The Barra Europe Equity Model (EUE3)

Research Notes

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1. Introduction

This document describes the Barra Europe Equity Risk Model, EUE3. It provides insight into all stages of the model estimation process and includes extensive back-tests, which demonstrate the model's performance in different market conditions over the evaluation period from 1996 to 2008.

Europe has experienced considerable political and economic changes over the last decade. Since 2000, the European Union (EU) has expanded from 15 to 27 member states. Eight Eastern European countries joined the EU in 2004 thereby ending a 60-year division of Europe. Cyprus and Malta also became members in 2004, followed by Romania and Bulgaria who joined in 2007. Turkey, Croatia, and the Yugoslav Republic of Macedonia currently are EU membership candidates. With a population of nearly half a billion, the expanded EU accounts for about 30% of the world-wide gross domestic product.

The euro was first adopted by 11 member states in 1999 for commercial and financial transactions only. Greece joined in 2001, the issuance of euro notes and coins followed in 2002. Since then, Cyprus, Malta, Slovenia, and Slovakia have also adopted the euro, while Denmark, Lithuania, Latvia, and Estonia follow the Exchange Rate Mechanism, ERM II, which keeps exchange rate fluctuations against the euro in a narrow bandwidth through coordinated interventions from the European and local central banks.

The Barra EUE3 model enables users to explore common characteristics of the expanded European region by providing a unified perspective on risk across all main European equity markets. It replaces EUE2, which was introduced in 2001 and covered 15 Western European countries. Compared to its predecessor, EUE3 offers broader coverage, a deeper data history, more flexibility, and better forecasting accuracy.

EUE3 gives users flexibility by offering a choice between three model versions with different levels of detail in the industry factors, namely base, UK derived, and Eastern Europe derived. The base version uses a pan-European market factor and a single set of industry factors across Europe, hence emphasizing regional commonalities. In contrast, the derived versions split the region into two parts by applying separate market and industry factors for the UK and Eastern European countries, respectively. The derived versions offer an extra level of granularity to users who desire to separately budget risk for the UK or Eastern Europe. All models are provided in long-horizon (EUE3L) and short-horizon (EUE3S) variants to cater for users with different investment horizons.

In summary, EUE3 offers the following main features:

- Increased coverage, 13 new countries: Croatia, Cyprus, Czech Republic, Estonia, Hungary, Iceland, Latvia, Lithuania, Poland, Romania, Russia, Slovenia, and Turkey.
- Industry factors based on the Global Industry Classification Standard (GICS®) facilitate the integration of EUE3 forecasts with other models and proprietary reports.

- Industry and country factors exhibit comparable volatility because both are modeled as offsets to a European market factor.
- A refined set of nine styles including a style factor for liquidity. All styles are centered region-wide to facilitate comparison of exposures across Europe.
- Division of the European region into core and non-core countries. Market, industry, and style factors are estimated only on the core country universe.
- Calculation of daily factor returns with a robust multi-step regression scheme that curtails the influence of asset return outliers on the factor return estimates.
- Covariance matrices use the same split half-lives as EUE2. Currency factors are modeled with the same approach as the equity factors to provide uniform responsiveness across both factor categories.
- Detection and progressive trimming of outlier factor returns. This feature improves the forecasting accuracy as it prevents past outliers from biasing current forecasts.
- Completely redesigned specific risk model uses daily specific returns and delivers more accurate forecasts than EUE2.
- History of monthly EUE3 risk forecasts is available back to January 1995. Daily updates are provided from July 2008 on, allowing users to rapidly assess the impact of recent market events on the forecasts.

The remainder of this document starts with an overview of regional factor risk modeling and then explains all parts of the model estimation process. This is followed by comprehensive back-testing results. The final chapter describes the derived model versions and highlights their differences compared to the EUE3 base version. Appendix A contains cross-reference tables for all important model parameters, and Appendix B provides details of the constituent descriptors of the EUE3 risk indices.

2. Forecasting European Equity Risk

EUE3 describes the risk of an asset as the volatility of its excess returns, $\mathrm{sdev}(r_t^{[ref]})$, measured in the investor's reference currency (numeraire). The excess return at time t, r_t, is the total asset return, R_t, minus the risk-free rate, r_{0t}. Given that a regional portfolio is likely to contain assets with different foreign currency exposures, investors will assume two types of risk:

- *Currency risk* – the excess return volatility, $\mathrm{sdev}(q_t)$, of a foreign cash position. The foreign cash excess return, q_t, is measured in the investor's reference currency:

$$q_t = (1 + Q_t)\left(1 + r_{0t}^{[loc]}\right) - \left(1 + r_{0t}^{[ref]}\right). \tag{2.1}$$

  Here Q_t denotes the currency return, $r_{0t}^{[loc]}$ the risk-free rate in the foreign currency, and $r_{0t}^{[ref]}$ the risk-free rate in the numeraire currency.

- *Equity risk* – the excess return volatility, $\mathrm{sdev}(r_t^{[loc]})$, of a foreign equity position measured in its local exposure currency. The local currency excess return is defined as:

$$r_t^{[loc]} = R_t^{[loc]} - r_{0t}^{[loc]}. \tag{2.2}$$

The total asset return in the numeraire currency can be approximated as the sum of foreign currency asset return and currency return:

$$R_t^{[ref]} = (1 + Q_t)\left(1 + R_t^{[loc]}\right) - 1 \approx Q_t + R_t^{[loc]}. \tag{2.3}$$

This approximation neglects the cross-term $Q_t R_t^{[loc]}$ which tends to be small. Similarly, the reference currency excess return of an asset can be approximated by the sum:

$$r_t^{[ref]} \approx q_t + r_t^{[loc]}. \tag{2.4}$$

Using this approximation, the reference currency risk becomes:

$$\mathrm{sdev}\left(r_t^{[ref]}\right) = \sqrt{\mathrm{var}(q_t) + \mathrm{var}\left(r_t^{[loc]}\right) + 2\mathrm{cov}(q_t, r_t^{[loc]})}. \tag{2.5}$$

In line with other Barra equity risk models, EUE3 linearly decomposes the local currency excess returns, $r_t^{[loc]}$, into a sum of K equity factor returns, $f_{k,t}$, which represent components that are common to many assets in the model universe, and a specific return, u_t, which is unique to a single asset:

$$r_t^{[loc]} = \sum_{k=1}^{K} X_{k,t} f_{k,t} + u_t. \tag{2.6}$$

Factor returns are not directly observed but need to be estimated by the model. Section 4.1 explains in detail how the factor returns are estimated in EUE3. The factor exposures, $X_{k,t}$, represent the sensitivities of the asset returns to the factors. EUE3 uses two different exposure types. Exposures to market, country, and industry factors are set to discrete values of zero or one that typically remain static over time. In contrast, exposures to style factors take continuous values that vary over time. Section 3.3 describes how EUE3 defines and standardizes the factor exposures.

The reference currency excess return can also be written as a linear factor decomposition by combining the currency factor return, q_t, with a currency factor exposure, $X_q = 1$:

$$r_t^{[ref]} = X_q q_t + \sum_{k=1}^{K} X_k f_{k,t} + u_t. \tag{2.7}$$

Henceforth the subscripts t indicating time dependence are omitted for brevity.

To examine how return and risk of portfolios are expressed in the factor model context, we define a portfolio P with a time-dependent weights vector, h_n^{P}, of N assets and write the excess returns of P as:

$$r^{\mathrm{P}[ref]} = \sum_{n=1}^{N} h_n^{\mathrm{P}} r_n^{[ref]}. \tag{2.8}$$

The factor decomposition of the portfolio excess return becomes:

$$r^{\mathrm{P}[ref]} = \sum_{l=1}^{L} X_l^{\mathrm{P}} q_l + \sum_{k=1}^{K} X_k^{\mathrm{P}} f_k + \sum_{n=1}^{N} h_n^{\mathrm{P}} u_n \,. \tag{2.9}$$

The first term in this expression runs over the L foreign currencies the portfolio is exposed to, the second term is the return attributable to local equity factors, and the last term is the portfolio specific return.

The portfolio exposures to each factor are simply the weighted asset exposures:

$$X_k^{\mathrm{P}} = \sum_{n=1}^{N} h_n^{\mathrm{P}} X_{nk}. \tag{2.10}$$

Using (2.9) the portfolio volatility becomes:

$$\mathrm{sdev}\left(r^{\mathrm{P}[ref]}\right) = \left[\sum_{\mathrm{k,j}=1}^{K+L} X_k^{\mathrm{P}} F_{kj} X_j^{\mathrm{P}} + \sum_{n=1}^{N} (h_n^{\mathrm{P}})^2 \, \Delta_{nn}\right]^{1/2}. \tag{2.11}$$

The first term in the above expression represents the common factor variance of P. It combines the covariance matrix of factor returns, $F_{kj} = \mathrm{cov}(f_k, f_j)$, and the portfolio exposures (2.10). For brevity, the K equity and L currency factors have been combined in a single sum. The second term is the portfolio specific return variance which combines the weights vector and the diagonal covariance matrix of specific returns, $\Delta_{nn} = \mathrm{var}(u_n)$.

Equation (2.11) is based on the following two assumptions:

- Specific returns and factor returns are uncorrelated, $\mathrm{cov}(f_k, u_n) = 0$ for all k, n.
- Specific returns of different assets are uncorrelated, $\Delta_{mn} = \mathrm{cov}(u_m, u_n) = 0$ if $m \neq n$.

The regression method used to model the factor returns implies that the first assumption is satisfied. The second assumption is common to all Barra factor models. It is equivalent to postulating that the specific returns are fully diversifiable, that is, specific risk of an equal-weighted portfolio with a large number of assets becomes negligible. Though the simplifying assumption of uncorrelated specific returns is applicable to assets of different issuers, it fails when multiple assets of the *same* issuer are considered. In this case, EUE3 relaxes the assumption and adds off-diagonal terms, $\Delta_{mn} > 0$, to the specific variance. For example, this so-called linked specific risk correction is applied to A and B shares of the same issuer.

To summarize, the "ingredients" of the EUE3 factor model are:

- The factor exposures, X_{nk}, of all n assets in the model universe.
- The factor covariance matrix, F_{kj}.
- The specific return variances, Δ_{nn}, of all assets in the model universe.
- The linked specific return covariances, Δ_{mn}, of multiple assets of the same issuer.

3. Model Scope and Factors

3.1. Coverage and Estimation Universe

EUE3 provides risk forecasts for a broad coverage universe spanning 29 European markets. Compared to its predecessor, EUE2, coverage has been significantly expanded by adding markets in Eastern Europe. EUE3 distinguishes between the following three country blocks:

- 16 *Core Markets* in Western Europe: Austria, Belgium, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, and the United Kingdom. The EUE3 data history for these countries extends back to January 1995.
- 5 larger markets in *Eastern Europe*: Czech Republic, Hungary, Poland, Russia, and Turkey. Risk forecasts for these countries are provided from January 1997.
- 8 smaller *Frontier Markets*, most of which are located in Eastern Europe: Croatia, Cyprus, Estonia, Iceland, Latvia, Lithuania, Romania, and Slovenia. Only a limited history of data is available for these markets, hence EUE3 provides forecasts from January 2005.

As of December 2008, the EUE3 coverage universe contained around 9500 assets, of which about 1500 were listed in Eastern European and Frontier Markets.

EUE3 also covers cross-listings and depositary receipts (ADR, GDR) of European companies. The country and currency exposures of cross-listings and depositary receipts match their respective parent assets. For example, the US-listed ADR of Vodafone Group is modeled with UK country exposure and GBP currency exposure. Similarly, the French cross-listing of Norsk Hydro has exposures to the Norway country factor and to the NOK currency factor. This treatment provides consistency between cross-listings and their parent assets. Users might argue that different liquidity of cross-listed and parent assets can result in pair arbitrage opportunities. Such effects do exist though they are short-lived; the volatility difference between cross-listed and parent assets does not scale with square root of time but reverts back to zero as the time horizon increases.

The estimation universe (ESTU) is the subset of the full coverage universe used for calculating the EUE3 factor returns. Not all assets in the coverage universe are suitable for inclusion in the estimation universe. Many small cap stocks trade infrequently and have stale daily prices. Further, in comparison to large caps, the returns of small cap stocks tend to be influenced more by firm-specific drivers than by factors that are common to an entire industry or country.

The EUE3 estimation universe is built to provide a well diversified representation of each country and industry factor while excluding illiquid assets. From June 1994 onwards, the *MSCI All Country (AC) Europe Investable Market Index* (IMI) is used as a primary selection criterion. Here an important feature of the MSCI AC Europe IMI must be taken into consideration. Before July 2002, the index includes a broad basket of small cap stocks. However, from July 2002 onwards, a float-cap adjustment and more restrictive liquidity filtering rules are applied. This leads to a drop from about 3000 MSCI AC Europe IMI constituents in 2001 to about 1500 assets in 2003. To ensure continuity in the estimation universe, EUE3 includes additional small cap stocks to the index after the float adjustment and conversely excludes some small cap index stocks before the adjustment. The Frontier Markets are not part of MSCI AC Europe IMI, therefore the estimation universe for these markets is constructed by selecting the largest and most liquid assets. Overall, the EUE3 estimation universe is built using the following set of rules:

- Exclude illiquid assets, micro-caps (cap less than USD 10mn), depositary receipts, cross-listings, and UK trusts. Between June 1994 and June 2002, exclude assets that are not in MSCI AC Europe IMI.
- Build an estimation universe candidate by including at least 93% of the aggregate cap of each country and industry. After June 2002, include MSCI AC Europe IMI constituents even if they were excluded in the previous step (depositary receipts and trusts that are constituents).
- Add further stocks to countries and industries with fewer than 15 assets.

Between 1996 and 2008, the estimation universe gradually increases from around 1800 to around 2400 assets. Table A.4 in the appendix details the content of the estimation universe for each country as of December 2008.

3.2. Factor Structure

The base version of EUE3 uses a total of 68 equity factors:

- A regional market factor
- 29 country factors
- 29 industry factors
- 9 style factors

Every asset has unit exposure to the regional market factor, one country factor, and one industry factor. Exposures to the style factors take continuous values.

Returns of the market, industry, and style factors are estimated using only the 16 Western European *core markets*. In contrast, the country factors are estimated on the entire estimation universe. The reasons for the distinction between core and non-core markets are the following:

- Eastern European and Frontier Markets exhibit considerably fewer regional commonalities than the Western European Markets. Managers of European portfolios would routinely substitute a German bank with a Spanish bank in a regional portfolio, but they would hesitate to replace a position in a Hungarian bank with a Turkish bank.
- Many users of EUE2 expressed a desire to include Eastern Europe while keeping the main focus on Western Europe. This reflects a hierarchical approach to regional investing; the core portfolio is built from Western European stocks and selectively augmented with Eastern European assets.

EUE3 industry factors are based on the Global Industry Classification Standard (GICS®), a widely used hierarchical industry scheme that provides four levels of detail. Each EUE3 industry factor spans either a single GICS industry group (level 2) or a set of GICS industries (level 3) within the same industry group. GICS industry groups are subdivided if they contain sufficient assets and if the factor returns of the subdivided industries exhibit significantly different behavior. For example, the Metals and Mining industry is separated from the Materials industry group. Table A.1 in the appendix details the mapping between EUE3 industry factors and GICS industry codes.

Table A.2 lists the aggregate weight and number of estimation universe assets of each EUE3 industry. 20 EUE3 industries have a one-to-one correspondence to the industry factors of the *Barra Global Equity Model (GEM2)*. This close relationship between the two Barra models is helpful as it facilitates comparison of forecasts and integrated reporting. EUE2 industry factors were derived from a custom industry classification scheme. There is no one-to-one mapping between the two sets of industry factors, though many of the EUE3 industry factors can to a large extent be associated with a single EUE2 industry. Table A.3 associates the EUE3 industries with the corresponding industry factors of the predecessor model.

EUE3 is also published in *derived versions*. These versions provide a deeper level of detail than the EUE3 base version by splitting the European region into two sub-regions. Derived models assign a market factor and 29 industry factors to each sub-region, therefore they use a total of 98 equity factors. More detailed information about the derived model versions can be found in Section 8.

3.3. Style Factors

EUE3 uses nine style factors to capture both fundamental and market-driven systematic sources of risk and return:

- Volatility
- Momentum
- Size
- Liquidity
- Value
- Earnings Yield
- Dividend Yield
- Growth
- Leverage

In line with other Barra models, EUE3 builds the style factor exposures by linearly combining multiple descriptors with similar characteristics. The definitions of all 25 descriptors used in EUE3 can be found in Appendix B. This section first summarizes the qualitative characteristics of the styles and the main differences to those used in EUE2. The second part explains how the style exposures are calculated.

The *Volatility* factor captures return and risk differences between high-beta and low-beta stocks. It complements the regional market factor which represents the volatility of a broadly diversified market portfolio with $\beta = 1$. The Volatility factor combines a historical sample beta, the 12-month cumulative range, and a short-term daily returns volatility. Comparing two industry portfolios helps to explain this. In December 2002, the Software portfolio had an exposure of $X^{\mathrm{P}} = +1.6$ whereas the Food & Beverages industry had $X^{\mathrm{P}} = -0.97$. The factor returns correlation between Volatility and the market factor was $\rho = +0.85$. The positive style factor exposure of the high-beta Software industry and the positive correlation translate into excess risk over the market portfolio, while the negative exposure of the low-beta Food & Beverage industry corresponds to a reduced risk exposure.

Momentum describes risk associated with the trending behavior of stock returns over a horizon of about a year. It combines two relative strength descriptors and a two-year historical alpha. As many strategies use momentum as an alpha factor, it is essential to understand the excess risk assumed by tilting a portfolio towards momentum. If Momentum is correlated with the market, this translates into a risk asymmetry between “past winners” (positive momentum) and “past losers”. For example, as of December 2004 the cap-weighted regional Real Estate portfolio was a “winner” with $X^{\mathrm{P}} = +0.6$ while the Airlines portfolio was a “loser” with $X^{\mathrm{P}} = -0.8$. The correlation between momentum and market factor returns was negative at $\rho = -0.2$. This implies that holding the Airlines portfolio was associated with higher excess risk than holding the Real Estate portfolio. In December 2006, the Real Estate portfolio still had positive momentum exposure, but the correlation between momentum and market factors turned positive, indicating that the Real Estate portfolio became significantly riskier.

The *Size* factor captures systematic return and risk differences between large cap and small cap stocks. Size blends two descriptors that represent total market cap and total assets, respectively. Most stocks in the EUE3 universe have negative size exposure. This is a consequence of centering the exposures such that the cap-weighted regional market portfolio has zero exposure. Size factor returns generally exhibit positive correlations with the market factor.

The *Liquidity* factor helps to assess systematic risk associated with infrequent trading. It contains three turnover descriptors that measure the amount of an issuer’s total cap traded over the last 1–12 months. Liquidity exposures show slight positive correlation with the Size style because small cap stocks tend to be less liquid than large caps. However, this correlation is not as high as might perhaps be expected; Table A.5 indicates an average exposure correlation of $+0.3$.

The *Value* style factor combines the price-to-book and price-to-sales descriptors. It indicates how “inexpensively” a company is currently traded by using its book value and past revenues as yardsticks. The Value style does *not* consider a company’s ability to generate positive cash flows and earnings. Some companies have positive exposure to Value because the market undervalues their earnings potential, while other companies have positive Value exposure because the market recognizes that they don’t generate positive cash flows due to a failed business model. Comparing the Airlines and Metals & Mining industries illustrates this point. In 2000, both industry portfolios had Value exposure of one. Five years later, the Value exposure of

Metals & Mining had decreased to zero; this industry was successful, and its success was reflected in higher share prices. In contrast, the 2005 Value exposure of the Airline industry remained at one; the market recognized this industry's difficulty to generate sustainably positive cash flows and reacted accordingly.

Earnings Yield is the main companion to Value. It relates the current market valuation of a company to its ability to generate positive earnings and cash flows. Earnings Yield blends four different descriptors; the inclusion of analyst estimates of future price-to-earnings gives this style a partly forward-looking character. This contrasts with Value factor, which is entirely based on backward-looking fundamentals. Compared to the Value factor, exposures to Earnings Yield tend to be more volatile because they can reflect isolated events that affect a company's earnings. Table A.5 shows that the average exposure correlation between Earnings Yield and Value is $+0.29$, indicating that these two factors describe related yet different phenomena. As an example, consider again the Airlines and Metals & Mining industries. In December 2001, Airlines had a negative exposure of -0.65 to Earnings Yield as earnings and expected future earnings were low after the September 11 attack. In contrast, the Metals & Mining industry generated robust cash flows, it had a positive Earnings Yield exposure of $+0.44$.

Dividend Yield is the only style factor in EUE3 composed of a single descriptor, the past 12-month dividends per share divided by the share price. This style factor is related to Earnings Yield but has sufficiently different characteristics to justify keeping it separate. To some extent, companies can decide whether they pay out earnings as dividends or reinvest them in future growth. This means that companies with high Earnings Yield do not need to have high Dividend Yield. The converse is also not true, as it is common for companies to smooth yields by paying dividends even when experiencing a temporary earnings drop.

The *Growth* style may seem a bit like the antipode to Value, but in fact captures a different source of return and risk. Growth combines five descriptors of fundamental company growth. Three of these are backward-looking over a period of five years, capturing past growth of assets, sales, and earnings. The other two are forward-looking, representing the analyst expectations of short-term and long-term future earnings growth. Table A.5 illustrates that Growth and Value are two different phenomena; the average exposure correlation between these two styles is nearly zero. Growth also has a negative exposure correlation of -0.3 with Dividend Yield. This is plausible because firms typically pay dividends when they run out of growth options.

Finally, the *Leverage* factor captures the relationship between a firm's stock returns and its level of indebtedness. This style factor uses the book leverage and market leverage descriptors. From 1995 to 2008, the average correlation between the Leverage and market factor returns was almost zero. However, this correlation can quickly turn positive in periods of high volatility. During the second half of 2008 this correlation rose as high as $+0.4$, indicating a significantly increased risk of highly levered positions.

The following list provides an overview of the most important style factor differences between EUE3 and EUE2.

- EUE3 introduces a new style factor, Liquidity, which has no equivalent in EUE2.
- The separation of the Value and Earnings Yield factors provides more detail than the combined Value factor of EUE2. The introduction of forward-looking earnings significantly enhances the explanatory power of the Earnings Yield style factor.
- The separate Market Sensitivity and Volatility style factors of EUE2 are merged into the EUE3 Volatility factor as they are highly collinear.
- The introduction of forward-looking descriptors to the EUE3 Growth factor enhances its explanatory power compared to its equivalent in EUE2, which uses only backward-looking growth descriptors.
- The Foreign Exposure style of EUE2 has been dropped as it gradually lost explanatory power over recent years.

The remainder of this section describes technical details about the descriptor and style exposures of EUE3. The exposures are calculated as follows:

- Collection and calculation of raw descriptor exposures.
- Standardization of descriptor exposures.
- Blending of descriptors into styles using optimal weights.
- Standardization of the resulting style exposures.
- Estimation of style exposures for assets with missing descriptor data.

Descriptor and style exposures are centered and standardized over the *core market* part of the estimation universe:

$$X_{nk}^{(std)} = \frac{X_{nk}^{(raw)} - \mu_k}{\sigma_k}. \tag{3.1}$$

where $X_{nk}^{(raw)}$ denotes the raw exposure of asset n to style or descriptor k, μ_k is the *cap-weighted* average raw exposure of the *core market* estimation universe, and σ_k is the equal-weighted standard deviation of the core universe exposures. Centering renders the cap-weighted Western European estimation universe style-neutral. Standardization facilitates the comparison of exposures across all styles and is also a prerequisite for combining several descriptors into a style factor.

Occasionally, some assets have very large descriptor exposures. Such exposure outliers are typically caused by non-recurring events such as exceptional write-offs. Outlier exposures need to be truncated to control the influence of these stocks in the factor return regressions.

Traditionally, the standardized exposures are hard-truncated to a range of ± 3. This approach works, but has the drawback of dropping the rank of exposures that undergo truncation: two stocks with exposures of $+3.5$ and $+20$ are both truncated to the same threshold value of $+3$. EUE3 introduces an improved truncation method which preserves the rank of all exposures. This is achieved by compressing the scale for exposures outside the range of ± 3:

$$\tilde{X}_{nk}^{(std)} = \begin{cases} 3 \cdot \left(1 - s_{(+)}\right) + X_{nk}^{(std)} \cdot s_{(+)} & ;\ X_{nk}^{(std)} > 3 \\ X_{nk}^{(std)} & ;\ \text{-}3 \leq X_{nk}^{(std)} \leq 3 \\ -3 \cdot \left(1 - s_{(-)}\right) + X_{nk}^{(std)} \cdot s_{(-)} & ;\ X_{nk}^{(std)} < \text{-}3 \end{cases} \tag{3.2}$$

$$s_{(+)} = \max\left[0, \min\left[1, \frac{0.5}{\max\limits_{n}\left(X_{nk}^{(std)}\right) - 3}\right]\right] \tag{3.3}$$

Here, $\tilde{X}$ are the truncated exposures, and X denotes exposures before truncation. The scale factor, $s_{(+)}$, compresses the positive tail of the exposures such that $\tilde{X}_{nk}^{(std)} \leq 3.5$. No truncation is performed if $\max(X_{nk}^{(std)}) \leq 3.5$. The scale compression factor, $s_{(-)}$, of the negative tail is calculated analogously. After truncation the descriptors are again centered and standardized. Overall, soft truncation improves the usability of the standardized descriptors as the rank information is preserved.

In some cases not all descriptor exposures are available for a stock. If some constituent descriptors of a style are missing, the remaining descriptors are used and their blending weights rescaled. If all constituents are missing, EUE3 uses cross-sectional regression to estimate the relationship between style exposures and log(market cap) for each GICS sector. The 10 GICS sectors are used here instead of the 29 EUE3 industries to avoid small-sample biases. This method to estimate and back-fill missing exposures is based on the notion that many industries exhibit a distinctive style signature. For example, a growth sector, such as Information Technology, contains many small firms with high fundamental growth and high P/E ratios; many technology companies have little debt and pay no dividends.

4. Factor Returns

4.1. Cross-Sectional Regression

EUE3 estimates factor returns by regressing daily local excess returns against the market, industry, country, and style factor exposures. The cross-sectional regression is weighted and includes all assets in the estimation universe; it yields factor returns that minimize the weighted squared sum of the residuals, $\sum_n w_n u_{n,t}^2$. Cross-sectional regression can be described as a geometric projection; it projects the excess returns from the N-dimensional asset space to the much smaller K-dimensional factor space.

The following topics are relevant to understanding the EUE3 regression calculations:

- Weighting of asset returns.
- Elimination of collinearity between exposure vectors.
- Treatment of return outliers.
- Correction for countries and industries with few assets.

Regression weights are set to $w_n = \sqrt{cap_n}$ and truncated at the 95th percentile for the largest stocks. Using equal weights would be detrimental: as large cap returns exhibit more common behavior than small cap returns it makes sense to favor them in the regression. Some users might also find cap-weighted regressions appealing as this choice of weights would mimic a cap-weighted index portfolio. However, placing too much emphasis on the largest stocks would overly reduce the effective size of the estimation universe. As such, weighting by square root of cap is a good compromise between cap-weighting and equal weighting.

The definitions of the EUE3 market, country, and industry factors makes them intuitive and convenient to use but creates two perfect collinearities between their exposures: the sum of all industry exposures and the sum of all country exposures each equal the market factor. The EUE3 base model uses 68 equity factors, but these two collinearities reduce the dimensionality of its factor space to K=66. To remove the collinearities, EUE3 applies two constraints to the regressions. These require that the cap-weighted country and industry factors each add up to zero:

$$\sum_{\substack{C \\ n \in core}} W_{nC} f_C = 0 \qquad\qquad \sum_{\substack{I \\ n \in core}} W_{nI} f_I = 0. \tag{4.1}$$

Here W_{nC} is the cap of asset n in country C and W_{nI} is the cap of asset n in industry I. Note that both sums include only assets in the 16 Western European core countries.

These constraints make the cap-weighted core market portfolio neutral to the industry and country factors. As described in section 3.3, the cap-weighted core market portfolio is also style-neutral. The constraints imply that the country, industry, and style factor returns become offsets to the return of a cap-weighted core market portfolio. In other words, the country factor returns indicate whether a country has outperformed or underperformed relative to the regional market, and the same interpretation also applies to the industry factors. For clients who migrate from EUE2 to EUE3, it should be highlighted that this new symmetric treatment of country and industry factors differs somewhat from EUE2 where the regional market factor was embedded in the industry factors. Consequently, the EUE2 industry factors had a much higher volatility than the EUE2 country factors. EUE3 eliminates this asymmetry between country and industry factors.

The EUE3 factor regression proceeds in two stages. The first stage estimates the returns of all factors with exposures in the core universe:

$$r_{n\in core}^{[loc]} = f_M + \sum_I X_{nI} f_I + \sum_{C\in core} X_{nC} f_C + \sum_S X_{nS} f_S + u_n, \tag{4.2}$$

where f_M denotes the market factor return and the subscripts I, C, S denote industry, country, and style factors, respectively. This regression uses the two constraints (4.1).

The second stage estimates the country factors of the non-core countries after subtracting the market, industry, and style factor returns obtained in (4.2):

$$r_{n\notin core}^{[loc]} - f_M - \sum_I X_{nI} f_I - \sum_S X_{nS} f_S = \sum_{C\notin core} X_{nC} f_C + u_n \tag{4.3}$$

To control the influence of outlier returns on the regression results, EUE3 applies a two-step truncation scheme. The idea is to first regress un-truncated returns, then identify residual return outliers, and finally rerun the regression after truncating the returns of outlier stocks. Outliers are defined as residual returns exceeding $|u_n| > 4\tilde{\sigma}_u$, where the robust cross-sectional standard deviation, $\tilde{\sigma}_u$, is defined as:

$$\tilde{\sigma}_u = 1.4826 \cdot \mathrm{med}(|u_n - \mathrm{med}(u_n)|) \tag{4.4}$$

If a residual return exceeds $4\sigma_u$, it is split into an ordinary component and an extraordinary component, ξ_n, which can also be interpreted as an idiosyncratic return jump:

$$\xi_n = \begin{cases} \mathrm{sgn}(u_n)(|u_n| - 4\tilde{\sigma}_u) & |u_n| > 4\tilde{\sigma}_u \\ 0 & \text{otherwise} \end{cases} \tag{4.5}$$

The jumps, ξ_n, are then subtracted from the input returns and the regressions run again:

$$r_n^{[loc]} - \xi_n = \sum_{k=1}^{K} X_{nk} \tilde{f}_k + \tilde{u}_n \qquad\qquad u_n = \tilde{u}_n + \xi_n \tag{4.6}$$

The factor returns, $\tilde{f}_k$, resulting from this second step are the final regression results. To calculate the final specific returns, u_n, the jumps, ξ_n, are added back to the residuals, $\tilde{u}_n$, of the second regression. The same outlier truncation scheme is applied to the regressions (4.2) and (4.3).

EUE3 includes some countries and industries that contain few assets at some point in the back-testing period. For instance, the Semiconductors industry has 6 assets in December 1995 and Latvia has 8 assets in December 2003. Factor returns of such *thin categories* are affected by small-sample biases; they tend to be overestimated because they are derived from a poorly diversified sample. The same problem also occurs in a category with more assets if the category is dominated by few large cap stocks.

A convenient measure of concentration is the effective number of assets per category, φ:

$$n_{eff}^{\varphi} = \frac{\left(\sum_{n\in\varphi} w_n\right)^2}{\sum_{n\in\varphi} w_n^2} \tag{4.7}$$

Here w_n are the regression weights. EUE3 defines a category as thin if $n_{eff}^{\varphi} < 10$. To correct for thin category biases, EUE3 adds synthetic assets that act as Bayesian priors. Two synthetic asset types are considered:

- Synthetic asset A: If a category contains many stocks but few large caps, i.e., if $N_{\varphi} > 10$ but $n_{eff}^{\varphi} < 10$, a synthetic asset with the equal-weighted average category return is added.
- Synthetic asset B: represents a super-category, Φ, a similar but broader group. For thin EUE3 industries the corresponding GICS sector is used as super-category, for thin EUE3 countries a set of neighboring countries with similar characteristics is used. The return of this synthetic asset is the regression-weighted average over the super-category.

Synthetic assets are given zero exposures to all factors except their own category factor. Accordingly, their returns are calculated net of all other factors. For example, the constituent returns of a thin country prior use only the residuals and the factor return of this country:

$$\rho_n = X_{n\varphi} f_{\varphi} + u_n \tag{4.8}$$

With these constituent returns, the return of synthetic asset A becomes:

$$r_A^{\varphi} = \left(\frac{1}{N_{\varphi}}\right) \sum_{n\in\varphi} \rho_n \tag{4.9}$$

where N_{φ} is the number of assets in the thin category. Similarly, the return of synthetic asset B is:

$$r_B^{\varphi} = \frac{\sum_{n\in\Phi} w_n \rho_n}{\sum_{n\in\Phi} w_n} \tag{4.10}$$

where Φ denotes a super-category.

The regression weights of the synthetic assets are proportional to $(n_{eff}^{\varphi})^{-1/2} \sum_{n\in\varphi} w_n$ so that more weight is given to the priors as the effective number of assets in a thin category decreases. After adding the synthetic assets to the estimation universe, another multivariate regression yields the final factor returns of the thin category factors.

4.2. Factor Return Characteristics

The two criteria most commonly used to assess the in-sample performance of a factor model are the t-statistic of the factor returns and the goodness of fit, R^2, which indicates the percentage of the squared input returns that can be explained by the model.

The t-statistic of a factor return is defined as $t = f_k/\mathrm{se}(f_k)$, where $\mathrm{se}(f_k)$ denotes the standard error of the return estimate. A value of $|t| > 2$ indicates that the estimate is different from zero with 95% confidence. The time average of the squared t-statistics, $\mathrm{avg}(t^2)$, indicates how frequently a factor return is distinguishable from zero. However, $\mathrm{avg}(t^2)$ may be deceptive if t^2 is low most of the time but very high at few time points. Such outliers can occur when a country or industry witnesses extreme events. For example, the Icelandic market was heavily affected by the credit crisis in the second half of 2008; several banks defaulted and this is reflected in a large negative October 2008 factor return. Accordingly, the corresponding t^2-statistic of the Iceland country factor is 21 whereas the average is a more modest 2.5.

Table A.6 in Appendix A presents t-statistics for the industry factors, Table A.7 shows data for the country factors, and Table A.8 illustrates the significance of the style factors. The three tables also show annualized historical factor volatilities and average annual factor returns over the back-testing period. As expected, the regional market factor has by far the highest statistical significance. The t^2-values of the country factors vary considerably. Emerging economies such as Russia and Turkey tend to have high average t^2 whereas smaller EU countries like Austria have factors with more marginal significance. When considering that the annualized country factor returns represent offsets to the regional market factor, we find in Table A.7 that UK and the Netherlands have underperformed relative to the regional market. In contrast, France and Spain have outperformed. The average annualized volatility of the 16 core market country factors is 9.6%, whereas smaller countries tend to have higher factor volatilities. This difference between smaller and larger countries results from the fact that, to a large extent, the large countries define the regional market average. As a result, their factor returns are less likely to deviate from the market average than the small country factor returns. The t^2-statistics of the 8 Frontier Markets are shown separately as only a limited data history is available for these markets.

Figure 4.1 uses the Italy country factor as an example to illustrate the gradually increasing market integration within Europe. Before 2000, the Italy factor had high t^2, and it showed relatively weak correlation with the other European countries. The t^2 peak in spring 1998 reflects a temporary bull market which occurred in anticipation of the Italian lira being substituted by the euro. After 2000, t^2 of the Italy factor gradually decreased, indicating that the Italian market became less distinguishable from the European regional average.

The EUE3 industry factors also show widely varying t^2-statistics. Industries that have experienced crises or temporary bull markets tend to have high t^2. For example, the highly significant t^2 of the Telecommunication Services industry is related to the technology bull market in 1999 and the subsequent crash of internet and communications stocks. The 29 industry factors have $\mathrm{avg}(t^2)$ of 6.2, and they are significant for about 40% of all months in the back-testing period. The average annualized industry factor volatility is about 10%. This is comparable to the average core country volatility, indicating that industry and country factors both have similar relevance in the model. The annualized industry returns in Table A.6 should again be interpreted as offsets to the regional market average. As would be expected, we see that the Airlines industry has underperformed the regional market while the Pharmaceuticals industry has outperformed.

Figure 4.1 Squared t-statistic of the Italy country factor. The decreasing values are indicative of a progressively increasing degree of regional integration.

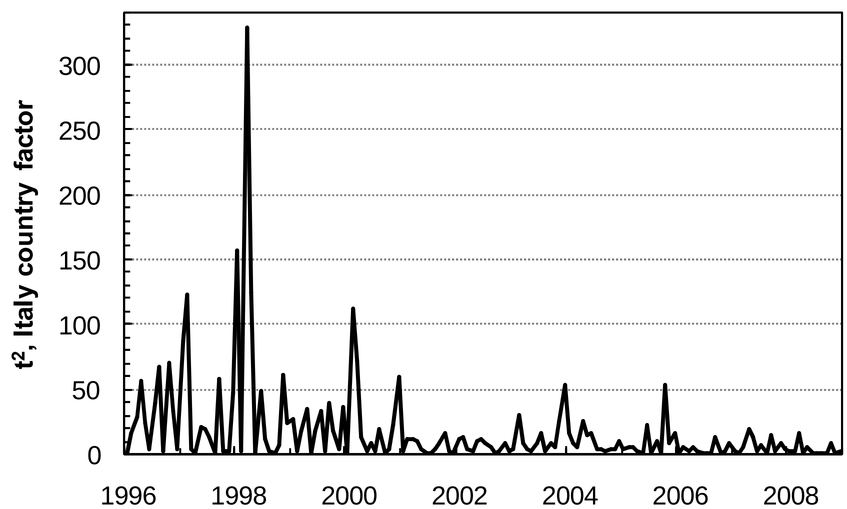


Table A.8 highlights the statistical significance of the EUE3 style factors. Data in this table is shown for two sub-periods. Volatility, Momentum, and Size have by far the highest average t^2-statistics, exhibiting persistent and strong style effects. Value and Earnings Yield rank next in importance with $\mathrm{avg}(t^2)$ of about 5, while the remaining styles are somewhat less significant. The Leverage style illustrates that factors should not be prematurely dismissed if they temporarily have low explanatory power. Table A.8 indicates that Leverage had $\mathrm{avg}(t^2)$ below 2 in the first half of the back-testing period but more than doubled its explanatory power in recent years. Considering the impact of the credit crisis in 2007/08 on highly levered firms, this result is as expected. The style factor volatilities are lower than the country or industry factors. In line with the ranking of t^2, the Volatility, Momentum, and Size factors have the highest historical volatilities. Momentum and Value had positive historical performance, whereas Volatility, Leverage, and Growth underperformed relative to a style-neutral market portfolio.

4.3. In-Sample Explanatory Power

The goodness of fit parameter, R^2, of the factor regression indicates the percentage of the squared input returns that can be explained by the model. It is defined as:

$$R^2 = 1 - \frac{\sum_n w_n u_n^2}{\sum_n w_n r_n^2} \tag{4.11}$$

Both sums use regression weights and include all assets in the estimation universe.

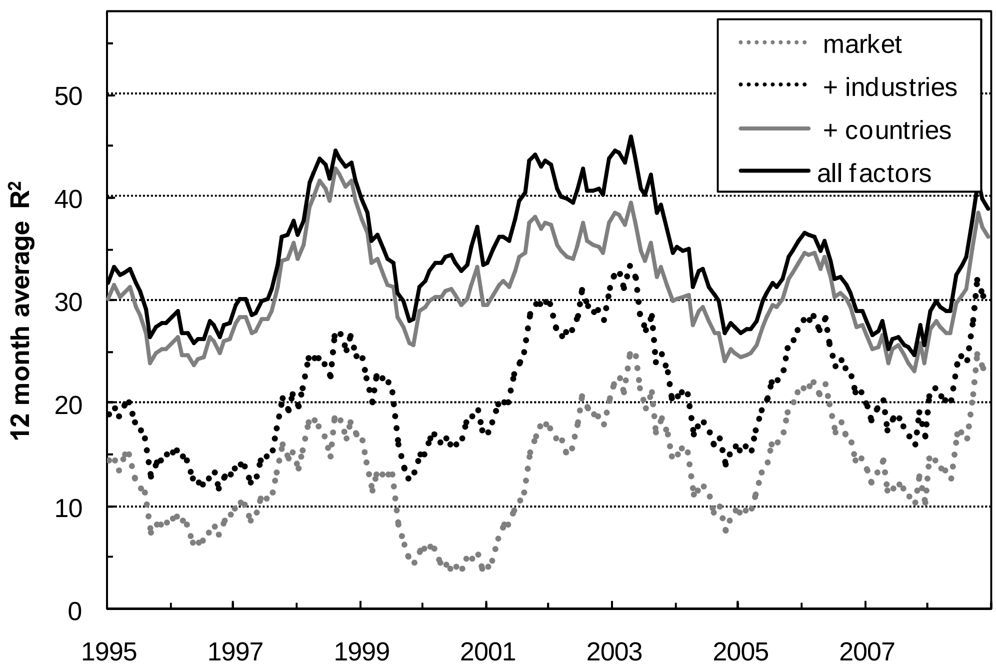
Before presenting R^2 data of EUE3, it is helpful to discuss the significance of R^2 in general terms. It is a common misconception that a risk model with higher average R^2 automatically gives better forecasts. This misconception is based on assuming that a model which explains more in-sample must be better at forecasting risk. Although in-sample explanatory power and out-of-sample forecasting performance are related, this correlation is less than perfect. In particular, it is easy to over-fit a model by adding lots of marginally relevant factors which will increase R^2 but not improve the forecasts. If additional factors describe transitory drivers of returns that don't persist over the forecasting horizon, adding such factors can in fact deteriorate the forecasts. Another potential flaw in the analysis of R^2 is to compare dissimilar data sets, for example, to compare R^2 of two models which were calculated with different estimation universes or different weights. Generally, the following qualitative guidelines help to put R^2 results in perspective:

- Reducing the size of the estimation universe will typically increase R^2 – it is easier to fit a small universe.
- The use of cap weights that emphasize few stocks will increase R^2, but this is a consequence of reducing the effective number of assets in the universe – see (4.7) for a definition of the effective number of assets.
- Over-fitting outlier returns can lead to deceptive R^2 increases because R^2 is sensitive to large r_n values. If a model tracks outliers, they don't appear in the residuals and therefore R^2 increases. However, factors that track single stock outliers are unlikely to be of value for out-of-sample forecasting.

Figure 4.2 shows the trailing 12-month average R^2 of EUE3. Results were obtained on the full EUE3 estimation universe and use regression weights. The continuous black line indicates that the full model has an average R^2 of 34.2% over the 14-year back-testing period. R^2 peaks in phases of high market volatility such as the Russia crisis or the technology bear market in 2001/2002. The subsequent market recovery from 2003 onwards is characterized by lower volatilities. In this phase the model has a lower $\mathrm{avg}(R^2)$, indicating that a smaller percentage of the asset volatilities can be attributed to common factors.

The additional R^2 time series in Figure 4.2 give valuable insight into the in-sample relevance of the four factor categories used in EUE3. These series are from regressions that omit one or several factor categories. The dotted gray line indicates the explanatory power of the EUE3 market factor, its $\mathrm{avg}(R^2)$ is 13.8%. While this indicates that the market factor is highly significant, it also shows that a simple market model misses a lot of systematic factor risk compared to the full EUE3 model. The dotted black trace in Figure 4.2 adds the 29 EUE3 industry factors to the market factor. It is clear that the industry factors have significant in-sample explanatory power; $\mathrm{avg}(R^2)$ increases by 7.5% compared to the market-only scenario. The continuous gray trace indicates that the country factors add further explanatory power; $\mathrm{avg}(R^2)$ increases by 9.9%. Finally, the style factors add an average 3% to the total R^2.

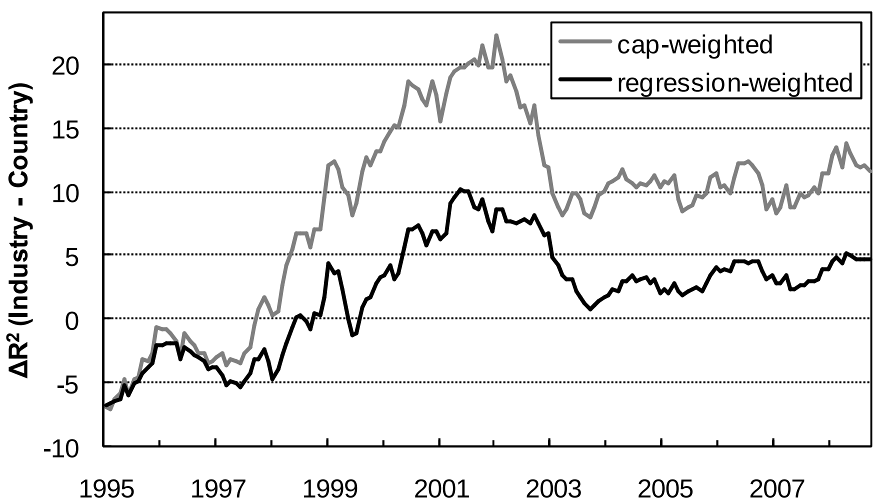
Figure 4.2 EUE3 12-month trailing average of R^2. The decomposition of R^2 into factor categories highlights the incremental explanatory power of the EUE3 market, industry, country, and style factors.



Comparing the explanatory power of industry and country factors helps to illustrate the extent of regional convergence in Europe. If industries have higher explanatory power than countries, this strongly suggests that a regional model is favorable to a set of single country models. If the industries have somewhat less explanatory power than the country factors, a regional model may still be advantageous because it offers the benefit of comparability: it highlights the joint properties of assets across the region.

Figure 4.3 compares R^2 of the EUE3 industry and country factors on the 16 Western European core markets used to estimate the industry factor returns. It shows the R^2 difference between a regression with market and industry factors, and a regression with market and country factors. The two time series were calculated using EUE3 regression weights (black line), and cap weights (gray line), respectively. It can be seen that the industries have higher average explanatory power than the countries. The advantage of the industry factors over the country factors increases over time, indicating a gradual increase of convergence within Western Europe. Figure 4.3 also shows that the cap-weighted industry factors outperform the countries by an even wider margin. This indicates that large cap firms, which tend to operate internationally, exhibit a higher degree of regional convergence than smaller firms.

Figure 4.3 Comparison of in-sample explanatory power of industry and country factors. 16 Western European core markets, regression-weighted and cap-weighted results.



The last part of this section presents comparative R^2 data to highlight differences between EUE3, its predecessor EUE2, and the Barra Global Equity Model (GEM2). These results are relevant to clients who migrate from EUE2 to EUE3 or who consider a regional model in addition to a global model. Comparisons use only the Western European part of the EUE3 estimation universe that is also covered in EUE2 and GEM2; the results are shown in Figure 4.4. The continuous gray line indicates that EUE3 has a 5.4% higher $\mathrm{avg}(R^2)$ than GEM2. This is as would be expected. GEM2 is estimated on a much broader universe, and therefore its industry and market factors represent global averages that fall short of capturing the level of regional detail offered by EUE3.

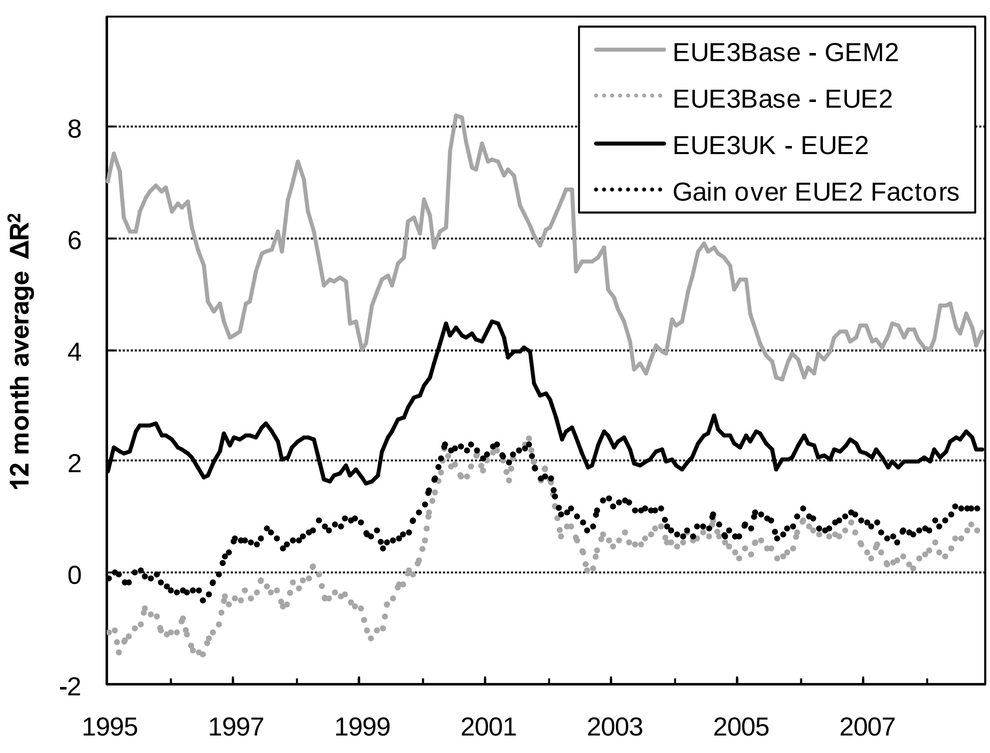
The dotted gray trace in Figure 4.4 compares the EUE3 base model with EUE2, indicating a moderate advantage of EUE3 with $\mathrm{avg}(\Delta R^2) =$ 0.3%. However, when considering this result, it is important to note that the EUE3 base version uses a *single* set of industries whereas EUE2 has *dual* industries for the UK and for continental Europe. As such, it is quite plausible that the additional industry factors would give EUE2 an advantage over EUE3. Indeed we note that EUE2 has somewhat higher R^2 than EUE3 before 2000, but on average the EUE3 base model outperforms EUE2 even with a single set of industries. This indicates that EUE3 offers considerable quality improvements in data and regression methodology.

A more like-for-like comparison between EUE3 and EUE2 uses the EUE3UK derived model (see chapter 8 for details on the derived models). This comparison is more appropriate because EUE3UK and EUE2 both use a separate set of UK industry factors. The continuous black trace in Figure 4.4 shows that EUE3UK exhibits a considerable R^2 gain over EUE2; the average ΔR^2 between these two models is 2.2%. This strong increase can result from three possible sources:

- Revised factor structure with new industries and styles.
- Revised regression methodology, improved thin industry correction and outlier truncation.
- Revised input data source based on global database.

It is interesting to isolate how much the revised factor structure contributes to the R^2 gain of EUE3UK over EUE2. To explore this question, EUE3UK regressions are run with the EUE2 industry and style factor exposures. These regressions use the same input data and regression methodology as EUE3UK but project onto the factor space of EUE2. The black dotted trace in Figure 4.4 shows the results of this analysis. The average ΔR^2 is now 0.9%, i.e. lower but still significantly positive, which indicates that the new factor space of EUE3 is a relevant improvement over EUE2.

Figure 4.4 Comparison of in-sample explanatory power across different models, trailing 12-month average R^2, Western Europe only.



5. Common Factor Risk

5.1. Modeling Covariance Matrices

Given a history of daily factor returns, EUE3 calculates daily covariance matrix forecasts with an exponentially weighted moving average (EWMA):

$$C_{kl}^{(d)} = cov(f_k, f_l)_t = \sum_{s=t-h}^{t} \lambda^{t-s}(f_{k,s} - \bar{f}_k)(f_{l,s} - \bar{f}_l) \Big/ \sum_{s=t-h}^{t} \lambda^{t-s} \tag{5.1}$$

Here h denotes the sample size and λ denotes the exponential weight, defined as $\lambda = 0.5^{1/\tau}$. Exponential weighting reduces the influence of past observations on the present forecasts; the half-life, τ, determines how quickly a past observation loses influence.

In line with other Barra models, EUE3 uses a shorter half-life for the estimation of factor variances than for the factor correlation estimates. This is based on the notion that correlations tend to be more stable over time than the factor variances. EUE3 is published in two variants that use different sets of half-lives.

Table 5.1 Covariance matrix parameters of EUE3S and EUE3L.

Model Variant	*Variance Half-life*	*Correlation Half-life*	*Sample Size*
EUE3S	90 days	180 days	540 days
EUE3L	250 days	500 days	1500 days

The shorter half-lives of the EUE3S matrices make them more responsive. EUE3S caters to users who look for accurate 1-month ahead risk forecasts and can accept a somewhat higher level of forecast variability. In contrast, EUE3L reacts somewhat less swiftly to recent events and exhibits a lower forecast variability. EUE3L is suitable for use in applications such as long-term buy-and-hold portfolio construction or strategic budgeting of sector and country allocations. However, users should be aware that the slower response of EUE3L tends to result in slightly less accurate forecasts. Chapter 7 provides details about the portfolio risk forecasting performance of both model variants.

All EUE3 risk forecasts are *monthly* volatility estimates. The use of *daily* factor returns in (5.1) necessitates scaling the covariance matrices to monthly horizon. This scaling step needs to account for possible serial correlation in subsequent factor returns, that is, $\mathrm{corr}(f_{k,t}, f_{k,t+\Delta}) \neq 0$. To correct for serial correlation, EUE3 estimates lagged sample covariance matrices and adds them to the contemporaneous matrix, $C^{(d)}$, see Newey and West[1] for details.

[1] Newey, W., and West, K., 1987. "A Simple, Positive Semi-Definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix." *Econometrica*, Vol. 55, No. 3: 703-708.

Specifically, the monthly covariance matrices, $C^{(m)}$, are calculated as:

$$C^{(m)} = 22\left[C^{(d)} + \sum_{\Delta=1}^{D}\left(1-\frac{\Delta}{D+1}\right)\left(C_{+\Delta}^{(d)} + C_{-\Delta}^{(d)}\right)\right] \quad (5.2)$$

Here $C^{(d)}$ denotes the daily contemporaneous covariance matrix introduced in (5.1), and $C_{+\Delta}^{(d)}$ is a Δ days lagged daily covariance matrix. EUE3 includes correction terms up to a maximum lag of $D = 15$.

Regional models need to accommodate that countries occasionally don't trade because of a local holiday. In such instances the daily country factor returns are undefined. Missing daily factor returns pose no problem for the calculation of factor variances but have a more significant effect on the correlation estimates. Here it would be detrimental to drop all factor returns for a day if a few factor returns are missing. Augmenting the factor returns is a preferable solution; EUE3 uses an expectation maximization (EM) algorithm[2] for this purpose. Missing daily factor returns are backfilled by application of the EM algorithm and the augmented returns history is then used to calculate the factor correlations. In addition to the augmentation step, a further modification may be applied to the factor returns to limit their influence on the covariance forecasts. This modification is described in section 5.3.

5.2. Factor Risk Forecasts

This section discusses the characteristics of the EUE3 factor risk forecasts. It provides insight into differences between the L and S variants and presents 1-month ahead bias test results. Bias tests are a standard method for assessing the out-of-sample performance of risk models. Section 7.1 describes in detail how bias tests are calculated and how their results should be interpreted. Essentially, bias tests determine how often the risk forecast of a test item (asset, factor, or portfolio) falls within a 95% confidence interval over a back-testing period. The confidence interval is based on a simplifying assumption of normally distributed returns. However, empirical returns tend to have excess kurtosis, which implies that less than 95% of all observations fall in the confidence interval even for perfect forecasts. To limit the influence of excess kurtosis on the bias tests we primarily use robust bias scores which are explained further in Section 7.1.

Table 5.2 presents robust and raw factor bias tests for the L and S variants of EUE3. The robust scores shown in this table highlight that EUE3S provides accurate factor risk forecasts across equity and currency factors; 92% of all samples are within confidence. EUE3L has more samples out of confidence. This loss of accuracy is attributed to the longer half-lives which lead to smoother forecasts that adapt less quickly to sudden changes.

[2] Dempster, A.P., Laird, N.M., and D.B. Rubin, 1977. "Maximum Likelihood from Incomplete Data via the EM Algorithm." *Journal of the Royal Statistical Society*, Vol. 39, No. 1: 1-38.

Bias tests results for EUE2 are also included in Table 5.2. Test results for the equity factors indicate that EUE3 provides considerably more accurate forecasts than EUE2. The currency block in the EUE2 covariance matrices uses a GARCH model and applies the same parameters to the L and S variants. Therefore, the test results are identical for the currency factors of EUE2S and L. Robust scores indicate that the EUE2 currency model is less accurate than EUE3S but has slightly more samples in confidence than EUE3L. Note that EUE3 covers currencies such as the Russian ruble and the Turkish lira which frequently exhibit return outliers. EUE2 does not cover these currencies and therefore has an advantage in the comparative factor bias tests. The difference between the robust and raw currency bias scores is an indication of the presence of return outliers. This difference is much larger for EUE3S than for EUE2S because of the inclusion of Eastern European currencies.

Table 5.2 Robust and raw (in brackets) 12-month rolling factor bias scores, Jan 1997 – Dec 2008. Percentage of all scores within the confidence interval.

		EUE3S		*EUE3L*		*EUE2S*		*EUE2L*	
Equity Factors	Within confidence	92.2	(89.3)	82.7	(79.7)	84.0	(79.7)	74.9	(71.4)
	Over-forecast	3.2	(3.2)	9.2	(9.2)	5.2	(5.2)	11.0	(11.0)
	Under-forecast	4.7	(7.6)	8.2	(11.1)	10.9	(15.1)	14.1	(17.7)
Currency Factors(*)	Within confidence	92.4	(85.8)	86.7	(82.2)	89.0	(87.9)	89.0	(87.9)
	Over-forecast	4.2	(4.2)	7.3	(7.3)	7.3	(7.3)	7.3	(7.3)
	Under-forecast	3.4	(10.0)	6.0	(10.5)	3.7	(4.8)	3.7	(4.8)

(*)excluding pre-euro currencies

While good forecasting accuracy is the primary objective of a risk model, it is not the only characteristic of interest. If clients use risk models to construct risk-optimized portfolios, the forecast variability can also play a significant role. We define the absolute and relative forecast variability as follows:

$$\overline{VA} = \frac{1}{K}\sum_{k} \underset{1\le t\le T}{\mathrm{med}}\left(\left|\sigma_{k,t} - \sigma_{k,t-1}\right|\right) \tag{5.3}$$

$$\overline{VR} = \frac{1}{K}\sum_{k} \underset{1\le t\le T}{\mathrm{med}}\left(\left|\sigma_{k,t} - \sigma_{k,t-1}\right|/\sigma_{k,t-1}\right) \tag{5.4}$$

The sums run over an ensemble of K factors or portfolios, and the median spans a back-testing period of T months.

Table 5.3 shows average values of these two variability indicators for EUE3 and EUE2. We find that the EUE3S exhibits about twice as much variability as the L variant. This variability increase is the cost of the enhanced responsiveness and accuracy of EUE3S. Comparing the equity factors of EUE3 with EUE2 indicates that both models have rather similar variability; EUE3 has slightly higher $\overline{VR}$ while EUE2 has somewhat higher $\overline{VA}$. The situation is different for the currency factors. EUE2 uses the same GARCH model for both variants, and this model shows a rather high variability. In contrast, the variability of the EUE3 currency factors is well aligned with the equity factors. This is most noticeable in EUE3L where there is a similar amount of smoothing in the currency and the equity factors.

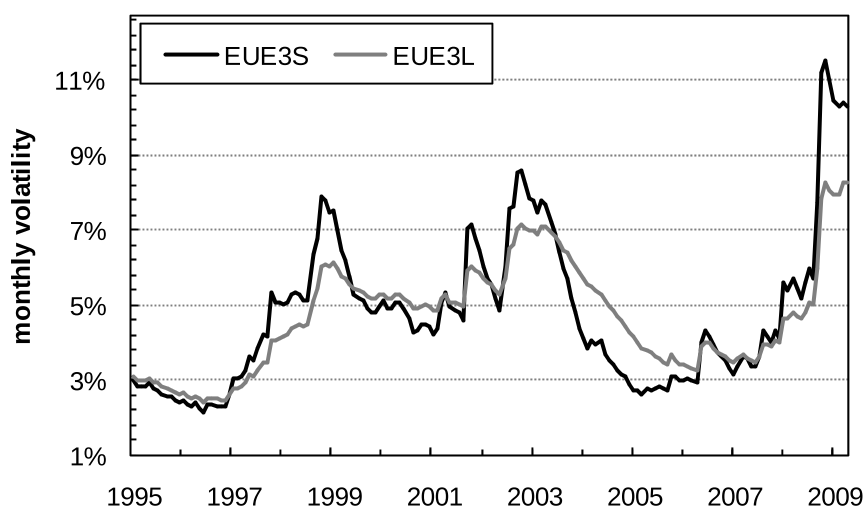
Table 5.3 Variability of factor volatility forecasts, Jan 1997 – Dec 2008.

Variability indicator		*EUE3S*	*EUE3L*	*EUE2S*	*EUE2L*
Equity Factors	abs. variability	0.08%	0.04%	0.14%	0.06%
	rel. variability	4.2%	1.9%	3.8%	1.6%
Currency Factors(*)	abs. variability	0.07%	0.03%	0.08%	0.08%
	rel. variability	4.3%	2.0%	4.6%	4.7%

(*)excluding pre-euro currencies

Figure 5.1 shows the time evolution of the EUE3 market factor volatility for the L and S variants. It illustrates the different responsiveness and variability of both model variants. This figure shows that the European markets have experienced different volatility regimes over the 14-year period covered in EUE3. A low volatility phase in the mid 1990's was followed by the deflation of the technology stocks bubble which brought high volatility in 2000 – 2002. The subsequent low volatility regime lasted until 2007, when the first signs of the credit crisis became apparent. Market shocks such as the August 1998 Russia crisis, the September 2001 events, and the credit crisis in October 2008 are also clearly discernible on Figure 5.1.

Figure 5.1 Volatility of the EUE3 market factor, comparison of S and L model variants.



5.3. Outlier Returns

Large factor returns can lead to a significant over-forecasting bias in subsequent factor risk forecasts. Large returns fall into two classes:

- Isolated *outliers,* which reflect shocks to a market or a market segment. The most extreme example of such an outlier in EUE3 is the Iceland country factor, which experienced a daily return of -42% in October 2008 as a consequence of the collapse of several large banks in the middle of the credit crisis.
- A cluster of large factor returns followed by a sequence of small returns. This is typical for *regime shifts* where a high volatility phase is followed by one of low volatility. The most extreme examples of such return patterns can be found in currency data. The Russian ruble had very high volatility in 1998 but stabilized after devaluation and the volatility rapidly declined.

EUE3 applies a correction to limit the influence of large factor returns on subsequent forecasts. A progressive outlier truncation scheme gradually reduces the amplitude of outliers as their age increases and they become less relevant for the current forecasts.

The EUE3 outlier truncation scheme is based on the notion that some time must pass before a large return observation can be identified either as an isolated outlier or as the start of a high volatility regime. Truncating large returns immediately after they are first observed can lead to an under-forecasting bias if these later prove to be early indicators of a persistent volatility change. EUE3 uses two sample volatilities to define its truncation threshold. In addition to the EWMA volatility estimate, σ_t, which uses the variance half-lives indicated in Table 5.1, EUE3 also calculates an EWMA estimate, $\tilde{\sigma}_t$, which uses a short half-life of 22 days. This fast-changing estimate provides a local perspective on volatility. The threshold for outlier truncation is set to 3.5 times the maximum of the two volatility estimates. Setting the threshold at 3.5 standard deviations was empirically found to maximize the benefits of the truncation.

$$F_{th} = 3.5 \cdot \max(\sigma_t, \tilde{\sigma}_t) \tag{5.5}$$

A large return that marks the onset of a high volatility phase may appear to be an outlier from a long-term perspective but not on the local scale of $\tilde{\sigma}_t$. This implies that the return will not exceed F_{th} and therefore not be altered. Factor returns with $|f_t| > F_{th}$ are modified to prepare forecasts at a later date $T > t$:

$$\tilde{f}_{t,T} = sgn(f_t) \cdot [\kappa|f_t| + (1-\kappa)F_{th}]; \qquad \kappa = \exp\left(\frac{t-T}{\tau}\right); \qquad \tau = 22 \text{ days} \tag{5.6}$$

As the age of an outlier increases, the coefficient κ decays to zero with the characteristic time τ, and the outlier factor return, f_t, is gradually pulled back to F_{th}. The modified factor return, $\tilde{f}_{t,T}$, is then used in (5.1).

Table 5.4 summarizes the impact of the outlier truncation on the factor volatility forecasts. The table shows robust RAD statistics which are explained in section 7.1. A perfect forecast would result in a RAD statistic of about 0.17, and higher RAD statistics indicate a higher average deviation of the forecasts.

Table 5.4 Robust RAD statistics of factor bias tests calculated over an averaging period of 156 months (Jan 1996 – Dec 2008).

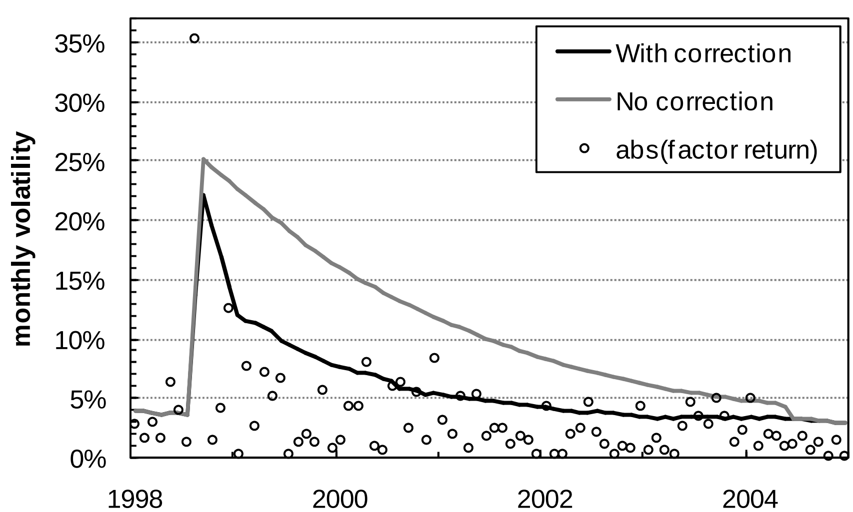
		EUE3S	*EUE3L*
Equity Factors	With outlier truncation	0.186	0.230
	No correction	0.186	0.234
Currency Factors(*)	With outlier truncation	0.198	0.234
	No correction	0.205	0.247

(*)excluding pre-euro currencies

As expected, this correction has a more significant influence on EUE3L. We also note that the accuracy gain is larger on the currency factors than on the equity factors. This is a result of the currency returns tending to have higher excess kurtosis than the equity returns.

Table 5.4 shows averages across all factors, i.e., it includes many factors that are only marginally affected by outliers. The effect of the correction is more significant for factors with strongly varying volatility. For instance, the correction makes the RAD statistic of the Russia country factor in EUE3L decrease from 0.29 to 0.25, and the RAD statistic of the Russian ruble factor decreases from 0.53 to 0.39. Figure 5.2 illustrates that the RUB volatility forecasts of EUE3L would have a long-term persistent upward bias without the correction.

Figure 5.2 RUB volatility estimates of EUE3L with and without correction. Circles represent absolute values of the monthly RUB factor returns.



6. Specific Risk

Specific risk is an idiosyncratic volatility component which cannot be explained by common factors. In the context of a factor model, specific risk is described as sdev(u_n), the standard deviation of the residual returns obtained from the factor regression. EUE3 introduces an entirely new specific risk model which generates considerably more accurate risk forecasts, as will be demonstrated in section 6.3.

6.1. Structural Models

EUE2 uses a structural approach to estimate specific risk. Structural models are built on the following two assumptions:

- Absolute values of the specific returns, $|u_n|$, are proportional to specific risk. This implies that specific risk can be estimated in two steps by separately forecasting $|u_n|$ and the scaling factor, $\text{sdev}(u_n)$ / $|u_n|$.
- Absolute values of the specific returns exhibit common behavior across a broad universe; assets which share certain characteristics have similar specific risk. For example, software stocks tend to have higher specific risk than utility stocks. This means that $|u_n|$ can be modeled by cross-sectional regression against explanatory variables such as industries, countries, and styles.

Robustness is the main strength of structural models. By their very nature, specific returns reflect individual company events such as non-recurring write-offs, acquisitions, or restructurings which can all give rise to return outliers. The influence of such outliers on the forecasts can be largely eliminated by using a structural model which extracts the common characteristics across a broad set of input data. Robustness is particularly relevant if specific risk is estimated from *monthly* residual returns because only limited input data per stock is available.

However, a significant drawback of structural models is that they tie the specific risk of a broad universe of several thousand assets to a small number of parameters. For instance, EUE2 uses 89 parameters to model the specific risk of 8000 stocks. As a result, this dimension reduction misses some detail. Further, if a structural model is estimated from monthly input data, it has limited ability to adapt to sudden changes in specific risk. For example, EUE2 estimates average specific risk with a half-life of 4 months and uses 5 years of equal-weighted monthly data to estimate systematic deviations from this average.

6.2. EUE3 Specific Risk Model

The EUE3 specific risk model primarily uses time series estimation. It exploits the fact that the daily calculation of factor and specific returns provides a sufficiently long data history to estimate $\text{sdev}(u_n)$ separately for each asset with an exponentially-weighted average. Time series estimates with daily data are 1-day forecasts and must be scaled to the monthly horizon. It is important to correct for serial correlation in this scaling step.

The main advantage of a time series-based specific risk model is its idiosyncratic character; risk is estimated individually for every stock. The challenge with using time series estimation is that not all stocks in a broad universe lend themselves to this modeling approach:

- Recent IPOs have only a short history of specific returns, which implies that their sample volatility has a high estimation error.
- Illiquid stocks can have a specific returns distribution that strongly deviates from a normal distribution. When these stocks trade they tend to "catch up" from several days of stale price data.
- Company-specific events can produce large outliers in the specific returns, which may severely bias a time series estimate if the estimate does not control for the influence of outliers.

In these cases EUE3 uses a structural model to provide robust estimates for stocks with specific return histories that are not well-behaved. The remainder of this section provides technical details about the time series and structural components of the EUE3 specific risk model. It also explains how the two components are combined to form the final forecasts.

Estimation begins with the calculation of a robust standard deviation for each asset:

$$\tilde{\sigma}_u = (1/1.35) \cdot (Q_3 - Q_1) \tag{6.1}$$

Here Q_1 and Q_3 are the first and third quartiles of the returns distribution. The sample size is limited to a maximum of 360 points.

Specific returns are then truncated to $\pm 10\tilde{\sigma}_u$ and an equal-weighted sample standard deviation, $\sigma_{u,eq}$, is calculated. The ratio, Z_u, between this sample standard deviation and the robust standard deviation is an indicator to decide whether time series estimation can be used for an asset:

$$Z_u = \left| \frac{\sigma_{u,eq} - \tilde{\sigma}_u}{\tilde{\sigma}_u} \right| \tag{6.2}$$

Outliers in the return series will affect σ_u but leave the robust estimate $\tilde{\sigma}_u$ unaltered. Large values of Z_u indicate a poorly conditioned return series.

Time series forecasts are used for assets with at least 180 days of returns history and $Z_u \leq 1$. If one of these conditions is not fulfilled, the model defines a blending coefficient, γ, and linearly combines the time series and structural forecasts:

$$\gamma = \left[\min\left(1, \max\left(0, \frac{h - 60}{120}\right)\right)\right] \cdot \left[\min\left(1, \max\left(0, \exp(1 - \mathrm{Z_u})\right)\right)\right] \tag{6.3}$$

$$\sigma_u = \gamma \sigma_\mathrm{u}^{(\mathrm{TS})} + (1 - \gamma) \sigma_\mathrm{u}^{(\mathrm{STR})} \tag{6.4}$$

Here h denotes the sample size, and $\sigma_u^{(TS)}$ and $\sigma_u^{(STR)}$ denote time series and structural specific risk forecasts. The first part of (6.3) reduces γ for assets with short time series histories, i.e. less than $h = 180$ points of data. The second part of (6.3) reduces γ for assets with $Z_u > 1$. Only structural forecasts are used if $h \leq 60$.

Time series forecasts of specific risk, $\sigma_u^{(TS)}$, use exponential averaging of daily specific returns:

$$\sigma_u^{(TS,d)} = \left(\sum_{s=t-h}^{t} \lambda^{t-s} (u_s - \bar{u})^2 \Bigg/ \sum_{s=t-h}^{t} \lambda^{t-s} \right)^{1/2} \qquad \lambda = 0.5^{1/\tau} \tag{6.5}$$

The serial correlation correction (5.2) is applied with lags of up to 10 days for scaling these estimates to a monthly horizon. The S and L variants of EUE3 use different specific risk half-lives.

Table 6.1 Specific risk model parameters of EUE3 S and L variants

Model Variant	*Specific Risk Half-Life*	*Sample Size*
EUE3S	65 days	360 days
EUE3L	180 days	360 days

The use of different half-lives is consistent with the factor risk part of EUE3. It renders the specific risk forecasts of EUE3S more responsive than the EUE3L forecasts. This significantly differs from EUE2, where the same specific risk forecasts were combined with the common factor risk forecasts of the S and L variants.

The time series forecasts are used as inputs for the structural specific risk model. This model regresses the logarithm of the time series forecasts against a set of explanatory variables. The regression uses $\sqrt{\text{cap}}$ weights:

$$\log\left(\sigma_{u,n}^{(TS)}\right) = \xi_M + \sum_I X_{nI}\xi_I + \sum_C X_{nC}\xi_C + \sum_S X_{nS}\xi_S + \varepsilon_n \tag{6.6}$$

The industry, country, and style exposures, X_n, are identical to those used in the factor regression, see equation (4.2). The structural specific risk model uses three styles of the factor model (Volatility, Liquidity, and Momentum) and adds the mean of the sample absolute specific returns, $|u_n|$, as a further explanatory variable.

Structural risk forecasts of each stock are calculated by combining the exposures, X_n, with the set of parameters, $\{\hat{\xi}_M, \hat{\xi}_I, \hat{\xi}_C, \hat{\xi}_S\}$, which were estimated in the regression (6.6):

$$\sigma_{u,n}^{(STR)} = \exp\left(\hat{\xi}_M + X_{nI}\hat{\xi}_I + X_{nC}\hat{\xi}_C + \sum_S X_{nS}\hat{\xi}_S \right) \cdot E_0 \tag{6.7}$$

The scale factor, E_0, is defined as the $\sqrt{\text{cap}}$-weighted average of the ratio between time series and structural specific risk forecasts. This scale factor is close to one, and its average over the back-testing period is 1.026. It adjusts for a tiny bias in the structural forecasts which results from the exponentiation of the least squares residuals.

The following table gives an indication of the relative importance of the time series and structural parts in the EUE3 specific risk model.

Table 6.2 Average percentage of assets that use the time series model, the structural model, or a combination of both models. Estimation universe and non-ESTU assets are shown separately.

(1995 - 2008)	$\gamma = 1$ *time series only*	$0 < \gamma < 1$ *combined*	$\gamma = 0$ *structural only*
ESTU	94.3%	5.6%	0.2%
non-ESTU	57.9%	38.9%	5.6%

These results highlight that the time series model is used far more than the structural model; 94% of all estimation universe assets use time series-based forecasts. The higher percentage of combined and structural forecasts for non-ESTU assets is plausible as many assets in this group are illiquid small caps that do not have well-behaved specific returns and therefore cannot be adequately modeled with the time series approach.

6.3. Specific Risk Forecasts

To gain insight into the forecasting accuracy of the EUE3 specific risk model, we calculate bias tests for individual stocks. Bias tests compare the risk forecasts at a given point in time with the realized specific returns one month out-of-sample. Section 7.1 describes in detail how bias tests are calculated and how their results should be interpreted.

Table 6.3 Robust and raw (in brackets) 12-month rolling specific risk bias scores, Jan 1997 – Dec 2008. Note that EUE2 uses the same specific risk model for the S and L variants.

	EUE3S		*EUE3L*		*EUE2S*		*EUE2L*	
Within confidence	92.8	(86.6)	89.6	(83.7)	85.2	(81.6)	85.2	(81.6)
Over-forecast	3.5	(3.5)	4.6	(4.6)	11.4	(11.4)	11.4	(11.4)
Under-forecast	3.8	(10.0)	5.9	(11.7)	3.4	(7.1)	3.4	(7.1)

Table 6.3 displays specific risk bias test results for the estimation universe assets of EUE3. The results illustrate that the shorter half-life of EUE3S improves the forecasting accuracy compared to EUE3L. Table 6.3 also includes bias test results for EUE2. Results are identical for the EUE2L and EUE2S because the specific risk model of EUE2 does not differentiate between L and S

variants. It can be seen that both variants of the EUE3 specific risk model provide more accurate forecasts than EUE2. This improvement is a result of the new approach that blends time series and structural components. If EUE3 forecasts only used the structural component, the bias test results would be comparable to EUE2.

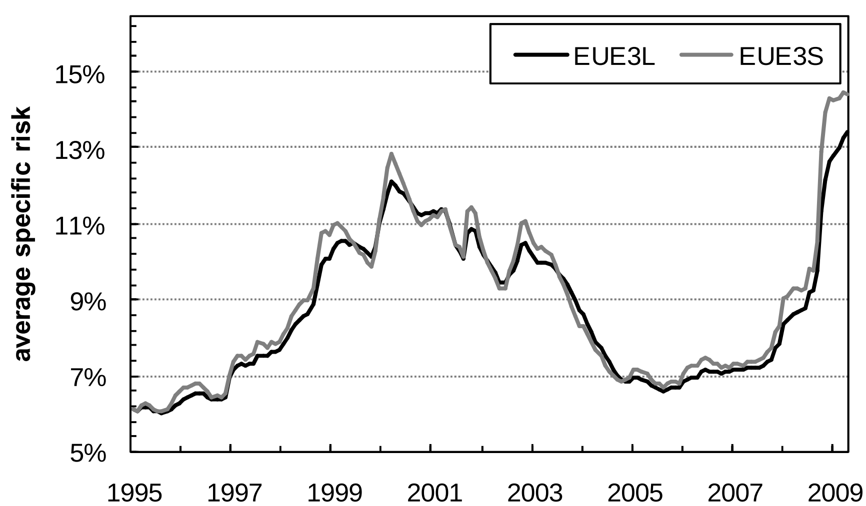
Absolute and relative variability indicators of the specific risk forecasts are shown in Table 6.4. EUE3L has a marginally lower variability than the EUE2 specific risk model. EUE3S uses a shorter half-life and therefore exhibits significantly higher forecast variability than EUE3L.

Table 6.4 Variability indicators of specific volatility forecasts, Jan 1997 – Dec 2008.

Variability indicator	*EUE3S*	*EUE3L*	*EUE2S*	*EUE2L*
abs. variability	0.51%	0.29%	0.35%	0.35%
rel. variability	6.8%	3.9%	4.4%	4.4%

Figure 6.1 shows the time dependence of the average specific risk level of EUE3. It illustrates the difference in responsiveness between EUE3S and EUE3L.

Figure 6.1 Equal-weighted average specific risk across the EUE3 estimation universe.



7. Forecasting Accuracy

7.1. Bias Tests

Bias tests compare risk forecasts with realized risk out of sample. As EUE3 provides monthly forecasts, we test the model's performance over a one-month horizon. Bias tests are calculated for a wide variety of portfolios and over different time periods to establish that a model performs consistently.

The first step of these tests is to calculate out-of-sample z-scores of the entities (assets, factors, portfolios) under investigation:

$$z_{t,q} = \frac{r_{t+q}}{\sigma_t} \tag{7.1}$$

Here σ_t is the risk forecast at time t and r_{t+q} is the realized return of the entity at $t+q$, where the look-ahead period, q, is typically set to one month.

If forecasts are perfect, the z-scores (7.1) have a standard deviation of one. A standard deviation below one is indicative of over-forecasting and a standard deviation above one indicates under-forecasting. The bias statistic, $b_{t,T}$, is defined as the sample standard deviation of the z-scores over a testing period, T:

$$b_{t,T} = \left(\frac{1}{T-1} \sum_{s=t-T+1}^{t} (z_{s,q} - \bar{z}_q)^2 \right)^{1/2} \tag{7.2}$$

The finite size of the testing period implies that, even for perfect forecasts, the bias statistic will deviate from one. Under the simplifying assumption that the returns, r_{t+q}, are normally distributed, a 95% confidence interval can be given for the bias statistic:

$$C_T = \left[1 - \sqrt{2/T}, 1 + \sqrt{2/T} \right] \tag{7.3}$$

This means that, with perfect forecasts and normal returns, 95% of a broad group of bias statistic values fall within the confidence interval, C_T.

It can be seen from (7.3) that the confidence interval gets narrower with increasing T. However, calculating the bias statistic over a long period does not necessarily increase accuracy. Instead, a phase where the model consistently over-forecasts can be followed by another phase where the model under-forecasts, producing an average deceptively close to one. Practitioners who care more about the forecasting accuracy over shorter periods cannot afford to use forecasts that are correct over a long-term horizon if these forecasts exhibit a persistent bias over 1-2 years. For this reason, we present bias test results that are based on rolling 12-month bias statistics.

The main disadvantage of short period bias statistics is a high sensitivity to outliers. Assuming $T = 12$ and $\bar{z}_q = 0$, the upper limit in (7.3) corresponds to a maximum of 21.8 for the sum of the squared z-scores in (7.2). If the sample has a single outlier with $z = 5$, all bias statistics containing this outlier will be out of confidence irrespective of the value of the other 11 z-scores used. If the sample contains a slightly less severe outlier with $z = 4$ but all other z-scores are exactly one, the bias statistic still falls out of confidence. As such we see that the outlier completely masks the perfect forecasts in its vicinity.

This shortcoming is related to the assumption of normally distributed returns in (7.3). If it was possible to precisely model the size and frequency of outliers, the confidence interval could be stretched accordingly to take into account the outlier distribution. An alternative way to limit the influence of outliers is by winsorizing the z-scores:

$$\tilde{z}_{t,q} = \max\left(-3, \min\left(+3, z_{t,q}\right)\right) \tag{7.4}$$

Robust bias statistics are calculated using winsorized z-scores, $\tilde{z}_{t,q}$. Robust bias statistics probe the ability of the model to predict the core of the returns distribution. They are much less affected by outliers than the raw bias statistics which use un-truncated z-scores. Simulations can provide further insight into how outliers affect the raw and robust bias statistics. For the raw bias statistics, we find that the inclusion of outliers rapidly decreases the confidence level of (7.3) from the theoretical 95% level to <90%. In contrast, the confidence level of the robust bias statistics decreases only marginally.

In spite of their shortcomings, raw bias statistics reflect the reality that outliers do occur in practice and cannot be ignored by portfolio managers. The difference between the results of raw and robust bias statistics provides valuable additional insight about the frequency and severity of outliers. When working with raw bias statistics it is important to consider that covariance models cannot predict the timing and size of future outliers and will always tend to under-forecast when an outlier is observed. The frequency and severity of outliers should be estimated separately rather than expecting them to be forecast by a covariance model.

A valuable extension of the bias statistic is the RAD statistic. Calculation of this statistic starts with the absolute deviation of a bias statistic from the ideal value of one:

$$AD_{t,T} = \left|b_{t,T} - 1\right| \tag{7.5}$$

By taking the absolute value, the differentiation between under-forecasting and over-forecasting is dropped; $AD_{t,T}$ penalizes both deviations from the ideal value of one. The RAD statistic is defined as the average of $AD_{t,T}$ over a back-testing period, $[t_0, t_1]$:

$$RAD_{t_0,t_1,T} = \frac{1}{t_1 - t_0} \sum_{t=t_0}^{t_1} AD_{t,T} \tag{7.6}$$

For a long back-testing period and assuming normally distributed returns, the RAD statistic is centered at about 0.17. The 95% confidence level for a test period of 150 months corresponds to $RAD \approx 0.22$. Outliers affect the RAD statistics in the same way as the bias statistics; the confidence level of 0.22 will be significantly lower than 95% in the presence of outliers. To address this we use winsorized z-scores to calculate robust RAD statistics which can, to a good approximation, be interpreted using the confidence interval derived with the assumption of normal returns.

7.2. Test Portfolios

Bias statistics and RAD statistics are calculated for a broad range of European portfolios to assess the forecasting performance of EUE3. The following portfolio groups are investigated:

- Cap-weighted and equal-weighted European industry portfolios. These portfolios contain all estimation assets within a single EUE3 industry.
- Cap-weighted and equal-weighted European country portfolios, containing all estimation assets within a single country.
- Small-cap and large-cap part of European country and industry portfolios. Assets within each category are sorted by their cap. The largest 50% of assets constitute the large-cap category portfolio, whilst the bottom 50% constitute the small-cap portfolio.
- Cap-weighted and equal-weighted top and bottom quintiles of the EUE3 style exposures. These portfolios are formed by ranking the exposures of all estimation assets to an EUE3 style factor and keeping the top and bottom 20% to form the quintiles.
- Cap-weighted and equal-weighted random portfolios with 20, 50, 100, and 200 assets. Portfolio constituent assets are picked at random and kept in the portfolio as long as an asset is in the estimation universe. Assets that drop out of the estimation universe are replaced by new random draws.

The industry, country, and style portfolios are aligned with the factor structure of EUE3; they test the ability of the model to handle strong factor bets. In contrast, the random portfolios ignore the EUE3 factor structure. As clients may decide not to align their portfolios with the EUE3 factors, it is important to test random portfolios in addition to the factor-aligned portfolios. Cap-weighted portfolios put more emphasis on large cap stocks and roughly mimic index portfolios. Equal-weighted portfolios are more diversified than their cap-weighted counterparts but they overweight small and mid-cap stocks.

Bias tests are run for total risk (volatility) and for active risk (tracking error). For active risk we benchmark against the EUE3 estimation universe. Cap-weighted test portfolios are benchmarked with the cap-weighted ESTU, and the equal-weighted ESTU is used for equal-weighted test portfolios.

7.3. Bias Test Results

Table A.9 and Table A.10 summarize the total risk bias test results. Table A.9 shows robust and raw bias statistics for each portfolio group; country and industry categories also include the small-cap and large-cap category portfolios. It provides clear evidence that EUE3S delivers high quality one-month forecasts. The robust bias statistics indicate that 89% – 93% of all portfolios fall within the confidence interval (7.3). Results are similarly accurate for total risk and for active risk. Equal-weighted portfolios that have more small cap exposure also perform well in the tests.

As mentioned in section 5.1, EUE3L uses longer half-lives and provides forecasts with less variability while accepting a slight loss of accuracy. Table A.9 indicates that between 82% and 90% of the EUE3L total risk forecasts fall within the confidence interval. Typically, about 5-6% more bias statistics are mis-forecast with EUE3L than with EUE3S. In addition to the robust bias statistics, this table also includes raw bias test results. As explained in the preceding section, the presence of outliers lowers the confidence level of (7.3) for the raw bias scores. We see that EUE3S has between 86% and 91% of all raw bias scores in confidence. That is, the presence of outliers lowers the confidence level by about 2 - 4%. The test portfolios are affected to a different extent by outliers, hence the difference between robust and raw bias statistics varies by portfolio.

Table A.10 shows robust and raw RAD statistics for all portfolio groups. In line with the high precision seen in the bias tests, the robust RAD statistics of EUE3S only marginally exceed the theoretical ideal value of 0.17. Robust RAD statistics of EUE3L fall between 0.20 and 0.26, again indicating a decrease of precision compared to the EUE3S. The raw RAD statistics are influenced by outliers in the same way as the raw bias statistics; therefore their values exceed the robust statistics.

Table A.11 and Table A.12 provide bias test and RAD statistics results for common factor risk. These tests compare predicted common factor risk with realized common factor return one month out of sample. While the total risk bias tests are of primary relevance for most users, common factor bias tests give complementary insight as they isolate the model’s ability to forecast factor volatilities and correlations. Results in Tables A.11 and A.12 closely correspond with the results for total risk shown in Tables A.9 and A.10, indicating high accuracy of EUE3S, and a slight loss of accuracy for EUE3L. The somewhat higher RAD statistics of the random portfolios in EUE3L is a consequence of temporarily over-forecasting market risk in 2003 and 2004 after the rapid volatility drop in late 2002. Random portfolios exhibit a risk profile which rather closely resembles the market factor because their exposures to non-market factors tend to be lower than for the country, industry, and style portfolios. Therefore the random portfolios are more influenced by a temporary forecasting bias in the market factor than the other portfolio groups.

For reference purposes, Tables A.13 – A.18 provide detailed total risk bias test results for the cap-weighted industry, country, style, and random portfolios. In addition to listing the percentage of all observations in confidence, Tables A.13 – A.15 also indicate how often the models under-forecast or over-forecast portfolio risk. The relatively high percentage of observations in confidence for some portfolios is simply a consequence of the rather small number of rolling bias statistics per portfolio; these cases should not be misinterpreted as a sign of the model doing exceptionally well. Tables A.16 – A.18 show portfolio-level details of the robust and raw RAD statistics and again confirm that EUE3 provides a high forecasting accuracy over the back-testing period from 1996 to 2008.

Tables A.19 and A.20 compare EUE3 against the predecessor model, EUE2, and against the Barra Global Equity Model, GEM2. All test portfolios used for these results span only countries in Western Europe and the tests start in 1997. These tables highlight that a regional model can provide more accurate forecasts for regional portfolios than a global model, and therefore, why clients may want to use different risk models for portfolios with a different country scope. The robust bias tests indicate that EUE3S has on average 4 - 7% more portfolios in confidence than GEM2S, the accuracy difference between the two L variants is of similar magnitude. The RAD statistics shown in Tables A.21 and A.22 confirm this difference in forecasting accuracy. Table A.21 highlights that for total risk, GEM2S and L both have 3% more portfolios in confidence than the EUE2 models. For active risk, GEM2 matches EUE2 in accuracy. The accuracy improvements of EUE3 over EUE2 are considerable. They are the combined result of a refined modeling approach and better input data. Overall, the comparative bias tests demonstrate the strong performance of EUE3 on a broad range of portfolios.

8. Derived Model Versions

8.1. Overview

Derived model versions offer clients the option to select different levels of granularity in the factor structure of EUE3. Users who build diversified European portfolios are well served by the single set of pan-European industry factors in the EUE3 base version because this structure matches their regional approach to industry allocation. However, some users may prefer a *home-centric* approach to portfolio construction; they may overweight their home market and take small cap exposure in home-market stocks, whilst concentrating only on the large caps in the rest of the region. This approach to portfolio construction treats the home-market portfolio as a separate entity. If a home market is broad enough to support its own set of industry factors, adding granularity to a regional model by splitting the industries can be beneficial for home-centric portfolios.

MSCI Barra provides three different versions of EUE3. In addition to the base version, users can select between two derived models which use *dual* market factors and *dual* sets of 29 industry factors; EUE3UK and EUE3EE. All models are published in short-horizon and long-horizon variants.

Model Version	*Market Factors*	*Industry Factors*
EUE3	1 Western Europe	29 Western Europe industries
EUE3UK	1 United Kingdom 1 Continental Europe	29 UK industries 29 Continental Europe industries
EUE3EE	1 Western Europe 1 Eastern Europe	29 Western Europe industries 29 Eastern Europe industries

EUE3UK is the derived model with UK market focus. The United Kingdom is the largest market in Europe. As of December 2008 it represented about 22% of the total estimation universe cap of EUE3. Even though the UK market exhibits increased convergence with the rest of Western Europe, this convergence is not perfect. Adding separate UK industries can therefore enhance the explanatory power of the model for UK-centric portfolios. As will be shown in section 8.3, this improvement is mostly relevant for portfolios with significant exposure to UK mid cap and small cap stocks. The predecessor model, EUE2, also used separate UK industry factors. Therefore, users that prefer a deeper level of detail in the UK market can replace EUE2 with EUE3UK, keeping the familiar dual industry structure, while benefiting from all improvements of the EUE3 model series.

The second derived model, EUE3EE, provides an enhanced level of detail for Eastern European stocks. The EUE3 base version explains the industry effects of Eastern European stocks by projecting Western European industry factor returns onto Eastern European stocks. This approach can, however, miss some characteristic differences between the Western European and Eastern European industries. EUE3EE may be considered by clients who build portfolios with significant exposure to Eastern Europe or who aim to analyze market risk in this region in detail. The performance of EUE3EE is equivalent to the EUE3 base version on a Western European universe; therefore there is no need for clients to subscribe the Eastern Europe derived model if they only invest in Western European markets.

Diversification within each industry factor also plays a role when deciding which model version is best for a given use case. Derived versions provide extra granularity within their home region, but this comes at the cost of adding some industries with only few constituents. For example, the UK Airlines industry had just two constituents in December 2008. To avoid over-fitting the returns of individual stocks within such thin industries, the EUE3 regression code applies a Bayesian correction which pulls the thin industry factor returns towards their complement industry. This makes the thin industry returns closely track the returns of their complement industry. For instance, the correlation between the UK Airlines and Continental Europe Airlines factors exceeded 85% in 2008. Such high correlations between industry factors may not pose problems for risk forecasting, but they may prove inconvenient in portfolio optimization. EUE3 derived models may be used by clients who wish to obtain the most detailed risk forecasts of portfolios with a home-market bias. For optimization, we believe that many clients may prefer working with the base version of EUE3.

The EUE3 model estimation code allows for flexibility in the choice of a home region. If there is future client demand for derived versions with other home regions, the EUE3 family may be expanded with the addition of further derived models. Potential home regions of interest might include the Nordic markets and/or the Eurozone (all countries that use the euro).

8.2. Regression with Dual Market and Industry Factors

The regression scheme of the derived models is identical to that of the base version (see section 4.1), except for one main difference. The dual market and industry factors of the derived models require two additional constraints to control for collinearities in the factor exposures. The UK-derived model uses the following four constraints:

$$\sum_{\substack{C \\ n\in core, n\notin UK}} W_{nC} f_C = 0 \qquad \sum_{\substack{I \\ n\in core, n\notin UK}} W_{nI} f_I = 0. \tag{8.1}$$

$$f_{UK} = 0 \qquad \sum_{\substack{I \\ n\in core, n\in UK}} W_{nI} f_I = 0.$$

Here W_{nC} is the cap of asset n in country C, W_{nI} is the cap of asset n in industry I, and f_C and f_I are country and industry factor returns. The UK country factor, f_{UK}, is replaced by the home market factor in EUE3UK. However, for reasons concerning compatibility with the other model versions, EUE3UK adds a placeholder that is uniformly set to zero.

The Eastern Europe derived model also uses four constraints:

$$\sum_{\substack{C \\ n \in East}} W_{nC} f_C = 0 \qquad \sum_{\substack{I \\ n \in East}} W_{nI} f_I = 0.$$

$$\sum_{\substack{C \\ n \in core, n \in West}} W_{nC} f_C = 0 \qquad \sum_{\substack{I \\ n \in core, n \in West}} W_{nI} f_I = 0. \tag{8.2}$$

Here the notation is the same as in (8.1). Country and industry factors separately sum to zero within Eastern Europe and within Western Europe.

Similar to the base model, the derived versions distinguish between core and non-core countries. Style factors are estimated only on the core countries. The UK-derived model estimates the continental Europe market and industry factors only on the core countries, whereas EUE3EE deviates from this scheme by estimating a set of industries on the Eastern European countries that are all outside the core universe.

A slightly modified thin category correction is used in the derived models. Here the complementary regional industry serves as a prior for thin home-region industries. For example, the Continental Europe Airlines industry is used as a prior in the thin category correction of the UK Airlines industry. If the complementary regional industry is also thin, the regression uses a pan-European sector as a second-stage prior.

Figures 8.1 and 8.2 compare trailing 12-month average R^2 data for the derived and base models. Figure 8.1 indicates that the additional factors of the UK-derived model lead to an average R^2 increase of 1.9% over the estimation universe. Note that for the period 1995 – 2000 the R^2 difference is significantly higher. This illustrates that in the late nineties, the UK market was still less integrated with the rest of Europe, but the degree of integration significantly increased from 2000 onwards. Figure 8.1 also shows that EUE3UK has a high R^2 gain for UK assets, which highlights the additional in-sample explanatory power of the derived model within its own home region. Figure 8.2 presents R^2 difference data for EUE3EE. The Eastern Europe derived model gains an average 1.1% over the entire estimation universe, and again the R^2 increase is much higher within Eastern Europe. R^2 comparisons over the estimation universe put the derived models at an advantage as these models use 29 more degrees of freedom than the base models to fit the same data. The adjusted R^2 coefficient compensates for this imbalance. We find that the average adjusted R^2 difference between EUE3UK and the base model is 0.9%. The adjusted R^2 difference between EUE3EE and the base model is only 0.1%, indicating that the addition of separate industries is more valuable for the broadly diversified UK market than for the somewhat heterogeneous Eastern European markets. There is no need to use adjusted R^2 for the tests which include only the home region because the base and derived models use the same number of factors to fit asset returns within their home region.

Figure 8.1 Trailing 12-month average R^2 difference between EUE3UK and the EUE3 base model calculated over the estimation universe (black trace) and over the UK part of the estimation universe (gray trace), respectively.

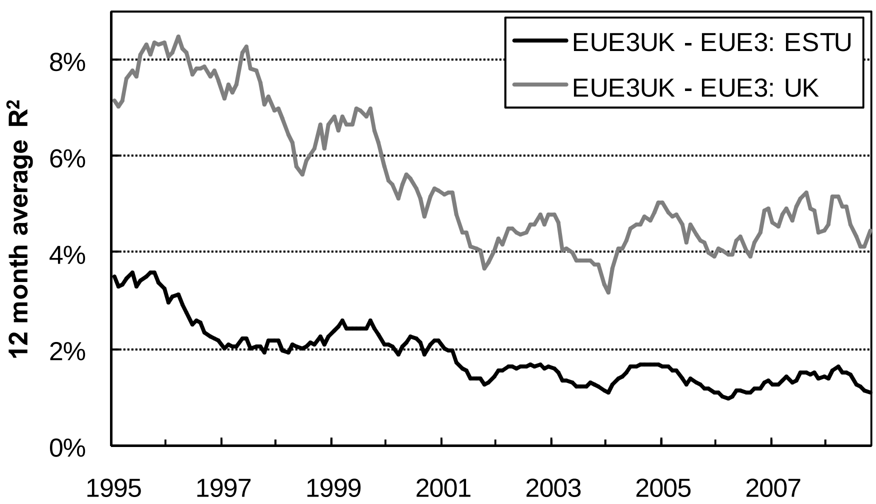
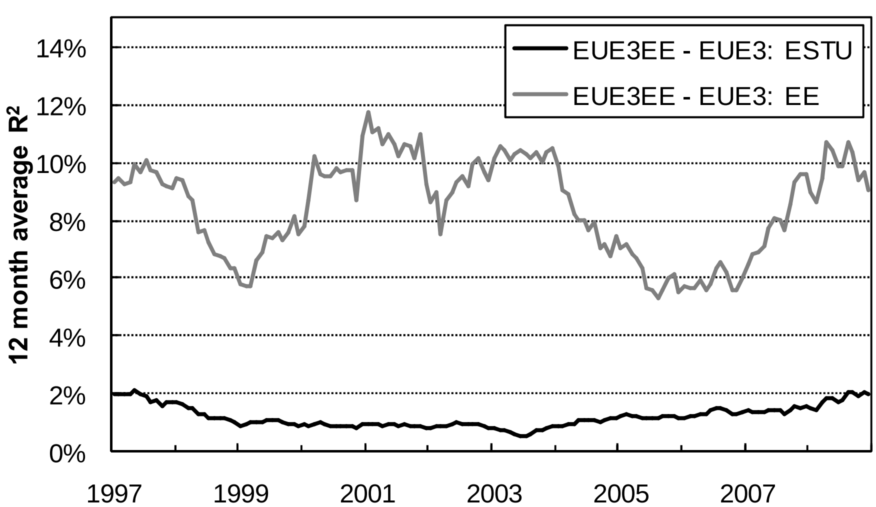


Figure 8.2 Trailing 12-month average R^2 difference between EUE3EE and the EUE3 base model, calculated over the estimation universe (black trace) and over the Eastern European part of the estimation universe (gray trace), respectively.



8.3. Forecasting Accuracy of Derived Models

Portfolio bias tests have been calculated for the derived model versions. Tables A.21 and A.22 summarize the rolling 12-month bias statistics and the RAD statistics of pan-European industry, country, style, and random portfolios. The results are directly comparable with the test scores of the EUE3 base model shown in Tables A.9 and A.10. We find that both derived versions perform consistently well on the pan-European portfolios. The accuracy differences between base and derived models are rather small for these portfolios. This indicates that the use of a derived model rather than the base model to forecast pan-European portfolios is more a matter of preference than a necessity.

Bias tests for UK and continental European industry portfolios give insight into the out-of-sample accuracy gain of EUE3UK compared to the base model. The benchmarks for these tests are the UK and Western Europe ex-UK parts of the EUE3 estimation universe. Results are shown in Tables A.23 and A.24. It can be seen that the UK-derived model, on average, outperforms the base model for these industry portfolios. Improvements are most significant in active risk of UK industry portfolios; here EUE3UKS has about 8-9% more observations in confidence than EUE3S. Somewhat smaller accuracy improvements are also seen for the active risk forecasts of the Europe ex-UK portfolios. Overall, these results highlight the strength of the UK-derived model: it can boost accuracy for UK-centric portfolios and can be used by institutional investors who build regional portfolios with a strong focus on the UK assets.

9. Conclusion

The Barra EUE3 model family offers portfolio managers and risk analysts a powerful combination of scope, accuracy, intuitive structure, and flexibility. The inclusion of Eastern Europe reflects recent economic and political changes; it recognizes the reality of a greater European region. The new factor structure treats countries and industries both as offsets to a regional market factor and provides a refined style palette through the introduction of the liquidity and earnings yield styles. Considerable technical refinements have been applied throughout the model from the dual-stage factor regression scheme to the entirely new specific risk model. The relevance of these refinements manifests in improved forecasting accuracy as highlighted in this report. The availability of base and derived models in L and S variants gives users flexibility to choose the model version that most closely matches their investment processes. Overall, the EUE3 model family is well positioned to be used in a broad range of different applications and to become the risk model of choice for European institutional investors.

Appendix A Tables and Figures

Table A.1 Mapping between EUE3 industry factors and GICS codes.

Industry Number	*EUE3 Industry Factor Name*	*GICS Code*
1	Energy Equipment & Services	101010
2	Oil, Gas & Consumable Fuels	101020
3	Other Materials	151010 151020 151030 151050
4	Metals & Mining	151040
5	Other Capital Goods	201010 201020 201040 201050 201070
6	Construction & Engineering	201030
7	Machinery	201060
8	Commercial Services	2020
9	Other Transport	203010 203030 203040 203050
10	Airlines	203020
11	Autos & Components	2510
12	Consumer Durables & Apparel	2520
13	Consumer Services	2530
14	Media	2540
15	Retailing	2550
16	Food & Staples Retailing	3010
17	Food, Beverages & Tobacco	3020
18	Household & Personal Products	3030
19	Healthcare Equipment & Services	3510
20	Pharmaceuticals	3520
21	Banks	4010
22	Diversified Financials	4020
23	Insurance	4030
24	Real Estate	4040
25	Software & Services	4510
26	Technology Hardware & Equipment	4520
27	Semiconductors	4530
28	Telecommunication Services	5010
29	Utilities	5510

Table A.2 EUE3 industry factors. Aggregate industry capitalization and number of stocks as of December 2008.

GICS Sector	*Industry Number*	*EUE3 Industry Factor Name*	*Stocks in ESTU*	*% of ESTU Cap*
Energy	1	Energy Equipment & Services *	38	0.7
	2	Oil, Gas & Consumable Fuels	96	11.7
Materials	3	Other Materials	136	3.6
	4	Metals & Mining	79	3.1
Industrials	5	Other Capital Goods	132	4.6
	6	Construction & Engineering	72	1.4
	7	Machinery	111	1.5
	8	Commercial Services *	90	1.2
	9	Other Transport *	77	1.9
	10	Airlines *	19	0.4
Consumer Discretionary	11	Autos & Components *	40	2.9
	12	Consumer Durables & Apparel *	94	1.7
	13	Consumer Services *	73	1.0
	14	Media *	95	2.3
	15	Retailing *	82	1.3
Consumer Staples	16	Food & Staples Retailing *	30	1.8
	17	Food, Beverages & Tobacco *	110	6.8
	18	Household & Personal Products *	15	1.3
Healthcare	19	Healthcare Equipment & Services *	61	1.2
	20	Pharmaceuticals	75	7.6
Financials	21	Banks *	130	10.4
	22	Diversified Financials *	166	4.2
	23	Insurance *	64	4.6
	24	Real Estate *	166	1.5
Information Technology	25	Software & Services	159	1.2
	26	Technology Hardware & Equipment	74	1.4
	27	Semiconductors *	16	0.3
Telecom	28	Telecommunication Services *	56	7.4
Utilities	29	Utilities *	88	11.2

* denotes industry with one-to-one correspondence to GEM2.

Table A.3 Mapping between EUE3 and EUE2 industries as of January 2008. EUE2 industries representing less than 10% of a EUE3 industry are not shown.

EUE3 Number	*EUE3 Industry Name*	*EUE2 Code*	*EUE2 Industry Name*	*Mapped percentage*
1	Energy Equipment & Services	ENG	Energy	86.9
2	Oil, Gas & Consumable Fuels	ENG	Energy	97.9
3	Other Materials	CHE	Chemicals	54.7
		CNS	Construction	28.4
		BAS	Basic Resources	11.1
4	Metals & Mining	BAS	Basic Resources	98.7
5	Other Capital Goods	IDE	Industrial Equipment	39.7
		ARO	Aerospace & Defense	22.5
		IDD	Industrial Diversified	13.4
6	Construction & Engineering	CNS	Construction	90.1
7	Machinery	IDD	Industrial Diversified	45.5
		IDE	Industrial Equipment	31.1
		TRA	Transportation	16.8
8	Commercial Services	SVC	Industrial Services	82.3
9	Other Transport	TRA	Transportation	88.1
10	Airlines	TRV	Travel	93.9
11	Autos & Components	ATO	Automobiles	98.9
12	Consumer Durables & Apparel	TEX	Textiles	80.2
13	Consumer Services	ENT	Entertainment & Leisure	73.3
		TRV	Travel	21.9
14	Media	MED	Media	95.3
15	Retailing	RET	Retail	84.5
16	Food & Staples Retailing	NCG	Non-cyclical Goods & Services	95.0
17	Food, Beverages & Tobacco	FOD	Food	64.7
		DST	Distillers & Brewers	20.9
		TOB	Tobacco	13.2
18	Household & Personal Products	NCG	Non-cyclical Goods & Services	95.4
19	Healthcare Equipment & Services	HCA	Healthcare	84.6
20	Pharmaceuticals	HCA	Healthcare	98.0
21	Banks	BAN	Banks	98.8
22	Diversified Financials	FSV	Financial Services	37.6
		BAN	Banks	34.4
		INV	Investment Trusts	18.0
23	Insurance	INS	Insurance	99.7
24	Real Estate	REA	Real Estate	90.7
25	Software & Services	SOF	Technology Software	84.9
26	Technology Hardware & Equipment	HAR	Technology Hardware	92.6
27	Semiconductors	HAR	Technology Hardware	94.4
28	Telecommunication Services	TEL	Telecommunications	97.3
29	Utilities	UTS	Utilities	95.8

Table A.4 EUE3 country scope. Aggregate country capitalization and number of stocks as of December 2008.

Western Europe

Country Code	*Country Factor Name*	*Number of stocks in ESTU*	*% of ESTU Capt*
AUT	Austria	42	0.9
BEL	Belgium	61	2.0
DEN	Denmark	60	1.6
FIN	Finland	60	1.7
FRA	France	238	17.7
GER	Germany	210	12.6
GRE	Greece	90	1.0
IRE	Ireland	29	0.5
ITA	Italy	168	6.2
NET	Netherlands	76	3.3
NOR	Norway	84	1.6
POR	Portugal	27	0.8
SPA	Spain	105	7.7
SWE	Sweden	123	3.0
SWI	Switzerland	126	9.7
UKI	United Kingdom	482	21.3

Eastern Europe

CZE	Czech-Republic	9	0.5
HUN	Hungary	15	0.2
POL	Poland	96	1.0
RUS	Russia	85	4.8
TUR	Turkey	109	1.4

Frontier Markets

CTA	Croatia	43	0.20
CYP	Cyprus	26	0.05
EST	Estonia	12	0.02
ICE	Iceland	10	0.02
LAT	Latvia	9	0.01
LTH	Lithuania	17	0.04
ROM	Romania	15	0.09
SIA	Slovenia	17	0.11

Table A.5 Regression-weighted average correlations of style and industry factor exposures, January 1996 – December 2008.

	Divyield	Earnyield	Growth	Leverage	Liquidity	Momentum	Size	Value	Volatility
Dividend Yield	1.00	0.40	-0.31	0.10	-0.01	-0.19	0.11	0.26	-0.24
Earnings Yield	0.40	1.00	-0.18	0.05	-0.03	-0.12	0.06	0.29	-0.10
Growth	-0.31	-0.18	1.00	-0.02	-0.01	0.01	-0.16	-0.07	0.25
Leverage	0.10	0.05	-0.02	1.00	0.03	-0.07	0.15	0.17	-0.03
Liquidity	-0.01	-0.03	-0.01	0.03	1.00	0.00	0.30	-0.08	0.26
Momentum	-0.19	-0.12	0.01	-0.07	0.00	1.00	0.04	-0.31	-0.03
Size	0.11	0.06	-0.16	0.15	0.30	0.04	1.00	-0.05	0.05
Value	0.26	0.29	-0.07	0.17	-0.08	-0.31	-0.05	1.00	0.00
Volatility	-0.24	-0.10	0.25	-0.03	0.26	-0.03	0.05	0.00	1.00
Energy Equip. & Svc.	-0.05	-0.01	0.06	0.00	0.03	0.02	-0.06	-0.02	0.05
Oil & Gas	0.00	0.10	-0.01	-0.09	-0.05	0.03	0.11	0.01	0.02
Other Materials	0.06	0.09	-0.05	-0.03	-0.03	-0.02	-0.03	0.08	-0.04
Metals & Mining	0.02	0.11	0.01	-0.05	0.00	0.02	-0.02	0.08	0.08
Other Capital Goods	0.01	0.03	0.00	-0.04	0.02	-0.01	-0.03	0.04	0.03
Construction & Eng.	0.02	0.04	0.02	0.01	0.01	0.02	-0.09	0.09	0.01
Machinery	0.03	0.04	-0.01	-0.06	0.00	0.01	-0.11	0.04	0.02
Commercial Svc.	-0.05	-0.04	0.04	-0.02	0.02	0.00	-0.12	-0.06	0.00
Other Transport	0.01	0.02	-0.03	0.04	-0.03	0.01	-0.04	-0.01	-0.06
Airlines	-0.04	0.00	0.03	0.05	0.00	-0.02	-0.01	0.09	0.04
Autos & Components	-0.01	0.10	0.01	0.01	0.00	0.00	0.06	0.14	0.05
Consumer Durables	-0.01	0.04	0.02	-0.08	-0.01	-0.01	-0.11	0.00	0.01
Consumer Services	0.00	-0.03	0.01	0.02	0.04	0.00	-0.07	-0.01	-0.03
Media	-0.03	-0.11	0.02	0.01	0.06	-0.04	-0.03	-0.13	0.02
Retailing	0.03	0.01	-0.01	-0.05	0.02	-0.01	-0.07	0.01	-0.04
Food Retailing	0.00	-0.02	-0.02	-0.01	0.04	0.00	0.03	0.05	-0.06
Food, Bevg. & Tobacco	0.02	0.00	-0.06	-0.01	0.01	0.00	0.00	-0.05	-0.15
Household Products	-0.03	-0.04	-0.03	-0.03	-0.02	0.00	0.00	-0.06	-0.04
Healthcare Equipm.	-0.08	-0.06	0.03	-0.03	-0.01	0.01	-0.08	-0.07	-0.04
Pharmaceuticals	-0.10	-0.11	0.00	-0.10	0.05	0.02	0.06	-0.16	-0.04
Banks	0.09	0.12	-0.01	0.23	-0.01	0.02	0.29	0.06	0.00
Diversified Financials	0.02	-0.02	-0.03	0.07	-0.06	0.00	0.01	0.02	0.03
Insurance	0.03	0.03	-0.03	0.06	0.01	-0.02	0.16	0.05	0.02
Real Estate	0.06	-0.09	-0.05	0.08	-0.09	0.01	-0.13	0.08	-0.14
Software & Services	-0.15	-0.16	0.14	-0.09	0.02	-0.01	-0.18	-0.11	0.18
Technology Hardware	-0.05	-0.07	0.04	-0.08	0.07	-0.03	-0.06	-0.05	0.12
Semiconductors	-0.09	-0.05	0.08	-0.05	0.07	-0.02	-0.03	-0.04	0.13
Telecom Services	-0.03	-0.06	0.03	0.00	0.04	-0.01	0.14	-0.08	0.05
Utilities	0.09	0.02	-0.06	0.04	-0.07	0.03	0.09	-0.01	-0.13

Table A.6 Characteristics of EUE3 market and industry factors. T-statistics of monthly regressions, average annualized volatility, average return, and Sharpe ratio.

Factor	% with |t|>2	Average t^2	Volatility (ann.)	Return (ann.)	Sharpe Ratio
	December 1995 - December 2008				
Market	93	281.3	16.97	3.00	0.18
Energy Equipm. & Svc.	34	4.7	16.82	2.81	0.17
Oil & Gas	50	12.1	12.44	1.94	0.16
Other Materials	36	6.1	7.30	-3.81	-0.52
Metals & Mining	52	11.5	14.03	-2.43	-0.17
Other Capital Goods	39	4.5	6.08	-2.62	-0.43
Construction & Eng.	30	3.4	7.88	-1.37	-0.17
Machinery	40	5.6	8.22	-3.89	-0.47
Commercial Services	21	3.0	6.91	-1.64	-0.24
Other Transport	23	2.8	6.18	0.06	0.01
Airlines	39	4.9	15.86	-7.79	-0.49
Autos & Components	36	5.6	11.07	-5.03	-0.45
Consumer Durables	37	5.0	8.02	-4.51	-0.56
Consumer Services	27	3.4	7.83	-2.19	-0.28
Media	41	8.3	10.59	0.54	0.05
Retailing	34	5.0	7.95	-4.17	-0.53
Food Retailing	33	4.2	9.96	-2.34	-0.23
Food, Bevg. & Tobacco	41	4.8	6.79	-1.85	-0.27
Household Products	27	3.1	8.97	0.65	0.07
Healthcare Equip. & Svc.	20	2.7	7.88	1.29	0.16
Pharmaceuticals	39	6.6	9.67	3.01	0.31
Banks	52	9.8	7.38	-2.35	-0.32
Diversified Financials	34	4.3	5.61	-0.88	-0.16
Insurance	50	7.7	8.74	-1.81	-0.21
Real Estate	40	7.7	8.64	-2.85	-0.33
Software & Services	39	9.4	14.00	4.52	0.32
Technology Hardware	41	5.8	11.07	0.12	0.01
Semiconductors	46	7.8	16.09	4.24	0.26
Telecom Services	55	13.7	13.10	8.17	0.62
Utilities	45	6.1	7.25	1.85	0.26

Table A.7 Characteristics of EUE3 country factors. T-statistics of monthly regressions, average annualized volatility, average return, and Sharpe ratio.

Factor	% with |t|>2	Average t^2	Volatility (ann.)	Return (ann.)	Sharpe Ratio
	December 1995 - December 2008				
Austria	22	2.7	8.65	-0.33	-0.04
Belgium	31	4.4	8.01	-1.07	-0.13
Czech Republic	38	5.0	17.00	-2.64	-0.16
Denmark	37	4.9	8.92	1.98	0.22
Finland	43	5.8	10.66	2.86	0.27
France	48	9.6	5.14	2.48	0.48
Germany	48	9.3	6.18	0.52	0.08
Greece	71	39.0	27.60	-6.26	-0.23
Hungary	31	5.1	21.00	-2.62	-0.12
Ireland	29	3.7	11.02	0.76	0.07
Italy	53	17.3	11.31	1.59	0.14
Netherlands	29	4.0	5.77	-2.52	-0.44
Norway	41	6.4	10.78	-1.58	-0.15
Poland	48	11.5	23.22	-2.71	-0.12
Portugal	31	4.2	11.77	-0.83	-0.07
Russia	78	42.8	41.04	3.94	0.10
Spain	49	9.8	9.04	3.12	0.34
Sweden	46	7.0	7.12	3.21	0.45
Switzerland	33	5.0	5.92	0.69	0.12
Turkey	81	76.2	45.37	-0.42	-0.01
United Kingdom	64	21.3	5.37	-1.92	-0.36
	December 2004 - December 2008				
Croatia	47	7.0	19.55	0.74	0.04
Cyprus	35	5.5	20.58	-2.34	-0.11
Estonia	24	3.5	19.90	-9.89	-0.50
Iceland	19	2.5	14.85	-9.17	-0.62
Latvia	27	2.7	17.71	-11.89	-0.67
Lithuania	24	2.8	18.43	-9.22	-0.50
Romania	53	11.2	20.79	-4.90	-0.24
Slovenia	28	3.7	17.11	-3.95	-0.23

Table A.8 Characteristics of EUE3 style factors. T-statistics of monthly regressions, average annualized volatility, average return, and Sharpe ratio for two sub-periods.

Factor	% with |t|>2	Average t^2	Volatility (ann.)	Return (ann.)	Sharpe Ratio
	December 1995 - December 2001				
Momentum	64	15.2	4.86	5.21	1.07
Size	48	10.8	3.25	0.86	0.27
Volatility	66	27.9	7.15	-1.96	-0.27
Liquidity	32	4.2	1.74	2.66	1.53
Earnings Yield	33	4.7	2.42	2.21	0.91
Value	30	4.1	2.08	2.22	1.07
Dividend Yield	19	2.2	1.54	1.64	1.07
Leverage	8	1.5	1.03	0.22	0.21
Growth	16	2.1	1.39	-0.93	-0.67

	% with |t|>2	Average t^2	Volatility (ann.)	Return (ann.)	Sharpe Ratio
	January 2002 - December 2008				
Momentum	60	14.9	3.50	5.67	1.62
Size	58	10.8	2.51	-0.83	-0.33
Volatility	68	26.1	6.24	-2.51	-0.40
Liquidity	31	3.5	1.68	0.09	0.05
Earnings Yield	42	6.6	2.30	1.14	0.50
Value	40	6.1	2.13	1.60	0.75
Dividend Yield	32	4.3	1.70	1.70	1.00
Leverage	25	3.3	1.29	-0.94	-0.73
Growth	27	3.7	1.84	-1.44	-0.78

Table A.9 Summary of EUE3 portfolio bias tests, total risk, Jan 1996 – Dec 2008. Results of robust and raw (in brackets) 12-months rolling bias statistics. Table shows percentage of all portfolios in confidence.

1996 - 2008	EUE3S		EUE3L	
cap-weighted	*Total Risk*	*Active Risk*	*Total Risk*	*Active Risk*
All Portfolios	**91.4** (89.7)	**91.6** (89.4)	**83.9** (80.9)	**85.9** (83.8)
Country Portfolios	92.5 (89.9)	91.7 (89.6)	86.0 (83.4)	86.0 (84.2)
Industry Portfolios	92.7 (90.4)	92.1 (90.0)	85.4 (82.1)	85.5 (83.6)
Style Portfolios*	90.2 (89.0)	89.4 (87.0)	82.0 (78.6)	81.8 (79.2)
Random Portfolios	90.2 (89.3)	93.1 (91.1)	82.0 (79.7)	90.3 (88.1)
equal-weighted	*Total Risk*	*Active Risk*	*Total Risk*	*Active Risk*
All Portfolios	**93.0** (87.8)	**91.6** (88.8)	**84.4** (80.0)	**86.8** (84.0)
Country Portfolios	92.8 (90.0)	90.8 (88.3)	86.2 (83.6)	86.9 (84.7)
Industry Portfolios	92.8 (88.2)	91.2 (90.0)	85.3 (81.3)	86.4 (84.8)
Style Portfolios*	93.8 (85.9)	91.9 (87.3)	84.6 (77.2)	85.0 (80.7)
Random Portfolios	92.8 (87.0)	92.5 (89.7)	81.7 (77.7)	88.8 (85.9)

*top and bottom quintiles

Table A.10 Summary of EUE3 portfolio bias tests, total risk, Jan 1996 – Dec 2008. Results of robust and raw (in brackets) RAD statistics.

1996 - 2008	EUE3S		EUE3L	
cap-weighted	*Total Risk*	*Active Risk*	*Total Risk*	*Active Risk*
All Portfolios	**0.18** (0.20)	**0.18** (0.19)	**0.24** (0.25)	**0.22** (0.23)
Country Portfolios	0.17 (0.19)	0.18 (0.19)	0.21 (0.24)	0.21 (0.22)
Industry Portfolios	0.18 (0.20)	0.18 (0.19)	0.23 (0.25)	0.22 (0.23)
Style Portfolios*	0.19 (0.20)	0.19 (0.20)	0.25 (0.26)	0.25 (0.26)
Random Portfolios	0.20 (0.20)	0.18 (0.19)	0.26 (0.26)	0.20 (0.21)
equal-weighted	*Total Risk*	*Active Risk*	*Total Risk*	*Active Risk*
All Portfolios	**0.19** (0.21)	**0.19** (0.20)	**0.23** (0.26)	**0.22** (0.24)
Country Portfolios	0.18 (0.20)	0.18 (0.20)	0.21 (0.24)	0.21 (0.22)
Industry Portfolios	0.19 (0.22)	0.20 (0.20)	0.24 (0.26)	0.23 (0.24)
Style Portfolios*	0.18 (0.22)	0.19 (0.21)	0.23 (0.28)	0.24 (0.26)
Random Portfolios	0.20 (0.22)	0.18 (0.20)	0.25 (0.28)	0.20 (0.22)

*top and bottom quintiles

Table A.11 Summary of EUE3 portfolio bias tests, factor risk, Jan 1996 – Dec 2008. Results of robust and raw (in brackets) 12-months rolling bias statistics. Table shows percentage of all portfolios in confidence.

1996 - 2008	EUE3S		EUE3L	
cap-weighted	*Factor Risk*	*Active F. Risk*	*Factor Risk*	*Active F. Risk*
All Portfolios	**91.1** (88.9)	**93.2** (91.1)	**84.1** (79.4)	**83.9** (81.7)
Country Portfolios	92.5 (90.3)	93.8 (91.5)	86.4 (82.8)	85.3 (83.4)
Industry Portfolios	92.6 (88.4)	93.3 (91.2)	83.3 (78.1)	84.5 (82.0)
Style Portfolios*	89.9 (88.5)	92.6 (90.5)	83.1 (78.3)	81.4 (79.1)
Random Portfolios	89.4 (88.6)	93.2 (91.0)	83.5 (78.2)	84.5 (82.4)
equal-weighted	*Factor Risk*	*Active F. Risk*	*Factor Risk*	*Active F. Risk*
All Portfolios	**93.4** (86.8)	**93.1** (89.9)	**83.0** (77.0)	**83.9** (80.5)
Country Portfolios	93.6 (90.6)	92.5 (91.0)	85.7 (82.5)	84.0 (82.4)
Industry Portfolios	92.7 (86.6)	93.1 (91.0)	84.0 (78.4)	85.0 (82.7)
Style Portfolios*	94.0 (85.4)	93.5 (89.2)	83.1 (74.7)	82.3 (77.4)
Random Portfolios	93.4 (84.7)	93.4 (88.6)	79.1 (72.5)	84.4 (79.6)

*top and bottom quintiles

Table A.12 Summary of EUE3 portfolio bias tests, factor risk, Jan 1996 – Dec 2008. Results of robust and raw (in brackets) 12-months rolling bias statistics. Table shows percentage of all portfolios in confidence.

1996 - 2008	EUE3S		EUE3L	
cap-weighted	*Factor Risk*	*Active F. Risk*	*Factor Risk*	*Active F. Risk*
All Portfolios	**0.19** (0.20)	**0.18** (0.19)	**0.25** (0.26)	**0.23** (0.25)
Country Portfolios	0.17 (0.19)	0.17 (0.19)	0.22 (0.25)	0.21 (0.22)
Industry Portfolios	0.19 (0.21)	0.17 (0.19)	0.25 (0.27)	0.23 (0.25)
Style Portfolios*	0.19 (0.20)	0.18 (0.20)	0.25 (0.27)	0.26 (0.27)
Random Portfolios	0.19 (0.20)	0.18 (0.19)	0.26 (0.27)	0.23 (0.25)
equal-weighted	*Factor Risk*	*Active F. Risk*	*Factor Risk*	*Active F. Risk*
All Portfolios	**0.19** (0.22)	**0.18** (0.20)	**0.24** (0.27)	**0.24** (0.26)
Country Portfolios	0.17 (0.20)	0.18 (0.19)	0.21 (0.25)	0.22 (0.23)
Industry Portfolios	0.19 (0.22)	0.18 (0.19)	0.24 (0.27)	0.23 (0.25)
Style Portfolios*	0.19 (0.23)	0.18 (0.20)	0.24 (0.28)	0.26 (0.29)
Random Portfolios	0.21 (0.24)	0.18 (0.20)	0.27 (0.29)	0.23 (0.26)

*top and bottom quintiles

Table A.13 Robust rolling 12-month bias tests of EUE3 industry portfolios. Table indicates the percentage of bias statistics in confidence. Percentage of over-forecasts (+) and under-forecasts (-) in small print.

1996 - 2008, cap-wgt	EUE3S				EUE3L			
	Total Risk		*Active Risk*		*Total Risk*		*Active Risk*	
Energy Equipm. & Svc.	91.1	+6.4 -2.6	94.9	+3.2 -1.9	86.0	+11.5 -2.6	96.2	+1.9 -1.9
Oil & Gas	94.9	+0.6 -4.5	91.1	+5.1 -3.8	87.3	+5.7 -7.0	89.8	+4.5 -5.7
Other Materials	91.1	+4.5 -4.5	86.6	+5.7 -7.6	80.3	+7.0 -12.7	77.1	+9.6 -13.4
Metals & Mining	91.7	+5.1 -3.2	94.3	+0.0 -5.7	90.5	+3.2 -6.4	91.1	+0.0 -8.9
Other Capital Goods	91.1	+5.7 -3.2	96.2	+1.9 -1.9	79.0	+7.6 -13.4	85.4	+4.5 -10.2
Construction & Eng.	89.8	+7.6 -2.6	91.1	+8.9 -0.0	86.6	+8.9 -4.5	84.7	+15.3 -0.0
Machinery	94.3	+0.6 -5.1	96.8	+0.0 -3.2	86.0	+3.2 -10.8	93.0	+0.0 -7.0
Commercial Services	96.2	+2.6 -1.3	98.7	+1.3 -0.0	81.5	+10.8 -7.6	92.4	+5.1 -2.6
Other Transport	91.7	+5.1 -3.2	100.0	+0.0 -0.0	86.0	+5.1 -8.9	100.0	+0.0 -0.0
Airlines	98.7	+1.3 -0.0	100.0	+0.0 -0.0	91.1	+7.6 -1.3	100.0	+0.0 -0.0
Autos & Components	97.5	+1.3 -1.3	97.5	+0.0 -2.6	85.4	+8.3 -6.4	97.5	+0.0 -2.6
Consumer Durables	91.7	+6.4 -1.9	98.7	+1.3 -0.0	82.8	+7.0 -10.2	91.7	+7.0 -1.3
Consumer Services	92.4	+4.5 -3.2	97.5	+2.6 -0.0	86.6	+8.9 -4.5	96.2	+3.2 -0.6
Media	93.0	+0.0 -7.0	84.7	+3.8 -11.5	79.0	+13.4 -7.6	70.1	+11.5 -18.5
Retailing	95.5	+3.2 -1.3	96.8	+1.9 -1.3	88.5	+10.2 -1.3	93.6	+4.5 -1.9
Food Retailing	93.0	+7.0 -0.0	94.9	+4.5 -0.6	87.3	+12.1 -0.6	90.5	+5.1 -4.5
Food, Bevg. & Tobacco	90.5	+9.6 -0.0	98.7	+0.0 -1.3	89.2	+10.2 -0.6	87.9	+3.2 -8.9
Household Products	98.7	+1.3 -0.0	98.1	+1.3 -0.6	95.5	+1.3 -3.2	98.1	+0.6 -1.3
Healthcare Equip. & Svc.	96.8	+3.2 -0.0	92.4	+5.7 -1.9	88.5	+11.5 -0.0	93.0	+3.8 -3.2
Pharmaceuticals	86.0	+14.0 -0.0	89.2	+1.9 -8.9	84.1	+15.9 -0.0	83.4	+3.8 -12.7
Banks	87.9	+9.6 -2.6	88.5	+3.2 -8.3	75.8	+13.4 -10.8	77.7	+5.1 -17.2
Diversified Financials	87.3	+11.5 -1.3	91.1	+0.6 -8.3	74.5	+17.2 -8.3	81.5	+7.6 -10.8
Insurance	96.8	+3.2 -0.0	94.9	+0.0 -5.1	75.8	+12.1 -12.1	77.1	+5.1 -17.8
Real Estate	99.4	+0.6 -0.0	90.5	+0.0 -9.6	99.4	+0.6 -0.0	80.3	+1.3 -18.5
Software & Services	91.1	+3.8 -5.1	90.5	+0.0 -9.6	77.7	+14.0 -8.3	75.2	+11.5 -13.4
Technology Hardware	100.0	+0.0 -0.0	91.1	+0.0 -8.9	92.4	+3.8 -3.8	86.0	+3.8 -10.2
Semiconductors	93.3	+0.0 -6.7	93.3	+0.0 -6.7	80.6	+8.2 -11.2	82.1	+3.7 -14.2
Telecom Services	96.8	+3.2 -0.0	94.9	+3.8 -1.3	80.3	+19.8 -0.0	80.3	+13.4 -6.4
Utilities	94.3	+5.7 -0.0	98.1	+0.6 -1.3	86.0	+14.0 -0.0	93.6	+1.3 -5.1
Average	**93.5**	+4.4 -2.1	**94.2**	+2.0 -3.9	**84.9**	+9.4 -5.7	**87.8**	+4.7 -7.5

Table A.14 Robust rolling 12-month bias tests of EUE3 country portfolios. Percentage of bias statistics in confidence and percentage of over/under-forecasts.

1996 - 2008, cap-wgt	EUE3S				EUE3L			
	Total Risk		*Active Risk*		*Total Risk*		*Active Risk*	
Austria	91.7	+3.2 -5.1	96.2	+3.8 -0.0	86.0	+4.5 -9.6	92.4	+5.7 -1.9
Belgium	94.3	+5.7 -0.0	89.8	+9.6 -0.6	81.5	+16.6 -1.9	78.3	+17.8 -3.8
Croatia	100.0	+0.0 -0.0	100.0	+0.0 -0.0	100.0	+0.0 -0.0	100.0	+0.0 -0.0
Cyprus	100.0	+0.0 -0.0	84.2	+15.8 -0.0	97.4	+0.0 -2.6	84.2	+15.8 -0.0
Czech Republic	91.4	+0.0 -8.6	98.3	+0.0 -1.7	91.4	+0.0 -8.6	94.8	+0.0 -5.2
Denmark	98.1	+1.9 -0.0	100.0	+0.0 -0.0	91.1	+1.3 -7.6	98.7	+1.3 -0.0
Estonia	83.9	+0.0 -16.1	80.7	+12.9 -6.5	71.0	+0.0 -29.0	74.2	+16.1 -9.7
Finland	98.1	+1.9 -0.0	88.5	+0.6 -10.8	89.8	+5.7 -4.5	88.5	+0.6 -10.8
France	86.6	+11.5 -1.9	82.2	+15.9 -1.9	82.2	+12.7 -5.1	79.0	+20.4 -0.6
Germany	94.9	+5.1 -0.0	94.9	+0.0 -5.1	81.5	+12.7 -5.7	77.1	+17.2 -5.7
Greece	94.9	+3.2 -1.9	93.6	+5.7 -0.6	79.0	+12.1 -8.9	71.3	+24.2 -4.5
Hungary	91.8	+5.2 -3.0	100.0	+0.0 -0.0	93.3	+6.7 -0.0	97.8	+2.2 -0.0
Iceland	100.0	+0.0 -0.0	93.8	+0.0 -6.3	75.0	+0.0 -25.0	56.3	+0.0 -43.8
Ireland	86.6	+13.4 -0.0	94.3	+3.2 -2.6	84.1	+14.7 -1.3	85.4	+9.6 -5.1
Italy	91.7	+5.1 -3.2	89.8	+8.9 -1.3	76.4	+15.9 -7.6	88.5	+11.5 -0.0
Latvia	86.1	+13.9 -0.0	88.9	+11.1 -0.0	83.3	+16.7 -0.0	86.1	+13.9 -0.0
Lithuania	100.0	+0.0 -0.0	100.0	+0.0 -0.0	89.5	+0.0 -10.5	100.0	+0.0 -0.0
Netherlands	89.8	+10.2 -0.0	86.6	+13.4 -0.0	88.5	+11.5 -0.0	85.4	+14.7 -0.0
Norway	95.5	+0.6 -3.8	86.6	+7.0 -6.4	92.4	+0.0 -7.6	79.6	+10.8 -9.6
Poland	97.0	+0.8 -2.2	97.8	+2.2 -0.0	96.3	+3.0 -0.8	95.5	+4.5 -0.0
Portugal	94.9	+3.2 -1.9	95.5	+4.5 -0.0	79.0	+10.8 -10.2	86.0	+11.5 -2.6
Romania	84.2	+0.0 -15.8	100.0	+0.0 -0.0	76.3	+0.0 -23.7	100.0	+0.0 -0.0
Russia	94.8	+0.8 -4.5	95.5	+2.2 -2.2	95.5	+0.8 -3.7	79.1	+17.2 -3.7
Slovenia	100.0	+0.0 -0.0	100.0	+0.0 -0.0	100.0	+0.0 -0.0	100.0	+0.0 -0.0
Spain	98.7	+0.6 -0.6	89.2	+3.8 -7.0	92.4	+6.4 -1.3	78.3	+10.8 -10.8
Sweden	93.0	+7.0 -0.0	92.4	+7.6 -0.0	88.5	+11.5 -0.0	89.2	+10.8 -0.0
Switzerland	87.9	+12.1 -0.0	94.3	+2.6 -3.2	82.8	+17.2 -0.0	94.3	+2.6 -3.2
Turkey	91.8	+1.5 -6.7	97.0	+0.8 -2.2	88.1	+4.5 -7.5	92.5	+6.0 -1.5
United Kingdom	86.0	+13.4 -0.6	87.9	+8.3 -3.8	79.6	+19.8 -0.6	81.5	+12.7 -5.7
Average	**93.2**	+4.1 -2.6	**93.0**	+4.8 -2.1	**86.6**	+7.1 -6.3	**86.7**	+8.9 -4.4

Table A.15 Robust rolling 12-month bias tests of EUE3 estimation universe, style portfolios, and random portfolios. Percentage of bias statistics in confidence and percentage of over/under-forecasts.

1996 - 2008	EUE3S				EUE3L			
	Cap-wgt		*Equal-wgt*		*Cap-wgt*		*Equal-wgt*	
ESTU Portfolio	87.9	+12.1 -0.0	95.5	+3.8 -0.6	80.3	+15.9 -3.8	86.6	+8.3 -5.1

1996 - 2008, cap-wgt	EUE3S				EUE3L			
	Total Risk		*Active Risk*		*Total Risk*		*Active Risk*	
Momentum Q1	86.6	+13.4 -0.0	83.4	+7.0 -9.6	79.0	+20.4 -0.6	78.3	+1.9 -19.8
Momentum Q5	90.5	+8.9 -0.6	96.2	+1.3 -2.6	75.8	+15.3 -8.9	67.5	+5.7 -26.8
Size Q1	87.3	+12.7 -0.0	86.6	+10.8 -2.6	79.0	+17.2 -3.8	87.3	+11.5 -1.3
Size Q5	96.8	+3.2 -0.0	100.0	+0.0 -0.0	94.3	+3.8 -1.9	96.8	+3.2 -0.0
Volatility Q1	92.4	+7.6 -0.0	94.3	+1.9 -3.8	83.4	+15.9 -0.6	77.7	+9.6 -12.7
Volatility Q5	94.3	+2.6 -3.2	89.8	+10.2 -0.0	84.1	+12.7 -3.2	79.0	+15.3 -5.7
Liquidity Q1	89.2	+10.8 -0.0	97.5	+0.6 -1.9	79.6	+17.8 -2.6	87.9	+7.0 -5.1
Liquidity Q5	93.6	+3.2 -3.2	96.8	+3.2 -0.0	90.5	+3.2 -6.4	93.0	+3.2 -3.8
Earnings Yield Q1	88.5	+11.5 -0.0	85.4	+1.9 -12.7	79.6	+12.1 -8.3	80.3	+4.5 -15.3
Earnings Yield Q5	94.9	+5.1 -0.0	79.6	+5.7 -14.7	81.5	+15.3 -3.2	70.7	+12.1 -17.2
Value Q1	94.9	+5.1 -0.0	96.2	+0.0 -3.8	84.1	+11.5 -4.5	88.5	+2.6 -8.9
Value Q5	80.9	+19.1 -0.0	80.3	+3.8 -15.9	72.0	+24.2 -3.8	69.4	+8.9 -21.7
Dividend Yield Q1	91.1	+8.9 -0.0	91.1	+0.0 -8.9	83.4	+12.7 -3.8	79.0	+1.3 -19.8
Dividend Yield Q5	92.4	+3.8 -3.8	89.8	+0.0 -10.2	83.4	+12.1 -4.5	80.9	+7.6 -11.5
Leverage Q1	93.0	+7.0 -0.0	93.0	+0.0 -7.0	79.6	+14.0 -6.4	93.0	+0.0 -7.0
Leverage Q5	91.7	+8.3 -0.0	94.9	+4.5 -0.6	81.5	+18.5 -0.0	89.8	+6.4 -3.8
Growth Q1	96.2	+3.8 -0.0	85.4	+5.1 -9.6	84.1	+10.8 -5.1	84.7	+3.8 -11.5
Growth Q5	89.8	+10.2 -0.0	88.5	+1.9 -9.6	85.4	+14.7 -0.0	85.4	+3.8 -10.8
Average, Style	**91.3**	+8.1 -0.6	**90.5**	+3.2 -6.3	**82.2**	+14.0 -3.8	**82.7**	+6.0 -11.3
20 assets, random	91.5	+6.8 -1.8	93.6	+2.7 -3.7	84.2	+11.1 -4.7	91.1	+3.1 -5.8
50 assets, random	91.1	+8.0 -1.0	93.2	+2.7 -4.1	81.7	+14.0 -4.2	90.3	+3.3 -6.5
100 assets, random	89.7	+9.8 -0.5	93.5	+2.8 -3.6	81.3	+15.0 -3.7	91.2	+3.0 -5.8
200 assets, random	88.4	+11.3 -0.3	92.1	+3.0 -4.9	80.9	+15.6 -3.6	88.4	+4.1 -7.6
Average, random	**90.1**	+9.0 -0.9	**93.1**	+2.8 -4.1	**82.0**	+13.9 -4.0	**90.2**	+3.4 -6.4

Table A.16 Robust and raw (in brackets) RAD statistics of EUE3 industry portfolios.

1996 - 2008, cap-wgt	EUE3S				EUE3L			
	Total Risk		*Active Risk*		*Total Risk*		*Active Risk*	
Energy Equipm. & Svc.	0.21	(0.22)	0.19	(0.19)	0.24	(0.26)	0.23	(0.24)
Oil & Gas	0.18	(0.18)	0.15	(0.18)	0.23	(0.23)	0.18	(0.22)
Other Materials	0.18	(0.18)	0.23	(0.23)	0.22	(0.22)	0.28	(0.30)
Metals & Mining	0.18	(0.20)	0.18	(0.19)	0.19	(0.22)	0.18	(0.20)
Other Capital Goods	0.20	(0.20)	0.17	(0.17)	0.25	(0.26)	0.20	(0.21)
Construction & Eng.	0.20	(0.20)	0.17	(0.18)	0.21	(0.22)	0.18	(0.18)
Machinery	0.19	(0.21)	0.14	(0.14)	0.22	(0.24)	0.17	(0.18)
Commercial Services	0.16	(0.16)	0.15	(0.15)	0.20	(0.20)	0.17	(0.17)
Other Transport	0.17	(0.18)	0.13	(0.13)	0.22	(0.24)	0.15	(0.15)
Airlines	0.14	(0.17)	0.15	(0.18)	0.21	(0.24)	0.17	(0.19)
Autos & Components	0.19	(0.22)	0.17	(0.19)	0.22	(0.24)	0.20	(0.21)
Consumer Durables	0.21	(0.24)	0.20	(0.20)	0.23	(0.26)	0.21	(0.21)
Consumer Services	0.16	(0.20)	0.16	(0.19)	0.19	(0.23)	0.19	(0.21)
Media	0.18	(0.20)	0.23	(0.26)	0.27	(0.29)	0.32	(0.38)
Retailing	0.18	(0.18)	0.15	(0.15)	0.21	(0.22)	0.18	(0.18)
Food Retailing	0.18	(0.19)	0.16	(0.18)	0.22	(0.23)	0.19	(0.20)
Food, Bevg. & Tobacco	0.18	(0.18)	0.17	(0.17)	0.23	(0.23)	0.24	(0.25)
Household Products	0.14	(0.15)	0.15	(0.15)	0.18	(0.20)	0.17	(0.17)
Healthcare Equip. & Svc.	0.18	(0.20)	0.20	(0.23)	0.22	(0.24)	0.23	(0.27)
Pharmaceuticals	0.23	(0.23)	0.21	(0.23)	0.27	(0.27)	0.23	(0.25)
Banks	0.21	(0.21)	0.21	(0.22)	0.30	(0.31)	0.26	(0.26)
Diversified Financials	0.21	(0.21)	0.20	(0.23)	0.28	(0.29)	0.22	(0.26)
Insurance	0.18	(0.20)	0.19	(0.20)	0.28	(0.29)	0.25	(0.26)
Real Estate	0.17	(0.19)	0.20	(0.22)	0.21	(0.22)	0.27	(0.29)
Software & Services	0.20	(0.21)	0.20	(0.23)	0.28	(0.30)	0.29	(0.32)
Technology Hardware	0.14	(0.14)	0.17	(0.19)	0.21	(0.21)	0.21	(0.22)
Semiconductors	0.20	(0.23)	0.17	(0.23)	0.30	(0.33)	0.27	(0.33)
Telecom Services	0.17	(0.17)	0.19	(0.20)	0.23	(0.23)	0.24	(0.26)
Utilities	0.19	(0.20)	0.14	(0.14)	0.22	(0.23)	0.17	(0.18)
Average	**0.18**	(0.20)	**0.18**	(0.19)	**0.23**	(0.25)	**0.22**	(0.23)

Table A.17 Robust and raw (in brackets) RAD statistics of EUE3 country portfolios.

1996 - 2008, cap-wgt	EUE3S				EUE3L			
	Total Risk		*Active Risk*		*Total Risk*		*Active Risk*	
Austria	0.19	(0.19)	0.17	(0.18)	0.24	(0.25)	0.19	(0.19)
Belgium	0.17	(0.17)	0.20	(0.20)	0.23	(0.23)	0.25	(0.25)
Croatia	0.09	(0.11)	0.10	(0.10)	0.08	(0.12)	0.08	(0.08)
Cyprus	0.13	(0.19)	0.17	(0.18)	0.15	(0.21)	0.19	(0.19)
Czech Republic	0.17	(0.20)	0.20	(0.22)	0.19	(0.22)	0.22	(0.25)
Denmark	0.14	(0.14)	0.13	(0.13)	0.19	(0.20)	0.15	(0.15)
Estonia	0.21	(0.21)	0.21	(0.21)	0.29	(0.31)	0.25	(0.25)
Finland	0.17	(0.17)	0.21	(0.22)	0.20	(0.20)	0.24	(0.25)
France	0.20	(0.21)	0.20	(0.20)	0.27	(0.27)	0.22	(0.22)
Germany	0.18	(0.18)	0.17	(0.18)	0.26	(0.27)	0.23	(0.23)
Greece	0.20	(0.27)	0.22	(0.25)	0.26	(0.33)	0.33	(0.36)
Hungary	0.14	(0.17)	0.13	(0.14)	0.17	(0.19)	0.16	(0.16)
Iceland	0.14	(0.21)	0.21	(0.27)	0.22	(0.35)	0.30	(0.39)
Ireland	0.18	(0.18)	0.20	(0.20)	0.21	(0.21)	0.24	(0.24)
Italy	0.21	(0.22)	0.20	(0.20)	0.25	(0.26)	0.26	(0.26)
Latvia	0.17	(0.18)	0.16	(0.16)	0.17	(0.17)	0.20	(0.20)
Lithuania	0.14	(0.16)	0.15	(0.18)	0.16	(0.19)	0.15	(0.17)
Netherlands	0.16	(0.16)	0.19	(0.19)	0.23	(0.23)	0.20	(0.20)
Norway	0.15	(0.19)	0.24	(0.24)	0.18	(0.23)	0.28	(0.28)
Poland	0.16	(0.16)	0.13	(0.13)	0.14	(0.15)	0.18	(0.18)
Portugal	0.18	(0.21)	0.18	(0.18)	0.26	(0.28)	0.23	(0.23)
Romania	0.25	(0.29)	0.12	(0.12)	0.29	(0.36)	0.15	(0.16)
Russia	0.17	(0.17)	0.22	(0.22)	0.21	(0.21)	0.24	(0.24)
Slovenia	0.12	(0.13)	0.12	(0.12)	0.14	(0.15)	0.12	(0.12)
Spain	0.16	(0.16)	0.21	(0.22)	0.22	(0.22)	0.26	(0.26)
Sweden	0.15	(0.15)	0.16	(0.16)	0.21	(0.21)	0.19	(0.19)
Switzerland	0.22	(0.22)	0.16	(0.16)	0.28	(0.29)	0.17	(0.17)
Turkey	0.18	(0.24)	0.17	(0.23)	0.21	(0.26)	0.22	(0.27)
United Kingdom	0.23	(0.23)	0.22	(0.23)	0.29	(0.29)	0.25	(0.26)
Average	**0.17**	(0.19)	**0.18**	(0.19)	**0.21**	(0.24)	**0.21**	(0.22)

Table A.18 Robust and raw (in brackets) RAD statistics of EUE3 estimation universe, style portfolios, and random portfolios.

1996 - 2008	EUE3S				EUE3L			
	Cap-wgt		*Equal-wgt*		*Cap-wgt*		*Equal-wgt*	
ESTU Portfolio	0.19	(0.19)	0.18	(0.22)	0.26	(0.26)	0.24	(0.28)

1996 - 2008, cap-wgt	EUE3S				EUE3L			
	Total Risk		*Active Risk*		*Total Risk*		*Active Risk*	
Momentum Q1	0.19	(0.19)	0.21	(0.21)	0.25	(0.25)	0.27	(0.29)
Momentum Q5	0.20	(0.20)	0.17	(0.17)	0.28	(0.28)	0.30	(0.30)
Size Q1	0.20	(0.20)	0.20	(0.20)	0.27	(0.27)	0.23	(0.24)
Size Q5	0.15	(0.23)	0.17	(0.21)	0.20	(0.28)	0.20	(0.24)
Volatility Q1	0.17	(0.17)	0.19	(0.19)	0.25	(0.25)	0.27	(0.28)
Volatility Q5	0.18	(0.19)	0.18	(0.18)	0.22	(0.24)	0.27	(0.27)
Liquidity Q1	0.18	(0.18)	0.19	(0.20)	0.27	(0.27)	0.27	(0.29)
Liquidity Q5	0.18	(0.19)	0.16	(0.16)	0.20	(0.21)	0.21	(0.21)
Earnings Yield Q1	0.21	(0.23)	0.19	(0.22)	0.27	(0.29)	0.23	(0.26)
Earnings Yield Q5	0.18	(0.18)	0.23	(0.23)	0.25	(0.25)	0.31	(0.33)
Value Q1	0.17	(0.17)	0.17	(0.17)	0.25	(0.25)	0.22	(0.22)
Value Q5	0.22	(0.22)	0.28	(0.31)	0.28	(0.28)	0.32	(0.36)
Dividend Yield Q1	0.21	(0.21)	0.21	(0.21)	0.28	(0.28)	0.28	(0.30)
Dividend Yield Q5	0.17	(0.19)	0.18	(0.19)	0.23	(0.25)	0.24	(0.26)
Leverage Q1	0.21	(0.22)	0.18	(0.18)	0.28	(0.28)	0.18	(0.19)
Leverage Q5	0.17	(0.17)	0.19	(0.20)	0.23	(0.23)	0.20	(0.20)
Growth Q1	0.18	(0.19)	0.18	(0.20)	0.24	(0.25)	0.22	(0.24)
Growth Q5	0.19	(0.19)	0.22	(0.25)	0.24	(0.24)	0.26	(0.29)
Average, Style	**0.19**	(0.20)	**0.19**	(0.20)	**0.25**	(0.26)	**0.25**	(0.26)
20 assets random	0.20	(0.21)	0.18	(0.19)	0.24	(0.25)	0.20	(0.21)
50 assets random	0.19	(0.20)	0.18	(0.20)	0.25	(0.25)	0.20	(0.22)
100 assets random	0.20	(0.20)	0.18	(0.19)	0.26	(0.26)	0.19	(0.20)
200 assets random	0.20	(0.20)	0.19	(0.20)	0.27	(0.27)	0.21	(0.22)
Average, Random	**0.20**	(0.20)	**0.18**	(0.19)	**0.26**	(0.26)	**0.20**	(0.21)

Table A.19 Comparison of EUE3 with EUE2 and GEM2. Robust and raw (in brackets) 12-months rolling bias statistics, January 1997 – December 2008. Table shows percentage of all portfolios in confidence.

1997 - 2008, cap-wgt ***Total Risk***	EUE3S	EUE3L	EUE2S	EUE2L	GEM2S	GEM2L
All Portfolios	**91.0** (89.7)	**81.4** (78.5)	**84.2** (79.7)	**74.5** (70.1)	**87.4** (82.6)	**77.8** (73.6)
Country Portfolios	91.9 (90.4)	83.0 (81.4)	85.4 (81.8)	77.6 (74.0)	88.6 (84.2)	79.1 (76.1)
Industry Portfolios	93.4 (91.2)	83.4 (79.8)	85.4 (79.9)	74.8 (70.4)	88.0 (83.4)	79.2 (75.5)
Style Portfolios*	90.0 (89.1)	80.4 (76.4)	82.9 (77.5)	72.6 (67.1)	86.4 (80.4)	76.2 (70.7)
Random Portfolios	88.8 (87.9)	79.0 (76.4)	83.3 (79.7)	73.1 (68.9)	86.5 (82.6)	76.6 (72.1)
Active Risk						
All Portfolios	**91.6** (89.6)	**84.7** (83.0)	**84.8** (81.7)	**80.4** (78.0)	**84.7** (81.7)	**80.7** (78.6)
Country Portfolios	91.0 (89.7)	83.6 (83.0)	90.3 (89.0)	84.7 (84.0)	89.1 (87.7)	81.4 (80.6)
Industry Portfolios	93.3 (91.3)	86.0 (84.6)	80.8 (76.1)	76.4 (73.0)	79.9 (74.9)	77.6 (74.7)
Style Portfolios*	89.4 (86.7)	80.3 (77.1)	77.3 (73.2)	71.2 (67.8)	77.9 (74.3)	74.9 (72.1)
Random Portfolios	92.7 (90.9)	89.1 (87.2)	91.0 (88.7)	89.2 (87.1)	91.9 (90.1)	88.9 (87.2)

*top and bottom quintiles

Table A.20 Comparison of EUE3 with EUE2 and GEM2. Robust and raw (in brackets) RAD statistics, January 1997 – December 2008.

1997 - 2008, cap-wgt ***Total Risk***	EUE3S	EUE3L	EUE2S	EUE2L	GEM2S	GEM2L
All Portfolios	**0.19** (0.20)	**0.26** (0.27)	**0.24** (0.26)	**0.30** (0.33)	**0.22** (0.24)	**0.27** (0.29)
Country Portfolios	0.18 (0.19)	0.25 (0.25)	0.23 (0.24)	0.28 (0.31)	0.21 (0.22)	0.26 (0.28)
Industry Portfolios	0.19 (0.20)	0.24 (0.26)	0.24 (0.27)	0.29 (0.33)	0.22 (0.24)	0.26 (0.28)
Style Portfolios*	0.19 (0.20)	0.27 (0.27)	0.25 (0.27)	0.32 (0.36)	0.22 (0.24)	0.28 (0.30)
Random Portfolios	0.20 (0.21)	0.27 (0.28)	0.25 (0.26)	0.31 (0.34)	0.22 (0.24)	0.28 (0.29)
Active Risk						
All Portfolios	**0.19** (0.20)	**0.23** (0.24)	**0.23** (0.25)	**0.26** (0.29)	**0.23** (0.25)	**0.25** (0.27)
Country Portfolios	0.19 (0.20)	0.24 (0.24)	0.20 (0.21)	0.24 (0.25)	0.21 (0.21)	0.25 (0.25)
Industry Portfolios	0.18 (0.20)	0.23 (0.24)	0.26 (0.29)	0.28 (0.33)	0.26 (0.30)	0.27 (0.31)
Style Portfolios*	0.20 (0.22)	0.26 (0.28)	0.28 (0.31)	0.32 (0.36)	0.27 (0.30)	0.29 (0.30)
Random Portfolios	0.18 (0.19)	0.21 (0.22)	0.20 (0.21)	0.21 (0.22)	0.19 (0.20)	0.21 (0.22)

*top and bottom quintiles

Table A.21 Bias tests of EUE3 derived model versions. Robust 12-months rolling bias statistics, January 1996 – December 2008. Table shows percentage of all portfolios in confidence.

1996 - 2008	EUE3 UK S				EUE3 UK L			
	Total Risk		*Active Risk*		*Total Risk*		*Active Risk*	
	Cap	*Equal*	*Cap*	*Equal*	*Cap*	*Equal*	*Cap*	*Equal*
All Portfolios	**90.8**	**92.7**	**91.3**	**91.7**	**82.6**	**83.2**	**85.0**	**86.5**
Country Portfolios	92.1	92.6	92.2	91.9	84.9	85.1	85.9	87.7
Industry Portfolios	92.3	92.3	91.5	90.6	83.9	83.6	83.9	84.8
Style Portfolios*	88.7	93.0	89.0	92.0	79.9	82.4	80.4	84.7
Random Portfolios	90.1	92.8	92.6	92.4	81.8	81.5	89.7	88.8
1996 - 2008	EUE3 EE S				EUE3 EE L			
	Total Risk		*Active Risk*		*Total Risk*		*Active Risk*	
	Cap	*Equal*	*Cap*	*Equal*	*Cap*	*Equal*	*Cap*	*Equal*
All Portfolios	**89.5**	**91.4**	**90.9**	**91.6**	**82.1**	**82.8**	**84.6**	**86.4**
Country Portfolios	92.1	92.2	92.2	91.3	85.0	84.6	85.8	86.9
Industry Portfolios	92.5	92.4	91.7	91.4	84.2	83.8	84.3	85.6
Style Portfolios*	88.9	93.1	88.9	92.0	79.8	82.2	80.2	84.2
Random Portfolios	84.6	88.1	90.9	91.8	79.4	80.5	88.2	88.9

*top and bottom quintiles

Table A.22 Bias tests of EUE3 derived model versions.
Robust RAD statistics, January 1996 – December 2008.

1996 - 2008	EUE3 UK S				EUE3 UK L			
	Total Risk		*Active Risk*		*Total Risk*		*Active Risk*	
	Cap	*Equal*	*Cap*	*Equal*	*Cap*	*Equal*	*Cap*	*Equal*
All Portfolios	**0.18**	**0.19**	**0.18**	**0.19**	**0.24**	**0.23**	**0.22**	**0.22**
Country Portfolios	0.17	0.18	0.18	0.18	0.21	0.21	0.21	0.21
Industry Portfolios	0.18	0.19	0.18	0.20	0.23	0.24	0.22	0.23
Style Portfolios*	0.19	0.18	0.19	0.19	0.25	0.23	0.25	0.24
Random Portfolios	0.20	0.20	0.18	0.18	0.26	0.25	0.20	0.21
1998 - 2008	EUE3 EE S				EUE3 EE L			
	Total Risk		*Active Risk*		*Total Risk*		*Active Risk*	
	Cap	*Equal*	*Cap*	*Equal*	*Cap*	*Equal*	*Cap*	*Equal*
All Portfolios	**0.19**	**0.19**	**0.19**	**0.19**	**0.25**	**0.24**	**0.23**	**0.22**
Country Portfolios	0.17	0.18	0.18	0.18	0.22	0.22	0.23	0.21
Industry Portfolios	0.19	0.19	0.18	0.20	0.24	0.24	0.22	0.23
Style Portfolios*	0.19	0.19	0.19	0.18	0.26	0.25	0.26	0.24
Random Portfolios	0.22	0.22	0.20	0.19	0.26	0.25	0.21	0.21

*top and bottom quintiles

Table A.23 Comparison of EUE base and UK derived models. Robust rolling 12-month bias tests of UK and ex-UK industry portfolios. Table shows percentage of bias statistics in confidence and percentage of over/under-forecasts.

1996-2008		S - Models				L - Models			
Model	*Industry Portfolios*	*Total Risk*		*Active Risk*		*Total Risk*		*Active Risk*	
EUE3 UK	UK cap-wgt	92.6	+4.1 -3.3	93.2	+2.3 -4.5	83.5	+9.9 -6.7	85.6	+6.4 -8.0
EUE3		90.4	+3.4 -6.2	84.0	+0.6 -15.4	83.3	+7.8 -8.8	79.4	+2.1 -18.4
EUE3 UK	ex-UK cap-wgt	92.7	+5.4 -2.0	94.4	+2.7 -3.0	83.8	+10.7 -5.5	88.4	+5.8 -5.8
EUE3		92.4	+5.3 -2.3	91.1	+4.6 -4.3	83.1	+10.6 -6.3	84.2	+9.4 -6.4
EUE3 UK	UK equal-wgt	93.9	+2.5 -3.6	93.8	+4.3 -1.9	85.6	+6.9 -7.5	86.3	+7.8 -5.9
EUE3		89.8	+2.0 -8.2	84.4	+1.9 -13.7	83.7	+4.2 -12.1	79.3	+2.7 -18.0
EUE3 UK	ex-UK equal-wgt	91.7	+6.1 -2.1	87.4	+11.3 -1.2	83.4	+12.1 -4.6	82.2	+15.6 -2.2
EUE3		91.5	+6.1 -2.4	87.2	+11.1 -1.7	82.8	+11.8 -5.4	82.4	+7.8 -5.6

Table A.24 Comparison of EUE base and UK derived models. Robust RAD statistics of UK and ex-UK industry portfolios.

1996-2008		S - Models		L - Models	
Model	*Industry Portfolios*	*Total Risk*	*Active Risk*	*Total Risk*	*Active Risk*
EUE3 UK	UK cap-wgt	0.19	0.18	0.23	0.22
EUE3		0.20	0.23	0.24	0.25
EUE3 UK	ex-UK cap-wgt	0.19	0.18	0.24	0.21
EUE3		0.19	0.18	0.25	0.22
EUE3 UK	UK equal-wgt	0.18	0.18	0.23	0.23
EUE3		0.20	0.23	0.25	0.27
EUE3 UK	ex-UK equal-wgt	0.19	0.18	0.24	0.25
EUE3		0.19	0.19	0.24	0.22

Appendix B Descriptor Details

The nine style factors of EUE3 combine a total of 25 descriptors. This appendix describes the definition of all descriptors and provides their weights in the style factors. The descriptors are grouped by the styles they constitute.

Style: **Size**

Definition: $0.5 \cdot LNCAP + 0.5 \cdot ASSI$

Components:

$LNCAP$ Log of the month-end issuer capitalization.

$ASSI$ Log of total assets; an indicator of fundamental firm size.

Style: **Liquidity**

Definition: $0.42 \cdot LSTOA + 0.36 \cdot LSTOQ + 0.22 \cdot LSTOM$

Components:

$LSTOA$ Log of annual share turnover; an indicator of the average liquidity of an asset over one year.

$$LSTOA = ln\left(\frac{1}{NP}\sum_{t=T-NP+1}^{T}\frac{VOL_t}{NOS_t}\right) \quad \text{(B.1)}$$

VOL_t = monthly volume of shares traded at time t
NOS_t = number of shares outstanding at time t
NP = number of periods in sum, here NP = 12

$LSTOQ$ Log of quarterly share turnover; defined as $LSTOA$ with NP = 3.

$LSTOM$ Log of monthly share turnover; defined as $LSTOA$ with NP = 1.

Style: **Momentum**

Definition: $0.24 \cdot HWALPHA + 0.36 \cdot MOM\_12\_1 + 0.4 \cdot MOM\_6\_1$

Components:

$HWALPHA$ Historical weekly alpha. Intercept, α_i, of a regression of weekly asset returns, r_{it}, against weekly returns of the cap-weighted EUE3 estimation universe, r_{mt}.

$$r_{it} = \alpha_i + \beta_i r_{mt} + e_{it} \quad \text{(B.2)}$$

The regression uses 104 weeks of data and exponential weighting with a half-life of 52 weeks.

	MOM_12_1	12-month relative strength, lagged by 1 month.

$$MOM\_12\_1 = \sum_{t=T-NP}^{T-1} ln(1+r_{it}) - \sum_{t=T-NP}^{T-1} ln(1+r_{0t}) \tag{B.3}$$

r_{it} = monthly asset return at time t
r_{0t} = monthly risk-free return at time t
NP = number of periods in sum, here NP = 12

MOM_6_1 — 6-month relative strength, lagged by 1 month. Defined as MOM_12_1 but uses NP = 6.

Style: **Volatility**

Definition: $0.53 \cdot HWBETA + 0.25 \cdot CMRA\_12\_0 + 0.22 \cdot DSTD\_65\_23$

Components:

$HWBETA$ — Historical weekly beta. Slope, β_i, of the regression (B.2).

$CMRA\_12\_0$ — Cumulative range, defined as

$$CMRA = \ln(1+\max(Z_t)) - \ln(1+\min(Z_t)) \tag{B.4}$$

$$Z_t = \sum_{s=1}^{t} ln(1+r_{is}) - \sum_{s=1}^{t} ln(1+r_{0s}) \tag{B.5}$$

r_{is} = monthly asset return at time s
r_{0s} = monthly risk-free return at time s

$DSTD\_65\_23$ — Daily asset volatility. Exponentially-weighted standard deviation of local daily asset returns. Uses 65 daily returns and a half-life of 23 days.

$$DSTD\_65\_23 = \left(\sum_{s=t-64}^{t} \lambda^{t-s}(r_{is} - \bar{r}_i)^2 \Big/ \sum_{s=t-64}^{t} \lambda^{t-s} \right)^{1/2} \tag{B.6}$$

$\lambda = 0.5^{1/\tau}$ and $\tau = 23$ days

Style: **Value**

Definition: $0.62 \cdot BTOP + 0.38 \cdot SATOP$

Components:

$BTOP$ — Book-to-price ratio. The last published book value of common equity divided by the current issuer capitalization.

$SATOP$ — Sales-to-price ratio. Sales over the last 12 months divided by the current issuer capitalization.

Style: **Earnings Yield**

Definition: $0.1 \cdot ETOP + 0.17 \cdot CETOP + 0.13 \cdot ROE + 0.6 \cdot EPIBS$

Components:

$ETOP$ Trailing earnings-to-price ratio. Net earnings over the last 12 months divided by the current issuer capitalization.

$CETOP$ Cash earnings-to-price ratio. Cash earnings over the last 12 months divided by the current issuer capitalization.

ROE Return on equity. Net earnings over the last 12 months divided by the last available book value of common equity.

$EPIBS$ Predicted earnings-to-price ratio. 12-month forward-looking earnings per share divided by current price. Forward-looking earnings per share are defined as follows:

$$EPS_{12F} = [M \cdot EPS_1 + (12 - M)EPS_2]/12 \quad \text{(B.7)}$$

EPS_1 = consensus of EPS forecasts for current fiscal year.
EPS_2 = consensus of EPS forecasts for next fiscal year.
M = number of months remaining to end of current fiscal year.

Style: **Dividend Yield**

Definition: $1.0 \cdot YILD$

Components:

$YILD$ Annualized dividend per share divided by the current price.

Style: **Leverage**

Definition: $0.59 \cdot BLEV + 0.41 \cdot MLEV$

Components:

$BLEV$ Book leverage. Computed as

$$BLEV = \frac{BV + PF + LD}{BV} \quad \text{(B.8)}$$

BV = most recent book value of common equity.
PF = most recent book value of preferred equity.
LD = most recent book value of long-term debt.

$MLEV$ Market leverage. Defined in the same way as $BLEV$ but BV is replaced by the most recent issuer capitalization.

Style: **Growth**

Definition: $0.23 \cdot AGRO + 0.07 \cdot SAGRO + 0.05 \cdot EGRO + 0.5 \cdot EGRSF + 0.15 \cdot EGRLF$

Components:

$AGRO$ Trailing growth of total assets. Slope coefficient of the reported total assets regressed against time over the last 5 years.

$SAGRO$ Trailing growth of annual sales. Slope coefficient of the reported annual sales regressed against time over the last 5 years.

$EGRO$ Trailing growth of annual net earnings. Slope coefficient of the reported annual net earnings regressed against time over the last 5 years.

$EGRSF$ Short-term predicted earnings growth, defined as

$$EGRSF = \frac{EPS_{12F} - EPS_{12B}}{|EPS_{12B}|} \tag{B.9}$$

EPS_{12F} = forward-looking earnings per share, see (B.7).
EPS_{12B} = backward-looking earnings per share, defined as:

$$EPS_{12B} = [M \cdot EPS_0 + (12 - M)EPS_1]/12 \tag{B.10}$$

EPS_0 = last reported earnings per share.
EPS_1 = consensus of EPS forecasts for current fiscal year.
M = number of months since end of last fiscal year.

$EGRLF$ Long-term (3-5 years) predicted earnings growth, consensus of analyst estimates.

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