prob1

prove PCA

$$\max_{f} Var(f^T X) = f^T \cdot Var(X) \cdot f \quad s.t\ f^T f = 1$$

$$= f^T \cdot \Sigma f$$

라그랑주 승수법

$$\Rightarrow L = f^T \Sigma f - \lambda(f^T f - 1)$$

where $\frac{\partial}{\partial f} L = 0$

$$\Rightarrow \frac{\partial}{\partial f} L = 2\Sigma f - 2\lambda f$$

$$\Rightarrow \Sigma f = \lambda f$$

$$\therefore \max_{f} Var(f^T X) = f^T \Sigma f$$

$$= f^T \lambda f$$

$$= \lambda \cdot f^T f$$

$$= \lambda$$