ESC HW3

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(a)

```
library(dplyr)
```

```
##
## Attaching package: 'dplyr'
```

```
## The following objects are masked from 'package:stats':
##
##     filter, lag
```

```
## The following objects are masked from 'package:base':
##
##     intersect, setdiff, setequal, union
```

```
library(tidyr)
library(ggplot2)
library(MCMCpack)
```

```
## Loading required package: coda
```

```
## Loading required package: MASS
```

```
##
## Attaching package: 'MASS'
```

```
## The following object is masked from 'package:dplyr':
##
##     select
```

```
## ##
## ## Markov Chain Monte Carlo Package (MCMCpack)
```

```
## ## Copyright (C) 2003-2019 Andrew D. Martin, Kevin M. Quinn, and Jong Hee Park
```

```
## ##
## ## Support provided by the U.S. National Science Foundation
```

```
## ## (Grants SES-0350646 and SES-0350613)
## ##
```

```
library(coda)

schools.list = lapply(1:8, function(i) {
  s.tbl = paste0('http://www.stat.washington.edu/people/pdhoff/Book/Data/hwdata/school', i, '.dat') %>%
    url %>%
    read.table

  data.frame(
    school = i,
    hours = s.tbl[, 1] %>% as.numeric
  )
})
```

```r
schools.raw = do.call(rbind, schools.list)
Y = schools.raw
head(Y)
```

```
##   school hours
## 1      1  2.11
## 2      1  9.75
## 3      1 13.88
## 4      1 11.30
## 5      1  8.93
## 6      1 15.66
```

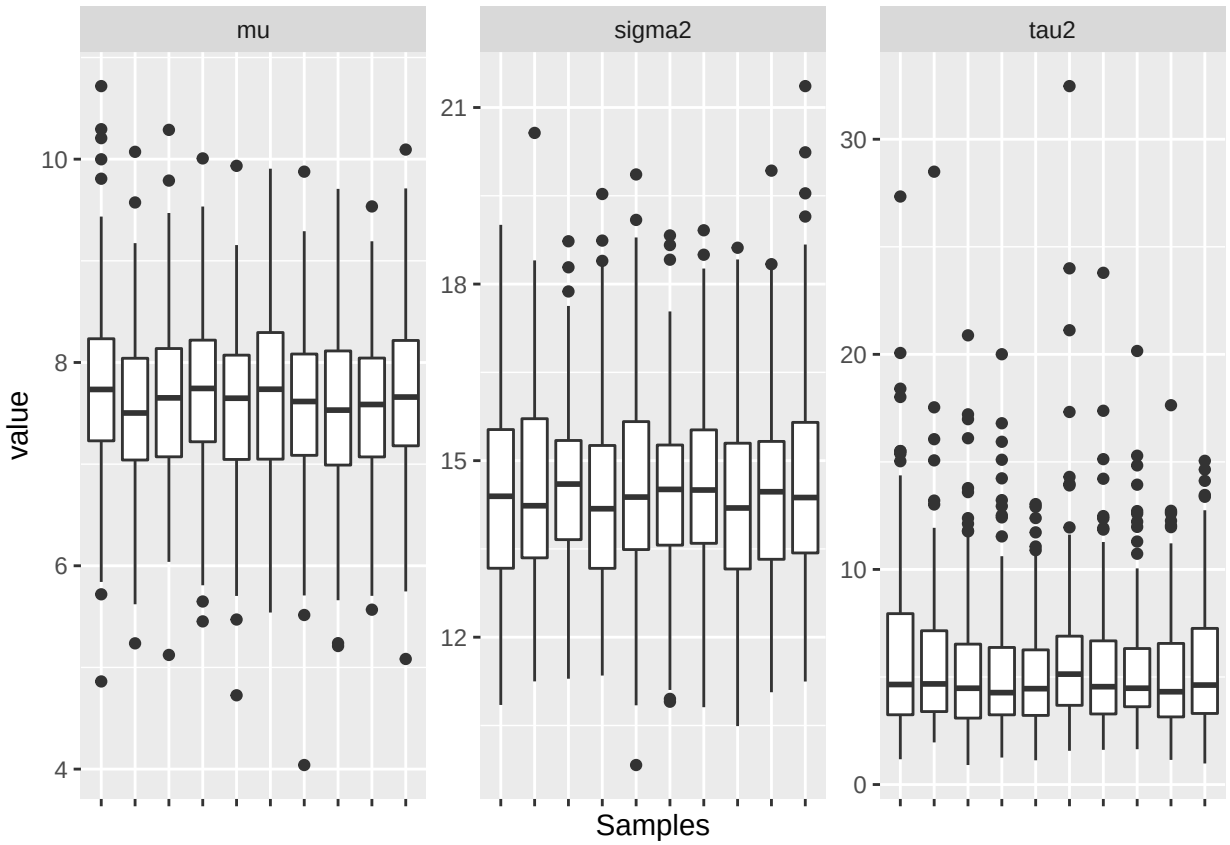
```r
mu0 <- 7
g20 <- 5
t20 <- 10
eta0 <- 2
s20 <- 15
nu0 <- 2
```

```r
#starting values
m=length(unique(Y[,1]))
n<-sv<-ybar<-rep(NA,m)

for (j in 1:m) {
  Y_j <- Y[Y[,1]==j,2]
  ybar[j] <- mean(Y_j)
  sv[j] <- var(Y_j)
  n[j] <- length(Y_j)
}
theta <- ybar
sigma2 <- mean(sv)
mu <- mean(theta)
tau2 <- var(theta)
```

```r
S = 1500
THETA = matrix(nrow = S, ncol = m)
SMT = matrix(nrow = S, ncol = 3)
colnames(SMT) = c('sigma2', 'mu', 'tau2')

for (s in 1:S) {
  # Sample thetas
  for (j in 1:m) {
    vtheta = 1 / (n[j] / sigma2 + 1 / tau2)
    etheta = vtheta * (ybar[j] * n[j] / sigma2 + mu / tau2)
    theta[j] = rnorm(1, etheta, sqrt(vtheta))
  }

  # Sample sigma2
  nun = nu0 + sum(n)
  ss = nu0 * s20
  # Pool variance
  for (j in 1:m) {
    ss = ss + sum((Y[Y[, 1] == j, 2] - theta[j])^2)
  }
  sigma2 = 1 / rgamma(1, nun / 2, ss / 2)
```

```
  # Sample mu
  vmu = 1 / (m / tau2 + 1 /g20)
  emu = vmu * (m * mean(theta) / tau2 + mu0 / g20)
  mu = rnorm(1, emu, sqrt(vmu))

  # Sample tau2
  etam = eta0 + m
  ss = eta0 * t20 + sum((theta - mu)^2)
  tau2 = 1 / rgamma(1, etam / 2, ss / 2)

  THETA[s, ] = theta
  SMT[s, ] = c(sigma2, mu, tau2)
}
```

```
smt.df <- data.frame(SMT)
colnames(smt.df) <- c('sigma2','mu','tau2')
smt.df$s <- 1:S
cut_size <- 10
smt.df <- smt.df %>%
  tbl_df %>%
  mutate(scut=cut(s, breaks=cut_size)) %>%
  gather('variable', 'value', sigma2:tau2)
ggplot(smt.df, aes(x=scut, y=value))+
  facet_wrap(~ variable, scales = 'free_y')+
  geom_boxplot()+
  theme(axis.text.x = element_blank())+
  xlab('Samples')
```




```
effectiveSize(SMT[,1])
```

```
##     var1
## 1369.569
```

```
effectiveSize(SMT[,2])
```

```
##     var1
## 1112.417
```

```
effectiveSize(SMT[,3])
```

```
##     var1
## 1155.915
```

(b)

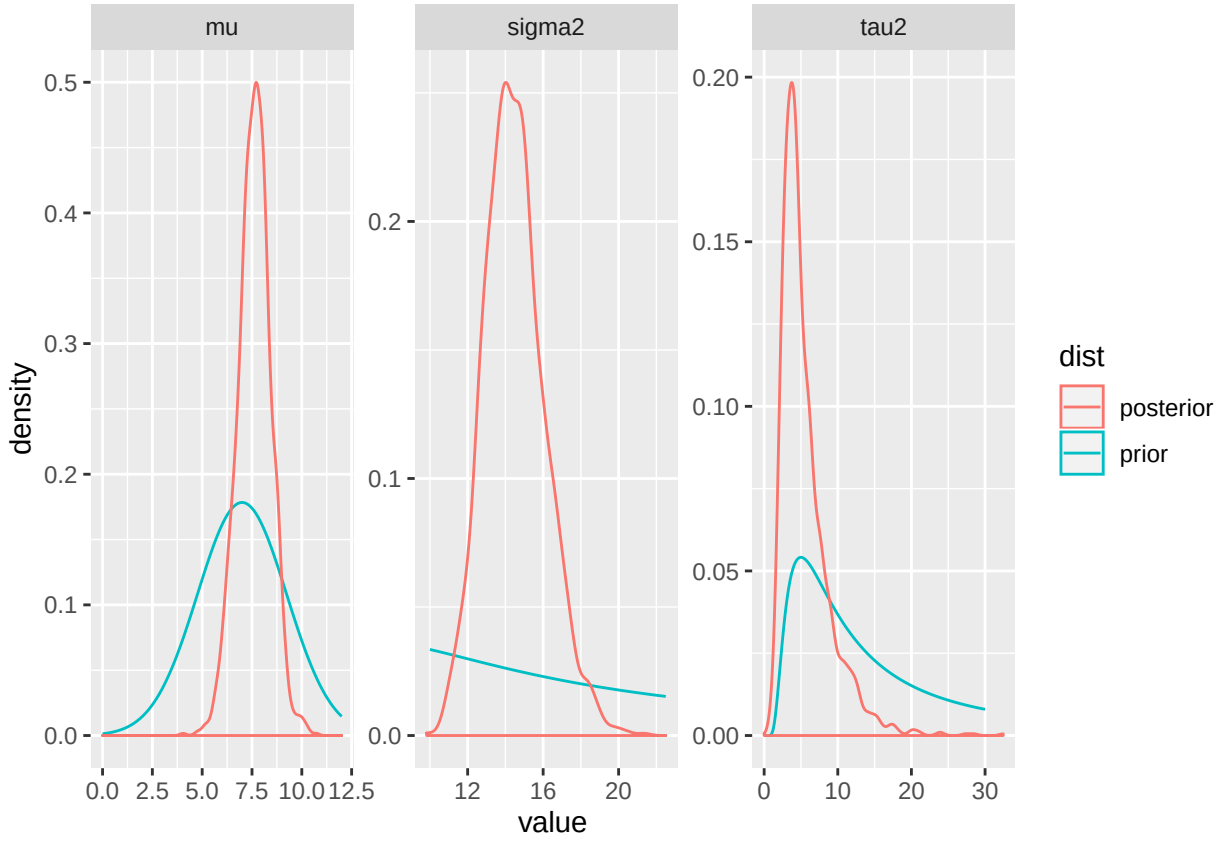
```
#posterior mean 95% CI
apply(SMT, MARGIN = 2, mean)
```

```
##    sigma2        mu      tau2
## 14.492879  7.611452  5.525548
```

```
t(apply(SMT, MARGIN = 2, FUN = quantile, probs=c(0.025,0.975)))
```

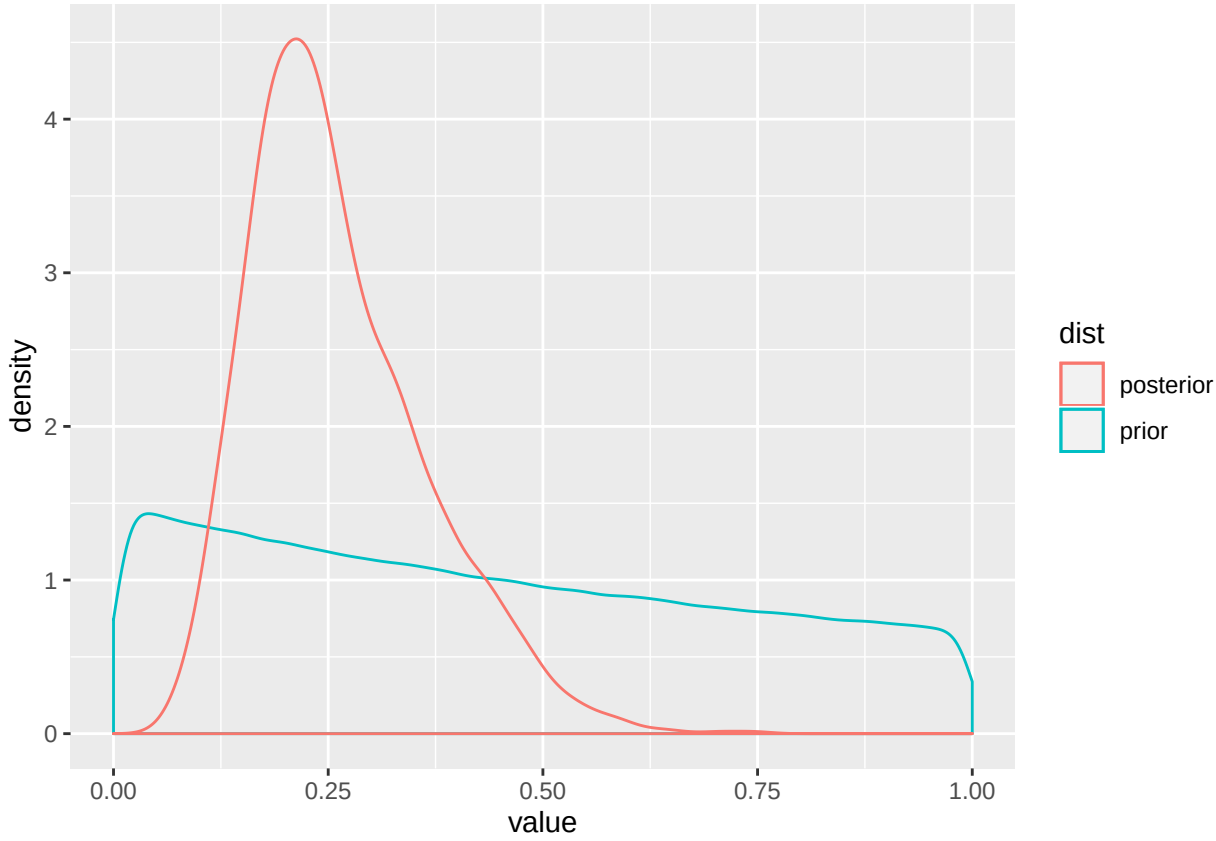
```
##             2.5%     97.5%
## sigma2 11.651678 18.030539
## mu      5.966113  9.170848
## tau2    1.918692 14.137111
```

```r
library(MCMCpack)
sigma2_prior <- data.frame(
  value = seq(10, 22.5, by = 0.1),
  density = dinvgamma(seq(10, 22.5, by = 0.1), nu0 / 2, nu0 * s20 / 2),
  variable = 'sigma2'
)
tau2_prior <- data.frame(
  value = seq(0, 30, by = 0.1),
  density = dinvgamma(seq(0, 30, by = 0.1), eta0 / 2, eta0 * t20 / 2),
  variable = 'tau2'
)
mu_prior <- data.frame(
  value = seq(0, 12, by = 0.1),
  density = dnorm(seq(0, 12, by = 0.1), mu0, sqrt(g20)),
  variable = 'mu'
)
priors <- rbind(sigma2_prior, tau2_prior, mu_prior)
priors$dist <- 'prior'
smt.df$dist <- 'posterior'

ggplot(priors, aes(x=value, y=density, color=dist))+
  geom_line()+
  geom_density(data=smt.df, mapping = aes(x=value, y=..density..))+
  facet_wrap(~variable, scales = 'free')
```



###(c)

```
t20_prior <- (1/rgamma(1e6, eta0/2, eta0*t20/2))
s20_prior <- (1/rgamma(1e6, nu0/2, nu0*s20/2))

R_prior <- data.frame(
  value = (t20_prior)/(t20_prior+s20_prior),
  dist = 'prior'
)
R_post <- data.frame(
  value = SMT[,'tau2']/(SMT[,'tau2'] + SMT[,'sigma2']),
  dist = 'posterior'
)

ggplot(R_prior, aes(x=value, y=..density.., color=dist))+
  geom_density(data=R_prior)+
  geom_density(data=R_post)
```




```
mean(R_post$value)
```

```
## [1] 0.2612659
```

(d)

```
theta7_6 <- THETA[,7]<THETA[,6]
mean(theta7_6)
```

```
## [1] 0.5246667
```

```
theta7_s <- (THETA[,7]<THETA[,-7]) %>%
  apply(MARGIN = 1, FUN = all)

mean(theta7_s)
```

```
## [1] 0.314
```

(e)

```
relationship = data.frame(
  sample_average = ybar,
  post_exp = colMeans(THETA),
  school = 1:length(ybar)
)

ggplot(relationship, aes(x = sample_average, y = post_exp, label = school)) +
  geom_text() +
  geom_abline(slope = 1, intercept = 0) +
  geom_hline(yintercept = mean(schools.raw[, 'hours']), lty = 2) +
  annotate('text', x = 10, y = 7.9, label = paste0("Pooled sample mean ", round(mean(schools.raw[, 'hou
  geom_hline(yintercept = mean(SMT[, 'mu']), color = 'red') +
  annotate('text', x = 10, y = 7.4, label = paste0("Posterior exp. mu ", round(mean(SMT[, 'mu']), 2)), c
```