FAJ
Financial Analysts Journal
Volume 67 • Number 5
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# Decomposing Global Equity Cross-Sectional Volatility

Jose Menchero, CFA, and Andrei Morozov

*The authors present an exact methodology for decomposing cross-sectional volatility into contributions from various factors. Treating country, industry, and style factors equally, they used their framework to investigate several relevant issues in the global equity markets, including the importance of country versus industry, emerging markets versus developed markets, and the strength of style factors vis-à-vis country and industry factors.*

Cross-sectional volatility (CSV) is the standard deviation of a set of asset returns over a single period. This dispersion measure is fundamentally important because it represents the opportunity set for active portfolio management—that is, if the dispersion of stock returns is very small, then all stocks behave similarly and there is little opportunity to outperform the market. Conversely, when CSV levels are high, performance differences among active managers are more pronounced, as documented by Ankrim and Ding (2002).

Another important characteristic of CSV is that it uses only the most recent information, unlike time-series volatility. This characteristic makes CSV analysis a powerful tool for identifying the latest trends in the global equity markets. For instance, Solnik and Roulet (2000) showed that under a set of simplifying assumptions, cross-sectional data could be used to provide an "instantaneous" measure of realized correlation.

Understanding the drivers of CSV can also provide timely and valuable information for portfolio managers. For example, a sudden increase in a particular factor group's CSV may signal a new trend or structural shift in the equity markets. Of course, active managers need to stay abreast of such trends and how they might affect their portfolios. Similarly, CSV analysis can yield new insights into the relative importance of industry versus country—a question of fundamental relevance to global equity portfolio managers. On the one hand, if country effects dominate, primacy should be given to the country allocation decision. On the other hand, if global economic integration is lessening the distinctions between countries, an industry-first investment process is more appropriate.

The industry-versus-country debate has received much attention over the years. In an early study, Grinold, Rudd, and Stefek (1989) used a global multifactor model to show that countries had greater explanatory power than industries over 1983–1988. One shortcoming of their approach, however, is that the effect of market beta is combined with the country factor rather than attributed to a style factor, which makes country effects appear stronger at the expense of style factors.

Heston and Rouwenhorst (1995) also studied the question of industries versus countries. They constructed a factor model for developed Europe containing 7 industries and 12 countries and found that country factors were far more volatile than their industry counterparts over 1978–1992. In a subsequent study, Cavaglia, Brightman, and Aked (2000) investigated a broader universe comprising 21 developed markets and 36 industries and found that industry effects had overtaken country effects in the late 1990s. A basic limitation of both studies, however, is that style factors were omitted from the analysis.

More recently, Puchkov, Stefek, and Davis (2005) used a two-tiered factor model to study the relative importance of country, global industry, global style, and purely local factors. They found that country factors nearly always dominated global industry factors in developed markets over 1992–2004. The exception was the subperiod 1999–2002, when the two effects were comparable. Under their approach, global style factors were found to be far weaker than either country or global industry factors. A shortcoming of their approach, however, is that it gives preferential treatment to country

*Jose Menchero, CFA, is executive director and Andrei Morozov is vice president of MSCI, Berkeley, California.*

factors at the expense of industry and style factors, which we discuss later in the article.

In our study, we devised a new framework for both analyzing cross-sectional volatility and investigating the relative importance of country, industry, and style factors. Our methodology provides an exact decomposition of CSV into contributions from various factors. These contributions can then be cleanly aggregated to answer questions regarding the net strength of entire categories of factors. Furthermore, our approach treats all factors (country, industry, and style) equally and thus represents a powerful new tool for investigating the relative importance of these drivers of global equity returns.

## Attribution of Time-Series Volatility (*x*-sigma-rho)

Before we present our decomposition of cross-sectional volatility, a brief discussion of the attribution of *time-series* volatility would be useful. In general, a portfolio's return over period $t$ can be attributed to a set of sources ($m$):

$$R_t = \sum_m x_m g_{mt}, \tag{1}$$

where $x_m$ is the portfolio's exposure to the source and $g_{mt}$ is the return to the source. As shown by Menchero and Davis (2011), the time-series volatility of the portfolio $\sigma(R)$ can be exactly attributed to the same sources via the $x$-sigma-rho formula:

$$\sigma(R) = \sum_m x_m \sigma(g_m) \rho(g_m, R), \tag{2}$$

where $\sigma(g_m)$ is the time-series volatility of source $m$ and $\rho(g_m, R)$ is the time-series correlation between source $m$ and the portfolio return. This intuitive formula states that there are three drivers of portfolio volatility: (1) the size of the position ($x$), (2) the stand-alone volatility of the source (sigma), and (3) the correlation between the source and the overall portfolio (rho). This risk attribution framework has also been generalized to the case of extreme risk (see Goldberg, Hayes, Menchero, and Mitra 2010).

## Attribution of Cross-Sectional Volatility (*x*-sigma-rho)

The $x$-sigma-rho formula can be further extended to decompose cross-sectional volatility. Suppose that the return to stock $n$ is expressed according to a multifactor model:

$$r_n = \sum_k f_k X_{nk} + u_n, \tag{3}$$

where $f_k$ is the return to factor $k$, $X_{nk}$ is the stock's exposure to the factor, and $u_n$ is the idiosyncratic return unexplained by the factors. CSV is defined as

$$\sigma(r) = \sqrt{\sum_n w_n (r_n - \bar{r})^2}, \tag{4}$$

where $w_n$ is the weight of stock $n$ and $\bar{r}$ is the mean return to the estimation universe. As shown in Appendix A, application of the $x$-sigma-rho formula yields

$$\sigma(r) = \sum_k f_k \, \sigma(X_k) \rho(X_k, r) + \sigma(u) \rho(u, r). \tag{5}$$

Equation 5 states the three drivers of CSV arising from factors: (1) the magnitude of the factor return, $f_k$; (2) the cross-sectional volatility of the factor exposures, $\sigma(X_k)$; and (3) the cross-sectional correlation between the factor exposures and the stock returns, $\rho(X_k, r)$. Note that in going from the time-series view to the cross-sectional perspective, the roles of factor exposures and factor returns are effectively interchanged.

Equation 5 also states the two drivers of CSV arising from the idiosyncratic component. The first driver is the CSV of the specific returns, $\sigma(u)$; the second driver is the cross-sectional correlation between the specific returns and the stock returns, $\rho(u, r)$.

## Global Factor Model

To investigate the relative importance of country, industry, and style factors, we constructed a global factor model that we labeled the base model to distinguish it from two alternative models that we discuss later in the article. Our base model contained 1 world factor, 48 country factors spanning both developed markets and emerging markets, 24 industry factors based on the Global Industry Classification Standard (GICS), and 8 style factors. We derived our base model from the Barra Global Equity Model (GEM2), as described by Menchero, Morozov, and Shepard (2010), but with several notable modifications.

First, to avoid introducing biases in favor of either country or industry factors, we standardized all style factors on a global basis. In contrast, for purposes of maximizing the explanatory power of the model, GEM2 standardizes some style factors on a global basis and others on a country basis. Second, we used 24 industry factors in our base model versus the 34 industry factors in GEM2. For our purposes, we adhered to a single level of the GICS hierarchy, with slightly fewer industries than countries. In contrast, the 34 GEM2 industry factors were designed to maximize the explanatory power of the model within a relatively parsimonious factor structure. Although using the GEM2 industry factors would obviously give added importance to the industry factors, doing so would not alter the main conclusions of our study. Finally, we used full-market-capitalization regression weighting versus the square-root market-cap weighting in GEM2. Square-root weighting is more appropriate for forecasting risk, whereas full-cap weighting is more appropriate for active managers seeking investment opportunities.

The stock returns within our global factor model are explained by

$$r_n = f_w + f_{c(n)} + f_{i(n)} + \sum_s X_{ns} f_s + u_n, \tag{6}$$

where

$f_w$ = the return to the world factor

$f_{c(n)}, f_{i(n)}$ = the country and industry factor returns to stock $n$

$X_{ns}$ = the stock exposure to style factor $s$

$f_s$ = the return to the style factor

$u_n$ = the specific return to the stock

We obtained our data on stock returns from MSCI. We determined industry exposures by official GICS membership and country exposures by membership in the MSCI All Country World Investable Market Index (ACWI IMI), a broad index that aims to reflect the full breadth of investment opportunities in both developed and emerging markets.

We took our eight style factors from GEM2: volatility, momentum, size, value, growth, leverage, liquidity, and nonlinear size. The volatility factor essentially represents market beta; momentum is the relative performance over the last 6–12 months; size is the log of market capitalization; value is determined by a combination of book-to-price, earnings-to-price, and dividend-to-price ratios; growth is defined primarily by analyst forecasts of long-term earnings growth; leverage is given by such financial leverage ratios as debt-to-assets; liquidity represents share turnover during the last year; nonlinear size captures return differences between mid-cap stocks and large-cap/small-cap stocks. (These factors are described in greater detail in Menchero, Morozov, and Shepard 2010.) We standardized our style factors to have a cap-weighted mean of zero and a standard deviation of 1 over the estimation universe (the MSCI ACWI IMI).

We performed monthly cross-sectional regressions over January 1993–April 2010. Note that the regression in Equation 6 contains two exact collinearities—namely, both the sum of country factor exposures and the sum of industry factor exposures replicate the world factor exposure. Therefore, two constraints must be applied to obtain a unique solution. Following the approach of Heston and Rouwenhorst (1995), we set the sum of cap-weighted country and industry factor returns to zero for every period.

Under this model specification, the pure world factor portfolio is represented by the cap-weighted estimation universe. The pure country factor portfolios are 100 percent long the particular country and 100 percent short the world portfolio and have zero exposure to every industry and style factor. Similarly, pure industry factor portfolios are 100 percent long the particular industry and 100 percent short the world portfolio and have zero exposure to every country and style factor. Finally, pure style factors have unit exposure to the particular style and zero exposure to every other style factor, as well as every country and industry factor. This regression approach cleanly disentangles the effects of multicollinearity and captures the pure effect for every factor. For a more in-depth discussion of factor portfolios, see Menchero (2010).

Note that the CSV contributions are independent of any scaling conventions in our model specification. For instance, arbitrarily doubling all the exposures for a particular factor would merely cause the factor return to decrease by half but would leave the CSV contribution unchanged.

## CSV Empirical Results

**Figure 1** reports the cap-weighted trailing 12-month total CSV over January 1994–April 2010, as well as the contributions from factors and stock-specific sources. Note that average CSV levels vary widely over time, ranging from a peak of 14 percent in the wake of the internet bubble to a low of less than 7 percent during the 2005–07 bull market.

Over the sample period, most of the CSV is explained by the idiosyncratic component, reflecting the high "noise" level characteristic of equity returns. The ratio of the CSV's factor contribution to total CSV gives the relative $R^2$ of the model:

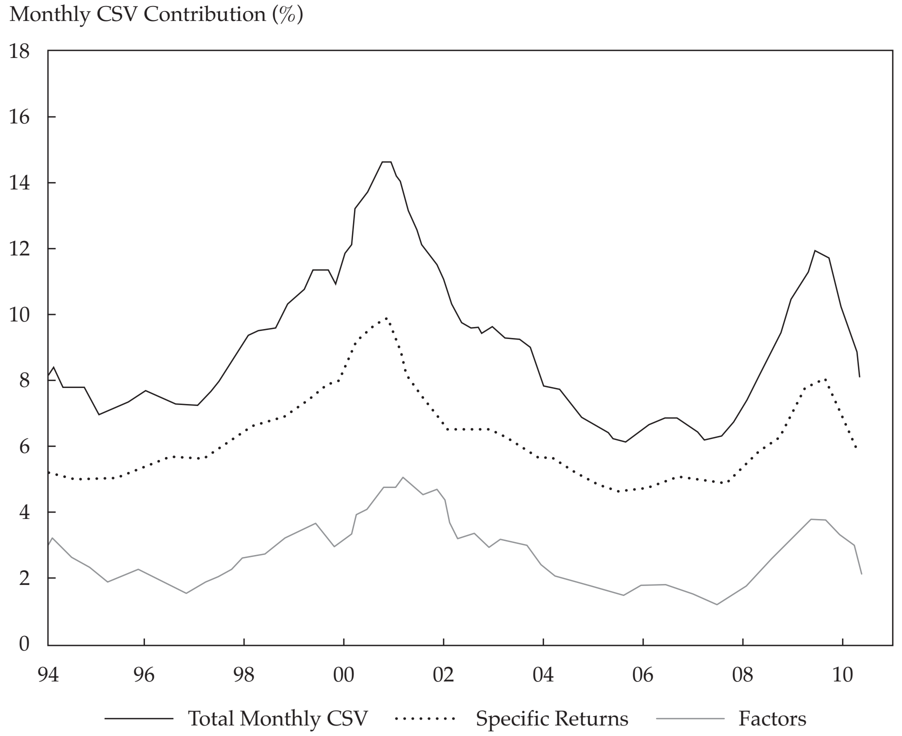
$$R_R^2 = 1 - \frac{\sum w_n u_n^2}{\sum w_n \left(r_n - \bar{r}\right)^2}, \tag{7}$$

where $\bar{r}$ is the mean return for the estimation universe. For example, Figure 1 shows that in January 1994, the explained CSV was about 3 percent, with a total CSV of roughly 8 percent. The relative $R^2$, therefore, was approximately 38 percent.

**Figure 2** reports the contributions to CSV by factor type. In the mid-1990s, country factors strongly dominated industry factors. Beginning in

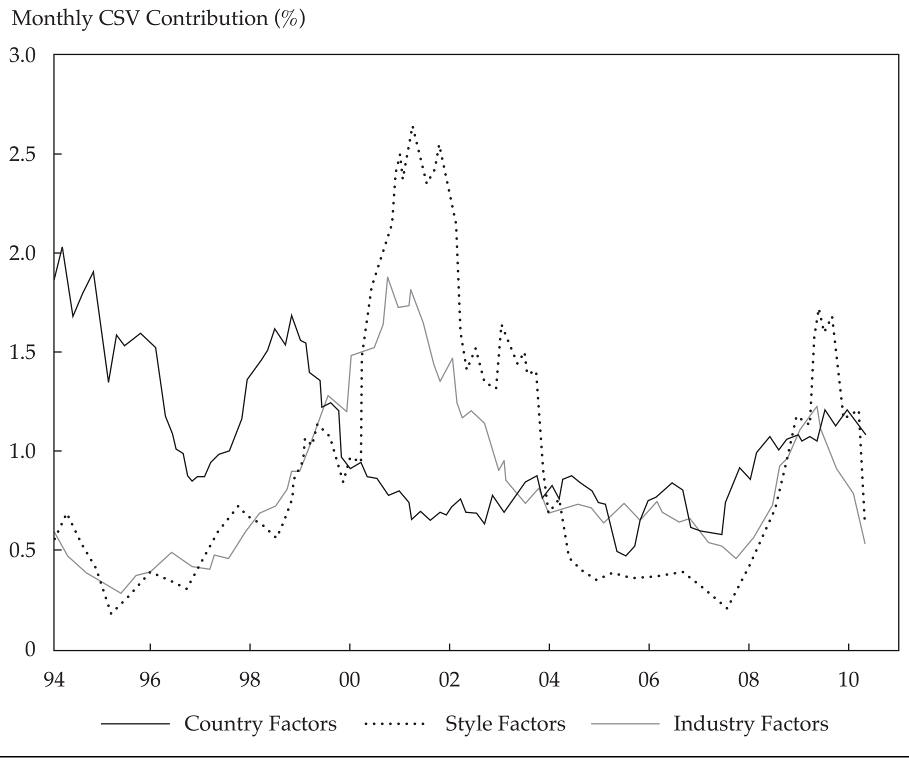
 

**Figure 1. Total CSV and Contributions from Factors and Specific Returns, January 1994–April 2010**




*Note:* Lines were smoothed by using 12-month moving averages.

**Figure 2. Contributions to CSV by Factor Type, January 1994–April 2010**




*Note:* See note to Figure 1.

1999, however, industries overtook countries in importance, consistent with the findings of Cavaglia, Brightman, and Aked (2000). Industries continued to dominate countries for several years, until about 2003. Although industries and countries were of comparable strength from 2003 to 2007, countries have mostly dominated industries since 2007.

Perhaps the most striking feature of Figure 2 is the contribution of style factors to CSV. The magnitude varies dramatically over time, ranging from lows of about 20 bps in 1995 and 2007 to highs well above 250 bps in 2001. In fact, over 2000–2004, style factors were the largest contributor to CSV, dominating both countries and industries. In contrast, over 2004–2007, styles were the weakest contributor, although they reasserted themselves as the financial crisis unfolded in 2008 and 2009.

A commonly held—but erroneous—view is that style factors are typically less important than either industry or country factors. The basis for this misperception is the empirical observation that style factors are usually less volatile than country or industry factors. To understand why style effects can be so strong despite their low volatility, consider the following observations. If we assume that collinearity among the factors is negligible, Equation 5 can be simplified to give

$$\sigma(r) \approx \frac{1}{\sigma(r)}\left[\sum_k f_k^2 \sigma^2(X_k) + \sigma^2(u)\right]. \tag{8}$$

This approximation says that an individual factor's contribution to CSV is proportional to the *product* of the squared factor return and the cross-sectional variance of factor exposures. Although style factors generally have smaller factor returns than country or industry factors, their cross-sectional variance is much larger.

More specifically, if $W_k$ is the weight of assets with exposure to country or industry factor $k$, the cross-sectional variance of factor exposures is given by

$$\sigma^2(X_k) = W_k - W_k^2. \tag{9}$$

A fairly large country or industry factor may have a weight of 10 percent, which leads to a cross-sectional variance of 0.09. A *typical* country factor, however, may have a weight of only 2 percent, which leads to a cross-sectional variance of about 0.02. In contrast, style factors have a cross-sectional variance of 1.0 by construction, which is more than 10 times that of a large country or industry factor and perhaps 50 times that of a typical one. Therefore, the large cross-sectional variance of style factors may more than compensate for their smaller returns.

The empirical results in Figure 2 (i.e., style factors sometimes dominate country and industry factors) contrast with those of Puchkov, Stefek, and Davis (2005), who found that global style factors are always weaker than country and industry factors. The main reason for this discrepancy is likely that our methodology estimates factor returns *directly* in a one-step regression that treats all factors equally, whereas the methodology of Puchkov et al. uses an indirect two-step procedure that effectively transfers explanatory power to country factors. More specifically, in the latter approach, local factor returns for industry and style factors are first computed within each country. In the second step, these local factor returns are used to estimate global factor returns for country, industry, and style factors. In particular, global style factor returns are computed as a weighted average of the local style factor returns, which serves to mask the strength of the global style factors while enhancing the strength of the country factors—that is, because the global style factors do not explain the portion of country return attributable to a style tilt, this component is picked up by the country factor instead.

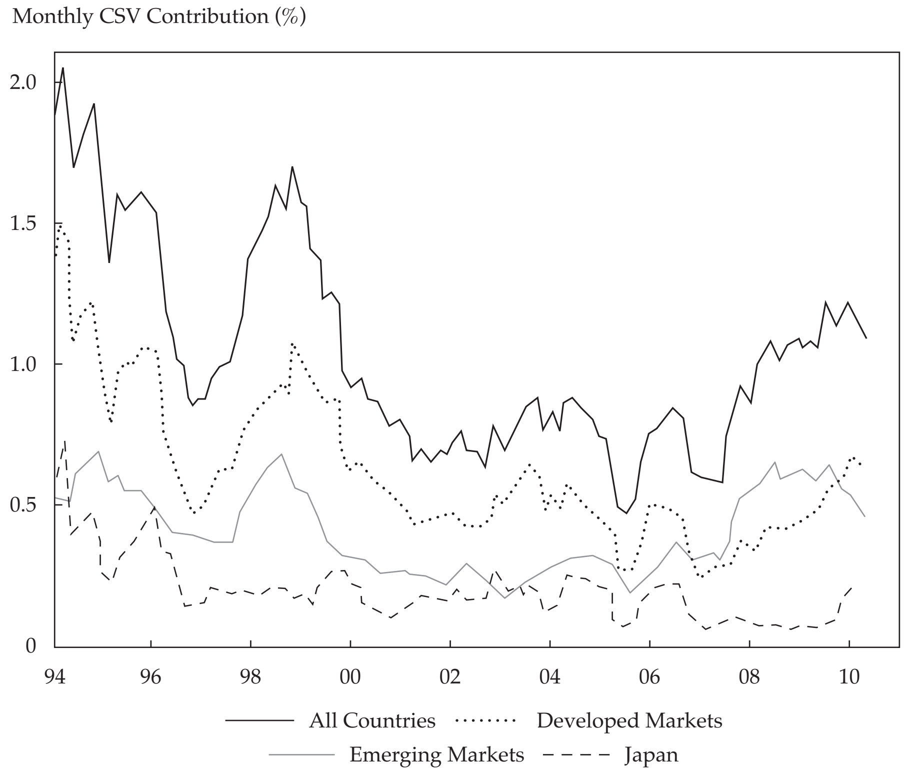
For international investors, another major question is the relative importance of developed markets versus emerging markets. Using our framework, we investigated this question by aggregating CSV contributions within each group of factors. **Figure 3** plots the total CSV contribution from countries and the contributions from developed markets and emerging markets. In the 1990s, developed markets consistently dominated emerging markets. Since 2005, however, emerging markets have increased in importance to be roughly on a par with developed markets. In fact, during the financial crisis of 2008, emerging markets dominated developed markets by a sizable margin.

One can also drill into each category of factors. Illustrating this point, Figure 3 also plots the CSV contribution from Japan, a particularly strong developed-market factor. Early on, Japan contributed nearly half the CSV attributed to developed-market factors. Since 1997, this proportion has decreased, although Japan has remained a major contributor to CSV.

**Figure 4** plots the total CSV contribution from industries, as well as the individual contributions from the semiconductor and energy industries. It clearly illustrates the strong buildup of the semiconductor factor in the late 1990s. In the aftermath of the internet bubble, the semiconductor factor reached a peak of nearly 30 bps, but it has since

**Figure 3. CSV Contributions from All Countries, Developed and Emerging Markets, and Japan, January 1994–April 2010**




*Note:* See note to Figure 1.

become considerably weaker. In contrast, the energy factor overtook semiconductors in 2005 and has strongly dominated ever since. In recent years, the energy factor has been one of the main industry drivers of CSV.

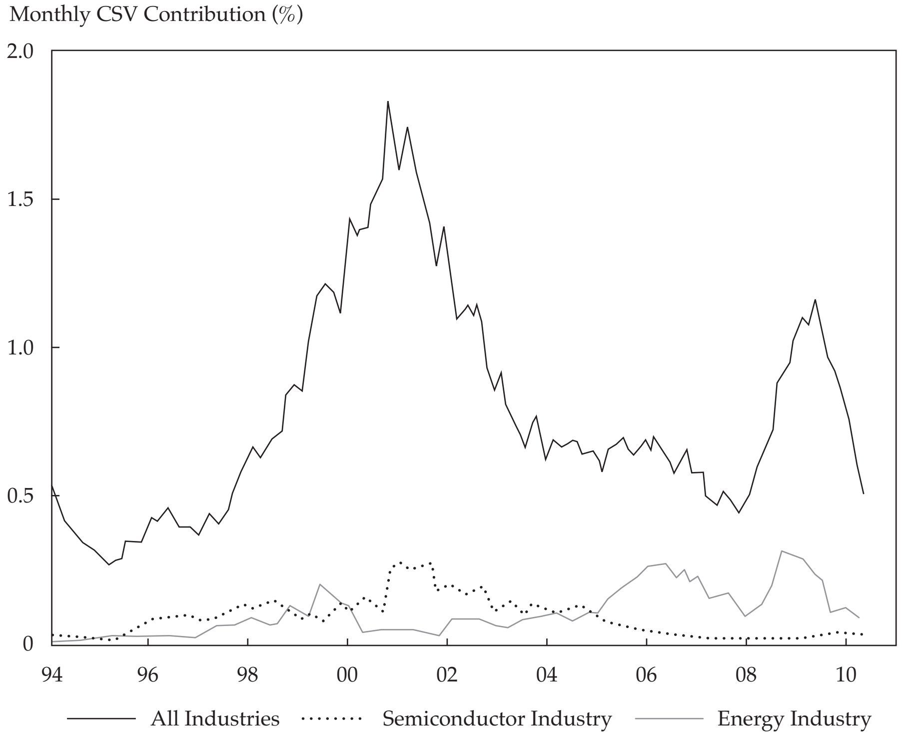
**Figure 5** presents the total CSV contribution from style factors, as well as the individual contributions from the volatility and momentum factors. For most periods—but especially during market turbulence—the volatility factor was the main contributor to CSV among the style factors. The few brief periods when the momentum factor was stronger than the volatility factor tended to be periods of relative calm.

An attractive feature of our methodology is that it allows for a direct apples-to-apples comparison of the strengths of individual country, industry, and style factors. For instance, as seen in Figure 5, the volatility factor made a maximum CSV contribution of about 170 bps, whereas momentum made a peak contribution of roughly 70 bps. In contrast, energy—a strong industry factor—rarely exceeded 30 bps. Japan, a particularly strong country factor, made a maximum CSV contribution of about 70 bps but has contributed in the range of 10–30 bps since 1997.

One might well question the robustness of our results with respect to factor specification. For example, why use 24 GICS industry groups rather than the 68 classifications at the industry level? Why use eight style factors rather than a more parsimonious set? Would our results change materially under different model specifications?

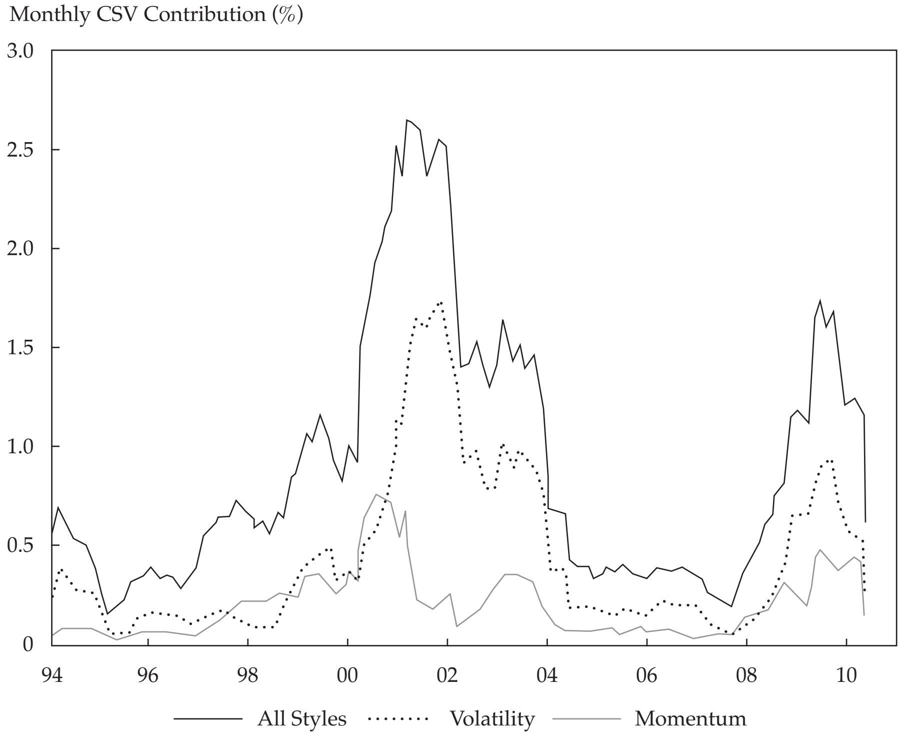
To address these questions, we constructed two alternative models and compared them with the base model that we used to generate our results thus far. The first alternative model that we constructed was the extended industries model, which was identical to the base model except that it contained 68 GICS industries rather than 24 industry groups. The second alternative model that we constructed was the restricted styles model, which was identical to the base model except that it included only four styles: volatility, momentum, size, and value. These factors essentially correspond to the four-factor model of Carhart (1997). The estimation universe and the regression-weighting scheme for the alternative models were identical to those for the base model.

**Figure 4. CSV Contributions from All Industries and the Semiconductor and Energy Industries, January 1994–April 2010**




*Note:* See note to Figure 1.

**Figure 5. CSV Contributions from All Style Factors and the Volatility and Momentum Factors, January 1994–April 2010**




*Note:* See note to Figure 1.

**Table 1** reports the percentage CSV contributions for the three models over January 1994–April 2010. Averaged over this analysis period, the total monthly CSV was 8.97 percent. In the base model, factors accounted for about 30 percent of the total CSV, which decomposes almost evenly into countries (10.94 percent), industries (9.10 percent), and styles (9.94 percent). In the extended industries model, the explanatory power of the factors increases to 32.91 percent of total CSV. Naturally, the contribution of industries is enhanced (12.39 percent), whereas the CSV contributions of countries (10.88 percent) and styles (9.63 percent) are slightly diminished. In the restricted styles model, the factors explain 29.48 percent of total CSV, which is 50 bps less than in the base model. The style factors explain 9.15 percent of CSV, slightly less than in the base model, whereas country and industry factors explain slightly more.

Although the CSV decomposition certainly shifts as the number of industry or style factors is varied, Table 1 demonstrates that the effect is quite small. In other words, the CSV decomposition is very robust, and none of our conclusions are materially affected by any reasonable choice of factors.

**Table 1. Contributions to CSV for Three Models, January 1994–April 2010**

| Factors | Base Model | Extended Industries Model | Restricted Styles Model |
|---|---|---|---|
| Country factors | 48 | 48 | 48 |
| Industry factors | 24 | 68 | 24 |
| Style factors | 8 | 8 | 4 |
| *Explained CSV* | | | |
| Country factors | 10.94% | 10.88% | 10.99% |
| Industry factors | 9.10 | 12.39 | 9.34 |
| Style factors | 9.94 | 9.63 | 9.15 |
| All factors | 29.98% | 32.91% | 29.48% |
| Specific returns | 70.02 | 67.09 | 70.52 |
| Total | 100.00% | 100.00% | 100.00% |

*Notes:* This table reports the percentage CSV contributions for the three models. The average monthly total CSV was 8.97 percent. We obtained all results by using monthly cap-weighted regressions, with MSCI ACWI IMI as the estimation universe. The base model included 48 country factors, 24 GICS industry factors, and 8 style factors; the extended industries model used 68 GICS industry factors rather than 24; the restricted styles model used 4 style factors rather than 8.

## Decomposing RMS Returns

Traditional managers are primarily interested in performance relative to a benchmark, and so CSV is of the greatest relevance. Hedge fund managers, in contrast, may be more interested in the drivers of *absolute* returns. To identify the drivers of absolute returns, we can use the root mean squared (RMS) return, defined as

$$s(r) = \sqrt{\textstyle\sum_n w_n r_n^2}. \tag{10}$$

Note that because the RMS measure uses absolute returns (as opposed to relative returns), it is strictly larger than CSV. In Appendix A, we show how to apply the $x$-sigma-rho formula to decompose RMS returns into contributions from individual factors.

**Figure 6** plots total RMS levels, as well as contributions from factors and stock-specific sources. The total RMS is qualitatively similar to the total CSV presented in Figure 1 except that total RMS levels are slightly higher than CSV levels, as expected. A more significant difference is that the factor contribution to RMS is substantially greater than the factor contribution to CSV. The underlying reason is that the world factor contributes to RMS but not to CSV. This result can be seen mathematically in Equation 5 because the CSV of the world factor is zero. In contrast, the RMS of the world factor is 1, which contributes to total RMS via Equation A17 (see Appendix A). In essence, the world factor explains the mean return to the estimation universe but cannot explain anything *relative to* the mean.

The total $R^2$ of the model is defined as

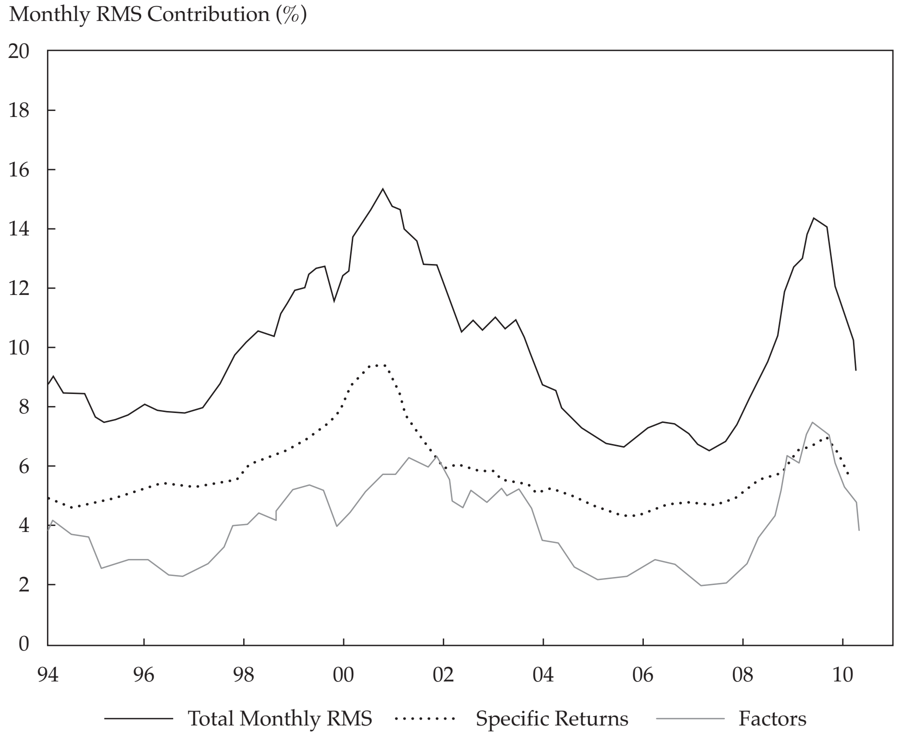
$$R_T^2 = 1 - \frac{\sum w_n u_n^2}{\sum w_n r_n^2}. \tag{11}$$

The ratio of the RMS's factor contribution to total RMS gives the total $R^2$ of the model. During periods of extreme market volatility (e.g., 2009), the factors contributed more to RMS than to the specific returns, which indicates that the total $R^2$ of the model exceeded 50 percent during those periods.

**Figure 7** plots the contribution to RMS returns from country, industry, and style factors, as well as the world factor. Comparing Figure 7 with Figure 2, we can identify many similarities. For instance, countries dominated industries over 1994–1999, and industries surpassed countries over 1999–2003. Moreover, styles dominated both countries and industries over 2000–2004 in both figures.

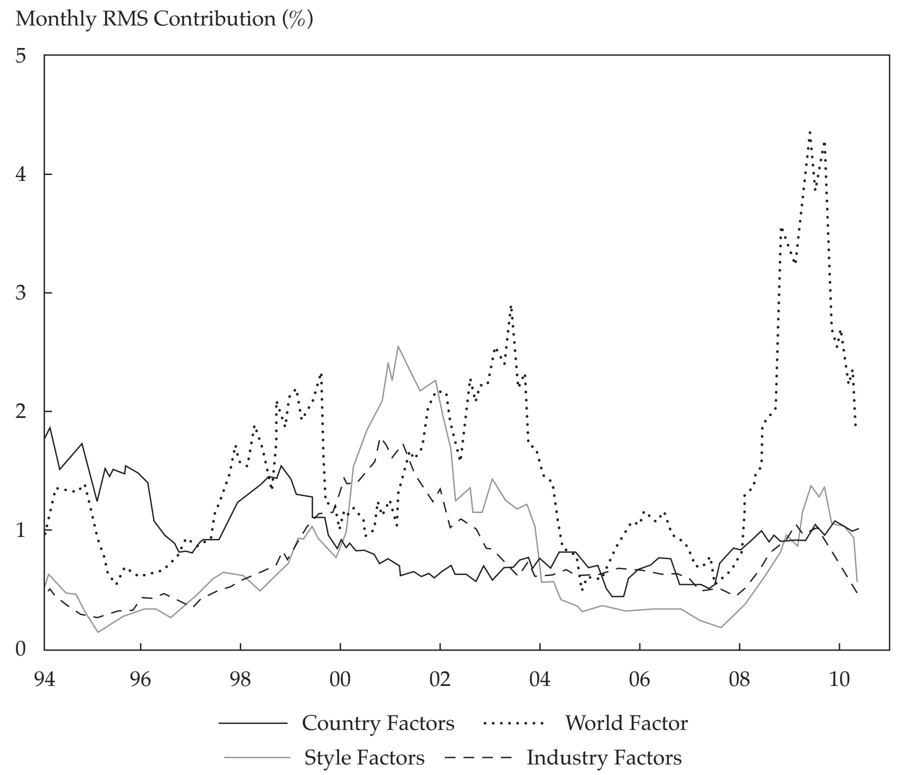
The distinctive feature of Figure 7 is that it shows the contribution of the world factor. On average, the world factor was on a par with the other factor categories in explaining RMS. During times of extreme market turbulence, however, the world factor easily dominated the other sources of absolute returns. For example, during the height of the financial crisis in 2009, the world factor dwarfed the other factor categories in terms of explaining RMS returns.

**Figure 6. Total RMS and Contributions from Factors and Specific Returns, January 1994–April 2010**




*Note:* See note to Figure 1.

**Figure 7. Contributions to RMS by Factor Type, January 1994–April 2010**




*Note:* See note to Figure 1.

## Conclusion

We have presented a new framework for analyzing the global equity markets. Our approach leverages the $x$-sigma-rho attribution methodology to provide an exact decomposition of cross-sectional volatility, which allows one to identify the drivers of CSV at the individual factor level and to aggregate contributions to find the relative importance of each factor category. We used our approach to investigate the relative importance of industry, country, and style factors, as well as to study the strength of emerging markets vis-à-vis developed markets. We also extended our methodology to decompose RMS returns, which are more relevant for absolute return managers.

*This article qualifies for 1 CE credit.*

## Appendix A. The Cross-Sectional *X*-Sigma-Rho Formula

Suppose that stock returns $r_n$ are driven by $K$ factors and an idiosyncratic term $u_n$:

$$r_n = \sum_{k=1}^{K} f_k X_{nk} + u_n, \tag{A1}$$

where $X_{nk}$ is the exposure of stock $n$ to factor $k$ and $f_k$ is the factor return. For notational efficiency, the idiosyncratic term can be conveniently incorporated into the factor component:

$$r_n = \sum_{k=1}^{K+1} f_k X_{nk}, \tag{A2}$$

where the idiosyncratic "factor return" is $f_{K+1} = 1$ and the "factor exposure" is $X_{n,K+1} = u_n$. The cross-sectional variance of stock returns is given by

$$\sigma^2(r) = \sum_n w_n (r_n - \bar{r})^2, \tag{A3}$$

where $w_n$ is the weight of stock $n$, used for the variance computation. Any weighting scheme is permissible, subject to the usual conditions that the weights be positive and add to unity. The mean stock return is defined as

$$\bar{r} = \sum_n w_n r_n, \tag{A4}$$

which can be expressed as

$$\bar{r} = \sum_{k=1}^{K+1} f_k \bar{X}_k, \tag{A5}$$

where

$$\bar{X}_k = \sum_n w_n X_{nk}. \tag{A6}$$

Therefore, the excess stock return can be expressed as

$$r_n - \bar{r} = \sum_{k=1}^{K+1} f_k \left(X_{nk} - \bar{X}_k\right). \tag{A7}$$

The cross-sectional variance in Equation A3 can thus be rewritten as

$$\sigma^2(r) = \sum_n w_n (r_n - \bar{r}) \sum_{k=1}^{K+1} f_k \left(X_{nk} - \bar{X}_k\right), \tag{A8}$$

which can be rearranged to give

$$\sigma^2(r) = \sum_{k=1}^{K+1} f_k \sum_n w_n \left(X_{nk} - \bar{X}_k\right)(r_n - \bar{r}). \tag{A9}$$

The cross-sectional variance of source exposures is given by

$$\sigma^2(X_k) = \sum_n w_n \left(X_{nk} - \bar{X}_k\right)^2, \tag{A10}$$

and the cross-sectional correlation between source exposures and stock returns is

$$\rho(X_k, r) = \frac{\sum_n w_n \left(X_{nk} - \bar{X}_k\right)(r_n - \bar{r})}{\sigma(X_k)\sigma(r)}. \tag{A11}$$

Substituting Equation A11 into Equation A9 provides the decomposition of cross-sectional volatility into its component sources:

$$\sigma(r) = \sum_{k=1}^{K+1} f_k \sigma(X_k) \rho(X_k, r). \tag{A12}$$

If we wish to separate the idiosyncratic term, we obtain

$$\sigma(r) = \sum_{k=1}^{K} f_k \sigma(X_k) \rho(X_k, r) + \sigma(u)\rho(u, r), \tag{A13}$$

which is Equation 5 (in the main text). Equation A13 represents an exact decomposition of cross-sectional volatility.

In some instances, one may wish to decompose not the cross-sectional volatility but rather the cross-sectional RMS return $s(r)$, defined by

$$s^2(r) = \sum_n w_n r_n^2. \tag{A14}$$

Similarly, the RMS for individual factors is given by

$$s^2(X_k) = \sum_n w_n X_{nk}^2. \tag{A15}$$

We can also define the "pseudocorrelation" as follows:

$$\tilde{\rho}\left(X_k, r\right) = \frac{\sum_n w_n X_{nk} r_n}{s\left(X_k\right) s\left(r\right)}. \quad \text{(A16)}$$

Like a standard correlation, the pseudocorrelation is bounded by ±1. The only difference between $\tilde{\rho}\left(X_k, r\right)$ and the standard correlation given by Equation A11 is that the latter uses demeaned variables whereas the former does not. The cross-sectional RMS return can be decomposed as

$$s\left(r\right) = \sum_{k=1}^{K} f_k s\left(X_k\right) \tilde{\rho}\left(X_k, r\right) + s\left(u\right) \tilde{\rho}\left(u, r\right), \quad \text{(A17)}$$

which is the RMS counterpart of Equation A13.

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