a). 같은 y_{ij} 에 대한 variability를 비교하는 것인데

첫번째 것에는 θ_j, σ^2 를 컨디션으로 주어 μ, τ^2 보다 더 구체적으로 (조금하다)

컨디션을 준 것이므로 $Var[y_{ij} \mid \theta_j, \sigma^2]$ 이 더 작은 것이다.

b) ∘ $Cov[y_{i1,j}, y_{i2,j} \mid \theta_j, \sigma^2]$

$y_{1,j} \sim y_{nj,j} \mid \theta_j, \sigma^2 \overset{iid}{\sim} N(\theta_j, \sigma^2)$ 이므로. zero 일 것.

∘ $Cov[y_{i1,j}\, y_{i2,j} \mid \cancel{\theta_j, \sigma^2}\ \mu, \tau^2]$

θ_j가 unknown 이지만, $y_{i1,j}$를 알게 되면 θ_j에 대한 belief를 update 하여

유사한. $y_{i2,j}$를 얻을 수 있게 되므로 positive 일 것.

c) ∘ $V[y_{ij} \mid \theta_j, \sigma^2] = \sigma^2$.

∘ $V[\bar{y}_{\cdot j} \mid \theta_j, \sigma^2] = \sigma^2/n_j$.

∘ $V[y_{ij} \mid \mu, \tau^2] = V[E[y_{ij} \mid \theta_j, \sigma^2] \mid \mu, \tau^2] + E[V[y_{ij} \mid \theta_j, \sigma^2] \mid \mu, \tau^2]$

$= V[\theta_j \mid \mu, \tau^2] + E[\sigma^2 \mid \mu, \tau^2]$

$= \tau^2 + \sigma^2$

∘ $V[\bar{y}_{\cdot j} \mid \mu, \tau^2] = V[E[\bar{y}_{\cdot j} \mid \theta_j, \sigma^2] \mid \mu, \tau^2] + E[V[\bar{y}_{\cdot j} \mid \theta_j, \sigma^2] \mid \mu, \tau^2]$

$= V[\theta_j \mid \mu, \tau^2] + E[\sigma^2/n_j \mid \mu, \tau^2]$

$= \tau^2 + \sigma^2/n_j$.

∘ $Cov[y_{i1,j}, y_{i2,j} \mid \theta_j, \sigma^2] = E[y_{i1,j} \times y_{i2,j} \mid \theta_j, \sigma^2] - E[y_{i1,j} \mid \theta_j, \sigma^2] \times E[y_{i2,j} \mid \theta_j, \sigma^2] = 0$.

∘ $Cov[y_{i1,j}\ y_{i2,j} \mid \mu, \tau^2] = \cancel{Cov[E[y_{i1,j}, y_{i2,j} \mid \theta_j, \sigma^2] \mid \mu, \tau^2]}$

$= \cancel{Cov}\ E[Cov[y_{i1,j}, y_{i2,j} \mid \theta_j, \sigma^2] \mid \mu, \tau^2]$

$+ Cov[E[y_{i1,j} \mid \theta_j, \sigma^2], E[y_{i2,j} \mid \theta_j, \sigma^2]]$

$= 0 + Cov[\theta_j, \theta_j] = \tau^2$.

$$p(\mu \mid \theta, \sigma^2, \tau^2, Y) = p(\mu, \theta, \sigma^2, \tau^2, Y) \Big/ \int p(\mu, Y, \theta, \sigma^2, \tau^2)\, d\mu.$$

$$= \frac{p(Y \mid \theta, \sigma^2, \tau^2, \mu) \times p(\theta \mid \sigma^2, \tau^2, \mu) \times p(\sigma^2, \tau^2, \mu)}{\int p(Y \mid \theta, \sigma^2, \tau^2, \mu) \times p(\theta \mid \sigma^2, \tau^2, \mu) \times p(\sigma^2, \tau^2, \mu)\, d\mu}$$

$$= \frac{p(Y \mid \theta, \sigma^2) \times p(\theta \mid \mu, \tau^2) \times p(\sigma^2) \times p(\tau^2) \times p(\mu)}{\int \;'' \qquad d\mu}$$

$$= p(\mu) \cdot p(\theta \mid \mu, \tau^2) \Big/ \int p(\mu) \cdot p(\theta \mid \mu, \tau^2)\, d\mu.$$

$$= \cancel{p(\mu) \cdot p(\theta \mid} \; p(\mu \mid \theta, \tau^2) \Big/ \int p(\mu \mid \theta, \tau^2)\, d\mu = p(\mu \mid \theta, \tau^2)$$

∴ μ는 σ^2, Y와 독립. (θ가 알려져 있다면).

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