ESC21SUMMER_HW4_woohyunchoi

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[1]:
```
import numpy as np
import pandas as pd

from sklearn.preprocessing import StandardScaler

import matplotlib.pyplot as plt
import seaborn as sns
```

[2]:
```
import warnings
warnings.filterwarnings('ignore')
```

[3]:
```
from sklearn.datasets import fetch_lfw_people
from sklearn.decomposition import PCA
```

0.1 HW : PCA from scratch

- iris   numpy   PCA   !
- We can find PCA makes visualization easier!

[4]:
```
df = pd.read_csv('https://raw.githubusercontent.com/uiuc-cse/data-fa14/gh-pages/
 ↪data/iris.csv')
df.head()
```

[4]:
```
   sepal_length  sepal_width  petal_length  petal_width species
0           5.1          3.5           1.4          0.2  setosa
1           4.9          3.0           1.4          0.2  setosa
2           4.7          3.2           1.3          0.2  setosa
3           4.6          3.1           1.5          0.2  setosa
4           5.0          3.6           1.4          0.2  setosa
```

[5]:
```
X = df.iloc[:,:-1]
label = df.iloc[:,-1]
X.head()
```

[5]:
```
   sepal_length  sepal_width  petal_length  petal_width
0           5.1          3.5           1.4          0.2
1           4.9          3.0           1.4          0.2
2           4.7          3.2           1.3          0.2
```

```
3            4.6          3.1           1.5           0.2
4            5.0          3.6           1.4           0.2
```

0.1.1 Using Covariance Matrix

[6]:
```python
# Step 1. Center Data
X_scaled = StandardScaler().fit_transform(X)
X_scaled[:5]
```

[6]:
```
array([[-0.90068117,  1.03205722, -1.3412724 , -1.31297673],
       [-1.14301691, -0.1249576 , -1.3412724 , -1.31297673],
       [-1.38535265,  0.33784833, -1.39813811, -1.31297673],
       [-1.50652052,  0.10644536, -1.2844067 , -1.31297673],
       [-1.02184904,  1.26346019, -1.3412724 , -1.31297673]])
```

[8]:
```python
# Step 2. Compute Covariance Matrix
cov_matrix = np.cov(X_scaled.T)
cov_matrix
```

[8]:
```
array([[ 1.00671141, -0.11010327,  0.87760486,  0.82344326],
       [-0.11010327,  1.00671141, -0.42333835, -0.358937  ],
       [ 0.87760486, -0.42333835,  1.00671141,  0.96921855],
       [ 0.82344326, -0.358937  ,  0.96921855,  1.00671141]])
```

[17]:
```python
# Step 3. Eigenvalue decomposition
eigvals, eigvecs = np.linalg.eig(cov_matrix)
eigvals
```

[17]:
```
array([2.93035378, 0.92740362, 0.14834223, 0.02074601])
```

[18]:
```python
# Ratio of explained variance per PC
explained_variances = []
for i in range(len(eigvals)):
    explained_variances.append(eigvals[i] / np.sum(eigvals))

print(np.sum(explained_variances), '\n', explained_variances)
```

```
0.9999999999999999
 [0.7277045209380132, 0.23030523267680658, 0.03683831957627386,
0.005151926808906299]
```

, PC variance 95%

[21]:
```python
eigvecs.shape
```

[21]:
```
(4, 4)
```

[22]:
```
X_scaled.shape
```

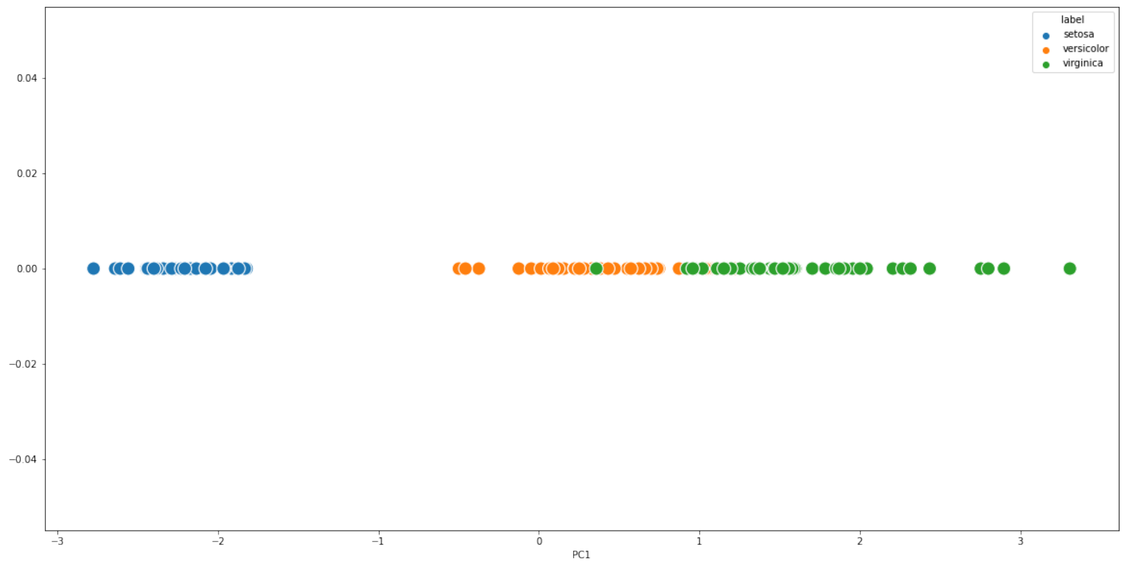
[22]:
```
(150, 4)
```

[25]:
```
# Visualization (Embedding)
pc1 = X_scaled @ eigvecs.T[0]
pc2 = X_scaled @ eigvecs.T[1]
res = pd.DataFrame(pc1, columns=['PC1'])
res['PC2'] = pc2
res['label'] = label
res.head()
```

[25]:
```
        PC1       PC2   label
0 -2.264542 -0.505704  setosa
1 -2.086426  0.655405  setosa
2 -2.367950  0.318477  setosa
3 -2.304197  0.575368  setosa
4 -2.388777 -0.674767  setosa
```

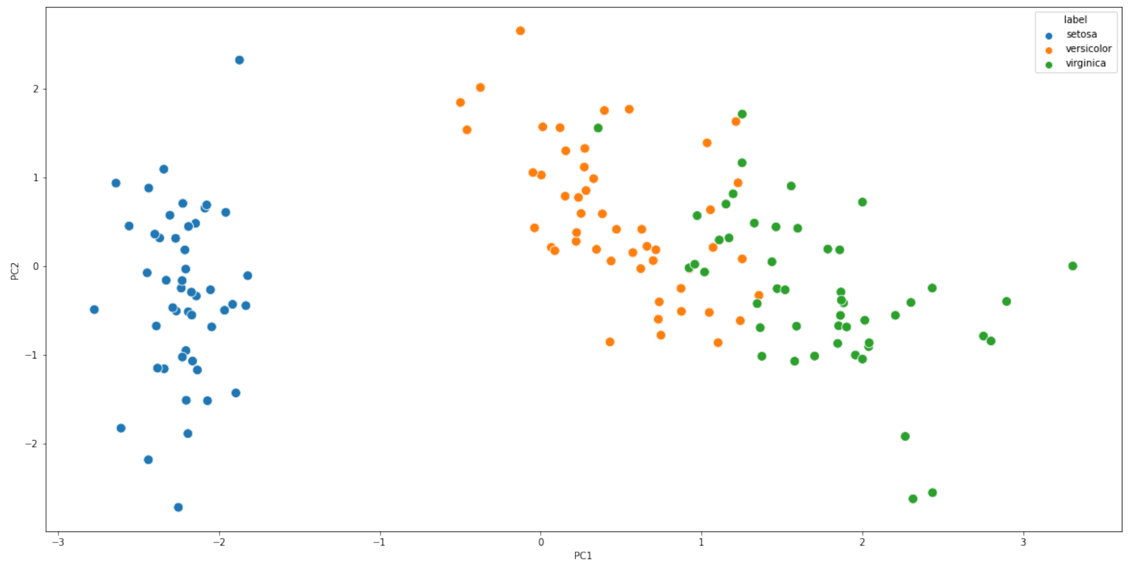
[26]:
```
# Projection on 1-dim subspace
plt.figure(figsize=(20, 10))
sns.scatterplot(res['PC1'], [0] * len(res), hue=res['label'], s=200)
```

[26]:
```
<AxesSubplot:xlabel='PC1'>
```



[27]:
```
# Projection on 2-dim subspace
plt.figure(figsize=(20, 10))
sns.scatterplot(res['PC1'], res['PC2'], hue=res['label'], s=100)
```

```
[27]: <AxesSubplot:xlabel='PC1', ylabel='PC2'>
```



Q. PC를 만드는 vector가 공분산행렬의 eigenvector 이고,
그 때의 variance가 eigenvalue

sol) Goal : maximize $var(\delta^T X)$ when $\|\delta\| = 1$

$L = var(\delta^T X) - \lambda(\delta^T \delta - 1)$

$= \delta^T \underset{\text{covariance matrix}}{\underset{\uparrow}{\Sigma}} \delta - \lambda(\delta^T \delta - 1)$

$\frac{\partial L}{\partial \delta} = 0 \Rightarrow \Sigma\delta = \lambda\delta$ $\quad \therefore \delta_i$'s are eigenvectors of Σ.

$var(\delta^T X) = \delta^T \Sigma \delta = \delta^T \lambda \delta = \lambda$

$\therefore$ The variance is eigenvalue