i. Derive the marginal posterior distribution for (alpha, beta).

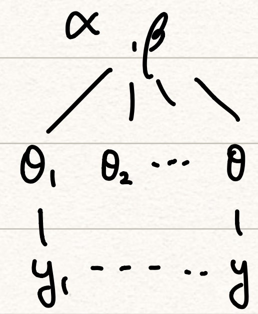
$$
\begin{aligned}
p(\theta, \alpha, \beta | y) &\propto p(y|\theta, \alpha, \beta) p(\theta, \alpha, \beta) \\
&= p(y|\theta) p(\theta, \alpha, \beta) \\
&= p(y|\theta) p(\theta|\alpha, \beta) p(\alpha, \beta)
\end{aligned}
$$

$$
p(\alpha, \beta | y) = \int_{\Theta} p(\theta, \alpha, \beta | y) \, d\theta
$$

$p(\alpha,\beta) = (\alpha+\beta)^{-\frac{5}{2}}$

$\theta_j | \alpha,\beta \sim Beta(\alpha,\beta)$

$y_j | \theta_j \sim Binom(n_j, \theta_j)$



$$
p(\theta, \alpha, \beta | y) = (\alpha+\beta)^{-\frac{5}{2}} \cdot \prod_{j=1}^{m} \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \theta_j^{\alpha-1}(1-\theta_j)^{\beta-1} \cdot \prod_{j=1}^{m} \binom{n_j}{y_j} \theta_j^{y_j}(1-\theta_j)^{n_j-y_j}
$$

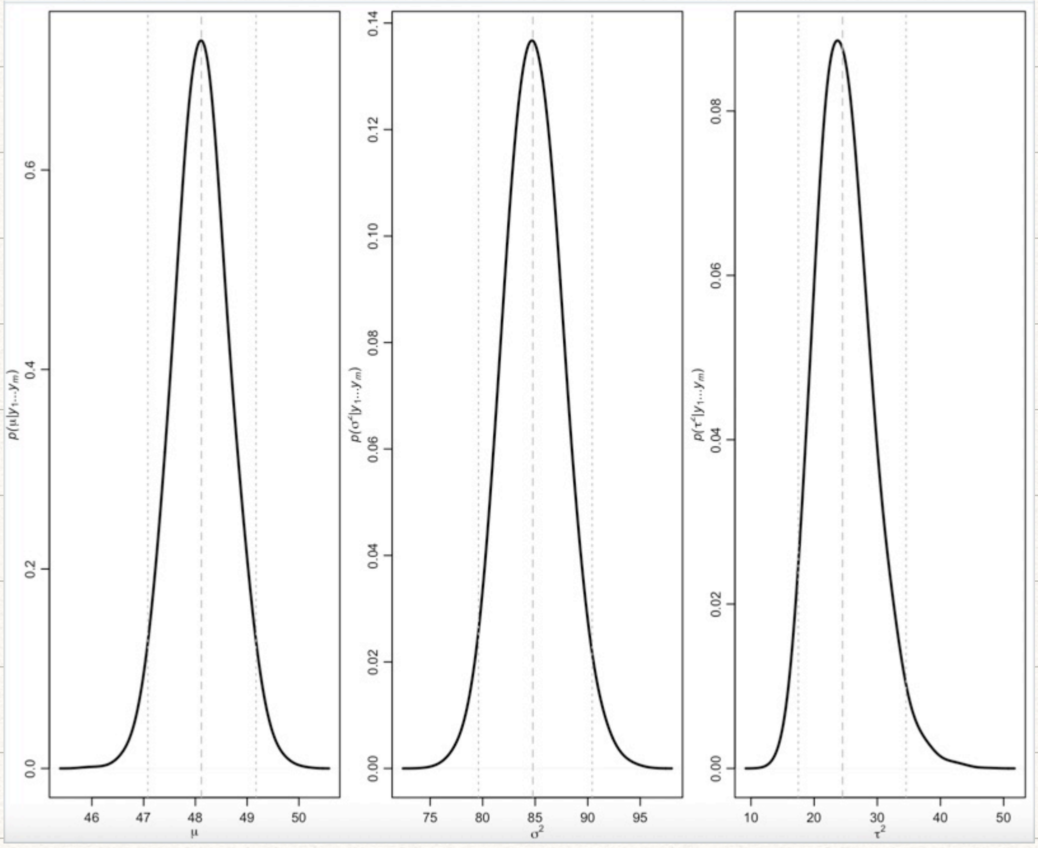
$$
\propto (\alpha+\beta)^{-\frac{5}{2}} \prod_{j=1}^{m} \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \theta_j^{\alpha-1}(1-\theta_j)^{\beta-1} \prod_{j=1}^{m} \theta_j^{y_j}(1-\theta_j)^{n_j-y_j}
$$

$$
p(\alpha,\beta | y_j) \propto (\alpha+\beta)^{-\frac{5}{2}} \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \int \theta_j^{\alpha+y_j-1}(1-\theta_j)^{\beta+n_j-y_j-1} \, d\theta
$$

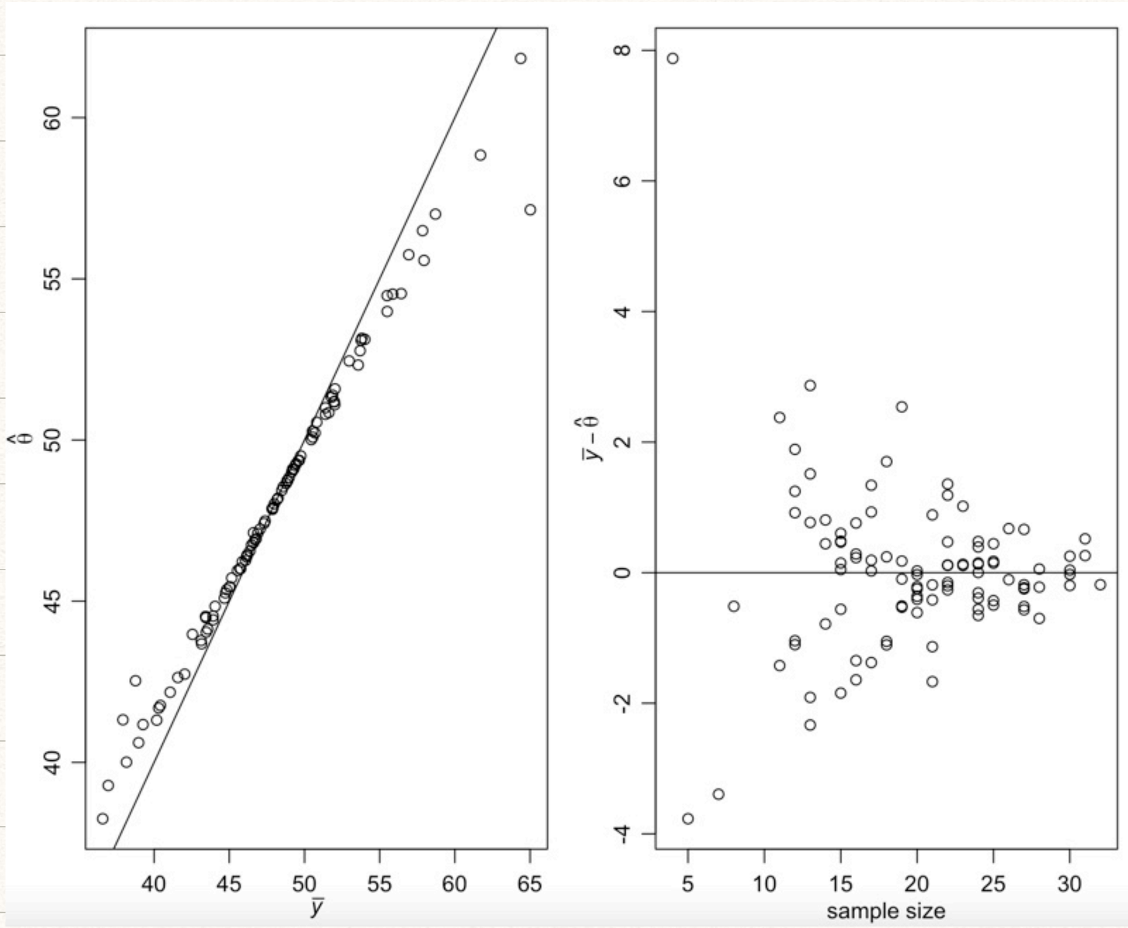
$$
= (\alpha+\beta)^{-\frac{5}{2}} \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \frac{\Gamma(\alpha+y_j)\Gamma(\beta+n_j-y_j)}{\Gamma(\alpha+\beta+n_j)}
$$

ii) line 45

(-5/2) * log(a+b) + sum(lgamma(a+b) - lgamma(a) ---



$p(\mu|y_1 \ldots y_m)$
μ
$p(\sigma^2|y_1 \ldots y_m)$
σ^2
$p(\tau^2|y_1 \ldots y_m)$
τ^2



$\hat{\theta}$
$\bar{y}$
$\bar{y} - \hat{\theta}$
sample size