Week3

HW1. DBA ex2.8

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1. BDA 2.8

(a) For n students, $\bar{y}=150$., $Y|\theta \sim N(\theta, 20^2)$

prior : $\theta \sim N(180, 40^2)$

Then using conjungacy,

our posterior $\theta|Y \sim N\left(\underbrace{\dfrac{\frac{1}{40^2}\times 180 + \frac{n}{20^2}\times\bar{y}}{\frac{1}{40^2}+\frac{n}{20^2}}}_{=\mu}, \underbrace{\dfrac{1}{\frac{1}{40^2}+\frac{n}{20^2}}}_{\sigma^2}\right)$

(b) Using posterior, our posterior predictive for $\tilde{y}$ would be :

$\tilde{y}|Y \sim N(\mu, \sigma^2+20^2)$ since $\tilde{y}|\theta \sim N(\theta, 20^2)$ also.

(c) For $n=10$, we compute the posterior and posterior predictive as follows

$\theta|y \sim N(150.73, (6.25)^2)$

$\tilde{y}|y \sim N(150.73, (20.95)^2)$

Thus from the distributions we computed, we get 95% C.I. as follows.

For θ, $150.73 \pm 1.96\times 6.25 \Rightarrow (138.48, 162.98)$

For $\tilde{y}$, $150.73 \pm 1.96\times 20.95 \Rightarrow (109.67 ; 191.79)$

(d) Likewise, we compute the posterior and posterior predictive as follows

$\theta|y \sim N(150.07, 2^2)$, $\tilde{y}|y \sim N(150.07, 20.1^2)$

Then for θ, 95% c.i. is $150.07 \pm 1.96\times 2 \Rightarrow (146.15, 153.99)$

For $\tilde{y}$, 95% c.i. is $150.07 \pm 1.96\times 20.1 \Rightarrow (110.67, 189.47)$

HW2 Derive the marginal posterior of sigma^2

2. $p(\sigma^2 | y) \propto p(\sigma^2)\, p(y|\sigma^2) = p(\sigma^2) \int p(y|\mu,\sigma^2)\, p(\mu|\sigma^2)\, d\mu$

Inv-gamma

$$\Rightarrow p(\sigma^2) \propto \left(\frac{1}{\sigma^2}\right)^{\frac{v_0}{2}+1} \exp\left(\frac{-v_0\sigma_0^2}{2\sigma^2}\right)$$

$\int p(y|\mu,\sigma^2)\, p(\mu|\sigma^2)\, d\mu$

$$= \int \left(\frac{1}{\sigma^2}\right)^{\frac{n}{2}} \exp\left(-\frac{1}{2\sigma^2}\left((n-1)s^2 + n(\bar{y}-\mu)^2\right)\right) \cdot \left(\frac{k_0}{\sigma^2}\right)^{\frac{1}{2}} \exp\left(-\frac{k_0}{2\sigma^2}(\mu-\mu_0)^2\right) d\mu$$

$$= \left(\frac{1}{\sigma^2}\right)^{\frac{n}{2}} \exp\left(-\frac{1}{2\sigma^2}(n-1)s^2\right) \cdot \left(\frac{1}{\sigma^2}\right)^{\frac{1}{2}} \cdot \int \exp\left(-\frac{1}{2\sigma^2}n(\bar{y}-\mu)^2 - \frac{k_0}{2\sigma^2}(\mu-\mu_0)^2\right) d\mu$$

$$\int \exp\left(-\frac{1}{2\sigma^2}\left((n+k_0)\mu^2 - 2(n\bar{y}+k_0\mu_0)\mu + k_0\mu_0^2 + n\bar{y}^2\right)\right) d\mu$$

$$= \int \exp\left(-\frac{n+k_0}{2\sigma^2}\left(\mu^2 - 2\left(\frac{n\bar{y}+k_0\mu_0}{n+k_0}\right)\mu + \left(\frac{n\bar{y}+k_0\mu_0}{n+k_0}\right)^2\right)\right)$$

$$\times \exp\left(-\frac{1}{2\sigma^2}\left(k_0\mu_0^2 + n\bar{y}^2 - \frac{(n\bar{y}+k_0\mu_0)^2}{n+k_0}\right)\right) d\mu$$

$\left(\mu - \frac{n\bar{y}+k_0\mu_0}{n+k_0}\right)^2$

$$= \frac{nk_0\mu_0^2 + nk_0\bar{y}^2 - 2n\bar{y}k_0\mu_0}{n+k_0} = \frac{nk_0(\mu_0-\bar{y})^2}{n+k_0}$$

$$\therefore p(\sigma^2|y) \propto \left(\frac{1}{\sigma^2}\right)^{\frac{v_0}{2}+1} \exp\left(\frac{-v_0\sigma_0^2}{2\sigma^2}\right) \times \left(\frac{1}{\sigma^2}\right)^{\frac{n}{2}} \exp\left(-\frac{1}{2\sigma^2}(n-1)s^2\right)$$

$$\times \left(\frac{1}{\sigma^2}\right)^{\frac{1}{2}} \times \int \exp\left(-\frac{n+k_0}{2\sigma^2}\left(\mu - \frac{n\bar{y}+k_0\mu_0}{n+k_0}\right)^2\right) d\mu$$

$$\times \exp\left(-\frac{1}{2\sigma^2}\left(\frac{nk_0(\bar{y}-\mu_0)^2}{n+k_0}\right)\right)$$

$\frac{n+k_0}{\sigma^2} \propto \left(\frac{1}{\sigma}\right)^{-\frac{1}{2}}$

$$\propto \left(\frac{1}{\sigma^2}\right)^{\frac{n+v_0}{2}+1} \times \exp\left(-\frac{1}{2\sigma^2}\left(v_0\sigma_0^2 + (n-1)s^2 + \frac{nk_0(\bar{y}-\mu_0)^2}{n+k_0}\right)\right)$$

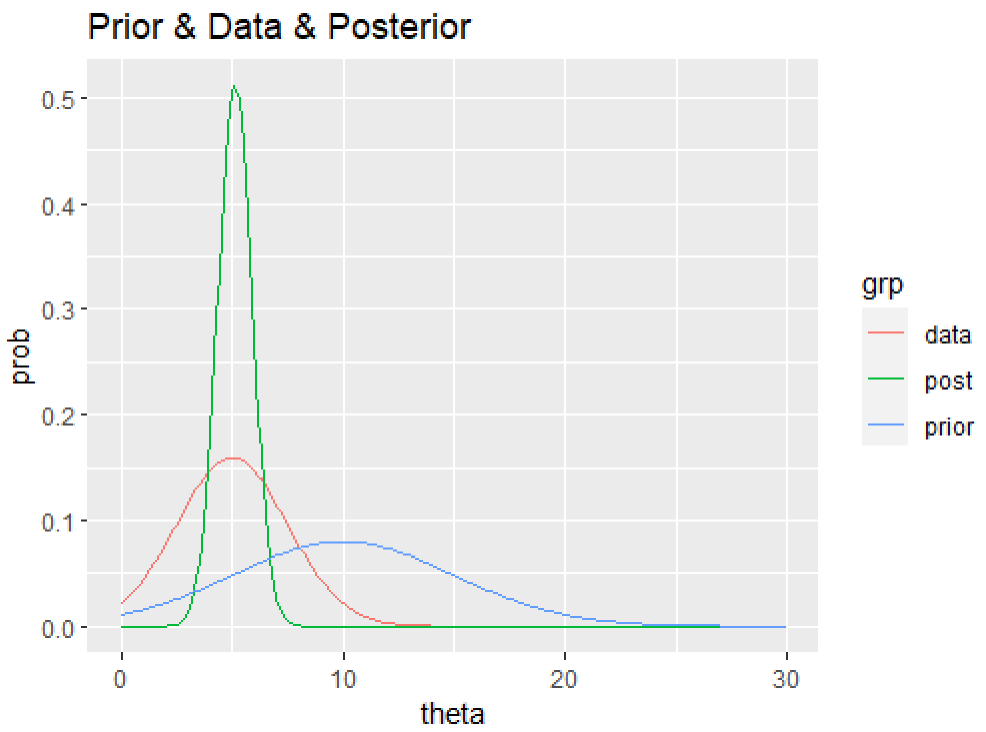
$$= (\sigma^2)^{-\frac{n+v_0}{2}-1} \exp\left(-\frac{v_n}{2\sigma^2} \times \left(\frac{1}{v_n}\left(v_0\sigma_0^2 + (n-1)s^2 + \frac{nk_0(\bar{y}-\mu_0)^2}{n+k_0}\right)\right)\right)$$

Thus $\sigma^2|y \sim \text{Inv-}\chi^2(\underbrace{v_0+n}_{v_n}, \sigma_n^2)$

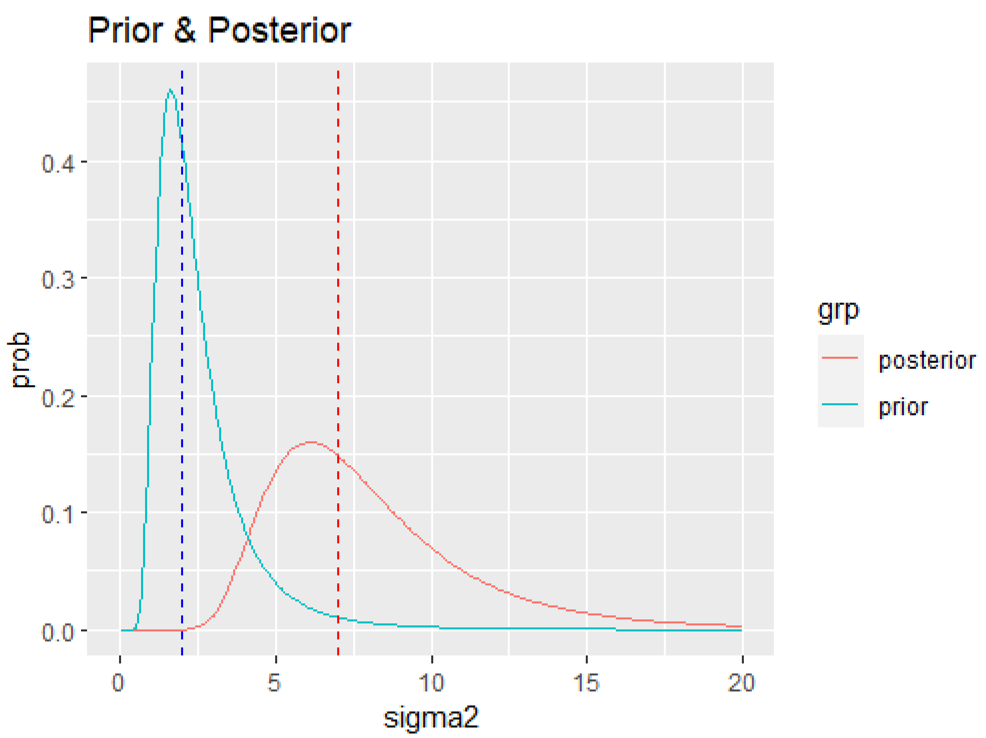
HW3. R visualization

one parameter

unknown mu



unknown sigma^2



two parameter

unknown mu and sigma^2