✏️ [1번 문제] ISL 3.5

5. Consider the fitted values that result from performing linear regression without an intercept. In this setting, the ith fitted value takes the form

$$\hat{y}_i = x_i \hat{\beta},$$

where

$$\hat{\beta} = \left(\sum_{i=1}^{n} x_i y_i \right) / \left(\sum_{i'=1}^{n} x_{i'}^2 \right). \tag{3.38}$$

Show that we can write

$$\hat{y}_i = \sum_{i'=1}^{n} a_{i'} y_{i'}.$$

What is $a_{i'}$?

$$\hat{\beta} = \frac{\Sigma x_i y_i}{\Sigma x_i} \qquad \therefore a_{i'} = \frac{x_i}{\Sigma x_i^2}$$

$$\hat{y} = \Sigma k_i y_i$$

원래 $k_i = \dfrac{x_i - \bar{x}}{\Sigma (x_i - \bar{x})^2}$

centering ⇒ (no intercept) $k_i' = \dfrac{x_i}{\Sigma x_i^2}$

Ex. 3.4 Show how the vector of least squares coefficients can be obtained from a single pass of the Gram–Schmidt procedure (Algorithm 3.1). Represent your solution in terms of the QR decomposition of $\mathbf{X}$.

$$X = Z\Gamma \qquad Q = ZD^{-1} \qquad R = D\Gamma$$

$$X = QR \quad \hat{\beta} = (X'X)^{-1} \cdot X'y = (R'Q'QR)^{-1} \cdot R'Q' \cdot y = R^{-1} \cdot (R')^{-1} \cdot R'Q'y = R^{-1} \cdot Q' \cdot y$$

$$\Rightarrow R\hat{\beta} = Q'y = \underbrace{\begin{bmatrix} (x_1 \cdot q_1) & \cdots & (x_p \cdot q_1) \\ & \ddots & \\ 0 & & (x_p \cdot q_p) \end{bmatrix}}_{R} \underbrace{\begin{bmatrix} \hat{\beta}_1 \\ \vdots \\ \hat{\beta}_p \end{bmatrix}}_{\hat{\beta}} = \underbrace{\begin{bmatrix} q_1 \cdot y \\ \vdots \\ q_p \cdot y \end{bmatrix}}_{Q'y}$$

$$= \begin{bmatrix} \hat{\beta}_1 (x_1 \cdot q_1) + \cdots + \hat{\beta}_p (x_p \cdot q_1) \\ \vdots \\ (x_p \cdot q_p) \cdot \hat{\beta}_p \end{bmatrix} = \begin{bmatrix} q_1 \cdot y \\ \vdots \\ q_p \cdot y \end{bmatrix}$$

위의 식을 풀어나가면 $\hat{\beta}_1 \ldots \hat{\beta}_p$ 를 구할수있다