Research Insight

Managing the Unique Risks of Leverage with the Barra Optimizer: Theory and Practice

Leverage and Volatility Trade-offs in Long-Short Portfolio Construction

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Introduction

Leverage is an intrinsic characteristic of a long-short portfolio, and it typically increases the portfolio volatility. Leverage also increases the possibility of margin calls, liquidity crunches, losses exceeding capital invested, and bankruptcies. For all these reasons, investors prefer to control leverage.

In contrast to traditional long-only investment strategies, where leverage is non-existent, long-short investment strategies involve leverage risk in addition to volatility risk. Recently, Jacobs and Levy (2012, 2013a, 2013b, and 2014) published a series of papers emphasizing the importance and uniqueness of the leverage risk. Their debate with Noble laureate Harry Markowitz (2013) has sparked renewed interests in the theory and application of long-short optimization.

In this Research Insight, we point out that MSCI has been a pioneer in the commercialization of long-short portfolio optimization since the early 1990s. The notion of "leverage aversion," as proposed by Jacobs and Levy (2012), is actually a long-standing feature of the Barra Optimizer. Jacobs and Levy's recent theoretical findings are a strong validation of an existing MSCI product.

In our view, the debate between Jacobs and Levy and Harry Markowitz is essentially about the pros and cons of a leverage penalty versus a leverage constraint. We point out that the Barra Optimizer supports both modeling choices and offers much more flexibility. We explain why long-short optimization may be difficult to solve and reveal how the Barra Optimizer handles this challenge. Using a plain-vanilla case, we demonstrate a strong correlation between the optimal levels of leverage and volatility, showing how both decrease as the risk aversion increases. Within the same case settings, we also examine the effects of a leverage constraint as well as a leverage penalty on the various aspects of the optimized portfolio. Our research reveals that a binding lower bound on leverage or a higher-than-optimal leverage target may force the solution portfolio to take on unnecessary risk and leverage, thus potentially hurting portfolio performance.

The Barra Optimizer is a Pioneer in Commercial Long-Short Optimization

Hedge Optimization, based on Barra Optimization Research's innovative approaches, was first offered to clients in the Barra AURORA System in the early 1990s. In 2001, it was renamed Long-Short Optimization in Barra Aegis 3.2. Since its first release, more Optimizer features have been added through research innovations. For example, short-rebates and risk target in long-short optimization were offered to clients in Barra Aegis 4.3, released in 2009. Most recently, the "weighted total leverage" features were introduced with the release of the Barra Open Optimizer 2.0 in 2013.

Today, the Barra Optimizer supports a broad array of leverage controls,[1] catering to different use cases by different portfolio managers. Not only can users choose to constrain the long or short leverage separately, or in parallel, they may also control the total leverage or leverage ratios. In addition, portfolio managers can even control the total leverage in a given industry or country by utilizing the newly supported "weighted total leverage" features.

This broad coverage of leverage concepts and related analytics, together with the option to use one or more control mechanisms—penalties, constraints, and soft bounds—provides portfolio managers with greater flexibility in managing leverage risk in a long-short portfolio.

[1] The definition of leverage in the Barra Optimizer may not be fully consistent with the conventional concept of leverage. For example, according to Barra Optimizer's definitions, a long-only portfolio would have 100 percent long leverage, 0 percent short leverage, and 100 percent total leverage. A 130/30 portfolio would have 130% long leverage, 30% short leverage, and 160% total leverage.

We provide more information on these leverage definitions and control mechanisms in the following subsections.

Granular Leverage Control in the Barra Optimizer

Jacobs and Levy (2012) define portfolio leverage as the sum of the absolute values of the weights of all the security holdings in a portfolio, minus one.[2] This definition is very close to the Barra Optimizer's definition of *Total Leverage*, with the slight difference being the constant one.

In addition to *Total Leverage*, the Barra Optimizer also supports *Long Leverage*, *Short Leverage*, *Long-Short Leverage Ratio*, *Net-Total Leverage Ratio*, *Weighted Long Leverage*, *Weighted Short Leverage*, and *Weighted Total Leverage*. Table 1 provides a brief description for each of these leverage terms.

Table 1. Leverage Calculations Available in the Barra Optimizer.

Term	Definition
Long Leverage	Sum of the weights of all long securities[3]
Short Leverage	Absolute value of the sum of weights of all short securities
Total Leverage	Long Leverage + Short Leverage
Short-Long Leverage Ratio[4]	Short Leverage / Long Leverage
Net-Total Leverage Ratio	(Long Leverage – Short Leverage) / Total Leverage
Weighted Long Leverage	A linear combination of all long securities' weights
Weighted Short Leverage	A linear combination of the absolute values of all short securities' weights
Weighted Total Leverage	A linear combination of the absolute values of all (long and short) securities' weights

Due to the portfolio balance constraint, the sum of the long leverage, short leverage and the cash position must be equal to one or a pre-specified constant in a long-short portfolio.[5] Given two of these three quantities, the third is implied. Therefore, the cash position and cash bounds play a vital role in long-short optimization. In fact, the portfolio's *Net Leverage*, which is defined as the long leverage minus the short leverage, is required to be one minus the cash position.

[2] In a fully-invested long-only portfolio, the sum of all asset weights equal to one.

[3] Cash and future weights are excluded in all the leverage calculations.

[4] We deliberately choose Long Leverage as the denominator to avoid the division-by-zero situation for long-only portfolios.

[5] The constraint "$C + L – S = c$, where C is the cash position, L is the long leverage, S is the short leverage, and c is a constant" is automatically enforced by the Barra Optimizer.

Choosing Penalties, Constraints and Soft Bounds

For a given leverage term, users may choose among the following three options to control its magnitude when constructing a long-short portfolio—(1) use a linear or quadratic penalty together with a leverage target;[6] (2) use a leverage constraint; or (3) use a soft bound on leverage.[7]

A leverage constraint is an upper, lower or equality bound set on a leverage term. It is a hard requirement that a solution portfolio must obey. By contrast, a soft bound on leverage specifies a desirable bound for a leverage term. The soft bound is a "nice to have" instead of a "must have" feature for the solution portfolio. Similarly, a leverage target specifies a desirable value for the given leverage term, and applies a user-specified linear or quadratic penalty on any deviations away from it.

Combinations of the above leverage control mechanisms[8] can be utilized in the Barra Optimizer. Whether to choose one or more depends on the manager's modeling objective and case circumstances. For more information on the technical definitions of leverage constraints, and the distinctions between constraints, soft bounds and penalties, please refer to the latest *Barra Optimizer Users' Guide 2.1*, Section 4.5, "By-Side Optimization—Leverage, Turnover by Side, and Cardinality by Side," and Section 3.9, "Constraint Flexibility—Penalties, Soft Bounds, and Hierarchy."

Note that a leverage term is defined at the asset group or portfolio level. It is the sum of asset weights or a ratio of two sums. Unlike the cash bound, an individual bound on securities, even if it stipulates a short position, is not a leverage constraint. Rather, it is a simple, standard constraint that is easy to be dealt with in the optimization process.

Leverage Aversion is Essentially a Penalty Term in the Barra Optimizer

Jacobs and Levy (2012) propose that long-short portfolio optimization include a term for leverage aversion in addition to risk aversion. From a theoretical standpoint, their proposal may be significant, as it forces portfolio managers to explicitly consider the trade-offs between leverage and volatility in portfolio construction. From the Barra Optimizer's technical perspective, however, the proposed measure of leverage aversion is simply a penalty on total leverage[9] added to the standard mean-variance objective function. Leverage aversion is merely one of the many existing vehicles managers could employ to manage the unique risks of leverage.

More specifically, Jacobs and Levy (2012, 2013a, 2013b) have suggested two different ways to specify the leverage aversion term in the Markowitz (1952) mean-variance utility, with the key difference depending on whether the portfolio's leverage[10] is multiplied by a constant or by the portfolio's total volatility. Jacobs and Levy argue that choosing a leverage level and imposing it on the portfolio with a constraint is "largely *ad hoc*" and is inferior to an explicit leverage aversion term added directly to the utility function.

While we agree that leverage constraints have their limitations and may not be appropriate for all occasions, we would like to point out that a leverage aversion term, which is essentially a quadratic penalty term on portfolio leverage, is not without its shortcomings. In particular, the magnitude of the penalty is difficult to choose and balance with the volatility risk term. In fact, the pros and cons of using a constraint versus a penalty term in the objective function are not unique to the portfolio leverage. These trade-offs hold true in other aspects of portfolio construction as well.

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[6] Penalties on leverage ratios (i.e., the short-long leverage ratio and the net-total leverage ratio) are not supported in the Barra Optimizer.

[7] This is also known as a "soft leverage constraint."

[8] Simultaneous constraint and soft bounds on the same leverage term over difference ranges are not currently supported, though.

[9] More precisely, it is a penalty on the square of any deviation of the total leverage from one. We often call such a penalty as a quadratic penalty.

[10] More precisely, it is the square of the portfolio's leverage that is being multiplied.

It is ultimately a manager's preference and decision to choose among leverage constraints, leverage penalties (e.g., aversion terms in the objective), or soft bounds on leverage when managing a long-short portfolio. Whatever the choice, managers may rely on the comprehensive and practical tools in the Barra Optimizer to examine and control leverage risk.

For example, Jacobs and Levy's mean-variance-leverage model (2012) specified with a constant is a quadratic programming problem and could be handled directly by the Barra Optimizer with ease. In a later model (Jacobs and Levy 2013a) the leverage aversion term is specified with the portfolio's total volatility, posing a quartic mathematical problem, and this cannot be solved directly with any quadratic optimization solver, including the Barra Optimizer. However, as the authors delineated, it can be handled with a "fixed-point iteration" in which the total volatility is treated as a constant during each quadratic programming sub-problem. Therefore, the Barra Optimizer can be utilized to sequentially solve this more complicated mean-variance-leverage model as well.

In short, Jacobs and Levy's recent theoretical insight on leverage aversion is a significant validation of a long-standing feature in the Barra Optimizer; this provides a bridge between theory and practice, and the Barra Optimizer may even be utilized to further advance the theory.

Modeling Long-Short Strategies

Most practical long-short investment strategies are characterized by one or more leverage control features. Below, we illustrate how to model these strategies using the Barra Optimizer.

Dollar-Neutral Strategies

A dollar-neutral portfolio consists of a cash position together with offsetting long and short positions in equal dollar amounts, at all times. By fixing both the upper and lower bounds on the cash asset to 100 percent, the portfolio holding constraint becomes "long leverage = short leverage," which will force the total long positions to offset the total short positions at all times. In other words, the characteristic leverage control of a dollar neutral strategy is that the constraint on a cash position must be equal to one.

Enhanced Equity Strategies

An enhanced active equity strategy seeks to improve upon the performance of an actively managed long-only portfolio by allowing limited leverage and shorting to be used in portfolio construction. It is usually characterized by two leverage constraints—(1) cash position must be equal to zero, and (2) long or short leverage must be equal to a user-defined amount. For example, the second characteristic constraint for a 130-30 strategy is a long leverage constraint requiring the long positions to be 130 percent of the portfolio value. Alternatively, it could also be a short leverage constraint requiring the short positions to be 30 percent.

General Hedge Strategies

A general hedge fund is a portfolio consisting of a cash position, some long positions and some short positions. The long leverage may or may not equal to short leverage, and the cash position may or may not be zero or 100 percent. However, the portfolio balance constraint must always be satisfied. Thus, constraining the cash position to be 10 percent, combined with a long leverage constraint requiring long positions not to exceed 120 percent, will effectively produce a general hedge portfolio with 10 percent cash, no more than 120 percent longs, and no more than 30 percent shorts.

How is Long-Short Optimization Performed?

In this section, we discuss why long-short optimization may be difficult, and explain the two approaches that the Barra Optimizer employs to handle the long-short optimization. Both approaches consist of integrated algorithms optimizing the long and short positions simultaneously. Each approach has its own strengths and weaknesses.

Why Long-Short Optimization May Be Difficult

Long-short optimization, which is characterized by at least one leverage constraint or one leverage penalty term, is considerably more difficult to solve than standard portfolio optimization. Why?

First, leverage features cannot be expressed by simple linear functions, in terms of the asset weights, until one knows exactly which assets are long and which ones are short.

Secondly, leverage features could be complex and non-convex, which may lead to local optimal or sub-optimal solutions. Two key indicators for non-convex optimization problems are:

- *A binding lower bound (either hard or soft) on a leverage constraint* (e.g., long leverage >=120 percent). Mathematically, a long leverage is a convex piece-wise linear function. A binding lower bound on a convex piece-wise linear function will make the constraint non-convex.
- *Too high a target associated with a leverage penalty.* A target on leverage (e.g., 170 percent for long leverage) is not a "must-be-obeyed" constraint. The Barra Optimizer will try to find a portfolio as close to the target as possible, weighing the impact of return, risk, penalty and other terms on utility. The effect of a high target is similar to that of a high lower bound on the leverage, which may make the optimization problem non-convex.

To keep a long-short portfolio construction problem simple, a general principle is to avoid *setting a lower bound or a high target on leverage whenever possible.* An excessively high lower bound or target may force the portfolio to take on unnecessarily high levels of leverage and risk.[11] *No lower bound, or a loose lower bound, on a leverage constraint is recommended.*

The Nonlinear Programming (NLP) Approach—Splitting Variables

A common and intuitive approach to handle the leverage controls is to split a non-cash asset variable into two—one for the "long" side and the other for the "short" side. More specifically, each non-cash variable is now defined as the difference between two non-negative split variables. As a result, one may easily express a leverage constraint as a simple linear constraint in terms of the new "long" and "short" variables only.

However, this approach of "splitting" variables presents its own difficulties as well. First, the number of decision variables will double, which may increase computational complexity and inefficiency, especially when the investment universe is large. Secondly, a solution feasible to the new set of decision variables may not be feasible to the original set, due to the fact that the "long" and '"short" side variables for the same asset may be simultaneously positive (which does not make practical sense).

For certain long-short optimization problems, the solutions feasible to the split variables are also guaranteed feasible to the original variables. In this case, the problem or portfolio is referred to as "trimable." (See Jacobs, Levy, and Markowitz (2006) for definitions and conditions for "trimability.") The rule of thumb is that if the problem is convex, the portfolio will be trimable and can be dealt with

[11] In general, for a given problem, there exists an optimal leverage level. To impose more leverage than this optimal level will not produce a higher return See results in Figures 2 and 3..

rather easily by splitting variables. Non-trimable problems often present difficulties and cannot be handled by efficient algorithms for long-only portfolio optimization problems.

Splitting variables is one of the approaches used by the Barra Optimizer to handle the long-short optimization. To discourage "long" and "short" side variables for the same asset from occurring simultaneously, a penalty term penalizing the sum of all cross products of each "long" and "short" pair is added to the utility. Since this penalty term is nonlinear in the new split variables, a non-linear optimization engine is required and actually used to deal with it.

A strength of this NLP approach to long-short optimization in the Barra Optimizer is its ease in handling other nonlinear terms in the utility or in the constraints; for example, risk budgeting constraints, the function-term transaction costs, and so on.

The Quadratic Programming (QP) Approach Using Piecewise Linear Functions

The Barra Optimizer's in-house quadratic programming solver is designed for solving quadratic mean-variance portfolio optimization problems. One of its prominent strengths is that it can handle piecewise linear constraints very efficiently.

When running a long-short optimization, the Barra Optimizer formulates leverage constraints or targets as piecewise linear constraints or functions. No splitting variables are necessary, so the problem size is kept small. All else equal, this should lead to a faster performance; due to non-splitting of variables in this approach, going long and short for the same asset at the same time does not occur, and a nonlinear penalty is not needed to penalize the cross terms. In addition, since this approach has been long supported in the Barra Optimizer, many of the features and functionalities developed originally for long-only strategies can be carried over to long-short strategies easily.[12]

Depending on the characteristics of the case involved, the Barra Optimizer uses either the QP approach or the NLP approach, or both, to solve a long-short portfolio optimization problem; the choosing of an approach is based on a set of branching rules. For example, the QP approach is the default for small convex problems, while the NLP approach is the default for any problem with a risk constraint. For complicated non-convex problems of small or medium size, both may be used to help enhance the success rate and solution quality.

[12] However, in contrast to the NLP approach, this QP programming approach cannot handle complicated non-linear features.

Leverage and Volatility Risk Trade-offs

Melas and Suryanarayaman (2008) have demonstrated analytically and empirically that there is an optimal level of leverage for a given level of active risk. Put another way, given the active volatility risk, the optimal leverage is fixed. In this section, we use examples to illustrate the relationships between leverage and volatility risk, and the various aspects of long-short optimization.

Case Settings

The portfolios reported in the tables and graphs in this section are all constructed using the Barra Optimizer, based on these common settings:

- Investment Universe: MSCI US Prime Market 750 (large and mid cap names) as of October 31, 2013 (containing 754 assets).
- Benchmark: Same as the Investment Universe.
- Risk Model: Barra USE4S.
- Invest all cash (i.e., cash asset bounds are locked to zero).
- Regular asset bounds are all set as [-1, 1].
- Same randomly-generated Alpha's across all cases. By construction, the Alpha's are all within the [-0.018, 0.018] range.
- Initial portfolio consists of 100 randomly-generated assets. By construction, the initial weights are all within the [-0.1, 0.1] range.
- Risk aversion (for both the factor risk and specific risk) is fixed to 1.0, except for the cases in Figure 1, where risk aversion is the variable on the X-axis.
- Transaction cost is ignored.
- The long leverage constraint is the one optional constraint that may be applied (other than the portfolio balance constraint that is present in all cases).

Optimal Levels of Volatility and Leverage Decrease as Risk Aversion Increases

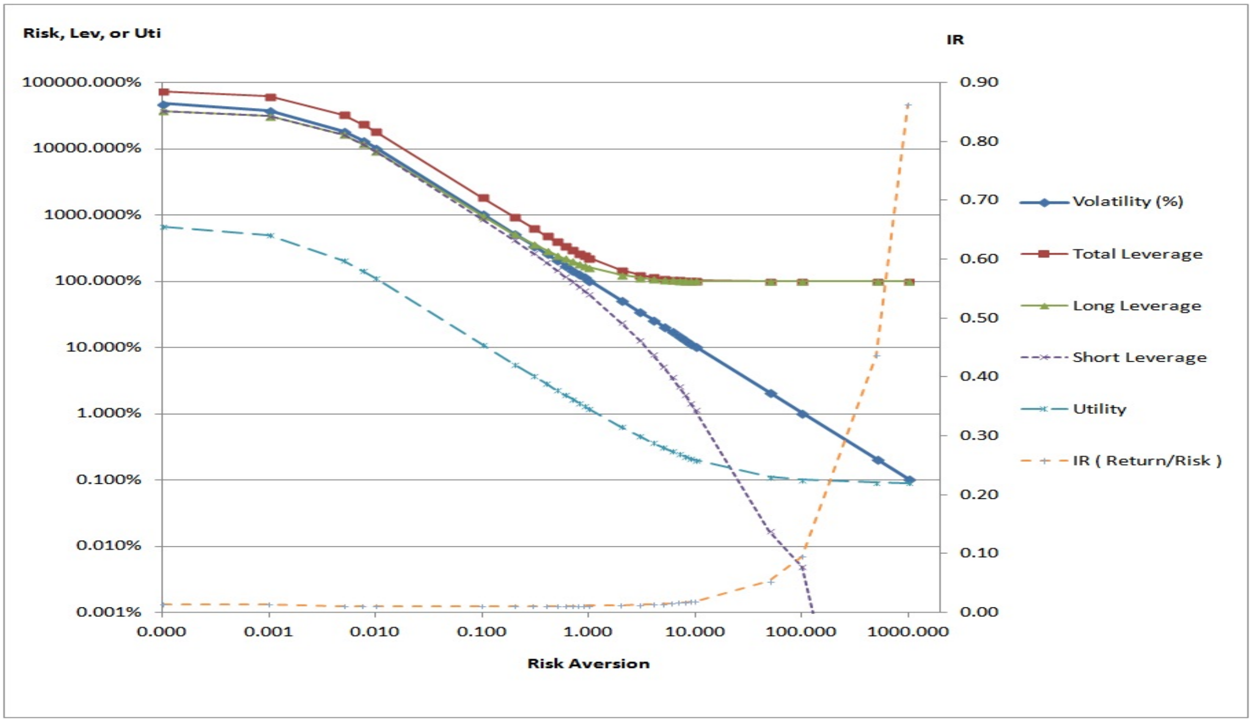
In our test case, we first relax the long leverage constraint, i.e., set its upper bound to infinity and lower bound to zero. Since there is essentially no leverage constraint now in the problem setting, the long-short optimization is reduced to a standard optimization. We adjust the risk aversion (i.e., re-setting both values for the factor risk and specific risk simultaneously), and let the optimizer decide on the optimal portfolio based on the mean-variance utility function (and the case settings described earlier).

Table 2 shows how the volatility, leverage, utility, and information ratio values of the optimal portfolio change with risk aversion. We see that as aversion to risk increases, the optimal volatility level decreases. So are the optimal levels of long leverage, short leverage, as well as the total leverage. The information ratio value decreases first as the optimal return decreases slightly faster than the optimal volatility, but it shoots up as risk aversion becomes very large and volatility approaches zero.

Table 2. The Effects of Risk Aversion on Optimal Levels of Volatility and Leverage.

Risk Aversion	Volatility (%)	Total Leverage	Long Leverage	Short Leverage	Utility	Return(%)	IR (Return/Risk)
0.0001	488.1	73931%	37016%	36916%	6.66310	690.14	0.01365
0.001	378.1	62498%	31299%	31199%	5.02399	645.33	0.01329
0.005	186.0	32742%	16421%	16321%	2.08417	381.38	0.01121
0.0075	134.9	23747%	11923%	11823%	1.45163	281.55	0.01076
0.01	103.7	18236%	9168%	9068%	1.10128	217.64	0.01062
0.1	10.5	1857%	978%	878%	0.11138	22.19	0.01060
0.2	5.26	940%	520%	420%	0.05614	11.14	0.01068
0.3	3.50	638%	369%	269%	0.03772	7.46	0.01077
0.4	2.63	488%	294%	194%	0.02852	5.61	0.01085
0.5	2.10	398%	249%	149%	0.02299	4.51	0.01094
0.6	1.75	340%	220%	120%	0.01931	3.77	0.01102
0.7	1.50	299%	199%	99%	0.01668	3.25	0.01111
0.8	1.31	268%	184%	84%	0.01471	2.85	0.01119
0.9	1.17	245%	172%	72%	0.01317	2.54	0.01128
1	1.05	227%	163%	63%	0.01194	2.30	0.01136
2	0.53	148%	124%	24%	0.00642	1.19	0.01221
3	0.35	125%	113%	13%	0.00458	0.83	0.01306
4	0.26	116%	108%	8%	0.00366	0.64	0.01392
5	0.21	110%	105%	5%	0.00310	0.53	0.01477
6	0.18	107%	104%	4%	0.00274	0.46	0.01562
7	0.15	105%	103%	3%	0.00247	0.41	0.01647
8	0.13	104%	102%	2%	0.00228	0.37	0.01732
9	0.12	103%	101.5%	1.5%	0.00212	0.33	0.01817
10	0.11	102.3%	101.1%	1.1%	0.00200	0.31	0.01902
50	0.02	100.03%	100.02%	0.02%	0.00112	0.13	0.05306
100	0.01	100.01%	100.005%	0.005%	0.00101	0.11	0.09561
500	0.002	100.00%	100.000%	0.0000%	0.00092	0.094	0.43620
1000	0.001	100.00%	100.000%	0.0000%	0.00091	0.092	0.86295

Figure 1. The Visual Plot of Table 1.



An important message here is that adjusting risk aversion provides an effective way to control volatility and leverage risk at the same time. A desirable leverage level may be achieved without using a leverage constraint or a leverage penalty, as proposed by Jacobs and Levy.

A Strict Lower Bound on Leverage Induces Unnecessary Risk and Limits Utility

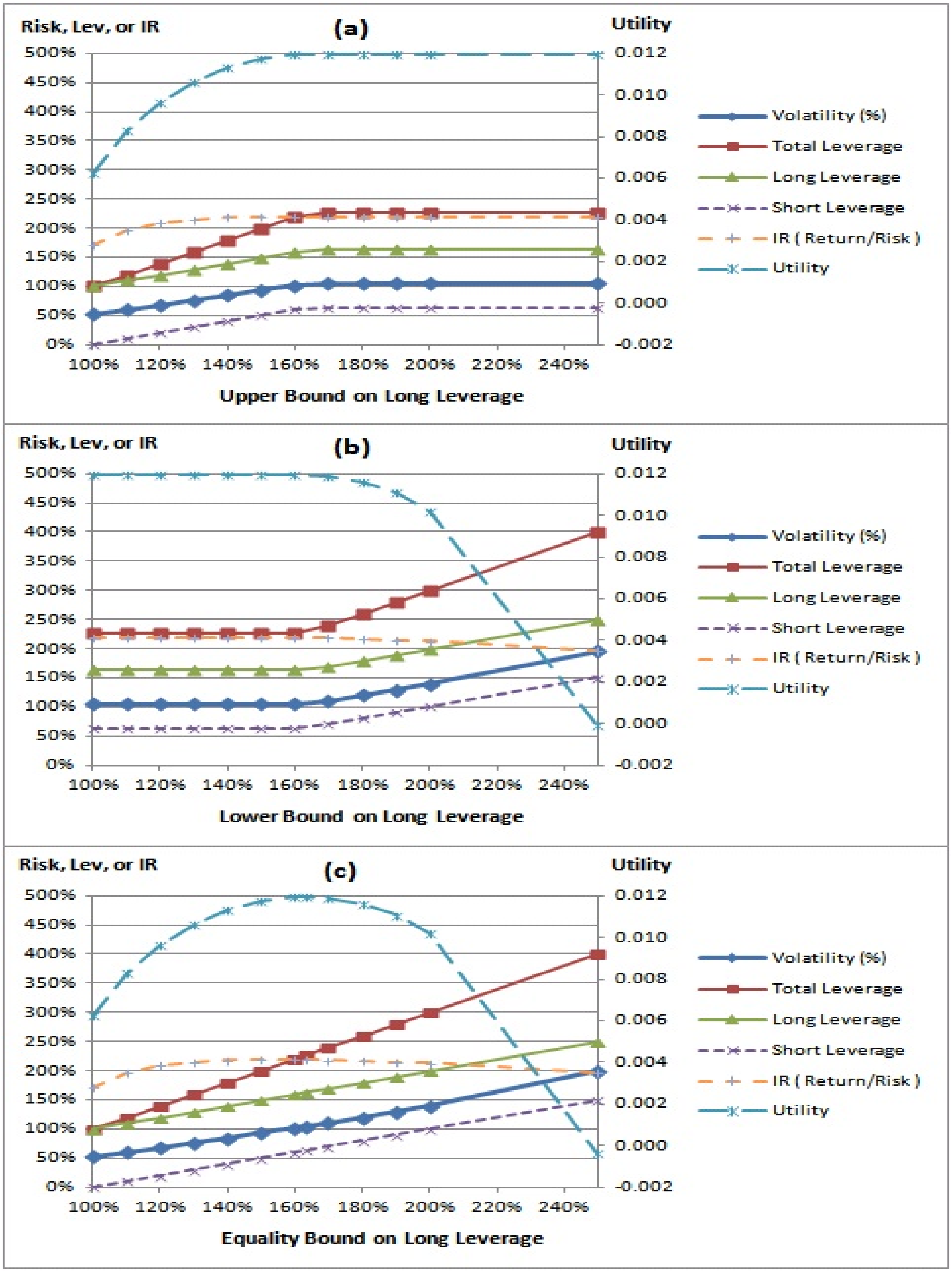
Figure 2 demonstrates the impact of a leverage constraint on the optimal portfolio. Here, as mentioned earlier, the risk aversion is fixed at 1.0. We know from the results in Figure 1 that the unconstrained[13] optimal long leverage level is 163 percent.

In Figure 2(a), we keep the lower bound of the long leverage constraint at zero, and adjust its upper bound from 100 percent to 250 percent. We see that when the upper bound is binding (i.e., less than 163 percent), the volatility, leverage levels, information ratio, and utility all increase with the upper bound. When it is not binding (i.e., greater than or equal to the unconstrained optimal level of 163 percent), the utility value stays at a peak level, but so does the volatility and leverage. The information ratio peaks before the upper bound reaches the unconstrained optimal level.

In Figure 2(b), we relax the upper bound of the long leverage constraint to infinity, and vary its lower bound from 100 percent to 250 percent. We see that when the lower bound is not binding, the volatility and leverage levels stay at the lowest possible value, with the utility value being the highest. As the lower bound becomes binding, the volatility and leverage levels increase while the utility and the information ratio decrease.

[13] "Unconstrained" here means no constraints other than the portfolio balance constraint and asset bounds.

Figure 2. Effects of Upper, Lower and Equality Bounds on Long Leverage.



In Figure 2(c), we adjust the equality bound of the long leverage constraint, which is essentially adjusting both its upper and lower bounds simultaneously, from 100 percent to 250 percent. Based on previous results in Figures 2(a) and 2(b), we know that the equality bound will always be binding. Why? Below the unconstrained optimal leverage level, the upper bound is binding, while at or above this level, the lower bound is binding. Thus, it is not surprising to see that the volatility and leverage levels keep increasing with the equality bound, but the utility value starts to decrease when the lower bound becomes binding . The information ratio peaks before the unconstrained optimal level and decreases after this level.

As expected, a binding constraint always has the negative effect of reducing the utility value. However, in the case of a binding upper bound on leverage, the lower utility value is associated with a lower volatility risk and leverage levels as well. On the other hand, in the case of a binding lower bound on leverage, the lower utility is accompanied by an undesirable higher volatility.

Again, a take-away message here is that unless absolutely necessary, lower bounds on leverage constraints should be avoided because they may induce higher volatility and lower utility at the same time.

When the Penalty is Large Enough, a Penalty Target on Leverage Acts Like an Equality Constraint

In this subsection, we relax the long leverage constraint again by setting its upper bound to infinity and lower bound to zero. However, we now add a quadratic penalty function to the utility with a target on long leverage, and a penalty scalar to control the magnitude of the penalty and the trade-offs among the return, volatility, and penalty terms. We vary the target from 100 percent to 250 percent.

Figure 3. The Effects of a Penalty Target.

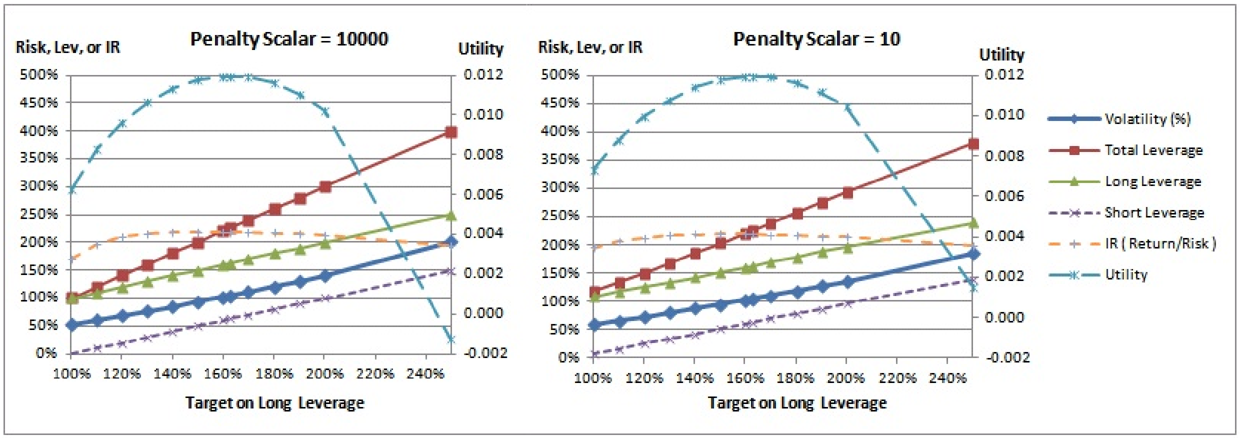


Figure 4. The Effects of a Penalty Scalar.

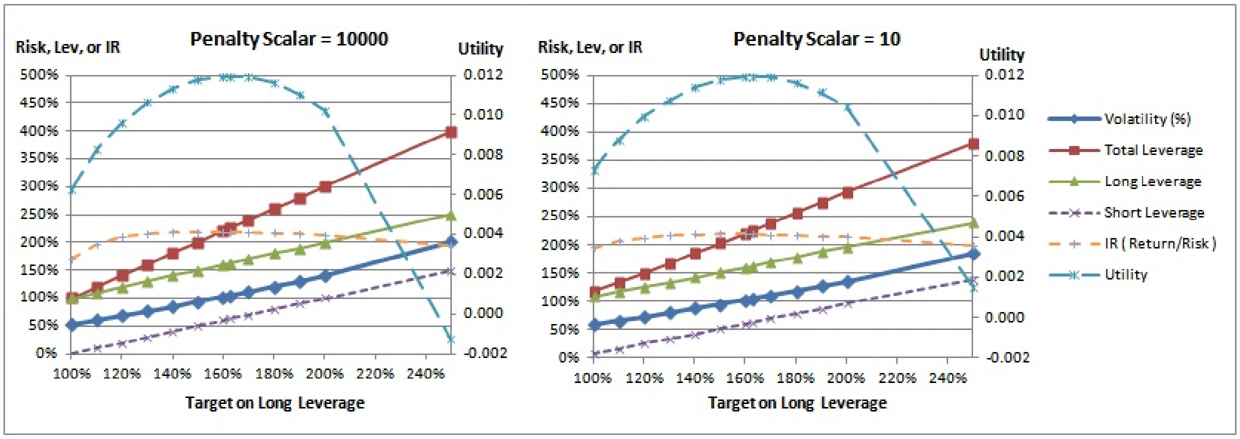


Figure 3 and Figure 4 illustrate that, when the penalty scalar is very large (e.g., 10,000), adjusting the target has almost the same effect as adjusting the equality bound, since the target is almost reached. As the penalty scalar gets smaller and smaller, more and more deviations away from the target occur, since doing so is deemed optimal by the optimizer—the increased risk-adjusted return more than offsets the scaled penalty on leverage. Note that when the target is below the unconstrained optimal leverage level, the deviations will be above the target, and vice versa. In other words, the optimal portfolio's leverage level will tend to be no further away from the unconstrained optimal level than the target.

Conclusion

Jacobs and Levy have recently published a series of papers promoting the notion of "leverage aversion" and the benefits of incorporating it explicitly in the traditional Markowitz Mean-Variance Optimization. We point out that "leverage aversion" is in practice a leverage penalty term in the Barra Optimizer, a feature available since 2009. Jacobs and Levy's recent theoretical insight on leverage aversion is a significant validation of a long-standing feature in the Barra Optimizer. It provides a bridge between theory and practice, and the Barra Optimizer may even be utilized to further advance the theory.

Leverage penalties and leverage constraints are two different modeling options to control the unique risks of leverage. The Barra Optimizer supports both, and more. Both leverage penalties and leverage constraints have their shortcomings. A desirable leverage level may be achieved by simply adjusting the risk aversion parameters, without using either a leverage penalty or a leverage constraint. An upper bound or a lower-than-optimal target on leverage may limit the potential of the managed portfolio, but at least its volatility risk will be commensurate with its utility. A lower bound or a higher-than-optimal target on leverage could hurt the utility of the managed portfolio and induce a higher volatility at the same time.

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References

Jacobs, Bruce I. and Kenneth N. Levy (2012) “Leverage Aversion and Portfolio Optimality” *Financial Analyst Journal*, Vol. 68, No. 5 (September/October), pp. 89–94.

Jacobs, Bruce I. and Kenneth N. Levy (2013a) “Leverage Aversion, Efficient Frontiers, and the Efficient Region” *The Journal of Portfolio Management*, Vol. 39, No. 3, pp. 54–64.

Jacobs, Bruce I. and Kenneth N. Levy (2013b) “A Comparison of the Mean-Variance-Leverage Optimization Model and the Markowitz General Mean-Variance Portfolio Selection Model” *The Journal of Portfolio Management*, Vol. 40, No. 1, (Fall), pp.1-5.

Jacobs, Bruce I. and Kenneth N. Levy (2014) “Traditional Optimization Is Not Optimal for Leverage-Averse Investors” *The Journal of Portfolio Management*, Vol. 40, No. 2, (Winter), pp.1-11.

Jacobs, Bruce I., Kenneth N. Levy, and Harry M. Markowitz (2006) “Trimability and Fast Optimization of Long-Short Portfolios”, *Financial Analysts Journal,* Vol. 62, No. 2, pp. 77–91.

Markowitz, Harry M. (2013) "How to Represent Mark-to-Market Possibilities with the General Portfolio Selection Model." *Journal of Portfolio Management,* Vol. 39, No. 4, pp. 1–3.

Markowitz, Harry M. (1952) "Portfolio Selection." *Journal of Finance,* Vol. 7, No. 1, pp. 77–91.

Lo, Andrew W. and Pankaj N. Patel (2008) “130-30: The New Long-Only”, *Journal of Portfolio Management*, Vol. 34, No. 2 (Winter), pp. 12-38.

Melas, Dimitris and Raghu Suryanaryanan (2008) “130/30 Implementation Challenges”, *MSCI Barra Horizon Newsletter* (Q1), pp. 9-27.

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New York	+ 1.212.804.3901
San Francisco	+ 1.415.836.8800
Sao Paulo	+ 55.11.3706.1360
Toronto	+ 1.416.628.1007

Europe, Middle East & Africa

Cape Town	+ 27.21.673.0100
Frankfurt	+ 49.69.133.859.00
Geneva	+ 41.22.817.9777
London	+ 44.20.7618.2222
Milan	+ 39.02.5849.0415
Paris	0800.91.59.17 (toll free)

Asia Pacific

China North	10800.852.1032 (toll free)
China South	10800.152.1032 (toll free)
Hong Kong	+ 852.2844.9333
Seoul	00798.8521.3392 (toll free)
Singapore	800.852.3749 (toll free)
Sydney	+ 61.2.9033.9333
Taipei	008.0112.7513 (toll free)
Tokyo	+ 81.3.5290.1555

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[1] *As of March 31, 2014, as reported on June 25, 2014, by eVestment, Lipper and Bloomberg* *July 2014*