SHOULD YOU CARE ABOUT ACTIVE SHARE?

A Portfolio Construction Study

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EXECUTIVE SUMMARY

”Active Share” — a popular measure of how much a portfolio’s composition differs from that of its benchmark — has been credited as a predictor of manager skill. Seminal academic research showed that managers with high Active Share and low Tracking Error relative to the benchmark displayed persistent outperformance. Conversely, managers with low Active Share and low Tracking Error were labelled “closet indexers” and have recently become the subject of increasing regulatory scrutiny.

Subsequent research has not unanimously supported the original results. Most has adopted a similar approach, looking only at Active Share and actual fund performance. However, this is like a field study that does not control for the environment, potentially leading to the conflation of different factors. Therefore, we instead simulated strategies and tested them under various conditions and constraints, to identify the true drivers of Active Share.

Through portfolio construction experiments and analytical results we find:

- Active Share is not *a priori* linked to management skill, regardless of whether the manager employs a stock-picking or a factor-timing approach. Among stock-picking managers, high Active Share *may* be linked to performance through conviction.
- Active Share depends on the choice of benchmark and on style tilts. With managers benchmarked to cap-weighted indexes, Active Share favors tilts towards small caps, but may penalize low volatility managers. A more concentrated benchmark usually leads to lower Active Share for the same level of Tracking Error.
- Active Share provides additional information compared to Tracking Error, but there is substantial commonality because the level of risk aversion is a driver of both.
- Active Share depends on active factor exposure. It is systematically higher for stock-picking managers who take on more idiosyncratic risk than factor investors.
- Requiring managers to run high Active Share may have unintended consequences. They may boost Active Share by choosing a different benchmark, investing outside the benchmark universe, taking on excessive Tracking Error or employing portfolio constraints. These measures could have no direct bearing on performance and may even be detrimental.

We cannot offer any ”silver bullet” single metric that would allow investors to easily identify skillful active managers. However, we find that Active Share needs to be complemented with Tracking Error-based analysis, using a suitable risk model and sensible benchmark, for a meaningful attribution of manager performance and risk. This will help indicate whether a manager is acting within their designated mandate, and not taking unintended bets or adding value beyond systematic factors.

INTRODUCTION

Active Share is a measure (between 0 and 1) of how a portfolio differs from its benchmark, calculated as half the absolute difference in weights between the two:

$$AS = \frac{1}{2}\sum_{i=1}^{N}|w_i - b_i|$$

K. J. Martijn Cremers and Antti Petajisto of Yale University,[1] who originated the concept, found significant outperformance for mutual fund managers showing high Active Share and low Tracking Error, which the authors categorized as "diversified stock pickers." Managers with both low Active Share and low Tracking Error were labelled "closet indexers" and showed the most consistent underperformance.

Some subsequent researchers have confirmed their original results, but others have not. Critics have noted:

- Active Share depends on benchmark selection, investment style and market volatility.
- Active Share favors managers who tilt toward small-cap stocks.
- High Tracking Error also is a predictor of outperformance.
- High Active Share correlates with increased downside risk.
- Outperforming high Active Share funds tend to be benchmarked to small-cap indexes; Active Share loses most of its predictive power when restricted to funds benchmarked to the same index.

Despite these critiques, Active Share has become part and parcel of the public debate on "closet indexing" particularly relative to active managers whose portfolios are very similar to the benchmark, but who charge higher active fees. Articles in the financial press regularly call on investors to scrutinize the Active Share score of portfolio managers in order to weed out closet indexers. In some countries – especially in Scandinavia — Active Share is attracting the interest of financial regulators. As a result, a number of asset managers in Europe have already announced the intention to report Active Share of their funds from, and there are frequent calls on others to do the same.

[1] Cremers, K. J. Martijn and Antti Petajisto. (2009) "How Active Is Your Fund Manager? A New Measure That Predicts Performance." *Review of Financial Studies*, Yale School of Management.

One reason for the increasing popularity of Active Share is its purported predictive power for manager performance and growing threats of regulators using it as a metric. Another reason is the measure's simplicity. Whereas the calculation of Tracking Error requires a history of fund and benchmark returns and possibly a risk model, Active Share requires only synchronous data on fund and benchmark holdings.

Active Share's simple definition however masks practical challenges in implementation. First, accurate and timely fund holdings data are more difficult to find than fund performance data, and stale holdings information may bias the value of Active Share.

Data quality aside, high Active Share may be a sign that the benchmark is inappropriate, or indeed irrelevant, to the portfolio. Managers with substantial holdings outside the benchmark universe will have high Active Share – as much as 100% if benchmark assets are avoided altogether — while Tracking Error is not sensitive to off-benchmark holdings. Further complications arise if the portfolio holds derivatives, fixed income assets or even cash. While alternative asset classes are easily accounted for in Tracking Error, it is challenging to incorporate them into Active Share. Lastly, Active Share is effectively limited to long-only portfolios.

Most studies in the literature attempt to empirically link Active Share and manager performance through fund holdings and performance data. "Top-down" empirical studies using large samples tend to aggregate managers using different benchmarks, working under different mandates and employing different investment styles, constraints and alpha signals. Besides combining very different management styles, aggregating results across very different market regimes will also affect results.

We therefore take a different tack in this paper and explore the meaning of Active Share through analytical relationships and simulated portfolio construction studies. For all studies in this paper, we use European equities as our investment universe and all backtests use the Barra European Equity Model (EUE4).

This "bottom-up" approach has the advantage of working in a controlled environment, where our testing can help tease apart the drivers of Active Share. In addition, empirical studies are, by definition, backward-looking and thus are not a reliable guide as to whether any link between low Active Share and underperformance today will prevail if Active Share is established as a yardstick for active management. In particular, managers may adapt to new expectations from clients and perhaps regulators by boosting Active Share through portfolio engineering, without boosting actual performance.

A drawback to simulations, however, is that we have to assume certain stylized investment strategies in order to derive analytical results. These stylized strategies may not encompass the whole variety of investment strategies found among managers.

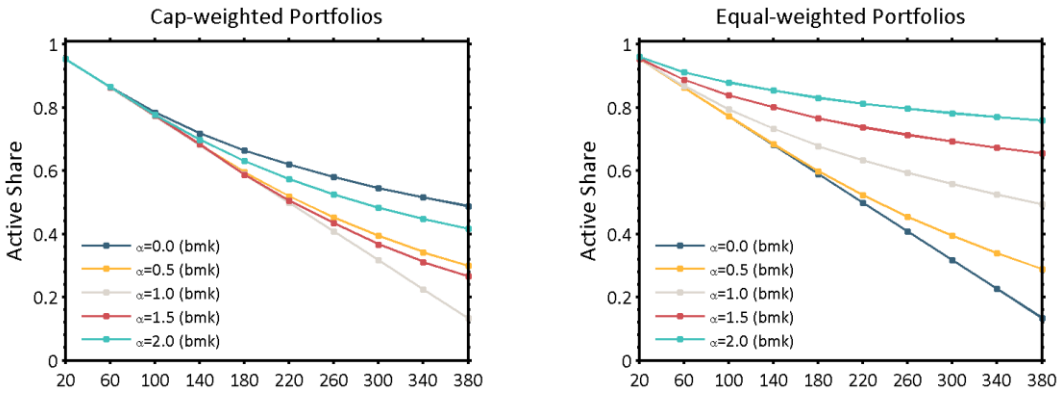
ACTIVE SHARE, BENCHMARK CHOICE AND MANAGEMENT STYLE

The choices of benchmark and management style are among the most important determinants of Active Share. In this section, we explore how Active Share depends on the number of names in the portfolio and on the match between the distribution of the portfolio and benchmark weights. Drilling deeper, we find that Active Share favors certain investment styles — particularly small-cap portfolios — and disadvantages others, such as low volatility.

THE CHOICE OF BENCHMARK AFFECTS ACTIVE SHARE

Practitioners sometimes assert that Active Share is determined by the concentration of the benchmark. We investigate this effect by constructing benchmarks of various concentrations and measuring the expected Active Share of randomly chosen portfolios against these benchmarks. Exhibit 1 shows the Active Share of Cap-Weighted (left panel) and Equal Weighted (right panel) randomly generated portfolios of between 20 and 380 names (x-axis) against benchmarks with weightings ranging from equal weighted (α=0) to cap-weight squared (α =2).

Exhibit 1: The Choice of Benchmark Affects Active Share



Active Share of long-only portfolios with 20, 60 …380 names selected randomly from benchmark universe. Benchmarks have varying concentration from flat to square-cap weighted. Selected portfolios are either cap weighted (left panel) or equal weighted (right panel).

We find that cap-weighted portfolios display minimal Active Share when compared to a cap-weighted benchmark. If we deviate from the cap-weighted benchmark, either by using a flatter (lower alpha) or a more concentrated (higher alpha) benchmark, a higher Active Share is obtained. Similarly, an equal-weighted portfolio has a lower Active Share when compared to an equal-weighted benchmark, while choosing more concentrated benchmarks leads to ever higher Active Share. Misalignment between the weight profile of the portfolio and the benchmark leads to high Active Share, but does not reflect the properties of the benchmark. Such a mismatch between portfolio and benchmark is often driven by the manager's tilt on styles – especially Size — as we see below.

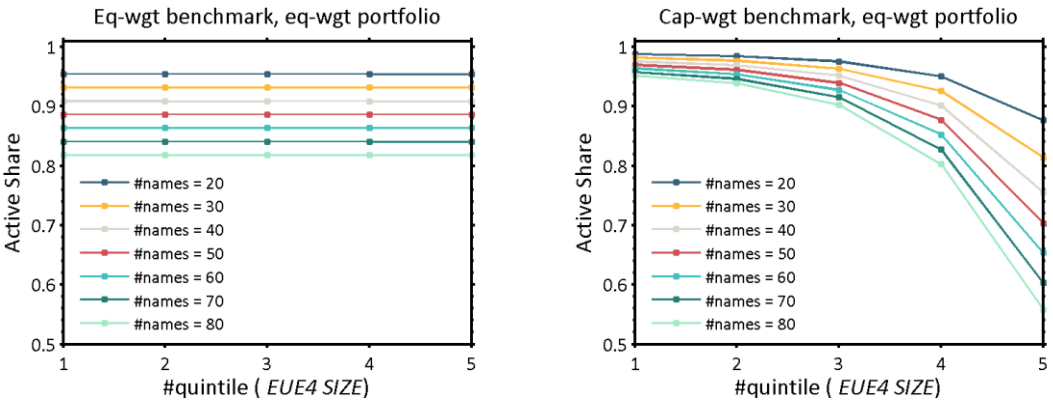
ACTIVE SHARE DEPENDS ON SIZE TILT AND INVESTMENT STYLE

Several authors[2] have noted that the Active Share of funds can correlate with their small-cap tilt. We have already seen that portfolios with a flatter weight profile – thus tilted towards small caps — have higher Active Share against cap-weighted benchmarks.

To isolate the effects of a Size tilt, we generate 1,000 random portfolios by choosing names from quintiles of EUE4 Size exposure. Portfolios are equal-weighted and the number of names ranges from 20 to 80. For example, portfolios with 20 names chosen from the first Size quintile will be least diversified and have the strongest small-cap tilt, whereas portfolios with 80 names chosen from the fifth quintile will be most diversified and have the strongest large-cap tilt.

[2] S. Sapra et al. (2013), T. Cohen et al. (2014)

Exhibit 2: Benchmark Matters for Size-tilted Portfolios



Active Share of random portfolios drawn from quintiles of EUE4 Size exposure. Left: equal-weighted benchmark. Right: cap-weighted benchmark. For each quintile in EUE4 Size exposure, we average 1,000 random portfolios of 20, 30… 80 names from that quintile.

For each portfolio, we calculate Active Share against both the original cap-weighted MSCI Europe Index and its equal-weighted version. We average Active Share over random samples for each value of portfolio size and Size tilt.

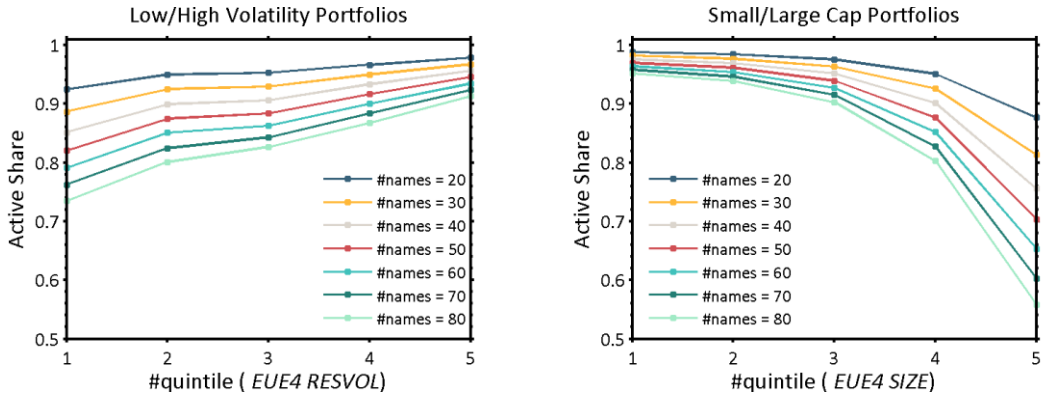
We observe the following:

1. Increasing the number of names decreases Active Share *for any benchmark*. For the equal-weighted benchmark, this is the only effect – the Size tilt makes no difference.
2. For the same number of names, a small-cap manager typically will have significantly larger Active Share than a large-cap manager. As a result, a large-cap manager must implement much more concentrated portfolios to reach the same level of Active Share.
3. For a given Size tilt, Active Share depends on the choice of benchmark. For example, for small-cap tilted portfolios chosen from the first Size quintile, benchmarking to

the equal-weighted index leads to lower Active Share whereas for large-cap tilted portfolios, the opposite holds[3].

Other style tilts may also affect Active Share through interactions with Size. For example, Residual Volatility tilts towards small caps, meaning that a low volatility manager will likely take a significant positive Size exposure. To demonstrate this effect, we modify our earlier experiment by replacing Size with the Residual Volatility factor.

Exhibit 3: Low Volatility Portfolios Achieved Lower Active Share than Small-Cap Portfolios



Active Share of equal weighted random portfolios against the cap weighted MSCI Europe Index. Stocks in the random portfolios are chosen from quintiles in EUE4 Size (Residual Volatility) in the left (right) panel.

Exhibit 3 shows that portfolios with lower Residual Volatility exposure have lower Active Share. Just as small-cap managers are at a natural advantage in achieving high Active Share, low volatility managers are at a natural disadvantage – assuming both are benchmarked to a cap-weighted index. Our experiment shows that empirical results relating Active Share to performance must be interpreted by considering style tilts – especially when small-cap and low volatility managers are prevalent in the sample.

ACTIVE SHARE, TRACKING ERROR AND RISK AVERSION

How are Active Share and Tracking Error related? Does Active Share provide any new information that is not available through Tracking Error? In this section, we establish a link between Active Share and Tracking Error that we use to explain the effect of Tracking Error

[3] Petajisto (2014) reports high Active Share and low Tracking Error continue to predict outperformance even if funds are first bucketed into small-, mid- and large-cap, but outperformance decreases with capitalization. In contrast, Cohen et al. (2014) find that among large-cap funds, the small-cap tilt is mostly responsible for driving Active Share and performance.

limits on Active Share, and show that Active Share is correlated with Tracking Error through risk aversion.

Motivated by Sapra et al. (2013), we find that if one assumes cross-sectional returns are driven by a factor model, then Active Share can be related to specific Tracking Error through the concentration of active portfolio weights and market dispersion. In the Appendix, we derive the following equation:

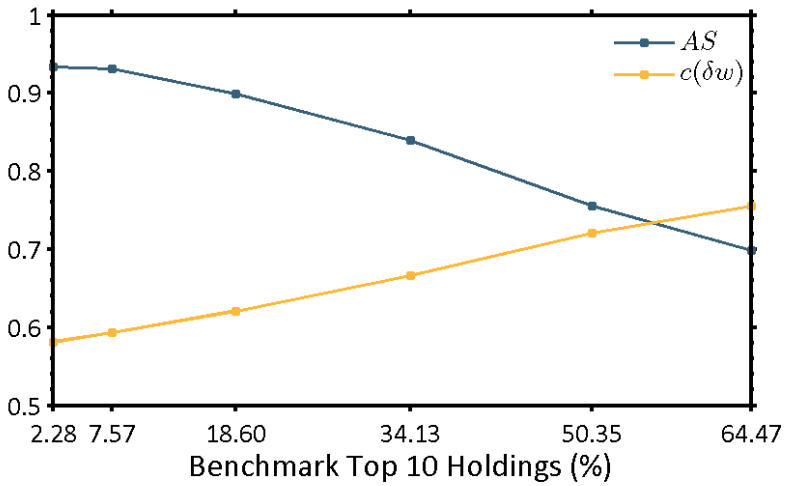
$$\mathrm{AS} = \frac{1}{2\sigma_\varepsilon}\left(\sqrt{\mathrm{N}} - \left(\sqrt{\mathrm{N}} - 1\right)\mathrm{c}(\delta \mathrm{w})\right)\frac{\mathrm{TE}_\varepsilon}{\mathrm{TE}}\,\mathrm{TE}$$

Active Share and Tracking Error are denoted AS and TE, respectively; TE_ε denotes the purely idiosyncratic component of specific risk and σ_ε denotes the average specific risk (dispersion) in the investment universe of size N. We also introduce the concentration of active weights $\mathrm{c}(\delta \mathrm{w})$: for a portfolio x, the measure $\mathrm{c}(\mathrm{x})$ can take values from 0 (least concentrated i.e., uniform weights) to 1 (most concentrated i.e., only one non-zero weight). To understand this formula, note that for fixed $\boldsymbol{TE_\varepsilon}$, Active Share will decrease with increasing concentration of active weights $\boldsymbol{c(\delta w)}$. This somewhat counterintuitive observation helps explain why funds benchmarked against more top-heavy indexes tend to have lower Active Share. In short, if the benchmark is more concentrated, the active portfolio will tend to be more concentrated as well. Assuming that funds are constrained by roughly the same level of Tracking Error, it follows that funds benchmarked against more concentrated indexes will have lower Active Share.

To test the hypothesis, we optimize long-only portfolios benchmarked against indexes of various concentrations but always targeting an annualized Tracking Error of 5%. We control the concentration of our simulated benchmark indexes by setting benchmark weights proportional to powers of market cap. We use the BARRA Open Optimizer on our test portfolios with the EUE4S model and an alpha signal[4] combining EUE4 style factors. Exhibit 4 plots the Active Share and the concentration of active weights against the concentration of the benchmark, which we measure as the proportion of benchmark weight in the top 10 holdings. For all benchmarks, Tracking Error was bound at 5%. Moving from least to most concentrated benchmark, the concentration of active weights increased and Active Share decreased, in accordance with our formula.

[4] The alpha signal goes long EUE4S Momentum and EUE4 'Value' (a combination of Earnings Yield, Book-to-Price and Dividend Yield) and goes short EUE4S Residual Volatility

Exhibit 4: Active Share of Portfolio Against Indexes of Varying Concentration.

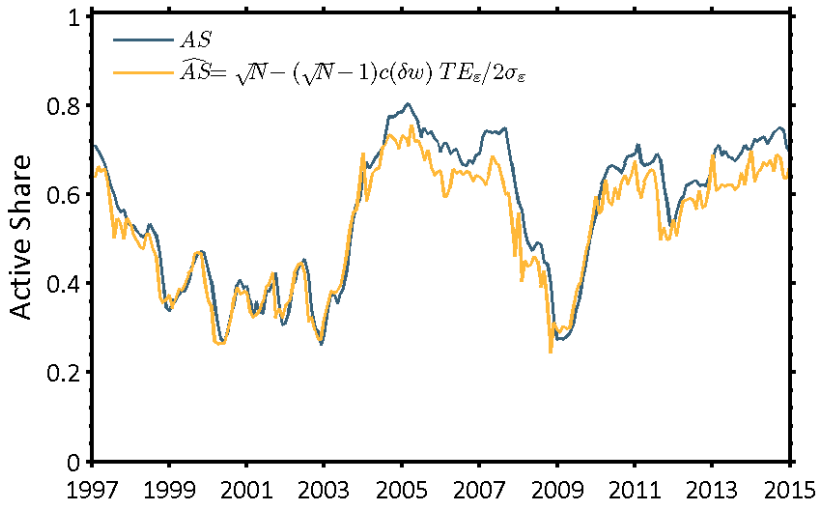


The result is intuitive: a manager benchmarked against a steeply weighted index can increase Active Share substantially by taking active positions in the largest benchmark holdings, which may result in prohibitively high Tracking Error. A more uniformly weighted index offers more freedom of movement for an active manager constrained by Tracking Error.

Our formula supports the taxonomy of fund managers presented in Petajisto (2013). Note that for a given level of Tracking Error, a portfolio with relatively uniform active weights and relatively large specific component of Tracking Error ratio will have high Active Share – these managers are the ”diversified stock pickers.” Increasing the factor component of Tracking Error leads to the lower Active Share typical of factor investors.

We also perform a more direct test of the decomposition formula by comparing the measured and predicted Active Share in an optimized portfolio backtest. We impose a long-only constraint, rebalance monthly and use an alpha signal built from EUE4S style factors.

Exhibit 5: Testing the Relationship between Active Share and Tracking Error

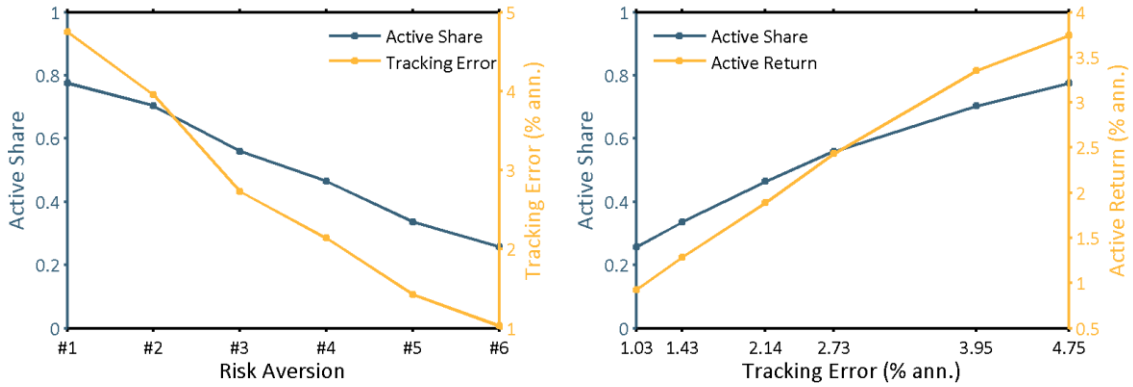


Specific Tracking Error and average specific risk are predicted by EUE4S; specific risk is averaging using index weights.

Exhibit 5 compares the backtested portfolio's actual and estimated Active Share. The overall fit is reasonable, even though we did not take into account the heteroscedasticity of specific risk and the estimation error in Tracking Error and market dispersion.

Next, we repeat our backtest by varying the level of risk aversion and leaving all other settings unchanged.

Exhibit 6: Average Active Share and Realized Tracking Error with Varying Risk Aversion



Averaging is performed over the backtest period from January 1997 until December 2014.

The left panel of Exhibit 6 compares the average Active Share and average Tracking Error at various levels of risk aversion. For the sampled range of risk aversions, Active Share and Tracking Error are equivalent reflections of the investor's risk aversion level (for very low risk aversion, Active Share would saturate at 1 while Tracking Error would continue to increase).

The right panel of Exhibit 6 shows that both Active Share and Tracking Error are correlated with active return. This observation has consequences for empirical studies of manager performance and Active Share. In a sample of managers with roughly equal skill who differ only in risk aversion, selecting for high Tracking Error and high Active Share would be equivalent and both would select for higher benchmark-adjusted performance, though not necessarily for higher risk-adjusted performance.

In fact, Kang et al. (2011) find that very active managers, as defined by high Tracking Error, outperform closet indexers, as defined by low Tracking Error. This observation may mean that more skillful managers have lower risk aversion and this is reflected both in their Tracking Error and their Active Share.[5]

[5] *Petajisto* claims that Active Share explains manager performance even when managers are grouped by Tracking Error], which would mean that the outperforming managers selected by high Active Share are not simply the high conviction managers.

DOES ACTIVE SHARE MEASURE MANAGER SKILL?

So far we have shown that Active Share is influenced by the choice of benchmark, investment style and risk aversion. But is it — as has widely been assumed —a measure of manager skill? To determine any relationship between Active Share and investment skill, we simulate scenarios where we control for skill directly. Active managers can add value both through skillful factor timing and through stock selection, so we simulate series of investors exhibiting varying degrees of investing ability within both investment styles.

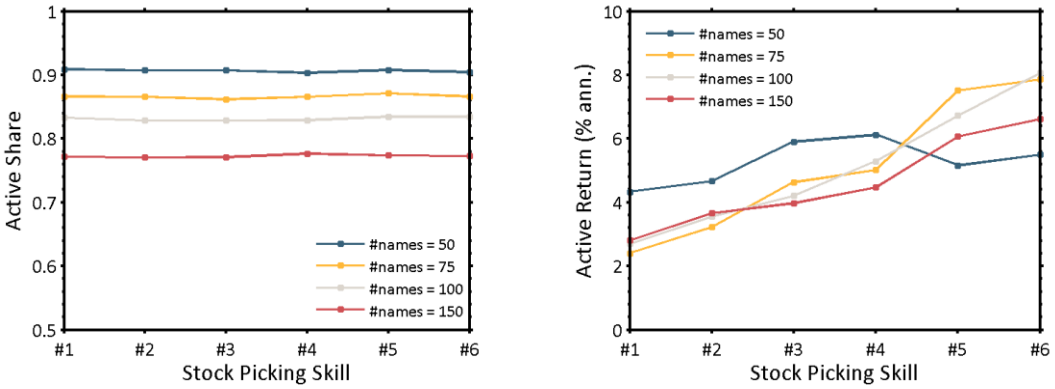
In both experimental setups, we find that benchmark-adjusted performance is correlated with skill, as expected, but that Active Share is unrelated to manager skill. Nevertheless, in the case of stock-picking managers, we can show a correlation between higher Active Share and performance for managers with stronger conviction, defined by how heavily they weight their ”best picks,” assuming that they actually have skill.

ACTIVE SHARE AND STOCK-PICKING MANAGERS

We define stock-picking managers as those who choose securities based on their estimates of the assets' future performance without any regard to their factor exposures; the holdings are equal weighted. The simulated managers vary in the number of names they select and in their stock-picking skill, defined as their “hit rate,” i.e., their probability of picking a stock that outperforms the benchmark in the subsequent period.

Portfolios are chosen randomly, with portfolio sizes between 50 and 150 names and hit rates from 51% to 56%. To mitigate sampling noise, we generate 100 random portfolios for each portfolio size and skill level, and average performance and Active Share.

Exhibit 7: Active Share and Performance as a Function of Stock-picking Skill



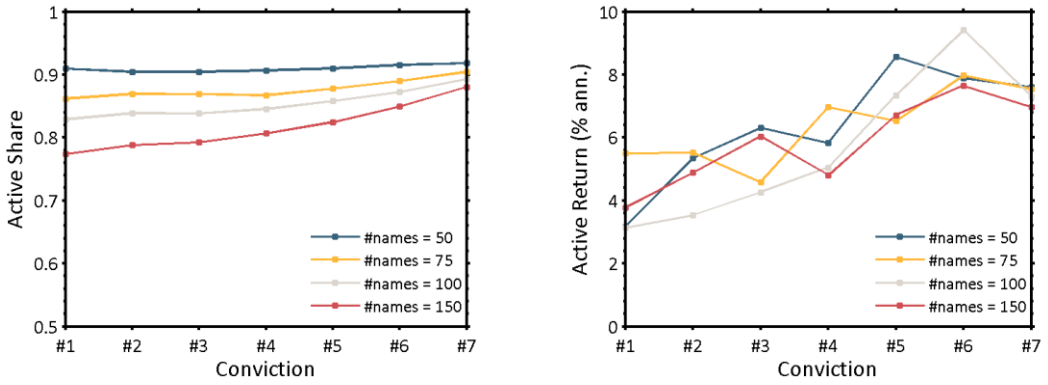
Left panel: Active Share depends on number of names, but not on stock-picking skill. Right panel: benchmark-adjusted return depends on both number of names and stock-picking skill.

In Exhibit 7, we plot the Active Share (left panel) and mean annualized active return (right panel) of the stock-picking portfolios. We find that for any given portfolio size, outperformance rises with increasing stock-picking skill, but Active Share depends only on portfolio size and is completely uncorrelated with investing skill.

We now modify our experimental setup to highlight a further dimension of portfolio manager behavior – conviction. Motivated by De Rossi, et al. (2014) and Cohen, et al. (2010), we envision a manager who places greater weight on a fixed, limited number of ”high-conviction” positions, while also including ”low-conviction” positions of lower weight. The degree of conviction is measured by the fraction of AUM that is invested in high-conviction positions. Crucially, we assume that the manager does have significant skill (having a 58% hit rate) in picking 15 high-conviction positions, but no discernible skill in picking the remaining low-conviction positions. The motivation for holding extra stocks might be tracking error control, asset gathering or insecurity.

Whatever their motives, managers are differentiated by how aggressively they overweight their high-conviction positions. In our model, the manager chooses the proportion (from 30% to 90%, irrespective of portfolio size) of the portfolio to allocate among high-conviction stocks, with balance allocated to low-conviction stocks. All positions are equal weighted.

Exhibit 8: Active Share and Performance as a Function of Conviction



Left panel: Active Share depends on level of risk aversion, but not on factor-timing skill. Right panel: benchmark-adjusted return depends on both risk aversion level and factor-timing skill.

Exhibit 8 shows that Active Share rises with increasing conviction, driven by the portfolio's increasing concentration. As we assumed skill in choosing the high-conviction positions, benchmark-adjusted performance also increases with increasing conviction.

In summary, conviction may provide a link between Active Share and performance if managers show any skill in picking a limited number of high-conviction stocks. Furthermore, one driver of low conviction may be the desire to achieve low Tracking Error through diversification, thereby linking low Active Share and low Tracking Error to poorer performance, with no change in stock-picking skill.

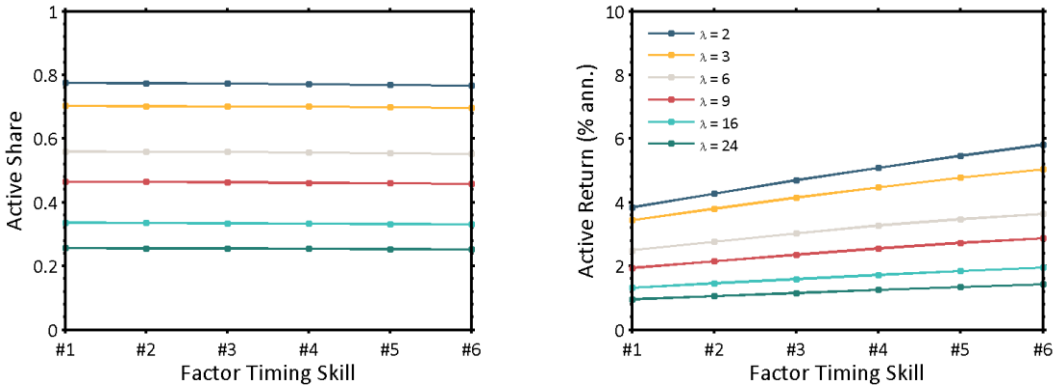
ACTIVE SHARE DOES NOT INDICATE INVESTMENT SKILL OF FACTOR-TIMING MANAGERS

Next, we move on to investors who add value through factor timing. We create six factor-timing managers who tilt on a linear combination of EUE4 Momentum, Earnings Yield, Book-to-Price, Dividend Yield and Residual Volatility as detailed in the Appendix. The investors differ in their risk aversion and their factor-timing skill, defined as their ability to tilt to factors in anticipation of their future performance. The baseline manager's alpha signal has a static allocation to factors, whereas increasingly skillful investors more and more heavily overweight factors that outperform in the future.

In Exhibit 9, we plot the Active Share (left panel) and mean annualized active return (right panel) of the factor-timing portfolios defined above. For any given level of risk aversion,

active performance increases with increasing factor-timing skill. By contrast, Active Share depends only on risk aversion and does not correlate with investing skill at all.

Exhibit 9: Active Share and Performance as a Function of Factor-Timing Skill



Left panel: Active Share depends on level of risk aversion, but not on factor-timing skill. Right panel: benchmark-adjusted return depends on both risk aversion level and factor-timing skill.

ACTIVE SHARE AND POTENTIAL REGULATION

Several European regulatory agencies are currently investigating "closet indexing" practices (i.e., active managers whose portfolios are very similar to the benchmark but who charge higher active fees) and are expressing interest in using Active Share as a regulatory tool to differentiate active managers from closet indexers. For example, the Swedish Shareholders Association filed a class-action lawsuit against the asset manager SwedBank Robur, accusing it of misrepresenting "closet tracking" products as active funds. Separately, several asset managers have decided to proactively publish the Active Share of their funds, either in anticipation of regulation and client demand or to differentiate themselves from other active managers[6].

Based on the results in this paper, we observe that while Active Share may add a useful dimension for investors trying to navigate active funds, it is no panacea.

[6] - http://www.ft.com/cms/s/0/0aba6a9c-7bb9-11e4-a695-00144feabdc0.html#axzz3fm4C4F1L and http://www.ft.com/cms/s/0/684a109a-7588-11e4-a1a9-00144feabdc0.html#axzz3fm4C4F1L

Some issues to highlight are:

Investing off benchmark. Implicitly, Active Share requires the fund manager to invest only within the benchmark investment universe. In reality, managers are not universally required to stay within their benchmarks in a legally binding manner. If managers' gaming of the system is to be avoided, management mandates also need to be amended to specify exactly how much managers' holdings can deviate from their benchmarks.

One-size-fits-all thresholds for Active Share. Academic papers and the financial press have touted concrete numerical thresholds, such as 60% Active Share, to define closet indexers. Based on our results concerning the effects of benchmark choice and investment style on Active Share, a single threshold value for all managers is likely to be both inaccurate and unfair. Another major drawback of Active Share as a uniform regulatory marker would be the unintended influence over choice of benchmark. If measuring high Active Share becomes a regulatory goal, managers will favor less challenging benchmarks. This may not be optimal for the investor, who is best served by a benchmark that challenges the manager on performance, e.g., measuring a concentrated value manager against a high exposure value index.

Targeting Active Share and Tracking Error. Targeting a fixed Active Share threshold might have unintended consequences in times of financial distress. Investors might reasonably wish to decrease both Active Share and Tracking Error in very volatile periods. Requiring all managers to target a fixed Active Share would likely require them to increase their risk exposure in crisis periods, which might not offer the best outcome for investors.

A possible framework for thinking about Active Share and the active management fee debate is that of information asymmetry. Investors know that a necessary – though by no means sufficient— condition of obtaining superior returns is substantial investment research. However, the amount of research that an active manager performs is largely private information: insofar as the manager is deemed 'active,' their process will be opaque.

From the perspective of investors, a lower bound on Active Share appears to be a way of ensuring a minimum level of active management. However, for the reasons discussed above, using Active Share alone is unlikely ensure that all managers perform the minimum required amount of investment research. From the perspective of fund managers, adopting Active Share is a way of "signaling" their ability or propensity to invest actively. Again, for the reasons outlined above, the investor should not take this at face value.

Comparison with passive investment suggests a partial solution for active management. Investors are able to achieve some degree of transparency by attributing the actively managed portfolio returns and holdings to factors carrying risk premia. If an active manager's strategy is consistently replicable by passive products available in the market (see "Can Alpha be Captured by Risk Premia?" by Bender, Hammond & Mok (2012)) then we can say with certainty that management is overpriced. Rather than impose measures of active management effort, investors may want to focus on not paying for services they can obtain more cheaply. Of course, factor attribution also comes with difficulties: the fact that an active manager can be matched with a passive strategy in the past does not imply that they can also be replaced by a passive strategy going forward. Nevertheless, attributing active manager performance to passive strategies paints a more detailed picture than any measure that simply measures "activeness."

CONCLUSION

Taking an alternative approach to many empirical studies in the literature, we chose to analyze Active Share and its relationship to manager performance through analytical results and portfolio construction exercises. Empirical studies on Active Share are inexact tools because managers may invest outside their benchmark, and hold non-equity assets, while the reporting of fund holdings may be lagged or sporadic.

The advantage of our simulation approach is in teasing apart the drivers of Active Share and performance through careful experimentation. In contrast, empirical samples tend to confound many variables that are hard to control for individually. Our analytical and simulation studies revealed:

- Active Share depends on benchmark and investment style. For cap-weighted benchmarks, small-cap tilted portfolios will have higher Active Share, while portfolios tilted towards low Residual Volatility will have lower Active Share.
- Active Share is related to specific Tracking Error and the concentration of active weights. For a given level of Tracking Error, a portfolio with more concentrated active weights can achieve lower Active Share. This can happen when the benchmark is more top-heavy.
- Risk drives both Tracking Error and Active Share. All other variables unchanged, a portfolio constructed with greater risk will have both higher Tracking Error and higher Active Share. An active portfolio with large factor exposure and timing will generally produce a lower Active Share than a largely idiosyncratic active tilt based on stock picking.
- Active Share is not directly related to investment skill. Explicitly controlling for skill in simulated stock-picking and factor-timing scenarios, we confirm that Active Share is not linked to the investor's ability to forecast asset or factor returns.

- For stock-picking investors, Active Share may be related to conviction. If managers have roughly equal skill, then those with higher conviction who overweight their "top picks" more aggressively can have both higher Active Share and performance.

Widespread adoption of Active Share — generated by changing client demands or regulatory requirements—may render Active Share useless as a predictor of performance going forward, as managers have plenty of tools at their disposal to increase their portfolios' Active Share *without becoming more skilled*.

Investors will continue to scrutinize active management fees and closet indexing. We argued that besides adopting a new measure of activeness, investors should ask whether a manager's performance is consistently replicable by cheaper passive or factor investing products. The attribution of manager performance to priced factors and purely idiosyncratic contributions gives a more detailed picture than Active Share.

REFERENCES

R. B. Cohen, C. Polk and B. Silli. (2010). *Best Ideas,* Working Paper.

T. Cohen, B. Leite, D. Nielson and A. Browder. (2014). *Active Share: A Misunderstood Measure in Manager Selection*, Fidelity Investment Insights.

Cremers, K. J. Martijn and Antti Petajisto. (2009). *How Active is Your Fund Manager?*, Review of Financial Studies, Vol. 22, No.9: 3329-3365.

A. Frazzini, J. Friedman and L. Pomorski. (2015). *Deactivating Active Share*, AQR.

X. Kang, F. Nielsen and G. Fachinotti. (2011). *Some Like it Hot*, MSCI Research Insight.

A. Petajisto. (2013). *Active Share and Mutual Fund Performance*, Financial Analysts Journal, 2013, 69(4):73-93.

G. de Rossi, G. Brar, I. Mirciov, J. Conomos, J. Lamplough, W. Fortmann (2014), B. Lau, T. Sharp, J. Rudolph and N. Kalamkar. (2014). *Risky Business*, Macquarie Research.

S. Sapra and M. Hunjan. (2013). *Active Share, Tracking Error and Manager Style*, PIMCO Quantitative Research.

T. Schlanger, C. B. Philips and K. P. LaBarge. (2012). *The search for outperformance: Evaluating 'active share'*, Vanguard Research.

M. Somerset Webb. (2014). *Shameful secrets of 'active' fund managers*, Financial Times, November 21, 2014.

APPENDIX

DERIVATION OF RELATIONSHIP BETWEEN ACTIVE SHARE AND TRACKING ERROR

In order to derive the relationship between Tracking Error and Active Share, we make the conventional assumption that asset returns are governed by a factor model:

$$r_{it} = X_{ik} f_{kt} + \varepsilon_{it}$$

Where r_{it} denotes return of the i^{th} asset, f_{kt} denotes returns of the k^{th} factor, ε_{it} is the specific return of the i^{th} asset and X_{ik} denotes the matrix of factor exposures. As a further simplification, we assume that all assets have uniform specific variance, i.e. $Var(\varepsilon_i) = \sigma_\varepsilon^2$ for all assets.

The factor structure of returns allows us to decompose Tracking Error into factor and idiosyncratic contributions:

$$TE^2 = \delta w'(X\ F\ X')\delta w + \sigma_\varepsilon^2 \sum_1^N \delta w_i^2 = TE_f^2 + \sigma_\varepsilon^2 \sum_1^N \delta w_i^2$$

We denoted TE_f^2 the factor driven part of tracking error. We observe that while Active Share is proportional to the sum of absolute active weights (the L^1 norm), tracking error is related to the sum of squared active weights (the L^2norm). It is well-known that the ratio of the L^1 norm and L^2norm of a vector is related to the *concentration* of the vector.

Let us introduce the quantity $c(x)$ as a measure of the concentration of an N-dimensional vector x:

$$c(x) = \frac{1}{\sqrt{N} - 1}\left(\sqrt{N} - \frac{\sum |x_i|}{(\sum x_i^2)^{1/2}}\right)$$

It is straightforward to check that $c(x)$ takes values between 0 and 1, with 0 corresponding to the least concentrated (uniform) vector $x = (1, 1, \dots, 1)$ and 1 corresponding to the most concentrated vector $x = (1, 0, \dots, 0)$. Using our concentration measure, we can now write Active Share as follows:

$$AS = \left(\sqrt{N} - \left(\sqrt{N} - 1\right)c(\delta w)\right)\frac{\sqrt{TE^2 - TE_f^2}}{2\sigma_\varepsilon} = \left(\sqrt{N} - \left(\sqrt{N} - 1\right)c(\delta w)\right)\frac{TE_\varepsilon}{2\sigma_\varepsilon}$$

It will be helpful in relating Active Share to Tracking Error to re-write our equation in the following form:

$$AS = \frac{1}{2\sigma_\varepsilon}\left(\sqrt{N} - \left(\sqrt{N} - 1\right)c(\delta w)\right)\frac{TE_\varepsilon}{TE}\,TE$$

The assumption of uniform (homoscedastic) specific risk is overly restrictive. We can generalize the formula by assuming that that heteroscedasticity can be parameterized as:

$$\sigma_i^2 = \eta_i \sigma_\varepsilon^2, \quad i = 1,..,N$$

where η_i may be taken to be proportional to the square-root of market cap, for example, which is the choice in many BARRA models. With this specification, our formula is modified only in that $c(x)$ must be calculated with the η-weighted L^2 norm $\left(\sum \eta_i x_i^2\right)^{1/2}$. In this case there is no guarantee that the concentration falls between 0 and 1.

In our empirical test of the formula, the concentration of active weights was calculated without accounting for heteroscedasticity.

We can estimate the (average) specific risk at time t with cross-sectional volatility,

$$\sigma_\varepsilon^2 \approx \sum_{i=1}^{N} \varepsilon_i^2/\eta_i$$

Here, $\varepsilon_i = r_i - \sum_i X_i f_i$ are the estimated residuals, and the estimator converges as $N \to \infty$. We take specific returns from the EUE4 model and weight them using MSCI Europe index weights.

FORMULATION OF EXPERIMENT: ACTIVE SHARE AND SKILL

In this experiment, we define factor timing investors whose alpha signals are linear combinations of the EUE4 factors Momentum, Earnings Yield, Book-to-Price, Dividend Yield and Residual Volatility. The baseline investor employs an alpha signal which is a static combination of factors. Investors with varying degrees of skill have an alpha signal which is a dynamically weighted combination of factors:

$$\alpha_t^\gamma = \sum_{k=1}^{5} w_t^{k,\gamma} X_{kt}$$

Factor timing ability is encoded into dynamic weights through the parameter γ as follows:

$$w_t^{k,\gamma} \sim \frac{\exp(\gamma\, R_{t,t+3}^k)}{\sum_k \exp(\gamma\, R_{t,t+3}^k)}$$

where $R^k_{t,t+3}$ is the return to factor k in the subsequent 3 months. By design, factors with superior future performance will be weighted heavier than the baseline allocation. For a larger value of γ, the factors with future outperformance will be overweighted more aggressively.

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