ESC Spring21 Week4 Exercise7.3

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data

```
bluecrab = as.matrix(read.table
        (url("http://www2.stat.duke.edu/~pdh10/FCBS/Exercises/bluecrab.dat")))
orangecrab = as.matrix(read.table
        (url("http://www2.stat.duke.edu/~pdh10/FCBS/Exercises/orangecrab.dat")))
```

(a) blue crab

```
n = nrow(bluecrab)
ybar = colMeans(bluecrab)
Mu0 = c(ybar)
Sigma = cov(bluecrab)
S0 = Lambda0 = Sigma
nu0 = 4

# Gibbs Sampler
inv=solve
S = 10000
MU = matrix(NA, nrow = S, ncol=2)
SIGMA = matrix(NA, nrow = S, ncol = 4)

for(s in 1:S){
  # update MU
  Lambdan = inv(inv(Lambda0) + n*inv(Sigma))
  Mun = Lambdan %*% (inv(Lambda0) %*% Mu0 + n*inv(Sigma) %*% ybar)
  Mu = MASS::mvrnorm(n=1, Mun, Lambdan)

  # update Sigma
  Sn = S0 + (t(bluecrab) - c(Mu)) %*% t(t(bluecrab) - c(Mu))
  # notation => Sn = S0 = Smu
  # Smu = sum(yi-mu)(y-mu)T
  Sigma = inv(rWishart(1,nu0+n,inv(Sn))[,,1])

  MU[s,] = Mu
  SIGMA[s,] = c(Sigma)
}
```

orange crab

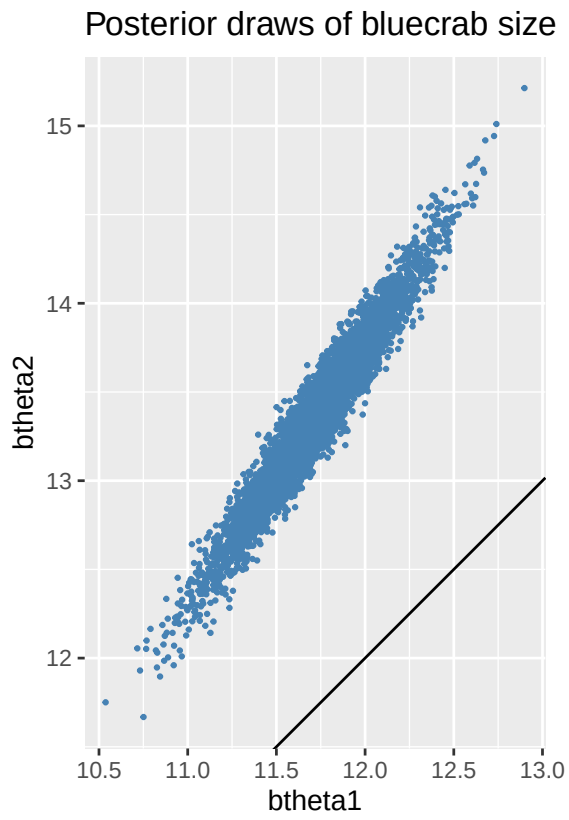
```
n = nrow(orangecrab)
ybar = colMeans(orangecrab)
Mu0 = c(ybar)
Sigma = cov(orangecrab)
```

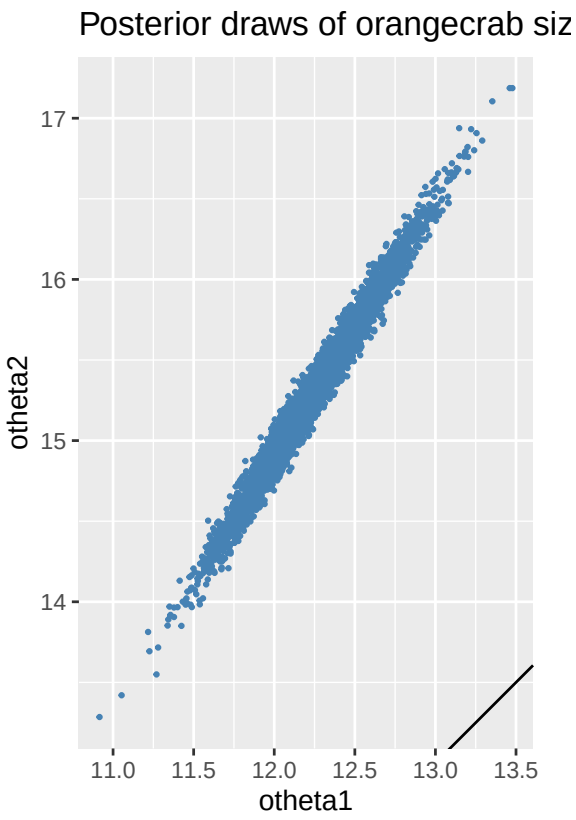
```
S0 = Lambda0 = Sigma
nu0 = 4

# Gibbs Sampler
inv=solve
S = 10000
MU1 = matrix(NA, nrow = S, ncol=2)
SIGMA1 = matrix(NA, nrow = S, ncol = 4)

for(s in 1:S){
  # update MU
  Lambdan = inv(inv(Lambda0) + n*inv(Sigma))
  Mun = Lambdan %*% (inv(Lambda0) %*% Mu0 + n*inv(Sigma) %*% ybar)
  Mu = MASS::mvrnorm(n=1, Mun, Lambdan)

  # update Sigma
  Sn = S0 + (t(orangecrab) - c(Mu)) %*% t(t(orangecrab) - c(Mu))
  # notation => Sn = S0 = Smu
  # Smu = sum(yi-mu)(y-mu)T
  Sigma = inv(rWishart(1,nu0+n,inv(Sn))[,,1])

  MU1[s,] = Mu
  SIGMA1[s,] = c(Sigma)
}
```

(b)

```
##
## Attaching package: 'dplyr'

## The following objects are masked from 'package:stats':
##
##     filter, lag

## The following objects are masked from 'package:base':
##
##     intersect, setdiff, setequal, union
```

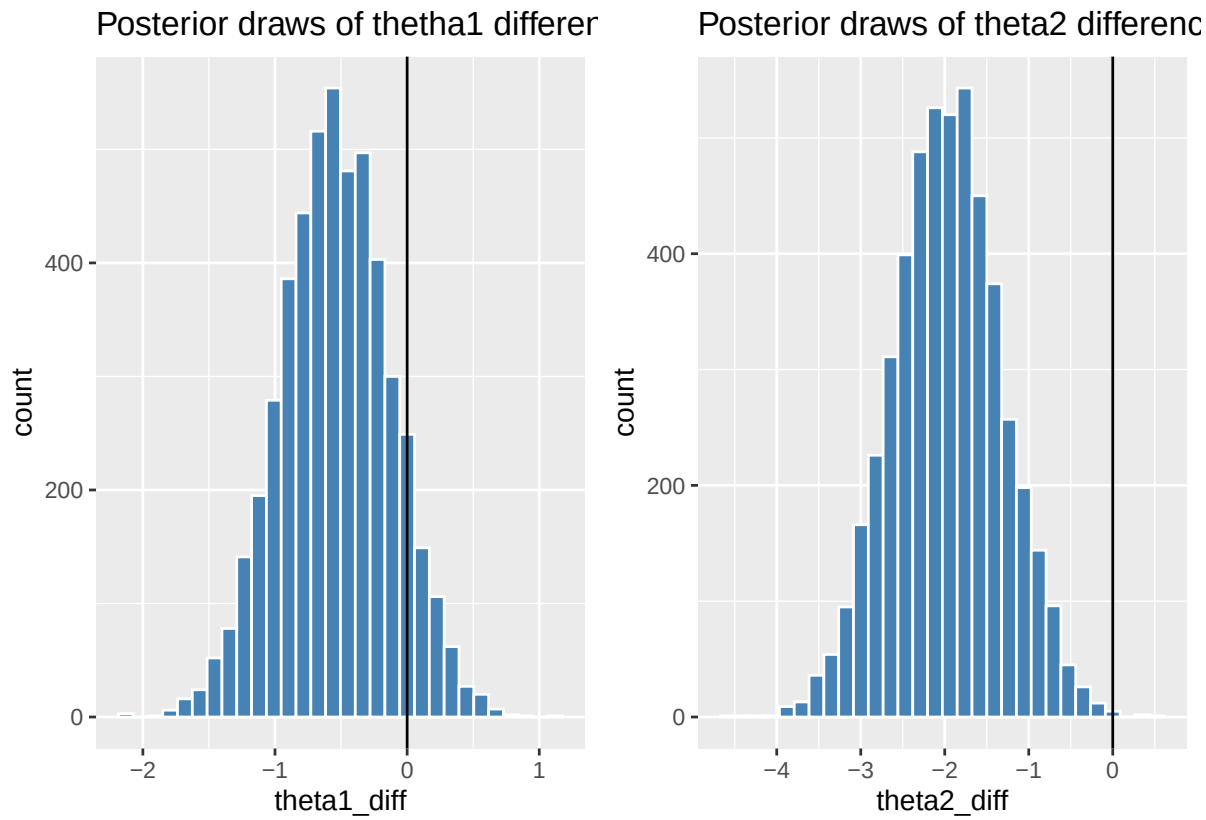
```
disp = tail(1:S,S/2)
title1 = "Posterior draws of bluecrab size"
p1 = data.frame(btheta1 = MU[disp,1], btheta2 = MU[disp,2]) %>%
  ggplot(aes(x=btheta1, y=btheta2)) + geom_point(size=0.5, color="steelblue")+
  geom_abline(slope=1,intercept=0)+coord_fixed(ratio=1)+labs(title=title1)

title2 = "Posterior draws of orangecrab size"
p2 = data.frame(otheta1 = MU1[disp,1], otheta2 = MU1[disp,2]) %>%
  ggplot(aes(x=otheta1, y=otheta2)) + geom_point(size=0.5, color="steelblue")+
  geom_abline(slope=1,intercept=0)+coord_fixed(ratio=1)+labs(title=title2)

ggarrange(p1, p2)
```





Blue crab□ Orange crab□ posterior theta □□□

```
title3 = "Posterior draws of thetha1 difference"
theta1_diff = MU[disp,1] - MU1[disp,1]
p3 = data.frame(theta1_diff = theta1_diff) %>%
  ggplot(aes(x=theta1_diff))+
    geom_histogram(color="white", fill="steelblue", bins=30)+
    geom_vline(xintercept=0)+labs(title=title3)

title4 = "Posterior draws of theta2 difference"
theta2_diff = MU[disp,2] - MU1[disp,2]
p4 = data.frame(theta2_diff = theta2_diff) %>%
  ggplot(aes(x=theta2_diff))+
    geom_histogram(color="white", fill="steelblue", bins=30)+
    geom_vline(xintercept=0)+labs(title=title4)

ggarrange(p3,p4)
```




```
mean(MU[disp,1] > MU1[disp,1])
```

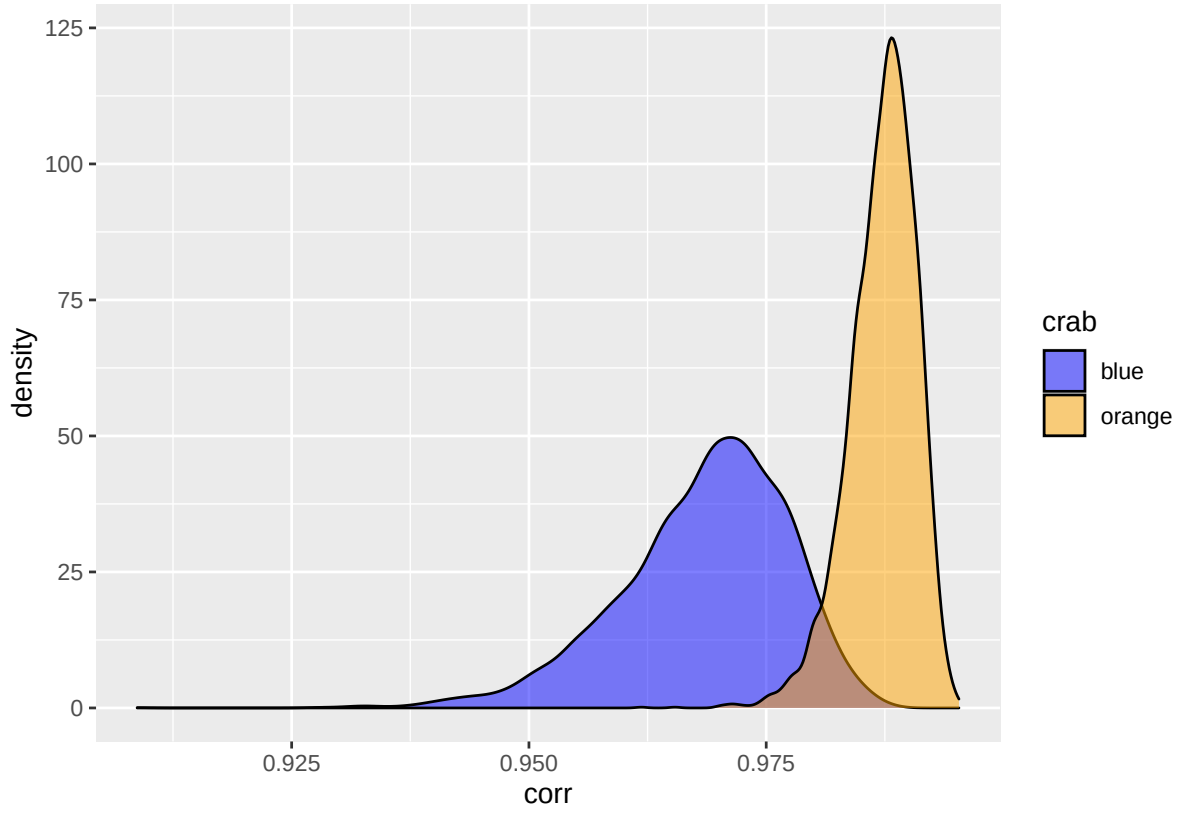
```
## [1] 0.1
```

```
mean(MU[disp,2] > MU1[disp,2])
```

```
## [1] 0.001
```

(c)

```
bcorr = apply(SIGMA, MARGIN = 1, FUN = function(SIGMA){
  SIGMA[2] / sqrt(SIGMA[1]*SIGMA[4])
})

ocorr = apply(SIGMA1, MARGIN = 1, FUN = function(SIGMA){
  SIGMA[2] / sqrt(SIGMA[1]*SIGMA[4])
})

p5 = data.frame(crab = c(rep('blue', length(bcorr)/2), rep('orange', length(ocorr)/2)),
                corr = c(bcorr[disp], ocorr[disp])) %>%
  ggplot(aes(x = corr, fill = crab))+
  geom_density(alpha = 0.5)+
  scale_fill_manual(values = c('blue','orange'))

ggarrange(p5)
```




```
mean(bcorr<ocorr)
```

```
## [1] 0.9902
```