
print(boruta signif)
 [1] "state"
                                                   "householdsize"
                            "population"
 [4] "racepctblack"
                            "racePctWhite"
                                                   "racePctAsian"
 [7] "racePctHisp"
                            "agePct65up"
                                                   "medIncome"
    "pctWInvInc"
                            "pctWSocSec"
                                                   "pctWPubAsst"
[13] "medFamInc"
                            "perCapInc"
                                                   "blackPerCap"
[16] "PctPopUnderPov"
                            "PctLess9thGrade"
                                                   "PctNotHSGrad"
                                                   "FemalePctDiv"
[19] "PctOccupManu"
                            "MalePctDivorce"
                            "PctKids2Par"
                                                   "PctYoungKids2Par"
[22] "PctFam2Par"
                            "PctWorkMomYoungKids"
                                                   "PctWorkMom"
[25] "PctTeen2Par"
[28] "PctKidsBornNeverMar"
                           "NumImmig"
                                                   "PctRecImmig10"
                            "PctLargHouseOccup"
                                                   "PersPerOccupHous"
[31] "PctLargHouseFam"
[34] "PersPerOwnOccHous"
                            "PctPersOwnOccup"
                                                   "PctPersDenseHous"
[37] "PctHousLess3BR"
                            "Hous Vacant"
                                                   "PctHousNoPhone"
                                                   "PctForeignBorn"
[40] "OwnOccMedVal"
                            "MedRent"
[43] "PctSameHouse85"
                            "PopDens"
```



Hyperparameter를 default 값으로 돌린 결과 21개의 변수가 선택됨

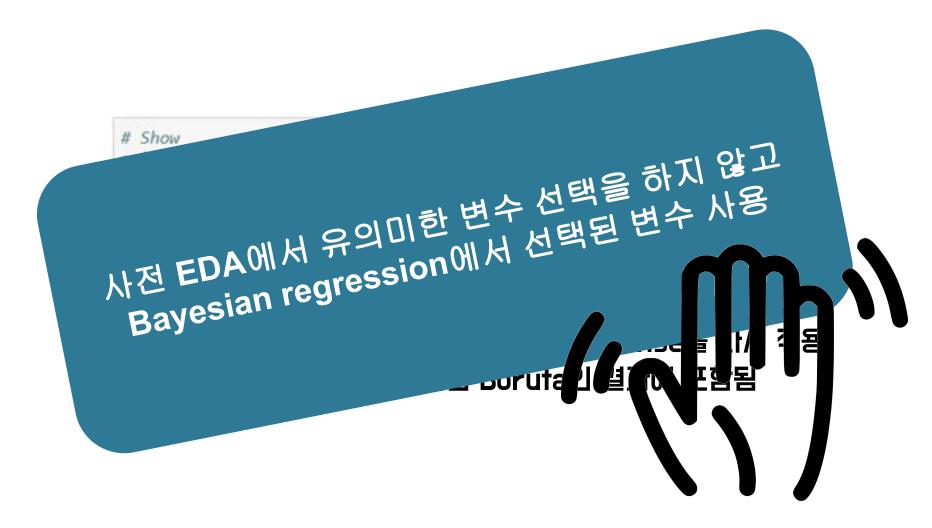
Variable Selection – Forward Stepwise Selection

```
# Show
print(shortlistedVars)

[1] "PctKids2Par" "PersPerOwnOccHous" "PctWorkMomYoungKids"
```

- 굉장히 적은 개수의 변수가 뽑히기 때문에 많은 정보를 상실
- 변수를 10-20개씩 group으로 나누어 stepwise를 다시 적용
- 선택된 세 가지 변수 모두 앞선 Boruta의 결과에 포함됨

Variable Selection – Forward Stepwise Selection

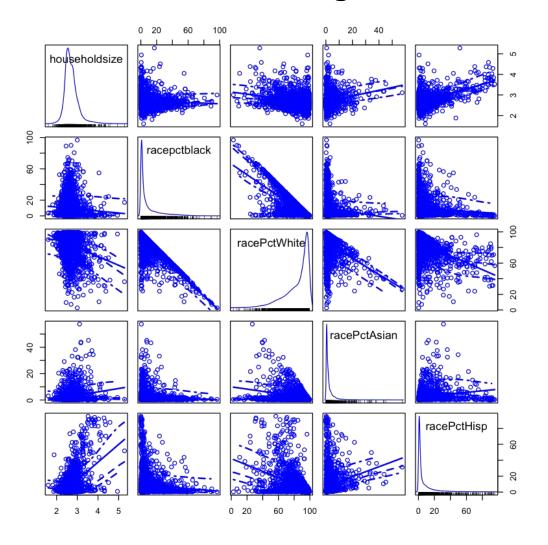


Divide into train and test set





Skewness and Scaling

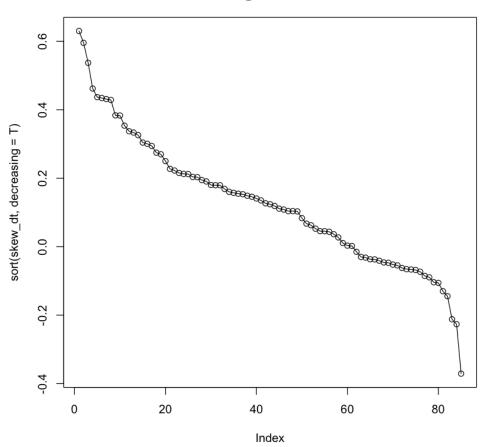


101개의 variables 중 model training에 필요 없는 variable들은 제외

Continuous variables에 한해 scaling, skewness 조정 후 MedNumBR과 불입

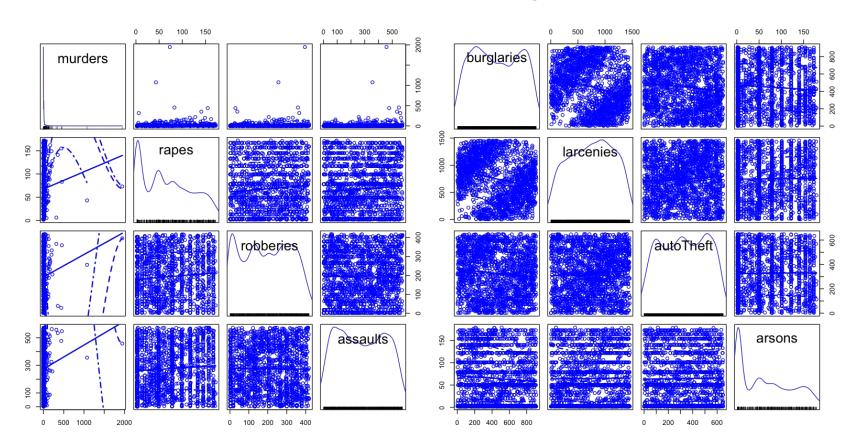
Skewness and Scaling

Checking skewness



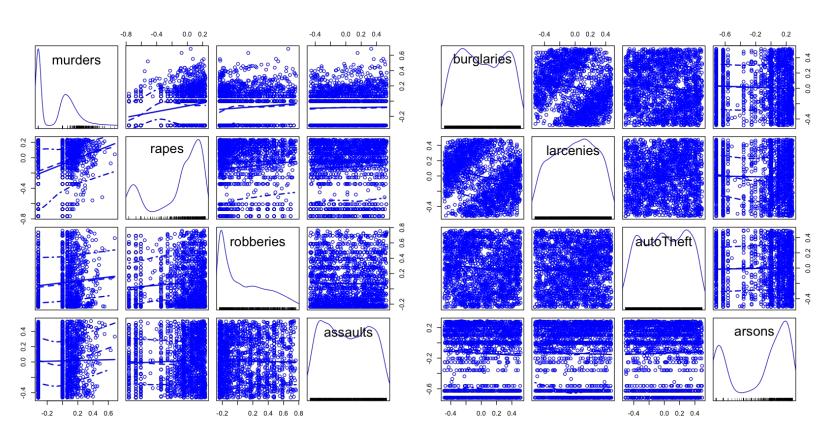
Skewness and Scaling

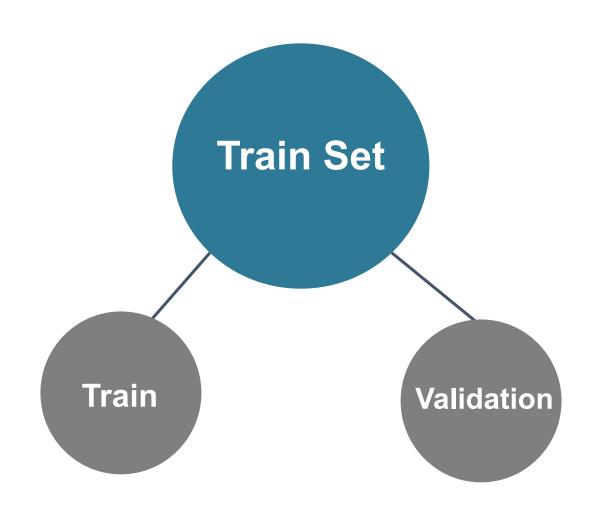
Scatter plot of training set



Skewness and Scaling

Scatter plot of scaled and skewed traing set





Start variable selection

Robberies Murders Assaults Rapes **AutoTheft Larcenies Burglaries Arsons**

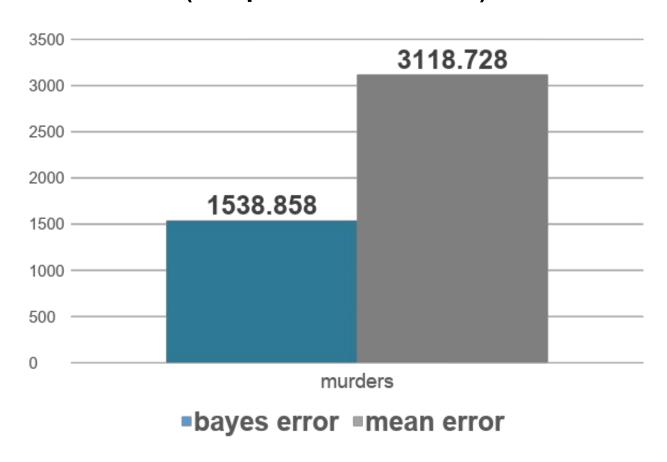
i) Bayesian regression code -'bayes_reg'

```
#Bayes Selection function
                                                                                                           #MCMC - beta, sigma
#input is target index(1~8)
                                                                                                           SIGMA = matrix(nrow=S, ncol=1); BETA = matrix(nrow=S, ncol=dim(X)[2])
                                                                                                           for(s in 1:5){
bayes_reg=function(target,sample_n){
                                                                                                               Xz = X[,Z[s,]==1, drop=F]
    t=which(colnames(scaled.skew.t)==target)
   Y=as.matrix(scaled.skew.t[,t]); colnames(Y)= 'Y'; N=length(Y)
    crim.lse = lm(Y\sim. -1, data=as.data.frame(cbind(Y, X))) # Y\sim. -1 as X already includes an intercept col
                                                                                                               # sigma given Y, X
                                                                                                               Hg = (g/(g+1)) * Xz %*% solve(t(Xz)%*%Xz) %*% t(Xz)
   MSE = sum(crim.lse$residuals^2) / (N-p)
                                                                                                               SSRg = t(Y) %*% (diag(1, dim(Xz)[1]) - Hg) %*% Y
                                                                                                               SIGMA[s,] = 1/rgamma(1, (v0+N)/2, (v0*s20 + SSRg)/2)
    #prior parameters
                                                                                                               # beta given Y, X, sigma
    g=N
                                                                                                               Vb = (g/(g+1)) * solve(t(Xz) %*% Xz) *SIGMA[s,]
    v0 = 1; s20 = MSE
                                                                                                               Eb = (g/(g+1)) * solve(t(Xz) %*% Xz) %*% t(Xz) %*% Y
                                                                                                               BETA[s,Z[s,]==1] = mvrnorm(1, Eb, Vb)
   #log likelihood of data
   lpy.X = function(Y, X, g=length(Y), v0=1, s20 = try(summary(lm(Y<math>\sim -1+X))$sigma^2, silent = T)){
                                                                                                              #output - beta / cutoff=0.5
       n = dim(X)[1]; p = dim(X)[2]
       if(p==0) {Hg = 0; s20 = mean(Y^2)} # if a model with no regressor is selected
                                                                                                              Z_post_mean = apply(Z, 2, mean, na.rm=T)
       if(p>0) {Hg = g/(g+1) * X %*% solve(t(X)%*%X) %*% t(X)}
                                                                                                              Beta Bay est = apply(BETA, 2, mean, na.rm=T)
       SSRg = t(Y) %*% ( diag(1, nrow=n) - Hg ) %*% Y
                                                                                                              final z=Z post mean[Z post mean>0.3]
       return(-.5*(n*log(pi)+p*log(1+g)-v0*log(v0*s20)+(v0+n)*log(v0*s20+SSRg))
                                                                                                              final z index=which(Z post mean>0.3)
              +lgamma((v0+n)/2)-lgamma(v0/2))
                                                                                                              final beta=Beta Bay est[final z index]
                                                                                                              output_dt=as.data.frame(cbind(final_z,final_beta))
                                                                                                              rownames(output_dt)=colnames(X[,final_z_index])
    #Gibbs sampling - z
   S=sample n
                                                                                                              colnames(output dt)=c('final z', 'final Beta')
    z = rep(1, dim(X)[2])
                                      # initial value for the model string Z
    lpy.c = lpy.X(Y,X[,z=1, drop=F]) # calculate current <math>logP(Y|X,z)
                                                                                                              newlist=list(output_dt=output_dt, Beta_Bay_est=Beta_Bay_est)
    Z = matrix(NA, S, dim(X)[2])
                                     # result slot
                                                                                                              return(newlist)
   M lpy = matrix(rep(NA, S),ncol=1) # result slot (optional)
  for(s in 1:5){ progress(s, 5-1)
               for(j in sample(1:dim(X)[2])){
                   zp = z; zp[j] = 1 - zp[j] # change 1 to 0, 0 to 1
                   lpy.p = lpy.X(Y,X[,zp==1, drop=F]) # calculate logP(Y|Xz) for proposed z
                   r = (lpy.c - lpy.p)*(-1)^(zp[j]==0)
                   z[j] = rbinom(1,1,1/(1+exp(r))) # change 1 to 0, 0 to 1 with prob. <math>1/(1+exp(r))
                   if(z[j] == zp[j])\{lpy.c = lpy.p\}
               Z[s,] = z; M_lpy[s] = lpy.c
```

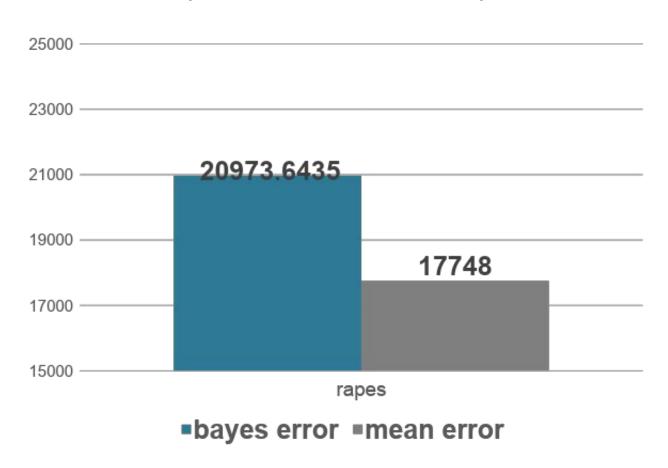
ii) Bayesian error code -'bayes_err'

```
##Real values are stored in 'real.v'
##Scaled and skew-adjusted values are stored in 'scaled.skew.v'
##1. Define coefficient vector
##2. Define design matrix
##3. Predict: X %*% coefficient
##4. Transform predictions: unscale ->log, square (skew.train.set)
##5. Compare with real.v
set.seed(111)
bayes_err=function(target, sample_n, skew_trans){
    output_list=bayes_reg(target,sample_n)
    ##0. Conduct Bayes Selection
    t=which(colnames(scaled.skew.t)==target)
    output crimes=output list[[1]]
    output_crimes=as.data.frame(output_crimes)
    index_crimes=which(colnames(scaled.skew.v) %in% rownames(output_crimes))
    ##1. Define coefficient vector
    Beta Bayes est=output list[[2]]
    {if(sum(rownames(output_crimes) %in%'intercept')==1) coef_crimes<-output_crimes[,2]</pre>
    else if(Beta_Bayes_est[1]=='NaN') coef_crimes<-output_crimes[,2]
        else coef_crimes<-c(Beta_Bayes_est[1],output_crimes[,2])</pre>
    ##2. Define design matrix
    X crimes=as.matrix(cbind(1,scaled.skew.v[,index crimes]))
    if(ncol(X_crimes) == length(coef_crimes)){
        X_crimes<-X_crimes
    }else X_crimes<-X_crimes[,-1]</pre>
    ##3. Predict: X %*% coefficient
    pred_crimes=X_crimes %*% coef_crimes
    ##4. Transform predictions: unscale ->log, square (skew.train.set)
    pred_crimes_real=pred_crimes*(max(skew.train.set[,t])-min(skew.train.set[,t]))+median(skew.train.set[,t])
    {if(skew_trans=='log') pred_crimes_real<-exp(pred_crimes_real)</pre>
    else if(skew_trans=='sq') pred_crimes_real<-sqrt(pred_crimes_real)
        else pred crimes real<-pred crimes real
    #pred_crimes_real= ifelse(skew_trans=='log',exp(pred_crimes_real),
            ifelse(skew_trans=='sq',sqrt(pred_crimes_real),pred_crimes_real))
    ##5. Compare with real.v
    err=sum((pred_crimes_real-real.v[,t])^2)/length(real.v)
    newlist2=list(output crimes,err)
    return(newlist2)
```

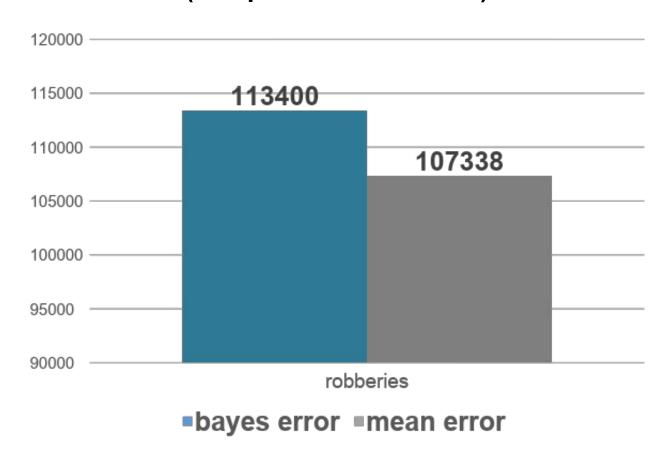
1. murders



2. rapes

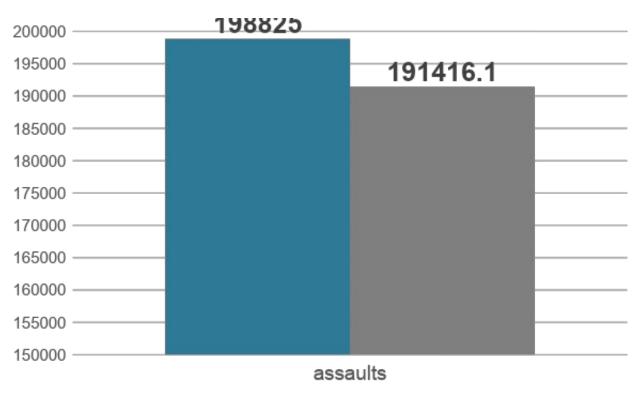


3. robberies



4. assaults

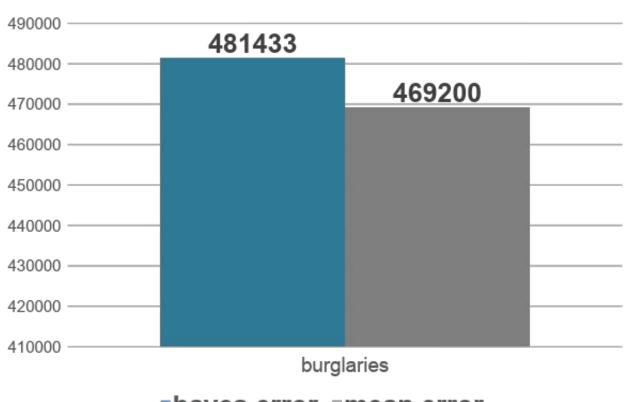
Bayes imputation error vs mean imputation error (sample number = 10)



bayes error mean error

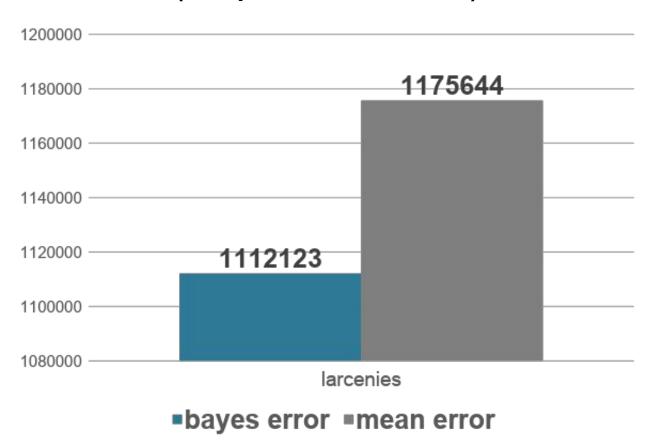
5. burglaries

Bayes imputation error vs mean imputation error (sample number = 10)



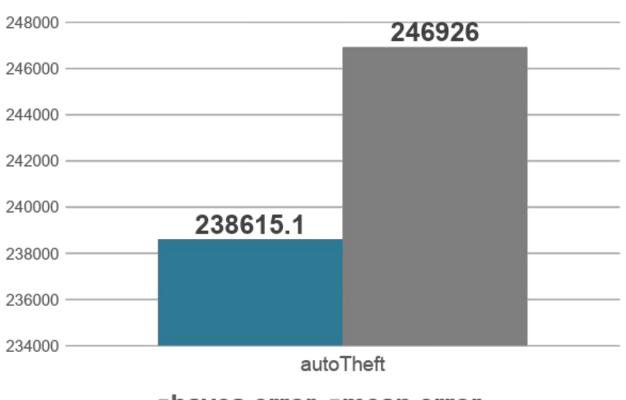
bayes error mean error

6. larcenies



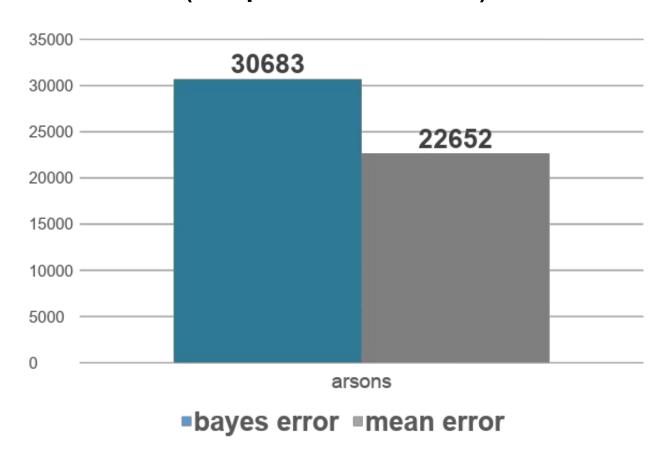
7. autoTheft

Bayes imputation error vs mean imputation error (sample number = 10)



■bayes error ■mean error

8. arsons



Other Methods – LASSO

기존의 Linear Regression에서 적절한 가중치와 편향을 찾아내는 것이 관건이었다면, LASSO(Least Absolute Shrinkage and Selection Operator)는 거기에 덧붙여서 추가 제약조건(L1 Norm)을 준다. 그 제약조건은 MSE가 최소가 되게 하는 가중치와 편향을 찾는 데 동시에 가중치들의 절대값들의 합, 즉 가중치의 절대값들이 최소가 되게 해야한다는 것이다. 다시 말해서 가중치의 모든 원소가 이미 되거나 이에 가깝게 되게 하는 것이다.

$$MSE + penalty$$

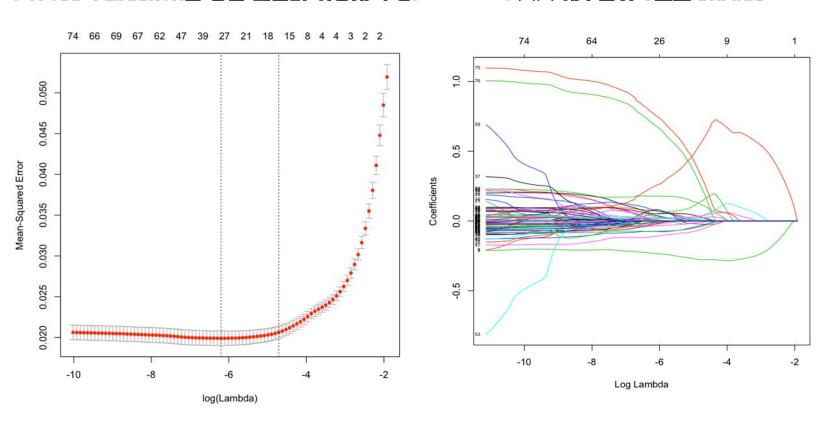
$$= MSE + \alpha \cdot L_1 - norm$$

$$= \frac{1}{n} \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 + \alpha \sum_{j=1}^{m} |w_j|$$

L1 Penalty

〈 Cross Validation을 통한 패널티 가증치 추정〉

〈 몇몇 계수들이 0으로 shrink〉



Robberies - sq

(Intercept) 0.0533646760907696 householdsize -0.0725768189916002 racepctblack 0.0673734278588427 racePctWhite -0.047254749872441 agePct12t21 -0.0113046911085456 indianPerCap 0.000741025051785238 OtherPerCap -0.0279703274022045 PctWorkMomYoungKi... -0.0181814396815018 PctWorkMom -0.0269957828894548 PctNotSpeakEnglWell 0.106059912575755 HousVacant 0.019157311184707 MedYrHousBuilt -0.0356171837452312 MedRentPctHousInc -0.0859597487008759 **PopDens** 0.095090494946478

대략 20~30개의 변수가 뽑힘

Other Methods – Sparse PCA

Sparse PCA

SPCA(Sparse Principal Component Analysis)란?

기존 PCA의 principal components는 모든 input variable의 linear combination이기 때문에 해석의 어려움이 발생

반면,

Sparse PCA의 principal components는 유의한 input variable의 linear combination

SPCA는 기존 PCA에 LASSO penalty를 접목. data의 dimension을 줄임

➡ input variable이 많은 high-dimensional data분석에 용이

Sparse PCA

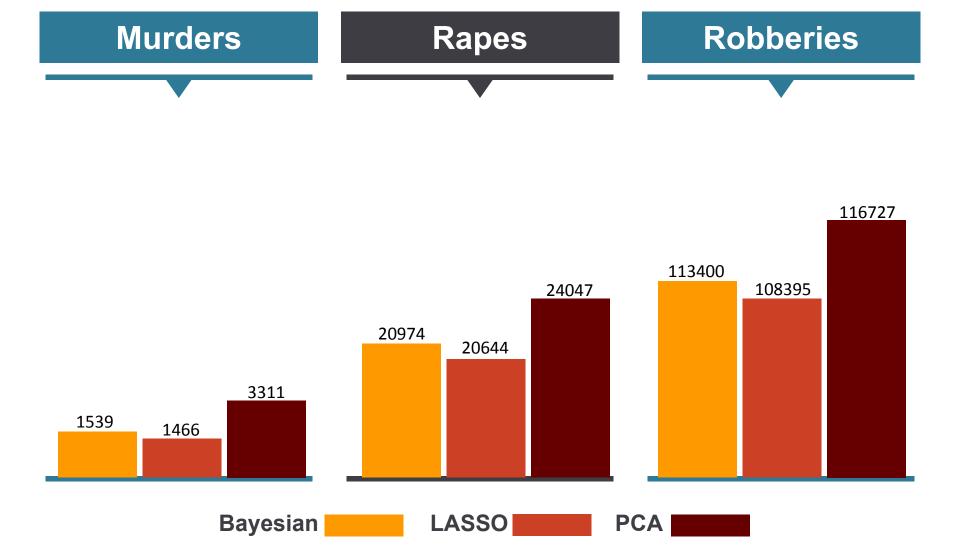
1.				
1.	medincome	-0.46538576497294	racePctWhite	0.472732222887109
	PctPopUnderPov	0.659971542326747	alePctDivorce	-0.0304452226885781
	PctHousNoPhone	0.31298619142701	PctKids2Par	0.872375347984423
	OwnOccMedVal	-0.21616843495342	BornNeverMar	-0.119479774139656
	MedRent	-0.450737733926516	rsDenseHous	-0.016827645713195
	weakent	-0.450/3//33920510		-0.010027040710100
2.			Loadings of PC1	
۷.	racePctHisp	-0.332920000282132	octNotHSGrad	-0.00894006241440504
	Numlmmig	-0.622739124884626	PctBSorMore	0.782459647714106
	PctRecImmig10	-0.701426117206124	PctEmplManu	-0.0419645047027797
	PctBornSameState	0.072587694995952	EmplProfServ	0.411262787750252
	PctUsePubTrans	-0.0639740915245425	ctOccupManu	-0.465595183329988
3.				
	MedYrHousBuilt	-0.807654893837169	HousVacant	-0.146429124872914
	PctBornSameState	0.388960893927252	ctHousOccup	0.0697599661948257
	PctSameHouse85	0.205911657532864	acantBoarded	-0.684899639952924
	PctSameCity85	0.353840254551932	/acMore6Mos	-0.688682572092002
	PctUsePubTrans	0.169706979841363	LandArea	-0.174128850551965
4.	M-1-D-1D:	0.440050000440704		
	MalePctDivorce	0.113952089419724	racePctHisp	0.01550348573795
	PctLargHouseOccup	-0.531599324332032	indianPerCap	0.975917578086567
	PersPerOccupHous	-0.825451483370874	alePctDivorce	0.196249360851691
	PersPerRentOccHous	-0.141274462277511	FemalePctDiv	0.0889681575797726
	PctBornSameState	-0.055573866120957	HousVacant	0.030255214654448
_				
5.	racePctHisp	-0.023973850024768	pctWWage	0.297927248752846
	OtherPerCap	-0.999248815201844	pctWSocSec	-0.733813927224357
	PctPopUnderPov	-0.0237752537878255	PctEmploy	0.43767524210203
	PctPersDenseHous	-0.0162385920708983	:tImmigRec10	0.425641171216214
	PctSameHouse85	0.00990481032533901	ctSameCity85	-0.00514300289171929
			GioaineCityos	-0.00314300269171929

PC 개수는 10개, 한 PC당 5개 변수

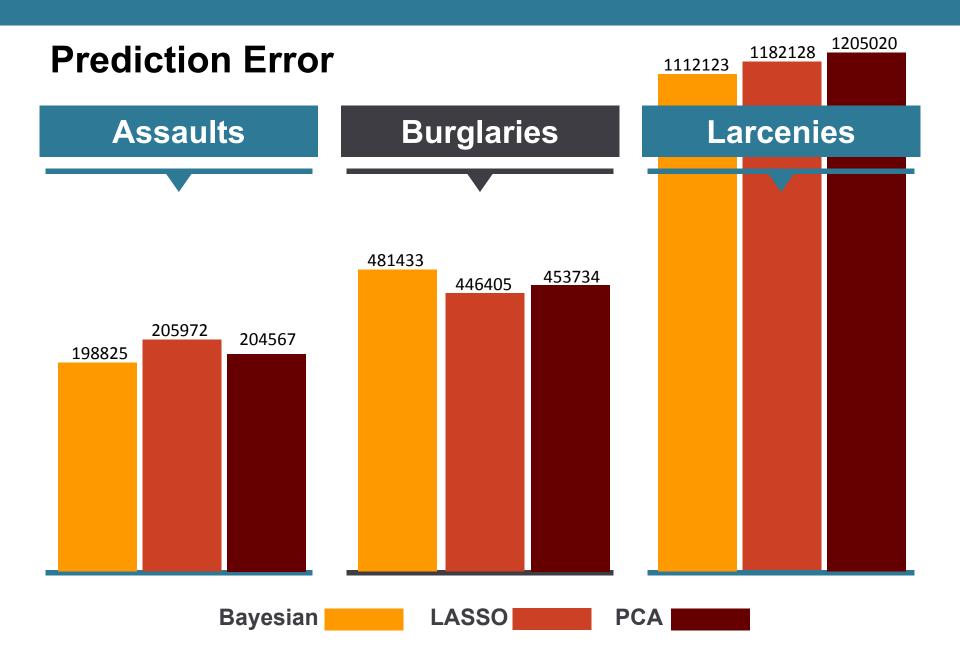
Conclusion – Model Selection

Model Selection

Prediction Error

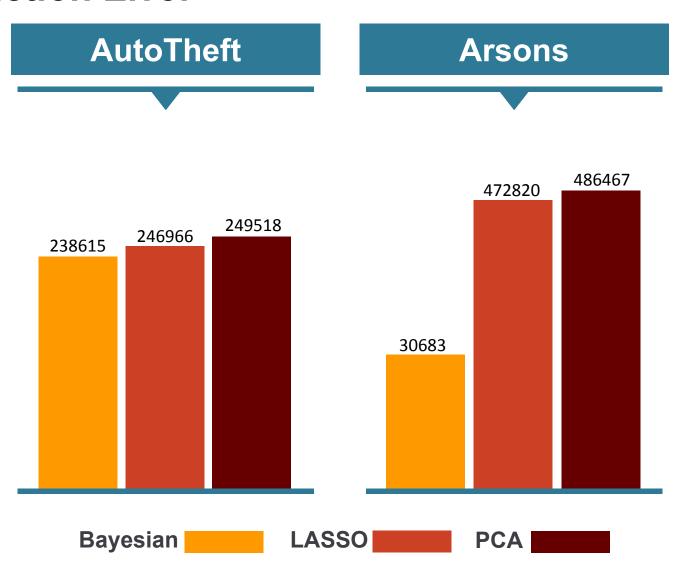


Model Selection



Model Selection

Prediction Error



Model Selection

Our Models

Bayesian Reg

LASSO

PCA

4

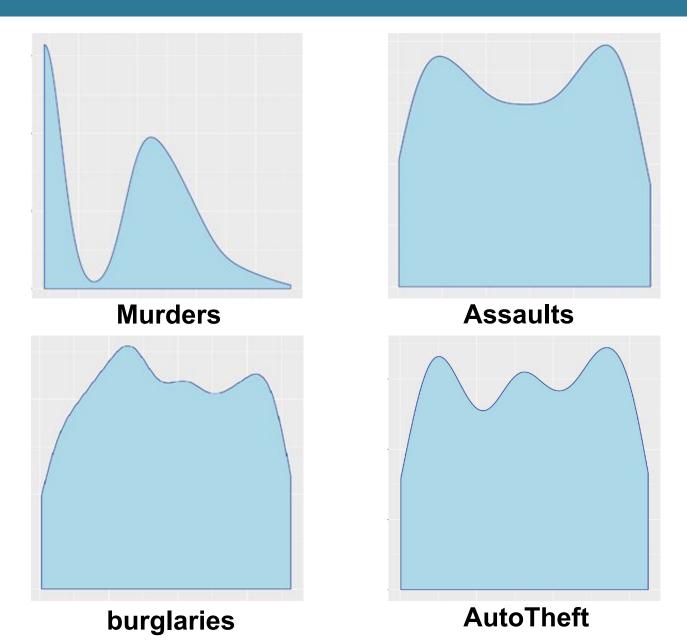
4

Assaults
Larcanies / AutoTheft/ Arsons

Murders / Rapes / Robberies
Burglaries

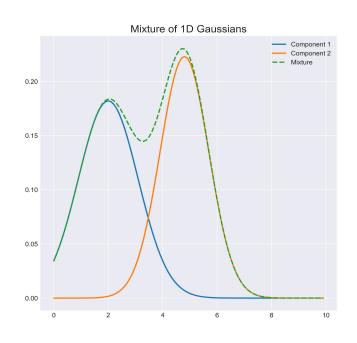
최종 모델은 Bayesian Regression으로 선택함

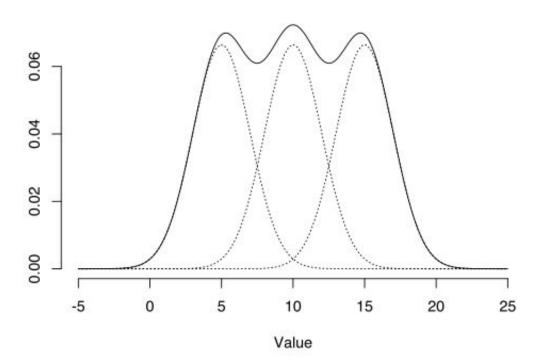
Conclusion – Suggestions



Mixture Gaussian Distributions Detected

- 하나의 정규분포 형태가 아닌 문제 발생





Mixture Gaussian Distributions Detected

- 하나의 정규분포 형태가 아닌 문제 발생

9.2 Bayesian estimation for a regression model

We begin with a simple semiconjugate prior distribution for β and σ^2 to be used when there is information available about the parameters. In situations where prior information is unavailable or difficult to quantify, an alternative "default" class of prior distributions is given.

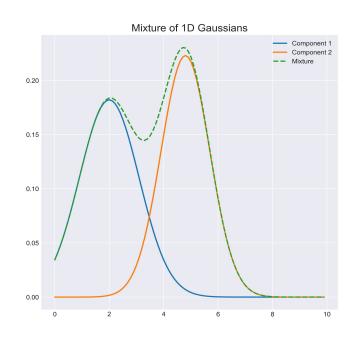
9.2.1 A semiconjugate prior distribution

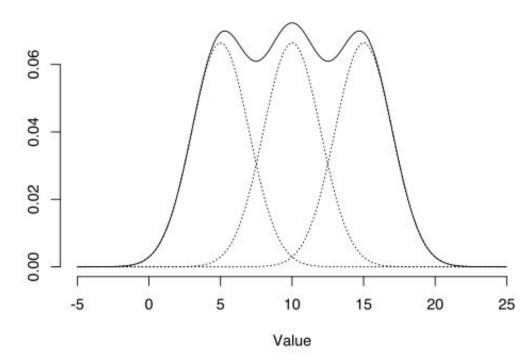
The sampling density of the data (Equation 9.3), as a function of β , is

$$p(\mathbf{y}|\mathbf{X}, \boldsymbol{\beta}, \sigma^2) \propto \exp\{-\frac{1}{2\sigma^2} SSR(\boldsymbol{\beta})\}$$
$$= \exp\{-\frac{1}{2\sigma^2} [\mathbf{y}^T \mathbf{y} - 2\boldsymbol{\beta}^T \mathbf{X}^T \mathbf{y} + \boldsymbol{\beta}^T \mathbf{X}^T \mathbf{X}\boldsymbol{\beta}]\}.$$

Mixture Gaussian Distributions Detected

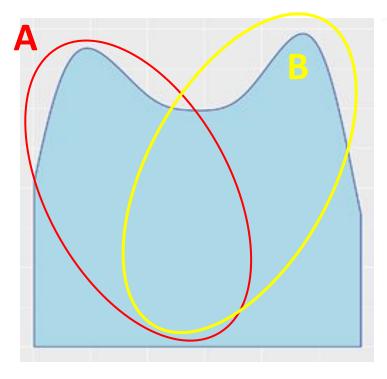
- 해결책: Y를 cluster로 나누어 cluster별 특징 확인





Mixture Gaussian Distributions Detected

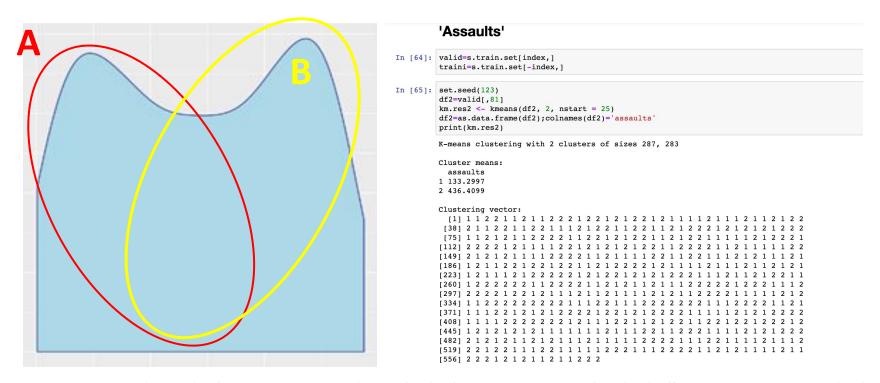
- 해결책: Y를 cluster로 나누어 cluster별 특징 확인



'Assaults' In [64]: valid=s.train.set[index,] traini=s.train.set[-index,] In [65]: set.seed(123) df2=valid[,81] $km.res2 \leftarrow kmeans(df2, 2, nstart = 25)$ df2=as.data.frame(df2);colnames(df2)='assaults' print(km.res2) K-means clustering with 2 clusters of sizes 287, 283 Cluster means: assaults 1 133.2997 2 436.4099 Clustering vector: [482] 2 1 2 1 2 1 1 2 1 2 1 1 1 2 1 1 1 1 2 2 2 2 2 1 1 1 2 2 1 1 1 1 2 1 1 1 2 [519] 2 2 1 2 2 1 1 1 2 2 1 1 1 1 2 2 2 1 1 1 1 2 2 2 1 1 1 2 2 2 2 1 1 2 1 2 1 2 1 1 1 1 2 1 2 [556] 2 2 2 1 2 1 2 1 1 2 1 1 2 2 2

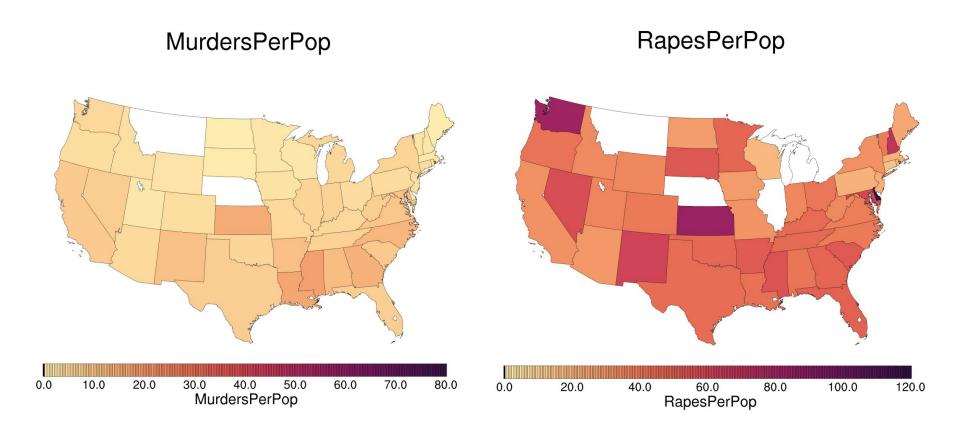
Mixture Gaussian Distributions Detected

- 데이터가 어느 클러스터에 속하는 지 결정하는 변수(latent variable zi)를 추가한 hierarchical 모형으로 Bayesian Reg을 시행한다면 더 나은 예측이 될 것이라 생각



(K-means를 사용하여 cluster는 나누었지만 cluster를 잘 나타내는 feature은 하지 못함)

Mixture Gaussian Distributions Detected



Visit our Github

Our Final Code

2019 FALL FINAL Group 2 최종.ipynb

Y Outputs

finalimputation2.csv



Thank you