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Product Insight

Portfolio Optimization with Custom Factor Weight

Fine-tuning the risk aversion to systematic factors

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Introduction

Portfolio managers often have different levels of risk aversion for stocks from different regions, industries, and investment styles. In a standard mean-variance portfolio optimization framework, managers may define a single risk aversion parameter, which is applied evenly across all sources of systematic risk. This paper demonstrates how managers can use the Custom Factor Weight feature in the Barra Aegis System to express different levels of risk aversion to different factors.

We first discuss how managers can fine-tune their portfolio construction process and align it with their view of the market. We then use a case study to demonstrate the benefits of this feature when a manager has a strong conviction on a specific alpha signal that overlaps with the risk model. Lastly, we provide details on the mathematical formulation of this methodology and address some frequently asked questions.

Factor Weighting and Optimization

The trade-off between risk factors and alpha signals is an important consideration during the optimization process. Applying Custom Factor Weight allows users to manage this trade-off effectively, and we discuss several cases where such an approach can be incorporated into the investment process.

1. Assign Risk Aversion Parameters to Risk Factors

Traditionally, managers only use a single risk aversion parameter to quantify an acceptable level of risk. This parameter would be applied to all factors equally. By varying the weight for each factor, equivalent to setting different risk aversion values for different factors, managers can incorporate their risk aversion to stocks from different countries, industries and investment styles.

2. Strengthen Bets on Alpha Factors with Higher Conviction Levels

In a standard portfolio optimization, the optimizer is seeking to balance expected portfolio return, as predicted by alpha factors, and portfolio risk as forecasted by a risk model. It is common that some factors, such as Value and Momentum, will be considered as both alpha and risk factors.

When alpha factors overlap with risk factors in optimization, the size of bets on alpha factors will be reduced because of the associated risk factors. Managers with different confidence levels on the forecasting power of these alpha factors may want their views to be reduced differently based on the strength of the factor they have determined. This is equivalent to setting a lower risk aversion to a particular factor.

For example, if a manager is more confident on alpha factor 1 than alpha factor 2 while both appear in the risk model, the manager may want to reduce the weight on factor 1 relative to factor 2, which is likely to tilt the optimized portfolio more towards factor 1. If the manager's original assessment that factor 1 has a higher forecasting power than factor 2 was correct, this procedure may help the manager improve the portfolio's performance.

3. Suppress Certain Factors from the Covariance Matrix

When alpha factors overlap with risk factors, some portfolio managers prefer to remove the shared factors from the risk model in order to mitigate the alpha-risk misalignment effect. This new feature allows portfolio managers to remove or suppress what they deem to be an alpha factor from the risk model without manipulating the risk model data directly.

A Use Case Illustration

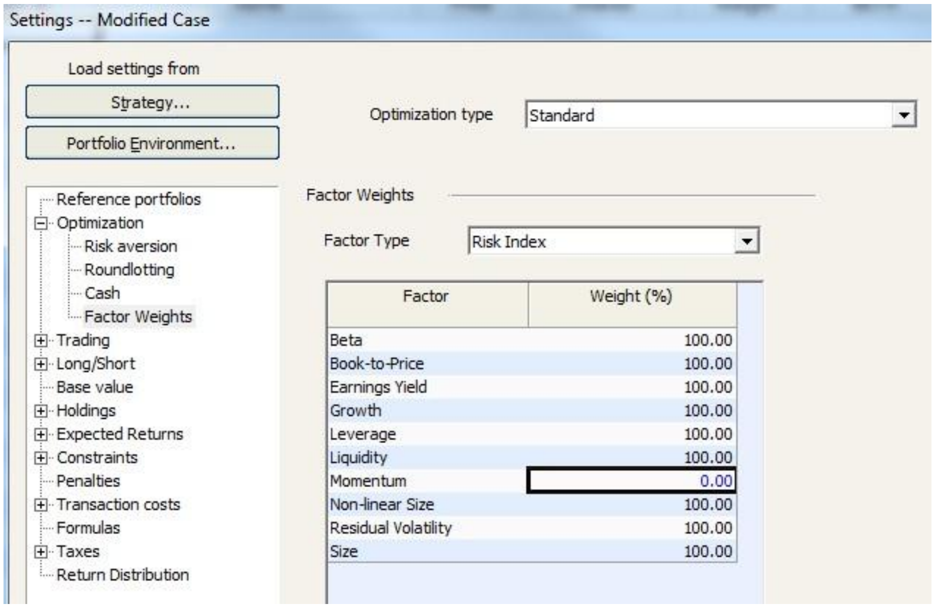
We will use a simple case study to illustrate the benefits of using this new feature when portfolio managers have different confidence levels on their alpha factors.

Optimization Settings

In the Base Case, the manager uses the Barra China Equity Model (CNE5) to build an enhanced index tracking portfolio with respect to MSCI China A Index. The manager prefers stocks with strong recent performance (positive Momentum) and small-cap companies (negative Size). To achieve this view, the manager forecasts the Momentum factor with a +6% return and the Size factor with a -6% return. Risk aversion is set to be 0.5 so that the forecast tracking error of the optimized portfolio is around 2% (refer to Liu and Xu [2010] on how to set the risk aversion parameter in portfolio optimization). The manager also enforces that no stock can deviate from its benchmark weight by more than 5%.

The manager then changes the factor weight for Momentum to zero (see Figure 1) as the manager is very confident on the outperformance of the Momentum factor. This modified strategy is defined as the Modified Case.

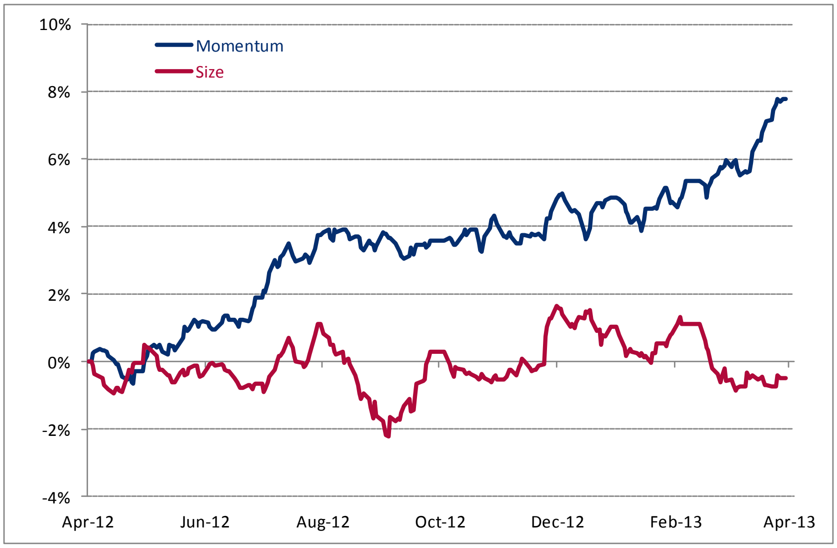
Figure 1: Setting the Factor Weight for Momentum to 0%



Backtest Result

Based on the settings described above, a backtest is run on both the Base Case and Modified Case from 2012 April to 2013 March (12 months with monthly rebalance), which was a period where the Momentum factor outperformed strongly while the Size factor's cumulative return was close to 0% (see Figure 2). The results will allow the manager to confirm whether the optimizer increases the bet on the Momentum factor as desired.

Figure 2: Daily Cumulative Returns for Momentum and Size factors from Apr 2012 – Mar 2013



As seen in Figure 3 and 4, the Base Case delivered a 3.42% active return with 1.81% tracking error over the 12-month test period (annualized), which results in an information ratio of 1.89. In contrast, the Modified Case had a 11.75% active return with 3.67% active risk over the same test period, which translates to an information ratio of 3.2. It is worth noting that the active return contribution from Risk indices has significantly increased from 3.15% in the Base Case to 7.13% in the Modified Case.

Figure 3: Performance Attribution of the Backtest Result of Base Case

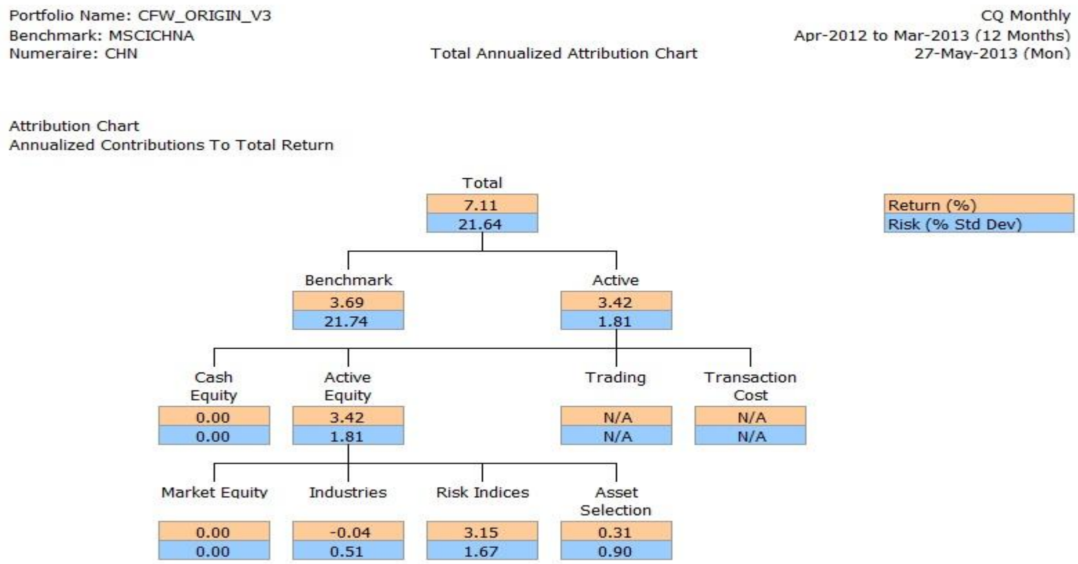
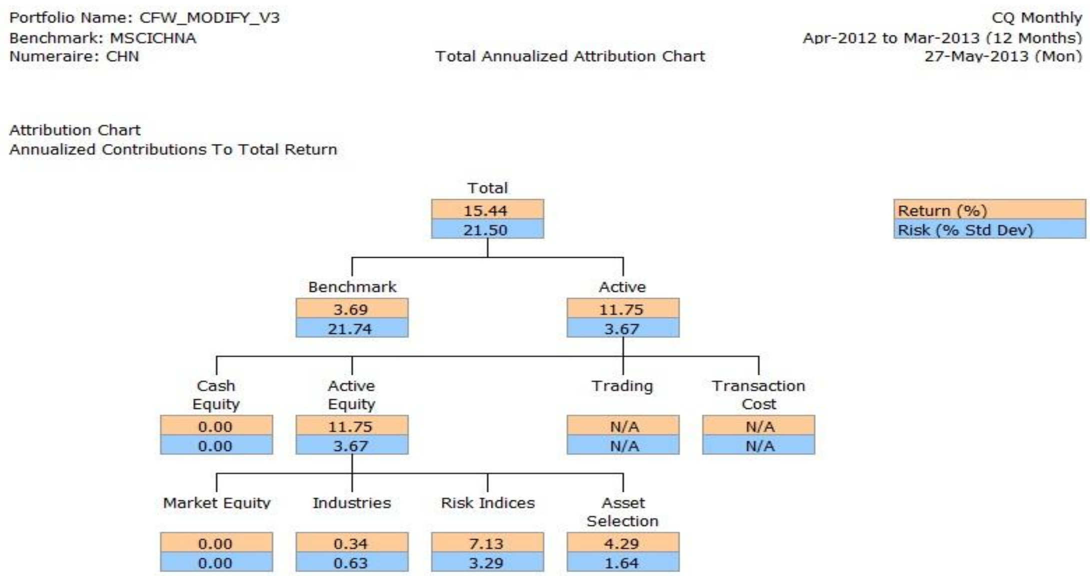


Figure 4: Performance Attribution of the Backtest Result of Modified Case



Let’s now check how style factor exposures have changed in the Modified Case and its impact on the factor return contribution.

Table 1: Comparison of Average Active Exposure and Return Contribution for Risk Indices

	Base Case		**Modified Case**	
Source of Return	Average Active Exposure	Return Contribution (%)	Average Active Exposure	Return Contribution (%)
Beta	-0.04	-0.16	-0.12	-0.64
Momentum	0.42	3.52	0.92	8.21
Size	-0.32	0.07	-0.17	0.12
Earnings Yield	0.15	-0.25	0.02	-0.19
Residual Volatility	-0.16	0.16	0.08	0.03
Growth	0.01	0.03	0.14	0.31
Book-to-Price	0.02	0.00	-0.15	0.07
Leverage	0.05	-0.09	-0.04	-0.13
Liquidity	-0.09	0.90	-0.01	0.14
Non-linear Size	0.22	-1.04	0.15	-0.80
Total		**3.15**		**7.13**

From the table 1, we can see that the average active exposure to Momentum factor increased from 0.42 in the Base Case to 0.92 in the Modified Case, while the active exposure to Size dropped slightly from -0.32 to -0.17. This is expected, as the optimizer ignored the risk contribution from the Momentum factor and any factor correlation involving Momentum. Therefore, the optimized portfolio will have a larger tilt towards Momentum and the exposure to the rest of the factors will be adjusted accordingly based on their correlation with the Momentum factor. Since the Momentum factor had a strong performance over the test period, the return contribution from the Momentum factor increased from 3.52% in the Base Case to 8.21% in the Modified Case, a 4.69% improvement in portfolio return. It is worth noting also that the active risk for the Modified Case was increased as well, although the information ratio was still significantly higher for the Modified Case.

Custom Factor Weight Impact on Optimization

Let’s take a look at the mathematical formulation of a portfolio optimization problem and examine the impact of introducing the Custom Factor Weight feature.

In a standard portfolio optimization problem in Aegis Portfolio Manager, we maximize the expected portfolio return offset by the forecast portfolio variance and transaction costs, subject to a set of user-defined constraints. Mathematically, this problem is represented in equation 1:

$$\text{maximize } h^T r - \lambda(h^T X F X^T h + \theta_{\frac{AS}{CF}} h^T D h) \tag{1}$$

$$\text{subject to: any constraints}$$

$$\text{e.g total turnover} \leq 20\%$$

Where:

h is a vector of the stock weights in the optimized portfolio

r is a vector of the forecast stock returns

X is a matrix of stock exposures to risk factors

F is the common factor covariance matrix

D is the specific risk matrix

λ is the risk aversion ratio

$\theta_{\frac{AS}{CF}}$ is the ratio of asset selection to common factor risk

With the new Custom Factor Weight feature, users have the flexibility to assign weights to all factors in the common factor covariance matrix. Mathematically, the objective function becomes:

$$\text{maximize } h^T r - \lambda(h^T X W F W^T X^T h + \theta_{\frac{AS}{CF}} h^T D h) \tag{2}$$

Where:

W is a diagonal matrix of $\text{diag}(w_1 \ldots w_i \ldots w_n)$

w_i is the weight for factor *i*.

There are a number of frequently asked questions about this new feature. We consider two questions in detail here.

Does It Affect both Common and Specific Risk

This new feature will only affect the common factor covariance matrix. In the augmented optimization problem, described by equation 2, the weight matrix *W* is multiplied only by the common factor covariance matrix *F*. It has no impact on the specific risk matrix *D*.

Does It Affect Factor Correlations

To illustrate how the common factor risk matrix is affected by *W*, let's consider a covariance matrix with only two factors:

$$F = \begin{bmatrix} \sigma_1^2 & \rho^2 \\ \rho^2 & \sigma_2^2 \end{bmatrix}$$

Where:

σ_1, σ_2 are the standard deviation of factor 1, 2

ρ is the correlation between factor 1 and 2

When we multiply F by W, we get:

$$WFW^T = \begin{bmatrix} w_1 & 0 \\ 0 & w_2 \end{bmatrix} \begin{bmatrix} \sigma_1^2 & \rho^2 \\ \rho^2 & \sigma_2^2 \end{bmatrix} \begin{bmatrix} w_1 & 0 \\ 0 & w_2 \end{bmatrix}^T$$
$$= \begin{bmatrix} (w_1\sigma_1)^2 & w_1 w_2 \rho^2 \\ w_1 w_2 \rho^2 & (w_2\sigma_2)^2 \end{bmatrix}$$

Hence, both factor risks and factor correlations are affected by our chosen weights in the resulting covariance matrix.

Summary

Custom Factor Weight is a powerful new feature in Barra Aegis Portfolio Manager, which allows users to customize the weights of individual risk factors in the covariance matrix during optimization. In this paper, we discussed how portfolio managers can derive value from this new feature by assigning different risk aversion levels to risk factors, as well as adjusting bets on alpha factors based on confidence level. We examined a sample use case, where manager had a strong confidence in the outperformance of the Momentum factor. During the sample period, the Momentum factor outperformed and adjusting the weight placed on its risk in optimization helped improve the performance of the optimized portfolio significantly.

References

Liu, Scott and Xu, Rong. “The Effects of Risk Aversion on Optimization,” MSCI Barra Research Insight, February 2010

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[1] *As of September 30, 2012, as published by eVestment, Lipper and Bloomberg on January 31, 2013* *Apr 2013*