1. $\beta \mid y, \sigma^2 \sim N\left(\frac{g}{g+1}\hat{\beta}_{mle}, \frac{g}{g+1} Var(\hat{\beta}_{mle})\right)$ 증명

$\tilde{X} = XH \rightarrow$ 사후분포 $\beta \leftarrow y, X$

$\tilde{\beta} \leftarrow y, \tilde{X}$ $\Big\} \rightarrow$ β와 $H\tilde{\beta}$의 사후분포 같아야 한다.

$\begin{cases} \beta_0 = 0 \\ \Sigma_0 = k(X^TX)^{-1} \text{ for any } k>0 \end{cases}$ 만족

이때, $k = g\sigma^2$ for any $g>0$

$$Var[\beta \mid y, X, \sigma^2] = \left(\Sigma_0^{-1} + \frac{X^TX}{\sigma^2}\right)^{-1}$$

$$E[\beta \mid y, X, \sigma^2] = \left(\Sigma_0^{-1} + \frac{X^TX}{\sigma^2}\right)^{-1}\left(\Sigma_0^{-1}\beta_0 + \frac{X^Ty}{\sigma^2}\right)$$

$\rightarrow$ under principle of invariance

$$Var[\beta \mid y, X, \sigma^2] = \left[\frac{X^TX}{g\sigma^2} + \frac{X^TX}{\sigma^2}\right]^{-1} = \frac{g}{g+1}\sigma^2(X^TX)^{-1}$$

$$E[\beta \mid y, X, \sigma^2] = \left[\frac{X^TX}{g\sigma^2} + \frac{X^TX}{\sigma^2}\right]^{-1}\frac{X^Ty}{\sigma^2}$$

$$= \frac{g}{g+1}(X^TX)^{-1}X^Ty$$

2. $SSR(g) \to SSR(\hat{\beta}_{mle})$ as $g \to \infty$

$$p(y|X,\sigma^2) = \left(\frac{1}{2\pi}\right)^{\frac{n}{2}} \left(\frac{1}{1+g}\right)^{\frac{p}{2}} \left(\frac{1}{\sigma^2}\right)^{\frac{n}{2}} \exp\left(-\frac{1}{2\sigma^2} SSR_g\right)$$

$$SSR_g = y^T y - m^T V^{-1} m = y^T\left(I - \frac{g}{g+1} X(X^T X)^{-1} X^T\right) y$$

as $g \to \infty$ $\quad SSR_g \to y^T(I - \underline{(X^T X)^{-1} X^T})\,\underline{y}$

$\hat{\beta}_{ols} = (X^T X)^{-1} X^T y$