Refining Portfolio Construction When Alphas and Risk Factors are Misaligned | March 2009

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Introduction

Portfolio managers often use factor models to forecast risk and exceptional return, or "alpha." Many use risk models based on one set of factors and alpha models based on another, overlapping set of factors. Risk factors are selected to explain portfolio volatility, while alpha factors are chosen to forecast outperformance.

Portfolio optimization requires forecasts of both risk and alpha. The practice of using different models for risk and return in portfolio optimization, though widespread, has raised some concern. Portfolio managers worry that discrepancies between risk and alpha factors may create unintended biases in their optimized portfolios. This has led some to wonder whether it is better to use a risk model that is more aligned with their alpha factors.

Some researchers and practitioners contend that it is important to include the alpha factors in the risk model used in optimization. Lee and Stefek (2008) show that better aligning risk factors with alpha factors may improve the information ratio of optimized portfolios. They propose four ways of modifying a risk model to reduce misalignment. That discussion focuses on situations in which alpha factors resemble—but are not identical to—some of the risk factors.

Another way to mitigate the problems arising from misaligned risk and alpha factors is to modify the optimization process itself (Stefek 2007). In particular, we can penalize the portion of the alpha that is not related to the risk factors, the "residual alpha," to counteract the optimizer's tendency to overemphasize this portion. In this Research Insight, we describe this idea in more detail. We first review how to decompose a set of alphas into two components—one that is related to risk model factors (the "spanned alpha"), and one that is not (the "residual alpha"). Then we show how penalizing the residual alpha in portfolio optimization may improve a portfolio's exposures and ex-ante information ratio.

Decomposing Alphas in Portfolio Construction

An asset covariance matrix Σ and a set of alphas α are the basic inputs for portfolio optimization. Under the multifactor framework, the covariance matrix can be written as:

$$\Sigma = X_R F_R X_R' + \Delta_R \tag{1}$$

Here, X_R is a matrix representing the asset exposures to the risk factors, F_R is the covariance of the risk factors, and Δ_R is the (diagonal) covariance matrix of the specific returns.

A manager's alpha may be decomposed into a part that is spanned by the risk exposures, α_R, and a part that is residual (orthogonal) to them, $\alpha_{R\perp}$.

$$\alpha = \underbrace{X_R\left(X_R'X_R\right)^{-1}X_R'\alpha}_{\alpha_R} + \underbrace{\left(I - X_R\left(X_R'X_R\right)^{-1}X_R'\right)\alpha}_{\alpha_{R\perp}} \tag{1}$$

A key point is that these components of alpha are viewed differently by a risk model. The spanned alpha is captured by the risk factors. A tilt in its direction incurs factor risk. In contrast, the residual alpha is outside the risk factors, since $X_R'\alpha_{R\perp} = 0$. Tilting the portfolio in this direction incurs no factor risk.

To see why using different risk and alpha factors may be problematic, consider the unconstrained active optimization problem:

$$MAX \quad \alpha'h \ - \frac{\lambda}{2}h'\Sigma h \tag{2}$$

where h is the vector of active weights, and λ is the risk aversion parameter.

The optimal portfolio is: $h^* = \frac{1}{\lambda}\Sigma^{-1}\alpha$. Substituting equation (1) here, and with some derivation, this can be rewritten to highlight the role of the risk factors:[1]

$$h^* = \frac{1}{\lambda\sigma_s^2}\cdot\alpha_{R\perp} \quad + \quad \frac{1}{\lambda\sigma_s^2}\cdot\left(\mathrm{I}\text{-}X_R\left(X_R'X_R + \sigma_s^2F_R^{-1}\right)^{-1}X_R'\right)\alpha_R \ . \tag{3}$$

The optimal solution is the sum of two terms. The first term is simply the residual alpha, scaled to adjust for specific risk. The second term is the contribution of the alpha spanned by the risk exposures. This component of alpha is not as directly represented in the optimal solution. It is not only scaled for specific risk, but also adjusted—twisted and shrunk—to mitigate the common factor risk that it bears. The optimizer favors $\alpha_{R\perp}$ over α_R!

Lee and Stefek (2008) show that misalignment of alpha and risk factors may result in inadvertent and unwanted bets that may hamper performance. They propose several ways of altering a risk model to reduce the misalignment. A more detailed explanation of all four methods can be found in that paper.

[1] For simplicity, assume that the risk model forecasts the same specific risk for all assets σ_s.

Penalizing the Residual Alpha

Another approach to mitigating the problems described above is to penalize the residual alpha in the portfolio optimization as illustrated in Stefek (2007). In this method, a quadratic penalty term is added to the optimizer's objective function:

$$Maximize \quad \alpha' h \text{ - } \lambda h'\Sigma h - \theta \cdot \left(h'\alpha_{R_\perp} \right)^2 \tag{4}$$

This counteracts the optimizer's tendency to overemphasize the portion of the alpha that is not explained by the risk factors, i.e., the residual alpha. The parameter θ allows the user to control the importance of the penalty term.

How should θ be set? Let's assume that there is some factor return and risk accruing to a tilt on the residual alpha $\alpha_{R\perp}$.[2] To achieve an optimal risk-reward tradeoff, θ should be chosen to approximate $\lambda\sigma^2_{\alpha_{R_\perp}}$, where $\sigma^2_{\alpha_{R_\perp}}$ is the variance of the return to the residual alpha. This associates the proper risk penalty with a bet on $\alpha_{R\perp}$.

On the other hand, suppose the residual alpha $\alpha_{R\perp}$ is essentially noise. This may happen if the manager's alpha and the risk model factors capture very similar properties but measure them a little differently. Then, θ should be chosen to be large enough to suppress any tilt in this direction.

Some Illustrations

First, let's look at how the parameter θ affects the portfolio's tilt on the spanned and residual alpha. We consider the case of a portfolio manager who bets on twelve-month price momentum, defined as the sum of the last twelve months' returns lagged one month from today's date. That is, the manager's alpha at the beginning of month t is $\alpha_t = r_{t-2} + r_{t-3} + \ldots r_{t-13}$, where r_t is the return over month t. For the risk model, we use the Barra US Equity Risk Model (USE3L). Our example is as of June 2007. In the risk model, exposure to the Momentum factor is defined as the sum of the last twelve months returns without a lag, i.e., $X_t = r_{t-1} + r_{t-2} + ... + r_{t-12}$.[3] Therefore, the definition of Momentum in the risk model is different from that of the Momentum alpha.

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[2] Here we assume that the return to residual alpha is uncorrelated with the risk model factor returns. Note that the residual alpha $\alpha_{R\perp}$ in the penalty term is standardized, consistent with the other risk factors.

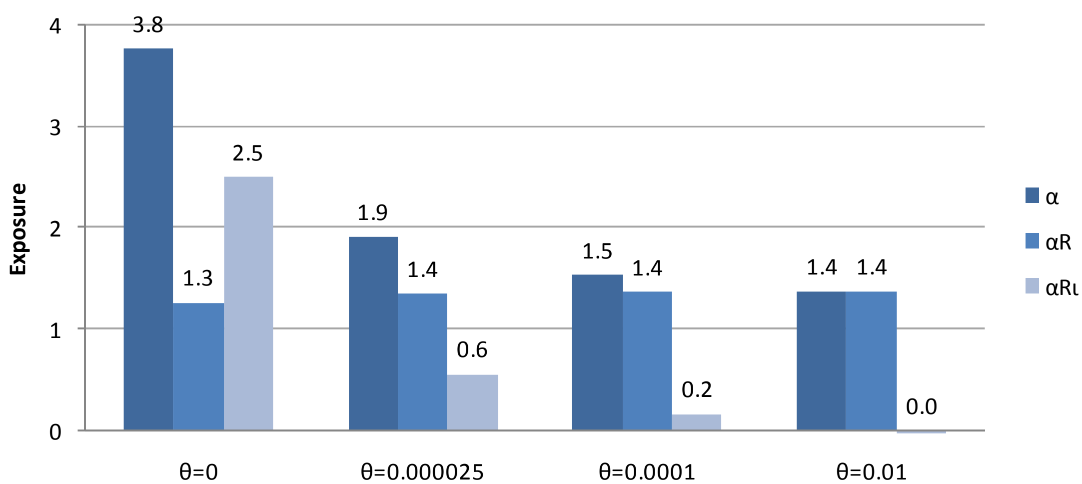
[3] Note that the Barra USE3 Momentum factor exposure is defined as a blend of relative strength and historical alpha for each stock. To simplify the comparison, we redefine the Momentum exposure in the risk model as the sum of the last twelve months of returns.

Decomposing alpha using equation (2), we find that roughly 89% is spanned by the risk model factors, while 11% is comprised by the residual component.[4]

We next maximize the objective function shown in equation (4). The MSCI US Mid Cap 450 Index serves as both the investment universe and the benchmark. We set $\lambda = 0.5$. We run four cases with different values for θ — 0, 0.000025, 0.0001, and 0.01. Then we compute the exposure of the optimized portfolio to the original alpha ($h^T \alpha$), the spanned alpha ($h^T \alpha_R$), and the residual alpha ($h^T \alpha_{R\perp}$).

Figure 1 displays the four portfolios' exposures to the spanned and residual alphas. Increasing θ decreases the resulting optimal portfolio's exposure to the residual alpha. When we set $\theta = 0$ (i.e., we leave out the penalty), the resulting portfolio is tilted significantly toward $\alpha_{R\perp}$, even though its relative fraction in α is small. Gradually increasing θ will tilt the portfolio away from residual alpha $\alpha_{R\perp}$ until there is no exposure left (in this case, when $\theta = 0.01$).[5]

Figure 1: Portfolio Active Exposures to Alpha Components



Next, we will illustrate how it affects performance in the same optimization setting.

Case 1: When the Residual Alpha is Noise

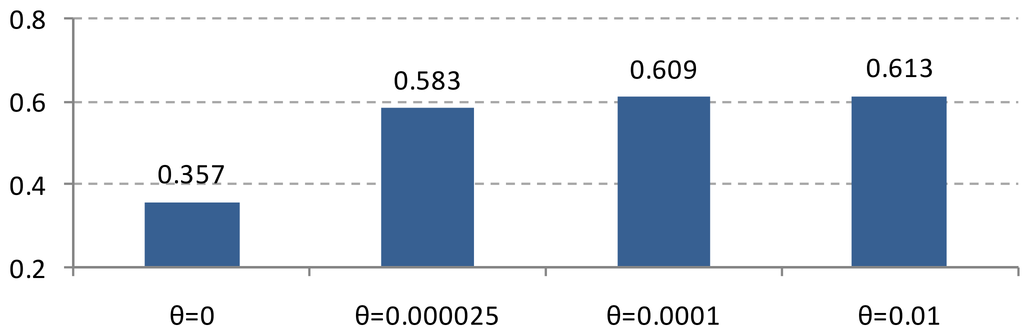
First, let's look at a case where the residual alpha is noise. For this case, we assume that the risk model is estimated without error (i.e., it is the "true" risk model), and Momentum, as it is defined in

[4] These estimates—89% and 11%—are found by calculating $\frac{\alpha' \alpha_R}{\alpha' \alpha}$ and $\frac{\alpha' \alpha_{R\perp}}{\alpha' \alpha}$, respectively.

[5] Note that the exposure to α also decreases as we increase θ with the reduction mostly coming from the residual alpha component.

the risk model, is the "true" alpha. In contrast, the manager's Momentum alpha is defined as it was previously, $\alpha_t = r_{t-2} + r_{t-3} + \ldots r_{t-13}$. Since the risk model definition of Momentum is the "true" alpha, the residual alpha is therefore noise. Thus, tilting on it contributes risk to the portfolio but provides no return. As we increasingly penalize the residual alpha (i.e., as we increase θ), the optimized portfolio achieves a higher information ratio.

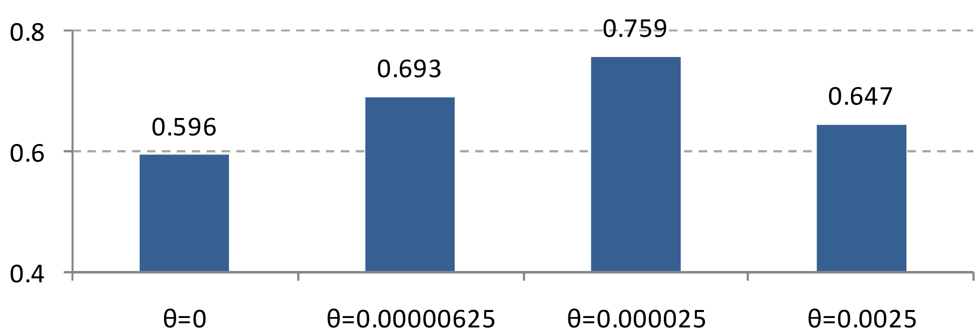
Figure 2: Information Ratio When Residual Alpha is Noise



Case 2: When the Residual Alpha Contains Return and Risk

Now, consider a case where the residual alpha contains return and risk. Again, the manager's Momentum alpha is defined as $\alpha_t = r_{t-2} + r_{t-3} + \ldots r_{t-13}$, and thus is not aligned with the Barra Momentum factor in the risk model. However, now we assume that the manager's Momentum alpha is the "true" alpha. Therefore, the residual alpha has some return associated with it. If we further assume that the "true" risk model is the Barra risk model plus this residual alpha factor's risk[6], then the risk of the residual alpha is "missing" in the portfolio optimization and is underpenalized. Increasing θ allows us to correct this. Figure 3 shows the results.

Figure 3: Information Ratio When Residual Alpha Contains Return and Risk



[6] The annual volatility of the residual alpha is assumed to be 0.71%. To keep the example simple, we also assume that the residual factor return is uncorrelated to returns of other risk factors in the risk model.

Notice that the IR is maximized when $\theta = 0.000025$. Recall that when the residual alpha has some factor return and risk associated with it, we choose $\theta = \lambda\sigma^2_{\alpha_{R\perp}}$, where $\sigma^2_{\alpha_{R\perp}}$ is the variance of the return to the residual alpha, to achieve the optimal risk-reward tradeoff. Doing so associates the proper risk with the residual alpha. In this example, $\sigma_{\alpha_{R\perp}}$ = 0.71% and $\lambda = 0.5$. Thus, the portfolio with the highest risk-reward tradeoff is achieved when we choose $\theta = 0.000025$.[7] If θ is too big or too small, the volatility of the residual alpha will be over- or underestimated, resulting in a suboptimal portfolio.

Summary

The misalignment of alpha and risk factors may result in inadvertent and unwanted bets that may hamper performance. Here, we discuss one way to mitigate these problems by modifying the optimization process, itself. Alphas can be decomposed into two distinct components—one that is spanned by the risk factors, and one that is residual (orthogonal). The objective function is modified to include a penalty term on the residual alpha. We show two examples, one where the residual alpha is known to contain only error, and another where the residual alpha contains information with known risk. In both cases, the method proposed helps to mitigate the mismatch between alpha and risk by assigning a suitable penalty to the residual alpha.

Reference

Lee, J-H and Stefek D. “Do Risk Factors Eat Alphas.” The Journal of Portfolio Management, Vol.34 No.4, Summer 2008, pp. 12-24.

Stefek D. “Getting the Most out of Portfolio Optimization - Guarding Against Estimation Error.” MSCI Barra Research Conference 2007.

[7] $\sqrt{\dfrac{\theta}{\lambda}} = 0.71\%$ (i.e., the annual volatility of the residual alpha)

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