HW 풀이 1번

```
lpfun = function(a, b, y, n) # marginal posterior
(-5/2)* log(a+b) + sum(lgamma(a+b) - lgamma(a) - lgamma(b) + lgamma(a+y) + lgamma(b+n-y) - lgamma(a+b+n))
lp = mapply(lpfun, cA, cB, MoreArgs = list(DF$y, DF$n))
df_marg = data.frame(alpha= cA, beta= cB, posterior = exp(lp)/sum(exp(lp)))
```

$$
\begin{aligned}
p(\theta,\alpha,\beta|y) &\propto p(\alpha,\beta)p(\theta|\alpha,\beta)p(y|\theta,\alpha,\beta) \\
&\propto p(\alpha,\beta)\prod_{j=1}^{J}\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\theta_j^{\alpha-1}(1-\theta_j)^{\beta-1}\prod_{j=1}^{J}\theta_j^{y_j}(1-\theta_j)^{n_j-y_j}. \qquad (5.6)
\end{aligned}
$$

Given (α,β), the components of θ have independent posterior densities that are of the form $\theta_j^A(1-\theta_j)^B$—that is, beta densities—and the joint density is

$$
p(\theta|\alpha,\beta,y) = \prod_{j=1}^{J}\frac{\Gamma(\alpha+\beta+n_j)}{\Gamma(\alpha+y_j)\Gamma(\beta+n_j-y_j)}\theta_j^{\alpha+y_j-1}(1-\theta_j)^{\beta+n_j-y_j-1}. \qquad (5.7)
$$

We can determine the marginal posterior distribution of (α,β) by substituting (5.6) and (5.7) into the conditional probability formula (5.5):

$$
p(\alpha,\beta|y) \propto p(\alpha,\beta)\prod_{j=1}^{J}\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\frac{\Gamma(\alpha+y_j)\Gamma(\beta+n_j-y_j)}{\Gamma(\alpha+\beta+n_j)}. \qquad (5.8)
$$

HW 풀이 1번

```r
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lp = mapply(lpfun, cA, cB, MoreArgs = list(DF$y, DF$n))
df_marg = data.frame(alpha= cA, beta= cB, posterior = exp(lp)/sum(exp(lp)))
```

$$\ln (\alpha+\beta)^{-5/2}\prod_{j=1}^{m}\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\theta_j^{\alpha-1}(1-\theta_j)^{\beta-1}\prod_{j=1}^{m}\theta_j^{y_j}(1-\theta_j)^{n_j-y_j} = \text{lpfun}$$

$$p(\psi|y) = \int_{\Theta} p(\theta,\psi|y)\,d\theta$$

$$p(\psi|y) = \frac{p(\theta,\psi|y)}{p(\theta|\psi,y)} = \text{posterior}$$

HW 풀이 2번

```
# sample new values of the thetas
for(j in 1:m)
{
  vtheta<-1/(n[j]/sigma2+1/tau2)
  #theta의 conditional posterior mean
  etheta<-vtheta*(ybar[j]*n[j]/sigma2+mu/tau2)
  theta[j]<-rnorm(1,etheta,sqrt(vtheta))
}
```

$$\{\theta_j | y_{1,j}, \ldots, y_{n_j,j}, \sigma^2\} \sim \text{normal}(\underbrace{\frac{n_j \bar{y}_j/\sigma^2 + 1/\tau^2}{n_j/\sigma^2 + 1/\tau^2}}_{\text{etheta}}, \underbrace{[n_j/\sigma^2 + 1/\tau^2]^{-1}}_{\text{vtheta}}).$$

HW 풀이 2번

```
#sample a new value of mu
#mu의 conditional posterior variance
vmu<- 1/(m/tau2+1/g20)
emu<- vmu*(m*mean(theta)/tau2 + mu0/g20)
mu<-rnorm(1,emu,sqrt(vmu))
```

$$\{\mu|\theta_1,\ldots,\theta_m,\tau^2\} \sim \text{normal}\left(\frac{m\bar{\theta}/\tau^2+\mu_0/\gamma_0^2}{m/\tau^2+1/\gamma_0^2},[m/\tau^2+1/\gamma_0^2]^{-1}\right)$$

emu

vmu

HW 풀이 2번

```
# sample a new value of tau2
etam<-eta0+m
#tau2의 conditional posterior
ss<- eta0*t20 + sum( (theta-mu)^2 )
tau2<-1/rgamma(1,etam/2,ss/2)
```

$$\{1/\tau^2|\theta_1,\ldots,\theta_m,\mu\} \sim \text{gamma}\left(\underbrace{\frac{\eta_0+m}{2}}_{\text{etam}}, \underbrace{\frac{\eta_0\tau_0^2+\sum(\theta_j-\mu)^2}{2}}_{\text{ss}}\right).$$

HW 풀이 2번

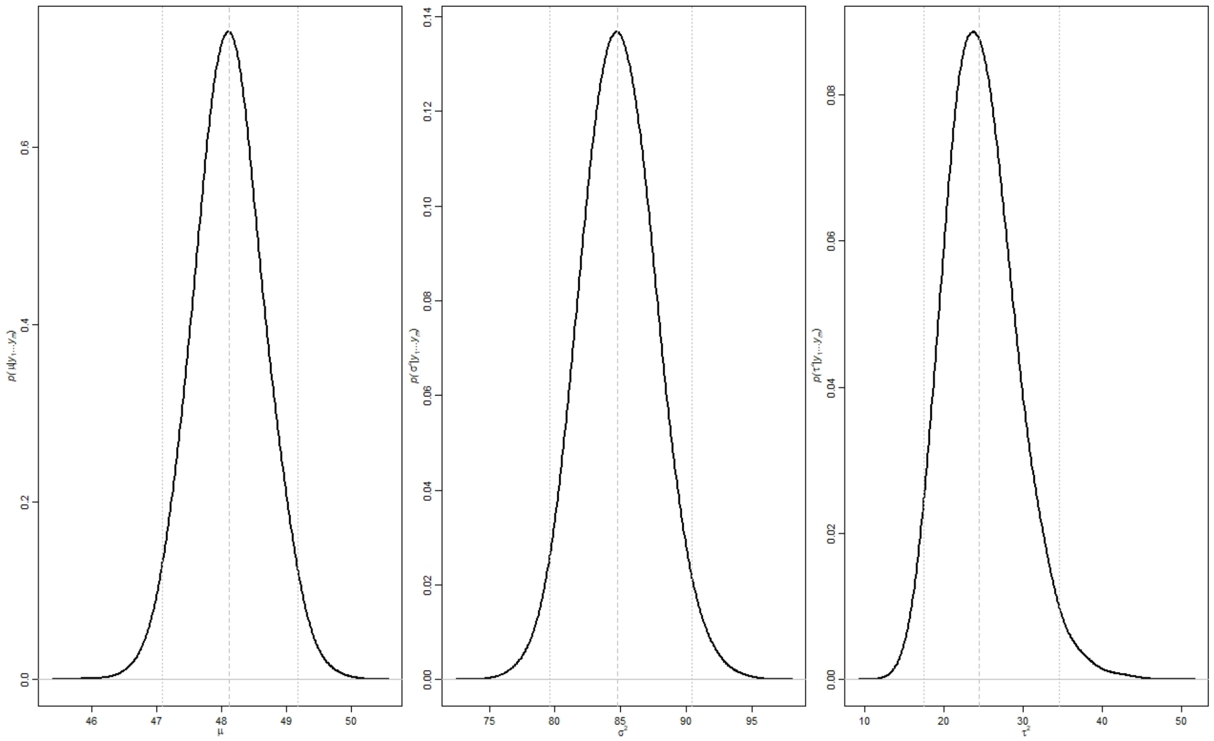

Figure 8.7

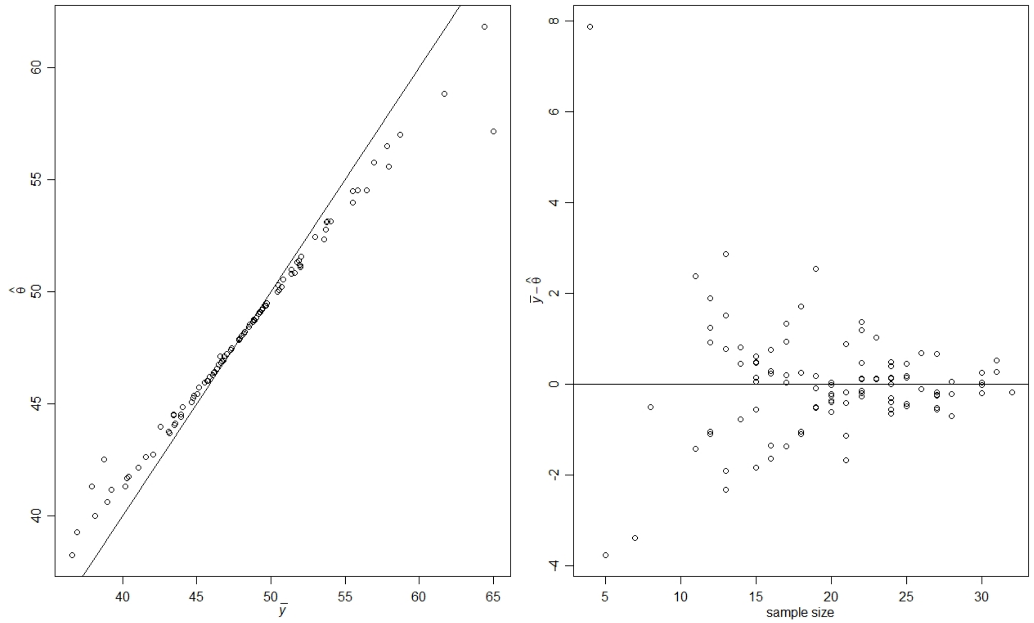

Figure 8.8