HW3

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❖ Assignment

- ✓ Read the textbook.
- ✓ Watch the video
  - https://youtu.be/sLNPd_nPGIc
- ✓ Prove that
  - $T(n) = O(n^2)$

    where $T(n) = \begin{cases} \Theta(1) & \text{if } n = 1 \\ 3T(n/4) + cn^2 & \text{if } n > 1 \end{cases}$
  - by using substitution method (The guess is also $T(n) = O(n^2)$).
  - Can this be also proved by the Master method(theorem)? Explain why.
  - Upload the answer to the HDLMS [assignments] menu.

① $n=1$, $T(n) = \theta(1)$ True

② $n>1$, $T(n) = c_1 n^2$ for some $c_1 \geq 0$

$$T(n) = 3T(n/4) + c_2 n^2 \quad \text{for some } c_2 \geq 0$$

$$\leq 3c_1 \cdot \frac{n^2}{16} + c_2 n^2$$

$$= \frac{3}{16} c_1 \cdot n^2 + c_2 n^2$$

$$= \left(\frac{3}{16} c_1 + c_2\right) n^2$$

$$= c_1 n^2 + \left(c_2 - \frac{13}{16} c_1\right) n^2$$

$$\left(\begin{aligned} c_2 - \frac{13}{16} c_1 \geq 0 \\ c_1 \leq \frac{16}{13} c_2 \end{aligned}\right)$$

$$\leq c_1 n^2 \quad \text{for some } \left(0 < c_1 \leq \frac{16}{13} c_2,\ 0 < c_2\right)$$

$$T(n) \leq c_1 n^2 \text{ for some } \left(0 < c_1 \leq \frac{16}{13} c_2,\ 0 < c_2\right)$$

$$T(n) = O(n^2)$$

Master method ?

$$T(n) \begin{cases} \theta(1) & \text{if } n = 1 \\ 3T(n/4) + cn^2 & \text{if } n > 1 \end{cases}$$

$a = 3$, $b = 4$, $f(n) = c_2 n^2$

1. $f(n) = c_2 n^2 = O(n^{\log_4 3 - \epsilon})$ for some constant $\epsilon > 0$

$$\log_4 3 - \epsilon \geq 2 \quad \text{(upper bound)}$$

$$\epsilon \leq \log_4 3 - 2 < 0 \quad \text{(False)}$$

2. $f(n) = c_2 n^2 = \theta(n^{\log_4 3})$

$$2 \neq \log_4 3 \quad \text{(False)}$$

3. $f(n) = c_2 n^2 = \Omega(n^{(\log_4 3 + \epsilon)})$ for some $\epsilon > 0$

$$2 \geq \log_4 3 + \epsilon$$

$$\epsilon \leq \log_4 \frac{16}{3} \quad \text{(True)}$$

$\Rightarrow T(n) = \theta(c_2 n^2) = O(c_2 n^2)$

So, it is possible to prove $T(n) \leq cn^2$ using master method