MSCI

Model Insight

The Barra Global Equity Model (GEM3)

Empirical Notes

Andrei Morozov

Jun Wang

Laszlo Borda

Jose Menchero

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1. Introduction

1.1. Model Highlights

This document provides empirical results and analysis for the new Barra Global Equity Model (GEM3). These notes include extensive information on factor structure, commentary on the performance of select factors, an analysis of the explanatory power of the model, and an examination of the statistical significance of the factors. Furthermore, these notes also include a thorough side-by-side comparison of the forecasting accuracy of the GEM3 Model and the GEM2 Model, its predecessor.

The GEM3 Model leverages the same methodological advances used in the Barra US Equity Model (USE4). These methodological details may be found in the companion document: *USE4 Methodology Notes*, described by Menchero, Orr, and Wang (2011).

Briefly, the main advances included in GEM3 are:

- An innovative Optimization Bias Adjustment that improves risk forecasts for optimized portfolios by reducing the effects of sampling error on the factor covariance matrix
- A Volatility Regime Adjustment designed to calibrate factor volatilities and specific risk forecasts to current market levels
- A new specific risk model based on daily asset-level specific returns
- A Bayesian adjustment technique to reduce specific risk biases due to sampling error
- A uniform responsiveness for factor and specific components, providing greater stability in sources of portfolio risk
- Increased coverage with the inclusion of 22 frontier markets
- A daily update for all components of the model

The GEM3 Model is offered in short-term (GEM3S) and long-term (GEM3L) versions. The two versions have identical factor exposures and factor returns, but differ in their factor covariance matrices and specific risk forecasts. The GEM3S Model is designed to be more responsive and provide more accurate forecasts at a monthly prediction horizon. The GEM3L model is designed for longer-term investors willing to trade some degree of accuracy for greater stability in risk forecasts.

2. Methodology Highlights

2.1. Optimization Bias Adjustment

One significant bias of risk models is the tendency to underpredict the risk of optimized portfolios, as demonstrated empirically by Muller (1993). More recently, Shepard (2009) derived an analytic result for the magnitude of the bias, showing that the underforecasting becomes increasingly severe as the number of factors grows relative to the number of time periods used to estimate the factor covariance matrix. The basic source of this bias is estimation error. Namely, spurious correlations may cause certain stocks to appear as good hedges in-sample, while these hedges fail to perform as effectively out-of-sample.

An important innovation in the GEM3 Model is the identification of portfolios that capture these biases and to devise a procedure for correcting these biases directly within the factor covariance matrix. As shown by Menchero, Wang, and Orr (2011), the *eigenfactors* of the sample covariance matrix are systematically biased. More specifically, the sample covariance matrix tends to underpredict the risk of low-volatility eigenfactors, while overpredicting the risk of high-volatility eigenfactors. Furthermore, removing the biases of the eigenfactors significantly reduces the biases of optimized portfolios.

In the context of the GEM3 Model, eigenfactors represent portfolios of the original pure factors. The eigenfactor portfolios, however, are special in the sense that they are mutually uncorrelated. Also note that the number of eigenfactors equals the number of pure factors within the model.

As described in the *USE4 Methodology Notes*, we estimate the biases of the eigenfactors by Monte Carlo simulation. We then adjust the predicted volatilities of the eigenfactors to correct for these biases. This procedure has the benefit of building the corrections directly into the factor covariance matrix, while fully preserving the meaning and intuition of the pure factors.

Special consideration must be taken into account when applying the adjustment within the context of a global model. As shown by Menchero, Wang, and Orr (2011), applying the adjustment procedure to the covariance matrix can induce small biases at the individual factor level. The magnitude of the bias grows, however, in proportion to the number of factors in the model. Since the global model contains many more factors than the USE4 model, applying the adjustment to the factor covariance matrix may induce significant biases in the volatility forecasts at the factor level of a global model. To overcome this difficulty, we apply the adjustment procedure directly to the *factor correlation matrix*. The individual factor volatilities are then scaled in to obtain the factor covariance matrix.

Second, a realistic portfolio optimization within a global context would entail some constraints on factor exposures. For instance, the typical global investor is not performing an unconstrained portfolio optimization across all emerging markets, frontier markets, and currencies. Applying the full adjustment would therefore lead to *overestimation* of risk for optimized portfolios. In order to avoid this problem, we reduce the magnitude of the adjustment by 50 percent. Our empirical findings show that this essentially eliminates the biases for realistic optimized portfolios, as shown in Section 5.2.

2.2. Volatility Regime Adjustment

Another major source of risk model bias is due to the fact that volatilities are not stable over time, a characteristic known as *non-stationarity*. Since risk models must look backward to make predictions about the future, they exhibit a tendency to underpredict risk in times of rising volatility, and to overpredict risk in times of falling volatility.

Another important innovation in the GEM3 Model is the introduction of a Volatility Regime Adjustment for estimating factor volatilities. As described in the *USE4 Methodology Notes*, the Volatility Regime Adjustment relies on the notion of a cross-sectional bias statistic, which may be interpreted as an *instantaneous* measure of risk model bias for that particular day. By taking a weighted average of this quantity over a suitable interval, the non-stationarity bias can be significantly reduced.

A natural consideration in a global model is to apply *separate* adjustments for different classes of factors (e.g., industries, countries, currencies, etc.). However, we do not find a benefit in risk forecasting accuracy from applying separate adjustments to different blocks of factors. Furthermore, the currencies and country factors in a global model are often characterized by extreme returns, making them less useful to include in the cross-sectional bias statistic. Therefore, the GEM3 model computes the Volatility Regime Adjustment using the World factor, industries, and styles, and applies this adjustment to the entire factor covariance matrix. Further details on the GEM3 factor covariance matrix estimation parameters are provided in Appendix D.

2.3. Specific Risk Model with Bayesian Shrinkage

The GEM3 specific risk model builds upon methodological advances introduced with the European Equity Model (EUE3), as described by Briner, Smith, and Ward (2009). The EUE3 model utilizes daily observations to provide timely estimates of specific risk directly from the time series of specific returns. A significant benefit of this approach is that specific risk is estimated individually for every stock, thus reflecting the idiosyncratic nature of this risk source.

A potential shortcoming of the pure time-series approach is that specific volatilities may not fully persist out-of-sample. In fact, as shown in the *USE4 Methodology Notes*, there is a tendency for time-series volatility forecasts to overpredict the specific risk of high-volatility stocks, and underpredict the risk of low-volatility stocks.

To reduce these biases, we apply a Bayesian shrinkage technique. We segment stocks into deciles based on their market capitalization. Within each size bucket, we compute the mean and standard deviation of the specific risk forecasts. We then pull or “shrink” the volatility forecast toward the mean within the size decile, with the shrinkage intensity increasing with the number of standard deviations away from the mean.

Just as factor volatilities are not stable across time, the same holds for specific risk. In the GEM3 Model, a Volatility Regime Adjustment is also applied to specific risk. We estimate the adjustment by computing the cross-sectional bias statistic for the specific returns. The parameters for estimating the GEM3 specific risk model are reported in Appendix D.

3. Factor Structure Overview

3.1. Estimation Universe

The coverage universe is the set of all securities for which the model provides risk forecasts. The estimation universe, by contrast, is the subset of stocks that is used to actually estimate the model. Judicious selection of the estimation universe is an important part of building a sound risk model. The estimation universe must be broad enough to accurately represent the investment opportunity set of investors, without being so broad as to include illiquid stocks that may introduce spurious return relationships into the model. Furthermore, the estimation universe must be sufficiently stable to ensure that factor exposures are well behaved across time. *Representation, liquidity, and stability* are the three primary issues that must be addressed when selecting a risk model estimation universe.

A well-constructed equity index must address these very same issues, and thus serves as an excellent basis for the estimation universe. The GEM3 estimation universe utilizes the MSCI All Country World *Investable Markets Index* (ACWI IMI), which aims to reflect the full breadth of investment opportunities within developed and emerging markets by targeting 99 percent of the float-adjusted market capitalization. The MSCI index construction methodology applies innovative rules designed to achieve index stability, while reflecting the evolving equity markets in a timely fashion. Moreover, liquidity screening rules are applied to ensure that only investable stocks that meet the index methodological requirements are included for index membership.

The GEM3 model also includes coverage of many markets that are not regarded as fully open to international investors, and hence outside of the MSCI ACWI IMI. Examples include the China domestic A-share market, the six Gulf Cooperation Council (GCC) markets, and frontier markets. These stocks are down-weighted in the regressions to ensure that these markets do not unduly influence the factor returns associated with the investable universe.

3.2. Country Factors

Countries are important variables for explaining the sources of equity return co-movement. The GEM3 Model includes a separate country factor for every market covered within the model. In **Table 3.1** we report the average weight of each country as well as the country weight as of 30-Sep-2011. We also report the largest stock within each country as of 30-Sep-2011, as well as the market capitalization in billions of US dollars. Table 3.1 is organized into three panels. Panel A contains results for 24 developed markets, Panel B contains results for 21 emerging markets, and Panel C contains results for 32 markets outside of ACWI IMI.

Table 3.1(A)

GEM3 developed market country factors. Weights were determined within the GEM3 estimation universe using total market capitalization. Averages were computed over the sample period (31-Dec-1996 to 30-Sep-2011).

Country name	Average Weight	30-Sep-2011 Weight	Largest Stock (30-Sep-2011)	Capitalization (Billions USD)
Australia	1.87	2.53	BHP BILLITON LIMITED	109.31
Austria	0.19	0.19	OMV AG (NPV(VAR))	9.89
Belgium	0.60	0.50	ANHEUSER	85.68
Canada	2.76	3.77	ROYAL BK CDA MONTREAL QUE	65.82
Denmark	0.39	0.39	NOVO-NORDISK B	47.37
Finland	0.56	0.34	NOKIA OYJ	21.34
France	4.12	3.46	TOTAL	105.17
Germany	3.13	2.65	SIEMENS	83.56
Greece	0.26	0.09	COCA COLA HELL BOT	6.50
Hong Kong	1.11	1.42	AIA Group Ord Shs	34.50
Ireland	0.22	0.12	CRH (ORD EUR0.32(DUBLI	11.19
Israel	0.23	0.29	TEVA PHARMACEUTICAL	33.32
Italy	1.89	1.11	ENI	71.04
Japan	10.44	8.60	TOYOTA MOTOR	120.24
Netherlands	1.53	0.75	UNILEVER CVA	54.65
New Zealand	0.08	0.07	FLETCHER BUILDING LTD	4.02
Norway	0.38	0.52	STATOIL	68.98
Portugal	0.20	0.17	GALP ENERGIA	14.22
Singapore	0.55	0.86	SINGAPORE TELECOMMUNIC	38.89
Spain	1.43	1.31	TELEFONICA SA (EUR1)	88.33
Sweden	1.01	1.02	HENNES & MAURITZ B	43.96
Switzerland	2.44	2.50	NESTLE	191.12
United Kingdom	7.86	6.50	HSBC HOLDINGS	137.92
United States	42.49	33.96	EXXON MOBIL CORP	357.78

Table 3.1(B)

GEM3 emerging market country factors. Weights were determined within the GEM3 estimation universe using total market capitalization. Averages were computed over the sample period (31-Dec-1996 to 30-Sep-2011).

Country name	Average Weight	30-Sep-2011 Weight	Largest Stock (30-Sep-2011)	Capitalization (Billions USD)
Brazil	1.02	2.11	PETROBRAS ON	83.80
Chile	0.22	0.38	S A C I FALABELLA	18.96
China International	1.30	3.08	CHINA MOBILE LTD	198.74
Colombia	0.09	0.32	ECOPETROL	80.97
Czech Republic	0.08	0.10	CEZ	20.74
Egypt	0.07	0.07	ORASCOM CONSTR IND	7.46
Hungary	0.06	0.04	MOL HUNGARIAN OIL	7.12
India	0.92	2.13	RELIANCE INDS	54.03
Indonesia	0.20	0.63	ASTRA INTERNATIONAL	29.31
Korea	1.23	2.13	SAMSUNG ELECTRONICS	105.03
Malaysia	0.46	0.74	MALAYAN BANKING BHD	18.74
Mexico	0.48	0.71	AMERICA MOVIL S. A. B. DE C. V.	60.82
Morocco	0.06	0.10	MAROC TELECOM	15.07
Peru	0.06	0.14	SOUTHERN COPPER CORP	21.24
Philipines	0.08	0.19	PHIL.LONG DISTANCE TEL	9.39
Poland	0.15	0.27	POWSZECHNA KASA OS	12.50
Russia	0.85	1.43	GAZPROM OAO (RUB5(REGD	114.48
South Africa	0.75	0.99	MTN GROUP	31.06
Taiwan	1.28	1.57	TSMC MFG. CO. LTD.	59.51
Thailand	0.24	0.50	PTT PUBLIC CO. LTD.	23.86
Turkey	0.24	0.45	T GARANTI BANKASI	16.35

Table 3.1(C)

GEM3 country factors for non-ACWI IMI markets. Weights were determined within the GEM3 estimation universe using total market capitalization. Averages were computed over the sample period (31-Dec-1996 to 30-Sep-2011).

Country name	Average Weight	30-Sep-2011 Weight	Largest Stock (30-Sep-2011)	Capitalization (Billions USD)
Argentina	0.090	0.083	YPF SOCIEDAD ANONIMA	13.45
Bahrain	0.023	0.014	ALUMINUM BAHRAIN	2.05
Bangladesh	0.042	0.054	GRAMEENPHONE LTD	2.94
Bosnia Herzegovina	0.008	0.008	JELSINGRAD FMG AD GRADISKA	1.03
Bulgaria	0.010	0.007	ENERGONI AD-SOFIA	1.17
China Domestic	2.781	7.108	PETROCHINA COMPANY LIM	250.27
Croatia	0.031	0.038	INA INDUSTRIJA NAFTE	7.52
Cyprus	0.007	0.002	CYPRUS LIMNI RESORTS	0.40
Estonia	0.010	0.004	TALLINK GRUPP AS	0.53
Iceland	0.029	0.005	OSSUR HF (ISK1)	0.74
Jamaica	0.010	0.012	1ST CARIBBEAN INTL BK	2.06
Jordan	0.033	0.042	ARAB BANK	6.03
Kazakhstan	0.041	0.032	KAZMUNAIGAZ	6.18
Kenya	0.015	0.018	EAST AFRICAN BREW	1.28
Kuwait	0.228	0.184	NATL BK OF KUWAIT (KWD0.10)	15.15
Latvia	0.004	0.002	LATVIJAS GAZE (LVL1)	0.40
Lebanon	0.015	0.017	BLOM BANK	1.76
Lithuania	0.012	0.008	LIETUVOS TELEKOMAS	0.64
Mauritius	0.006	0.009	MAURITIUS COMMERICAL BANK	1.46
Nigeria	0.049	0.066	DANGOTE CEMENT PLC	9.70
Oman	0.020	0.025	BANK MUSCAT	2.70
Pakistan	0.032	0.054	OIL & GAS DEVELOPMENT	6.50
Qatar	0.127	0.229	QATAR NATIONAL BK (QAR10)	24.86
Romania	0.033	0.027	PETROM S.A.	4.94
Saudi Arabia	0.556	0.545	SAUDI BASIC INDUST	73.99
Serbia	0.007	0.007	NAFTNA INDUSTRIJA SRBIJE	1.34
Slovenia	0.020	0.013	KRKA DD (SIT4000)	2.44
Sri Lanka	0.007	0.019	JOHN KEELLS HOLDINGS	1.57
Tunisia	0.008	0.014	POULINA GP HLDG	1.03
Ukraine	0.033	0.024	UKRNAFTA	2.99
United Arab Emirates	0.133	0.120	DP WORLD	8.47
Vietnam	0.024	0.035	MASAN GROUP CORP	2.97

3.3. Industry Factors

Industries represent another important source of equity return co-movement. One of the strengths of the GEM3 Model is that it uses the Global Industry Classification Standard (GICS®) for the industry factor structure. The GICS scheme is hierarchical, with 10 sectors at the top level, 24 industry groups at the next level, followed with increasing granularity at the industry and sub-industry levels. GICS applies a consistent global methodology to classify stocks based on careful evaluation of the firm's business model and economic operating environment.

It is important that the industry factor structure for each country reflect the unique characteristics of the local market. For instance, some countries may require fine industry detail in some sectors, while a coarser structure may be appropriate for other sectors. When building Barra risk models, special care is taken in customizing the industry factor structure to the local market. Within each sector, we analyze which combinations of industries and sub-industries best reflect the market structure, while also considering the economic intuition and explanatory power of such groupings.

The result of this investigative process is the set of 34 GEM3 industry factors, presented in **Table 3.2**. Industries that qualify as factors tend to exhibit high volatility and have significant weight. Also reported in Table 3.2 are the average weights (from 31-Dec-1996 to 30-Sep-2011) and end-of-period industry weights.

Table 3.2

GEM3 industry factors. Weights were determined within the GEM3 estimation universe using total market capitalization. Averages were computed over the sample period (31-Dec-1996 to 30-Sep-2011).

GICS Sector	GEM3 Code	GEM3 Industry Factor Name	Average Weight	30-Sep-2011 Weight
Energy	1	Energy Equipment and Services	0.87	1.17
	2	Oil Gas and Consumable Fuels	5.93	7.32
	3	Oil and Gas Exploration and Production	1.32	2.23
Materials	4	Chemicals	2.26	2.90
	5	Construction Containers Paper	1.25	1.02
	6	Aluminum Diversified Metals	1.40	2.21
	7	Gold and Precious Metals	0.52	1.22
	8	Steel	0.93	1.48
Industrials	9	Capital Goods	7.43	7.86
	10	Commercial and Professional Services	1.28	0.96
	11	Transportation Non-Airline	1.86	2.00
	12	Airlines	0.36	0.31
Consumer Discretionary	13	Automobiles and Components	2.47	2.79
	14	Consumer Durables and Apparel	2.11	1.86
	15	Hotels Restaurants and Leisure	1.38	1.75
	16	Media	3.16	1.98
	17	Retailing	3.08	3.02
Consumer Staples	18	Food and Staples Retailing	1.96	2.28
	19	Food Beverage and Tobacco	4.69	5.95
	20	Household and Personal Products	1.45	1.54
Health Care	21	Health Care Equipment and Services	2.16	2.29
	22	Biotechnology	0.84	0.72
	23	Pharmaceuticals and Life Sciences	5.81	4.79
Financials	24	Banks	10.36	8.93
	25	Diversified Financials	5.28	3.61
	26	Insurance	4.39	3.42
	27	Real Estate	2.24	3.33
Information Technology	28	Internet Software and Services	0.70	0.96
	29	IT Services and Software	3.32	3.98
	30	Communications Equipment	2.16	1.03
	31	Computers Electronics	3.62	3.08
	32	Semiconductors	2.31	1.94
Telecom Services	33	Telecommunication Services	6.75	5.53
Utilities	34	Utilities	4.35	4.49

In **Table 3.3**, we report the underlying GICS codes that map to each of the GEM3 industry factors. This table helps illustrate the customization that takes place within each sector. Taking the Health Care sector as an example, we see that this sector is divided into three industry factors. The first factor (Health Care Equipment & Services) is taken from GICS Industry Group 3510. The other industry group (3520) is divided into two risk factors. One factor is obtained by separating the Biotechnology industry (352010); the other factor is obtained by combining the Pharmaceuticals Industry (352020) and the Life Sciences Industry (352030) into a separate factor. In each case, the industry structure is guided by a combination of financial intuition and empirical analysis.

Table 3.3

Mapping of GEM3 industry factors to GICS codes.

Code	GEM3 Industry Factor Name	GICS Codes
1	Energy Equipment & Services	101010
2	Oil, Gas & Consumable Fuels	10102010, 10102030, 10102040, 10102050
3	Oil & Gas Exploration & Production	10102020
4	Chemicals	151010
5	Construction, Containers, Paper	151020, 151030, 151050
6	Aluminum, Diversified Metals	15104010, 15104020
7	Gold, Precious Metals	15104030, 15104040
8	Steel	15104050
9	Capital Goods	2010
10	Commercial & Professional Services	2020
11	Transportation Non-Airline	203010, 203030, 203040, 203050
12	Airlines	203020
13	Automobiles & Components	2510
14	Consumer Durables & Apparel	2520
15	Consumer Services	2530
16	Media	2540
17	Retailing	2550
18	Food & Staples Retailing	3010
19	Food, Beverage & Tobacco	3020
20	Household & Personal Products	3030
21	Health Care Equipment & Services	3510
22	Biotechnology	352010
23	Pharmaceuticals, Life Sciences	352020, 352030
24	Banks	4010
25	Diversified Financials	4020
26	Insurance	4030
27	Real Estate	4040
28	Internet Software & Services	451010
29	IT Services, Software	451020, 451030
30	Communications Equipment	452010
31	Computers, Electronics	452020, 452030, 452040
32	Semiconductors	4530
33	Telecommunication Services	5010
34	Utilities	5510

In **Table 3.4** we report the largest firm within each industry, as well as the total market capitalization as of 30-Sep-2011. The two largest firms were Exxon Mobil and Apple, each with a market capitalization exceeding $350 billion.

Table 3.4

Largest stock within each industry as of 30-Sep-2011. Market capitalizations are reported in billions of US dollars.

GEM3 Code	GEM3 Industry Factor Name	Largest Stock (30-Sep-2011)	Capitalization, (Billions USD)
1	Energy Equipment and Services	SCHLUMBERGER LTD	81.1
2	Oil Gas and Consumable Fuels	EXXON MOBIL CORP	357.8
3	Oil and Gas Exploration and Production	CNOOC LTD	74.6
4	Chemicals	SAUDI BASIC INDUST	74.0
5	Construction Containers Paper	HOLCIM N	17.5
6	Aluminum Diversified Metals	BHP BILLITON LIMITED	109.3
7	Gold and Precious Metals	BARRICK GOLD CORP	47.1
8	Steel	Vale Sa	74.8
9	Capital Goods	GENERAL ELECTRIC CO	161.6
10	Commercial and Professional Services	WASTE MGMT INC DEL	15.4
11	Transportation Non-Airline	UNITED PARCEL SERVICE INC	46.6
12	Airlines	SINGAPORE AIRLINES	10.5
13	Automobiles and Components	TOYOTA MOTOR	120.2
14	Consumer Durables and Apparel	LVMH	67.8
15	Hotels Restaurants and Leisure	MCDONALDS CORP	91.1
16	Media	WALT DISNEY CO	57.0
17	Retailing	AMAZON COM INC	97.7
18	Food and Staples Retailing	WAL MART STORES INC	180.2
19	Food Beverage and Tobacco	NESTLE	191.1
20	Household and Personal Products	PROCTER & GAMBLE CO	176.4
21	Health Care Equipment and Services	UNITEDHEALTH GROUP INC	50.0
22	Biotechnology	AMGEN INC	51.1
23	Pharmaceuticals and Life Sciences	JOHNSON & JOHNSON	174.6
24	Banks	ICBC -A	163.4
25	Diversified Financials	JPMORGAN CHASE & CO	119.7
26	Insurance	BERKSHIRE HATHAWAY [B]	75.4
27	Real Estate	SIMON PPTY GROUP INC NEW	32.3
28	Internet Software and Services	GOOGLE INC [A]	130.1
29	IT Services and Software	INTERNATIONAL BUSINESS MACHS	212.0
30	Communications Equipment	CISCO SYS INC	85.2
31	Computers Electronics	APPLE INC	352.5
32	Semiconductors	INTEL CORP	113.1
33	Telecommunication Services	CHINA MOBILE LTD	198.7
34	Utilities	GDF SUEZ	67.8

3.4. Style Factors

Investment style represents another major source of systematic risk for equity portfolios. Style factors are constructed from financially intuitive stock attributes called *descriptors*, which serve as effective predictors of equity return covariance.

In order to facilitate comparison across style factors, the factors are standardized to have a cap-weighted mean of 0 and an equal-weighted standard deviation of 1. The cap-weighted estimation universe, therefore, is *style neutral*.

The GEM3 Model contains 11 style factors. The GEM3 style factors are based on the GEM2 style factors, with two major differences. First, the GEM2 Volatility factor is broken out into two GEM3 style factors: Beta and Residual Volatility. Second, the GEM2 Value factor is separated into three style factors: Dividend Yield, Book-to-Price, and Earnings Yield. The factors are described in Appendix A, together with descriptor definitions and descriptor weights. Here we provide a brief qualitative description of the factors:

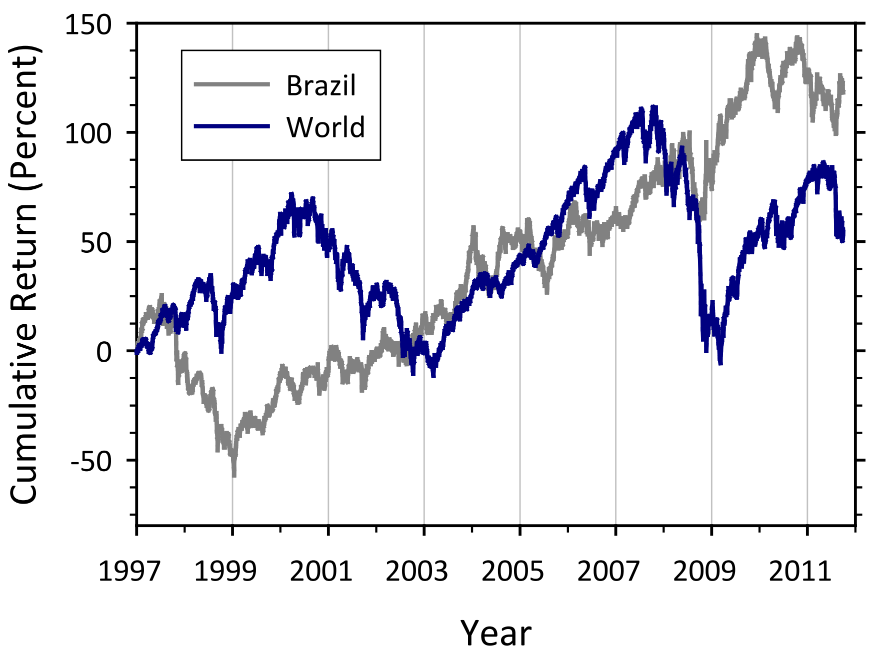
- The *Beta* factor is typically the most important style factor. It captures market risk that cannot be explained by the World factor. We compute Beta by time-series regression of excess stock returns against the cap-weighted estimation universe, as described in Appendix A. To better understand how Beta relates to the World factor, consider a fully invested long-only portfolio that is tilted toward high-beta stocks. Intuitively, this portfolio has greater market risk than a portfolio with a beta of 1. This additional market risk is captured through positive exposure to the Beta factor. Since the time-series correlation between the World factor and the Beta factor is typically very high, these two sources of risk are additive in this example. If, by contrast, the portfolio were invested primarily in low-beta stocks, then the risk from the Beta and the World factors would have been partially offset, as expected.
- The *Momentum* factor is often the second strongest factor in the model, although sometimes it may surpass Beta in importance. Momentum differentiates stocks based on their performance over the trailing 6-12 months. When computing Momentum exposures we exclude recent returns in order to avoid the effects of short-term reversal.
- The *Siz*e factor represents another strong source of equity return covariance, and captures return differences between large-cap stocks and small-cap stocks. We measure Size by the log of market capitalization.
- The *Earnings Yield* factor describes return differences based on a company's earnings relative to its price. Earnings Yield is considered by many investors to be a strong value signal. The most important descriptor in this factor is the analyst-predicted 12-month forward earnings-to-price ratio.
- The *Residual Volatility* factor is composed of three descriptors: (a) the volatility of daily excess returns, (b) the volatility of daily residual returns, and (c) the cumulative range of the stock over the last 12 months. Since these descriptors tend to be highly collinear with the Beta factor, the Residual Volatility factor is orthogonalized with respect to the Beta factor, as described by Menchero (2010).
- The *Growth* factor differentiates stocks based on their prospects for sales or earnings growth. The most important descriptor in this factor is the analyst predicted long-term earnings growth. Other descriptors include sales and earnings growth over the trailing five years.
- The *Dividend Yield* factor explains return differences attributable to dividend payouts of the firm. This factor is defined by the trailing 12-month dividend divided by the current price.

- The *Book-to-Price* factor is also considered by some to be an indicator of value. This factor is given by the last reported book value of common equity divided by current market capitalization.
- The *Leverage* factor captures return differences between high-leverage and low-leverage stocks. The descriptors within this style factor include market leverage, book leverage, and debt-to-assets ratio.
- The *Liquidity* factor describes return differences due to relative trading activity. The descriptors for this factor are based on the fraction of total shares outstanding that trade over a recent window.
- The *Non-Linear Size* (NLS) factor captures non-linearities in the payoff to the Size factor across the market-cap spectrum. This factor is based on a single raw descriptor: the cube of the Size exposure. However, since this raw descriptor is highly collinear with the Size factor, it is orthogonalized with respect to Size. This procedure does not affect the fit of the model, but does mitigate the confounding effects of collinearity, while preserving an intuitive meaning for the Size factor. As described by Menchero (2010), the NLS factor roughly captures the risk of a "barbell portfolio" that is long mid-cap stocks and short small-cap and large-cap stocks.

3.5. Performance of Select Factors

It is helpful to consider the performance of individual factors. In **Figure 3.1**, we report cumulative returns to the GEM3 World factor and the Brazil factor. The World factor return essentially represents the excess return (i.e., above the risk-free rate) of the cap-weighted world portfolio. Figure 3.1 clearly illustrates the main features of the global equity market since 1997. For instance, the three bull markets of the sample period are clearly visible, as is the bear market after the Internet Bubble, and the market crash of 2008.

Figure 3.1
Cumulative returns of GEM3 World factor and Brazil factor.

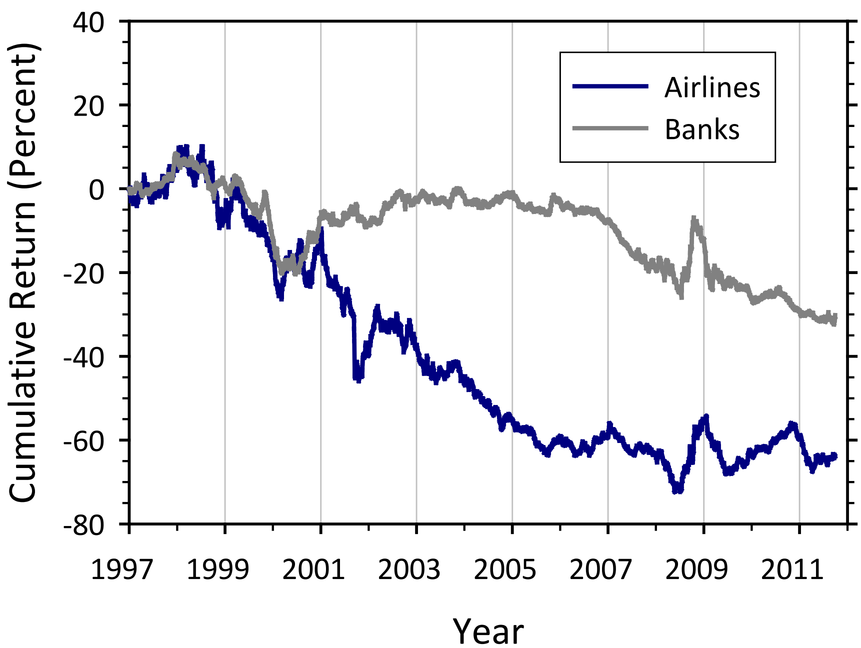


The country factor return represents the performance of the pure country relative to the overall market, net of all industry and style effects. In other words, the pure country factor portfolio is dollar neutral and has zero exposure to every industry and style. In Figure 3.1 we report the cumulative return of the Brazil factor. We see that from 1997-1999, Brazil severely underperformed, dropping by roughly 50 percent. Since January 1999, however, Brazil has experienced a strong bull market.

In **Figure 3.2** we report the cumulative returns for the Airlines factor and Banks factor. Both factors had similar performance for the first four years of the sample, but subsequently diverged. The Airlines factor portfolio was down by more than 60 percent since 1997, whereas the Banks factor was down by roughly 30 percent over the same period.

Figure 3.2

Cumulative returns of Airlines factor and Banks factor.

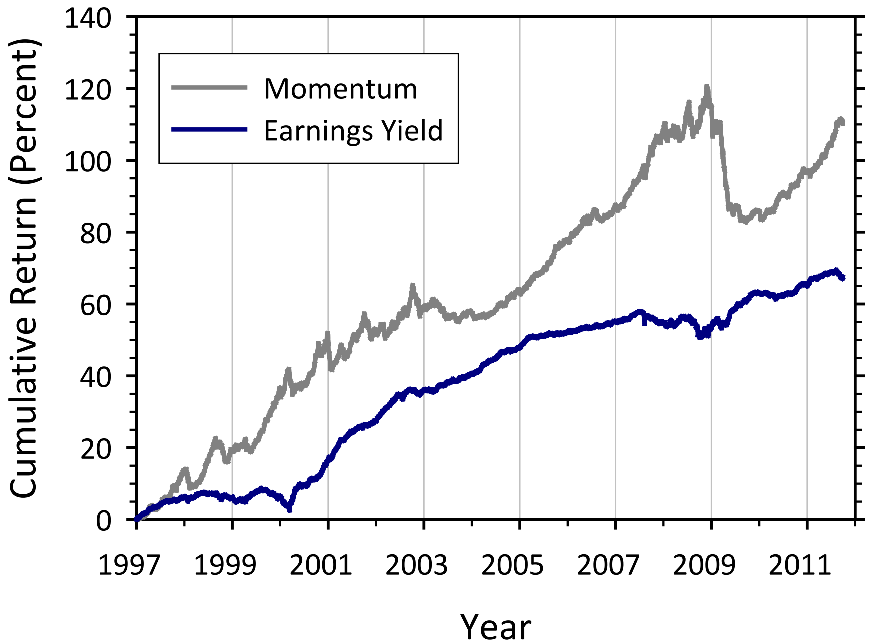


Style factor returns represent the returns of pure factor portfolios that have exposure only to the style in question. In other words, they have net zero weight in every industry and country, and have zero exposure to every other style factor. A more detailed discussion of pure factor portfolios is provided by Menchero (2010).

In **Figure 3.3**, we report the cumulative returns to the Earnings Yield and Momentum factors, which represent two common strategies often used by quantitative investors. Overall, Earnings Yield performed very well over the past 16 years, consistent with the notion of a "value premium." As described by Basu (1977), this reflects the tendency of stocks that are priced low relative to fundamentals to outperform. A notable exception, however, occurred during the Internet Bubble in 1999, when Earnings Yield performed poorly. The Earnings Yield factor also performed poorly from the Quant Meltdown in August 2007 until the end of 2008.

Figure 3.3

Cumulative returns of Earnings Yield and Momentum factors.

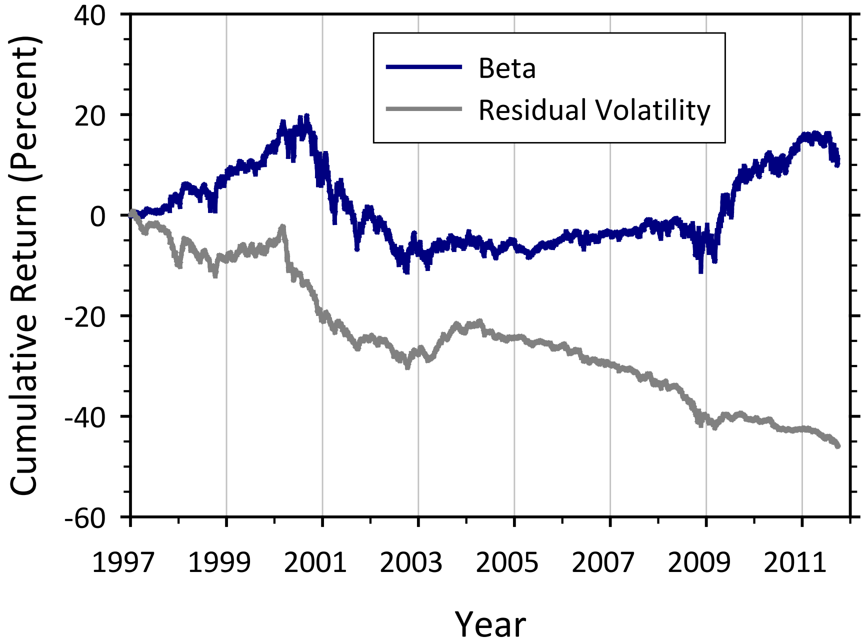


Momentum also performed well since 1997, consistent with the empirical observation noted by Jegadeesh and Titman (1993) that stocks with strong performance over the previous 6-12 months continue to outperform. There were, however, two major periods of underperformance. The first occurred in late 2002, which coincided with the market rebound following the 2000-2002 bear market. The second major downturn for the Momentum factor began in March 2009, which again coincided with a market recovery — this time after the crash of 2008.

In **Figure 3.4** we report cumulative returns to the Beta and Residual Volatility factors. Qualitatively, the Beta factor showed many of the main features as the World factor in Figure 3.1, consistent with the high correlation between these factors. For instance, the Beta factor performed well during the bull market of 1997-2000, and performed poorly during the bear market of 2001-2003. However, Beta remained essentially flat during the 2003-2007 bull market. Furthermore, Beta did not experience a large decline during the market crash of 2008. Since 2009, the Beta factor has performed well.

Figure 3.4

Cumulative returns of Beta and Residual factors.



The Residual Volatility factor exhibited a consistent downward drift over the sample period. This is consistent with the empirical findings of Ang *et al.*, who showed that stocks with high specific risk have lower returns than stocks with lower specific risk.

4. Model Characteristics and Properties

4.1. Country Factors

One requirement of a high-quality factor structure is that the factor returns be statistically significant. This helps prevent weak or noisy factors from finding their way into the model. We measure statistical significance by the *t*-statistic of the factor return. Assuming normality, absolute *t*-statistics greater than 2 are considered significant at the 95-percent confidence level. In other words, if the factor truly had no explanatory power (i.e., it was pure noise), then by chance we would observe $|t|>2$ about 5 percent of the time.

In **Table 4.1** we report mean absolute *t*-statistics for the GEM3 country factors, as well as the percentage of observations with $|t|>2$. Note that the *t*-statistics reported in Table 4.1 were computed using *monthly* cross-sectional regressions, even though we run daily cross-sectional regressions for purposes of constructing the factor covariance matrix. This distinction is important, because what is ultimately relevant is the explanatory power of the factors at the prediction horizon of the model.

For developed markets (Panel A), the two factors with the highest statistical significance were Japan and USA. For emerging markets (Panel B), the most significant factors were Taiwan and Korea, whereas for markets outside ACWI IMI (Panel C) the most significant factor was China Domestic.

Also reported in Table 4.1 are the returns, volatilities, and Sharpe ratios for the country factors. These quantities were computed using *daily* factor returns and stated on an annualized basis. For developed markets, the top-performing factor on a risk-adjusted basis was Canada, with a Sharpe ratio of 0.31. The worst-performing developed market was Netherlands, which had a Sharpe ratio of -0.32.

Table 4.1 also reports the correlations of the daily factor returns with the estimation universe. The USA factor had the largest positive correlation, whereas most of other countries exhibited negative correlations. This suggests that the USA market “drives” the global equity market (i.e., when the USA performs well, the global market return tends to be positive). By contrast, emerging markets and frontier markets exhibit overwhelmingly negative correlations with the estimation universe.

Table 4.1 (A)

Country factor summary statistics for developed markets. The two columns pertaining to *t*-statistics were computed using monthly cross-sectional regressions. The last four columns were computed based on daily factor returns. The sample period is from 31-Dec-1996 to 30-Sep-2011 (177 months of returns).

Country Name	Factor Start Date	Average Absolute *t*-stat	Percent Observ. |*t*|>2	Annual. Factor Return	Annual. Factor Volatility	Factor Sharpe Ratio	Correl. with ESTU
Australia	31-Dec-1996	2.67	49.2	-1.44	14.42	-0.10	-0.32
Austria	31-Dec-1996	1.50	28.8	0.54	14.11	0.04	-0.14
Belgium	31-Dec-1996	1.80	35.6	-2.13	11.56	-0.18	-0.09
Canada	31-Dec-1996	2.84	57.1	2.88	9.26	0.31	-0.10
Denmark	31-Dec-1996	2.06	46.3	-2.56	13.42	-0.19	-0.12
Finland	31-Dec-1996	2.04	40.7	2.90	14.00	0.21	-0.01
France	31-Dec-1996	3.51	63.8	0.09	11.37	0.01	0.02
Germany	31-Dec-1996	3.31	63.8	-3.11	11.69	-0.27	0.03
Greece	31-Dec-1996	4.09	68.4	-6.85	26.56	-0.26	-0.14
Hong Kong	31-Dec-1996	4.37	67.2	-0.43	20.67	-0.02	-0.20
Ireland	31-Dec-1996	1.79	38.4	-1.17	16.01	-0.07	-0.14
Israel	31-Dec-1996	2.47	50.3	1.42	17.82	0.08	-0.14
Italy	31-Dec-1996	3.74	61.6	-1.62	12.65	-0.13	-0.01
Japan	31-Dec-1996	11.60	89.3	-2.12	18.42	-0.12	-0.23
Netherlands	31-Dec-1996	2.31	49.7	-3.85	11.88	-0.32	0.05
New Zealand	31-Dec-1996	1.13	15.8	-0.37	13.22	-0.03	-0.38
Norway	31-Dec-1996	2.18	45.2	-1.87	16.30	-0.11	-0.08
Portugal	31-Dec-1996	1.86	35.0	-2.05	13.90	-0.15	-0.13
Singapore	31-Dec-1996	3.23	55.4	0.26	19.29	0.01	-0.18
Spain	31-Dec-1996	3.00	58.2	0.56	12.04	0.05	0.02
Sweden	31-Dec-1996	2.68	56.5	2.21	13.80	0.16	0.02
Switzerland	31-Dec-1996	2.52	51.4	-1.34	10.74	-0.12	0.01
United Kingdom	31-Dec-1996	4.25	72.3	-2.45	10.44	-0.23	-0.07
United States	31-Dec-1996	8.26	83.6	1.00	10.50	0.10	0.25
Average		3.30	53.5		14.34		

Table 4.1 (B)

Country factor summary statistics for emerging markets. The two columns pertaining to *t*-statistics were computed using monthly cross-sectional regressions. The last four columns were computed based on daily factor returns. The sample period is from 31-Dec-1996 to 30-Sep-2011 (177 months of returns).

Country Name	Factor Start Date	Average Absolute *t*-stat	Percent Observ. |*t*|>2	Annual. Factor Return	Annual. Factor Volatility	Factor Sharpe Ratio	Correl. with ESTU
Brazil	31-Dec-1996	4.26	71.2	5.48	19.46	0.28	-0.01
Chile	31-Dec-1996	2.16	42.9	1.87	12.16	0.15	-0.16
China International	31-Dec-1996	5.22	67.2	2.07	28.44	0.07	-0.17
Colombia	31-Dec-1996	1.82	37.3	9.97	18.27	0.55	-0.18
Czech Republic	31-Dec-1996	1.50	28.8	0.46	19.77	0.02	-0.13
Egypt	31-Dec-1996	2.39	42.4	-1.29	24.16	-0.05	-0.28
Hungary	31-Dec-1996	1.46	25.4	-8.26	23.58	-0.35	-0.05
India	31-Dec-1996	5.79	78.0	7.82	24.55	0.32	-0.21
Indonesia	31-Dec-1996	3.50	62.7	-1.70	27.97	-0.06	-0.22
Korea	31-Dec-1996	6.96	76.8	3.73	30.56	0.12	-0.18
Malaysia	31-Dec-1996	4.79	69.5	-3.71	23.75	-0.16	-0.21
Mexico	31-Dec-1996	2.46	50.8	1.37	14.15	0.10	0.00
Morocco	1-Jun-1997	1.55	23.7	5.20	15.54	0.33	-0.33
Peru	1-Jul-1997	1.65	33.9	2.43	17.13	0.14	-0.14
Philipines	31-Dec-1996	2.51	45.2	-5.70	24.14	-0.24	-0.31
Poland	31-Dec-1996	2.30	50.3	-5.03	22.15	-0.23	-0.14
Russia	31-Dec-1996	4.68	72.9	11.49	41.49	0.28	-0.01
South Africa	31-Dec-1996	3.05	58.8	-0.46	13.56	-0.03	-0.21
Taiwan	31-Dec-1996	7.31	78.5	-0.63	24.50	-0.03	-0.26
Thailand	31-Dec-1996	3.97	65.0	-0.07	26.36	0.00	-0.19
Turkey	31-Dec-1996	5.22	73.4	2.84	37.74	0.08	-0.11
Average		3.55	55.0		23.31		

Table 4.1 (C)

Country factor summary statistics for markets outside ACWI IMI. The two columns pertaining to *t*-statistics were computed using monthly cross-sectional regressions. The last four columns were computed based on daily factor returns. The sample period is from 31-Dec-1996 to 30-Sep-2011 (177 months of returns).

Country Name	Factor Start Date	Average Absolute *t*-stat	Percent Observ. $\|t\|>2$	Annual. Factor Return	Annual. Factor Volatility	Factor Sharpe Ratio	Correl. with ESTU
Argentina	31-Dec-1996	2.29	47.2	-3.41	24.09	-0.14	-0.06
Bahrain	1-Jun-2001	1.05	12.2	-0.34	13.31	-0.03	-0.41
Bangladesh	1-Jun-2010	4.71	43.5	5.79	35.96	0.16	-0.20
Bosnia Herzegovina	1-May-2011	0.64	8.3	13.93	14.40	0.97	-0.44
Bulgaria	1-Feb-2006	1.07	12.0	-4.80	22.88	-0.21	-0.37
China Domestic	31-Dec-1996	14.76	88.7	8.15	28.65	0.28	-0.25
Croatia	1-Dec-2001	1.49	24.8	4.87	20.65	0.24	-0.27
Cyprus	1-Feb-2003	1.36	19.82	-8.29	19.94	-0.42	-0.28
Estonia	1-Jan-2003	1.11	12.5	4.00	19.66	0.20	-0.27
Iceland	1-Oct-2001	1.43	28.3	-1.48	21.30	-0.07	-0.26
Jamaica	1-Jun-2009	0.78	5.7	2.58	21.97	0.12	-0.43
Jordan	31-Dec-1996	1.39	21.3	0.43	19.55	0.02	-0.30
Kazakhstan	1-Aug-2006	1.82	33.3	-2.14	41.13	-0.05	-0.15
Kenya	1-Feb-2003	1.39	26.1	14.08	20.56	0.68	-0.34
Kuwait	1-Jun-2001	3.35	58.0	5.22	19.87	0.26	-0.33
Latvia	1-Dec-2002	0.82	4.4	3.84	22.87	0.17	-0.30
Lebanon	1-Feb-2003	1.31	18.9	10.70	23.50	0.46	-0.29
Lithuania	1-Dec-2002	1.36	23.0	3.80	19.84	0.19	-0.29
Mauritius	1-Feb-2003	0.76	6.3	12.47	19.82	0.63	-0.31
Nigeria	1-Mar-2001	2.52	45.5	4.19	22.65	0.19	-0.36
Oman	1-Jun-2001	1.17	15.3	2.39	19.57	0.12	-0.35
Pakistan	31-Dec-1996	2.36	45.5	5.88	30.87	0.19	-0.24
Qatar	1-Jun-2001	3.24	49.6	16.37	26.26	0.62	-0.26
Romania	1-Dec-2002	1.70	33.9	0.14	27.48	0.01	-0.19
Saudi Arabia	1-Jun-2001	5.72	67.9	7.17	31.70	0.23	-0.20
Serbia	1-Nov-2009	1.03	4.8	-16.48	20.50	-0.80	-0.35
Slovenia	1-Sep-2000	1.10	18.6	-6.29	17.42	-0.36	-0.28
Sri Lanka	31-Dec-1996	1.57	29.2	-2.24	24.60	-0.09	-0.26
Tunisia	1-Feb-2005	1.02	13.8	16.75	14.41	1.16	-0.43
Ukraine	1-Jan-2007	2.50	56.3	-19.53	30.60	-0.64	-0.21
United Arab Emirates	1-Jun-2001	3.33	50.4	7.49	26.82	0.28	-0.24
Vietnam	1-Aug-2007	2.43	47.4	-4.59	34.33	-0.13	-0.34
Average		2.27	30.4		23.66		

4.2. Industry Factors

In **Table 4.2** we report mean absolute *t*-statistics for the GEM3 World factor and industry factors, as well as the percentage of observations with $|t|>2$. Again, the *t*-statistics reported in Table 4.2 were computed using *monthly* cross-sectional regressions, even though we run daily cross-sectional regressions for purposes of constructing the factor covariance matrix.

From Table 4.2, we see that the World factor was by far the strongest factor during the sample period. On average, it had an absolute *t*-statistic of 26.13, and was significant about 95 percent of the months. Of the industry factors, Semiconductors had the highest statistical significance. Across all industries, the *t*-statistics were significant in 58.6 percent of the observations. This is higher than the mean statistical significance of the country factors, and reflects the strength of the industry factors.

Table 4.2

Industry factor summary statistics. The first two columns pertain to *t*-statistics, and were computed using monthly cross-sectional regressions. The last four columns were computed based on daily factor returns. The sample period is from 31-Dec-1996 to 30-Sep-2011 (177 months of returns).

Factor Name	Average Absolute *t*-stat	Percent Observ. $\lvert t \rvert > 2$	Annual. Factor Return	Annual. Factor Volatility	Factor Sharpe Ratio	Correl. with ESTU
World	26.13	94.9	2.93	16.01	0.18	0.996
Energy Equipment & Services	5.30	81.4	-0.29	23.19	-0.01	-0.02
Oil, Gas & Consumable Fuels	4.19	71.2	0.53	8.81	0.06	-0.01
Oil & Gas Exploration & Production	5.17	74.0	0.33	15.06	0.02	-0.01
Chemicals	2.50	54.2	-0.31	5.15	-0.06	0.08
Construction, Containers, Paper	2.24	49.7	-5.49	4.89	-1.12	0.01
Aluminum, Diversified Metals	3.85	68.9	0.79	10.27	0.08	0.06
Gold, Precious Metals	5.36	74.0	8.94	22.64	0.40	-0.16
Steel	3.44	63.3	-4.32	8.33	-0.52	0.08
Capital Goods	3.45	63.3	-2.42	3.39	-0.71	0.06
Commercial & Professional Services	1.81	40.1	-1.51	4.17	-0.36	-0.13
Transportation Non-Airline	1.88	40.7	-0.10	4.59	-0.02	0.04
Airlines	2.53	52.0	-6.46	12.90	-0.50	-0.01
Automobiles & Components	3.19	60.5	-2.36	6.86	-0.34	0.05
Consumer Durables & Apparel	2.42	50.3	-4.29	4.65	-0.92	-0.05
Consumer Services	2.29	49.2	-0.64	5.76	-0.11	-0.11
Media	2.49	50.3	-0.26	5.36	-0.05	-0.10
Retailing	3.56	66.1	0.63	6.86	0.09	-0.11
Food & Staples Retailing	1.80	37.9	1.46	5.44	0.27	-0.20
Food, Beverage & Tobacco	2.41	57.1	1.38	4.03	0.34	-0.23
Household & Personal Products	1.70	33.3	3.46	7.12	0.49	-0.15
Health Care Equipment & Services	2.88	58.2	3.05	7.02	0.43	-0.26
Biotechnology	3.58	59.9	10.90	16.14	0.68	-0.07
Pharmaceuticals, Life Sciences	2.83	50.3	5.43	6.62	0.82	-0.17
Banks	3.98	65.5	-2.27	5.43	-0.42	0.15
Diversified Financials	3.01	59.9	-2.11	5.27	-0.40	0.14
Insurance	2.93	57.6	-4.24	6.33	-0.67	0.06
Real Estate	2.88	58.8	-4.06	5.68	-0.72	-0.02
Internet Software & Services	3.13	53.7	6.63	16.89	0.39	0.06
IT Services, Software	3.08	57.6	2.11	7.01	0.30	0.01
Communications Equipment	3.66	67.2	1.89	11.13	0.17	0.10
Computers, Electronics	3.91	66.7	-0.70	6.55	-0.11	0.01
Semiconductors	5.78	79.1	5.03	15.13	0.33	0.08
Telecommunication Services	3.47	58.8	1.94	6.03	0.32	-0.02
Utilities	3.16	61.6	-1.64	5.69	-0.29	-0.15
Average	3.23	58.6		8.54		

Also reported in Table 4.2 are the returns, volatilities, and Sharpe ratios for the industry factors. These quantities were computed using *daily* factor returns and stated on an annualized basis. The World factor had an annualized return of 2.93 percent and a volatility of 16.01 percent, leading to a Sharpe ratio of 0.18 over the sample period. The best-performing industries tended to be concentrated in the Consumer Staples and Health Care sectors, whereas the worst-performing industries tended to be in the Industrials and Financials sectors.

Table 4.2 also reports the correlations of the daily factor returns with the estimation universe. Particularly noteworthy is the 99.6 percent correlation between the World factor and the estimation universe, indicating the essential equivalence of the two. By contrast, most industry factors, being dollar-neutral portfolios, had relatively small correlations with the estimation universe. Industry factors within the Consumer Staples, Health Care, and Utilities tended to have negative correlations, whereas Banks and Diversified Financials were positive. It is important to stress that these correlations represent averages over the entire sample period. Within different sub-periods or market regimes, the correlations may deviate significantly from these reported values.

4.3. Style Factors

In **Table 4.3**, we report summary statistics for the GEM3 style factors, during the sample period. The sample is broken up into two roughly equal sub-periods. Note that the statistical significance of the style factors, on the whole, was on par with that for the industry factors. As measured by statistical significance, the strongest factors were generally Beta and Momentum, followed by Residual Volatility. In the first sample period (31-Dec-1996 to 31-May-2004), Momentum, Earnings Yield, and Liquidity performed extremely well, while Residual Volatility and Size performed poorly. These trends largely held for the second sample period (31-May-2004 to 30-Sep-2011), with the exception of Liquidity which was flat.

Most style factors, being dollar-neutral portfolios, had relatively small correlation with the estimation universe. The glaring exception is the Beta factor, which had a correlation of 0.83 in the first sample period and 0.90 during the second. The Residual Volatility factor also had a sizeable positive correlation with the estimation universe during both sub-periods. Again, it is important to stress that the correlations reported in Table 4.3 represent averages, and that the actual correlations in different market regimes may deviate from these reported values.

Also reported in Table 4.3 is the factor stability coefficient, described in the *USE4 Methodology Notes*. Briefly, this coefficient is computed as the cross-sectional correlation of factor exposures from one month to the next. Although there is no strict lower limit for what is considered acceptable, a useful rule of thumb is that values below 0.80 are regarded as too unstable for model inclusion, while those above 0.90 are considered sufficiently stable. From Table 4.3, we see that the average factor stability coefficient for style factors was 0.96 during both sample periods.

Table 4.3 also reports the Variance Inflation Factor (VIF). As explained in the *USE4 Methodology Notes*, VIF measures the degree of collinearity among the factors. Excessive collinearity can lead to increased estimation error in the factor returns and non-intuitive correlations among factors. Although there exists no strict upper bound, VIF scores above 5 are generally considered problematic. As shown in Table 4.3, all GEM3 style factors were well below this level during both sample periods.

Table 4.3

Style factor summary statistics. The first two columns pertain to *t*-statistics, and were computed using monthly cross-sectional regressions. The next four columns were computed based on daily factor returns. The factor stability coefficient and Variance Inflation Factor were computed on monthly data using square root of market-cap weighting. The entire sample period, comprising 177 months (31-Dec-1996 to 30-Sep-2011), is divided into two sub-periods.

A. 31-Dec-1996 to 31-May-2004 (89 months)

Factor name	Average Absolute *t*-stat	Percent Observ. |*t*|>2	Annual. Factor Return	Annual. Factor Volatility	Factor Sharpe Ratio	Correl. with ESTU	Factor stability coeff.	Variance inflation factor
Beta	8.17	85.4	-0.83	7.32	-0.11	0.83	0.95	2.91
Momentum	7.04	82.0	6.29	3.04	2.07	-0.07	0.90	1.89
Size	4.29	65.2	-0.93	2.19	-0.43	0.39	1.00	3.25
Earnings Yield	3.54	62.9	5.07	1.57	3.22	-0.19	0.95	1.62
Residual Volatility	6.10	77.5	-3.49	3.72	-0.94	0.58	0.96	2.76
Growth	2.42	41.6	0.49	1.61	0.31	0.25	0.96	1.34
Dividend Yield	2.42	50.6	1.99	1.33	1.50	0.11	0.98	1.73
Book-to-Price	2.39	46.1	2.35	1.32	1.78	0.11	0.97	1.93
Leverage	2.02	39.3	-0.79	0.98	-0.81	-0.09	0.95	1.32
Liquidity	3.26	69.7	5.35	1.46	3.68	0.35	0.97	1.54
Non-linear Size	2.51	50.6	-0.06	1.67	-0.03	0.19	0.98	1.23
Average	4.01	61.0	1.40	2.38	0.93	0.22	0.96	1.96

B. 31-May-2004 to 30-Sep-2011 (88 months)

Factor name	Average Absolute *t*-stat	Percent Observ. |*t*|>2	Annual. Factor Return	Annual. Factor Volatility	Factor Sharpe Ratio	Correl. with ESTU	Factor stability coeff.	Variance inflation factor
Beta	7.61	79.5	2.16	6.63	0.33	0.90	0.94	2.24
Momentum	6.28	78.4	4.04	2.77	1.46	-0.09	0.89	1.67
Size	3.51	67.0	-1.04	1.84	-0.56	0.35	1.00	2.78
Earnings Yield	3.14	61.4	2.05	1.63	1.26	-0.01	0.95	1.50
Residual Volatility	4.31	64.8	-4.69	3.02	-1.55	0.60	0.96	2.85
Growth	1.82	35.2	0.62	0.78	0.79	-0.02	0.93	1.23
Dividend Yield	2.78	45.5	1.33	1.32	1.01	-0.01	0.97	1.57
Book-to-Price	2.86	59.1	0.61	1.25	0.49	0.16	0.98	1.63
Leverage	2.49	52.3	-1.33	0.92	-1.44	0.01	0.96	1.38
Liquidity	2.42	53.4	-0.09	1.25	-0.07	0.27	0.98	1.51
Non-linear Size	2.06	43.2	1.10	1.48	0.74	0.08	0.99	1.26
Average	3.57	58.2	0.43	2.08	0.22	0.20	0.96	1.78

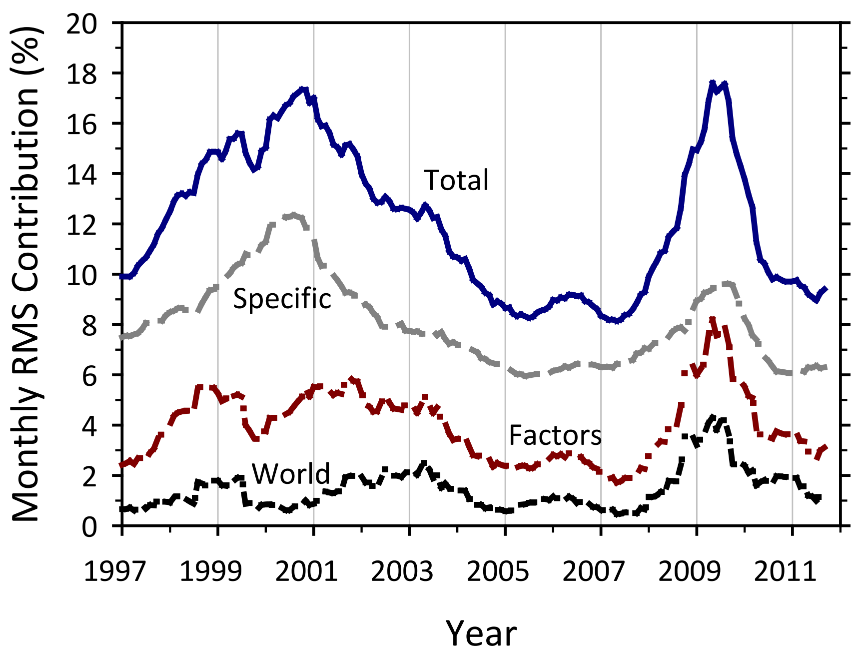
4.4. Cross-Sectional Dispersion

It is informative to study the cross-sectional dispersion of monthly stock returns. As discussed by Menchero and Morozov (2011), dispersion can be measured in one of two ways. The first is by cross-sectional volatility (CSV), which measures the dispersion relative to the *mean* return. The second way is by root mean square (RMS) return, which measures the dispersion relative to *zero* return. The main difference between the two is that the World factor makes no contribution to CSV, whereas it does contribute to RMS levels.

In **Figure 4.1**, we plot the trailing 12-month total RMS return. The two most prominent features corresponded to the Internet Bubble and the 2008/2009 financial crisis. Note that the Internet Bubble peak was much broader — the buildup and aftermath spanned several years — whereas the financial-crisis peak was relatively short in duration.

Figure 4.1

Total monthly cross-sectional dispersion as measured by root mean square (RMS) return. Also displayed are the stock-specific and factor contributions, and the contribution from the World factor. Lines were smoothed using 12-month moving averages.

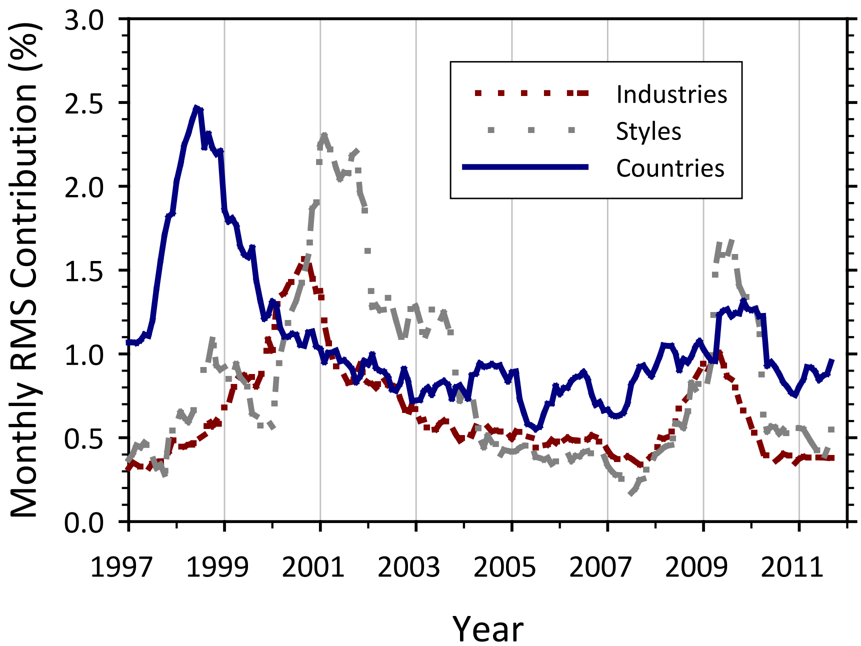


As discussed by Menchero and Morozov (2011), and shown in Appendix B, the RMS return can be decomposed and attributed to individual factors or groups of factors. Figure 4.1 shows the net RMS contributions from stock-specific sources, all factors, and the World factor. The stock-specific contribution dominated throughout the sample period, especially during the Internet Bubble period. During the financial crisis, factors became nearly as important as the stock-specific contribution. Note that the contribution of the World factor became particularly important during late 2008 and early 2009 as stocks moved together in aggregate.

In **Figure 4.2**, we report the factor RMS contributions from countries, industries, and styles. We see that all three sources were of comparable importance in explaining the cross section of RMS returns. However, the relative importance of these factors varied over time. For instance, country factors were the largest contributor to RMS return from 1997-2000, whereas styles dominated from 2001-2004. Industries dominated during a brief period in 2000.

Figure 4.2

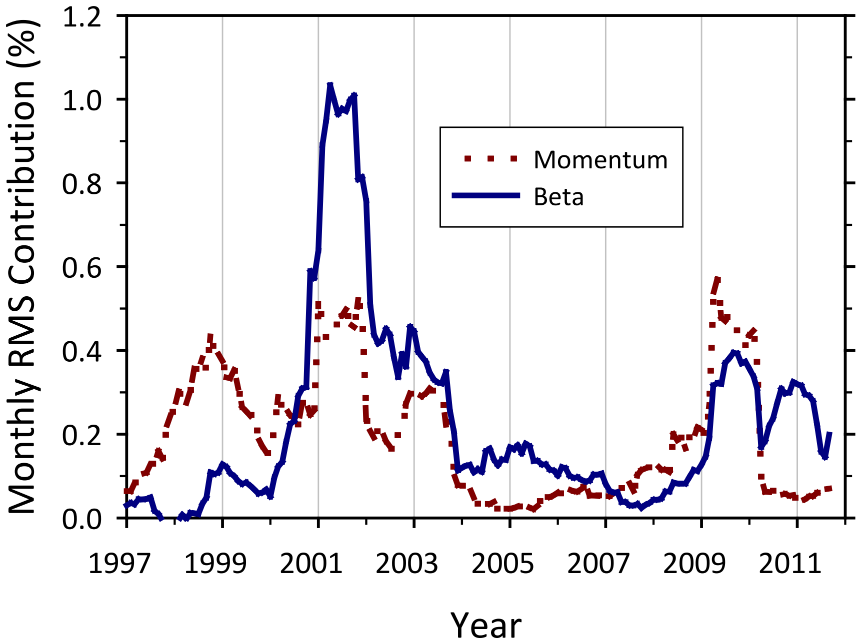
Contributions to monthly root mean square (RMS) return from countries, industries, and styles. Lines were smoothed using 12-month moving averages. All three sources are important contributors to cross-sectional dispersion.



In **Figure 4.3** we report RMS contributions from the Beta and Momentum factors. Particularly noteworthy is the large peak attributed to Beta from 2001-2003. The Momentum factor dominated in the late 1990s, as well as during 2009, following the rebound from the market crash of 2008.

Figure 4.3

Contributions to monthly root mean square (RMS) return from Beta and Momentum factors. Lines were smoothed using 12-month moving averages.

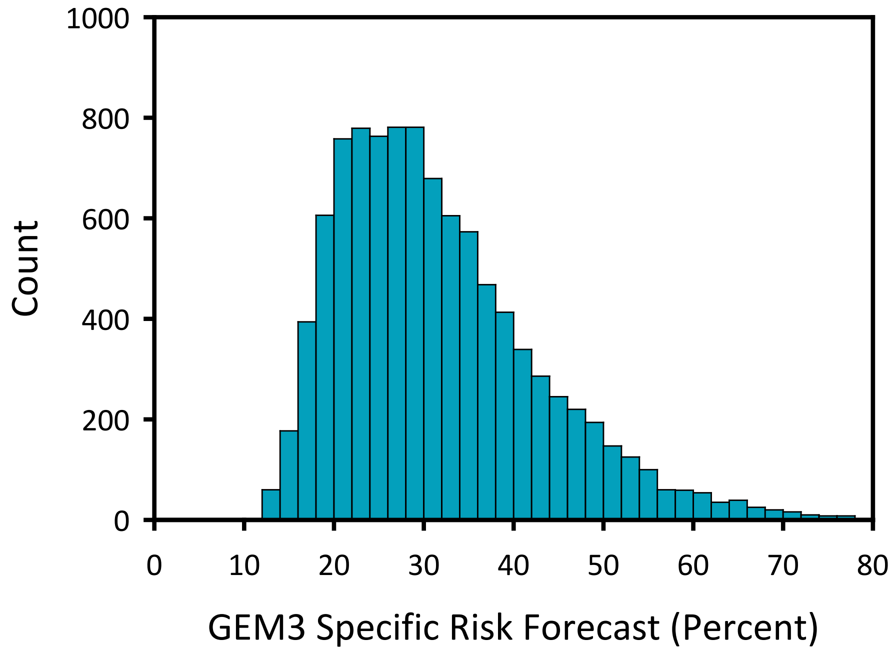


4.5. Specific Risk

The distribution of specific volatilities is an important characteristic to examine. In **Figure 4.4** we plot the histogram of GEM3S specific risk forecasts for analysis date 30-Sep-2011. Most stocks had specific risk forecasts within the range 15-60 percent, although the most volatile stocks had forecasts exceeding 70 percent. The mean specific risk forecast on 30-Sep-2011 was about 30 percent.

Figure 4.4

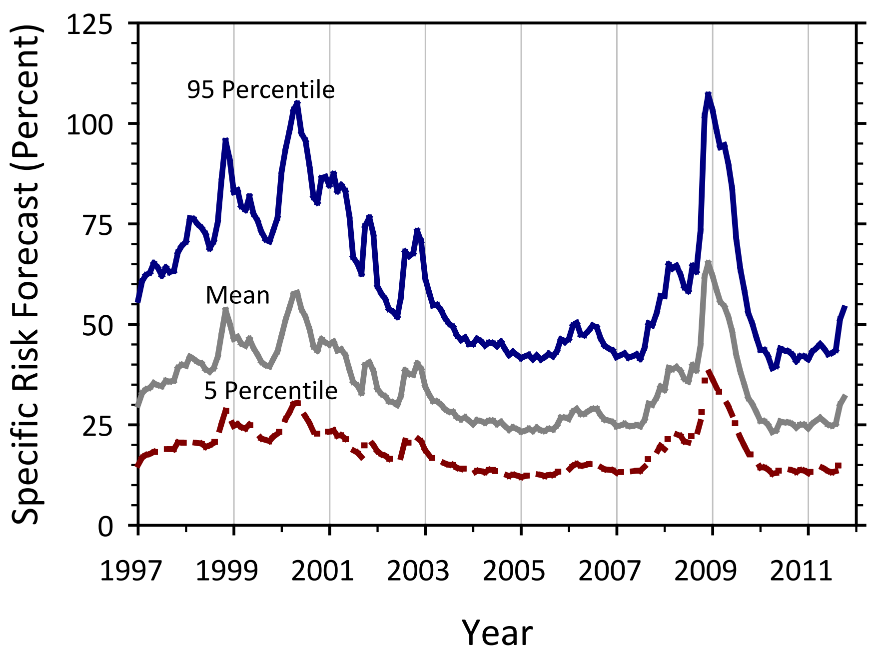
Histogram of GEM3S specific-risk forecasts as of 30-Sep-2011.



It is also interesting to study how the distribution of specific risk varied over time. In **Figure 4.5**, we plot the 5-percentile, mean, and 95-percentile values for the specific risk distribution. We see that the 5-percentile specific volatility historically ranged from about 15-30 percent, with the maximum occurring in late 2008. The mean specific risk varied within the range of 30-60 percent. Again, the peak happened in late 2008, although the Internet Bubble period saw comparable levels of specific risk. The 95-percentile specific volatility ranged from a low of 45 percent in 2007, to highs in excess of 100 percent during the Internet Bubble and financial crisis.

Figure 4.5

Specific risk levels versus time for GEM3S.



5. Forecasting Accuracy

5.1. Overview of Testing Methodology

In this section, we describe our methodology for evaluating and comparing the accuracy of risk model forecasts. We aim for a systematic and quantitative approach, yet one that is also visually intuitive.

The foundation of our approach rests on the bias statistic, described in Appendix C. Conceptually, the bias statistic is an out-of-sample measure that represents the ratio of realized risk to predicted risk. The ideal bias statistic for perfect risk forecasts should be close to 1. However, even for perfect risk forecasts, the bias statistic will never be *exactly* 1 due to sampling error. Nevertheless, we may define a confidence interval that is expected to contain 95 percent of the observations under the hypothesis of perfect risk forecasts. If the bias statistic falls outside of the confidence interval, we infer that the risk forecast was not accurate.

When determining the size of the confidence interval, standard practice is to assume that returns are normally distributed. In reality, however, stock returns tend to have fat tails (i.e., positive excess kurtosis). As shown in Appendix C, fewer than 95 percent of the observations are expected to fall within the standard confidence interval when kurtosis is taken into account.

We are interested in testing the full sample period. One potential shortcoming of the bias statistic is that over long windows, we may have sub-periods of overforecasting and underforecasting, yet obtain a bias statistic close to 1 over the entire window. In other words, forecasting errors may cancel out over the long term, even though the risk forecasts may be poor over shorter periods. For a portfolio manager who may be devastated by a single year of poor performance, it is small consolation knowing that a risk forecast is good *on average*.

For this reason, we focus on 12-month rolling windows. By plotting the mean rolling 12-month bias statistic across time for a collection of portfolios, we quickly visualize the magnitude of the average biases and can judge whether they were persistent or regime-dependent.

It is not enough, however, knowing the average bias statistic. We must also understand the extremes. We also compute, therefore, the 5-percentile (P5) and 95-percentile (P95) bias statistics across time. Assuming normally distributed returns and perfect risk forecasts, on average 5 percent of the rolling 12-month bias statistics will fall below 0.66 by pure chance. Therefore, if the P5 bias statistic falls significantly below this level, we infer that we are likely overpredicting the risk of at least some of the portfolios with bias statistics below 0.66. Similarly, if the P95 bias statistic lies well above 1.34, we infer that we are underpredicting the risk of some portfolios with bias statistics above 1.34. It is worth pointing out, however, that if we relax the normality assumption and allow for fat-tailed distributions, then for perfect risk forecasts the P5 bias statistic tends to fall below 0.66, and the P95 value generally lies above 1.34.

Another measure that provides insight into the accuracy of risk forecasts is the *mean rolling absolute deviation*, or MRAD. As described in Appendix C, this is computed by averaging the absolute deviation of the bias statistics from 1 for a collection of portfolios. Conceptually, MRAD penalizes any deviation from the ideal bias statistic of 1, whether due to overforecasting or underforecasting.

Assuming normally distributed returns and perfect risk forecasts, the expected value of MRAD is 0.17. Real financial returns, of course, tend to have fat tails. As shown in Appendix C, kurtosis levels within the range of 3.5 to 4.0 lead to MRAD values of approximately 0.19 for perfect risk forecasts. When comparing MRAD values across two models, it is crucial to keep in mind the lower bound of MRAD. For instance, assuming a 0.19 lower bound, reducing MRAD from 0.23 to 0.21 constitutes a 50 percent reduction in *excess* MRAD.

It is also important to recognize that MRAD is a *statistical* measure. As such, by pure chance the MRAD may dip below the level of 0.17. Indeed, consider a portfolio that has been overforecast for many months, leading to a bias statistic less than 1. Eventually, the risk model may begin underforecasting the risk of that same portfolio. When the transition from overforecasting to underforecasting occurs, the bias statistic must necessarily cross through 1, thereby producing an MRAD value close to zero. For a *large collection* of portfolios, however, it is highly improbable that the bias statistics of all portfolios will cross through 1 simultaneously. Consequently, for a sufficiently diverse set of portfolios, the MRAD is unlikely to dip significantly below 0.17 for any sustained period of time.

Our testing approach therefore relies principally on these four measures: the mean bias statistic, the P5 and P95 bias statistics, and the MRAD. All are computed and plotted on a rolling 12-month basis. These plots allow us to quickly evaluate the accuracy of risk forecasts in a visually intuitive manner.

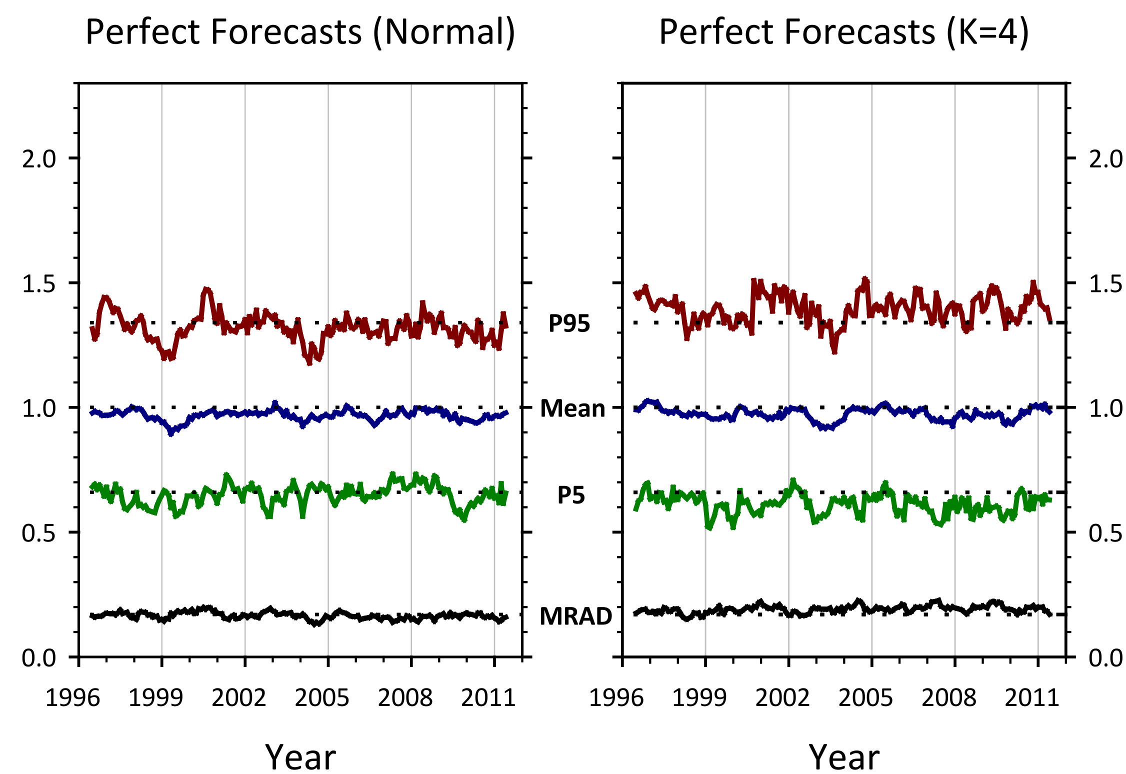
In order to develop a better understanding for how these measures behave in the ideal case of stationary returns and perfect risk forecasts, we perform two separate simulations for 100 sets of returns over a representative period of 191 months (from July 1995 through May 2011). In the first simulation, the returns were drawn from a standard normal distribution. In the second simulation, the returns were drawn from a *t*-distribution with standard deviation of 1 and kurtosis of 4. In all simulations, the predicted volatilities were equal to 1 (i.e., perfect risk forecasts).

In **Figure 5.1** we plot MRAD and bias statistics for the mean, P5 and P95 levels. The dashed horizontal lines represent the ideal positions of the curves for the case of perfect forecasts and normal distributions. On the left panel (normal distribution), we see that the realized curves indeed lie close to their ideal positions. In particular, the MRAD is closely centered at the 0.17 level. Note that the degree of “noise” in the lines depends on the number of portfolios in the sample. That is, the more portfolios that we use, the smaller the observed variability.

On the right panel of Figure 5.1 we plot MRAD and bias statistics for perfect risk forecasts and a kurtosis of 4. The effect of higher kurtosis is to increase the frequency of observations with bias statistics above 1.34 or below 0.66. In this case, the mean of the P5 line is shifted down to 0.61, whereas the P95 line moves upward to a mean of 1.40. This has the effect of increasing MRAD to approximately 0.19.

Figure 5.1

Simulated results for the rolling 12-month mean bias statistic, the 5-percentile and 95-percentile bias statistics, and MRAD. The left plot is for 100 portfolios with simulated returns drawn from a normal distribution with standard deviation of 1. The right plot is for 100 portfolios with simulated returns drawn from a *t*-distribution with a standard deviation of 1 and a kurtosis of 4. The risk forecasts in each case were perfect (i.e., predicted volatility of 1). The dashed horizontal lines indicate the ideal positions assuming normally distributed returns and perfect risk forecasts.



It is worth reiterating that Figure 5.1 represents the idealized case of perfect risk forecasts and stationary returns. In reality, risk forecasts are never perfect and returns are not stationary. Nevertheless, Figure 5.1 serves as a useful baseline for understanding the empirical backtesting results that follow.

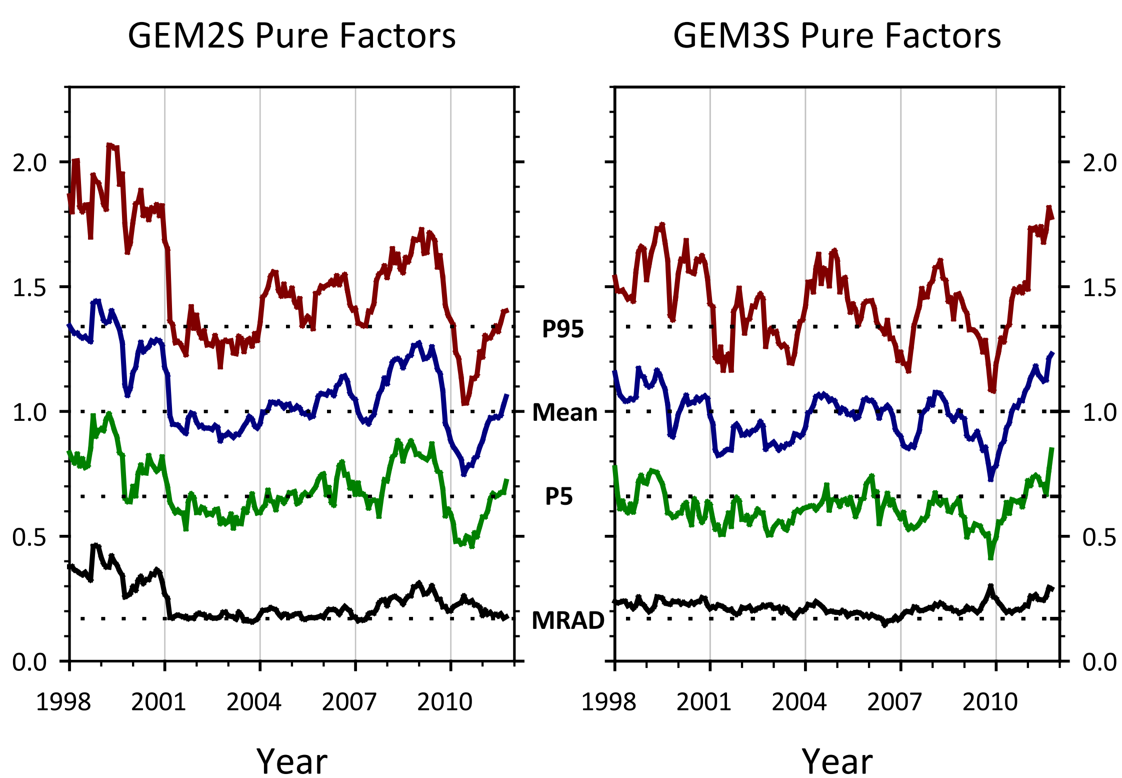
5.2. Backtesting Results

In this section we perform side-by-side comparisons for the GEM2 and GEM3 Models. We plot MRAD and bias statistics for a variety of test portfolios using both short-horizon and long-horizon models. The analysis period is 177 months, running from January 1997 through September 2011. Rolling 12-month quantities are therefore plotted starting in January 1998.

In **Figure 5.2** we report MRAD and bias statistics for the GEM2S and GEM3S pure factors. We see that the mean bias statistics for GEM3S were generally centered more closely to the ideal value of 1. For instance, whereas GEM2S generally underpredicted risk during the Internet Bubble, GEM3 had mean bias statistics closer to 1. The GEM2S Model also underpredicted risk during the 2008 Financial Crisis, whereas GEM3S had bias statistics closer to 1.

Figure 5.2

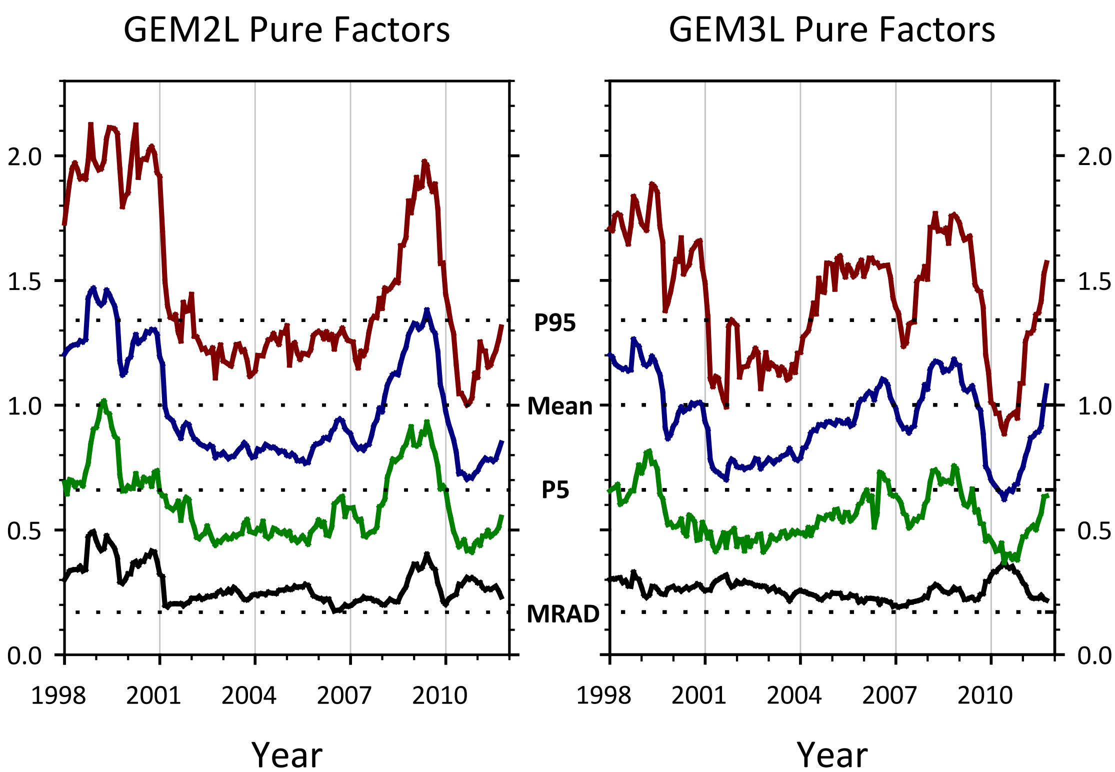
Comparison of GEM2S Model and GEM3S Model for pure factors. Each plot reports rolling 12-month values for the mean bias statistic, the 5-percentile and 95-percentile bias statistics, and MRAD. The dashed horizontal lines indicate the ideal positions assuming normally distributed returns and perfect risk forecasts.



In **Figure 5.3** we report MRAD and bias statistics for the GEM2L and GEM3L pure factors. We see that GEM2L underpredicted risk during the Internet Bubble but overpredicted risk for several years after (roughly 2002-2007). Similarly, GEM2L underpredicted risk leading into the 2008 Financial Crisis and overpredicted risk for several years thereafter. The GEM3L Model generally did a better job predicting volatility levels, with the exception of 2009 during which it overpredicted risk by an even larger margin than GEM2L.

Figure 5.3

Comparison of GEM2L Model and GEM3L Model for pure factors. Each plot reports rolling 12-month values for the mean bias statistic, the 5-percentile and 95-percentile bias statistics, and MRAD. The dashed horizontal lines indicate the ideal positions assuming normally distributed returns and perfect risk forecasts.

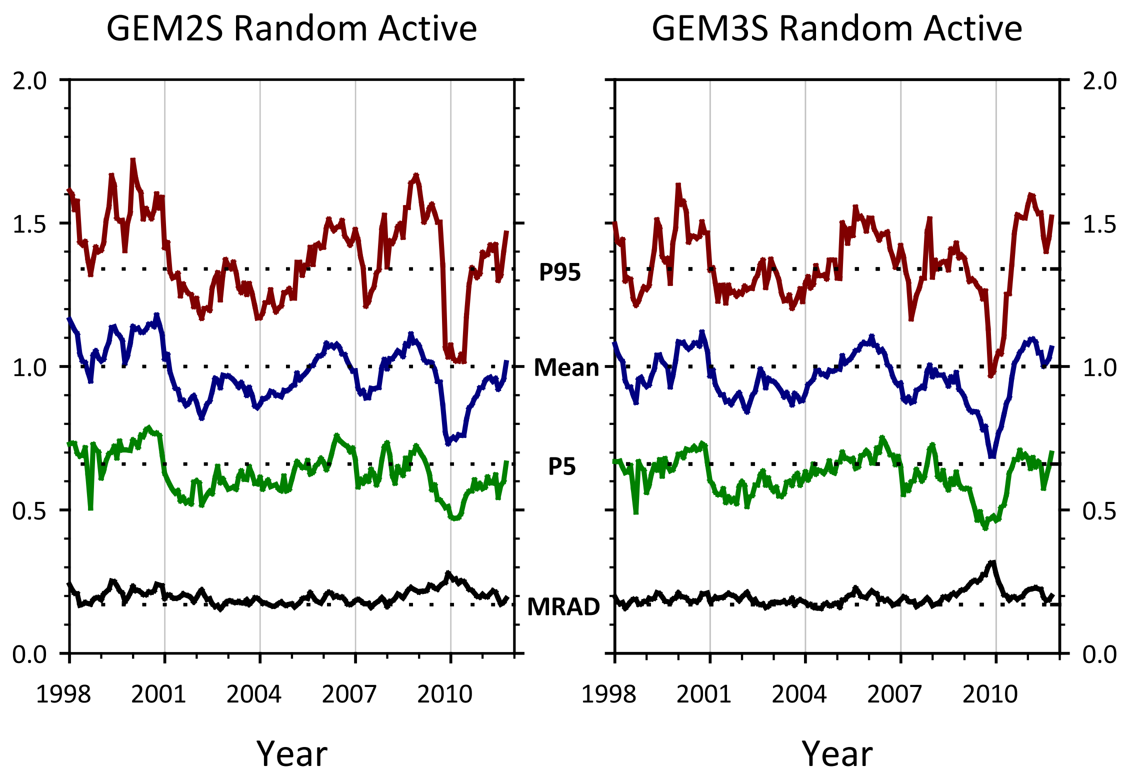


It is also instructive to compare the GEM3L results in Figure 5.3 with the corresponding results for GEM3S in Figure 5.2. For the more responsive GEM3S Model, we see that the MRAD and bias statistics were consistently closer to their ideal values.

In **Figure 5.4** we plot MRAD and bias statistics for 100 random active portfolios using the short-horizon models. The random active portfolios were constructed by going long 500 cap-weighted randomly selected stocks and shorting the cap-weighted GEM3 estimation universe. Overall, both models do an excellent job predicting portfolio risk over the sample period. After the 2008 Financial Crisis, however, both models overpredict risk. Nonetheless, in GEM3S the Volatility Regime Adjustment quickly corrected for the overprediction bias, so that by late 2010 the average bias statistics were again close to 1. On average, GEM3S MRAD values were slightly lower in their GEM2S counterparts.

Figure 5.4

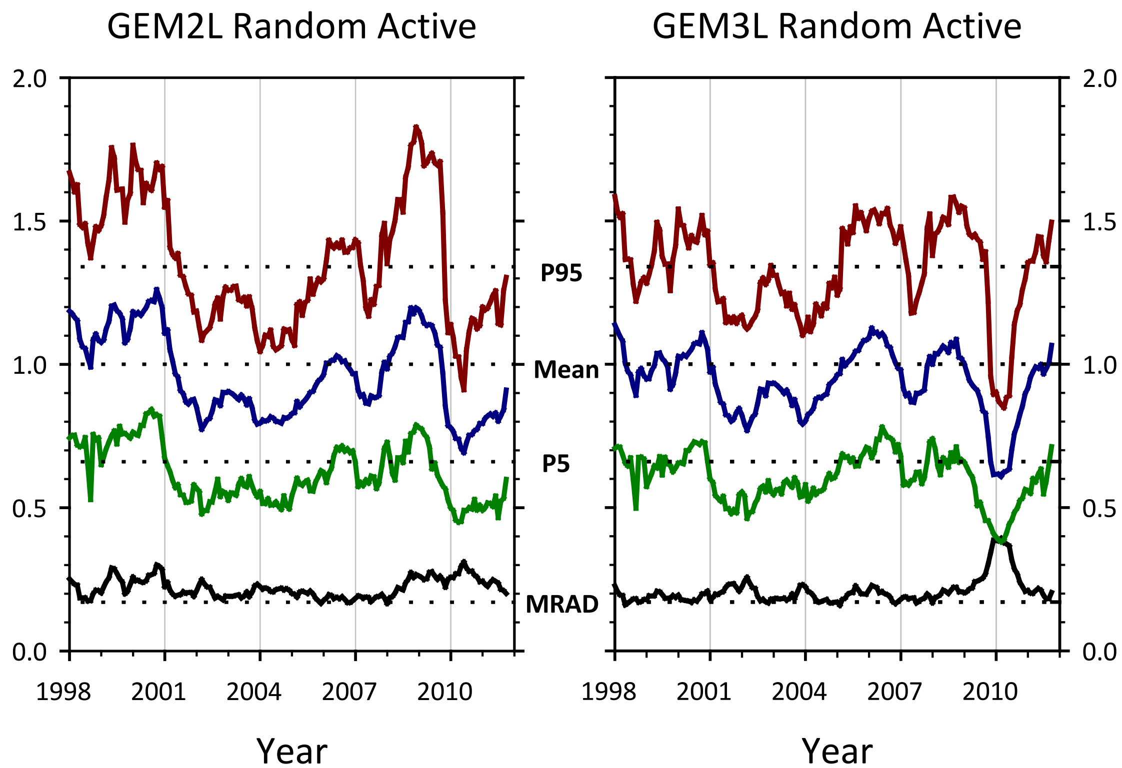
Comparison of GEM2S Model and GEM3S Model for 100 random active portfolios. The portfolios were generated by randomly selecting 500 stocks, which were then capitalization weighted. These portfolios were then run against the estimation universe to form dollar-neutral active portfolios. Each plot reports rolling 12-month values for the mean bias statistic, the 5-percentile and 95-percentile bias statistics, and MRAD. The dashed horizontal lines indicate the ideal positions assuming normally distributed returns and perfect risk forecasts.



In **Figure 5.5** we plot MRAD and bias statistics for the same 100 random active portfolios, except now using the long-horizon models. Mean bias statistics for GEM3L were generally closer to 1 over the sample period, except during early 2010. Even so, by the end of 2010 the GEM3L bias statistics were again close to 1 whereas the GEM2L Model was still overpredicting risk on average.

Figure 5.5

Comparison of GEM2L Model and GEM3L Model for 100 random active portfolios. The portfolios were generated by randomly selecting 500 stocks, which were then capitalization weighted. These portfolios were then run against the estimation universe to form dollar-neutral active portfolios. Each plot reports rolling 12-month values for the mean bias statistic, the 5-percentile and 95-percentile bias statistics, and MRAD. The dashed horizontal lines indicate the ideal positions assuming normally distributed returns and perfect risk forecasts.

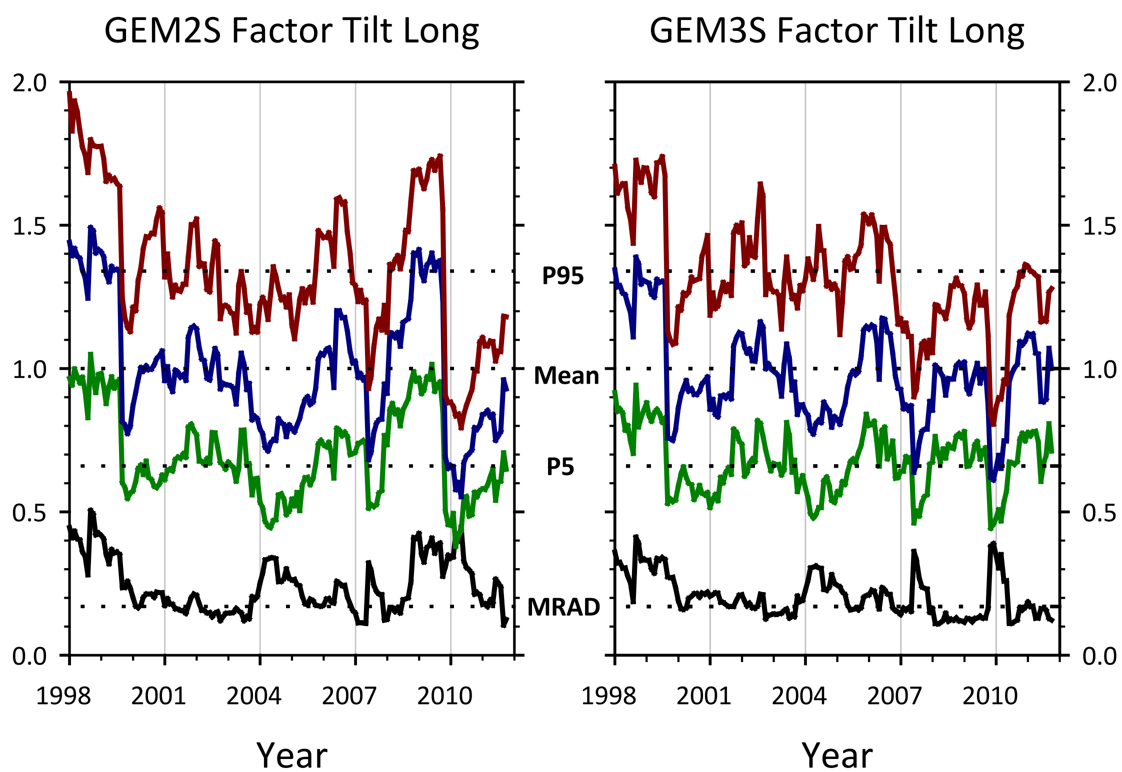


In **Figure 5.6** we plot MRAD and bias statistics for 117 long-only factor-tilt portfolios using the short-horizon models. The portfolios were constructed by cap-weighting the 24 GEM3 developed markets, the 21 GEM3 emerging markets, the 34 industries, and the top and bottom quintiles for the 8 GEM2 styles and the 11 GEM3 styles.

Overall, bias statistics for GEM3S were closer to 1 over the sample period. This was especially true during the Financial Crisis.

Figure 5.6

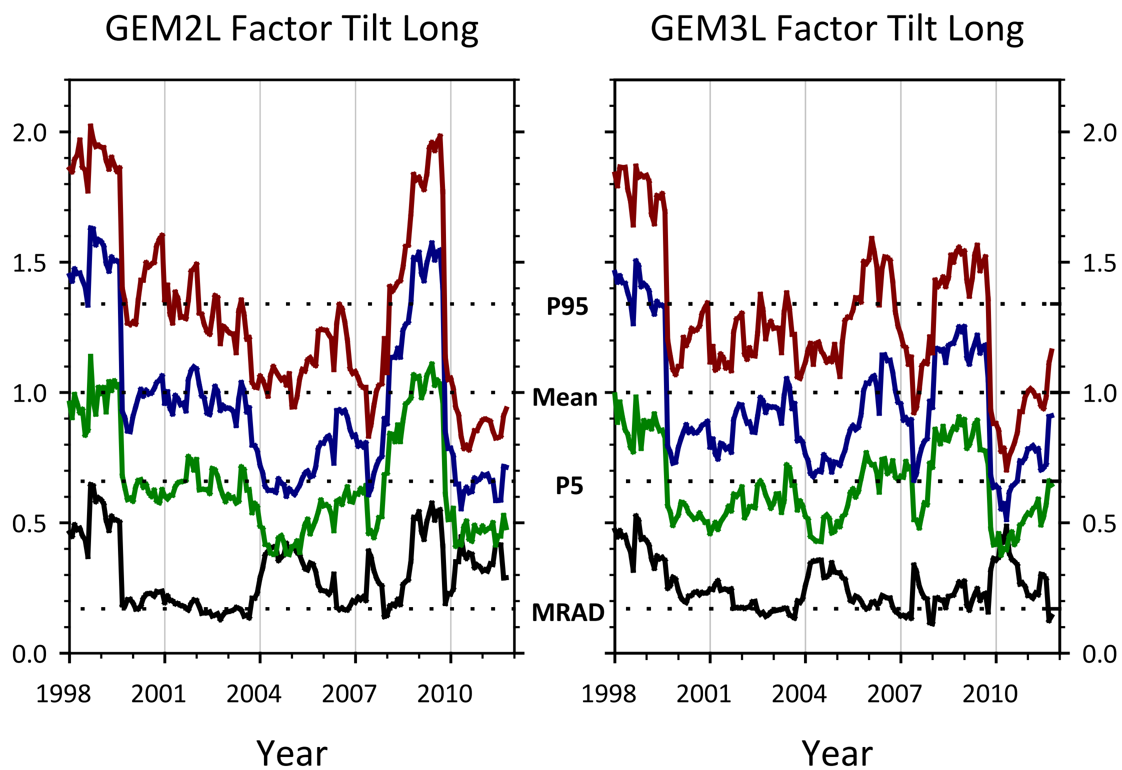
Comparison of GEM2S Model and GEM3S Model for country, industry, and style-tilt long-only portfolios. Each plot reports rolling 12-month values for the mean bias statistic, the 5-percentile and 95-percentile bias statistics, and MRAD. The dashed horizontal lines indicate the ideal positions assuming normally distributed returns and perfect risk forecasts.



In **Figure 5.7** we plot MRAD and bias statistics for the same 117 long-only factor-tilt portfolios as in Figure 5.6, except now using the long-horizon models. The GEM2L Model overpredicted risk by a wide margin from 2004-2008. Entering the financial crisis of 2008, it underpredicted risk, followed by overprediction as the crisis subsided. The GEM3L Model exhibited the same main features, except that the magnitudes of the biases were considerably smaller. That is, the Volatility Regime Adjustment helped to reduce the dramatic swings of overprediction and underprediction.

Figure 5.7

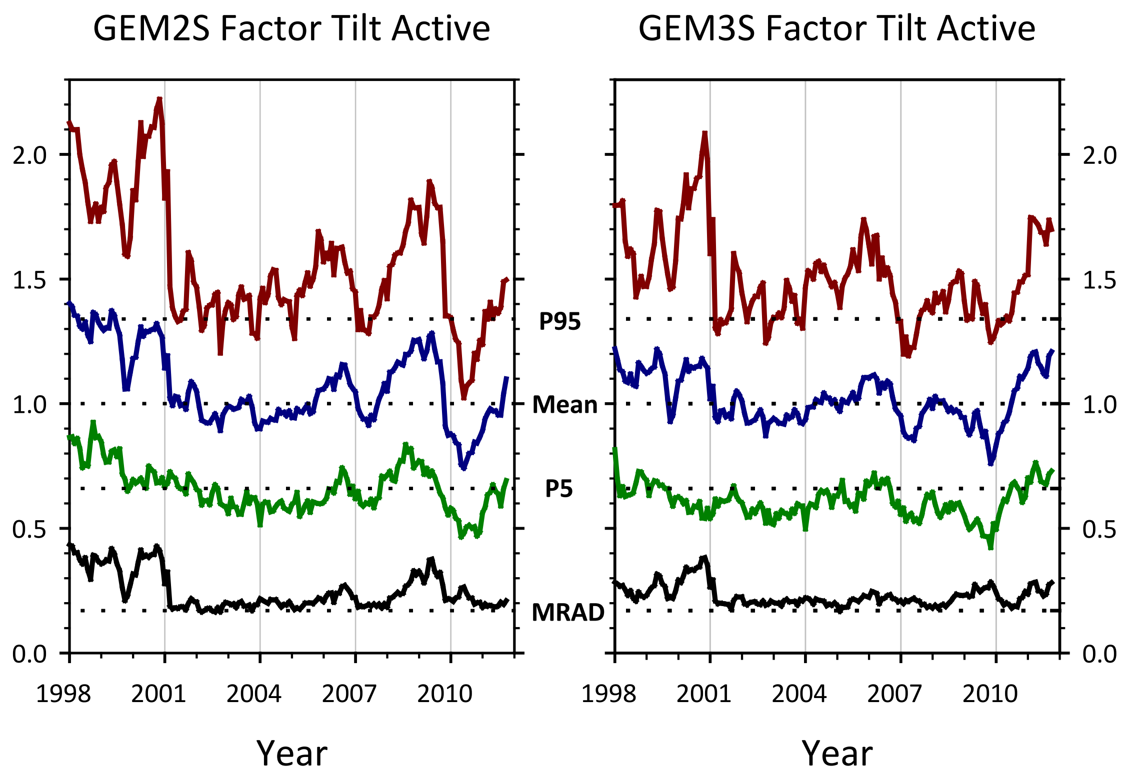
Comparison of GEM2L Model and GEM3L Model for country, industry, and style-tilt long-only portfolios. Each plot reports rolling 12-month values for the mean bias statistic, the 5-percentile and 95-percentile bias statistics, and MRAD. The dashed horizontal lines indicate the ideal positions assuming normally distributed returns and perfect risk forecasts.



In **Figure 5.8** we plot MRAD and bias statistics for the 117 active factor-tilt portfolios using the short-horizon models. The portfolios were constructed by going long the factor-tilt portfolios of Figure 5.6 and shorting the GEM3 estimation universe. The GEM2S Model tended to significantly underpredict the risk of these portfolios during the Internet Bubble, followed by overprediction in 2003-2004. The GEM2S Model also significantly overpredicted risk in the wake of the financial crisis. The GEM3S Model, by contrast, produced MRAD and bias statistics that were very stable and close to their ideal values over the entire sample period. In particular, the underforecasting during the Internet Bubble was very mild, as was the overforecasting following the financial crisis.

Figure 5.8

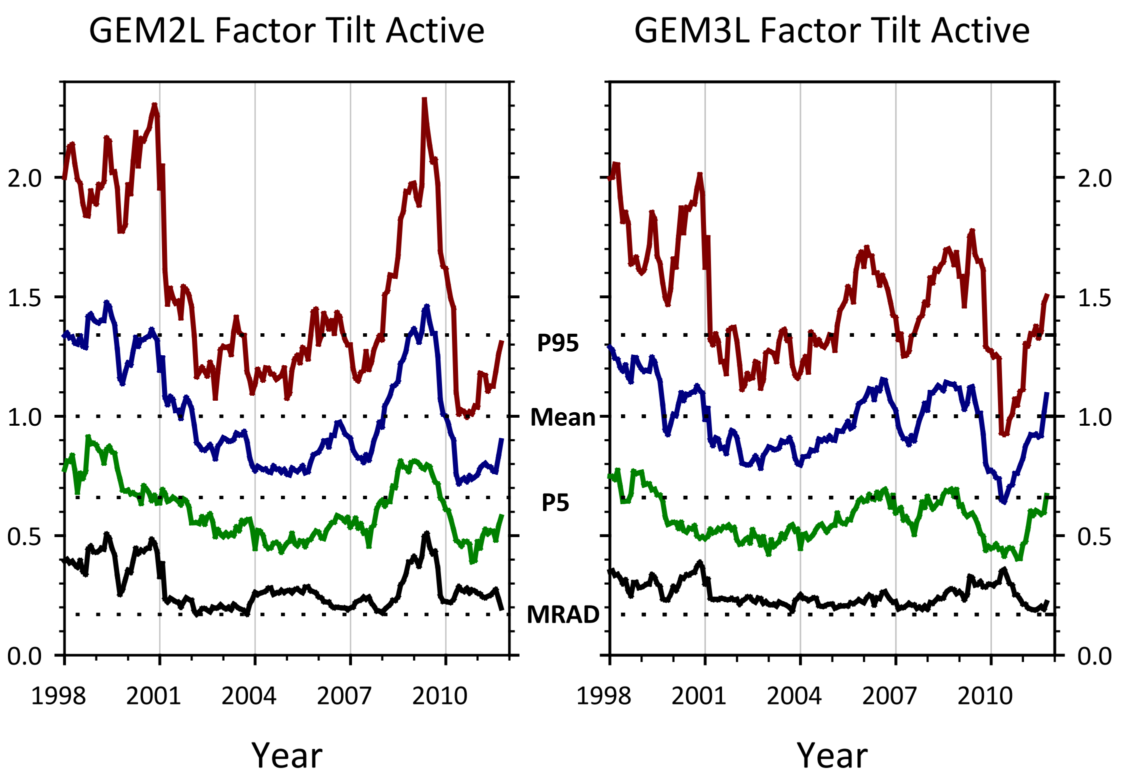
Comparison of GEM2S Model and GEM3S Model for country, industry, and style-tilt active portfolios. Each plot reports rolling 12-month values for the mean bias statistic, the 5-percentile and 95-percentile bias statistics, and MRAD. The dashed horizontal lines indicate the ideal positions assuming normally distributed returns and perfect risk forecasts.



In **Figure 5.9** we plot MRAD and bias statistics for the same 117 active factor-tilt portfolios, except now using the long-horizon models. The GEM2L Model significantly underpredicted the risk of these portfolios during the Internet Bubble, followed by a five-year period of overprediction as volatility levels subsided. Entering the financial crisis, the GEM2S Model underpredicted risk, followed by overprediction in the aftermath of the crisis. The GEM3L Model generally gave more accurate risk forecasts throughout the sample period, with the exception of a brief period in 2010 during which the model significantly overpredicted risk.

Figure 5.9

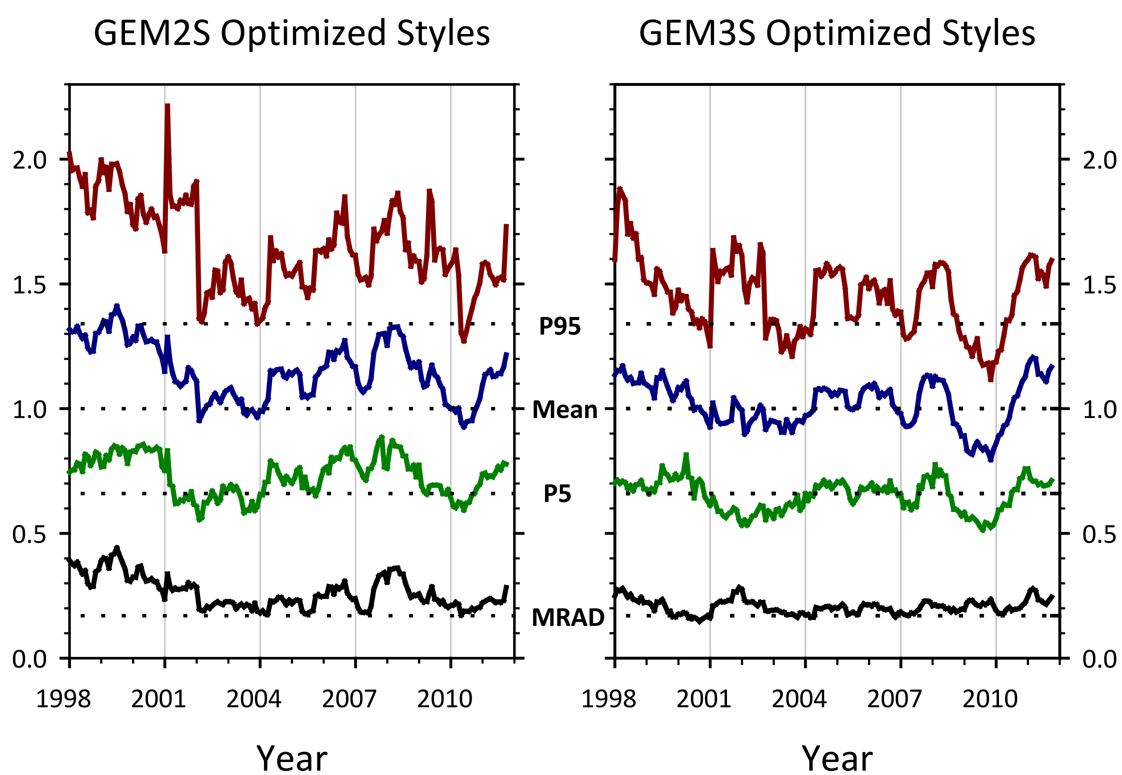
Comparison of GEM2L Model and GEM3L Model for country, industry, and style-tilt active portfolios. Each plot reports rolling 12-month values for the mean bias statistic, the 5-percentile and 95-percentile bias statistics, and MRAD. The dashed horizontal lines indicate the ideal positions assuming normally distributed returns and perfect risk forecasts.



In **Figure 5.10** we plot MRAD and bias statistics for optimized style-tilt portfolios using the short-horizon models. We constructed 160 GEM2S optimized portfolios by using the 8 GEM2 style factors as "alpha signals" and then forming the minimum volatility portfolio (with alpha equal to 1) for 20 draws of 500 randomly selected stocks. We constructed 220 GEM3S optimized portfolios similarly, except using the 11 GEM3 style factors as the "alpha signals" on the same set of randomly selected stocks. The mean bias statistics for the GEM2S model were greater than 1 for virtually the entire sample period, indicating underprediction of risk for these optimized portfolios. By contrast, the mean bias statistics for GEM3S were close to 1 on average, indicating that the Optimization Bias Adjustment was effective at reducing the underforecasting biases for these optimized portfolios.

Figure 5.10

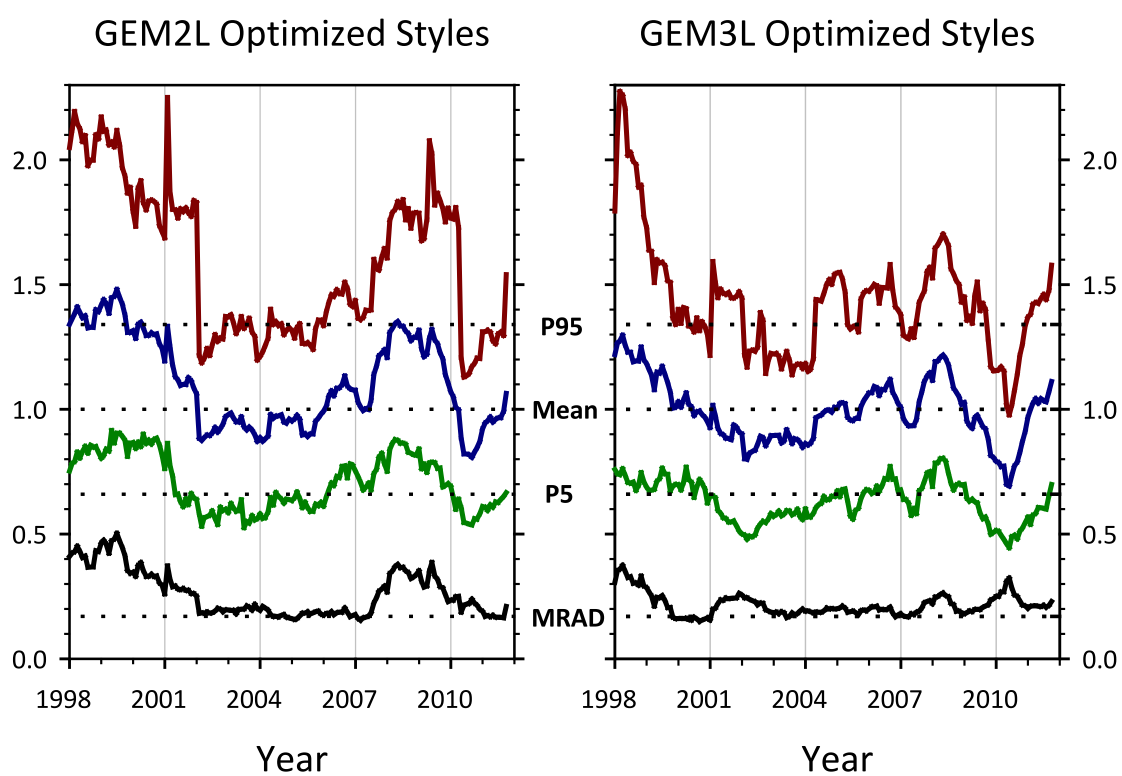
Comparison of GEM2S Model and GEM3S Model for optimized portfolios. The portfolios were constructed by minimizing risk subject to the unit alpha constraint, where the alpha signals were taken from the style factors of each model. Twenty sets of optimizations were performed using 500 randomly selected stocks for each style factor. Each plot reports rolling 12-month values for the mean bias statistic, the 5-percentile and 95-percentile bias statistics, and MRAD. The dashed horizontal lines indicate the ideal positions assuming normally distributed returns and perfect risk forecasts.



In **Figure 5.11** we plot MRAD and bias statistics for optimized style-tilt portfolios constructed in the same fashion as in Figure 5.10, except now using the long-horizon models. The bias statistics for the GEM2L Model were shifted upward throughout most of the sample period, indicating underprediction of risk for these optimized portfolios. The mean bias statistics for the GEM3L Model were closer to 1 on average, although there was a tendency to overpredict risk in 2010.

Figure 5.11

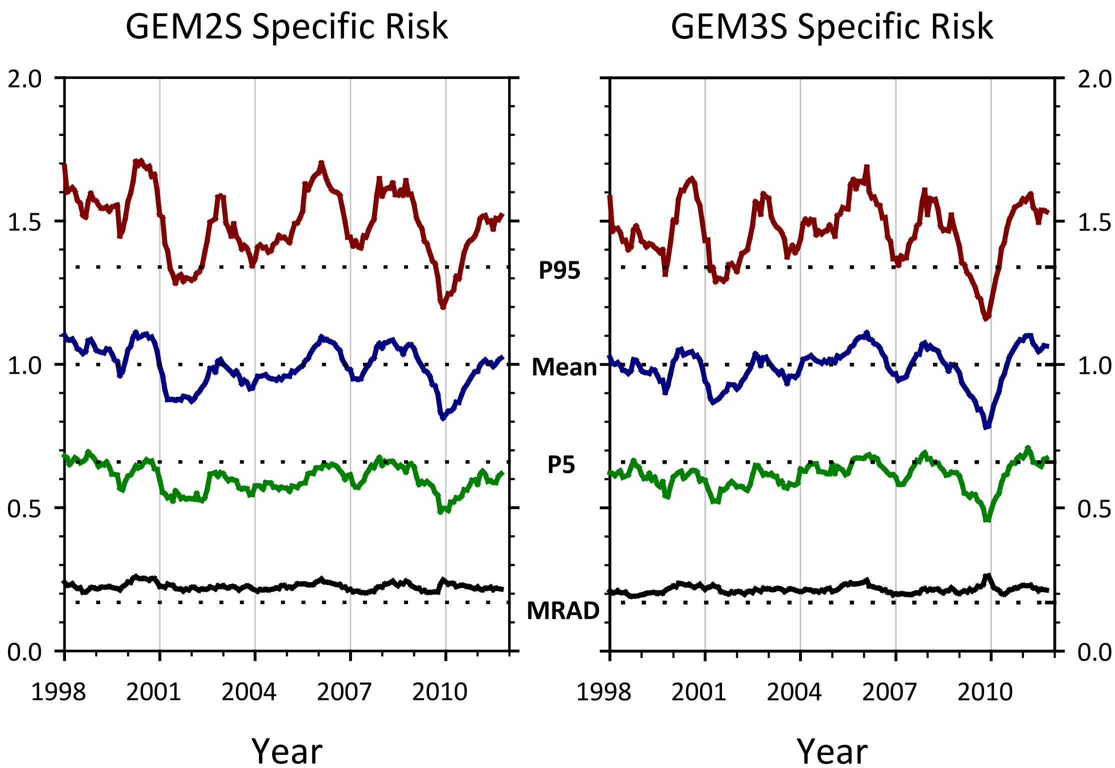
Comparison of GEM2L model and GEM3L model for optimized portfolios. The portfolios were constructed by minimizing risk subject to the unit alpha constraint, where the alpha signals were taken from the style factors of each model. Twenty sets of optimizations were performed using 500 randomly selected stocks for each style factor. Each plot reports rolling 12-month values for the mean bias statistic, the 5-percentile and 95-percentile bias statistics, and MRAD. The dashed horizontal lines indicate the ideal positions assuming normally distributed returns and perfect risk forecasts.



In **Figure 5.12** we plot cap-weighted MRAD and bias statistics for the specific returns of all stocks in the GEM3 estimation universe using the short-horizon models. Both models do an excellent job predicting specific volatility levels. Qualitatively, the main features are similar.

Figure 5.12

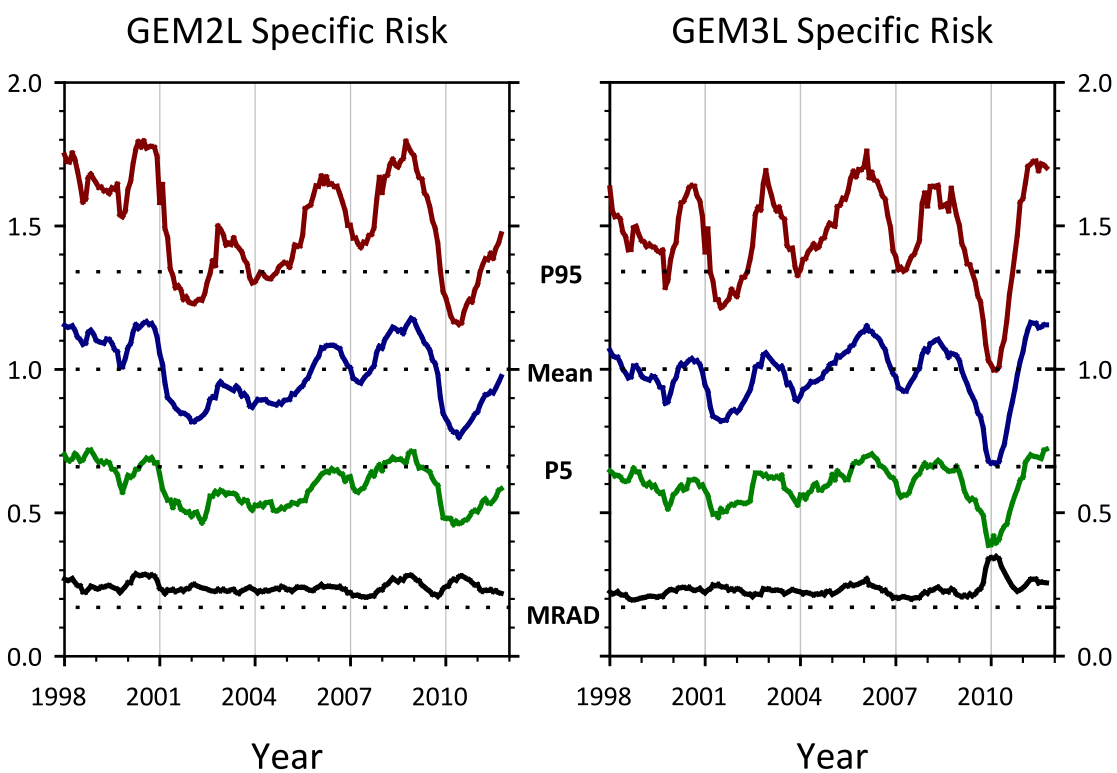
Comparison of GEM2S Model and GEM3S Model for specific risk. Results were capitalization weighted. Each plot reports rolling 12-month values for the mean bias statistic, the 5-percentile and 95-percentile bias statistics, and MRAD. The dashed horizontal lines indicate the ideal positions assuming normally distributed returns and perfect risk forecasts.



In **Figure 5.13** we plot cap-weighted MRAD and bias statistics for the same specific returns as in Figure 5.12, except now using the long-horizon models. The GEM3L Model generally has more accurate bias statistics, except immediately following the Financial Crisis, during which time GEM3L overpredicted specific risk. Again, however, the overprediction was fairly short-lived, as the Volatility Regime Adjustment produced bias statistics close to 1 by early 2011.

Figure 5.13

Comparison of GEM2L Model and GEM3L Model for specific risk. Results were capitalization weighted. Each plot reports rolling 12-month values for the mean bias statistic, the 5-percentile and 95-percentile bias statistics, and MRAD. The dashed horizontal lines indicate the ideal positions assuming normally distributed returns and perfect risk forecasts.



In **Table 5.1** we present summary MRAD and mean bias statistic numbers for the GEM2 and GEM3 Models for the test cases presented in Figures 5.2 to 5.13. For every class of portfolios, the GEM3 Model produced more accurate risk forecasts as measured by the MRAD statistic. The average outperformance by the MRAD measure was 250 bps for the short-horizon model and 283 bps for the long-horizon models. If we take 0.19 as the lower bound of MRAD for perfect risk forecasts and realistic levels of kurtosis, then the GEM3S Model represents a reduction in excess MRAD of more than 50 percent relative to the GEM2S Model.

Table 5.1

Summary of mean bias statistics and MRAD for GEM2 and GEM3 Models.

Figures	MRAD (GEM2S)	Mean B (GEM2S)	MRAD (GEM3S)	Mean B (GEM3S)	MRAD Diff (bp)	(S-Model) Portfolio Type
5.2	0.2369	1.07	0.2145	0.98	225	Pure Factors
5.4	0.1988	0.98	0.1952	0.96	36	Random Active
5.6	0.2433	1.01	0.2057	0.98	376	Factor Tilts Long
5.8	0.2486	1.07	0.2298	1.02	188	Factor Tilts Active
5.10	0.2633	1.16	0.2057	1.02	576	Optimized Styles
5.12	0.2244	0.99	0.2146	0.99	98	Specific Risk
Average	0.2359	1.05	0.2109	0.99	250	
Figures	MRAD (GEM2L)	Mean B (GEM2L)	MRAD (GEM3L)	Mean B (GEM3L)	MRAD Diff (bp)	(L-Model) Portfolio Type
5.3	0.2775	1.00	0.2584	0.94	190	Pure Factors
5.5	0.2202	0.96	0.2078	0.94	124	Random Active
5.7	0.3033	0.97	0.2541	0.95	492	Factor Tilts Long
5.9	0.2856	1.02	0.2521	0.98	335	Factor Tilts Active
5.11	0.2633	1.11	0.2165	0.99	468	Optimized Styles
5.13	0.2409	0.99	0.2318	0.98	91	Specific Risk
Average	0.2651	1.01	0.2368	0.96	283	

6. Conclusion

The new Barra Global Equity Model (GEM3) is the result of extensive research efforts in combination with client consultations. The GEM3 Model incorporates many methodological innovations and advances designed to address long-standing problems in risk modeling. For instance, the Optimization Bias Adjustment addresses the issue of underestimation of risk for optimized portfolios, and leads to better conditioning of the covariance matrix. The Volatility Regime Adjustment calibrates volatilities to current market levels and represents a key determinant of risk forecasts, especially during times of market turmoil. Another enhancement is the use of a Bayesian adjustment technique which aims to reduce biases in specific risk forecasts.

This document provides a thorough empirical analysis of the GEM3 Model. The factor structure is described in transparency and detail, for countries, industries and styles. The performance of select factors is presented and discussed. Key metrics are reported at the individual factor level, including statistical significance, performance, volatility, and correlation.

We also analyze the explanatory power of the GEM3 Model. In addition, we study the contributions to cross-sectional dispersion from the World factor, countries, industries, and styles. We find that each category of factors was of comparable importance in explaining the observed cross-sectional dispersion of equity returns.

Lastly, we systematically compare the forecasting accuracy of the GEM3S and GEM3L Models versus their GEM2 counterparts over a roughly 15-year backtesting window. We consider several classes of portfolios, including pure factors, random active portfolios, factor-tilt portfolios (both long-only and dollar-neutral), and optimized portfolios. We also compare the accuracy of specific risk forecasts between the two models. For every portfolio type considered, we find that the GEM3S and GEM3L Models provided more accurate risk forecasts than their GEM2 counterparts during the sample period.

Appendix A: Descriptors by Style Factor

Beta

Definition: *1.0 · BETA*

BETA Beta (β)

Computed as the slope coefficient in a time-series regression of excess stock return, $r_t - r_{ft}$, against the cap-weighted excess return of the estimation universe R_t,

$$r_t - r_{ft} = \alpha + \beta R_t + e_t . \tag{A1}$$

The returns are aggregated over two-day windows to reduce the effects of non-synchronicity. The regression coefficients are estimated over the trailing 252 trading days of returns with a half-life of 63 trading days.

Momentum

Definition: *0.6 · RSTR + 0.4 · HALPHA*

RSTR Relative strength

First, non-lagged relative strength for day τ is computed as the sum of excess log returns over the trailing $T = 504$ trading days,

$$RS(\tau) = \sum_{t=\tau}^{\tau+T} w_t \left[\ln(1+r_t) - \ln(1+r_{ft}) \right], \tag{A2}$$

where r_t is the stock return on day t, r_{ft} is the risk-free return, and w_t is an exponential weight with a half-life of 126 trading days.

Relative strength is computed as the equal-weighted average of non-lagged relative strength over the previous 11 to 21 trading days:

$$RSTR = \frac{1}{11} \sum_{\tau=11}^{21} RS(\tau) . \tag{A3}$$

This treatment introduces a lag while avoiding undue jumps in relative strength exposures when large returns enter the estimation window.

HALPHA Historical alpha

First, non-lagged values of historical alpha are computed by the time-series regression of Equation A1. Historical alpha is then computed as the equal-weighted average of non-lagged values over the previous 11 to 21 trading days. This treatment introduces a lag while avoiding undue jumps in historical alpha exposures when large returns enter the estimation window.

Size

Definition: *$1.0 \cdot LNCAP$*

LNCAP *Log of market cap*

Given by the logarithm of the total market capitalization of the firm.

Earnings Yield

Definition: *$0.643 \cdot EPFWD + 0.214 \cdot CETOP + 0.143 \cdot ETOP$*

EPFWD *Predicted earnings-to-price ratio*

Given by the 12-month forward-looking earnings divided by the current market capitalization. Forward-looking earnings are defined as a weighted average between the average analyst-predicted earnings for the current and next fiscal years.

CETOP *Cash earnings-to-price ratio*

Given by the trailing 12-month cash earnings divided by current price.

ETOP *Trailing earnings-to-price ratio*

Given by the trailing 12-month earnings divided by the current market capitalization. Trailing earnings are defined as the last reported fiscal-year earnings plus the difference between current interim figure and the comparative interim figure from the previous year.

Residual Volatility

Definition: *0.60 · DASTD + 0.30 · CMRA + 0.10 · HSIGMA*

DASTD *Daily standard deviation*

Computed as the volatility of daily excess returns over the past 252 trading days with a half-life of 42 trading days.

CMRA *Cumulative range*

This descriptor differentiates stocks that have experienced wide swings over the last 12 months from those that have traded within a narrow range. Let $Z(T)$ be the cumulative excess log return over the past T months, with each month defined as the previous 21 trading days

$$Z(T) = \sum_{\tau=1}^{T} \left[\ln(1+r_\tau) - \ln(1+r_{f\tau}) \right], \tag{A4}$$

where r_τ is the stock return for month τ (compounded over 21 days), and $r_{f\tau}$ is the risk-free return. The cumulative range is given by

$$CMRA = \ln(1+Z_{\max}) - \ln(1+Z_{\min}), \tag{A5}$$

where $Z_{\max} = \max\{Z(T)\}$, $Z_{\min} = \min\{Z(T)\}$, and $T = 1,\ldots,12$.

HSIGMA *Historical sigma* (σ)

Computed as the volatility of residual returns in Equation A1,

$$\sigma = \text{std}(e_t). \tag{A6}$$

The volatility is estimated over the trailing 252 trading days of returns with a half-life of 63 trading days.

Note: The Residual Volatility factor is orthogonalized to Beta to reduce collinearity.

Growth

Definition: *0.70 · EGRLF + 0.15 · EGRO + 0.15 · SGRO*

EGRLF *Long-term predicted earnings growth*

Long-term (3-5 years) earnings growth forecasted by analysts.

EGRO *Earnings growth (trailing five years)*

Annual reported earnings per share are regressed against time over the past five fiscal years. The slope coefficient is then divided by the average annual earnings per share to obtain the earnings growth.

SGRO *Sales growth (trailing five years)*

Annual reported sales per share are regressed against time over the past five fiscal years. The slope coefficient is then divided by the average annual sales per share to obtain the sales growth.

Dividend Yield

Definition: *1.0 · YILD*

YILD *Dividend-to-price ratio*

Given as the trailing 12-month dividend per share divided by current price.

Book-to-Price

Definition: *1.0 · BTOP*

BTOP *Book-to-price ratio*

Last reported book value of common equity divided by current market capitalization.

Leverage

Definition: *0.50 · MLEV + 0.10 · DTOA + 0.40 · BLEV*

MLEV *Market leverage*

Computed as

$$MLEV = \frac{ME + PE + LD}{ME}, \tag{A7}$$

where ME is the market value of common equity on the last trading day, PE is the most recent book value of preferred equity, and LD is the most recent book value of long-term debt.

DTOA *Debt-to-assets*

Computed as

$$DTOA = \frac{TD}{TA}, \tag{A8}$$

where TD is the book value of total debt (long-term debt and current liabilities), and TA is most recent book value of total assets.

BLEV *Book leverage*

Computed as

$$BLEV = \frac{BE + PE + LD}{BE}, \tag{A9}$$

where BE is the most recent book value of common equity, PE is the most recent book value of preferred equity, and LD is the most recent book value of long-term debt.

Liquidity

Definition: *0.20 · STOM + 0.35 · STOQ + 0.45 · STOA*

STOM <u>*Share turnover, one month*</u>

Computed as the log of the share turnover over the previous month,

$$STOM = \ln\left(\frac{V}{S}\right), \tag{A10}$$

where V is the trading volume for the month, and S is the number of shares outstanding.

STOQ <u>*Average share turnover, trailing 3 months*</u>

Let $STOM_\tau$ be the share turnover for month τ. The quarterly share turnover is defined by

$$STOQ = \ln\left[\frac{1}{T}\sum_{\tau=1}^{T}\exp\left(STOM_\tau\right)\right], \tag{A11}$$

where $T = 3$ months.

STOA <u>*Average share turnover, trailing 12 months*</u>

Let $STOM_\tau$ be the share turnover for month τ. The annual share turnover is defined by

$$STOA = \ln\left[\frac{1}{T}\sum_{\tau=1}^{T}\exp\left(STOM_\tau\right)\right], \tag{A12}$$

where $T = 12$ months.

Non-linear Size

Definition: *1.0 · NLSIZE*

NLSIZE <u>*Cube of Size*</u>

First, the standardized Size exposure (i.e., log of market cap) is cubed. The resulting factor is then orthogonalized to the Size factor on a regression-weighted basis. Finally, the factor is winsorized and standardized.

Appendix B: Decomposing RMS Returns

We decompose excess stock returns r_n into a systematic component, due to factors, and a stock-specific component u_n. The factor returns f_k are estimated each period by cross-sectional regression

$$r_n = \sum_k X_{nk} f_k + u_n \ , \tag{B1}$$

where X_{nk} is the exposure of stock n to factor k. The specific returns are assumed to be uncorrelated with one another as well as to the other factors.

The total *R*-squared of a regression measures the cross-sectional variation explained by the factors,

$$R_T^2 = 1 - \frac{\sum_n v_n u_n^2}{\sum_n v_n r_n^2}, \tag{B2}$$

where v_n is the regression weight of stock n (proportional to square-root of market capitalization). The root mean square (RMS) return, computed as

$$RMS = \sqrt{\sum_n v_n r_n^2}, \tag{B3}$$

measures the cross-sectional dispersion from zero return. As described by Menchero and Morozov (2011), the RMS return can be exactly decomposed into the return sources of Equation B1 using a cross-sectional version of the *x-sigma-rho* formula,

$$RMS = \sum_k f_k \, \sigma(X_k) \rho(X_k, r) + \sigma(u) \rho(u, r) \ , \tag{B4}$$

where $\sigma(X_k)$ is the RMS dispersion of factor k, and $\rho(X_k, r)$ is the cross-sectional correlation between factor k and the asset returns. The last term in Equation B4 represents the contribution to RMS coming from stock-specific sources.

Appendix C: Review of Bias Statistics

A commonly used measure to assess a risk model's accuracy is the bias statistic. Conceptually, the bias statistic represents the ratio of realized risk to forecast risk.

Let R_{nt} be the return to portfolio n over period t, and let σ_{nt} be the beginning-of-period volatility forecast. Assuming perfect forecasts, the *standardized* return,

$$b_{nt} = \frac{R_{nt}}{\sigma_{nt}}, \tag{C1}$$

has an expected standard deviation of 1. The bias statistic for portfolio n is the *realized* standard deviation of standardized returns,

$$B_n = \sqrt{\frac{1}{T-1}\sum_{t=1}^{T}\left(b_{nt} - \bar{b}_n\right)^2}, \tag{C2}$$

where T is the number of periods in the observation window.

Assuming normally distributed returns and perfect risk forecasts, for sufficiently large T the bias statistic B_n is approximately normally distributed about 1, and roughly 95 percent of the observations fall within the confidence interval,

$$B_n \in \left[1-\sqrt{2/T}, 1+\sqrt{2/T}\right]. \tag{C3}$$

If B_n falls outside this interval, we reject the null hypothesis that the risk forecast was accurate.

If returns are not normally distributed, however, then fewer than 95 percent of the observations will fall within the confidence interval, even for perfect risk forecasts. In **Figure C1**, we show simulated results for the percentage of observations actually falling within this interval, plotted versus observation window length T, for several values of kurtosis k.

For the normal case (kurtosis $k=3$), except for the smallest values of T, the confidence interval indeed captures about 95 percent of the observations. As the kurtosis increases, however, the percentage falling within the interval drops significantly. For instance, at a kurtosis level of 5, only 86 percent of bias statistics fall inside the confidence interval for an observation window of 120 periods.

C2. Rolling-Window Bias Statistics

The purpose of bias-statistic testing is to assess the accuracy of risk forecasts, typically over a long sample period. Let T be the length of the observation window, which corresponds to the number of months in the sample period. One possibility is to select the entire sample period as a single window, and to compute the bias statistic as in Equation C2. This would be a good approach if financial data were stationary, as sampling error is reduced by increasing the length of the window. In reality, however, financial data are not stationary. It is possible to significantly overpredict risk for some years, and underpredict it for others, while ending up with a bias statistic close to 1.

Often, a more relevant question is to study the accuracy of risk forecasts over 12-month periods. For this purpose, we define the rolling 12-month bias statistic for portfolio n,

$$B_n^\tau = \sqrt{\frac{1}{11}\sum_{t=\tau}^{\tau+11}(b_{nt} - \bar{b}_n)^2}\ , \tag{C4}$$

Where τ denotes the first month of the 12-month window. The 12-month windows are rolled forward one month at a time until reaching the end of the observation window. If T is the number of periods in the observation window, then each portfolio will have $T-11$ (overlapping) 12-month windows.

It is useful to consider, for a collection of N portfolios, the mean of the rolling 12-month bias statistics,

$$\bar{B}^\tau = \frac{1}{N}\sum_n B_n^\tau\ . \tag{C5}$$

We also define $B^\tau(5\%)$ and $B^\tau(95\%)$ to be the 5-percentile and 95-percentile values for the rolling 12-month bias statistics at a given point in time. Assuming normal distributions and perfect risk forecasts, these values should be centered about 0.66 and 1.34, respectively. Plotting these quantities versus time for different classes of portfolios is a visually powerful way of understanding the predictive accuracy of the risk model.

Another useful measure to consider is the 12-month *mean rolling absolute deviation* (MRAD), defined as

$$\mathrm{MRAD}^\tau = \frac{1}{N}\sum_n \left| B_n^\tau - 1 \right|. \tag{C6}$$

This penalizes every deviation away from the ideal bias statistic of 1. Smaller MRAD numbers, of course, are preferable to larger ones. A lower limit for this statistic can be obtained by assuming the ideal case of normally distributed returns and perfect risk forecasts, which leads to an expected value of 0.17 for the 12-month MRAD.

It is interesting to consider how MRAD depends on kurtosis levels. In **Figure C2** we report simulated results for 12-month MRAD assuming perfect risk forecasts. For normally distributed returns, as discussed, the expected MRAD value is 0.17. At higher kurtosis levels, however, the expected MRAD for perfect forecasts increases significantly. For instance, even at moderate kurtosis levels in the range of 3.5 to 4.0, the 12-month MRAD for perfect risk forecasts rises to approximately 0.19.

Figure C1

Percent of observations falling within the confidence interval $1 \pm \sqrt{2/T}$, where T is the number of periods in the observation window. Results were simulated using a normal distribution $k = 3$, and using a t-distribution with kurtosis values $k = 5$ and $k = 10$. The standard deviations were equal to 1 in all cases.

For the normal distribution, the percentage of observations inside the confidence interval quickly approaches 95 percent. As kurtosis is increased, however, the proportion within the confidence interval declines considerably.

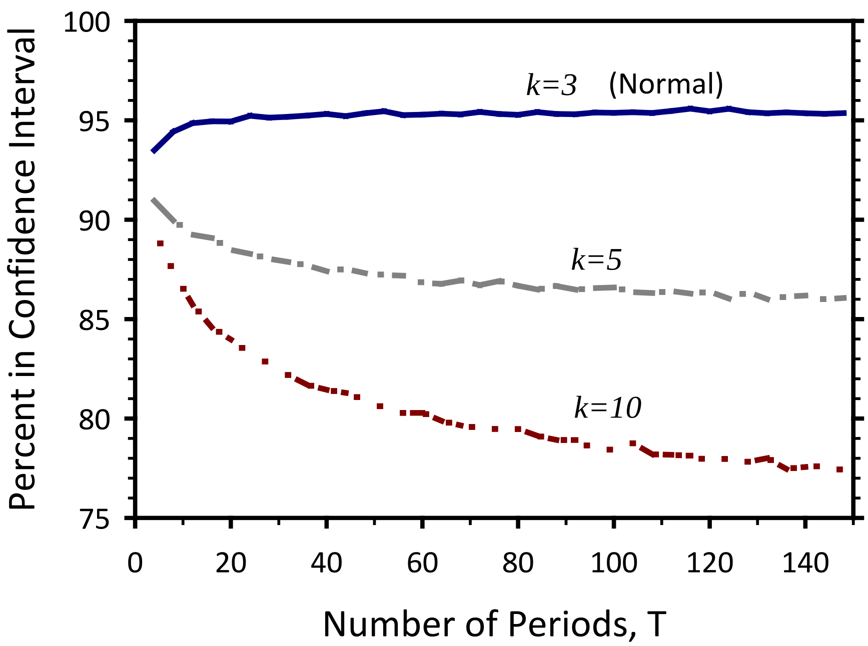
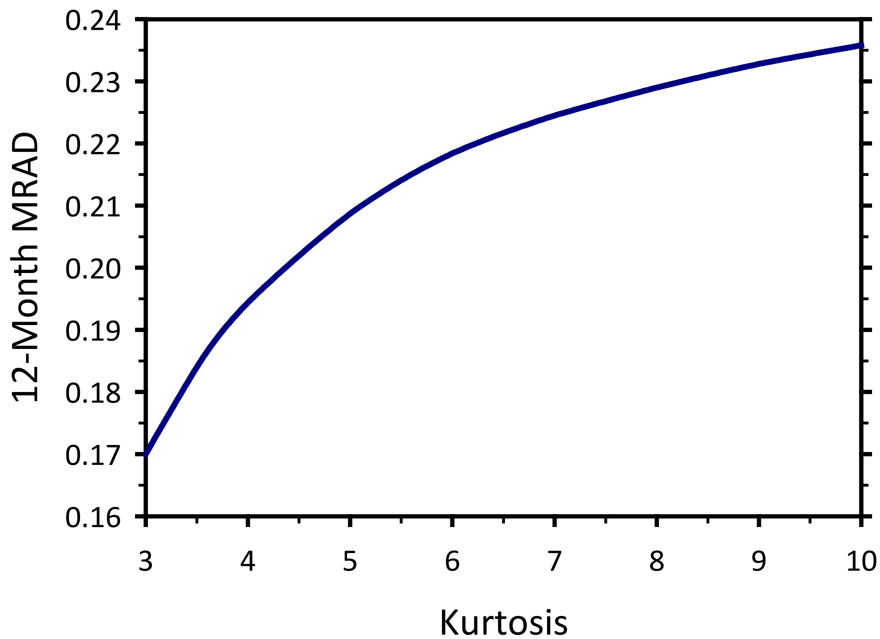


Figure C2

Plot of 12-month MRAD versus kurtosis levels for perfect risk forecasts. Results were simulated using a *t*-distribution.



Appendix D: Model Estimation Parameters

Table D1 contains factor covariance matrix estimation parameters for the GEM3 Model. Parameters are described in *USE4 Methodology Notes*, by Menchero, Orr, and Wang (2011).

Table D1

Factor covariance matrix parameters for the GEM3 model. All values are reported in trading days.

Model	Factor Volatility Half-Life	Newey-West Volatility Lags	Factor Correlation Half-Life	Newey-West Correlation Lags	Factor CSV Half-Life
GEM3S	84	10	504	3	42
GEM3L	252	10	504	3	168

Table D2 contains specific risk model estimation parameters for the GEM3 Model. The parameters are described in *USE4 Methodology Notes*.

Table D2

Specific risk parameters for the GEM3 model. Except for the dimensionless shrinkage parameter q, all values are reported in trading days.

Model	Specific Volatility Half-Life	Newey-West Auto-Corr. Lags	Newey-West Auto-Corr. Half-Life	Bayesian Shrinkage Parameter q	Specific CSV Half-Life
GEM3S	84	5	252	0.15	42
GEM3L	252	5	252	0.15	168

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[1] *As of June 30, 2011, based on eVestment, Lipper and Bloomberg data.*