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HALL EFFECTS ON HYDROMAGNETIC NATURAL CONVECTION FLOW IN A VERTICAL MICRO-POROUS-CHANNEL WITH INJECTION/SUCTION

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ABSTRACT. In this work, the hydromagnetic and thermal characteristics of natural convection flow in a vertical parallel plate micro-porous-channel with suction/injection is analytically studied in the presence of Hall current by taking the temperature jump and the velocity slip at the wall into account. The governing equations, exhibiting the physics of the flow formation are displayed and the exact analytical solutions have been obtained for momentum and energy equations under relevant boundary conditions. The impact of distinct admissible parameters such as Hartmann number, Hall current parameter, permeability parameter, suction/injection parameter, fluid wall interaction parameter, Knudsen number and wall-ambient temperature ratio on the flow formation is discussed with the aid of line graphs. In particular, as rarefaction parameter on the micro-porous-channel surfaces increases, the fluid velocity increases and the volume flow rate decreases for injection/suction.

1. INTRODUCTION

The steady or unsteady hydromagnetic convection flow in the presence of suction/injection is one of the notable present-day problems. For example, the process of suction/injection has a significant role in the field of space science and aerodynamics. Suction is applied to chemical processes to discard reactants and injection is used to add reactants. Sanea [1] studied the mixed convection heat transfer along with continuously moving heated vertical plate with suction or injection. Venkateswarlu et al. [2] discussed the thermodynamics analysis of Hall current and Soret number on hydromagnetic couette flow in a rotating system with a convective boundary condition. Makinde and Chinyoka [3] examined the analysis of the unsteady flow of a variable viscosity reactive fluid in a slit with wall suction or injection. Venkateswarlu et al. [4] studied the Dufour and heat source effects on radiative MHD slip flow of a viscous

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fluid in a parallel porous plate channel in the presence of chemical reaction. Rundora and Makinde [5] investigated the effect of suction/injection on unsteady reactive variable viscosity non-Newtonian fluid flow in a channel filled with porous medium and convective boundary conditions. Venkateswarlu et al. [6] examined the effects of chemical reaction and heat generation on MHD boundary layer flow of a moving vertical plate with suction and dissipation. Jha et al. [7] presented the natural convection flow in vertical micro-channel with suction/injection. In other related work, Jha et al. [8] examined the role of suction/injection on MHD natural convection flow in a vertical micro-channel. Venkateswarlu et al. [9] discussed the influence of thermal radiation and heat generation on steady hydromagnetic flow in a vertical micro-porous-channel in the presence of suction/injection.

The fluid flow and heat transfer through a porous medium have been extensively studied in the past because of its applicability to nuclear waste disposal, solid matrix heat exchanger, thermal insulation and other practical applications. Hayat and Abbas [10] studied the effects of Hall current and heat transfer on the flow in a porous medium with slip condition. Venkateswarlu et al [11] presented the influence of heat generation and viscous dissipation on hydromagnetic fully developed natural convection flow in a vertical micro-channel. In another study, Venkateswarlu et al. [12] examined Soret and Dufour Effects on the radiative hydromagnetic flow of a chemically reacting fluid over an exponentially accelerated inclined porous plate in presence of heat absorption and viscous dissipation. Beg et al. [13] presented the numerical study of magnetohydrodynamic viscous plasma flow in rotating porous media with Hall currents and inclined magnetic field influence. Venkateswarlu et al. [14] presented the influence of Hall current and heat source on MHD flow of a rotating fluid in a parallel porous plate channel. Ghara et al. [15] considered the Hall Effect on oscillating flow due to eccentrically rotating porous disk and a fluid at infinity. Venkateswarlu and Padma [16] studied the unsteady MHD free convective heat and mass transfer in a boundary layer flow past a vertical permeable plate with thermal radiation and chemical reaction. Tripathy et al. [17] studied the chemical reaction effect on MHD free convective surface over a moving vertical plate through a porous medium. Venkateswarlu et al. [18] examined the influence of slip condition on radiative MHD flow of a viscous fluid in a parallel porous plate channel in the presence of heat absorption and chemical reaction. Jha and Aina [19] considered the MHD natural convection flow in a vertical micro-porous-annulus in the presence of a radial magnetic field. Venkateswarlu and Phani kumar [20] presented the Soret and heat source effects on MHD flow of a viscous fluid in a parallel porous plate channel in the presence of slip condition. Jha and Aina [21] discussed the MHD natural convection flow in a vertical porous micro-channel formed by non-conducting and conducting plates in the presence of an induced magnetic field. Venkateswarlu et al. [22] discussed the influence of Hall current and thermal diffusion on radiative hydromagnetic flow of a rotating fluid in presence of heat absorption.

The objective of the present article is to study the impact of suction or injection on steady hydromagnetic natural convection flow in a vertical parallel plate micro-porous-channel in the presence of Hall current, which has not been accounted for in the existing literature. This study benefits the design of micro-heat exchangers.

2. FORMATION OF THE PROBLEM

The geometry of the problem under consideration in this study is presented schematically in Fig. 1. Consider the steady fully developed hydromagnetic natural convection flow of an incompressible, viscous and electrically conducting fluid in a micro-porous-channel between two infinite vertical parallel plates under the influence of Hall current. The distance between two infinite vertical parallel plates is a . Choose a Cartesian coordinate system such that x -axis is parallel to the gravitational acceleration g , y -axis is normal to the channel walls and z -axis is perpendicular to the xy -plane. A uniform magnetic field of strength B_0 is applied perpendicular to the direction of the flow and neglecting the induced magnetic field by assuming a small magnetic Reynolds number. The micro-porous-channel plates are heated with one plate maintained at a temperature T_1 while the other plate at a temperature T_2 where $T_1 > T_2$. A constant suction/injection velocity is taken into account. It is assumed that the fluid has a constant physical properties. Following Jha et al. [8, 23], under Boussinesq's approximation, the governing equations for the present physical situation in dimensional can be written as follows:

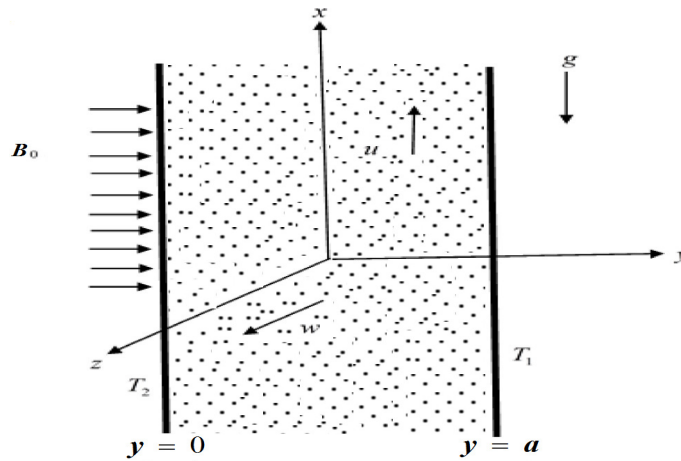


Figure 1: Flow configuration and coordinate system.

Continuity equation:

$$\frac{dv}{dy} = 0$$

Momentum equations:

$$V_0 \frac{du}{dy} = \nu \frac{d^2u}{dy^2} - \frac{\sigma B_0^2}{\rho} \left[\frac{u + mw}{1 + m^2} \right] + g\beta(T - T_0) - \frac{\nu}{K_1} u \quad (2.1)$$

$$V_0 \frac{dw}{dy} = \nu \frac{d^2w}{dy^2} + \frac{\sigma B_0^2}{\rho} \left[\frac{mu - w}{1 + m^2} \right] - \frac{\nu}{K_1} w \quad (2.2)$$

Energy equation:

$$V_0 \frac{dT}{dy} = \alpha \frac{d^2T}{dy^2} \quad (2.3)$$

Here u – fluid velocity in x – direction, v – fluid velocity in y – direction, w – fluid velocity in z – direction, B_0 – uniform magnetic field, ρ – fluid density, ν – kinematic viscosity of the fluid, σ – fluid electrical conductivity, g – gravitational acceleration, β – thermal expansion coefficient, T – fluid temperature, T_0 – reference temperature, V_0 – constant suction or injection velocity, $m = \omega_e \tau_e$ – Hall current parameter, ω_e – cyclotron frequency, τ_e – electron collision time, K_1 – dimensional permeability parameter and α – thermal diffusivity of the fluid respectively.

We should in prior warn the reader that our model is not the same as that by Jha et al. [23], in which the suction/injection parameter and permeability parameter were not taken into account. Interpretation of energy equation was different in both papers. The corresponding boundary conditions which describe the velocity slip and temperature jump at the fluid wall interface in dimensional form can be written as

$$\left. \begin{aligned} u &= \frac{2-F_v}{F_v} \lambda \frac{du}{dy}, w = \frac{2-F_v}{F_v} \lambda \frac{dw}{dy}, T = T_2 + \frac{2-F_t}{F_t} \frac{2\gamma}{\gamma+1} \frac{\lambda}{Pr} \frac{dT}{dy} \text{ at } y = 0 \\ u &= -\frac{2-F_v}{F_v} \lambda \frac{du}{dy}, w = -\frac{2-F_v}{F_v} \lambda \frac{dw}{dy}, T = T_1 - \frac{2-F_t}{F_t} \frac{2\gamma}{\gamma+1} \frac{\lambda}{Pr} \frac{dT}{dy} \text{ at } y = a \end{aligned} \right\}$$

Here F_v –tangential momentum accommodation coefficient, λ –molecular mean free path, T_1 –hot wall temperature, T_2 –cold wall temperature, F_t –tangential thermal accommodation coefficient, $\gamma = \frac{c_p}{c_v}$ –ratio of specific heats, c_v –specific heat at constant volume, c_p –specific heat at constant pressure, Pr –Prandtl number, and a –distance between two plates respectively.

The following non-dimensional quantities are introduced

$$\left. \begin{aligned} \eta &= \frac{y}{a}, \theta = \frac{T-T_0}{T_1-T_0}, U = \frac{u\nu}{g\beta(T_1-T_0)a^2}, W = \frac{w\nu}{g\beta(T_1-T_0)a^2}, Pr = \frac{\nu}{\alpha} \\ \xi &= \frac{T_2-T_0}{T_1-T_0}, \beta_v = \frac{2-F_v}{F_v}, \beta_t = \frac{2-F_t}{F_t} \frac{2\gamma}{\gamma+1} \frac{1}{Pr}, Kn = \frac{\lambda}{a}, ln = \frac{\beta_t}{\beta_v} \end{aligned} \right\} \quad (2.4)$$

Equations (2.1), (2.2) and (2.3) can be transformed to the following non-dimensional form

$$\frac{d^2U}{d\eta^2} - S \frac{dU}{d\eta} + \theta - M \left[\frac{U + mW}{1 + m^2} \right] - \frac{1}{K} U = 0 \quad (2.5)$$

$$\frac{d^2W}{d\eta^2} - S \frac{dW}{d\eta} + M \left[\frac{mU - W}{1 + m^2} \right] - \frac{1}{K} W = 0 \quad (2.6)$$

$$\frac{d^2\theta}{d\eta^2} - SPr \frac{d\theta}{d\eta} = 0 \quad (2.7)$$

Here $S = \frac{V_0 a}{\nu}$ is the suction/injection parameter, $M = \frac{\sigma B_0^2 a^2}{\rho \nu}$ is the magnetic parameter, $m = \omega_e \tau_e$ is the Hall current parameter and $K = \frac{K_1}{a^2}$ is the permeability parameter respectively.

The corresponding initial and boundary conditions can be written as

$$\left. \begin{aligned} U &= \beta_v Kn \frac{dU}{d\eta}, W = \beta_v Kn \frac{dW}{d\eta}, \theta = \xi + \beta_v Kn ln \frac{d\theta}{d\eta} \text{ at } \eta = 0 \\ U &= -\beta_v Kn \frac{dU}{d\eta}, W = -\beta_v Kn \frac{dW}{d\eta}, \theta = 1 - \beta_v Kn ln \frac{d\theta}{d\eta} \text{ at } \eta = 1 \end{aligned} \right\}$$

By defining the complex velocity $\psi = U + iW$, Eqs. (2.5)-(2.6) can be written as

$$\frac{d^2\psi}{d\eta^2} - S\frac{d\psi}{d\eta} + \theta - \left[\frac{M}{1+im} + \frac{1}{K} \right] \psi = 0 \quad (2.8)$$

The initial and boundary conditions in complex form can be expressed as

$$\left. \begin{aligned} \psi &= \beta_v Kn \frac{d\psi}{d\eta}, \theta = \xi + \beta_v Kn ln \frac{d\theta}{d\eta} \text{ at } \eta = 0 \\ \psi &= -\beta_v Kn \frac{d\psi}{d\eta}, \theta = 1 - \beta_v Kn ln \frac{d\theta}{d\eta} \text{ at } \eta = 1 \end{aligned} \right\} \quad (2.9)$$

Referring to the values of F_v and F_t given in Goniak Duffa [24], Eckert and Drake [25] the value of β_v is near unity and the value of β_t ranges from near 1 to more than 100 for actual wall surface conditions, also β_t is near 1.667 for several engineering applications, corresponding to $F_v = 1, F_t = 1, \gamma = 1.4$ and $Pr = 0.71$.

Given the micro—porous—channel slip velocity, dimensional volume flow rate δ can be written as

$$\delta = \nu \int_0^a [u(y) + iw(y)] dy \quad (2.10)$$

In non—dimensional form the volume flow rate Q can be written as

$$Q = \frac{\delta}{g\beta(T_1 - T_0)a^3} \quad (2.11)$$

Using the non-dimensional quantities in Eq. (2.4) and the Eq. (2.10) into Eq. (2.11), we obtain

$$Q = \int_0^1 [U(\eta) + iW(\eta)] d\eta$$

3. SOLUTION OF THE PROBLEM

The system of ordinary differential equations is solved analytically with corresponding boundary conditions. Equation (2.7), subject to the boundary conditions (2.9), has the following analytical solution:

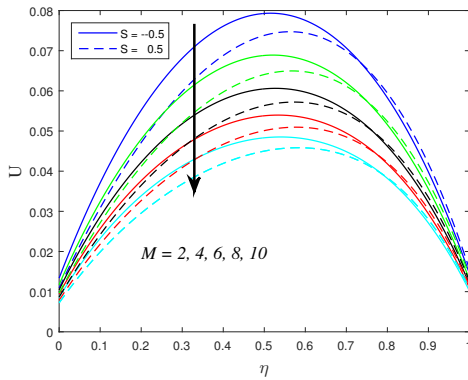
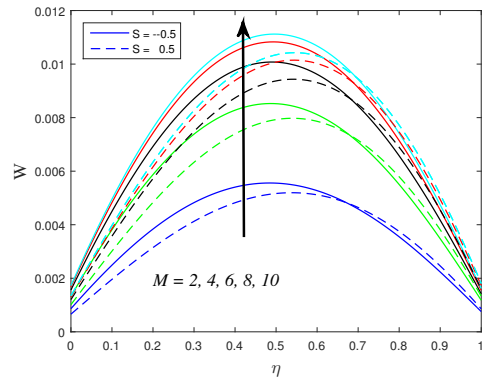
$$\theta(\eta) = a_3 + a_4 \exp(m_1\eta) \quad (3.1)$$

Substituting Eq. (3.1) into the momentum Eq. (2.8) and solving it by using the boundary conditions (2.9), we obtain

$$\psi(\eta) = a_{14} \exp(m_2\eta) + a_{15} \exp(m_3\eta) - a_7 \exp(m_1\eta) + a_6$$

$$U(\eta) = \Re[a_{14} \exp(m_2\eta) + a_{15} \exp(m_3\eta) - a_7 \exp(m_1\eta) + a_6]$$

$$W(\eta) = \Im[a_{14} \exp(m_2\eta) + a_{15} \exp(m_3\eta) - a_7 \exp(m_1\eta) + a_6]$$

Figure 2: Influence of M on U .Figure 3: Influence of M on W .

3.1. Volume flow rate. From the micro—porous—channel slip velocity, the complex volume flow rate Q can be expressed as

$$Q = a_{14}a_{17} + a_{15}a_{18} - a_7a_{16} + a_6$$

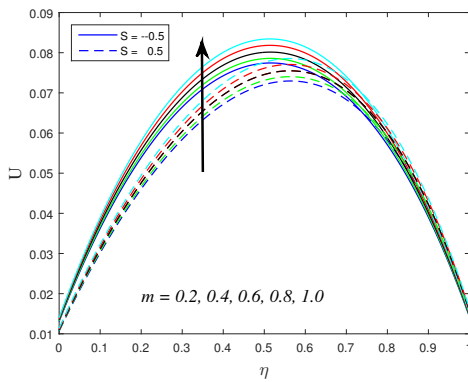
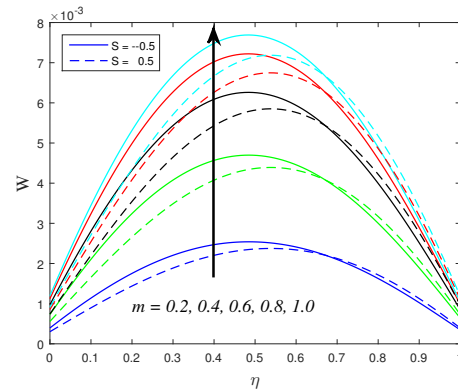
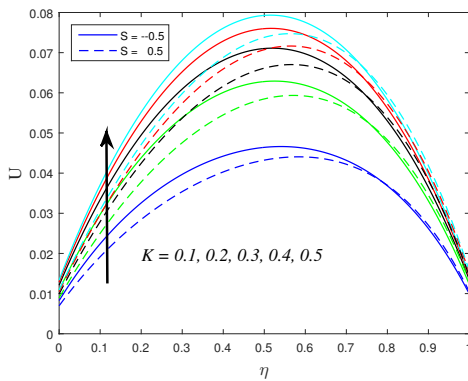
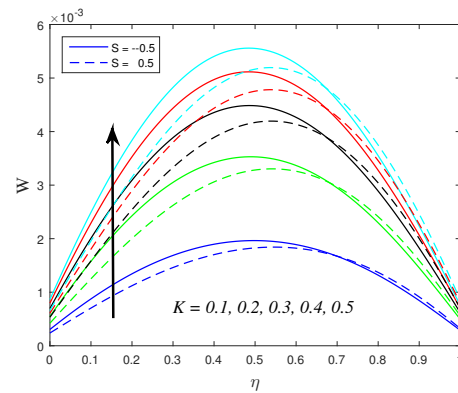
$$Q_x = \Re[a_{14}a_{17} + a_{15}a_{18} - a_7a_{16} + a_6]$$

$$Q_z = \Im[a_{14}a_{17} + a_{15}a_{18} - a_7a_{16} + a_6].$$

4. RESULTS AND DISCUSSION

In this article, the impact of various principal flow parameters like Hartmann number M , Hall current parameter m , permeability parameter K , rarefaction parameter $\beta_v Kn$, injection/suction parameter S , fluid wall interaction parameter ln and wall ambient temperature ratio ξ on primary velocity U , secondary velocity W , volume flow rate due to primary flow Q_x , volume flow rate due to secondary flow Q_z are displayed with the aid of line graphs (Figs. 2–26) and discussed consequently. The parametric study has been performed over the reasonable ranges $2 \leq M \leq 10$, $0.2 \leq m \leq 1.0$, $0.1 \leq K \leq 0.5$, $0 \leq \beta_v Kn \leq 0.1$, $0.8 \leq ln \leq 4$ with fixed value of $M = 2$, $m = 0.5$, $K = 0.5$, $Pr = 0.71$, $\beta_v Kn = 0.05$, $ln = 1.667$ and $\xi = 0.5$. In this study $S = -0.5$ corresponds to injection and $S = 0.5$ corresponds to suction.

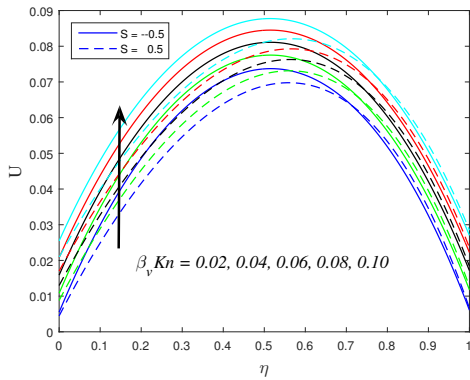
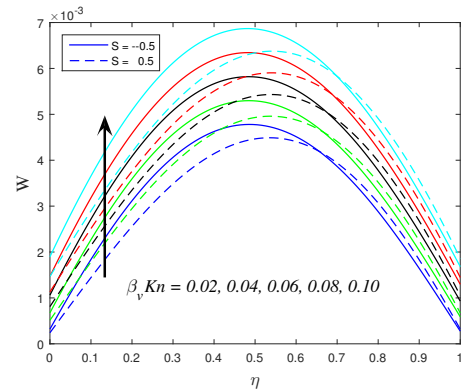
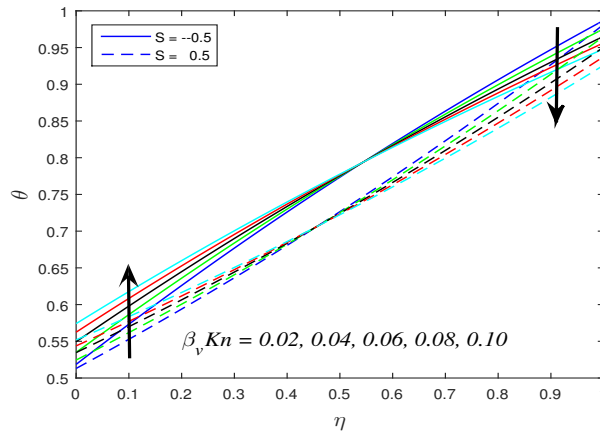
Figures 2 and 3 show that, the combined effects of Hartmann number and injection/suction parameter in primary and secondary directions of the fluid velocity. It is noticed that increasing the values of the Hartmann number has a tendency to slow down the primary velocity of the fluid in the micro—porous—channel whereas the reverse direction is recognized in the case of secondary velocity for injection/suction. This is because the application of the magnetic field generates a resistive force related to the drag force that acts in the opposite direction of the

Figure 4: Influence of m on U .Figure 5: Influence of m on W .Figure 6: Influence of K on U .Figure 7: Influence of K on W .

fluid motion, consequently primary velocity decreases and secondary velocity increases with an increase in Hartmann number for injection/suction.

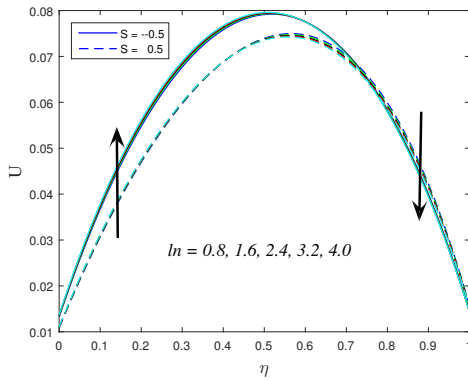
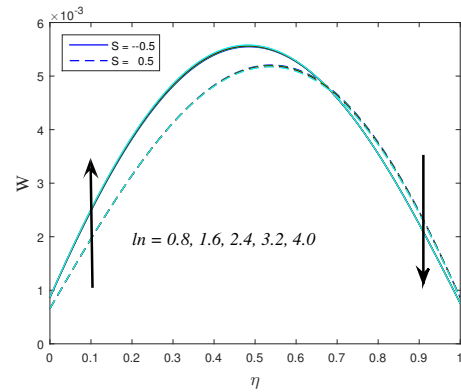
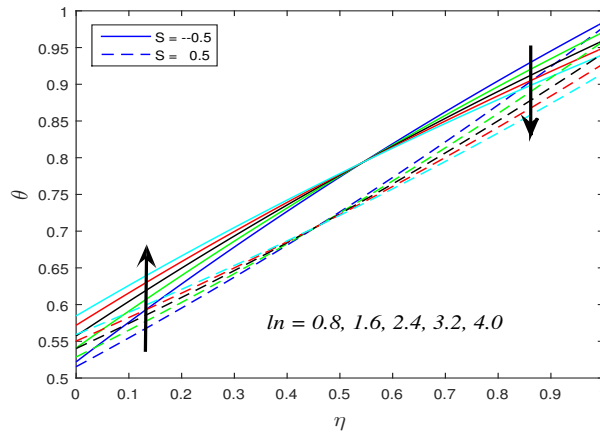
Figures 4 and 5 illustrate the variation of primary velocity and secondary velocity with Hall current parameter under injection/suction. It is interesting to observe that in both primary and secondary flow directions, an increase in Hall current parameter yields a noticeable increase in the fluid velocity for injection/suction. However, the increase is more prevalent along the secondary flow direction. The influence of Hall current parameter on the primary velocity and secondary velocity significantly pronounced in the presence of injection.

Figures 6 and 7 show that, the effects of permeability parameter as well as injection/suction parameter on the primary velocity and secondary velocity. It is seen that for injection/suction,

Figure 8: Influence of $\beta_v Kn$ on U .Figure 9: Influence of $\beta_v Kn$ on W .Figure 10: Influence of $\beta_v Kn$ on θ .

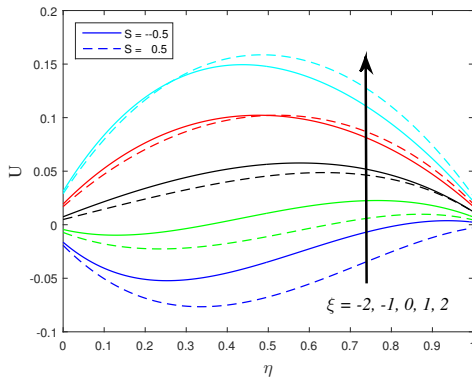
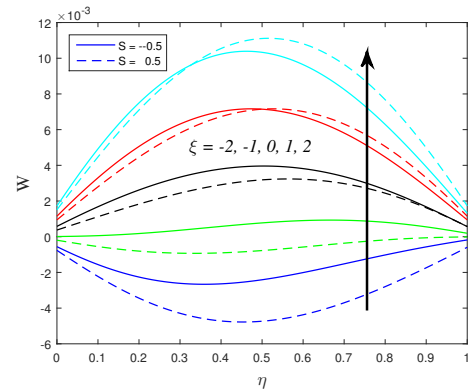
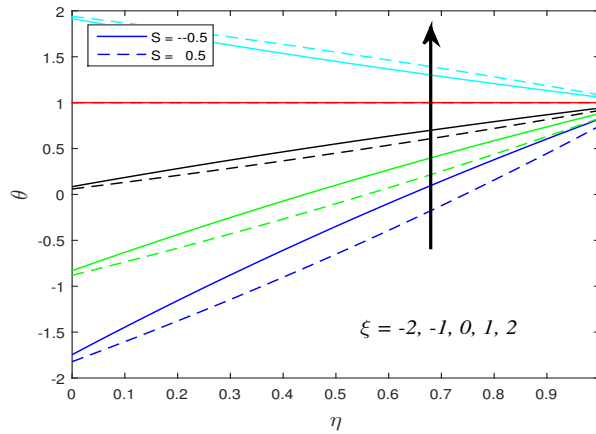
increasing the value of the permeability parameter leads to an enhancement in the fluid primary velocity and secondary velocity. Hence the complex velocity is an increasing function of permeability parameter in the presence of injection/suction.

Figures 8, 9, and 10 depict the influence of rarefaction parameter as well as injection/suction parameter on the fluid primary velocity, secondary velocity, and temperature respectively. It is noticed that for injection/suction, increasing the value of rarefaction parameter leads to an enhancement in the fluid primary velocity and secondary velocity. These effects are more pronounced in the case of injection. It is interesting to mention that the strength of the temperature

Figure 11: Influence of ln on U .Figure 12: Influence of ln on W .Figure 13: Influence of ln on θ .

field is directly proportional to the rarefaction parameter near the micro-porous-channel left wall whereas it is inversely proportional to the rarefaction parameter near the micro-porous-channel right wall for injection/suction. In addition, it is noticed that for injection/suction, there exists a point of intersection within the micro-porous-channel where the effect of the rarefaction parameter becomes insignificant on the temperature profile.

Figures 11, 12, and 13 demonstrate the combined effect of fluid wall interaction parameter and injection/suction parameter on the fluid primary velocity, secondary velocity, and temperature respectively. It is noticed that there is an enhancement in the primary velocity, secondary

Figure 14: Influence of ξ on U .Figure 15: Influence of ξ on W .Figure 16: Influence of ξ on θ .

velocity, and temperature profiles near the micro-porous-channel left wall and a reduction in these profiles near the micro-porous-channel right wall with an increase in fluid wall interaction parameter in presence of injection/suction. In addition, it is observed that for injection/suction, there exists a point of intersection within the micro-porous-channel where the effect of the fluid wall interaction parameter becomes insignificant on the velocity and temperature profiles.

Figures 14, 15, and 16 depict the influence of fluid wall ambient temperature difference ratio on the primary velocity, secondary velocity, and temperature profiles respectively in the

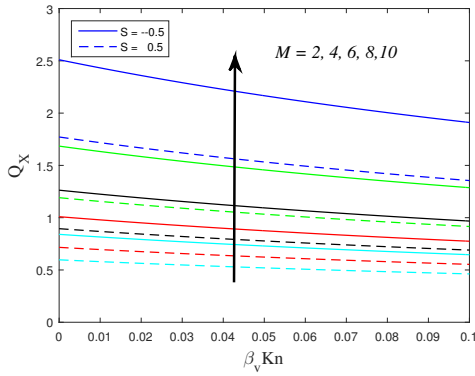


Figure 17: Q_x versus $\beta_v Kn$ for different values of M .

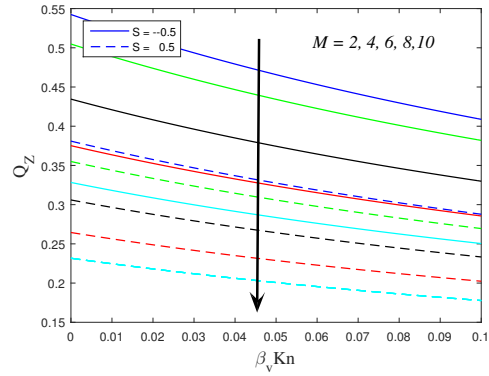


Figure 18: Q_z versus $\beta_v Kn$ for different values of M .

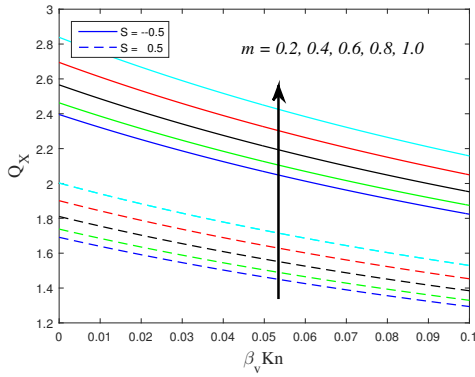


Figure 19: Q_x versus $\beta_v Kn$ for different values of m .

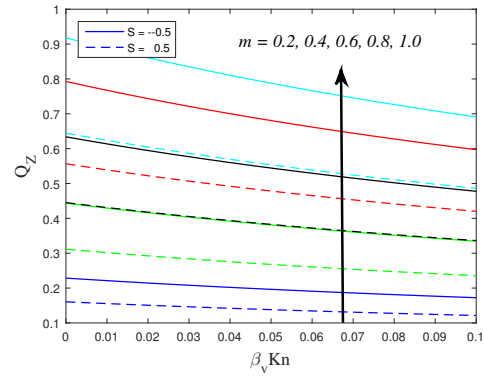


Figure 20: Q_z versus $\beta_v Kn$ for different values of m .

presence of injection/suction. It is interesting to mention that the strength of the primary velocity, secondary velocity, and temperature profiles are directly proportional to the fluid wall ambient temperature difference ratio for injection/suction. Therefore the complex velocity is an increasing function of fluid wall ambient temperature difference ratio for injection/suction.

Figures 17 and 18 illustrate the combined effects of Hartmann number, rarefaction parameter and injection/suction parameter on the primary volume flow rate and secondary volume flow rate respectively. It is observed that simultaneously increasing in the Hartmann number and the rarefaction parameter causes a reduction in primary volume flow rate and secondary volume

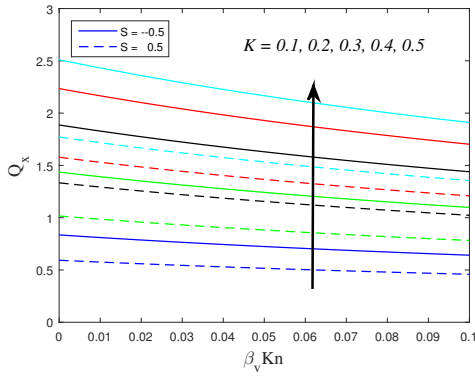


Figure 21: Q_x versus $\beta_v Kn$ for different values of K .

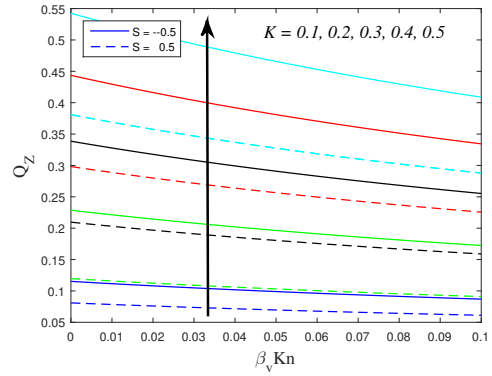


Figure 22: Q_z versus $\beta_v Kn$ for different values of K .

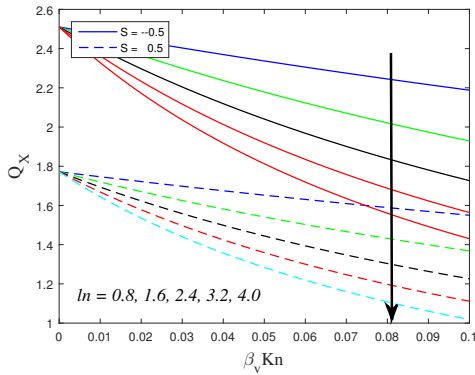


Figure 23: Q_x versus $\beta_v Kn$ for different values of ln .

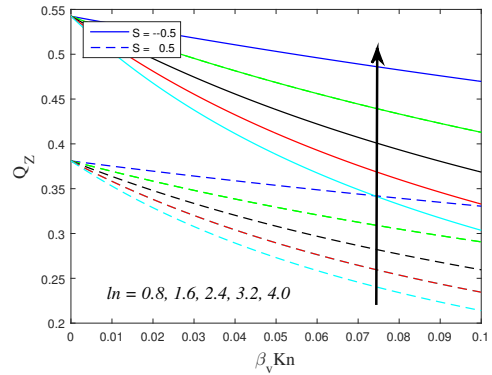


Figure 24: Q_z versus $\beta_v Kn$ for different values of ln .

flow rate for injection/suction. It is observed that in primary and secondary cases, the volume flow rate decreases with an increase in the rarefaction parameter.

It is observed from Figs. 19 and 20 that, there is an increase in the primary component of volume flow rate and secondary component of volume flow rate with an increase in the Hall current parameter for injection/suction. It is noticed that in both primary and secondary cases, the volume flow rate a decreasing function of rarefaction parameter for injection/suction.

It is noticed from Figs. 21 and 22 that, there is an increase in primary component of volume flow rate and secondary component of volume flow rate with the increase in the permeability parameter for injection/suction. An increase in the rarefaction parameter decreases the volume

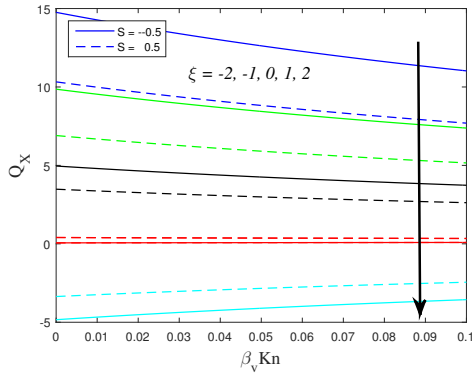


Figure 25: Q_x versus $\beta_v Kn$ for different values of ξ .

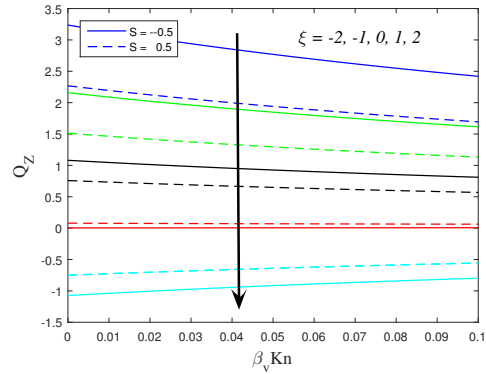


Figure 26: Q_z versus $\beta_v Kn$ for different values of ξ .

flow rate along the primary and secondary flow directions for all values of the permeability parameter for injection/suction.

Figures 23 and 24 display the combined effect of fluid wall interaction parameter and rarefaction parameter on the primary volume flow rate and secondary volume flow rate respectively. It is clear that, simultaneously increasing in the fluid wall interaction parameter and rarefaction parameter reduced the volume flow rate along the primary direction and secondary direction for injection/suction whereas an increase in rarefaction parameter yields to decrease in volume flow rate along with the primary flow direction as well as secondary flow direction for injection/suction.

Figures 25 and 26 depict the impact of rarefaction parameter as well as fluid wall ambient temperature difference ratio on the primary volume flow rate and secondary volume flow rate in the presence of injection/suction respectively. It is clear that in both primary and secondary cases, the volume flow rate is a decreasing function of fluid wall ambient temperature difference ratio for injection/suction. Furthermore, it is found that an increase in the rarefaction parameter leads to a decrease in the volume flow rate for injection/suction.

5. CONCLUSIONS

Main observations of presented research are listed below:

- An increase in the values of Hall current parameter and permeability parameter leads to enhancement in the fluid velocity and volume flow rate in primary and secondary directions for injection/suction.
- An increase in the values of fluid wall interaction parameter and rarefaction parameter leads to a decrease in the volume flow rate in primary and secondary directions.

- Primary velocity decreases and secondary velocity increases with an increase in the values of Hartmann number whereas the volume flow rate decreases in primary and secondary directions for injection/suction.

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NOMENCLATURE

V_0 – Constant suction/injection velocity
 u – Fluid velocity in x –direction
 v – Fluid velocity in y –direction
 w – Fluid velocity in z –direction
 g – Acceleration due to gravity
 c_p – Specific heat at constant pressure
 c_v – Specific heat at constant volume
 S – Suction/injection parameter
 m – Hall current parameter
 Q – Complex volume flow rate
 Q_x – Primary volume flow rate
 Q_z – Secondary volume flow rate
 K_1 – Dimensional permeability parameter
 K – Permeability parameter
 a – Distance between two parallel plates
 u_0 – Mean velocity
 B_0 – Uniform magnetic field
 F_v – Momentum accommodation coefficient
 F_t – Thermal accommodation coefficient
 Kn – Knudsen number
 ln – Fluid wall interaction parameter
 M – Hartmann number
 Pr – Prandtl number
 q_w – Heat flux
 T – Fluid temperature
 T_0 – Reference temperature
 T_1 – Hot wall temperature
 T_2 – Cold wall temperature
 U – Primary velocity
 W – Secondary velocity

Greek Symbols

- β – Coefficient of thermal expansion
 ν – Kinematic coefficient of viscosity
 δ – Non–dimensional volume flow rate
 ρ – Fluid density
 σ – Fluid electrical conductivity
 ψ – Non–dimensional complex velocity
 τ_w –Shear stress
 α – Thermal diffusivity of the fluid
 γ – Ratio of specific heats $\frac{c_p}{c_v}$
 ω_e –Cyclotron frequency
 τ_e –Electron collision time
 λ Molecular mean free path
 β_t, β_v – Non–dimensional variables
 ξ – Wall ambient temperature ratio
 η – A scaled coordinate
 θ – A scaled temperature

APPENDIX

$$\begin{aligned}
 m_1 &= SPr, a_1 = 1 - \beta_v K n l n m_1, a_2 = (1 + \beta_v K n l n m_1) \exp(m_1), a_3 = \frac{a_2 \xi - a_1}{a_2 - a_1}, \\
 a_4 &= \frac{1 - \xi}{a_2 - a_1}, a_5 = \frac{M}{1 + im} + \frac{1}{K}, m_2 = \frac{S - \sqrt{S^2 + 4a_5}}{2}, m_3 = \frac{S + \sqrt{S^2 + 4a_5}}{2}, a_6 = \frac{a_3}{a_5}, \\
 a_7 &= \frac{a_4}{m_1^2 - S m_1 - a_5}, a_8 = 1 - \beta_v K n m_2, a_9 = 1 - \beta_v K n m_3, a_{10} = (1 - \beta_v K n m_1) a_7 - a_6, \\
 a_{11} &= (1 + \beta_v K n m_2) \exp(m_2), a_{12} = (1 + \beta_v K n m_3) \exp(m_3), \\
 a_{13} &= (1 + \beta_v K n m_1) a_7 \exp(m_1) - a_6, a_{14} = \frac{a_{10} a_{12} - a_9 a_{13}}{a_8 a_{12} - a_9 a_{11}}, a_{15} = \frac{a_8 a_{13} - a_{10} a_{11}}{a_8 a_{12} - a_9 a_{11}}, \\
 a_{16} &= \frac{\exp(m_1) + 1}{m_1}, a_{17} = \frac{\exp(m_2) - 1}{m_2}, a_{18} = \frac{\exp(m_3) - 1}{m_3}.
 \end{aligned}$$

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