1. $\hat{y}_i = x_i \hat{\beta}$ where $\hat{\beta} = \left(\sum_{i=1}^{n} x_i y_i\right) / \left(\sum_{i=1}^{n} x_i^2\right)$

Sol) $\hat{y}_{i'} = x_{i'} \left(\sum_{i=1}^{n} x_i y_i\right) / \left(\sum_{i=1}^{n} x_i^2\right)$

$$= x_{i'} \cdot \sum_{i'=1}^{n} \left\{ \frac{x_{i'}}{\sum_{i=1}^{n} x_i^2} y_{i'} \right\}$$

$$= \sum_{i'=1}^{n} \frac{x_i x_{i'}}{\sum_{i=1}^{n} x_i^2} y_{i'}$$

$$= a_{i'}$$

2. In algorithm 3.1

(i) $z_0 = x_0 = 1$

(ii) $j = 1, 2, \cdots, p$ 에 대하여 regress x_j on $z_0 \cdots z_{j-1}$

$\Rightarrow \hat{\gamma}_{l,j} = \langle z_l, x_j \rangle / \langle z_l, z_l \rangle \quad l = 0, \cdots, j-1$

$z_j = x_j - \sum_{k=0}^{j-1} \hat{\gamma}_{k,j} z_k$

(iii) $\hat{\beta}_p = \dfrac{\langle z_p, y \rangle}{\langle z_p, z_p \rangle}$

$x_j = z_j + \sum_{k=0}^{j-1} \hat{\gamma}_{k,j} z_k$

$$X = [z_0 \ \cdots \ z_p] \begin{bmatrix} 1 & \hat{\gamma}_{0,1} & \cdots & \hat{\gamma}_{0,p} \\ 0 & 1 & & \vdots \\ \vdots & \vdots & & \hat{\gamma}_{p-1,p} \\ 0 & 0 & \cdots & 1 \end{bmatrix}$$

$= Z\Gamma$ upper triangular

D : diagonal matrix with $D_{jj} = \|z_j\|$ 라 하면

$$X = Z D^{-1} \ D\Gamma$$

$$= Q \quad R$$

$N \times (p+1)$	$(p+1) \times (p+1)$
$Q^T Q = I$: orthogonal	upper triangular

$$\hat{\beta} = (X^T X)^{-1} X^T y = (R^T \cancel{Q^T Q} R)^{-1} R^T Q^T y = R^{-1} \cancel{(R^T)^{-1} R^T} Q^T y$$

$$\hat{y} = X\hat{\beta} = Q \cancel{R R^{-1}} Q^T y = Q Q^T y$$

ESC21SUMMER_HW1_woohyunchoi

July 21, 2021

[1]:
```
# Data Import
import ssl
import pandas as pd
ssl._create_default_https_context = ssl._create_unverified_context #Github
 ↪           .                    !
data = pd.read_csv('https://github.com/YonseiESC/ESC-21SUMMER/blob/main/week1/
 ↪HW/week1_data.csv?raw=True')
y = data['mpg']
x = data.drop(['mpg'],axis=1)

import numpy as np
```

[2]:
```
x
```

[2]:

	cylinders	displacement	horsepower	weight	acceleration	year
0	8	307.0	130	3504	12.0	70
1	8	350.0	165	3693	11.5	70
2	8	318.0	150	3436	11.0	70
3	8	304.0	150	3433	12.0	70
4	8	302.0	140	3449	10.5	70
..	…	…	…	…	…	…
392	4	140.0	86	2790	15.6	82
393	4	97.0	52	2130	24.6	82
394	4	135.0	84	2295	11.6	82
395	4	120.0	79	2625	18.6	82
396	4	119.0	82	2720	19.4	82

[397 rows x 6 columns]

[3]:
```
x.info()
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 397 entries, 0 to 396
Data columns (total 6 columns):
 #   Column        Non-Null Count  Dtype
---  ------        --------------  -----
 0   cylinders     397 non-null    int64
 1   displacement  397 non-null    float64
```

```
 2   horsepower    397 non-null    object
 3   weight        397 non-null    int64
 4   acceleration  397 non-null    float64
 5   year          397 non-null    int64
dtypes: float64(2), int64(3), object(1)
memory usage: 18.7+ KB
```

horsepower why object...?

[4]:
```
x.horsepower[32]
```

[4]:
```
'?'
```

[5]:
```
sum(x.horsepower == '?')
```

[5]:
```
5
```

[15]:
```
idx = x.horsepower[x.horsepower == '?'].index
x = x.drop(idx)
y = y.drop(idx)
x.horsepower = pd.to_numeric(x.horsepower)
# delete the rows which have '?' and convert horsepower to numeric
```

[7]:
```
#data = data.apply(pd.to_numeric, errors = 'coerce').dropna()
```

[8]:
```
x.info()
```

```
<class 'pandas.core.frame.DataFrame'>
Int64Index: 392 entries, 0 to 396
Data columns (total 6 columns):
 #   Column        Non-Null Count  Dtype
---  ------        --------------  -----
 0   cylinders     392 non-null    int64
 1   displacement  392 non-null    float64
 2   horsepower    392 non-null    int64
 3   weight        392 non-null    int64
 4   acceleration  392 non-null    float64
 5   year          392 non-null    int64
dtypes: float64(2), int64(4)
memory usage: 21.4 KB
```

[9]:
```
X = np.array(x)
Y = np.array(y)
```

[10]:
```
# numpy       .
def YourOwnRegression(x,y):
    # beta  y              .
    beta = np.linalg.inv(x.T@x)@x.T@y
```

```
    yhat = x@beta
    return(beta, yhat)
```

[11]:
```
#
YourOwnRegression(X,Y)
```

[11]:
```
(array([-0.5226089 ,  0.01022108, -0.020873  , -0.00639456, -0.05202195,
         0.61025869]),
 array([15.93081361, 14.45720405, 16.11263762, 15.93670419, 16.10071197,
        10.51021782, 10.27543153, 10.53128391,  9.67525181, 13.49634208,
        15.59946068, 15.17857818, 14.95056695, 18.23756934, 23.85149163,
        20.70116218, 21.04691637, 22.47738545, 25.4075601 , 27.85849004,
        21.93938914, 23.54965655, 23.61026333, 24.5700506 , 22.02475307,
         7.48992449,  8.73758029,  8.68094775,  6.39448743, 26.01781879,
        25.50668622, 25.43458909, 22.95714525, 17.50355598, 18.56684981,
        18.98997897, 18.64504732, 11.74185816, 10.43958016, 12.2762232 ,
        12.39844199,  7.02172716,  8.71466792,  6.09084578, 20.89073634,
        24.7795066 , 18.89340516, 20.0843144 , 25.76559234, 26.24104628,
        26.30753902, 26.59195154, 28.28090927, 29.28279009, 28.2609542 ,
        27.13912286, 25.6470868 , 26.69569542, 26.07663641, 24.9880403 ,
        26.2074288 , 11.93647037, 11.52899822, 12.73330182, 13.0723569 ,
        15.65493241,  9.60277136, 10.60920749, 10.79899231, 10.95329347,
        25.45996983, 14.19610698, 13.24891771, 11.63170439, 13.0781754 ,
        21.23759877, 24.50545484, 21.19494316, 26.4550875 , 25.15275892,
        25.40418994, 24.27095036, 26.56185652, 26.71587043, 13.39863077,
        16.2651911 , 14.74101165, 13.99404444, 15.68483539,  8.35588404,
        12.1559116 , 12.08192325, 12.63929331,  9.52710571,  8.09046672,
        15.38896808, 20.7055219 , 19.98343495, 22.03288824, 21.95534612,
        22.05191343, 28.9278806 ,  8.64356655,  8.94866353, 10.06261966,
        10.76884679, 23.08246175, 26.05144779, 26.01877494, 25.52924044,
        27.53361915, 26.19737345, 24.2284252 , 26.29152354, 14.1399229 ,
        11.80551478, 29.17763168, 27.47281611, 24.48514487, 22.211482  ,
        18.18001246, 23.66132966, 21.80938422, 16.18633836, 21.37003802,
        22.92450763, 20.27483103, 29.01987066, 26.1144079 , 29.60478409,
        25.79483636, 17.42925127, 18.20305761, 18.16965645, 13.93463354,
        10.62233937, 11.89477047, 10.6524927 , 12.92654367, 27.29002111,
        29.11881818, 26.97024946, 31.35712023, 28.84708125, 28.08036911,
        28.1370547 , 27.59838633, 25.57416431, 26.13565935, 28.85620844,
        21.24635801, 20.04621563, 20.64756809, 22.47771079, 11.68048944,
        13.0191654 , 12.19083021, 11.60515866, 16.62277687, 17.11534294,
        18.13462735, 17.75872575, 22.48538772, 20.67124626, 21.09405268,
        28.38999264, 25.6178386 , 23.45163703, 25.94471506, 25.06438254,
        28.04340891, 25.7120626 , 22.52118358, 30.02318129, 21.7091604 ,
        24.86417853, 23.27558145, 23.28386212, 24.73315284, 31.11419428,
        27.02526368, 28.70906066, 26.57792197, 28.30139996, 28.71233562,
        14.76463858, 14.84864669, 16.74288177, 14.99473679, 21.98169626,
        21.41233074, 23.42150596, 23.09392064, 29.90480163, 29.05099549,
```

```
30.6230356 , 31.72965517, 19.18911918, 20.22456108, 19.35231654,
22.55428679, 31.49394493, 30.08735698, 29.08175554, 27.10777245,
22.52930717, 16.43688261, 21.62943493, 23.04180738, 17.16039851,
13.38138443, 16.19537407, 17.04478509, 17.57825889, 30.44250197,
29.8139795 , 31.85863268, 28.62881666, 30.99577532, 17.43872127,
16.22016388, 15.84658308, 14.94255744, 20.751491  , 21.24312777,
19.93366366, 20.83473714, 15.57784903, 15.61161917, 14.64046595,
14.81182982, 31.10306759, 26.25259415, 28.89497299, 26.0337752 ,
30.58653253, 30.07282718, 30.94607365, 29.52523606, 24.56685474,
26.54846444, 25.84824622, 31.61605985, 32.87446415, 31.51916094,
30.71307435, 32.82411068, 21.45671946, 19.17701507, 20.11026212,
21.03033425, 23.4041659 , 24.95271522, 26.90952532, 21.84884557,
23.53592664, 22.19836586, 24.23446653, 20.34711601, 22.00581861,
20.92208997, 20.65519199, 22.5275121 , 16.94488046, 30.45340282,
27.78764395, 29.22915666, 30.00329119, 28.04418604, 26.49072908,
26.10686219, 28.54732333, 25.25241894, 22.73501516, 25.65655504,
20.72705133, 31.43713179, 30.57609185, 23.48396909, 25.27825721,
26.3338669 , 23.74246252, 22.72920671, 19.07725247, 19.90688066,
18.75951203, 19.18011155, 15.71621531, 17.98573432, 20.31674699,
18.70734857, 32.50985671, 32.22227707, 32.45712629, 27.83310428,
22.24218693, 19.15384865, 24.39039603, 21.78419026, 30.97738025,
31.20867101, 31.71621295, 31.22474446, 27.87898482, 27.26090106,
26.50623701, 28.75362625, 31.67091587, 32.82504694, 31.90800579,
32.48874873, 28.41207605, 27.03039926, 26.14656464, 23.51339355,
31.28036395, 28.07931906, 29.22595587, 29.68156125, 30.99424682,
31.82881074, 27.47919606, 31.75096112, 32.18671672, 30.48403252,
26.14671839, 24.9076443 , 33.71404606, 31.67048481, 33.75190878,
25.44572433, 29.75593179, 29.36847327, 30.7931412 , 30.22783049,
29.42774108, 29.5118248 , 27.68686195, 31.04492087, 34.83572405,
34.04069724, 34.82406496, 32.80271279, 33.2140812 , 32.94723852,
33.3257177 , 32.15965292, 33.06580622, 30.68952944, 32.1263293 ,
31.98760285, 30.97773269, 28.97782474, 29.22077581, 25.39617051,
24.96456589, 26.39147031, 25.8288513 , 23.70096728, 21.82763916,
26.04576044, 23.80012019, 29.58125213, 29.40946443, 31.00734511,
30.01258259, 30.59867295, 29.19007052, 28.28783476, 34.02222172,
33.56574566, 33.94865969, 33.35582651, 33.00291086, 32.77391248,
32.62463215, 32.35858312, 34.13676256, 34.1003472 , 33.88249939,
26.77440244, 27.64530275, 30.34069094, 28.04577073, 29.6541942 ,
31.74589579, 27.85165521, 28.93428241, 32.95667079, 32.29831861,
29.77500853, 29.05306756]))
```

[12]:
```
y_result = YourOwnRegression(X,Y)[1]
```

[13]:
```
import matplotlib.pyplot as plt
```

[14]:
```
plt.plot(y)
plt.plot(y_result, 'r')
```

```
plt.show()
```