25. Reinforcement Learning

STA3142 Statistical Machine Learning

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** Slides adapted from EECS498/598 @ Univ. of Michigan by Justin Johnson*

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Recap: Machine Learning Tasks

- Supervised Learning
  - Classification
  - Regression
- Unsupervised Learning
  - Clustering
  - Density estimation
  - Embedding / Dimensionality reduction
- Reinforcement Learning
  - Learning to act
    (e.g., robot control, decision making, etc.)

Recap: Supervised Learning

Supervised Learning

Data: (x, y)

x is data, y is label

Goal: Learn a *function* to map x -> y

Examples: Classification, regression, image captioning, etc.

Classification

Cat

This image is CC0 public domain

Recap: Supervised Learning

- Given a dataset $D = \{(x_1, y_1), \dots, (x_n, y_n)\}$, where
  - $x_i \in \mathcal{X}$: input (feature)
  - $y_i \in \mathcal{Y}$: output (label)
- A black box ML algorithm produces a prediction function $h: \mathcal{X} \rightarrow \mathcal{Y}$, such that $h(x)$ can predict the y values for all x
  - Not only for all training data $x_i \in D$,
    but also for unseen test data $x^* \in \mathcal{X}$.
- Labels could be discrete or continuous
  - Discrete labels: **classification**
  - Continuous labels: **regression**

Recap: Unsupervised Learning

Unsupervised Learning

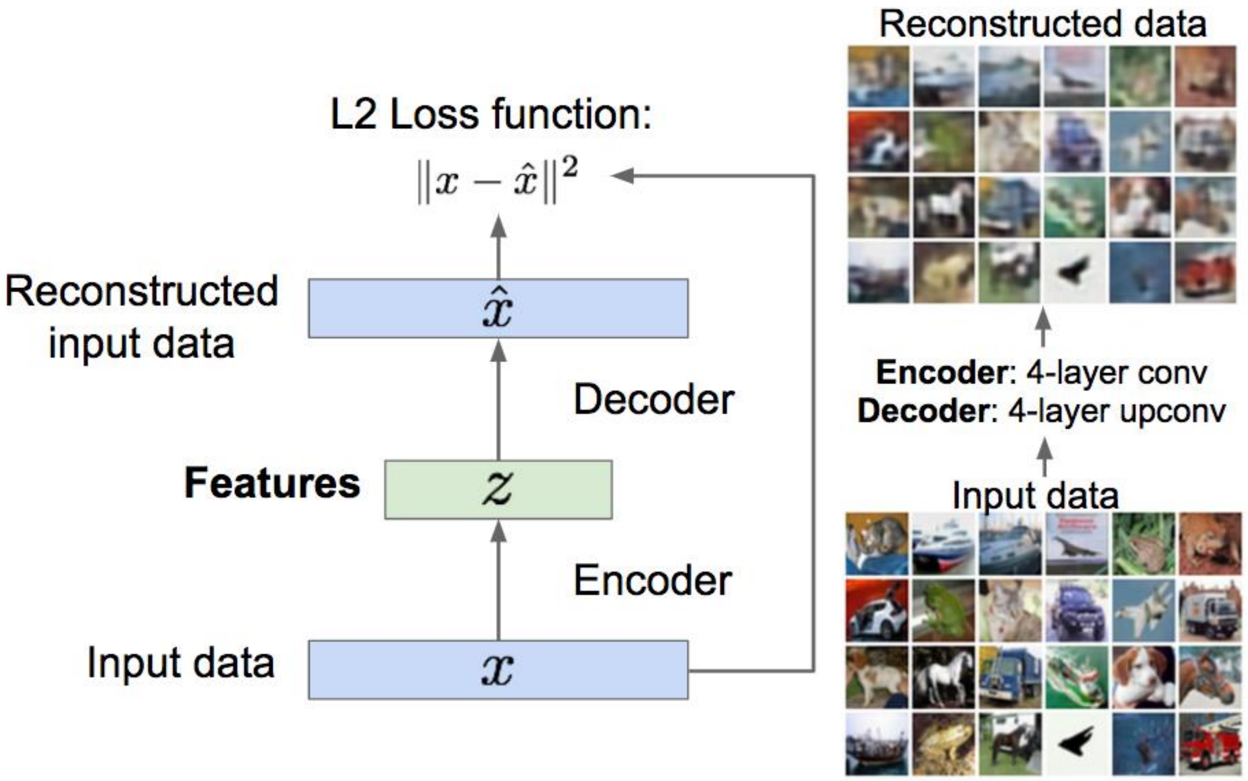
Data: x

Just data, no labels!

Goal: Learn some underlying hidden *structure* of the data

Examples: Clustering, dimensionality reduction, feature learning, density estimation, etc.

Feature Learning
(e.g., autoencoders)



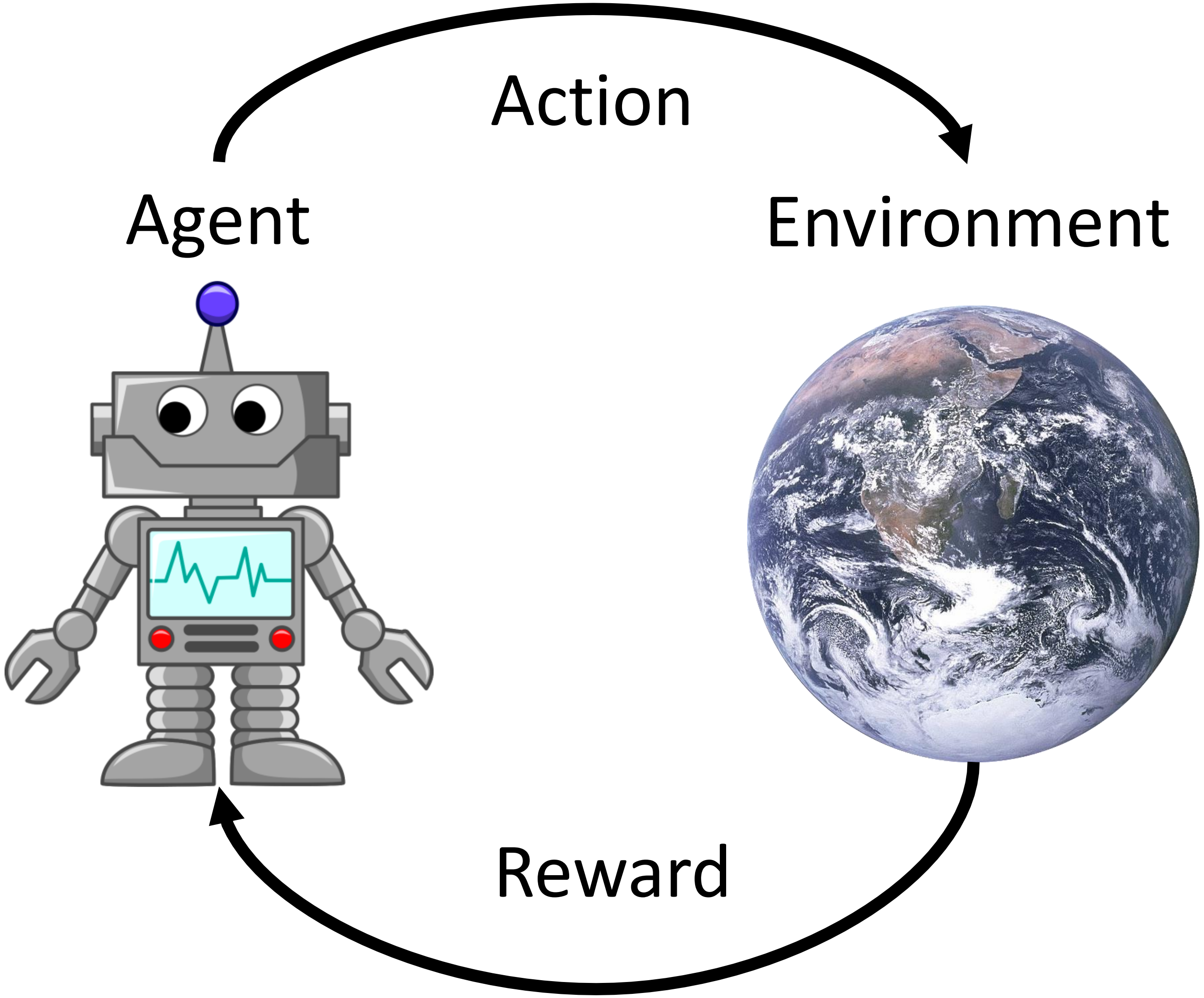
Unsupervised Learning

- Given a dataset $D = \{x_1, \dots, x_n\}$ without any labels, learning the underlying **structure** or **distribution** of the data
  - Clustering
  - Dimensionality reduction
  - Feature learning
  - Density estimation
  - Generating data
- “Learning without teacher (supervision)”

Reinforcement Learning

Problems where an **agent** performs **actions** in **environment**, and receives **rewards**

Goal: Learn how to take actions that maximize reward



Earth photo is in the public domain
Robot image is in the public domain

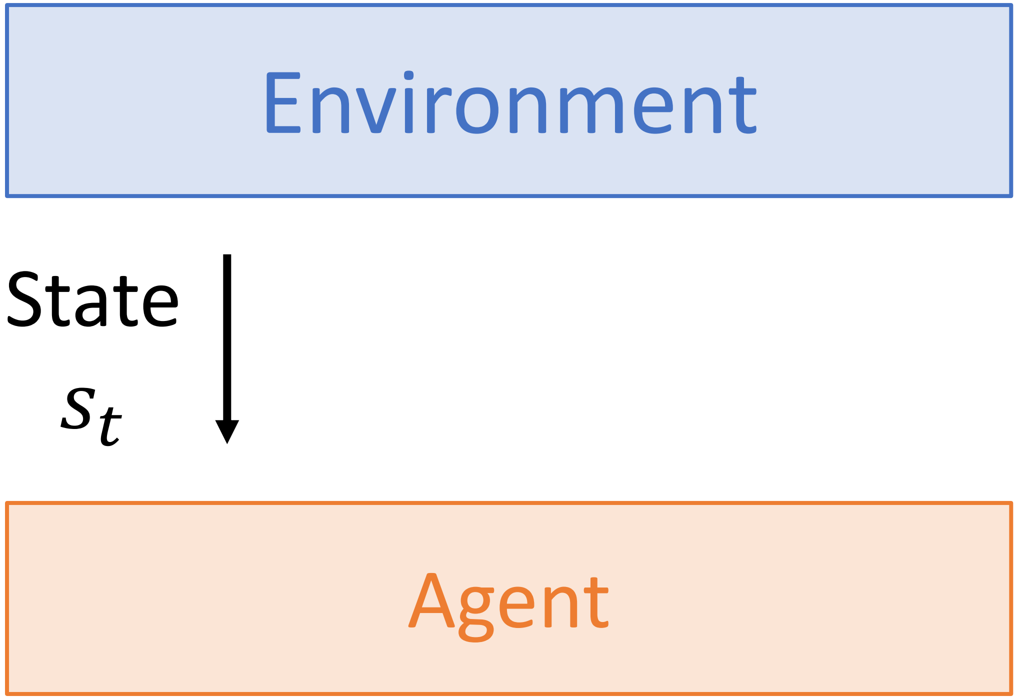
Overview

- What is reinforcement learning?
- Algorithms for reinforcement learning
    - Q-Learning
    - Policy Gradients

This is just a taste! Can easily teach entire courses on (deep) RL:

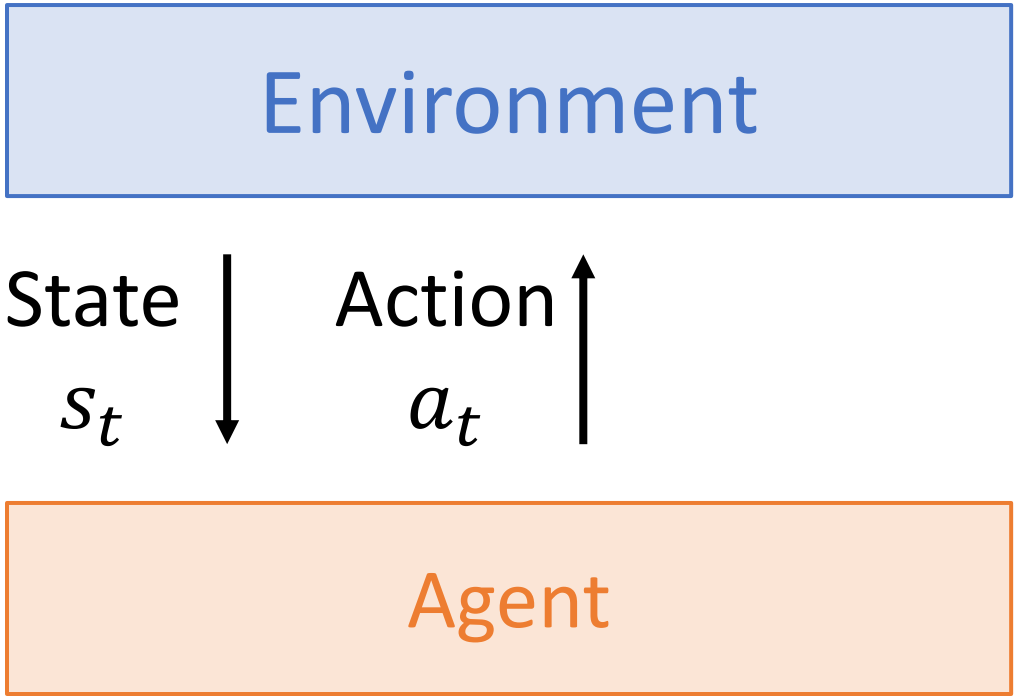
- Berkeley CS 285
- Stanford CS 234
- Yonsei STA3145

Reinforcement Learning



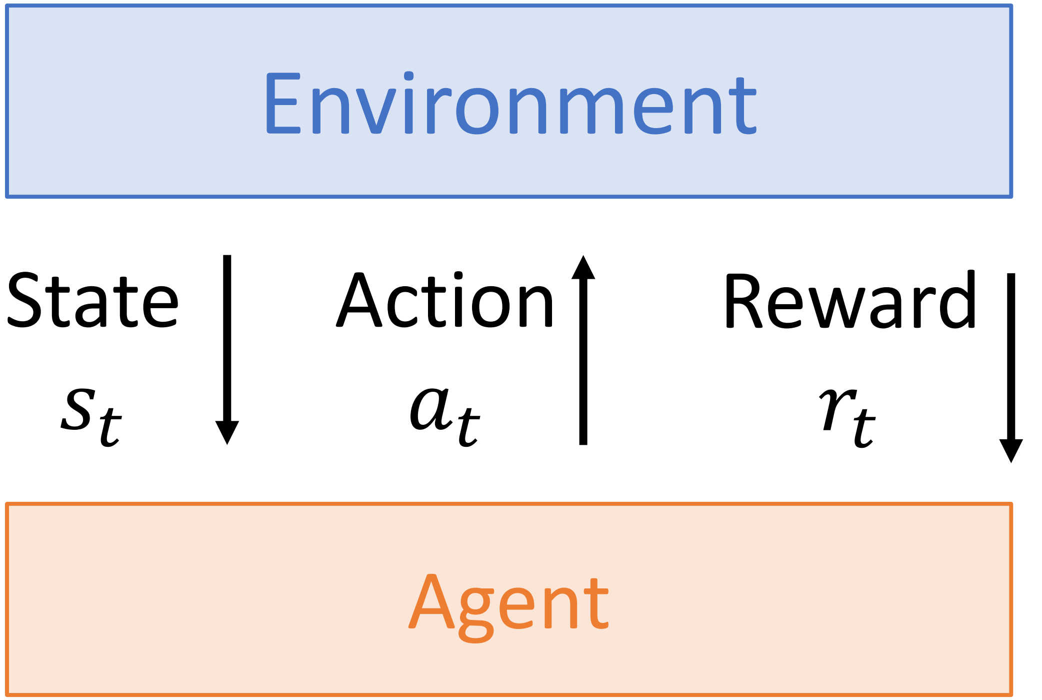
The agent sees a **state**; may be noisy or incomplete

Reinforcement Learning



The makes an **action** based on what it sees

Reinforcement Learning



Environment

State s_t

Action a_t

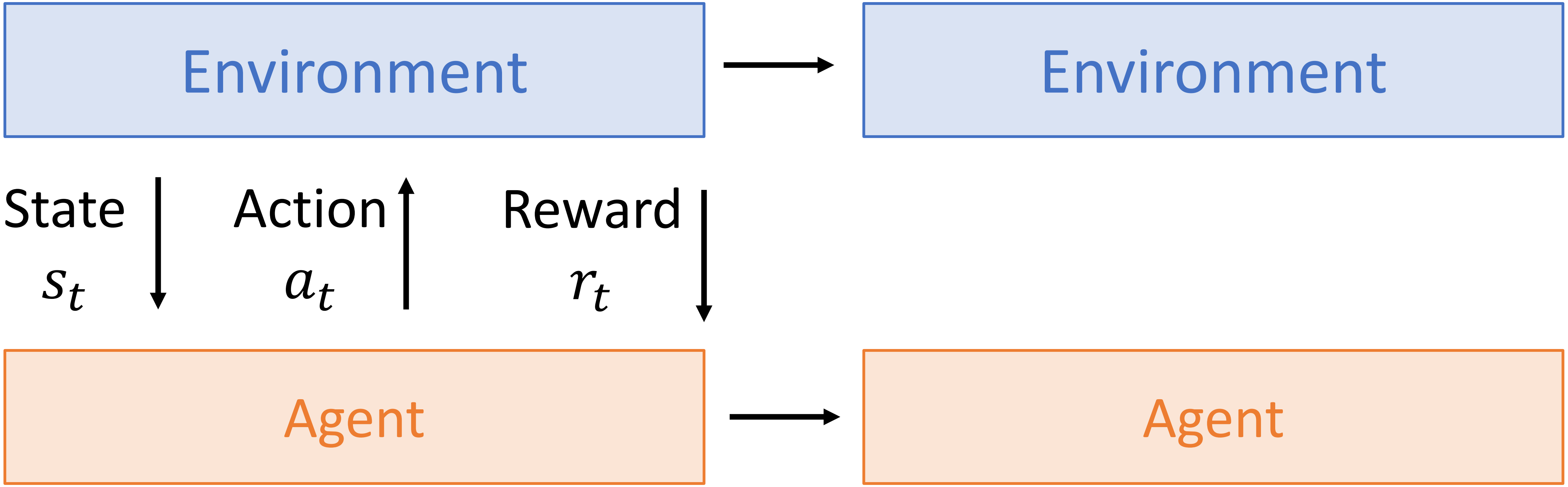
Reward r_t

Agent

Reward tells the agent how well it is doing

Reinforcement Learning

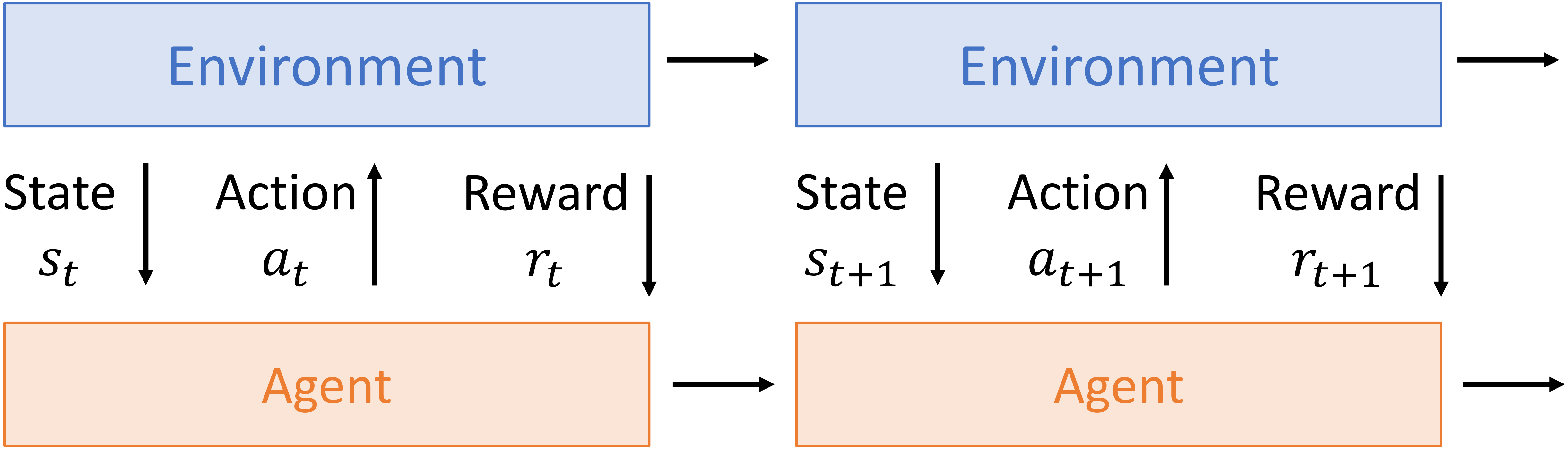
Action causes change to environment



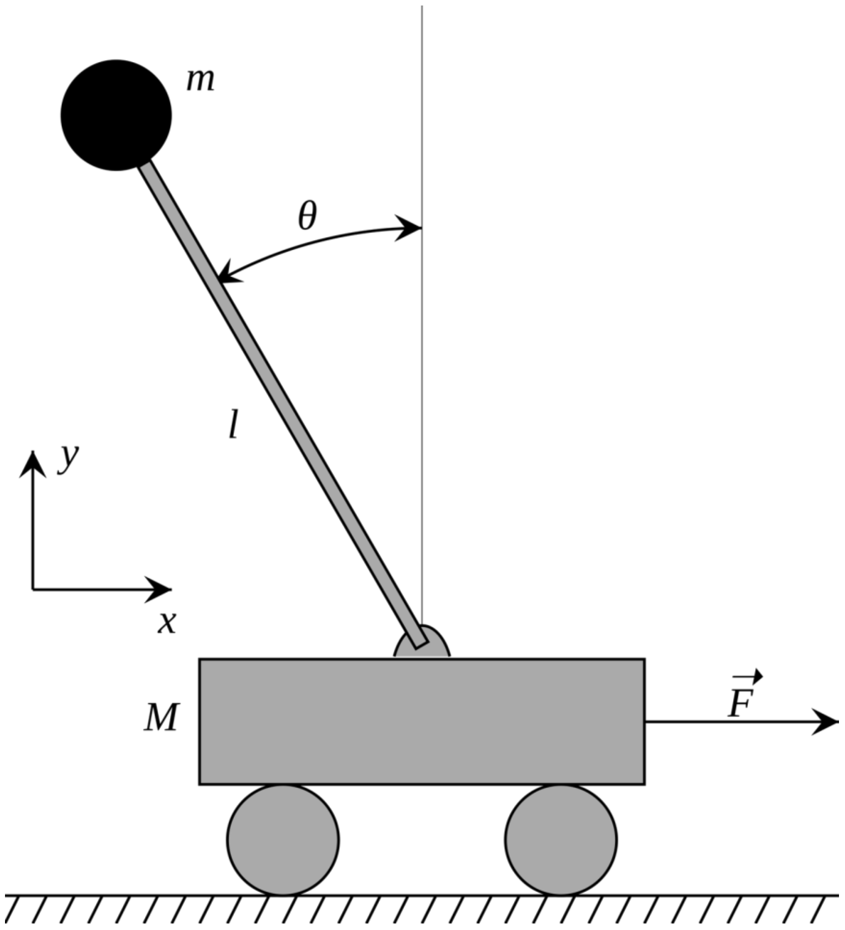
Agent learns

Reinforcement Learning

Process repeats



Example: Cart-Pole Problem



This image is CC0 public domain

Objective: Balance a pole on top of a movable cart

State: angle, angular speed, position, horizontal velocity

Action: horizontal force applied on the cart

Reward: 1 at each time step if the pole is upright

Example: Robot Locomotion

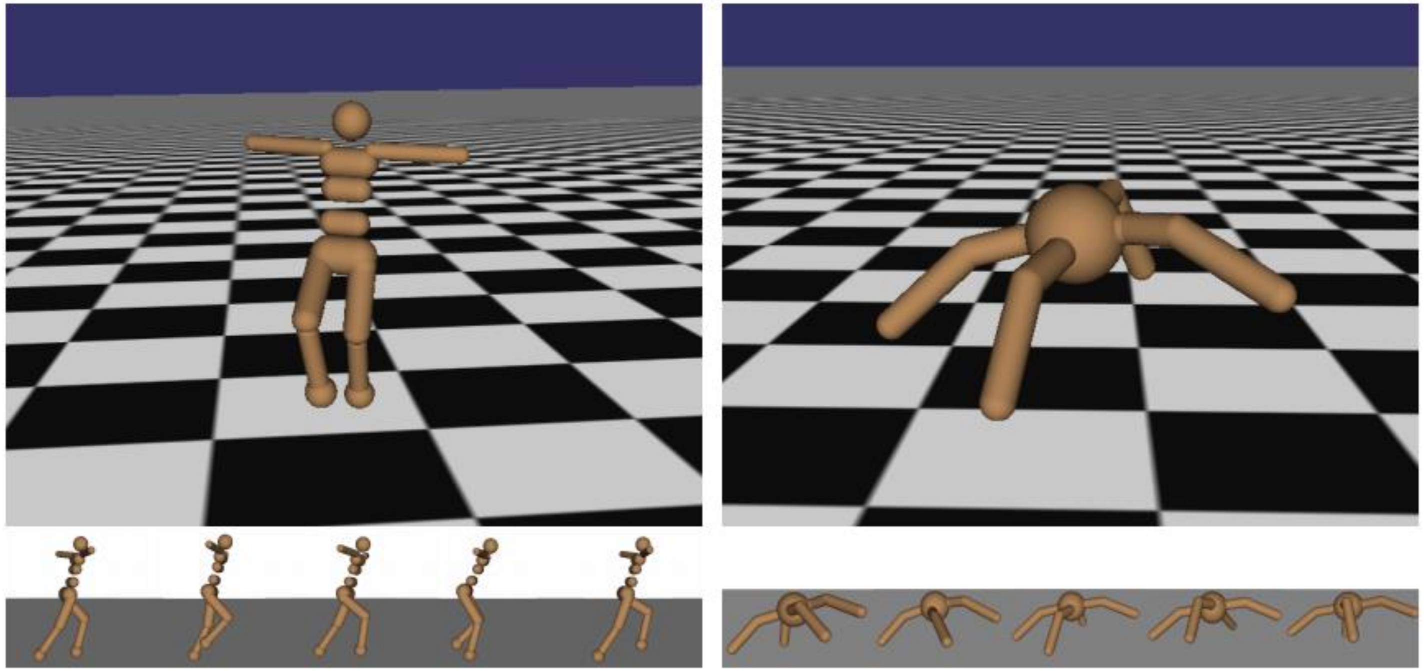


Figure from: Schulman et al, “High-Dimensional Continuous Control Using Generalized Advantage Estimation”, ICLR 2016

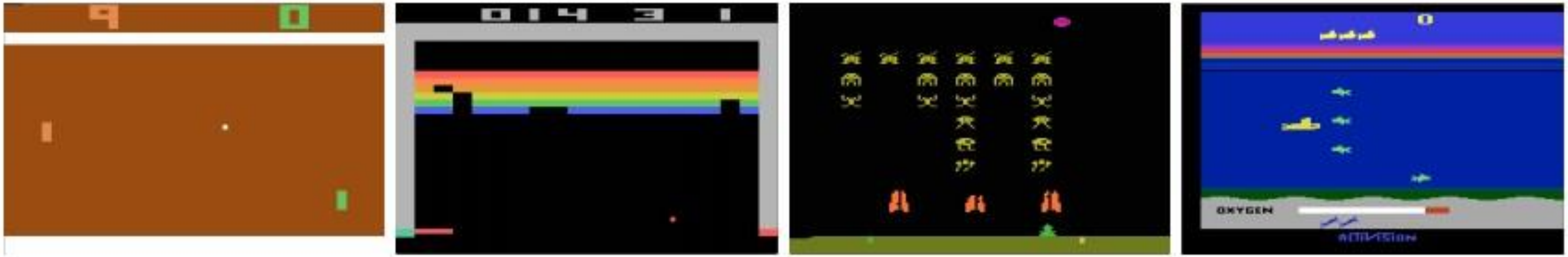
Objective: Make the robot move forward

State: Angle, position, velocity of all joints

Action: Torques applied on joints

Reward: 1 at each time step upright + forward movement

Example: Atari Games

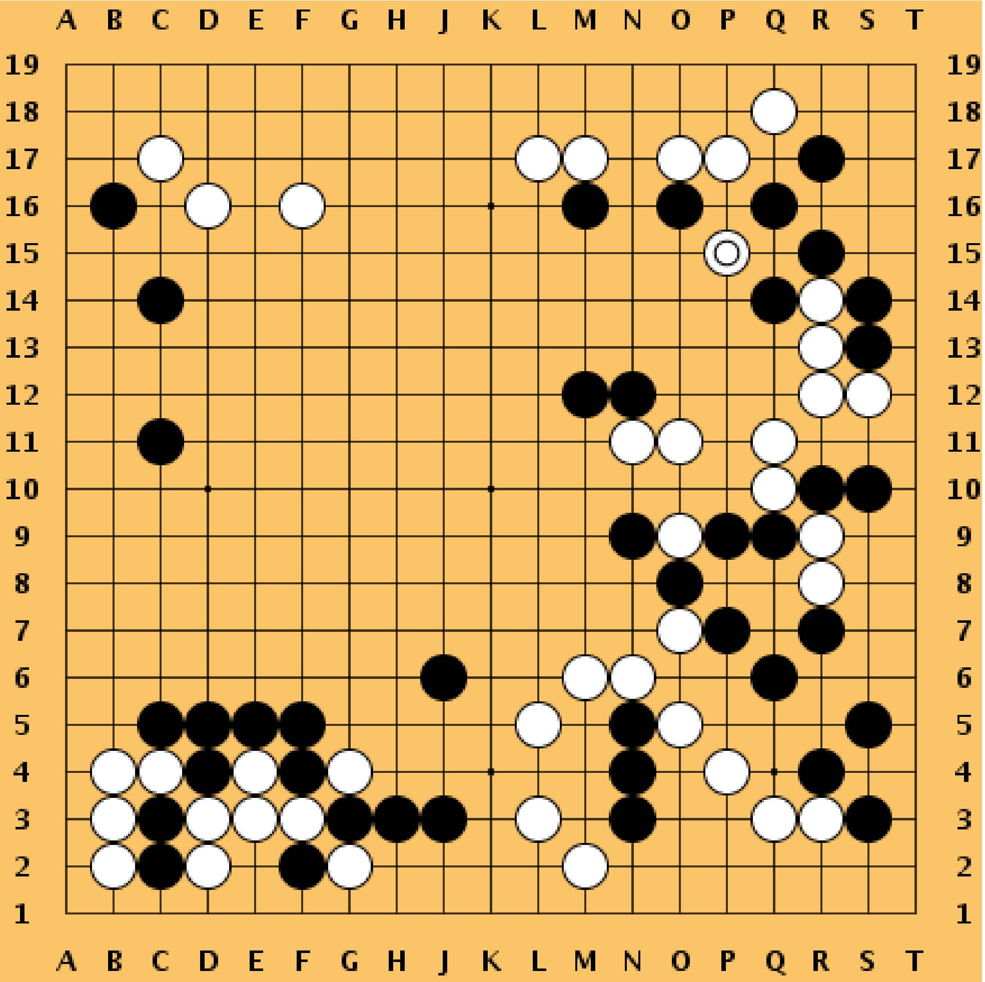


Objective: Complete the game with the highest score

State: Raw pixel inputs of the game screen
Action: Game controls e.g., Left, Right, Up, Down
Reward: Score increase/decrease at each time step

Mnih et al, “Playing Atari with Deep Reinforcement Learning”, NeurIPS Deep Learning Workshop, 2013

Example: Go



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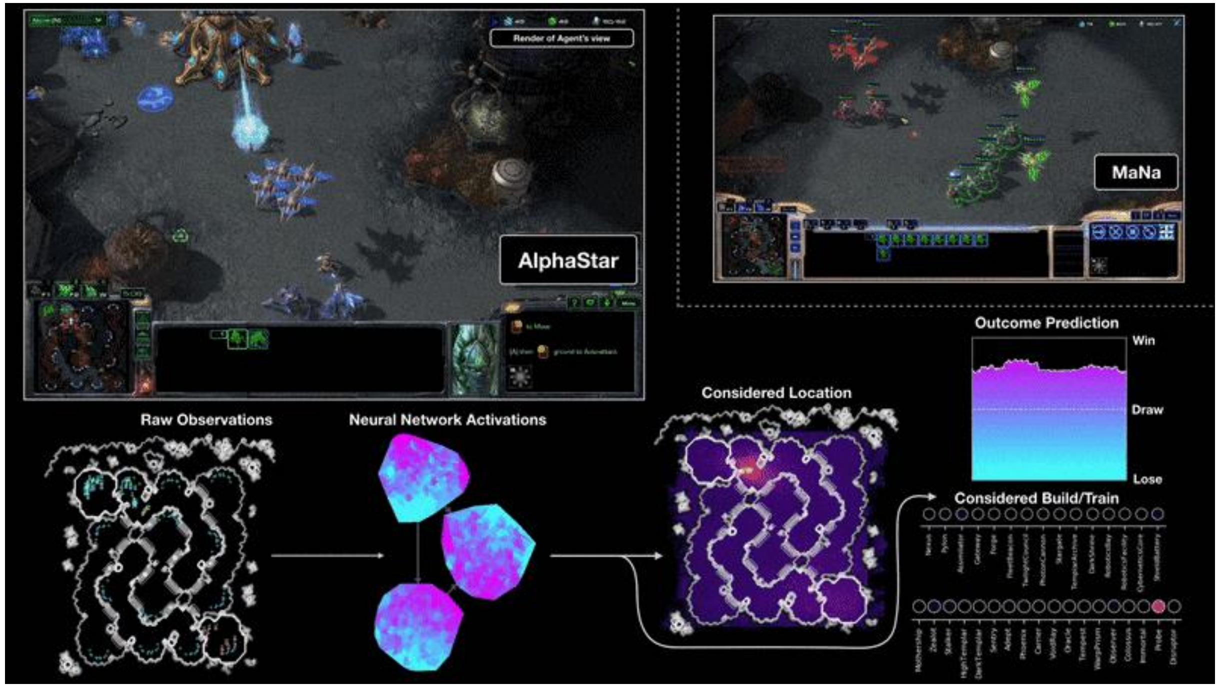
Objective: Win the game!

State: Position of all pieces

Action: Where to put the next piece down

Reward: On last turn: 1 if you won, 0 if you lost

Example: Real-Time Video Games



Objective: Win the game!

State: Real-time status

Action: What/where to do (under limited APM)

Reward: At the end: 1 if you won, 0 if you lost

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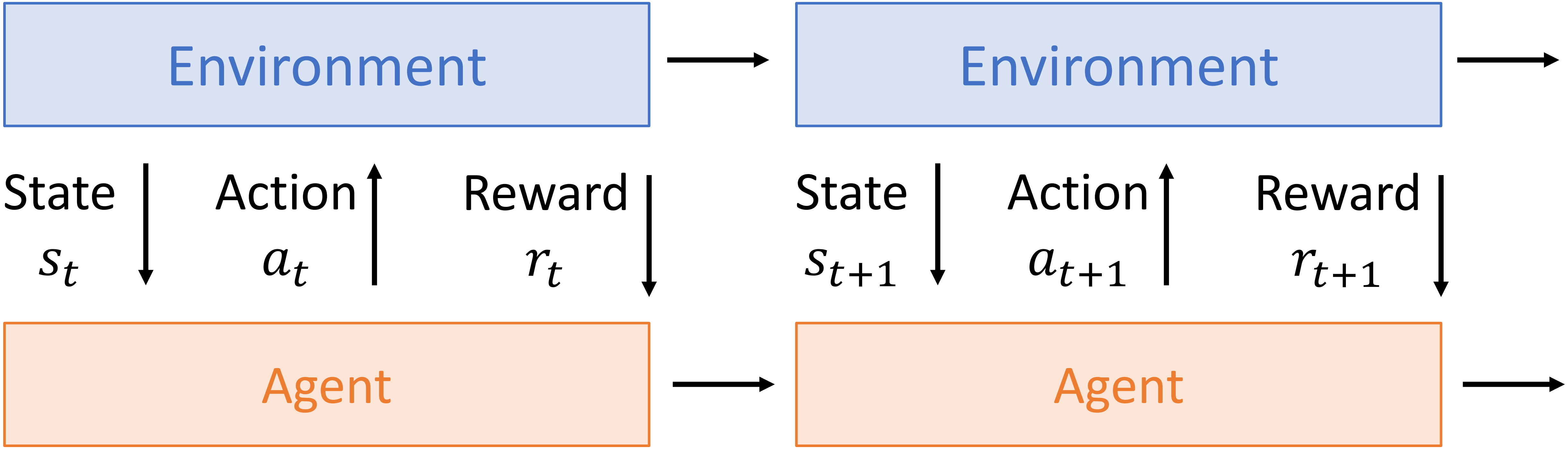
Reinforcement Learning vs. Supervised Learning



Environment

Environment

State s_t

Action a_t

Reward r_t

State s_{t+1}

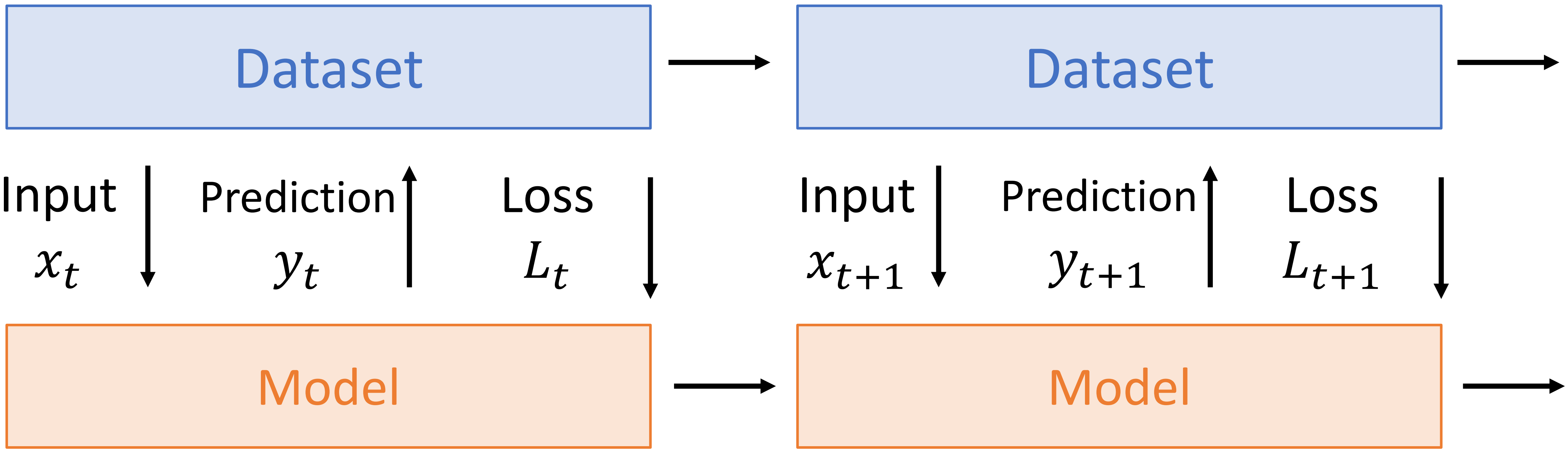
Action a_{t+1}

Reward r_{t+1}

Agent

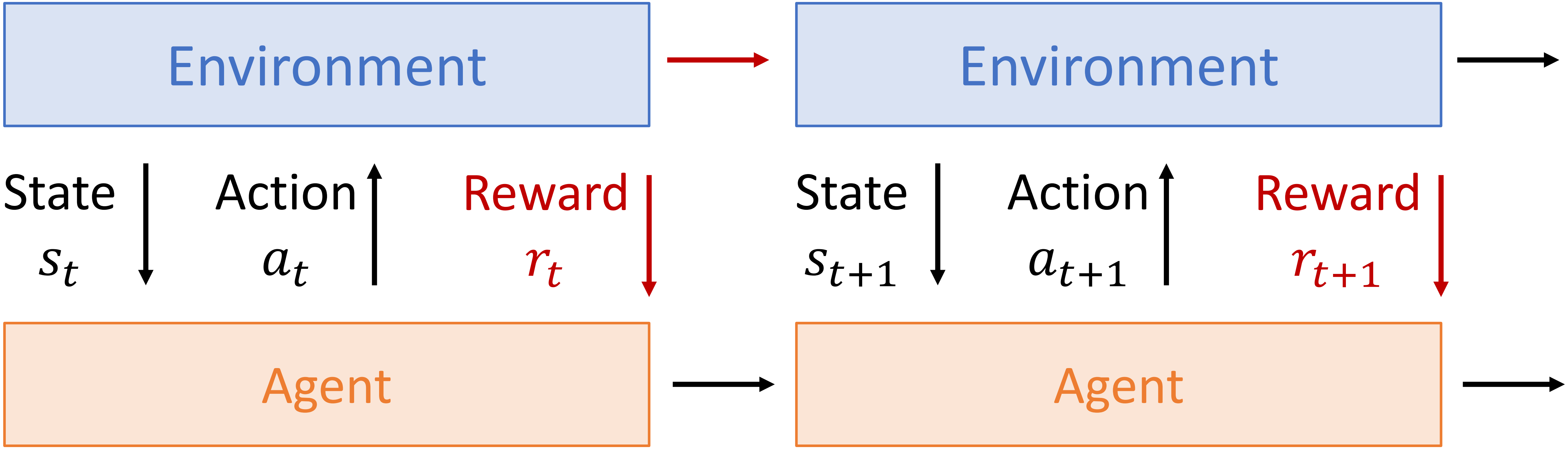
Agent

Reinforcement Learning vs. Supervised Learning



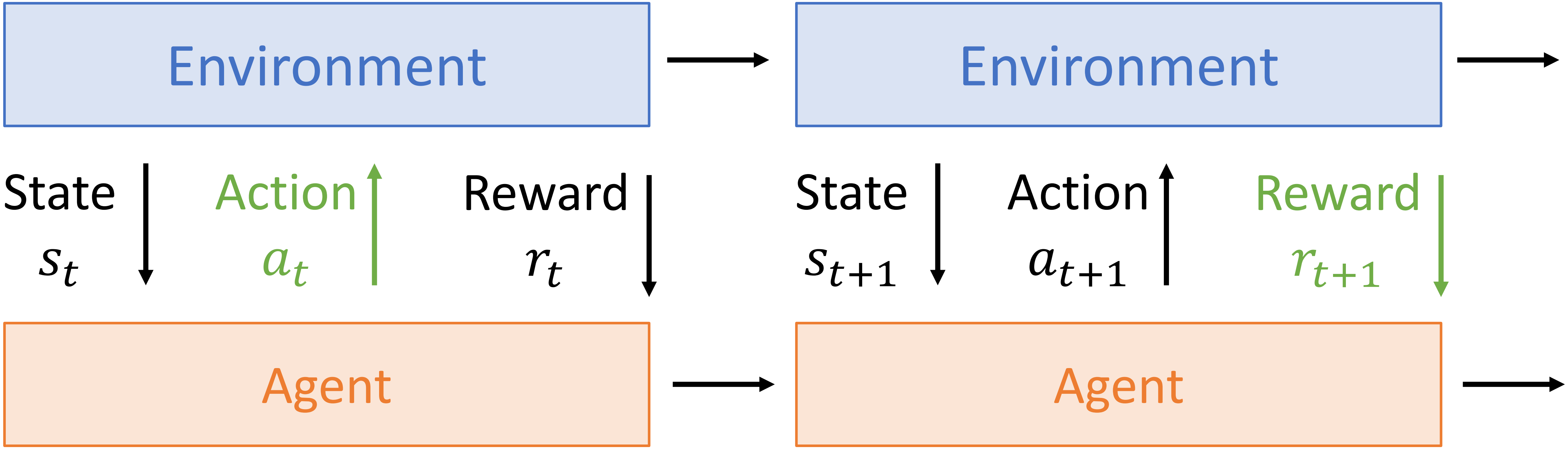
Why is RL different from normal supervised learning?

Reinforcement Learning vs. Supervised Learning



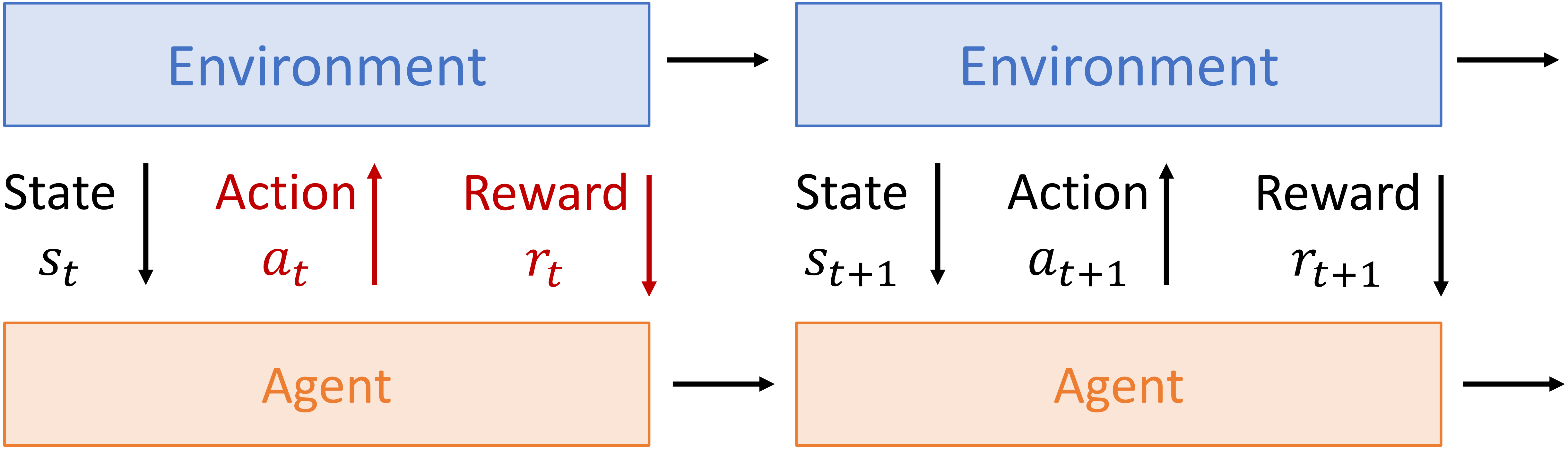
Stochasticity: Rewards and state transitions may be random

Reinforcement Learning vs. Supervised Learning



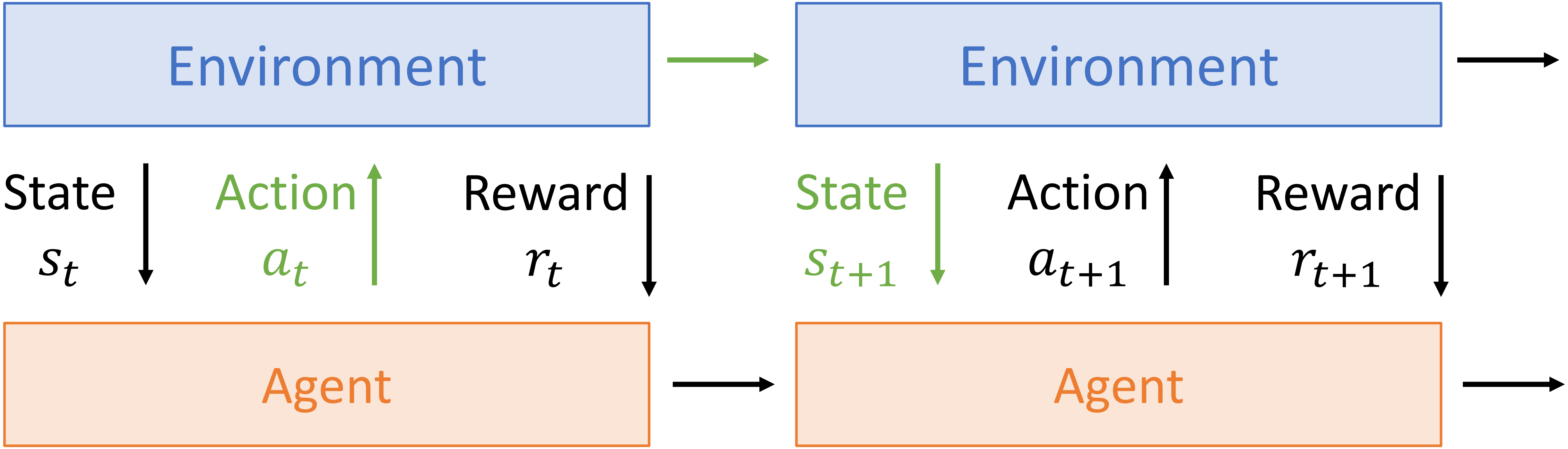
Credit assignment: Reward r_t may not directly depend on action a_t

Reinforcement Learning vs. Supervised Learning



Nondifferentiable: Can’t backprop through world; can’t compute $\partial r_t / \partial a_t$

Reinforcement Learning vs. Supervised Learning



Nonstationary: What the agent experiences depends on how it acts

Markov Decision Process (MDP)

Mathematical formalization of the RL problem: A tuple (S, A, R, P, γ)

S: Set of possible states
A: Set of possible actions
R: Distribution of reward given (state, action) pair
P: Transition probability: distribution over next state given (state, action)
γ: Discount factor (tradeoff between future and present rewards)

Markov Property: The current state completely characterizes the state of the world. Rewards and next states depend only on current state, not history.

Markov Decision Process (MDP)

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Agent executes a **policy** π giving distribution of actions conditioned on states
Goal: Find policy π^* that maximizes cumulative discounted reward: $\sum_t \gamma^t r_t$

Markov Decision Process (MDP)

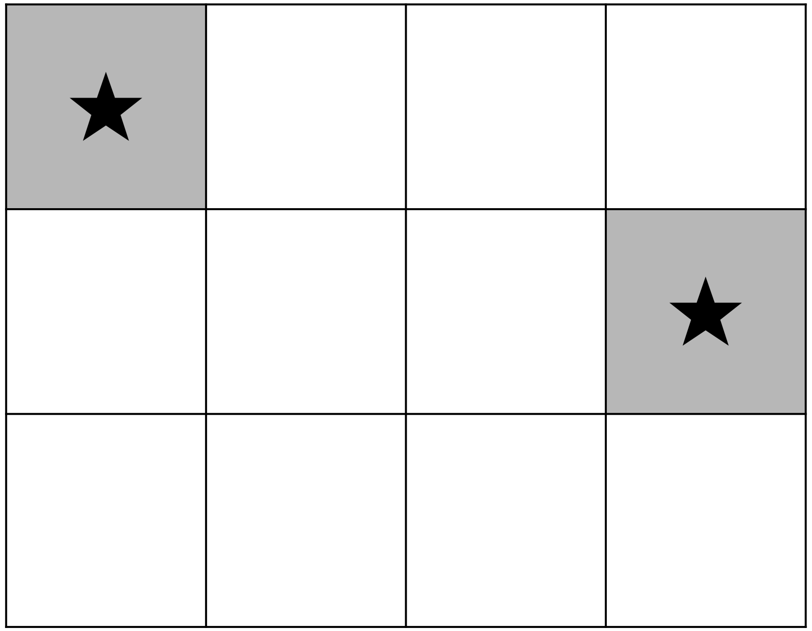
- At $t = 0$, environment samples an initial state $s_0 \sim p(s_0)$
- Then, for $t = 0$ until done:
- Agent selects action $a_t \sim \pi(a \mid s_t)$
- Environment samples reward $r_t \sim R(r \mid s_t, a_t)$
- Environment samples next state $s_{t+1} \sim P(s \mid s_t, a_t)$
- Agent receives reward r_t and next state s_{t+1}

A simple MDP: Grid World

Actions:

1. Right
2. Left
3. Up
4. Down

States



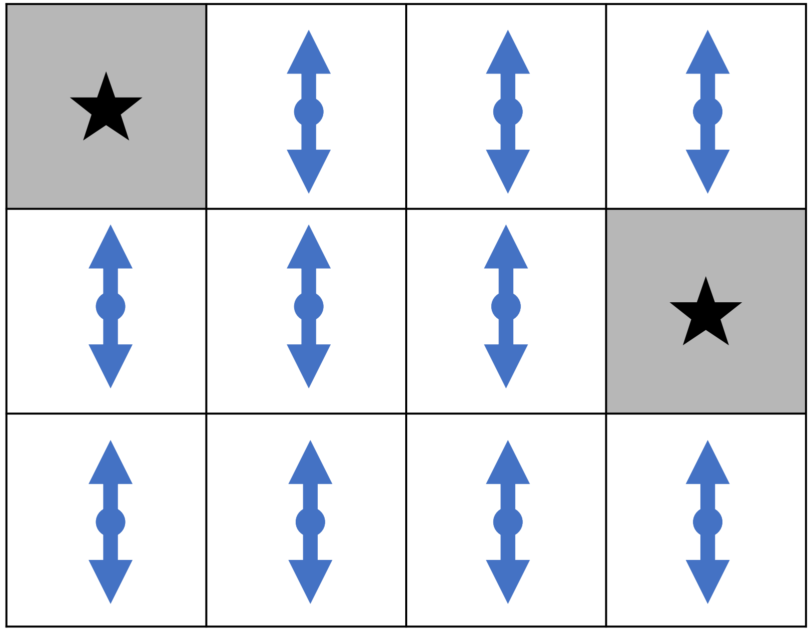
Reward

Set a negative "reward" for each transition (e.g., r = -1)

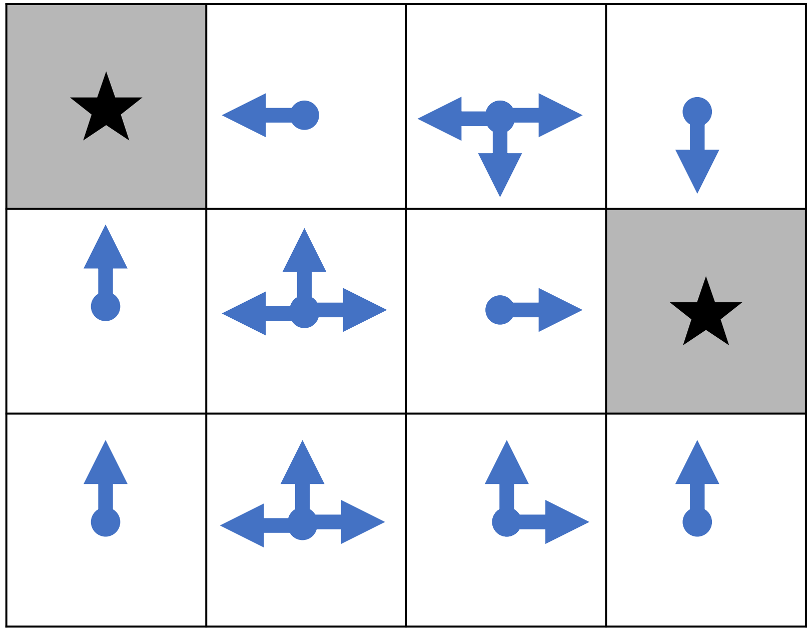
Objective: Reach one of the terminal states in as few moves as possible

A simple MDP: Grid World

Bad policy



Optimal Policy



Markov Decision Process (MDP)

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Markov Property: The current state completely characterizes the state of the world. Rewards and next states depend only on current state, not history.

Cf. Partially Observable MDP (POMDP)

Mathematical formalization of the RL problem: A tuple $(S, A, R, P, \Omega, O, \gamma)$

S: Set of possible states

A: Set of possible actions

R: Distribution of reward given (state, action) pair

P: Transition probability: distribution over next state given (state, action)

γ: Discount factor (tradeoff between future and present rewards)

Ω: Set of observations

O: Conditional observation probabilities given (state, action)

Markov Property: The current state completely characterizes the state of the world. Rewards and next states depend only on current state, not history.

Finding Optimal Policies

Goal: Find the optimal policy π^* that maximizes (discounted) sum of rewards.

Problem: Lots of randomness! Initial state, transition probabilities, rewards

Solution: Maximize the expected sum of rewards

$$\pi^* = \arg\max_{\pi} \mathbb{E}\left[\sum_{t \geq 0} \gamma^t \, r_t \mid \pi\right]$$

$$\begin{aligned} s_0 &\sim p(s_0) \\ a_t &\sim \pi(a \mid s_t) \\ s_{t+1} &\sim P(s \mid s_t, a_t) \end{aligned}$$

Value Function and Q-Function

Following a policy π produces **sample trajectories** (or paths) $s_0, a_0, r_0, s_1, a_1, r_1, \dots$

How good is a state? The **value function** at state s, is the expected cumulative reward from following the policy from state s:

$$V^\pi(s) = \mathbb{E}\left[\sum_{t\geq 0} \gamma^t r_t \mid s_0 = s, \pi\right]$$

How good is a state-action pair? The **Q-function** at state s and action a, is the expected cumulative reward from taking action a in state s and then following the policy:

$$Q^\pi(s,a) = \mathbb{E}\left[\sum_{t\geq 0} \gamma^t r_t \mid s_0 = s, a_0 = a, \pi\right]$$

Bellman Equation

Optimal Q-function: $Q^*(s,a)$ is the Q-function for the optimal policy π^*
It gives the max possible future reward when taking action a in state s:

$$Q^*(s,a) = \max_{\pi} \mathbb{E}\left[\sum_{t\geq 0} \gamma^t r_t \mid s_0 = s, a_0 = a, \pi\right]$$

Q^* encodes the optimal policy: $\pi^*(s) = \arg\max_{a\prime} Q^*(s,a')$

Bellman Equation: Q^* satisfies the following recurrence relation:

$$Q^*(s,a) = \mathbb{E}_{r,s\prime}\left[r + \gamma \max_{a\prime} Q^*(s',a')\right]$$

Where $r \sim R(s,a), s' \sim P(s,a)$

Intuition: After taking action a in state s, we get reward r and move to a new state s'. After that, the max possible reward we can get is $\max_{a\prime} Q^*(s',a')$

Solving for the optimal policy: Value Iteration

Bellman Equation: Q^* satisfies the following recurrence relation:

$$Q^*(s,a) = \mathbb{E}_{r,s'}\left[r + \gamma \max_{a'} Q^*(s',a')\right]$$

Where $r \sim R(s,a), s' \sim P(s,a)$

Idea: If we find a function $Q(s,a)$ satisfying the Bellman Equation, then it must be Q^*.
Start with a random Q, and use the Bellman Equation as an update rule:

$$Q_{i+1}(s,a) = \mathbb{E}_{r,s'}\left[r + \gamma \max_{a'} Q_i(s',a')\right]$$

Where $r \sim R(s,a), s' \sim P(s,a)$

Amazing fact: Q_i converges to Q^* as $i \rightarrow \infty$
Problem: Need to keep track of $Q(s,a)$ for all (state, action) pairs – impossible if infinite
Solution: Approximate $Q(s,a)$ with a neural network, use Bellman Equation as a loss!

Solving for the optimal policy: Deep Q-Learning

Bellman Equation: Q^* satisfies the following recurrence relation:

$$Q^*(s,a) = \mathbb{E}_{r,s'}\left[r + \gamma \max_{a'} Q^*(s',a')\right]$$

Where $r \sim R(s,a), s' \sim P(s,a)$

Train a neural network (with weights θ) to approximate Q^*: $Q^*(s,a) \approx Q(s,a;\theta)$

Use the Bellman Equation to tell what Q should output for a given state and action:

$$y_{s,a,\theta} = \mathbb{E}_{r,s'}\left[r + \gamma \max_{a'} Q(s',a';\theta)\right]$$

Where $r \sim R(s,a), s' \sim P(s,a)$

Use this to define the loss for training Q: $L(s,a) = \left(Q(s,a;\theta) - y_{s,a,\theta}\right)^2$

Problem: Nonstationary! “Target” for $Q(s,a)$ depends on the current weights θ!

Problem: How to sample batches of data for training?

Case Study: Playing Atari Games

Objective: Complete the game with the highest score

State: Raw pixel inputs of the game screen
Action: Game controls e.g., Left, Right, Up, Down
Reward: Score increase/decrease at each time step

Mnih et al, “Playing Atari with Deep Reinforcement Learning”, NeurIPS Deep Learning Workshop, 2013

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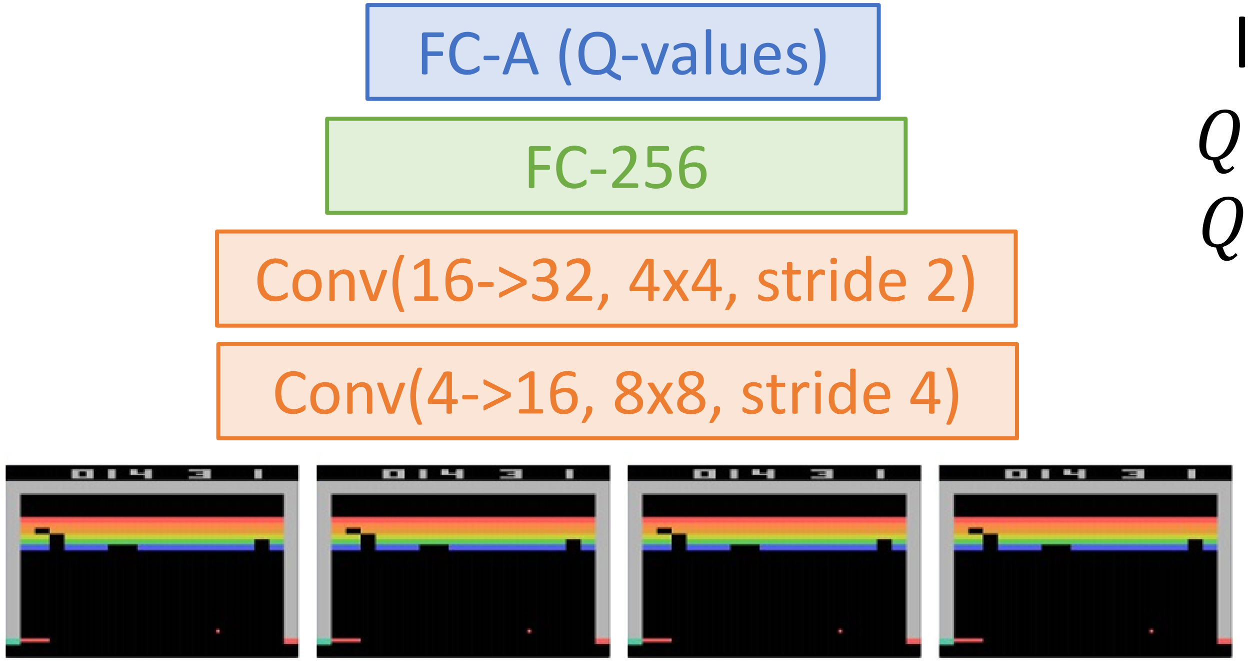
Case Study: Playing Atari Games

$Q(s, a; \theta)$
Neural network
with weights θ

Network output:
Q-values for all actions



FC-A (Q-values)

FC-256

Conv(16->32, 4x4, stride 2)

Conv(4->16, 8x8, stride 4)

With 4 actions: last layer gives values $Q(s_t, a_1), Q(s_t, a_2), Q(s_t, a_3), Q(s_t, a_4)$

Network input: state s_t: 4x84x84 stack of last 4 frames (after RGB->grayscale conversion, downsampling, and cropping)

https://www.youtube.com/watch?v=V1eYniJ0Rnk

Q-Learning

Q-Learning: Train network $Q_\theta(s, a)$ to estimate future rewards for every (state, action) pair

Problem: For some problems, this can be a hard function to learn.
For some problems, it is easier to learn a mapping from states to actions

Q-Learning vs. Policy Gradients

Q-Learning: Train network $Q_\theta(s, a)$ to estimate future rewards for every (state, action) pair

Problem: For some problems, this can be a hard function to learn.
For some problems, it is easier to learn a mapping from states to actions

Policy Gradients: Train a network $\pi_\theta(a \mid s)$ that takes state as input, gives distribution over which action to take in that state

Objective function: Expected future rewards when following policy π_θ**:**

$$J(\theta) = \mathbb{E}_{r \sim p_\theta}\left[\sum_{t \geq 0} \gamma^t r_t\right]$$

Find the optimal policy by maximizing: $\theta^* = \arg\max_\theta J(\theta)$ **(Use gradient ascent!)**

Policy Gradients

Objective function: Expected future rewards when following policy π_θ:

$$J(\theta) = \mathbb{E}_{r\sim p_\theta}\left[\sum_{t\geq 0} \gamma^t \, r_t\right]$$

Find the optimal policy by maximizing: $\theta^* = \arg\max_\theta J(\theta)$ **(Use gradient ascent!)**

Problem: Nondifferentiability! Don't know how to compute $\frac{\partial J}{\partial \theta}$

General formulation: $J(\theta) = \mathbb{E}_{x\sim p_\theta}[f(x)]$ Want to compute $\frac{\partial J}{\partial \theta}$

Policy Gradients: REINFORCE Algorithm

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General formulation: $J(\theta) = \mathbb{E}_{x \sim p_\theta}[f(x)]$ Want to compute $\frac{\partial J}{\partial \theta}$

$$\frac{\partial J}{\partial \theta} = \frac{\partial}{\partial \theta} \mathbb{E}_{x \sim p_\theta}[f(x)] = \frac{\partial}{\partial \theta} \int_X p_\theta(x) f(x) dx$$

Policy Gradients: REINFORCE Algorithm

General formulation: $J(\theta) = \mathbb{E}_{x\sim p_\theta}[f(x)]$ Want to compute $\frac{\partial J}{\partial \theta}$

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Policy Gradients: REINFORCE Algorithm

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$$\frac{\partial}{\partial \theta} \log p_\theta(x) = \frac{1}{p_\theta(x)} \frac{\partial}{\partial \theta} p_\theta(x)$$

Policy Gradients: REINFORCE Algorithm

General formulation: $J(\theta) = \mathbb{E}_{x \sim p_\theta}[f(x)]$ Want to compute $\frac{\partial J}{\partial \theta}$

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$$\frac{\partial}{\partial \theta} \log p_\theta(x) = \frac{1}{p_\theta(x)} \frac{\partial}{\partial \theta} p_\theta(x) \Rightarrow \frac{\partial}{\partial \theta} p_\theta(x) = p_\theta(x) \frac{\partial}{\partial \theta} \log p_\theta(x)$$

Policy Gradients: REINFORCE Algorithm

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$$\frac{\partial J}{\partial \theta} = \int_X f(x) p_\theta(x) \frac{\partial}{\partial \theta} \log p_\theta(x) \; dx$$

Policy Gradients: REINFORCE Algorithm

General formulation: $J(\theta) = \mathbb{E}_{x \sim p_\theta}[f(x)]$ Want to compute $\frac{\partial J}{\partial \theta}$

$$\frac{\partial J}{\partial \theta} = \frac{\partial}{\partial \theta} \mathbb{E}_{x \sim p_\theta}[f(x)] = \frac{\partial}{\partial \theta} \int_X p_\theta(x) f(x) dx = \int_X f(x) \frac{\partial}{\partial \theta} p_\theta(x) dx$$

$$\frac{\partial}{\partial \theta} \log p_\theta(x) = \frac{1}{p_\theta(x)} \frac{\partial}{\partial \theta} p_\theta(x) \Rightarrow \frac{\partial}{\partial \theta} p_\theta(x) = p_\theta(x) \frac{\partial}{\partial \theta} \log p_\theta(x)$$

$$\frac{\partial J}{\partial \theta} = \int_X f(x) p_\theta(x) \frac{\partial}{\partial \theta} \log p_\theta(x) \, dx = \mathbb{E}_{x \sim p_\theta}\left[f(x) \frac{\partial}{\partial \theta} \log p_\theta(x) \right]$$

Approximate the expectation via sampling!

Policy Gradients: REINFORCE Algorithm

Goal: Train a network $\pi_\theta(a \mid s)$ that takes state as input, gives distribution over which action to take in that state

Define: Let $x = (s_0, a_0, s_1, a_1, \dots)$ be the sequence of states and actions we get when following policy π_θ. It's random: $x \sim p_\theta(x)$

$$p_\theta(x) = \prod_{t \geq 0} P(s_{t+1} \mid s_t, a_t) \pi_\theta(a_t \mid s_t)$$

Policy Gradients: REINFORCE Algorithm

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Define: Let $x = (s_0, a_0, s_1, a_1, \dots)$ be the sequence of states and actions we get when following policy π_θ. It's random: $x \sim p_\theta(x)$

$$p_\theta(x) = \prod_{t \geq 0} P(s_{t+1} \mid s_t, a_t)\pi_\theta(a_t \mid s_t) \Rightarrow \log p_\theta(x) = \sum_{t \geq 0} (\log P(s_{t+1} | s_t, a_t) + \log \pi_\theta(a_t | s_t))$$

Policy Gradients: REINFORCE Algorithm

Goal: Train a network $\pi_\theta(a \mid s)$ that takes state as input, gives distribution over which action to take in that state

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Expected reward under π_θ:

$$J(\theta) = \mathbb{E}_{x \sim p_\theta}[f(x)]$$

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Sequence of states and actions when following policy π_θ

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Reward we get from state sequence x

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Gradient of predicted action scores with respect to model weights. Backprop through model π_θ!

Policy Gradients: REINFORCE Algorithm

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1. Initialize random weights θ
2. Collect trajectories x and rewards $f(x)$ using policy π_θ
3. Compute $\partial J / \partial \theta$
4. Gradient ascent step on θ
5. GOTO 2

Policy Gradients: REINFORCE Algorithm

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Intuition:

When $f(x)$ is high: Increase the probability of the actions we took.

When $f(x)$ is low: Decrease the probability of the actions we took.

Recap: Q-Learning and Policy Gradients

Q-Learning: Train network $Q_\theta(s, a)$ to estimate future rewards for every (state, action) pair
Use Bellman Equation to define loss function for training Q:

$$y_{s,a,\theta} = \mathbb{E}_{r,s'}\left[r + \gamma \max_{a'} Q(s', a'; \theta)\right] \qquad \text{Where } r \sim R(s, a), s' \sim P(s, a)$$

$$L(s, a) = \left(Q(s, a; \theta) - y_{s,a,\theta}\right)^2$$

Policy Gradients: Train a network $\pi_\theta(a \mid s)$ that takes state as input, gives distribution over which action to take in that state. Use REINFORCE rule for computing gradients:

$$J(\theta) = \mathbb{E}_{x \sim p_\theta}[f(x)] \qquad \frac{\partial J}{\partial \theta} = \mathbb{E}_{x \sim p_\theta}\left[f(x) \sum_{t \geq 0} \frac{\partial}{\partial \theta} log\, \pi_\theta(a_t | s_t)\right]$$

Improving policy gradients: Subtract **baseline** function $E[B] = 0$ to reduce variance of gradient estimator

Other Approaches

Actor-Critic: Train an actor that predicts actions (like policy gradient) and a critic that predicts the future rewards we get from taking those actions (like Q-Learning)

Sutton and Barto, "Reinforcement Learning: An Introduction", 1998; Degris et al, "Model-free reinforcement learning with continuous action in practice", 2012; Mnih et al, "Asynchronous Methods for Deep Reinforcement Learning", ICML 2016

Model-Based: Learn a model of the world's state transition function $P(s_{t+1}|s_t, a_t)$ and then use planning through the model to make decisions

Imitation Learning: Gather data about how experts perform in the environment, learn a function to imitate what they do (supervised learning approach)

Inverse Reinforcement Learning: Gather data of experts performing in environment; learn a reward function that they seem to be optimizing, then use RL on that reward function

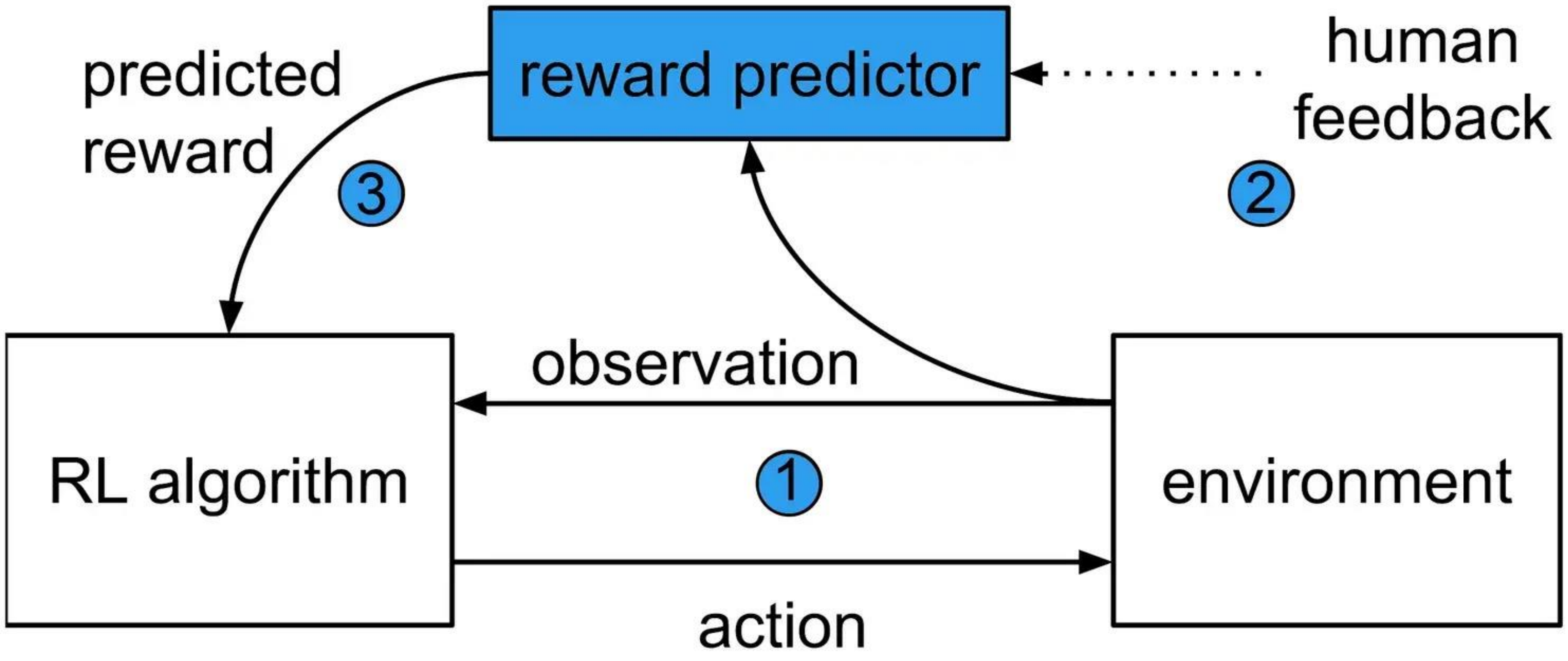
Ng et al, "Algorithms for Inverse Reinforcement Learning", ICML 2000

Adversarial Learning: Learn to fool a discriminator that classifies actions as real/fake

Ho and Ermon, "Generative Adversarial Imitation Learning", NeurIPS 2016

Human-in-the-Loop Reinforcement Learning

- Encourage to follow human preference
- E.g., reinforcement learning from human feedback (RLHF)



Source: https://www.deepmind.com/blog/learning-through-human-feedback
Christiano et al, “Deep reinforcement learning from human preferences”, NeurIPS 2017

Case Study: Playing Games

AlphaGo: (January 2016)

- Used imitation learning + tree search + RL
- Beat 18-time world champion Lee Sedol

AlphaGo Zero (October 2017)

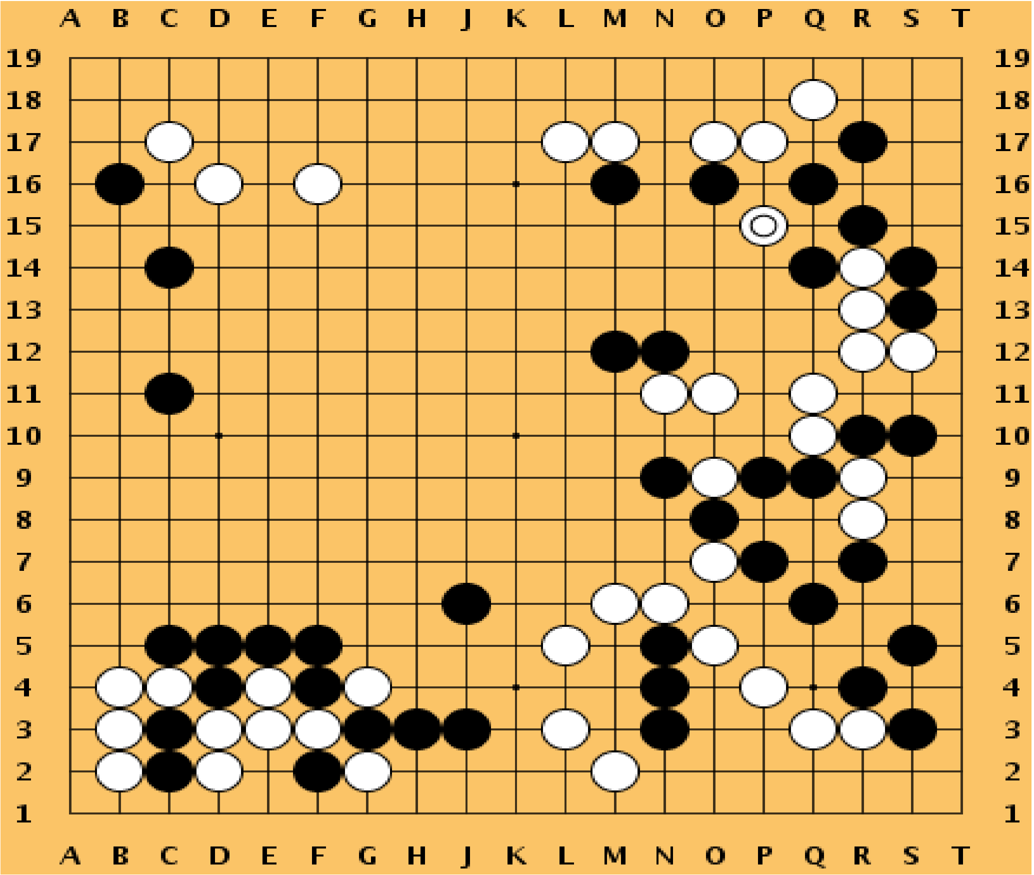
- Simplified version of AlphaGo
- No longer using imitation learning
- Beat (at the time) #1 ranked Ke Jie

Alpha Zero (December 2018)

- Generalized to other games: Chess and Shogi

MuZero (November 2019)

- Plans through a learned model of the game



Silver et al, “Mastering the game of Go with deep neural networks and tree search”, Nature 2016
Silver et al, “Mastering the game of Go without human knowledge”, Nature 2017
Silver et al, “A general reinforcement learning algorithm that masters chess, shogi, and go through self-play”, Science 2018
Schrittwieser et al, “Mastering Atari, Go, Chess and Shogi by Planning with a Learned Model”, arXiv 2019

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November 2019: Lee Sedol announces retirement

"With the debut of AI in Go games, I've realized that I'm not at the top even if I become the number one through frantic efforts"

"Even if I become the number one, there is an entity that cannot be defeated"

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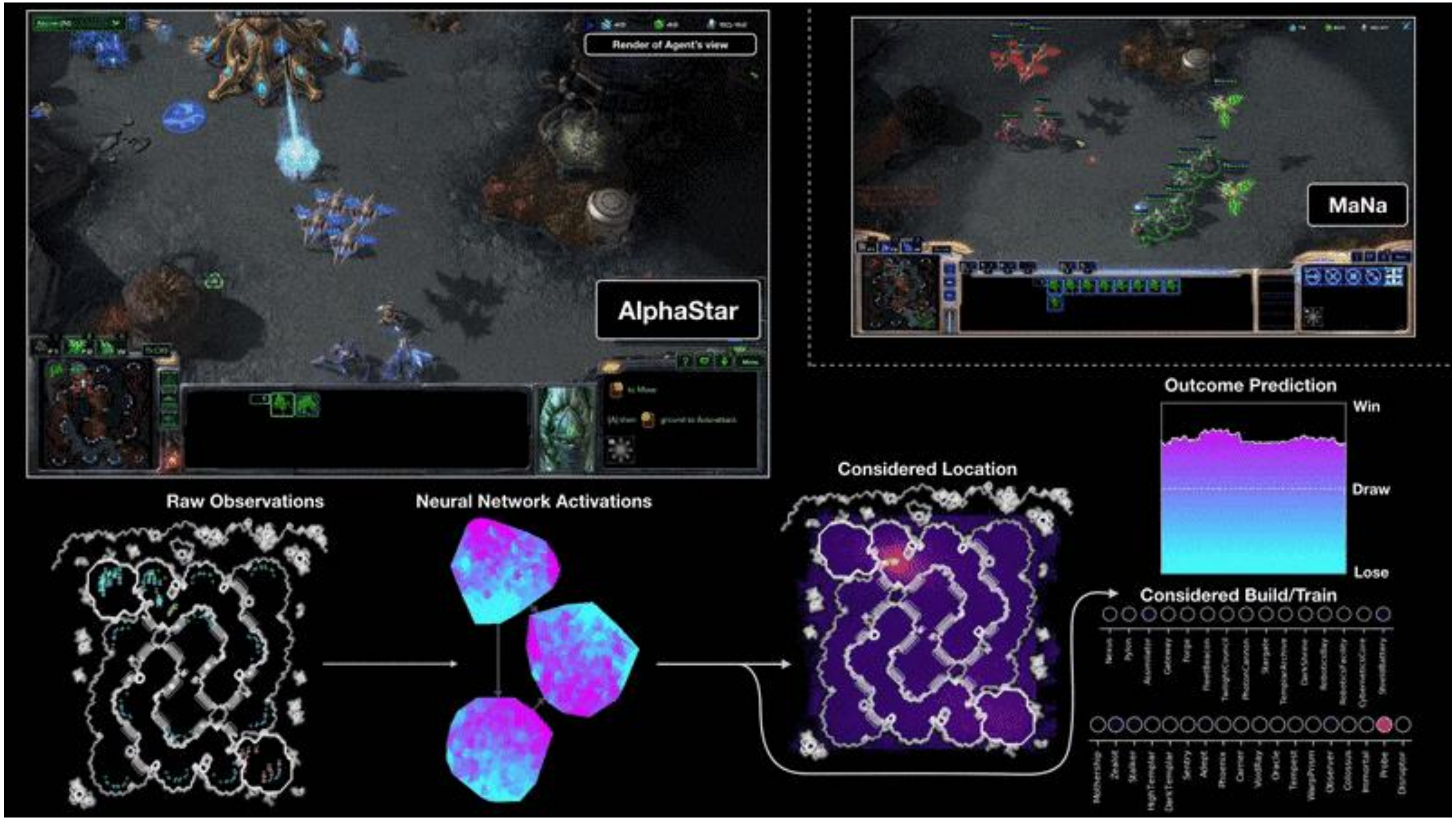
Quotes from: https://en.yna.co.kr/view/AEN20191127004800315
Image of Lee Sedol is licensed under CC BY 2.0

Case Study: Playing Games

StarCraft II: AlphaStar (October 2019)
Vinyals et al, "Grandmaster level in StarCraft II using multi-agent reinforcement learning", Science 2018

Dota 2: OpenAI Five (April 2019)
Dota 2 with Large Scale Deep Reinforcement Learning
https://arxiv.org/abs/1912.06680

AlphaStar: Grandmaster level in StarCraft II



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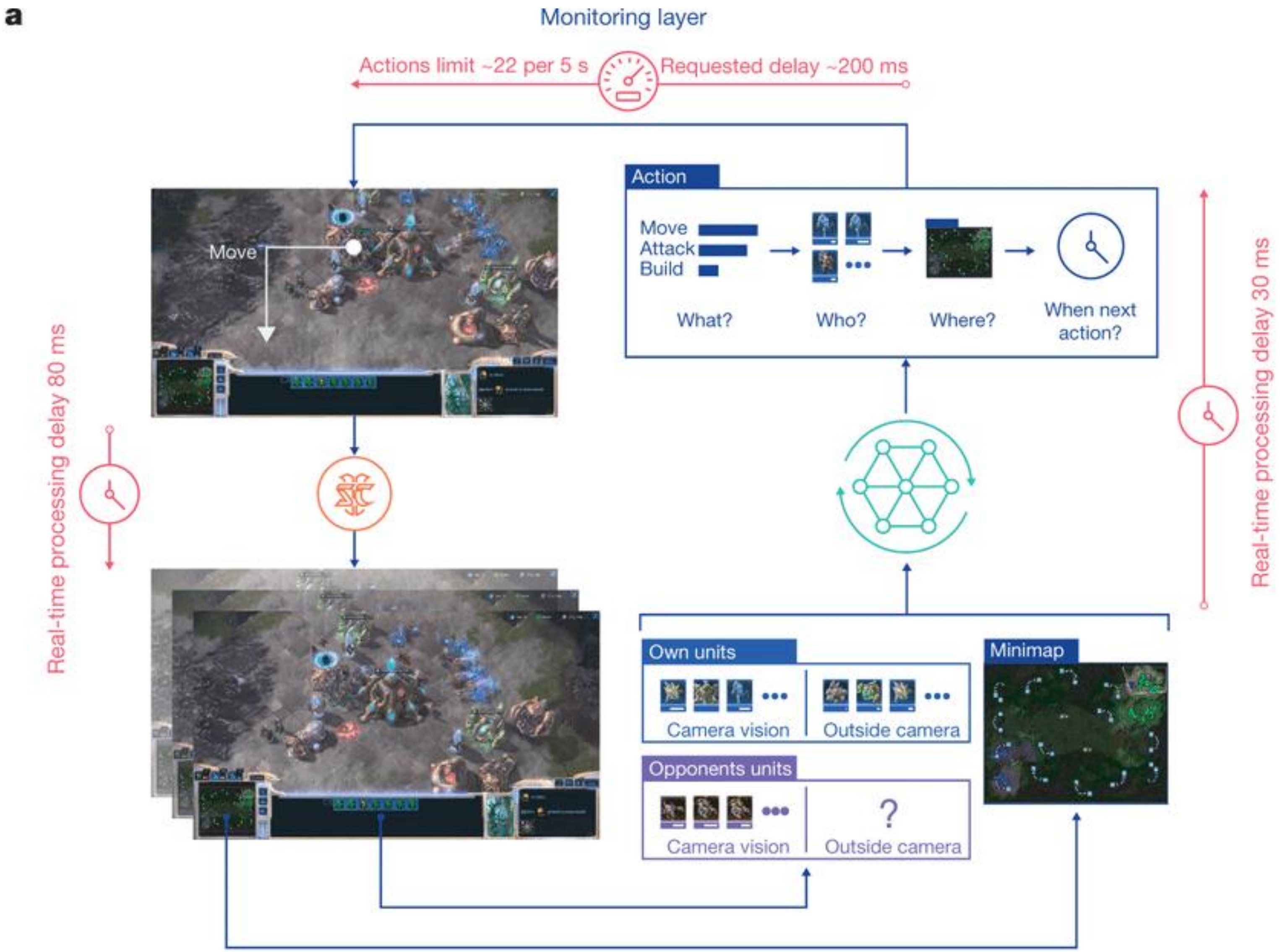

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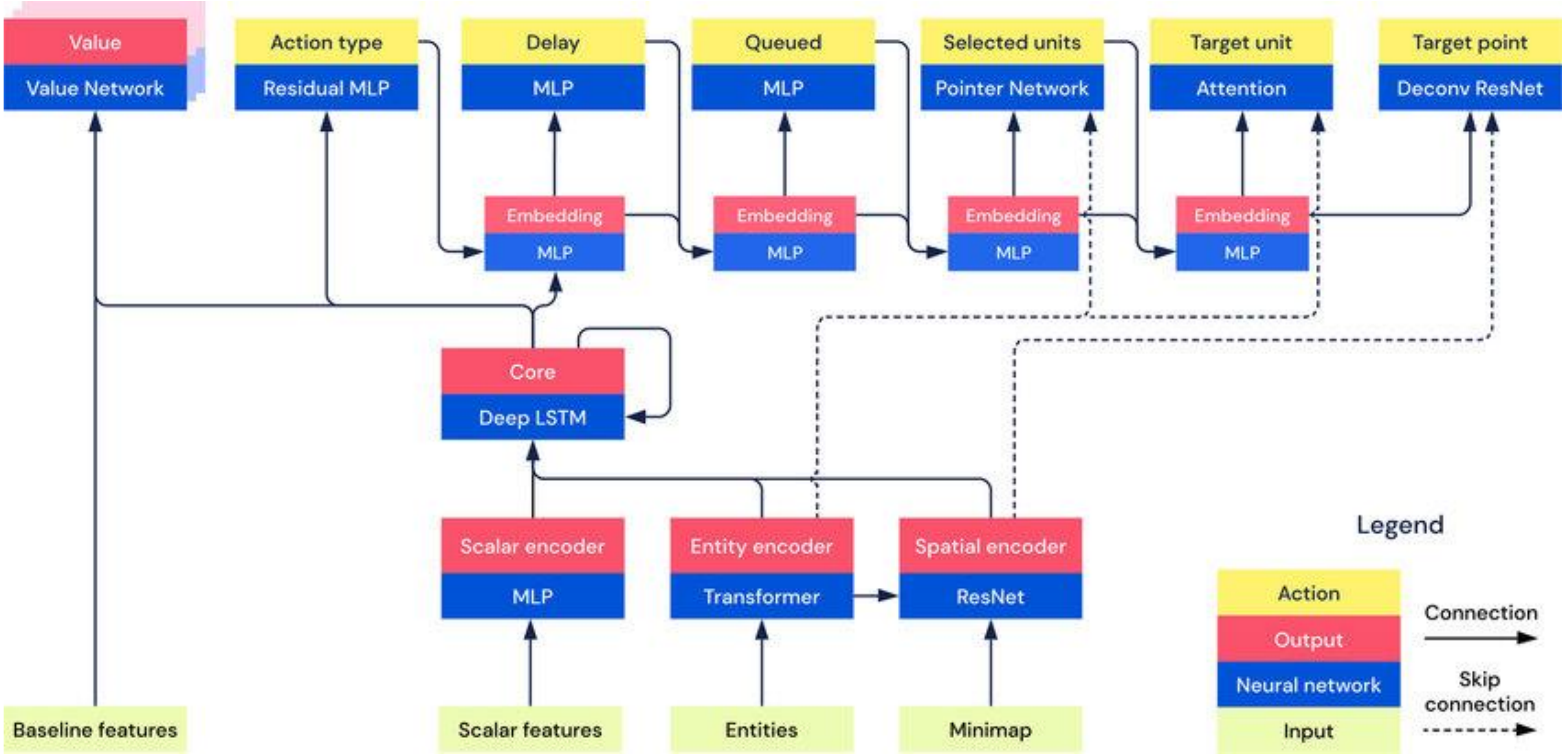
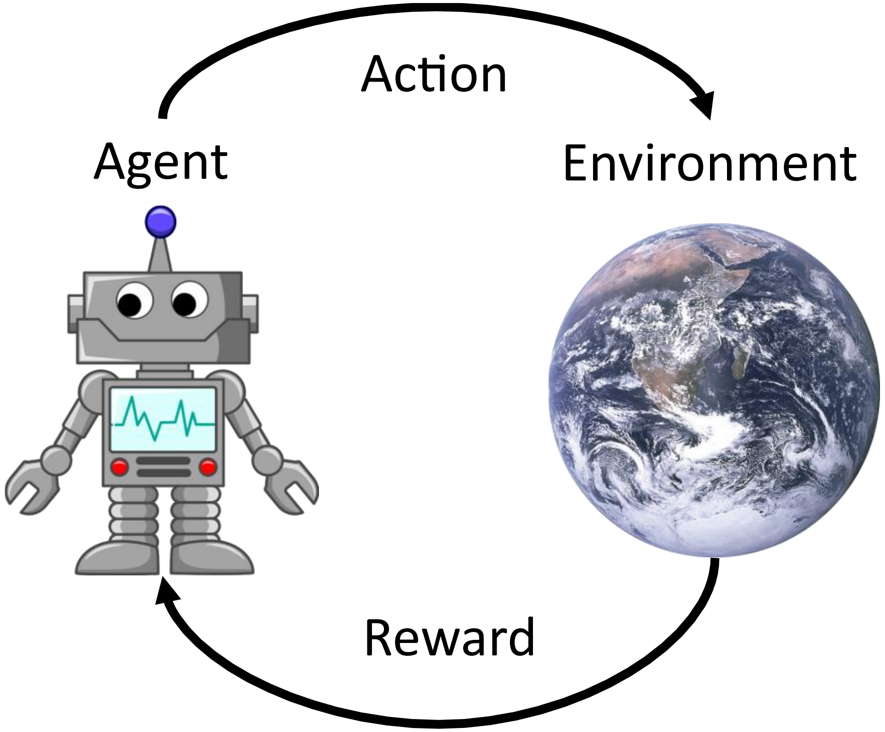


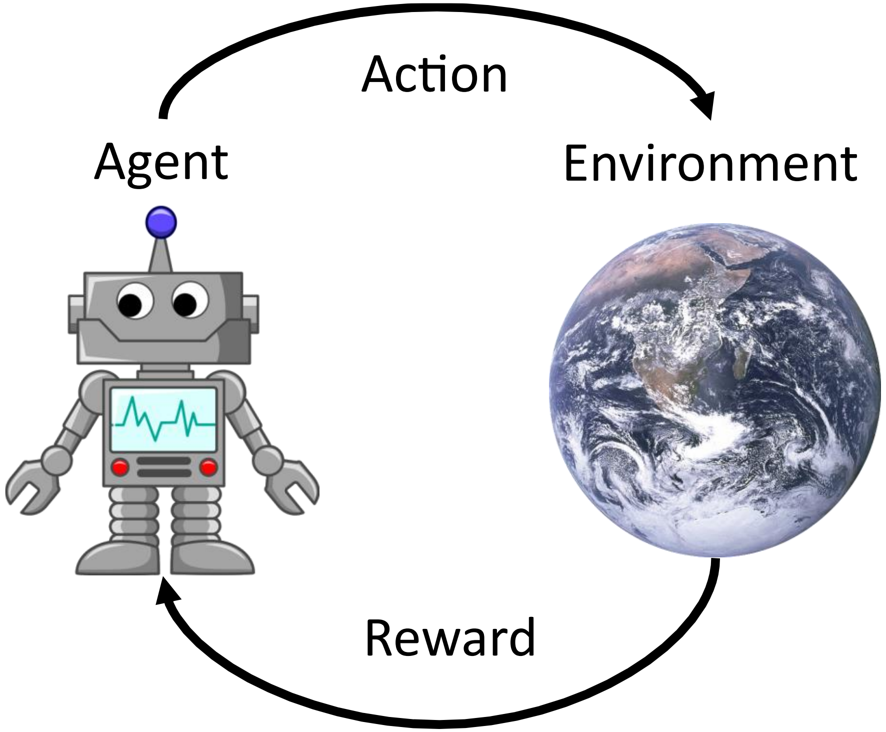
Reinforcement Learning: Interacting with World



Normally we use RL to train **agents** that interact with a (noisy, nondifferentiable) **environment**

Summary: Reinforcement Learning

RL trains **agents** that interact with an **environment** and learn to maximize **reward**



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Next: ML Advice