1.

i) hyperparameter의 marginal posterior

가정

$$y_j|\theta_j \sim B(n_j, \theta_j) \Rightarrow p(y_j|\theta_j) = \binom{n_j}{y_j} \theta_j^{y_j} (1-\theta_j)^{n_j - y_j}$$

$$\theta_j|\alpha, \beta \sim Beta(\alpha, \beta) \Rightarrow p(\theta_j|\alpha, \beta) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \theta_j^{\alpha-1} (1-\theta_j)^{\beta-1}$$

$$p(\alpha, \beta) \propto (\alpha+\beta)^{-5/2}$$

$$\Rightarrow p(\theta, \alpha, \beta | y) \propto p(y|\theta) p(\theta|\alpha, \beta) p(\alpha, \beta)$$

$$= \prod_{j=1}^{m} \left\{ \binom{n_j}{y_j} \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \theta_j^{\alpha+y_j-1} (1-\theta_j)^{\beta+n_j-y_j-1} \right\} \cdot (\alpha+\beta)^{-5/2}$$

$\therefore$ marginal : $p(\alpha, \beta|y) = \int_\theta p(\theta, \alpha, \beta|y)\, d\theta.$

$$\propto (\alpha+\beta)^{-5/2} \cdot \prod_{j=1}^{m} \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \int_\theta \theta_j^{\alpha+y_j-1} (1-\theta_j)^{\beta+n_j-y_j-1}\, d\theta$$

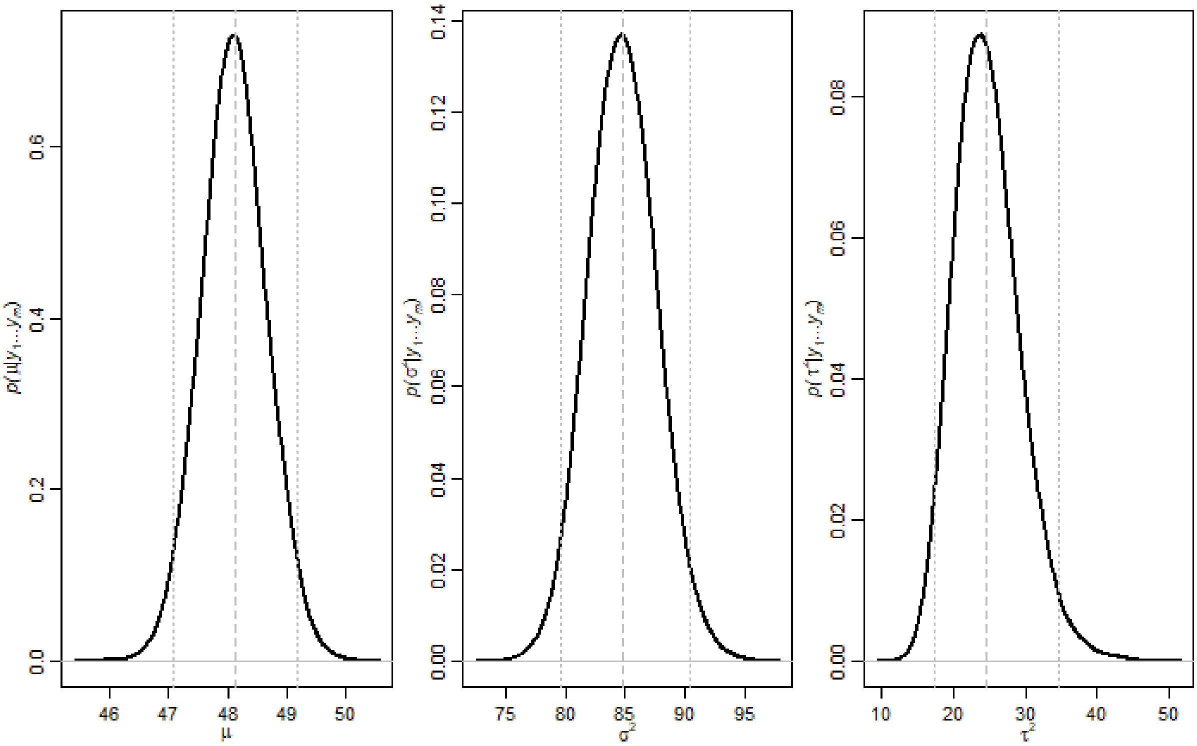
$$= \boxed{(\alpha+\beta)^{-5/2} \cdot \prod_{j=1}^{m} \left\{ \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \cdot \frac{\Gamma(\alpha+y_j)\Gamma(\beta+n_j-y_j)}{\Gamma(\alpha+\beta+n_j)} \right\}}$$

ii)

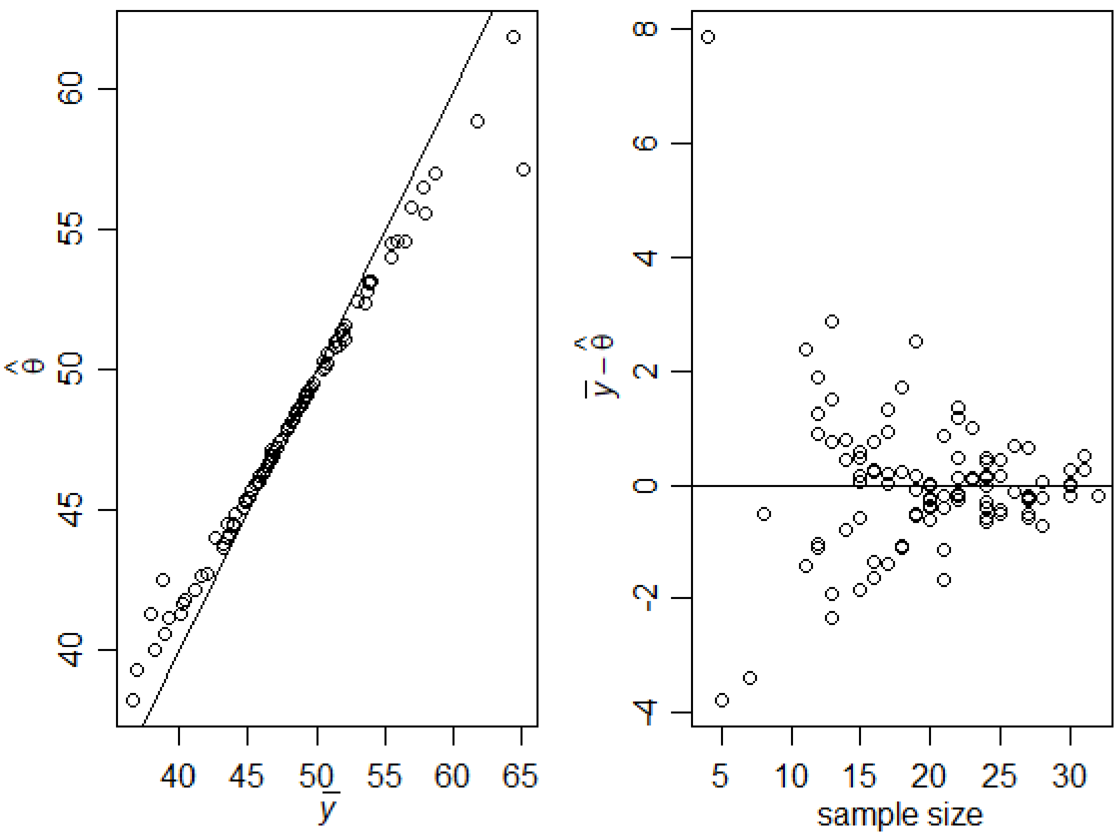
lpfun = function(a, b, y, n) # marginal posterior

(-5/2)* log(a+b) + sum(lgamma(a+b) - lgamma(a) - lgamma(b) + lgamma(a+y) + lgamma(b+n-y) - lgamma(a+b+n))

2.



$p(\mu|y_1 \ldots y_m)$
μ
$p(\sigma^2|y_1 \ldots y_m)$
σ^2
$p(\tau^2|y_1 \ldots y_m)$
τ^2



$\hat{\theta}$
$\bar{y}$
$\bar{y} - \hat{\theta}$
sample size