2번문제

1. likelihood $x|\theta \sim B(n, x)$

$$P(x|\theta) = \prod_{i=1}^{n} {}_nC_{x_i} (\theta)^{x_i} (1-\theta)^{n-x_i}$$

$$= \prod_{i=1}^{n} {}_nC_{x_i} \times \theta^{\Sigma x_i} \cdot (1-\theta)^{n^2 - \Sigma x_i}$$

$$= \prod_{i=1}^{n} {}_nC_{x_i} \times \exp\left( \Sigma x_i \cdot \log\theta + (n^2 - \Sigma x_i) \cdot \log(1-\theta) \right)$$

$$= \underbrace{\prod_{i=1}^{n} {}_nC_{x_i}}_{h(x)} \times \underbrace{\exp\left(n^2 \times \log(1-\theta)\right)}_{g(\theta)} \times \exp\left( \underbrace{\Sigma x_i}_{\eta(x)} \cdot \underbrace{\log\left(\frac{\theta}{1-\theta}\right)}_{k(\theta)} \right)$$

→ 지수족의 형태 만족.

$$P(x|\theta) \propto \theta^{\Sigma x_i} \cdot (1-\theta)^{n^2 - \Sigma x_i}$$

2. prior $\theta \sim Beta(a, b)$

$$P(\theta) \propto \theta^{a-1} (1-\theta)^{b-1}$$

3. posterior

$$P(\theta|x) \propto \theta^{\Sigma x_i + a - 1} \cdot (1-\theta)^{n^2 - \Sigma x_i + b - 1}$$

$$\therefore \theta|x \sim Beta(\Sigma x_i + a, n^2 - \Sigma x_i + b)$$

3번문제

$X \sim NB(r, p)$

$r \to \infty$, $p \to 1$, $r(1-p) \to \lambda$ (condition)

$$p(x) = \left(\begin{matrix} r-1+x \end{matrix}\right) (1-p)^r p^x$$

$$P(x) = \binom{\;}{x} (1-p)^{r} p$$

$$= \frac{(r-1+x)!}{x! \times (r-1)!} \times \left(\frac{r}{r+\lambda}\right)^{r} \times \left(\frac{\lambda}{r+\lambda}\right)^{x}$$

$$= \frac{(r-1+x)!}{x!\,(r-1)!} \times e^{-\lambda} \times \frac{\lambda^{x}}{(r+\lambda)^{x}}$$

$$= e^{-\lambda} \times \frac{\lambda^{x}}{x!}$$

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variance of $NB(r, p)$

$$= \frac{pr}{(1-p)^2}$$

variance of $pois(\lambda)$

$$= \lambda = \frac{pr}{1-p}$$

→ 음이항 분포의 분모가 더 큰 것을 볼 수 있다.

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$Y \sim B(n, p)$

$$P(y) = \binom{n}{y} \cdot p^{y} \cdot (1-p)^{n-y}$$

$$= \frac{n!}{y!(n-y)!} \cdot p^{y} \cdot (1-p)^{n-y}$$

$$= \frac{n!}{y!(n-y)!} \cdot \left(\frac{\lambda}{n}\right)^{y} \cdot \left(\frac{n-\lambda}{n}\right)^{n-y}$$

$$= \frac{\lambda^{y}}{y!} \left(1 - \frac{y}{n}\right)^{n-y}$$

$$= \frac{\lambda^{y}}{y!} \cdot e^{-ny} = \frac{\lambda^{y}}{y!} \cdot e^{-\lambda}$$