[1번 문제] ISL 3.5

5. Consider the fitted values that result from performing linear regression without an intercept. In this setting, the ith fitted value takes the form

$$\hat{y}_i = x_i \hat{\beta},$$

where

$$\hat{\beta} = \left(\sum_{i=1}^{n} x_i y_i\right) \Big/ \left(\sum_{i'=1}^{n} x_{i'}^2\right). \tag{3.38}$$

Show that we can write

$$\hat{y}_i = \sum_{i'=1}^{n} a_{i'} y_{i'}.$$

What is $a_{i'}$?

$$\hat{y}_i = x_i \hat{\beta}$$

$$= x_i \times \frac{\sum_{i=1}^{n} x_i y_i}{\sum_{i=1}^{n} x_{i'}^2} = x_i \times \frac{\sum_{i'=1}^{n} x_{i'} \cdot y_{i'}}{\sum_{j=1}^{n} x_j^2}$$

$$= \sum_{i'=1}^{n} \left(\frac{x_i \times x_{i'}}{\sum_{j=1}^{n} x_j^2} \times y_{i'} \right)$$

$$\boxed{a_{i'} = \frac{x_i x_{i'}}{\sum_{j=1}^{n} x_j^2}}$$

✏️ [2번 문제] ESL 3.4

Ex. 3.4 Show how the vector of least squares coefficients can be obtained from a single pass of the Gram–Schmidt procedure (Algorithm 3.1). Represent your solution in terms of the QR decomposition of **X**.

$$X = QR$$

$$\begin{aligned}
\hat{\beta} &= (X^T X)^{-1} X^T y \\
&= ((QR)^T QR)^{-1} (QR)^T y \\
&= (R^T \underbrace{\cancel{Q^T}\cancel{Q}}_{\boxed{I}} R)^{-1} R^T Q^T y \\
&= R^{-1} (R^T)^{-1} R^T Q^T y \\
&= R^{-1} Q^T y
\end{aligned}$$

$$R\hat{\beta} = Q^T y$$

$$\hookrightarrow \begin{bmatrix} (x_1 \cdot q_1) & (x_2 \cdot q_1) & \cdots & (x_p \cdot q_1) \\ 0 & (x_2 \cdot q_2) & \cdots & (x_p \cdot q_2) \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & (x_p \cdot q_p) \end{bmatrix} \begin{bmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \\ \vdots \\ \hat{\beta}_p \end{bmatrix}$$