연세대학교 통계 데이터 사이언스 학회 ESC 23-2 Fall Final Project

Parallel Analysis

[ESC Final Project 2조] 김민주 김종민 김준엽 박정현 조수연 조준태

Simulation

Simulation of parallel analysis

▶ 1조 논문에서 사용하는 data를 활용하여 simulation 진행

-> Model 1~4 중 Model 3, Model 2에 대해 Parallel analysis 적용

▶ Simulation의 목표?

1. PA 결과가 유의미하게 나타나는지, 즉 true K를 잘 찾아내는지 확인
2. P가 N보다 큰 high dimension data에 대해 PA가 어떤 결과를 보이는지 확인

▶ Simulation의 목표?

Factor model을 $y = Bf + \epsilon$ 의 형태로 정의 & True number of common factor K=5

= PA가 잘 적용되었을 때 latent factor가 5가 나올 것이라고 기대

Simulation

Pseudocode

Noreen이 제시한, 고유값들의 순열 p value를 계산하는 코드

High level outline of a program for computing permutation p values of eigenvalues

Initialization:

- Read data into an $N \times P$ matrix **x**.
- Compute the eigenvalues, and store them in a vector of length P.
- Initialize an integer vector **nge** of length P to zero (intended to count how often permutation eigenvalues exceed the observed ones).
- Allocate auxiliary data structures for permuted data, their eigenvalues, etc.

Repeat R times:

- Permute columns 2, ..., P of **x**, leaving the first column unchanged, using a separate random permutation for each column (see Noreen, 1989, for code).
- Obtain the eigenvalues of the permuted data.
- For $i=1, ..., P$, increase the count **nge**$[i]$ by one if the i-th eigenvalue is greater than or equal the observed i-th eigenvalue.

Finally:

- Obtain p-values for each eigenvalue: $(\mathbf{nge}[i]+1)/(R+1)$, $(i=1, ..., P)$.

▶ 주어진 데이터의 고유값에 대한 순열 p-value를 계산하는 과정

= 고유값이 우연히 발생한 것인지, 아니면 실제로 데이터에 내재된 구조를 반영하는 것인지 확인하는데 사용

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초기화

: 데이터 행렬 X를 정의한 후 X의 고유값을 계산하고, 각 고유값에 대한 순열 검정의 카운트를 저장할 벡터 nge를 생성

Simulation

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R번 반복

: 데이터의 열(column)을 무작위로 섞어 (permute) 새로운 데이터를 생성하고, 이 데이터의 고유값 계산하는 과정을 R번 반복

Simulation

Pseudocode

Noreen이 제시한, 고유값들의 순열 p value를 계산하는 코드

High level outline of a program for computing permutation p values of eigenvalues

$$\mathbf{X} = \begin{bmatrix} X_{1,1} & X_{1,2} & X_{1,3} & \cdots & X_{1,P} \\ X_{2,1} & X_{2,2} & X_{2,3} & \cdots & X_{2,P} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ X_{N,1} & X_{N,2} & X_{N,3} & \cdots & X_{N,P} \end{bmatrix} \longrightarrow \mathbf{X}_\pi = \begin{bmatrix} X_{1,1} & X_{\pi_2(1),2} & X_{\pi_3(1),3} & \cdots & X_{\pi_P(1),P} \\ X_{2,1} & X_{\pi_2(2),2} & X_{\pi_3(2),3} & \cdots & X_{\pi_P(2),P} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ X_{N,1} & X_{\pi_2(N),2} & X_{\pi_3(N),3} & \cdots & X_{\pi_P(N),P} \end{bmatrix}$$

순열로 생성된 데이터의 각 고유값이 원래 데이터의 해당 고유값보다 크거나 같은 경우, 해당 위치의 nge를 1 증가

Simulation

Pseudocode

Noreen이 제시한, 고유값들의 순열 p value를 계산하는 코드

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Repeat R times:

- Permute columns 2, ..., P of **x**, leaving the first column unchanged, using a separate random permutation for each column (see Noreen, 1989, for code).
- Obtain the eigenvalues of the permuted data.
- For i=1, ..., P, increase the count **nge**[i] by one if the i-th eigenvalue is greater than or equal the observed i-th eigenvalue.

Finally:

- Obtain p-values for each eigenvalue: $(\mathbf{nge}[i]+1)/(R+1)$, ($i$=1, ..., P).

p-value 계산

: (nge[i]+1)/(R+1)을 통해 각 고유값에 대한 permutation p-value를 계산

➔ 각 고유값이 우연히 발생할 확률을 의미 (p-value 작을수록 해당 고유값이 데이터 구조를 반영하는 것, 크면 중요하지 않은 것으로 간주함)

Simulation Code

Data generation

```
set.seed(2023)
p=50              # number of variables
#p=100
#p=300
#p=500
#p=1000
n<-300             # number of observations
f=5               # number of latent factor

# Model 3
beta<-matrix(rnorm(p*f,0,1),p,f); diagc=diag(rep(sqrt(1*f),p))      # error bound = 1

# Gaussian: Data generation of factor model(y=Bf+e)
fm<-matrix(rnorm(f*n,0,1),f,n); epsilon<-matrix(rnorm(n*p,0,1),p,n)
Data<-beta%*%fm+diagc%*%epsilon
X=Data-matrix(rep(t(apply(Data,1,mean)),n),ncol=n)
X <- t(X)
```

▶ 1조 의 R source code에서 가져온 것으로, 해당 데이터 X로 PA 진행

-> Model 1~4 중 Model 3, Model 2에 대해 Parallel analysis 적용

Simulation Code

Data Permutation, Eigenvalues, p-values

```r
X_cor <- cor(X)
X_cor
N<-nrow(X); P<-ncol(X)
R=99 # number of permutation

obs_eigenvalues <- eigen(X_cor)$values
obs_eigenvalues <- sort(obs_eigenvalues, decreasing=TRUE)
nge <- rep(0, p)

# Repeat R times
A<- matrix(0, nrow=R+1, ncol=p)
A[1,] <- obs_eigenvalues
for (r in 1:R) {
  for (j in 2:p) {
    X[,j] <- X[sample(N),j] # Permute columns 2, ..., P of x
  }
  permutX_cor<-cor(X)
  permuted_eigenvalues <- eigen(permutX_cor)$values # Obtain the eigenvalues of the permuted data
  permuted_eigenvalues <- sort(permuted_eigenvalues, decreasing=TRUE)
  A[r+1,] <- permuted_eigenvalues
  nge <- nge + (permuted_eigenvalues >= obs_eigenvalues) # Increase the count if i-th permuted eigenvalue >= observed i-th eigenvalue
}
# Finally
p_values <- (nge + 1) / (R + 1) # Obtain p-values for each eigenvalue
p_values
```

▶ X의 상관행렬에서 observed eigenvalues를 구하고
크기가 큰 순서부터 나열

Simulation Code

Data Permutation, Eigenvalues, p-values

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X_cor <- cor(X)
X_cor
N<-nrow(X); P<-ncol(X)
R=99 # number of permutation

obs_eigenvalues <- eigen(X_cor)$values
obs_eigenvalues <- sort(obs_eigenvalues, decreasing=TRUE)
nge <- rep(0, p)

# Repeat R times
A<- matrix(0, nrow=R+1, ncol=p)
A[1,] <- obs_eigenvalues
for (r in 1:R) {
  for (j in 2:p) {
    X[,j] <- X[sample(N),j] # Permute columns 2, ..., P of x
  }
  permutX_cor<-cor(X)
  permuted_eigenvalues <- eigen(permutX_cor)$values # Obtain the eigenvalues of the permuted data
  permuted_eigenvalues <- sort(permuted_eigenvalues, decreasing=TRUE)
  A[r+1,] <- permuted_eigenvalues
  nge <- nge + (permuted_eigenvalues >= obs_eigenvalues) # Increase the count if i-th permuted eigenvalue >= observed i-th eigenvalue
}
# Finally
p_values <- (nge + 1) / (R + 1) # Obtain p-values for each eigenvalue
p_values
```

▶ X의 각 열 (2열~p열)을 permute하여 순열 데이터를 생성하고 permuted eigenvalues를 계산

-> *이 permutation을 R(=99)번 반복하여 permuted eigenvalues의 분포인 permutation quantile 얻음*

이론 상 $N!^{\{p-1\}}$ 번 permutation을 진행해야 하지만, 논문에서 R=9면 충분 & 99면 정확한 결과 얻을 수 있다고 언급하여 R=99로 설정

Simulation Code

Data Permutation, Eigenvalues, p-values

```r
X_cor <- cor(X)
X_cor
N<-nrow(X); P<-ncol(X)
R=99 # number of permutation

obs_eigenvalues <- eigen(X_cor)$values
obs_eigenvalues <- sort(obs_eigenvalues, decreasing=TRUE)
nge <- rep(0, p)

# Repeat R times
A<- matrix(0, nrow=R+1, ncol=p)
A[1,] <- obs_eigenvalues
for (r in 1:R) {
  for (j in 2:p) {
    X[,j] <- X[sample(N),j] # Permute columns 2, ..., P of x
  }
  permutX_cor<-cor(X)
  permuted_eigenvalues <- eigen(permutX_cor)$values # Obtain the eigenvalues of the permuted data
  permuted_eigenvalues <- sort(permuted_eigenvalues, decreasing=TRUE)
  A[r+1,] <- permuted_eigenvalues
  nge <- nge + (permuted_eigenvalues >= obs_eigenvalues) # Increase the count if i-th permuted eigenvalue >= observed i-th eigenvalue
}
# Finally
p_values <- (nge + 1) / (R + 1) # Obtain p-values for each eigenvalue
p_values
```

▶ Permuted eigenvalue가 observed eigenvalue보다 크거나 같은 경우 nge에 1 더함

▶ 이렇게 구한 nge를 바탕으로 각 고유값의 p-value 계산

Simulation Code

Result of Significant factors, Permutation quantiles

```r
sig_level=0.05
sig_factor=0
for (i in 1:p){
  if (p_values[i]<sig_level){
    sig_factor=sig_factor+1
  }
}
sig_factor
head(obs_eigenvalues)
permutation_quantiles<-apply(A, 2, function(x) quantile(x, p = c(0,0.25,0.5,0.75,0.9,0.95,0.99)))
result_model3_gaussian_p1000 <- cbind(obs_eigenvalues, t(permutation_quantiles), p_values)
rownames(result_model3_gaussian_p1000) <- c(seq(1,p,1))
result_model3_gaussian_p1000
```

▶ 유의수준(0.05)보다 작은 p-value의 개수를 sig_factor, 즉 factor의 수로 정의: 참값인 5가 나와야 좋은 것!

▶ A의 열마다 quantile 함수를 적용하여 permutation quantiles 계산

마지막으로 obs_eigenvalues, permutation_quantiles, p_values를 합쳐서 결과로 제시

Simulation Result

Model 3 Gaussian p=50

```
> result_model3_gaussian_p50
```

	obs_eigenvalues	0%	25%	50%	75%	90%	95%	99%	p_values
1	7.2868062	1.7877902	1.8670088	1.9014876	1.9407681	1.9747422	1.9920189	2.0650518	0.01
2	5.2901862	1.7093876	1.7803265	1.8097188	1.8334139	1.8512314	1.8880833	1.9697749	0.01
3	4.6662330	1.6538492	1.7156863	1.7362412	1.7655735	1.7909047	1.8115017	1.8593184	0.01
4	4.0061752	1.6103486	1.6498711	1.6755201	1.6986065	1.7209271	1.7405501	1.7860091	0.01
5	3.6956789	1.5570922	1.6060476	1.6278414	1.6535260	1.6678883	1.6855985	1.7422204	0.01
6	1.2083506	1.2083506	1.5543264	1.5752735	1.5958574	1.6172194	1.6288094	1.6424813	1.00
7	1.1249165	1.1249165	1.5147596	1.5293114	1.5438254	1.5585140	1.5647483	1.5797164	1.00
8	1.0560002	1.0560002	1.4659544	1.4840736	1.5013572	1.5181825	1.5251593	1.5343860	1.00
9	1.0158217	1.0158217	1.4362746	1.4470083	1.4643205	1.4812923	1.4894861	1.5038000	1.00
10	0.9936122	0.9936122	1.3932392	1.4070820	1.4228880	1.4393956	1.4436492	1.4512759	1.00
11	0.9163169	0.9163169	1.3588146	1.3682915	1.3813808	1.3958283	1.4015860	1.4064508	1.00
12	0.8679801	0.8679801	1.3235548	1.3350508	1.3500538	1.3582290	1.3678084	1.3868978	1.00
13	0.8411562	0.8411562	1.2896948	1.2990394	1.3152973	1.3265949	1.3353769	1.3436599	1.00
14	0.8083091	0.8083091	1.2595314	1.2717333	1.2795223	1.2897648	1.2986422	1.3058202	1.00
15	0.7840128	0.7840128	1.2248259	1.2356664	1.2497070	1.2602652	1.2659231	1.2695070	1.00
16	0.7579229	0.7579229	1.1955724	1.2089146	1.2194133	1.2256167	1.2319600	1.2433109	1.00
17	0.7346055	0.7346055	1.1618646	1.1739746	1.1844534	1.1969036	1.2008779	1.2026588	1.00
18	0.7126399	0.7126399	1.1349928	1.1479122	1.1574892	1.1687241	1.1706624	1.1850520	1.00
19	0.6908061	0.6908061	1.1061741	1.1192298	1.1282743	1.1405607	1.1452794	1.1486712	1.00
20	0.6793842	0.6793842	1.0777109	1.0905655	1.1009686	1.1108857	1.1171840	1.1243801	1.00

Simulation Result

Model 3 Gaussian p=100

```
> result_model3_gaussian_p100
```

	obs_eigenvalues	0%	25%	50%	75%	90%	95%	99%	p_values
1	12.34840934	2.23342619	2.3683881	2.3971759	2.4355400	2.4806547	2.5081378	2.6467302	0.01
2	10.39717443	2.20009492	2.2698610	2.2965783	2.3336607	2.3758470	2.4023494	2.5356576	0.01
3	9.39098539	2.12687874	2.2089520	2.2299769	2.2532241	2.2697447	2.2854300	2.3768293	0.01
4	7.88687147	2.10047979	2.1464691	2.1685490	2.1888446	2.2153710	2.2243189	2.3078501	0.01
5	6.14954476	2.04089145	2.0937473	2.1124062	2.1307805	2.1446679	2.1613403	2.2213810	0.01
6	1.45703353	1.45703353	2.0378156	2.0587737	2.0744210	2.0993908	2.1099379	2.1265178	1.00
7	1.41585485	1.41585485	1.9942010	2.0140194	2.0330153	2.0524501	2.0584542	2.0865115	1.00
8	1.38951724	1.38951724	1.9599751	1.9772382	1.9951941	2.0056508	2.0171306	2.0343431	1.00
9	1.34411649	1.34411649	1.9167617	1.9328956	1.9484039	1.9591935	1.9622193	1.9820847	1.00
10	1.30910591	1.30910591	1.8707212	1.8870067	1.9046560	1.9146983	1.9255006	1.9335999	1.00
11	1.29359839	1.29359839	1.8320081	1.8467317	1.8669416	1.8841178	1.8871161	1.8982728	1.00
12	1.23777353	1.23777353	1.7975023	1.8118301	1.8245032	1.8365098	1.8463940	1.8572529	1.00
13	1.23004835	1.23004835	1.7656544	1.7786398	1.7932529	1.8007456	1.8101753	1.8299432	1.00
14	1.18451927	1.18451927	1.7308993	1.7451754	1.7616264	1.7728627	1.7822550	1.7911049	1.00
15	1.13495267	1.13495267	1.6933648	1.7071448	1.7229366	1.7409441	1.7480704	1.7589153	1.00
16	1.09285074	1.09285074	1.6629958	1.6762608	1.6912687	1.6980275	1.7031310	1.7226923	1.00
17	1.07291233	1.07291233	1.6335104	1.6468627	1.6590924	1.6696246	1.6739682	1.6940574	1.00
18	1.04426744	1.04426744	1.6066058	1.6186908	1.6314660	1.6423926	1.6470458	1.6570637	1.00
19	0.99082355	0.99082355	1.5723052	1.5873210	1.6008497	1.6142502	1.6224546	1.6304698	1.00
20	0.97612060	0.97612060	1.5457375	1.5600257	1.5705645	1.5836143	1.5916447	1.6052294	1.00

Simulation Result

Model 3 Gaussian p=300

```
> result_model3_gaussian_p300
```

	obs_eigenvalues	0%	25%	50%	75%	90%	95%	99%	p_values
1	3.378712e+01	3.796490e+00	3.854671e+00	3.892765e+00	3.939491e+00	3.984546e+00	4.001719e+00	4.369343e+00	0.01
2	3.105267e+01	3.667142e+00	3.755029e+00	3.792401e+00	3.831099e+00	3.863047e+00	3.885236e+00	4.232425e+00	0.01
3	2.664284e+01	3.603572e+00	3.684471e+00	3.713192e+00	3.738984e+00	3.761152e+00	3.780066e+00	4.046617e+00	0.01
4	2.573251e+01	3.562506e+00	3.610970e+00	3.639311e+00	3.665679e+00	3.692306e+00	3.707761e+00	3.955787e+00	0.01
5	2.153761e+01	3.486420e+00	3.549458e+00	3.577650e+00	3.601802e+00	3.626872e+00	3.645837e+00	3.856063e+00	0.01
6	2.347100e+00	2.347100e+00	3.501924e+00	3.519520e+00	3.540888e+00	3.555975e+00	3.567520e+00	3.604374e+00	1.00
7	2.159033e+00	2.159033e+00	3.433547e+00	3.461423e+00	3.487973e+00	3.505493e+00	3.513989e+00	3.530820e+00	1.00
8	2.108443e+00	2.108443e+00	3.379579e+00	3.401725e+00	3.432220e+00	3.453451e+00	3.464675e+00	3.475964e+00	1.00
9	2.065801e+00	2.065801e+00	3.338435e+00	3.356272e+00	3.377681e+00	3.392593e+00	3.405206e+00	3.422802e+00	1.00
10	2.012128e+00	2.012128e+00	3.293667e+00	3.314519e+00	3.333143e+00	3.348732e+00	3.354285e+00	3.374731e+00	1.00
11	1.970086e+00	1.970086e+00	3.255860e+00	3.272464e+00	3.294549e+00	3.306527e+00	3.316909e+00	3.352092e+00	1.00
12	1.936499e+00	1.936499e+00	3.204651e+00	3.227853e+00	3.241716e+00	3.252517e+00	3.266808e+00	3.287826e+00	1.00
13	1.915287e+00	1.915287e+00	3.165052e+00	3.188366e+00	3.202272e+00	3.216240e+00	3.227924e+00	3.260446e+00	1.00
14	1.896057e+00	1.896057e+00	3.127401e+00	3.145732e+00	3.162373e+00	3.176692e+00	3.188233e+00	3.200464e+00	1.00
15	1.848653e+00	1.848653e+00	3.085577e+00	3.101234e+00	3.122043e+00	3.136058e+00	3.143301e+00	3.149283e+00	1.00
16	1.846276e+00	1.846276e+00	3.044551e+00	3.066377e+00	3.081624e+00	3.099124e+00	3.103938e+00	3.118489e+00	1.00
17	1.826220e+00	1.826220e+00	3.005719e+00	3.029854e+00	3.042826e+00	3.058967e+00	3.068517e+00	3.088382e+00	1.00
18	1.805076e+00	1.805076e+00	2.970052e+00	2.987794e+00	3.007014e+00	3.020299e+00	3.029070e+00	3.043000e+00	1.00
19	1.754102e+00	1.754102e+00	2.935635e+00	2.953746e+00	2.969859e+00	2.988417e+00	2.995431e+00	3.018553e+00	1.00
20	1.741149e+00	1.741149e+00	2.901352e+00	2.913775e+00	2.931101e+00	2.944422e+00	2.955251e+00	2.966471e+00	1.00

Simulation Result

Model 3 Gaussian p=500

```
> result_model3_gaussian_p500
```

	obs_eigenvalues	0%	25%	50%	75%	90%	95%	99%	p_values
1	5.447718e+01	5.009519e+00	5.129224e+00	5.169071e+00	5.214707e+00	5.257505e+00	5.284307e+00	5.807696e+00	0.01
2	4.987497e+01	4.885571e+00	4.992326e+00	5.034016e+00	5.080555e+00	5.113095e+00	5.137031e+00	5.666497e+00	0.01
3	4.539259e+01	4.819504e+00	4.901542e+00	4.939648e+00	4.977332e+00	5.002510e+00	5.011278e+00	5.463115e+00	0.01
4	4.469094e+01	4.743249e+00	4.823771e+00	4.858076e+00	4.898445e+00	4.932800e+00	4.948362e+00	5.391543e+00	0.01
5	3.531808e+01	4.699391e+00	4.757925e+00	4.787777e+00	4.813802e+00	4.841466e+00	4.864450e+00	5.210631e+00	0.01
6	2.919390e+00	2.919390e+00	4.693567e+00	4.720939e+00	4.741772e+00	4.763966e+00	4.772941e+00	4.816388e+00	1.00
7	2.853962e+00	2.853962e+00	4.631302e+00	4.655733e+00	4.681248e+00	4.700904e+00	4.712550e+00	4.742342e+00	1.00
8	2.843370e+00	2.843370e+00	4.567043e+00	4.588955e+00	4.617391e+00	4.641496e+00	4.657422e+00	4.663888e+00	1.00
9	2.782457e+00	2.782457e+00	4.515872e+00	4.543449e+00	4.562287e+00	4.579360e+00	4.597002e+00	4.612026e+00	1.00
10	2.759148e+00	2.759148e+00	4.469450e+00	4.495588e+00	4.517194e+00	4.527341e+00	4.539726e+00	4.552950e+00	1.00
11	2.665609e+00	2.665609e+00	4.413009e+00	4.440610e+00	4.460891e+00	4.484620e+00	4.497050e+00	4.504716e+00	1.00
12	2.648443e+00	2.648443e+00	4.370260e+00	4.391943e+00	4.414465e+00	4.434435e+00	4.440014e+00	4.452989e+00	1.00
13	2.614015e+00	2.614015e+00	4.319181e+00	4.340312e+00	4.365662e+00	4.383010e+00	4.395617e+00	4.399687e+00	1.00
14	2.557595e+00	2.557595e+00	4.281612e+00	4.299839e+00	4.320963e+00	4.333811e+00	4.341754e+00	4.356904e+00	1.00
15	2.549857e+00	2.549857e+00	4.232463e+00	4.251940e+00	4.274899e+00	4.296312e+00	4.303822e+00	4.315371e+00	1.00
16	2.530922e+00	2.530922e+00	4.192492e+00	4.212969e+00	4.230309e+00	4.251381e+00	4.257337e+00	4.276235e+00	1.00
17	2.474377e+00	2.474377e+00	4.153704e+00	4.167880e+00	4.185941e+00	4.199722e+00	4.219056e+00	4.245670e+00	1.00
18	2.442782e+00	2.442782e+00	4.116397e+00	4.129444e+00	4.148914e+00	4.159233e+00	4.164115e+00	4.204228e+00	1.00
19	2.425531e+00	2.425531e+00	4.075078e+00	4.087502e+00	4.103381e+00	4.116593e+00	4.122924e+00	4.130073e+00	1.00
20	2.410878e+00	2.410878e+00	4.032819e+00	4.049802e+00	4.070334e+00	4.080726e+00	4.094799e+00	4.095959e+00	1.00

Simulation Result

Model 3 Gaussian p=500

```
> p_values
  [1] 0.01 0.01 0.01 0.01 0.01 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00
 [27] 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00
 [53] 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00
 [79] 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00
[105] 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00
[131] 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00
[157] 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00
[183] 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00
[209] 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00
[235] 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00
[261] 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00
[287] 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 0.01 0.01 0.01 0.01 0.96 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00
[313] 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 0.99 0.99 0.99 0.99
[339] 0.99 0.99 0.99 0.99 0.99 0.99 0.99 0.98 0.97 0.98 0.98 0.97 0.97 0.96 0.96 0.96 0.95 0.95 0.95 0.95 0.94 0.90 0.92 0.92 0.92 0.92
[365] 0.92 0.89 0.89 0.89 0.89 0.88 0.87 0.87 0.88 0.88 0.88 0.88 0.86 0.85 0.87 0.86 0.84 0.84 0.84 0.84 0.84 0.84 0.84 0.82 0.82 0.82
[391] 0.82 0.81 0.81 0.82 0.82 0.81 0.81 0.81 0.81 0.82 0.80 0.80 0.79 0.78 0.77 0.77 0.80 0.80 0.79 0.79 0.79 0.77 0.76 0.76 0.75 0.74
[417] 0.71 0.69 0.66 0.65 0.64 0.61 0.62 0.64 0.69 0.66 0.68 0.65 0.64 0.64 0.66 0.64 0.63 0.65 0.63 0.62 0.60 0.57 0.56 0.53 0.48 0.48
[443] 0.46 0.46 0.47 0.44 0.41 0.40 0.40 0.36 0.37 0.34 0.33 0.28 0.22 0.20 0.18 0.19 0.21 0.19 0.15 0.15 0.13 0.13 0.10 0.04 0.03 0.02
[469] 0.02 0.02 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01
[495] 0.01 0.01 0.16 1.00 1.00 1.00
```

```
> sig_level=0.05
> sig_factor=0
> for (i in 1:p){
+   if (p_values[i]<sig_level){
+     sig_factor=sig_factor+1
+   }
+ }
> sig_factor
[1] 40
```

Simulation Result

Model 3 Gaussian p=1000

```
> result_model3_gaussian_p1000
```

	obs_eigenvalues	0%	25%	50%	75%	90%	95%	99%	p_values
1	1.148764e+02	7.718249e+00	7.827475e+00	7.877916e+00	7.945357e+00	7.994096e+00	8.025182e+00	9.193161e+00	0.01
2	1.006694e+02	7.543651e+00	7.689417e+00	7.735940e+00	7.770901e+00	7.820209e+00	7.843213e+00	8.846822e+00	0.01
3	8.634562e+01	7.476022e+00	7.573014e+00	7.624444e+00	7.663905e+00	7.698240e+00	7.712764e+00	8.567675e+00	0.01
4	8.353625e+01	7.394909e+00	7.476218e+00	7.521566e+00	7.558118e+00	7.597924e+00	7.610818e+00	8.429549e+00	0.01
5	7.179708e+01	7.302212e+00	7.401208e+00	7.430357e+00	7.468201e+00	7.503815e+00	7.518284e+00	8.190339e+00	0.01
6	4.463017e+00	4.463017e+00	7.324826e+00	7.353071e+00	7.391548e+00	7.414817e+00	7.433089e+00	7.458773e+00	1.00
7	4.391811e+00	4.391811e+00	7.248312e+00	7.279652e+00	7.315314e+00	7.340879e+00	7.345356e+00	7.383675e+00	1.00
8	4.345809e+00	4.345809e+00	7.181044e+00	7.206296e+00	7.234069e+00	7.257924e+00	7.283264e+00	7.301729e+00	1.00
9	4.256965e+00	4.256965e+00	7.113901e+00	7.137643e+00	7.167913e+00	7.191461e+00	7.209752e+00	7.256785e+00	1.00
10	4.229590e+00	4.229590e+00	7.053319e+00	7.077587e+00	7.104552e+00	7.131592e+00	7.146828e+00	7.164619e+00	1.00
11	4.227864e+00	4.227864e+00	6.997591e+00	7.021875e+00	7.047008e+00	7.065175e+00	7.081853e+00	7.112942e+00	1.00
12	4.086095e+00	4.086095e+00	6.943632e+00	6.964993e+00	6.992644e+00	7.012014e+00	7.022309e+00	7.047386e+00	1.00
13	4.078085e+00	4.078085e+00	6.891464e+00	6.914668e+00	6.926598e+00	6.954070e+00	6.968837e+00	7.001858e+00	1.00
14	4.036456e+00	4.036456e+00	6.839244e+00	6.861065e+00	6.879145e+00	6.903528e+00	6.910218e+00	6.919433e+00	1.00
15	4.015351e+00	4.015351e+00	6.778494e+00	6.799976e+00	6.823832e+00	6.840229e+00	6.844644e+00	6.868165e+00	1.00
16	3.971700e+00	3.971700e+00	6.724396e+00	6.745897e+00	6.776415e+00	6.791806e+00	6.796580e+00	6.810045e+00	1.00
17	3.923120e+00	3.923120e+00	6.676421e+00	6.694005e+00	6.715844e+00	6.737759e+00	6.751355e+00	6.772449e+00	1.00
18	3.906276e+00	3.906276e+00	6.625835e+00	6.646953e+00	6.668200e+00	6.685602e+00	6.692552e+00	6.711126e+00	1.00
19	3.872927e+00	3.872927e+00	6.578230e+00	6.599213e+00	6.620286e+00	6.641679e+00	6.659393e+00	6.682651e+00	1.00
20	3.811589e+00	3.811589e+00	6.524694e+00	6.553458e+00	6.570430e+00	6.590861e+00	6.607685e+00	6.620752e+00	1.00

Simulation Result

```
> p_values
   [1] 0.01 0.01 0.01 0.01 0.01 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00
  [26] 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00
  [51] 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00
  [76] 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00
 [101] 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00
 [126] 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00
 [151] 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00
 [176] 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00
 [201] 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00
 [226] 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00
 [251] 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00
 [276] 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 0.01
 [301] 0.01 0.01 0.01 0.01 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00
 [326] 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00
 [351] 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00
 [376] 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00
 [401] 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00
 [426] 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 0.99 1.00 0.99 0.99 0.99 0.99 0.99 0.99 0.99 0.99 0.99 0.99 0.99 0.98 0.98
 [451] 0.98 0.98 0.97 0.97 0.97 0.97 0.97 0.97 0.97 0.97 0.97 0.97 0.97 0.97 0.97 0.97 0.97 0.96 0.96 0.96 0.96 0.96 0.96 0.96 0.96
 [476] 0.96 0.96 0.96 0.96 0.96 0.96 0.96 0.96 0.96 0.96 0.96 0.96 0.96 0.96 0.96 0.96 0.96 0.96 0.96 0.96 0.96 0.96 0.96 0.96 0.96
 [501] 0.96 0.96 0.96 0.96 0.96 0.96 0.96 0.96 0.96 0.96 0.96 0.96 0.96 0.96 0.96 0.96 0.96 0.96 0.96 0.96 0.96 0.96 0.96 0.95 0.95
 [526] 0.95 0.95 0.95 0.95 0.95 0.95 0.95 0.95 0.95 0.95 0.95 0.95 0.95 0.95 0.95 0.95 0.95 0.95 0.95 0.95 0.95 0.95 0.94 0.93 0.93
 [551] 0.93 0.93 0.92 0.93 0.94 0.93 0.92 0.92 0.92 0.92 0.92 0.91 0.90 0.90 0.90 0.90 0.90 0.90 0.90 0.90 0.90 0.90 0.90 0.90 0.90
 [576] 0.90 0.90 0.90 0.90 0.90 0.90 0.90 0.90 0.89 0.89 0.89 0.89 0.89 0.89 0.87 0.87 0.87 0.86 0.86 0.86 0.86 0.85 0.84 0.84 0.84
 [601] 0.83 0.83 0.83 0.82 0.81 0.80 0.80 0.79 0.79 0.78 0.78 0.78 0.78 0.78 0.79 0.78 0.78 0.78 0.78 0.78 0.78 0.78 0.78 0.77 0.76
 [626] 0.76 0.75 0.75 0.75 0.75 0.75 0.75 0.75 0.75 0.75 0.74 0.74 0.74 0.74 0.73 0.73 0.73 0.73 0.70 0.70 0.71 0.71 0.70 0.69 0.68
 [651] 0.69 0.68 0.64 0.64 0.62 0.63 0.64 0.64 0.64 0.64 0.64 0.64 0.64 0.64 0.63 0.63 0.63 0.63 0.63 0.63 0.63 0.62 0.62 0.62 0.61
 [676] 0.60 0.60 0.59 0.58 0.58 0.58 0.57 0.57 0.57 0.57 0.57 0.57 0.56 0.55 0.55 0.55 0.55 0.54 0.54 0.52 0.52 0.52 0.52 0.52 0.52
 [701] 0.50 0.51 0.48 0.49 0.48 0.48 0.47 0.48 0.47 0.49 0.49 0.48 0.46 0.46 0.46 0.46 0.46 0.47 0.47 0.46 0.47 0.46 0.46 0.46 0.46
 [726] 0.46 0.46 0.44 0.44 0.44 0.44 0.44 0.44 0.43 0.43 0.43 0.42 0.42 0.42 0.41 0.41 0.41 0.41 0.41 0.41 0.42 0.41 0.41 0.41 0.41
 [751] 0.40 0.40 0.40 0.40 0.40 0.40 0.40 0.40 0.40 0.40 0.40 0.40 0.40 0.40 0.40 0.40 0.38 0.38 0.38 0.38 0.37 0.37 0.37 0.37 0.36
 [776] 0.35 0.35 0.35 0.35 0.35 0.35 0.34 0.34 0.32 0.30 0.28 0.29 0.27 0.27 0.26 0.26 0.26 0.25 0.23 0.23 0.22 0.20 0.19 0.19 0.19
 [801] 0.19 0.19 0.18 0.18 0.17 0.17 0.17 0.17 0.17 0.17 0.16 0.16 0.16 0.16 0.16 0.16 0.15 0.14 0.12 0.12 0.12 0.12 0.12 0.11 0.10
 [826] 0.10 0.10 0.08 0.07 0.07 0.08 0.08 0.08 0.07 0.08 0.07 0.08 0.07 0.08 0.06 0.06 0.05 0.05 0.05 0.05 0.05 0.05 0.05 0.04 0.03
 [851] 0.04 0.04 0.03 0.03 0.03 0.03 0.03 0.03 0.03 0.03 0.03 0.03 0.03 0.03 0.03 0.02 0.02 0.02 0.02 0.02 0.02 0.02 0.02 0.02 0.02
 [876] 0.02 0.02 0.02 0.02 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01
 [901] 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01
 [926] 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01
 [951] 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01
 [976] 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 1.00 1.00 1.00 1.00 1.00 1.00
```

```
> sig_factor
[1] 156
```

Simulation Result

Model 3 Uniform p=50

```
> result_model3_uniform_p50
```

	obs_eigenvalues	0%	25%	50%	75%	90%	95%	99%	p_values
1	7.4717860	1.7685131	1.8497579	1.8891296	1.9243103	1.9534058	1.9697702	2.0501525	0.01
2	5.7018843	1.6925976	1.7783372	1.8052347	1.8281285	1.8514473	1.8601869	1.9418665	0.01
3	4.7001980	1.6547035	1.7159192	1.7381753	1.7625861	1.7805854	1.7967639	1.8450343	0.01
4	3.8614535	1.5894585	1.6539441	1.6706517	1.6933781	1.7167878	1.7423455	1.7945926	0.01
5	3.6412319	1.5360053	1.5991882	1.6231387	1.6426357	1.6681959	1.6888067	1.7279252	0.01
6	1.2390278	1.2390278	1.5499463	1.5705981	1.5933620	1.6074502	1.6181909	1.6324678	1.00
7	1.1520624	1.1520624	1.5096184	1.5240446	1.5482181	1.5628645	1.5748054	1.5952183	1.00
8	1.0866329	1.0866329	1.4699945	1.4906726	1.5026057	1.5208665	1.5330204	1.5446759	1.00
9	1.0030647	1.0030647	1.4313430	1.4437560	1.4633181	1.4727200	1.4777544	1.4858173	1.00
10	0.9477485	0.9477485	1.3953904	1.4070824	1.4280994	1.4381631	1.4468961	1.4557933	1.00
11	0.9157292	0.9157292	1.3595540	1.3715512	1.3889682	1.4015177	1.4107790	1.4150357	1.00
12	0.8656234	0.8656234	1.3257558	1.3373812	1.3547592	1.3712166	1.3745721	1.3801390	1.00
13	0.8504347	0.8504347	1.2891488	1.3037872	1.3151392	1.3328607	1.3365578	1.3422433	1.00
14	0.8236365	0.8236365	1.2560175	1.2702324	1.2834301	1.2967134	1.3037989	1.3090049	1.00
15	0.7856570	0.7856570	1.2230689	1.2370168	1.2509771	1.2601411	1.2665899	1.2794148	1.00
16	0.7477880	0.7477880	1.1952182	1.2059570	1.2163263	1.2252603	1.2301964	1.2525175	1.00
17	0.7280622	0.7280622	1.1684893	1.1787542	1.1864261	1.2023402	1.2085135	1.2185981	1.00
18	0.7055539	0.7055539	1.1358994	1.1481307	1.1601789	1.1700265	1.1816102	1.1912565	1.00
19	0.6820873	0.6820873	1.1090528	1.1235061	1.1339965	1.1413759	1.1472472	1.1676799	1.00
20	0.6537448	0.6537448	1.0784014	1.0928063	1.1075618	1.1159863	1.1216552	1.1319297	1.00

Simulation Result

Model 3 Uniform p=100

```
> result_model3_uniform_p100
   obs_eigenvalues          0%       25%       50%       75%       90%       95%       99% p_values
1      11.50116642 2.28652785 2.3651308 2.4053714 2.4368113 2.4838952 2.5164000 2.6516553     0.01
2      10.87537795 2.17967388 2.2674753 2.3062748 2.3320660 2.3591642 2.3739265 2.5467694     0.01
3       9.41023008 2.12914205 2.2104888 2.2292488 2.2550387 2.2894612 2.3046669 2.4140152     0.01
4       8.81165437 2.08525165 2.1435481 2.1711595 2.1913304 2.2110575 2.2255863 2.3192974     0.01
5       6.70989868 2.04445159 2.0933517 2.1136183 2.1331983 2.1540827 2.1660379 2.2322594     0.01
6       1.46158308 1.46158308 2.0431403 2.0624999 2.0879012 2.1049569 2.1124513 2.1252056     1.00
7       1.37651442 1.37651442 1.9997565 2.0155659 2.0323804 2.0461048 2.0608104 2.0705232     1.00
8       1.29631967 1.29631967 1.9590892 1.9709148 1.9891665 2.0102339 2.0213947 2.0336295     1.00
9       1.26633371 1.26633371 1.9140518 1.9325814 1.9458137 1.9608847 1.9691598 1.9943522     1.00
10      1.20926004 1.20926004 1.8782651 1.8955474 1.9093189 1.9204858 1.9277447 1.9333760     1.00
11      1.19788106 1.19788106 1.8452831 1.8583666 1.8713751 1.8820948 1.8892844 1.9054381     1.00
12      1.18947848 1.18947848 1.8075343 1.8180174 1.8303809 1.8424204 1.8512492 1.8680175     1.00
13      1.16539524 1.16539524 1.7690346 1.7840183 1.7970222 1.8114706 1.8218415 1.8321172     1.00
14      1.10973971 1.10973971 1.7344437 1.7455948 1.7632851 1.7766935 1.7844677 1.7890615     1.00
15      1.08528843 1.08528843 1.7019166 1.7146119 1.7274070 1.7395137 1.7500633 1.7585810     1.00
16      1.04824449 1.04824449 1.6699019 1.6840434 1.6975071 1.7055548 1.7116648 1.7189912     1.00
17      1.03971703 1.03971703 1.6348623 1.6479224 1.6624550 1.6778732 1.6855347 1.6912330     1.00
18      1.02084470 1.02084470 1.6068213 1.6213652 1.6396224 1.6504191 1.6568778 1.6685221     1.00
19      0.99679613 0.99679613 1.5769116 1.5905310 1.6063758 1.6204509 1.6273314 1.6301037     1.00
20      0.99567896 0.99567896 1.5456695 1.5565973 1.5716343 1.5821536 1.5928496 1.6050766     1.00
```

Simulation Result

Model 3 Uniform p=300

```
> result_model3_uniform_p300
```

	obs_eigenvalues	0%	25%	50%	75%	90%	95%	99%	p_values
1	3.286498e+01	3.766630e+00	3.854078e+00	3.910859e+00	3.944864e+00	3.981082e+00	3.990306e+00	4.379727e+00	0.01
2	2.947287e+01	3.666221e+00	3.762554e+00	3.791128e+00	3.832661e+00	3.868725e+00	3.893555e+00	4.197702e+00	0.01
3	2.671780e+01	3.605414e+00	3.679666e+00	3.716216e+00	3.749834e+00	3.767426e+00	3.796101e+00	4.059494e+00	0.01
4	2.623133e+01	3.533555e+00	3.605775e+00	3.642848e+00	3.672117e+00	3.700691e+00	3.711156e+00	3.964249e+00	0.01
5	2.334433e+01	3.464924e+00	3.548120e+00	3.574542e+00	3.600889e+00	3.620183e+00	3.630375e+00	3.836343e+00	0.01
6	2.282679e+00	2.282679e+00	3.481048e+00	3.510021e+00	3.536047e+00	3.557352e+00	3.561225e+00	3.582170e+00	1.00
7	2.209636e+00	2.209636e+00	3.434875e+00	3.460041e+00	3.482911e+00	3.500061e+00	3.507675e+00	3.538070e+00	1.00
8	2.140296e+00	2.140296e+00	3.387808e+00	3.405894e+00	3.429180e+00	3.442166e+00	3.453496e+00	3.495799e+00	1.00
9	2.130282e+00	2.130282e+00	3.333022e+00	3.357041e+00	3.380320e+00	3.397467e+00	3.416684e+00	3.440197e+00	1.00
10	2.092273e+00	2.092273e+00	3.281411e+00	3.301601e+00	3.327958e+00	3.345836e+00	3.359178e+00	3.378523e+00	1.00
11	2.066339e+00	2.066339e+00	3.239914e+00	3.263229e+00	3.281397e+00	3.308076e+00	3.322000e+00	3.334820e+00	1.00
12	2.022076e+00	2.022076e+00	3.198055e+00	3.220810e+00	3.239921e+00	3.255281e+00	3.261221e+00	3.265627e+00	1.00
13	1.992371e+00	1.992371e+00	3.152544e+00	3.172477e+00	3.194775e+00	3.211952e+00	3.222650e+00	3.236356e+00	1.00
14	1.938954e+00	1.938954e+00	3.116182e+00	3.135416e+00	3.159960e+00	3.176901e+00	3.180451e+00	3.198679e+00	1.00
15	1.895762e+00	1.895762e+00	3.080388e+00	3.094001e+00	3.112873e+00	3.129382e+00	3.145402e+00	3.167500e+00	1.00
16	1.865300e+00	1.865300e+00	3.041515e+00	3.059746e+00	3.076485e+00	3.093843e+00	3.100790e+00	3.131436e+00	1.00
17	1.839350e+00	1.839350e+00	3.007349e+00	3.023061e+00	3.039477e+00	3.055681e+00	3.057759e+00	3.074993e+00	1.00
18	1.821658e+00	1.821658e+00	2.973231e+00	2.989077e+00	3.002758e+00	3.015645e+00	3.022390e+00	3.030888e+00	1.00
19	1.783811e+00	1.783811e+00	2.939407e+00	2.953830e+00	2.967136e+00	2.984827e+00	2.994067e+00	3.004097e+00	1.00
20	1.742291e+00	1.742291e+00	2.899081e+00	2.918655e+00	2.932919e+00	2.944399e+00	2.962840e+00	2.973604e+00	1.00

Simulation Result

Model 3 Uniform p=500

	obs_eigenvalues	0%	25%	50%	75%	90%	95%	99%	p_values
1	5.455013e+01	4.965693e+00	5.114245e+00	5.168601e+00	5.212555e+00	5.260090e+00	5.302767e+00	5.875684e+00	0.01
2	4.902114e+01	4.877320e+00	4.990690e+00	5.025341e+00	5.071067e+00	5.105894e+00	5.129432e+00	5.610698e+00	0.01
3	4.810843e+01	4.774184e+00	4.899048e+00	4.939172e+00	4.970944e+00	5.002595e+00	5.018188e+00	5.514708e+00	0.01
4	4.271704e+01	4.721257e+00	4.824929e+00	4.854205e+00	4.876817e+00	4.915669e+00	4.940873e+00	5.341386e+00	0.01
5	3.707547e+01	4.669796e+00	4.754907e+00	4.781246e+00	4.809927e+00	4.839046e+00	4.847562e+00	5.195808e+00	0.01
6	3.003934e+00	3.003934e+00	4.690082e+00	4.717862e+00	4.743319e+00	4.771605e+00	4.792850e+00	4.803334e+00	1.00
7	2.893944e+00	2.893944e+00	4.627678e+00	4.651777e+00	4.671864e+00	4.708051e+00	4.724867e+00	4.744035e+00	1.00
8	2.873517e+00	2.873517e+00	4.571399e+00	4.591548e+00	4.613872e+00	4.642392e+00	4.649348e+00	4.662726e+00	1.00
9	2.774615e+00	2.774615e+00	4.524477e+00	4.544976e+00	4.562959e+00	4.583286e+00	4.590614e+00	4.613307e+00	1.00
10	2.693192e+00	2.693192e+00	4.473554e+00	4.490600e+00	4.511092e+00	4.528800e+00	4.544041e+00	4.567737e+00	1.00
11	2.669521e+00	2.669521e+00	4.421763e+00	4.449392e+00	4.466056e+00	4.483476e+00	4.489784e+00	4.505529e+00	1.00
12	2.632300e+00	2.632300e+00	4.374882e+00	4.395440e+00	4.412816e+00	4.429207e+00	4.444178e+00	4.455114e+00	1.00
13	2.613501e+00	2.613501e+00	4.324608e+00	4.343341e+00	4.362948e+00	4.382188e+00	4.389177e+00	4.400835e+00	1.00
14	2.577512e+00	2.577512e+00	4.282599e+00	4.297355e+00	4.319353e+00	4.331960e+00	4.349388e+00	4.375240e+00	1.00
15	2.512303e+00	2.512303e+00	4.234830e+00	4.259796e+00	4.277618e+00	4.291831e+00	4.300786e+00	4.313964e+00	1.00
16	2.463680e+00	2.463680e+00	4.186947e+00	4.216305e+00	4.230216e+00	4.249143e+00	4.256390e+00	4.275596e+00	1.00
17	2.442768e+00	2.442768e+00	4.144397e+00	4.166723e+00	4.185585e+00	4.203341e+00	4.210999e+00	4.218493e+00	1.00
18	2.426456e+00	2.426456e+00	4.106127e+00	4.127051e+00	4.141184e+00	4.158102e+00	4.164978e+00	4.176100e+00	1.00
19	2.407402e+00	2.407402e+00	4.066334e+00	4.085422e+00	4.108821e+00	4.121989e+00	4.128362e+00	4.142328e+00	1.00
20	2.370671e+00	2.370671e+00	4.025527e+00	4.050277e+00	4.066465e+00	4.081120e+00	4.092695e+00	4.097526e+00	1.00

ESC

Simulation Result

Model 3 Uniform p=500

```
> sig_factor
[1] 39
```

```
> p_values
  [1] 0.01 0.01 0.01 0.01 0.01 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00
 [27] 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00
 [53] 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00
 [79] 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00
[105] 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00
[131] 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00
[157] 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00
[183] 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00
[209] 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00
[235] 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00
[261] 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00
[287] 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 0.01 0.01 0.01 0.02 0.95 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00
[313] 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00
[339] 0.99 0.99 0.99 0.99 1.00 1.00 0.99 0.99 0.99 0.99 0.99 0.98 0.97 0.97 0.97 0.96 0.96 0.96 0.96 0.96 0.96 0.96 0.96 0.96 0.95 0.94
[365] 0.95 0.94 0.94 0.93 0.93 0.93 0.93 0.93 0.93 0.93 0.93 0.93 0.93 0.93 0.94 0.94 0.94 0.92 0.92 0.92 0.92 0.92 0.92 0.91 0.92 0.91
[391] 0.90 0.90 0.88 0.90 0.89 0.87 0.87 0.87 0.86 0.85 0.85 0.85 0.87 0.85 0.86 0.87 0.85 0.85 0.83 0.84 0.82 0.83 0.82 0.82 0.81 0.78
[417] 0.78 0.78 0.77 0.76 0.80 0.79 0.78 0.78 0.78 0.78 0.78 0.77 0.76 0.76 0.74 0.75 0.74 0.73 0.72 0.72 0.72 0.72 0.72 0.69 0.68 0.67
[443] 0.66 0.64 0.61 0.60 0.58 0.56 0.55 0.54 0.51 0.48 0.46 0.45 0.42 0.41 0.39 0.34 0.33 0.31 0.29 0.28 0.21 0.18 0.12 0.10 0.10 0.03
[469] 0.02 0.02 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01
[495] 0.01 0.01 0.01 0.92 1.00 1.00
```

Simulation Result

Model 3 Uniform p=1000

```
> result_model3_uniform_p1000
```

	obs_eigenvalues	0%	25%	50%	75%	90%	95%	99%	p_values
1	1.058304e+02	7.697557e+00	7.815664e+00	7.866408e+00	7.933950e+00	7.979406e+00	8.032608e+00	9.065200e+00	0.01
2	9.846374e+01	7.563951e+00	7.673342e+00	7.714750e+00	7.768511e+00	7.811886e+00	7.840330e+00	8.894418e+00	0.01
3	8.989680e+01	7.497997e+00	7.565720e+00	7.612957e+00	7.660238e+00	7.706836e+00	7.750259e+00	8.692889e+00	0.01
4	8.607223e+01	7.413760e+00	7.489836e+00	7.519817e+00	7.569564e+00	7.607417e+00	7.638633e+00	8.565093e+00	0.01
5	8.232174e+01	7.323763e+00	7.390442e+00	7.444128e+00	7.479580e+00	7.520652e+00	7.543480e+00	8.388322e+00	0.01
6	4.499162e+00	4.499162e+00	7.319353e+00	7.355593e+00	7.380415e+00	7.414047e+00	7.421964e+00	7.460823e+00	1.00
7	4.439319e+00	4.439319e+00	7.255523e+00	7.279757e+00	7.318373e+00	7.346229e+00	7.365858e+00	7.383900e+00	1.00
8	4.280903e+00	4.280903e+00	7.179951e+00	7.217795e+00	7.245357e+00	7.269409e+00	7.309890e+00	7.327888e+00	1.00
9	4.234863e+00	4.234863e+00	7.120634e+00	7.152510e+00	7.179753e+00	7.208154e+00	7.231307e+00	7.258313e+00	1.00
10	4.135676e+00	4.135676e+00	7.065884e+00	7.083626e+00	7.118499e+00	7.142036e+00	7.156599e+00	7.173652e+00	1.00
11	4.111341e+00	4.111341e+00	7.002108e+00	7.027428e+00	7.047225e+00	7.067118e+00	7.085147e+00	7.118077e+00	1.00
12	4.093428e+00	4.093428e+00	6.945364e+00	6.974543e+00	6.996813e+00	7.014412e+00	7.031370e+00	7.061448e+00	1.00
13	4.043709e+00	4.043709e+00	6.874769e+00	6.906276e+00	6.930469e+00	6.946560e+00	6.963274e+00	6.992679e+00	1.00
14	4.019078e+00	4.019078e+00	6.832600e+00	6.853898e+00	6.873626e+00	6.904720e+00	6.913712e+00	6.937120e+00	1.00
15	3.969990e+00	3.969990e+00	6.781530e+00	6.795369e+00	6.826416e+00	6.846717e+00	6.862495e+00	6.873810e+00	1.00
16	3.937112e+00	3.937112e+00	6.725764e+00	6.751334e+00	6.774879e+00	6.794236e+00	6.802074e+00	6.844827e+00	1.00
17	3.918722e+00	3.918722e+00	6.667060e+00	6.699322e+00	6.721422e+00	6.739724e+00	6.750845e+00	6.768352e+00	1.00
18	3.879545e+00	3.879545e+00	6.627745e+00	6.649007e+00	6.676690e+00	6.695598e+00	6.701706e+00	6.719455e+00	1.00
19	3.861073e+00	3.861073e+00	6.568875e+00	6.594195e+00	6.619258e+00	6.638088e+00	6.643135e+00	6.658866e+00	[illegible]
20	3.818149e+00	3.818149e+00	6.523527e+00	6.549551e+00	6.571729e+00	6.592505e+00	6.599823e+00	6.616025e+00	[illegible]

Simulation Result

Model 3 Uniform p=1000

```
> p_values
   [1] 0.01 0.01 0.01 0.01 0.01 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00
  [26] 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00
  [51] 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00
  [76] 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00
 [101] 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00
 [126] 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00
 [151] 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00
 [176] 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00
 [201] 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00
 [226] 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00
 [251] 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00
 [276] 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 0.01
 [301] 0.01 0.01 0.01 0.01 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00
 [326] 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00
 [351] 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00
 [376] 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00
 [401] 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 0.99 0.99 0.99 0.98 0.96 0.95
 [426] 0.95 0.94 0.93 0.93 0.93 0.92 0.92 0.90 0.90 0.90 0.89 0.87 0.86 0.86 0.86 0.86 0.86 0.86 0.86 0.86 0.86 0.86 0.85 0.84 0.84
 [451] 0.83 0.83 0.83 0.83 0.82 0.82 0.81 0.81 0.80 0.81 0.81 0.81 0.81 0.80 0.80 0.81 0.81 0.81 0.81 0.80 0.80 0.79 0.78 0.78 0.77
 [476] 0.76 0.76 0.76 0.76 0.75 0.75 0.75 0.72 0.74 0.73 0.74 0.72 0.73 0.73 0.73 0.72 0.72 0.71 0.70 0.71 0.70 0.70 0.70 0.69 0.70
 [501] 0.70 0.70 0.70 0.69 0.69 0.69 0.69 0.69 0.69 0.68 0.68 0.69 0.69 0.69 0.69 0.68 0.68 0.67 0.67 0.66 0.66 0.66 0.66 0.66 0.66
 [526] 0.66 0.66 0.66 0.66 0.66 0.66 0.66 0.65 0.64 0.63 0.63 0.63 0.64 0.62 0.61 0.61 0.60 0.60 0.59 0.59 0.60 0.58 0.59 0.58 0.58
 [551] 0.57 0.57 0.56 0.55 0.55 0.55 0.54 0.54 0.54 0.53 0.53 0.53 0.53 0.53 0.53 0.53 0.53 0.53 0.53 0.53 0.52 0.52 0.52 0.52 0.51
 [576] 0.49 0.48 0.48 0.47 0.47 0.47 0.48 0.46 0.46 0.46 0.48 0.46 0.47 0.46 0.46 0.45 0.45 0.47 0.47 0.45 0.45 0.45 0.45 0.45 0.45
 [601] 0.45 0.45 0.44 0.44 0.44 0.44 0.44 0.44 0.44 0.43 0.44 0.43 0.43 0.42 0.43 0.43 0.42 0.43 0.42 0.43 0.42 0.42 0.42 0.42 0.42
 [626] 0.42 0.42 0.42 0.42 0.42 0.40 0.40 0.39 0.38 0.38 0.37 0.37 0.37 0.37 0.36 0.36 0.36 0.35 0.35 0.35 0.35 0.35 0.34 0.34 0.33
 [651] 0.33 0.33 0.33 0.33 0.33 0.31 0.30 0.30 0.30 0.29 0.28 0.28 0.27 0.27 0.27 0.27 0.27 0.26 0.25 0.25 0.25 0.24 0.24 0.24 0.24
 [676] 0.24 0.23 0.21 0.20 0.19 0.17 0.17 0.17 0.17 0.17 0.15 0.14 0.14 0.15 0.15 0.14 0.14 0.14 0.13 0.13 0.12 0.12 0.12 0.11 0.12
 [701] 0.11 0.10 0.10 0.10 0.08 0.08 0.08 0.08 0.08 0.08 0.08 0.08 0.08 0.08 0.08 0.08 0.08 0.08 0.08 0.08 0.08 0.07 0.06 0.06 0.06
 [726] 0.06 0.06 0.06 0.07 0.06 0.07 0.07 0.07 0.06 0.06 0.07 0.07 0.07 0.06 0.06 0.06 0.05 0.05 0.05 0.04 0.04 0.04 0.04 0.05 0.05
 [751] 0.04 0.04 0.06 0.04 0.04 0.05 0.05 0.05 0.04 0.04 0.03 0.03 0.03 0.03 0.03 0.03 0.03 0.03 0.03 0.03 0.03 0.02 0.02 0.02 0.02
 [776] 0.02 0.02 0.02 0.02 0.02 0.01 0.01 0.02 0.02 0.02 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01
 [801] 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01
 [826] 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01
 [851] 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01
 [876] 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01
 [901] 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01
 [926] 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01
 [951] 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01
 [976] 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 1.00 1.00 1.00 1.00 1.00
```

Simulation Result

set.seed(2023) Result

N=300	P=50	P=100	P=300	P=500	P=1000
Model3 Gaussian	5	5	5	40	156
Model3 Uniform	5	5	5	39	255
Model2 Gaussian	5	5	5	41	178
Model2 Uniform	5	5	6	62	119

N>P에서는 PA가 유의한 Factor의 개수를 잘 뽑아 냈지만, N<P의 high-dimensional case의 경우 의미가 없었다!

ESC

Contribution

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END