

# 1. Forward Stepwise Selection

RStudio

File Edit Code View Plots Session Build Debug Profile Tools Help

```
1 # mlbench 패키지의 BostonHousing 데이터에 대하여 Forward stepwise selection을 해봅시다.
2
3
4 install.packages("mlbench")
5 library(mlbench)
6
7 data("BostonHousing")
8 head(BostonHousing)
9 dim(BostonHousing)
10 sum(is.na(BostonHousing))
11
12
13 install.packages("leaps")
14 library(leaps)
15
16
17 regfit.fwd = regsubsets(medv~., data = BostonHousing, nvmax=13, method="forward")
18 summary(regfit.fwd)
19 |
20
21
22 coef(regfit.fwd, 3)
23 coef(regfit.fwd, 4)
24 summary(regfit.fwd)$adjr2
25
26
27 # 1번. 예측변수가 4개인 모델에서 새로 선택된 변수 이름은? dis
28 # 그 변수의 추정된 계수 값과 해당 모델의 결정계수 R^2는 무엇인가요? -0.5519263, 0.6878351
29
```

```
Console Terminal R Markdown Jobs
R 4.1.0 - C:/Users/User/Desktop/
11 ( 1 ) "x" "x" "x" "x" "x" "x" "x" "x" "x" "x" "x" "x" "x" "x"
12 ( 1 ) "x" "x" "x" "x" "x" "x" "x" "x" "x" "x" "x" "x" "x" "x"
13 ( 1 ) "x" "x" "x" "x" "x" "x" "x" "x" "x" "x" "x" "x" "x" "x"
>
>
>
> coef(regfit.fwd, 3)
(Intercept)      rm      ptratio      lstat
18.5671115    4.5154209   -0.9307226   -0.5718057
> coef(regfit.fwd, 4)
(Intercept)      rm      dis      ptratio      lstat
24.4713576    4.2237922   -0.5519263   -0.9736458   -0.6654360
> summary(regfit.fwd)$adjr2
[1] 0.5432418 0.6371245 0.6767036 0.6878351 0.7051702 0.7123567 0.7182560 0.7222072 0.7239046 0.7288066
[11] 0.7348058 0.7343282 0.7337897
>
>
> # 1번. 예측변수가 4개인 모델에서 새로 선택된 변수 이름은? dis
> # 그 변수의 추정된 계수 값과 해당 모델의 결정계수 R^2는 무엇인가요? -0.5519263, 0.6878351
```

## 2. ESL 7.4

$$OP = Err_{in} - \bar{err}$$

$$W = E_y(OP) = \frac{2}{N} \sum_{i=1}^N Cov(\hat{y}_i, y_i)$$

$$\textcircled{1} Err_{in} = \frac{1}{N} \sum_{i=1}^N E_{Y_0} (Y_i^0 - \hat{f}(x_i))^2$$

$$Y_i^0 - \hat{f}(x_i) = Y_i^0 - f(x_i) + f(x_i) - E\hat{f}(x_i) + E\hat{f}(x_i) - \hat{f}(x_i)$$

$$\begin{aligned} (a+b+c)^2 \\ = a^2 + b^2 + c^2 \\ + 2ab + 2bc + 2ca \end{aligned}$$

$$\begin{aligned} Err_{in} &= \frac{1}{N} \sum_{i=1}^N E_{Y_0} (Y_i^0 - f(x_i) + f(x_i) - E\hat{f}(x_i) + E\hat{f}(x_i) - \hat{f}(x_i))^2 \\ &= \frac{1}{N} \sum_{i=1}^N \left[ E_{Y_0} (Y_i^0 - f(x_i))^2 + \underbrace{E_{Y_0} (f(x_i) - E\hat{f}(x_i))^2}_{\rightarrow (f(x_i) - E\hat{f}(x_i))^2} + \underbrace{E_{Y_0} (E\hat{f}(x_i) - \hat{f}(x_i))^2}_{\rightarrow (E\hat{f}(x_i) - \hat{f}(x_i))^2} \right. \\ &\quad + 2E_{Y_0} (Y_i^0 - f(x_i))(f(x_i) - E\hat{f}(x_i)) + 2E_{Y_0} (Y_i^0 - f(x_i))(E\hat{f}(x_i) - \hat{f}(x_i)) \\ &\quad \left. + 2E_{Y_0} (f(x_i) - E\hat{f}(x_i))(E\hat{f}(x_i) - \hat{f}(x_i)) \right] \end{aligned}$$

$$\textcircled{2} \bar{err} = \frac{1}{N} \sum_{i=1}^N (y_i - \hat{f}(x_i))^2 \rightarrow 2(f(x_i) - E\hat{f}(x_i))(E\hat{f}(x_i) - \hat{f}(x_i))$$

$$y_i - \hat{f}(x_i) = y_i - f(x_i) + f(x_i) - E\hat{f}(x_i) + E\hat{f}(x_i) - \hat{f}(x_i)$$

$$\begin{aligned} \bar{err} &= \frac{1}{N} \sum_{i=1}^N (y_i - f(x_i) + f(x_i) - E\hat{f}(x_i) + E\hat{f}(x_i) - \hat{f}(x_i))^2 \\ &= \frac{1}{N} \sum_{i=1}^N \left[ (y_i - f(x_i))^2 + (f(x_i) - E\hat{f}(x_i))^2 + (E\hat{f}(x_i) - \hat{f}(x_i))^2 \right. \\ &\quad \left. + 2(y_i - f(x_i))(f(x_i) - E\hat{f}(x_i)) + 2(y_i - f(x_i))(E\hat{f}(x_i) - \hat{f}(x_i)) \right. \\ &\quad \left. + 2(f(x_i) - E\hat{f}(x_i))(E\hat{f}(x_i) - \hat{f}(x_i)) \right] \end{aligned}$$

$$E_y(OP) = E_y(Err_{in} - \bar{err})$$

$$\begin{aligned} &= \frac{1}{N} \sum_{i=1}^N E_y \left[ \cancel{E_{Y_0} (Y_i^0 - f(x_i))^2} - \cancel{(y_i - f(x_i))^2} \right. \\ &\quad + \cancel{2E_{Y_0} (Y_i^0 - f(x_i))(f(x_i) - E\hat{f}(x_i))} - \cancel{2(y_i - f(x_i))(f(x_i) - E\hat{f}(x_i))} \\ &\quad \left. + \cancel{2E_{Y_0} (Y_i^0 - f(x_i))(E\hat{f}(x_i) - \hat{f}(x_i))} - \cancel{2(y_i - f(x_i))(E\hat{f}(x_i) - \hat{f}(x_i))} \right] \end{aligned}$$

$$\begin{aligned} E_y(OP) &= \frac{1}{N} \sum E(-2(y_i - f(x_i))(E\hat{f}(x_i) - \hat{f}(x_i))) = \frac{2}{N} \sum E((y_i - f(x_i))(f(x_i) - E\hat{f}(x_i))) \\ &= \frac{2}{N} \sum Cov(y_i, \hat{y}_i) \end{aligned}$$

$$\begin{aligned} \textcircled{1} E_y [E_{Y_0} (Y_i^0 - f(x_i))^2] &= E_y (y_i - f(x_i))^2 & f(x_i) &= E(Y_i^0) & Y &= f(X) + \epsilon \\ &= E_{Y_0} (Y_i^0 - f(x_i))^2 - E_y (y_i - f(x_i))^2 = 0 & \hat{f}(x_i) &= E(\hat{y}_i) \end{aligned}$$

$$\textcircled{2} E_y [(y_i - f(x_i))(f(x_i) - E\hat{f}(x_i))] = (E(y_i) - f(x_i))(f(x_i) - E\hat{f}(x_i)) = 0$$

$$\textcircled{3} E_y [E_{Y_0} (Y_i^0 - f(x_i))(f(x_i) - E\hat{f}(x_i))] = (E(Y_i^0) - f(x_i))(f(x_i) - E\hat{f}(x_i)) = 0$$

$$\textcircled{4} 2E_y [E_{Y_0} ((Y_i^0 - f(x_i))(E\hat{f}(x_i) - \hat{f}(x_i)))] = -2E_y [(Y_i^0 - E(Y_i^0))(\hat{y}_i - E\hat{y}_i)] = 0 \quad Y_i^0, \hat{y}_i \text{ independent} \rightarrow Cov = 0$$