

The Barra Europe Equity Model (EUE4)

Empirical Notes

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1. Introduction

This document provides empirical results and analysis for the new Barra Europe Equity Model (EUE4). It includes extensive information on factor structure, commentary on the performance of select factors, an analysis of the explanatory power of the model, and an examination of the statistical significance of the factors. Furthermore, these notes also include a side-by-side comparison of forecasting accuracy of the EUE4 Model and the EUE3 Model, its predecessor.

EUE4 leverages the same methodological advances used in the Barra US Equity Model (USE4). Detailed description of these methodological enhancements may be found in *USE4 Methodology Notes*, by Menchero, Orr, and Wang (2011).

The main advances in EUE4 include:

- Enhanced style factors reflecting the latest research
- Addition of new frontier markets
- Volatility Regime Adjustment designed to calibrate factor volatilities and specific risk forecasts to current market levels
- Optimization Bias Adjustment to factor covariance matrices - this reduces the effects of sampling error on factor covariance matrices, therefore improves risk forecasts for optimized portfolios
- New Daily-Horizon model to cater to investors with short investment horizons
- Improved specific risk model, including Volatility Regime Adjustment and Bayesian Adjustment

Similar to EUE3, EUE4 is offered in three versions, namely Base, Eastern Europe Derived, and UK Derived. In addition to the long-horizon (L) and short-horizon (S) variants in which each version of EUE3 is provided, each version of EUE4 is also provided in a daily-horizon variant (EUE4D). The three variants of each version have identical factor exposures and daily factor returns, but differ in factor covariance matrices and specific risk forecasts. The EUE4S model is designed to be more responsive and provides more accurate forecast at a monthly prediction horizon. The EUE4L model is designed for longer-term investors willing to trade some degree of accuracy for greater stability in risk forecast. The EUE4D model provides investors with a tactical risk forecast at a daily horizon.

2. Methodology Highlights

2.1. Volatility Regime Adjustment

One major source of risk model bias is due to the fact that volatilities are not stable over time, a characteristic known as *non-stationarity*. Since risk models must look backward to make predictions about the future, they exhibit a tendency to underpredict risk in times of rising volatility, and to overpredict risk in times of falling volatility.

Another important innovation in the EUE4 Model is the introduction of a Volatility Regime Adjustment for estimating factor volatilities. As described in the *USE4 Methodology Notes*, the Volatility Regime Adjustment relies on the notion of a cross-sectional bias statistic, which may be interpreted as an *instantaneous* measure of risk model bias for that particular day. By taking a weighted average of this quantity over a suitable interval, the non-stationarity bias can be significantly reduced.

A natural consideration in a regional model is to apply *separate* adjustments for different classes of factors (e.g., industries, countries, currencies, etc.). However, we did not find that to be beneficial. Furthermore, the currencies and country factors, especially Emerging and Frontier Market factors in a regional model are often characterized by extreme returns, making them less useful to be included in the cross-sectional bias statistic. Therefore, EUE4 –similarly to GEM3– computes the Volatility Regime Adjustment using the Market factor, industries, and styles, and applies this adjustment to the entire factor covariance matrix. Further details on the EUE4 factor covariance matrix estimation parameters are provided in Appendix D.

2.2. Optimization Bias Adjustment

Another significant bias of risk models is the tendency to underpredict the risk of optimized portfolios, as demonstrated empirically by Muller (1993). More recently, Shepard (2009) derived an analytic result for the magnitude of the bias, showing that the underforecasting becomes increasingly severe as the number of factors grows relative to the number of time periods used to estimate the factor covariance matrix. The basic source of this bias is estimation error. Namely, spurious correlations may cause certain stocks to appear as good hedges in-sample, while these hedges fail to perform as effectively out-of-sample.

An important innovation in the EUE4 Model is the identification of portfolios that capture these biases and to devise a procedure for correcting these biases directly within the factor covariance matrix. As shown by Menchero, Wang, and Orr (2011), the *eigenfactors* of the sample covariance matrix are systematically biased. More specifically, the sample covariance matrix tends to underpredict the risk of low-volatility eigenfactors, while overpredicting the risk of high-volatility eigenfactors. Furthermore, removing the biases of the eigenfactors essentially removes the biases of optimized portfolios.

In the context of the EUE4 Model, eigenfactors represent portfolios of the original pure factors. The eigenfactor portfolios, however, are special in the sense that they are mutually uncorrelated. Also note that the number of eigenfactors equals the number of pure factors within the model.

As described in the *USE4 Methodology Notes*, we estimate the biases of the eigenfactors by Monte Carlo simulation. We then adjust the predicted volatilities of the eigenfactors to correct for these biases. This procedure has the benefit of building the corrections directly into the factor covariance matrix, while fully preserving the meaning and intuition of the pure factors. The application of this correction procedure is a common feature of USE4 and all models released after USE4.

Special consideration must be taken when applying the adjustment within the context of a regional model. As shown by Menchero, Wang, and Orr (2011), applying the adjustment to the factor covariance matrix can induce small biases at the individual factor level. The magnitude of the bias grows, however, in proportion to the number of factors in the model. Since the European model contains many more factors than the USE4 model, applying the adjustment to the factor covariance matrix may induce significant biases in the volatility forecasts at the factor level of a regional model. To overcome this difficulty, we apply the adjustment procedure directly to the *factor correlation matrix*. The individual factor volatilities are then scaled in to obtain the factor covariance matrix.

Second, a realistic portfolio optimization within a regional or global context would entail some constraints on factor exposures. For instance, a typical global investor does not perform an unconstrained portfolio optimization across all emerging markets, frontier markets, and currencies. Applying the full adjustment would therefore lead to overestimation of risk of optimized portfolios. To avoid this problem, we reduce the magnitude of the adjustment by 50 percent. A similar reduction of the adjustment was applied in GEM3. Our empirical findings show that this essentially eliminates the biases for realistic optimized portfolios, as shown in Section 6.2.

2.3. Specific Risk Model with Bayesian Shrinkage

The EUE4 specific risk model builds upon methodological advances introduced in EUE3, as described in “The Barra European Equity Model (EUE3),” *Research Notes* by Briner, Smith, and Ward (2009). The EUE3 model utilizes time series of daily specific returns to provide timely estimates of specific risk. A significant benefit of this approach is that specific risk is estimated individually for every stock, thus reflecting the idiosyncratic nature of this risk source.

A potential shortcoming of the pure time-series approach is that specific volatilities may not fully persist out-of-sample. In fact, as shown in the *USE4 Methodology Notes*, there is a tendency for time-series volatility forecasts to overpredict the specific risk of high-volatility stocks, and underpredict the risk of low-volatility stocks.

To reduce these biases, we apply a Bayesian shrinkage technique. We segment stocks into deciles based on their market capitalization. Within each size bucket, we compute the mean and standard deviation of specific risk forecasts. We then pull or “shrink” the volatility forecast toward the mean within each size decile, with the shrinkage intensity increasing with the number of standard deviations away from the mean.

Just as factor volatilities are not stable across time, the same holds for specific risk. In the EUE4 Model, a Volatility Regime Adjustment is also applied to specific risk. We estimate the adjustment by computing the cross-sectional bias statistic for the specific returns. The parameters for estimating the EUE4 specific risk model are reported in Appendix D.

2.4. Daily Forecast Horizon

EUE4D is designed to provide accurate forecast at a daily horizon. This is achieved by using shorter half-lives in both factor covariance matrix and specific risk estimation, by removing Newey-West serial correlation correction from both factor covariance matrix and specific risk estimation, and by using a weaker Bayesian shrinkage intensity in the specific risk model.

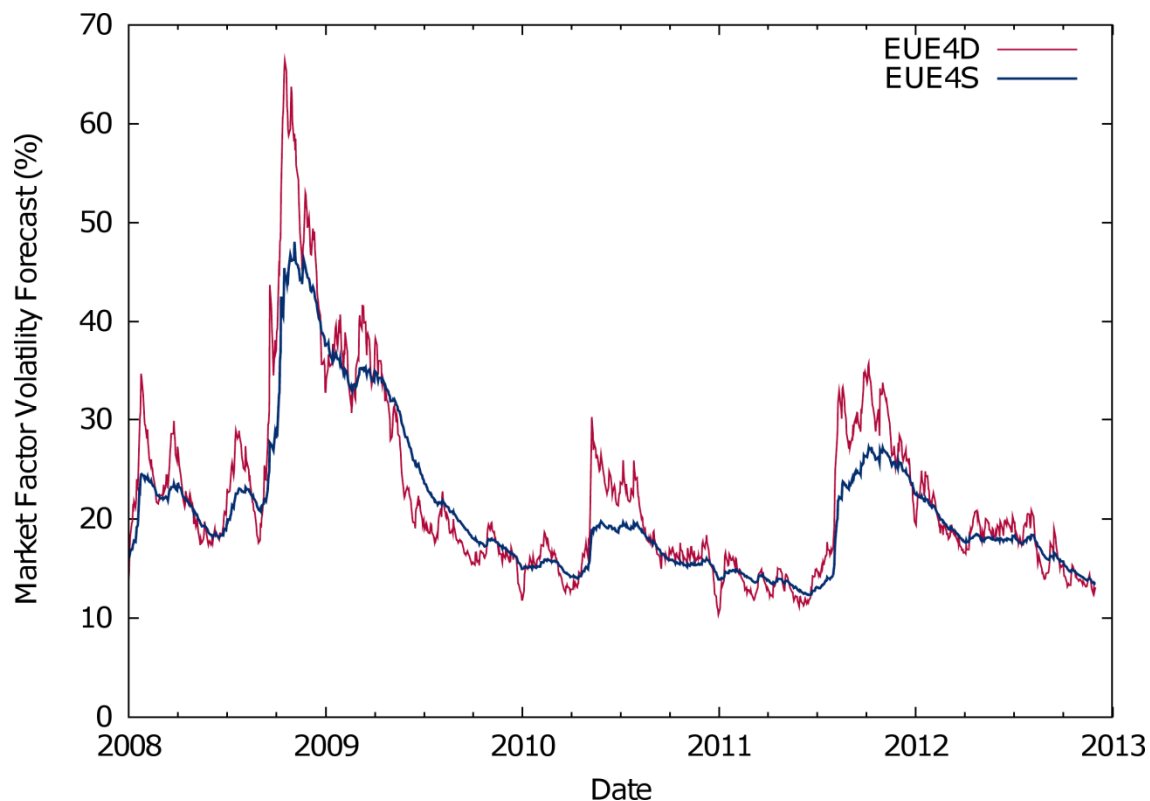
Figure 2.1 compares the Market factor volatility forecast given by EUE4D and EUE4S. One can see that the EUE4D forecast is generally significantly higher than the EUE4S forecast immediately after a major market shock. As volatility subsides following the market shock, the EUE4D forecast goes down more

quickly than the EUE4S forecast. Some may perceive the EUE4D forecast as being too “jumpy.” However, the empirical test results in Section 6 show that the Volatility Regime Adjusted EUE4D forecast is in fact more accurate than the EUE4S forecast on a daily horizon.

2.5. Model backtesting

Developing a sound risk model requires a reliable means of evaluating the accuracy of risk forecasts providing both portfolio and time resolution, i.e. evaluating the risk forecasts for a single portfolio across time, or for a set of portfolios within a single time period. To satisfy that criterion we use a testing methodology which penalizes both under and over forecasts and is not prone to “error cancellation” when averaged across time and/or test portfolios. In the course model quality tests we perform side-by-side comparison for the EUE3 and EUE4 Models based on 580 portfolios, including long only and active portfolios for a testing period of up to 215 months. We carefully analyze the effect of outlier returns and present results both without filtering and with the unanticipated noise of outliers filtered out. The raw and filtered results may be considered as complementing each other.

Figure 2.1: Market Factor Volatility Forecast: EUE4S vs. EUE4D.



3. Factor Structure Overview

3.1. Estimation Universe

The coverage universe is the set of all securities for which the model provides risk forecasts. The estimation universe, by contrast, is the subset of stocks that is used to actually estimate the model. Judicious selection of the estimation universe is an important part of building a sound risk model. The estimation universe must be broad enough to accurately represent the investment opportunity set of investors, without being so broad as to include illiquid stocks that may introduce spurious return relationships into the model. Furthermore, the estimation universe must be sufficiently stable to ensure that factor exposures are well behaved across time. *Representation, liquidity, and stability*, therefore, are the three primary issues that must be addressed when selecting a risk model estimation universe.

A well-constructed equity index must address these very same issues, and therefore serves as an excellent basis for the estimation universe. The EUE4 estimation universe utilizes the MSCI All Country Europe *Investable Markets Index* (AC Europe IMI), which aims to reflect the full breadth of investment opportunities within developed and emerging markets by targeting 99 percent of the float-adjusted market capitalization. The MSCI index construction methodology applies innovative rules designed to achieve index stability, while reflecting the evolving equity markets in a timely fashion. Moreover, liquidity screening rules are applied to ensure that only investable stocks that meet the index methodological requirements are included for index membership. For further details we refer the reader to the *MSCI Global Investable Market Indices Methodology Book*.

The EUE4 model also includes coverage of frontier markets that are outside of the AC Europe IMI. These markets are listed in the table 3.1C. For these markets, MSCI FM Index constituents are automatically included in the estimation universe. However, some of these countries have less than ten MSCI FM Index constituents. We thus add stocks with relatively higher liquidity to bring the count up to ten per country. Some countries still have less than five stocks in the estimation universe after this procedure. We then fill the gap with less liquid and relatively larger-cap stocks. As a result, each country has at least five stocks in the estimation universe.

3.2. Market and Country Factors

Every stock is assigned an exposure of 1 to the Market factor. Mathematically, the Market factor represents the intercept term in the cross-sectional regression. Economically, it describes the aggregate up-and-down movement of the European equity market. Typically, the Market factor is the dominant source of total risk for a diversified long-only portfolio.

Stocks are also assigned an exposure of 1 to their country factors. Countries are important variables for explaining the sources of equity return co-movement. The EUE4 Model includes a separate country factor for every market covered within the model. In **Table 3.1** we report the average weight of each country as well as the country weight as of 30-Nov-2012. We also report the largest stock within each country as of 30-Nov-2012, as well as the market capitalization in billions of Euros. Table 3.1 is organized into three panels. Panel A contains results for 16 developed markets, Panel B contains results for 5 emerging markets, and Panel C contains results for 13 markets outside of AC Europe IMI.

Table 3.1(A)

EUE4 developed market country factors. Weights were determined within the EUE4 estimation universe using total market capitalization. Averages were computed over the sample period (31-Dec-1994 to 30-Nov-2012).

Country name	Average Weight	30-Nov-2012 Weight	Largest Stock (30-Nov-2012)	Capitalization, (Billions EUR)
Austria	0.65%	0.81%	OMV AG	8.99
Belgium	2.20%	2.35%	ANHEUSER	108.24
Denmark	1.44%	1.83%	NOVO-NORDISK B	55.26
Finland	1.89%	1.26%	SAMPO A	13.72
France	14.65%	14.18%	TOTAL	90.95
Germany	11.53%	11.94%	SAP AG STA	73.73
Greece	0.81%	0.27%	COCA COLA HELL BOT	6.55
Ireland	0.77%	0.57%	CRH	10.15
Italy	6.42%	4.28%	ENI	66.07
Netherlands	5.66%	3.29%	UNILEVER CVA	50.03
Norway	1.39%	2.30%	STATOIL	59.96
Portugal	0.68%	0.46%	GALP ENERGIA	9.09
Spain	4.93%	4.64%	INDITEX	65.70
Sweden	3.79%	4.44%	HENNES & MAURITZ B	36.41
Switzerland	8.97%	10.35%	NESTLE	166.11
United Kingdom	29.06%	27.45%	HSBC HOLDINGS	142.72

Table 3.1(B)

EUE4 emerging market country factors. Weights were determined within the EUE4 estimation universe using total market capitalization. Averages were computed over the sample period (31-Dec-1994 to 30-Nov-2012).

Country name	Average Weight	30-Nov-2012 Weight	Largest Stock (30-Nov-2012)	Capitalization, (Billions EUR)
Czech Republic	0.29%	0.32%	CEZ	13.73
Hungary	0.20%	0.18%	MOL HUNGARIAN OIL	6.83
Poland	0.52%	1.09%	POWSZECHNA KASA OS	10.58
Russia	2.91%	5.30%	GAZPROM OAO	81.65
Turkey	0.87%	2.10%	T GARANTI BANKASI	15.33

Table 3.1(C)
EUE4 country factors for non-ACWI IMI markets. Weights were determined within the EUE4 estimation universe using total market capitalization. Averages were computed over the sample period (31-Dec-1994 to 30-Nov-2012).

Country name	Average Weight	30-Nov-2012 Weight	Largest Stock (30-Nov-2012)	Capitalization, (Billions EUR)
Bosnia Herzegovina	0.02%	0.02%	BH TELECOM	0.61
Bulgaria	0.03%	0.01%	BULGARTABAK HLDG (BGN1)	0.29
Croatia	0.11%	0.13%	INA INDUSTRIJA NAFTE	5.57
Cyprus	0.01%	0.00%	HELLENIC BANK	0.11
Estonia	0.03%	0.02%	TALLINK GRUPP AS	0.51
Iceland	0.10%	0.02%	MAREL (ISK1)	0.60
Kazakhstan	0.14%	0.13%	KAZMUNAIGAZ - UKI GDR	6.05
Latvia	0.01%	0.01%	LATVIJAS GAZE (LVL1)	0.33
Lithuania	0.04%	0.02%	LIETUVOS TELEKOMAS	0.56
Romania	0.12%	0.10%	PETROM S.A.	4.99
Serbia	0.02%	0.02%	NAFTNA INDUSTRIJA SRBIJE AD NOVI SAD	1.01
Slovenia	0.06%	0.05%	KRKA DD (SIT4000)	1.65
Ukraine	0.11%	0.05%	NORTH ORE MINING A	1.43

3.3. Industry Factors

Industries represent another important source of equity return co-movement. One of the strengths of the EUE4 Model is that it uses the Global Industry Classification Standard (GICS®) for the industry factor structure. The GICS scheme is hierarchical, with 10 sectors at the top level, 24 industry groups at the next level, followed with increasing granularity at the industry and sub-industry levels. GICS applies a consistent global methodology to classify stocks based on careful evaluation of the firm's business model and economic operating environment. GICS codes are reviewed annually to ensure that the classifications are timely and accurate.

It is important that the industry factor structure of a model reflect commonalities among different markets covered by the model, and at the same time, provide enough resolution, or explanatory power, to distinguish the behavior of different industries. When combining different GICS branches to form an industry factor structure, special care must also be taken to avoid forming over-crowded or over-thin industries, and to preserve economic intuition of the original GICS structure.

This investigation had been done in the course of the development of EUE3 and we reuse the industry scheme in EUE4. The resulting set of 29 EUE3/EUE4 industry factors is presented in **Table 3.2**. Industries that qualify as factors tend to exhibit high volatility and have significant weight. Also reported in Table 3.2 are the average weights from 30-Dec-1994 to 30-Nov-2012 and end-of-period weights.

Table 3.2
EUE4 industry factors. Weights were determined within the EUE4 estimation universe using total market capitalization. Averages were computed over the sample period (31-Dec-1994 to 30-Nov-2012).

GICS Sector	EUE4 Code	EUE4 Industry Factor Name	Average Weight	30-Nov-2012 Weight
Energy	1	Energy Equipment & Services	0.55%	1.62%
	2	Oil, Gas and Consumable Fuels	8.49%	8.29%
Materials	3	Other Materials	4.46%	4.67%
	4	Metals & Mining	2.62%	4.47%
Industrials	5	Other Capital Goods	4.38%	4.99%
	6	Construction & Engineering	1.07%	1.11%
	7	Machinery	1.72%	2.84%
	8	Commercial & Professional Services	1.24%	1.60%
	9	Transportation (non-Airlines)	1.83%	1.65%
	10	Airlines	0.39%	0.32%
Consumer Discretionary	11	Automobiles & Components	2.59%	3.13%
	12	Consumer Durables and Apparel	2.19%	3.01%
	13	Hotels, Restaurants & Leisure	1.25%	1.20%
	14	Media	2.99%	2.00%
	15	Retailing	2.15%	2.36%
Consumer Staples	16	Food, Drug & Staples Retailing	2.03%	1.56%
	17	Food, Beverage and Tobacco	6.44%	10.49%
	18	Household & Personal Products	1.16%	2.02%
Health Care	19	Health Care Equipment & Services	1.00%	1.36%
	20	Pharmaceuticals & Life Sciences	7.96%	8.65%
Financials	21	Banks	12.89%	9.47%
	22	Diversified Financials	5.09%	3.46%
	23	Insurance	5.76%	4.77%
	24	Real Estate	1.38%	1.73%
Information Technology	25	IT Services and Software	1.45%	1.87%
Technology	26	Technology Hardware & Equipment	2.37%	0.95%
	27	Semiconductors	0.54%	0.71%
Telecom Services	28	Telecommunication Services	7.90%	5.16%
Utilities	29	Utilities	6.13%	4.53%

In **Table 3.3**, we report the underlying GICS codes that map to each of the EUE4 industry factors. This table helps illustrate the customization that takes place within each sector. In each case, the industry structure is guided by a combination of financial intuition and empirical analysis.

Table 3.3
Mapping of EUE4 industry factors to GICS codes.

EUE4 Code	EUE4 Industry Factor Name	GICS Codes
1	Energy Equipment & Services	101010
2	Oil, Gas and Consumable Fuels	101020
3	Other Materials	151010 151020 151030 151050
4	Metals & Mining	151040
5	Other Capital Goods	201010 201020 201040 201050 201070
6	Construction & Engineering	201030
7	Machinery	201060
8	Commercial & Professional Services	2020
9	Transportation (non-Airlines)	203010 203030 203040 203050
10	Airlines	203020
11	Automobiles & Components	2510
12	Consumer Durables and Apparel	2520
13	Hotels, Restaurants & Leisure	2530
14	Media	2540
15	Retailing	2550
16	Food, Drug & Staples Retailing	3010
17	Food, Beverage and Tobacco	3020
18	Household & Personal Products	3030
19	Health Care Equipment & Services	3510
20	Pharmaceuticals & Life Sciences	3520
21	Banks	4010
22	Diversified Financials	4020
23	Insurance	4030
24	Real Estate	4040
25	IT Services and Software	4510
26	Technology Hardware & Equipment	4520
27	Semiconductors	4530
28	Telecommunication Services	5010
29	Utilities	5510

In **Table 3.4** we report the largest firm within each industry, as well as the total market capitalization as of 30-Nov-2012.

Table 3.4
Largest stock within each industry as of 30-Nov-2012. Market capitalizations are reported in billions of Euro.

EUE4 Code	EUE4 Industry Factor Name	Largest Stock (30-Nov-2012)	Capitalization (Billions EUR)
1	Energy Equipment & Services	TENARIS SA	18.0
2	Oil, Gas and Consumable Fuels	BP	101.2
3	Other Materials	BASF N ORD SHS	63.3
4	Metals & Mining	RIO TINTO PLC	53.8
5	Other Capital Goods	SIEMENS	72.5
6	Construction & Engineering	VINCI	19.5
7	Machinery	ATLAS COPCO AB-A FR	16.6
8	Commercial & Professional Services	SGS SA	13.5
9	Transportation (non-Airlines)	DEUTSCHE POST	19.3
10	Airlines	RYANAIR HLDGS	6.8
11	Automobiles & Components	VOLKSWAGEN STA	46.1
12	Consumer Durables and Apparel	LVMH	68.5
13	Hotels, Restaurants & Leisure	COMPASS GROUP	16.7
14	Media	BRITISH SKY BROADCASTING GROUP	15.7
15	Retailing	INDITEX	65.7
16	Food, Drug & Staples Retailing	TESCO	32.2
17	Food, Beverage and Tobacco	NESTLE	166.1
18	Household & Personal Products	L'OREAL	62.9
19	Health Care Equipment & Services	FRESENIUS MEDCARE STA	15.9
20	Pharmaceuticals & Life Sciences	NOVARTIS AG RS	128.8
21	Banks	HSBC HOLDINGS	142.7
22	Diversified Financials	UBS N	46.1
23	Insurance	ALLIANZ SE	45.5
24	Real Estate	UNIBAIL RODAMCO	16.6
25	IT Services and Software	SAP AG STA	73.7
26	Technology Hardware & Equipment	ERICSSON TEL. AB-B	21.6
27	Semiconductors	ASML HOLDING	20.2
28	Telecommunication Services	VODAFONE GROUP	98.0
29	Utilities	GDF SUEZ	40.2

3.4. Style Factors

Investment style represents another major source of systematic risk for equity portfolios. Style factors are constructed from financially intuitive stock attributes called *descriptors*, which serve as effective predictors of equity return covariance.

In order to facilitate comparison across style factors, the factors are standardized to have a cap-weighted mean of 0 and an equal-weighted standard deviation of 1. The cap-weighted estimation universe, therefore, is *style neutral*.

The EUE4 Model contains 11 style factors. The EUE4 style factors are based on the EUE3 style factors, with two major differences. First, the EUE3 Volatility factor is broken out into two EUE4 style factors: Beta and Residual Volatility. Second, the *Non-Linear Size* (NLS) factor is introduced. Another difference is

that the one of the descriptors of EUE3 Value factor, Book-to-Price is separated as an independent style factor while the other descriptor of Value, Sales-to-Price is not used in EUE4. The factors are described in Appendix A, together with descriptor definitions and descriptor weights. Here we provide a brief qualitative description of the factors:

- The *Beta* factor is typically the most important style factor. It captures market risk that cannot be explained by the Market factor. We compute Beta by time-series regression of excess stock returns against the cap-weighted estimation universe, as described in Appendix A. To better understand how Beta relates to the Market factor, consider a fully invested long-only portfolio that is tilted toward high-beta stocks. Intuitively, this portfolio has greater market risk than a portfolio with a beta of 1. This additional market risk is captured through positive exposure to the Beta factor. Since the time-series correlation between the Market factor and the Beta factor is typically very high, these two sources of risk are additive in this example. If, by contrast, the portfolio were invested primarily in low-beta stocks, then the risk from the Beta and the Market factors would have been partially offset, as expected.
- The *Momentum* factor is often the second strongest factor in the model, although sometimes it may surpass Beta in importance. Momentum differentiates stocks based on their performance over the trailing 6-12 months. When computing Momentum exposures we exclude recent returns in order to avoid the effects of short-term reversal.
- The *Size* factor represents another strong source of equity return covariance, and captures return differences between large-cap stocks and small-cap stocks. We measure Size by the log of market capitalization.
- The *Earnings Yield* factor describes return differences based on a company's earnings relative to its price. Earnings Yield is considered by many investors to be a strong value signal. The most important descriptor in this factor is the analyst-predicted 12-month forward earnings-to-price ratio.
- The *Residual Volatility* factor is composed of three descriptors: (a) the volatility of daily excess returns, (b) the volatility of daily residual returns, and (c) the cumulative range of the stock over the last 12 months. Since these descriptors tend to be highly collinear with the Beta factor, the Residual Volatility factor is orthogonalized with respect to the Beta factor, as described by Menchero (2010).
- The *Growth* factor differentiates stocks based on their prospects for sales or earnings growth. The most important descriptor in this factor is the analyst predicted long-term earnings growth. Other descriptors include sales and earnings growth over the trailing five years.
- The *Dividend Yield* factor explains return differences attributable to dividend payouts of the firm. This factor is defined by the trailing 12-month dividend divided by the current price.
- The *Book-to-Price* factor is also considered by some to be an indicator of value. This factor is given by the last reported book value of common equity divided by current market capitalization.
- The *Leverage* factor captures return differences between high-leverage and low-leverage stocks. The descriptors within this style factor include market leverage, book leverage, and debt-to-assets ratio.
- The *Liquidity* factor describes return differences due to relative trading activity. The descriptors for this factor are based on the fraction of total shares outstanding that trade over a recent window.
- The *Non-Linear Size* factor captures non-linearities in the payoff to the Size factor across the market-cap spectrum. This factor is based on a single raw descriptor: the cube of the Size exposure. However, since this raw descriptor is highly collinear with the Size factor, it is orthogonalized with respect to Size. This procedure does not affect the fit of the model, but does mitigate the confounding effects of collinearity, while preserving an intuitive meaning for the Size factor. As

described by Menchero (2010), the Non-Linear Size factor roughly captures the risk of a “barbell portfolio” that is long mid-cap stocks and short small-cap and large-cap stocks.

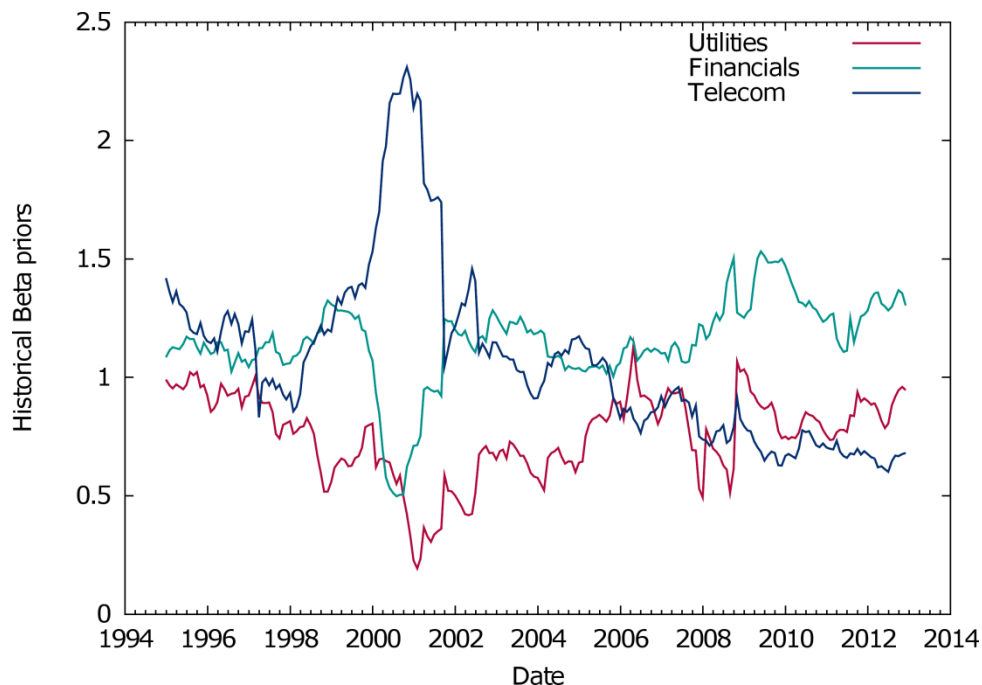
3.5. Historical Beta priors

Typically, Historical Beta (the descriptor of *Beta* factor) is computed as the slope coefficient in a time-series regression of excess stock return against the cap-weighted excess return of the estimation universe. (For more details see Appendix A) This approach is robust for liquid assets with sufficiently long history but has two drawbacks:

- It tends to underestimate the Beta of illiquid assets which trade rarely and,
- It is not practical to estimate the Beta of recent IPO-s.

To circumvent those difficulties, Beta priors were introduced in EUE4 to blend the Beta of recent IPO-s and illiquid assets. The prior is the cap-weighted GICS® sector average Beta of liquid stocks with sufficiently long return history. In **Figure 3.1** we show those priors for three GICS sectors as a function of time.

Figure 3.1
Historical Beta priors versus time for Utilities, Financials and Telecom sectors.



For each asset, we compute the density of non-zero returns, ρ in the time series window and, if necessary, blend the raw Beta obtained via time series regression with the sector prior,

$$\beta_{blended} = w_{prior} \beta_{prior} + (1 - w_{prior}) \beta_{raw} ,$$

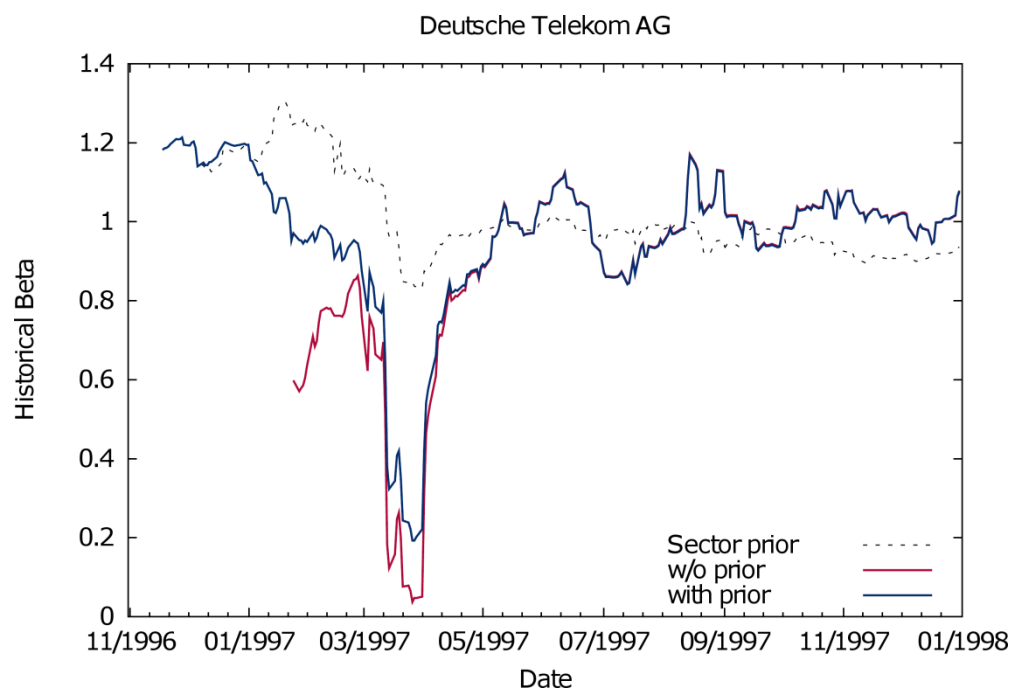
where the weight of prior is determined using the return density as

$$w_{prior} = \begin{cases} 1 & ; \rho < 0.15 \\ 1 - \frac{\rho - 0.15}{0.6} & ; 0.15 \leq \rho \leq 0.75. \\ 0 & ; 0.75 < \rho \end{cases}$$

In **Figure 3.2** we show the blending-in effect for an IPO.

Figure 3.2

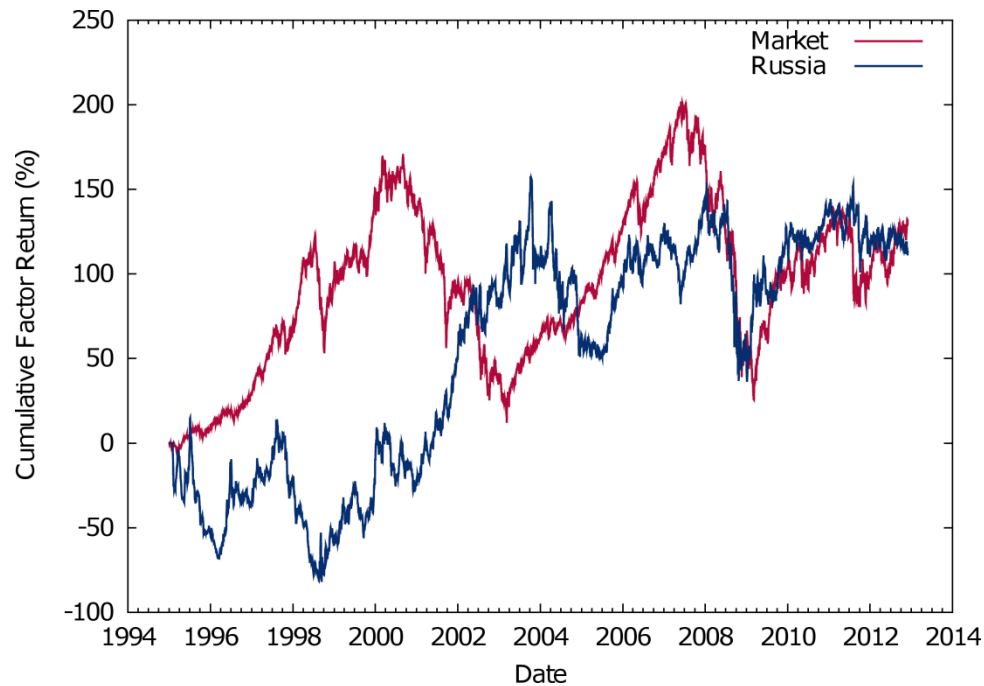
The effect of Beta blending on the Beta descriptor of Deutsche Telekom AG shortly after its IPO (to date, it was the second largest IPO in Europe). The dashed line depicts the prior in the Telecom sector. Right after the company's IPO, its Historical Beta coincides with the sector prior. As time proceeds, the return density in the time series window increases resulting in decreasing weight of the sector prior in the stock's Historical Beta.



3.6. Performance of Select Factors

It is helpful to consider the performance of individual factors. In **Figure 3.3**, we report cumulative returns to the EUE4 Market factor and Russia country factor. The Market factor return essentially represents the excess return (i.e., above the risk-free rate) of the cap-weighted European portfolio. Figure 3.3 clearly illustrates the main features of the European equity market since 1995. For instance, the three bull markets of the sample period are clearly visible, as is the bear market after the Internet Bubble, and the market crash of 2008.

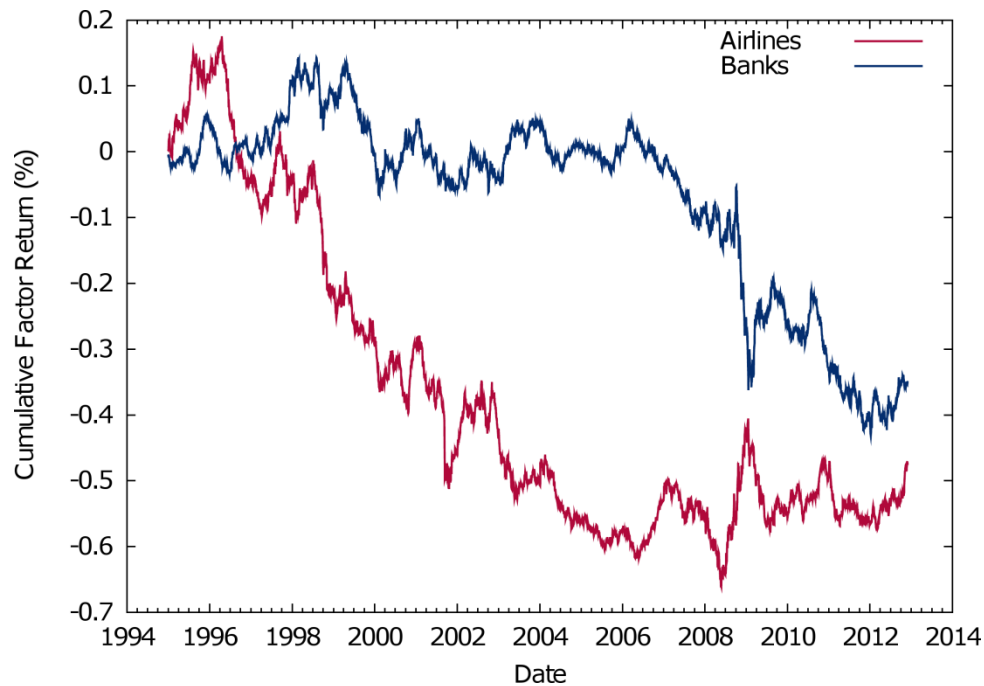
Figure 3.3
Cumulative returns of EUE4 Market factor and Russia country factor.



The country factor return represents the performance of the pure country relative to the overall market, net of all industry and style effects. In other words, the pure country factor portfolio is Euro neutral and has zero exposure to every industry and style. In **Figure 3.3** we also report the cumulative return of the Russia factor. We see that in 1998, Russia severely underperformed, dropping by roughly 70 percent. Between 1998-2004, however, Russia has experienced a strong bull market.

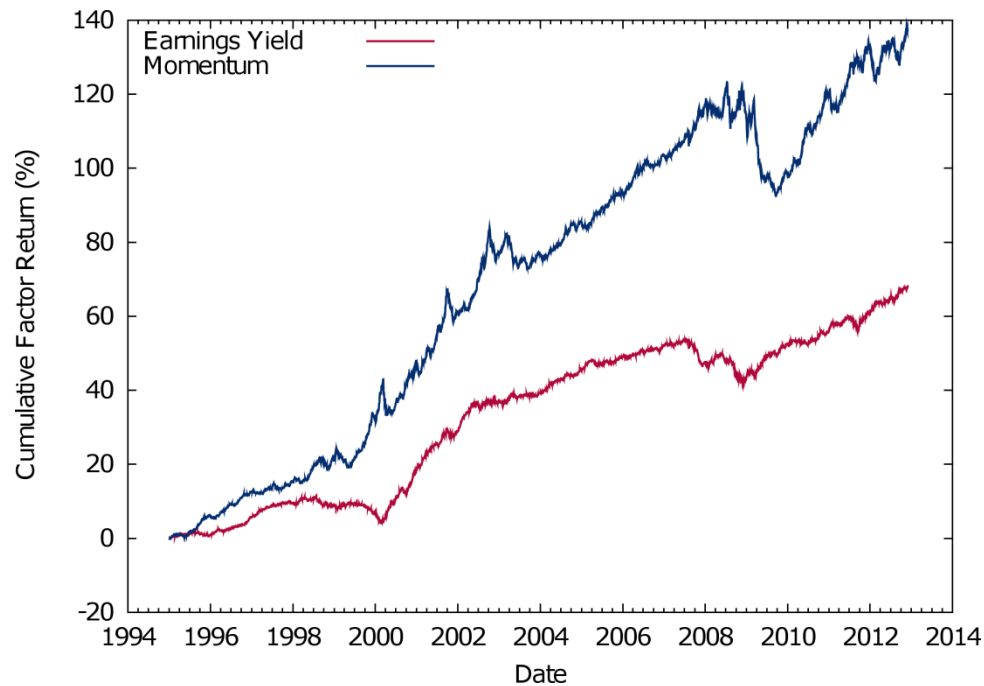
In **Figure 3.4** we report the cumulative returns for the Airlines factor and Banks factor. The Airlines factor portfolio was down by more than 50 percent since 1995, whereas the Banks factor was down by roughly 35 percent over the same period.

Figure 3.4
Cumulative returns of Airlines factor and Banks factor.



In **Figure 3.5**, we report the cumulative returns to the Earnings Yield and Momentum factors, which represent two common strategies often used by quantitative investors. Overall, Earnings Yield performed very well over the past 18 years, consistent with the notion of a “value premium.” As described by Basu (1977), this reflects the tendency of stocks that are priced low relative to fundamentals to outperform. A notable exception, however, occurred during the Internet Bubble in 1999, when Earnings Yield performed poorly. The Earnings Yield factor also performed poorly from the Quant Meltdown in August 2007 until the end of 2008.

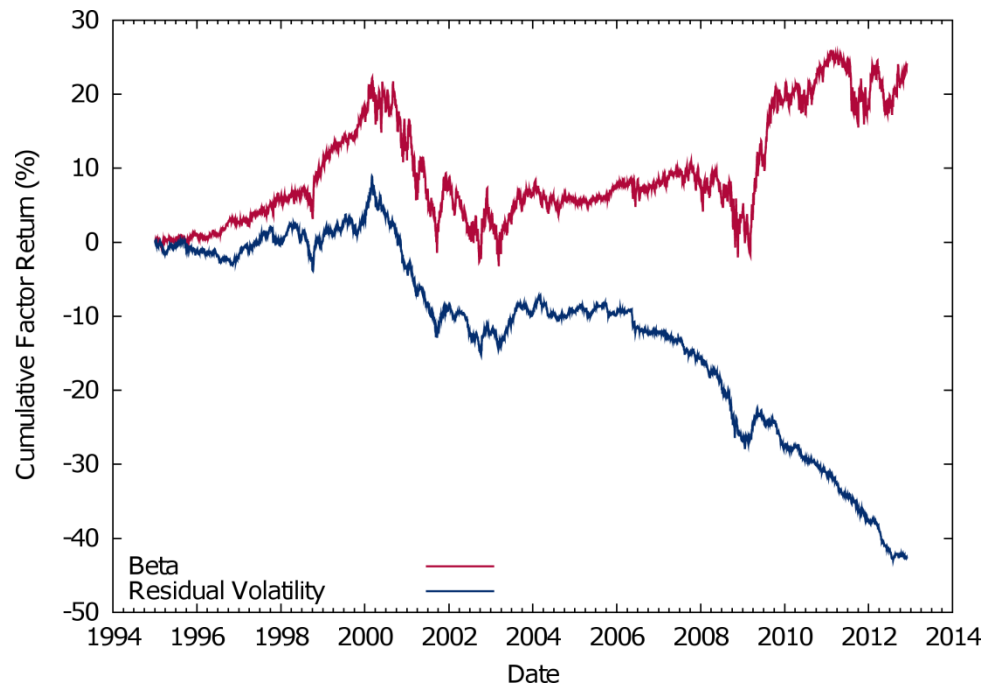
Figure 3.5
Cumulative returns of Earnings Yield and Momentum factors.



Momentum also performed well since 1997, consistent with the empirical observation noted by Jegadeesh and Titman (1993) that stocks with strong performance over the previous 6-12 months continue to outperform. There were, however, two major periods of underperformance. The first occurred in late 2002, which coincided with the market rebound following the 2000-2002 bear market. The second major downturn for the Momentum factor began in March 2009, which again coincided with a market recovery — this time after the crash of 2008.

In **Figure 3.6** we report cumulative returns to the Beta and Residual Volatility factors. Qualitatively, the Beta factor showed many of the main features as the Market factor in Figure 3.1, consistent with the high correlation between these factors. For instance, the Beta factor performed well during the bull market of 1997-2000, and performed poorly during the bear market of 2001-2003. However, Beta remained essentially flat during the 2003-2007 bull market. Furthermore, Beta did not experience a large decline during the market crash of 2008. Since 2009, the Beta factor has performed well.

Figure 3.6
Cumulative returns of Beta and Residual factors.



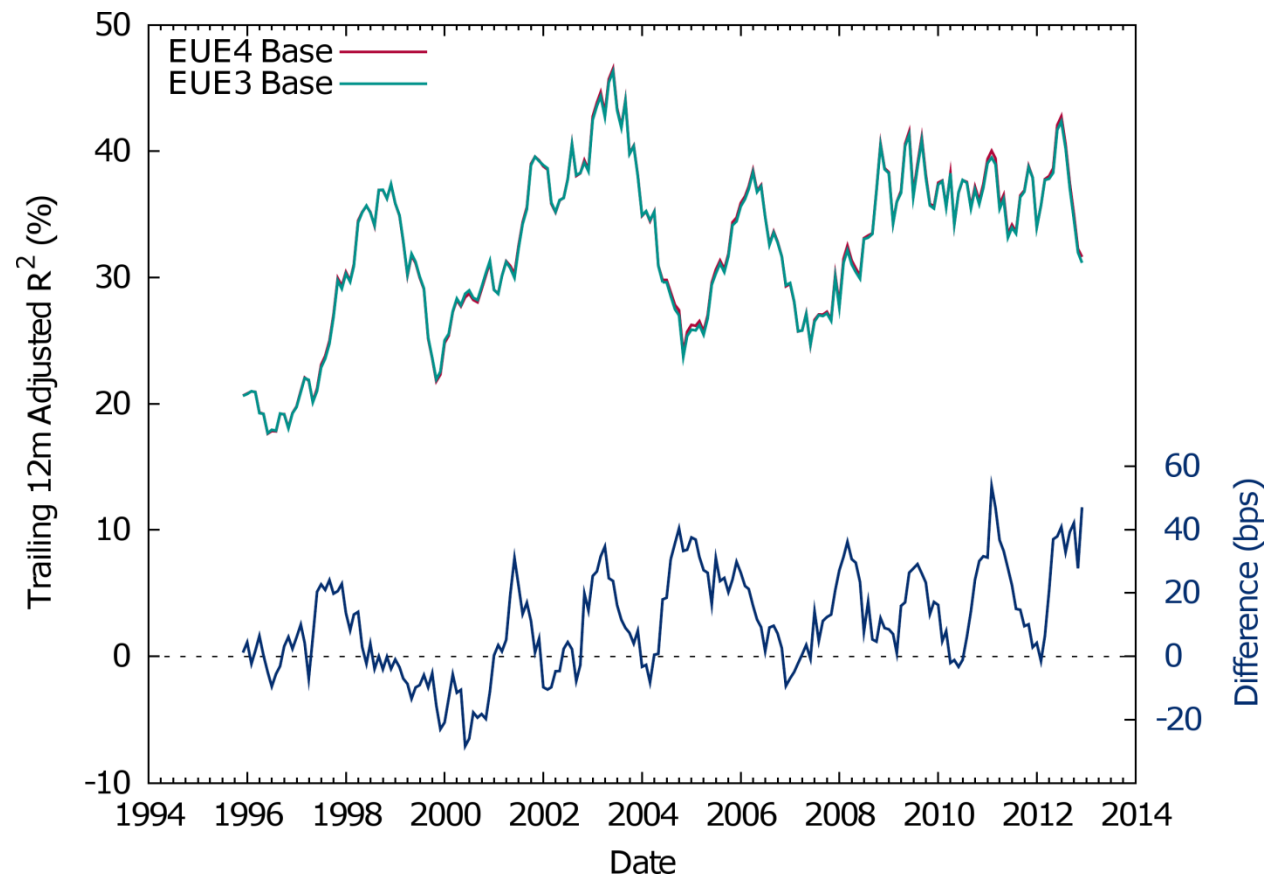
The Residual Volatility factor exhibited a consistent downward drift over the sample period. This is consistent with the empirical findings of Ang *et al.*, who showed that stocks with high specific risk have lower returns than stocks with lower specific risk.

The explanatory power, measured in R -squared or adjusted R -squared, is an important metric of model quality. A special care is needed when we compare R -squared values across different models: The value of R -squared can be susceptible to any change in regression weighting scheme, estimation universe and the time period under consideration. If each of these variables is carefully controlled then we can perform a meaningful comparison between models.

In Figure 3.7 we report the explanatory power of EUE4 and EUE3 base models. In order to ensure a fair comparison, the estimation universe (MSCI AC Europe IMI) and regression weighting scheme (square root of market capitalization) were identical for the two sets of regressions. Clearly, both models track each other closely in terms of explanatory power. In terms of adjusted R -squared we see 10 bps improvement of explanatory power for EUE4 as compared to its predecessor.

Figure 3.7
Trailing 12-month adjusted R^2 for EUE4 and EUE3 models. Results were computed based on monthly

cross-sectional regressions using a common estimation universe (MSCI AC Europe IMI) and regression weighting scheme (square root of market capitalization). The difference is also plotted, with the scale indicated on the right axis.



4. Model Characteristics and Properties

4.1. Country Factors

One requirement of a high-quality factor structure is that the factor returns be statistically significant. This helps prevent weak or noisy factors from finding their way into the model. We measure statistical significance by the t -statistic of the factor return. Assuming normality, absolute t -statistics greater than 2 are considered significant at the 95-percent confidence level. In other words, if the factor truly had no explanatory power (e.g., pure noise), then by chance we would observe $|t| > 2$ about 5 percent of the time.

In **Table 4.1** we report mean absolute t -statistics for the EUE4 country factors, as well as the percentage of observations with $|t| > 2$. Note that the t -statistics reported in Table 4.1 were computed using *monthly* cross-sectional regressions, even though we run daily cross-sectional regressions for purposes

of constructing the factor covariance matrix. This distinction is important, because what is ultimately relevant is the explanatory power of the factors at the prediction horizon of the model.

For developed markets (Panel A), the two factors with the highest statistical significance were United Kingdom and Greece. For emerging markets (Panel B), the most significant factors were Turkey and Russia, whereas for markets outside AC Europe IMI (Panel C) the most significant factor was Ukraine.

Also reported in Table 4.1 are the returns, volatilities, and Sharpe ratios for the country factors. These quantities were computed using *daily* factor returns and stated on an annualized basis. For developed markets, the top-performing factor on a risk-adjusted basis was Sweden, with a Sharpe ratio of 0.31. The worst-performing developed market was Italy, which had a Sharpe ratio of -0.28.

Table 4.1 also reports the correlations of the daily factor returns with the returns of the estimation universe portfolio. The Netherlands factor had the largest positive correlation, whereas Austria, Denmark, Greece, Italy and Portugal factors exhibited significant negative correlations. By contrast, emerging markets and frontier markets exhibit overwhelmingly negative correlations with the estimation universe.

Table 4.1 (A)

Country factor summary statistics for developed markets. The two columns pertaining to *t*-statistics were computed using monthly cross-sectional regressions. The last four columns were computed based on daily factor returns. The sample period is from 30-Dec-1994 to 30-Nov-2012 (215 months of returns).

Factor name	Factor start date	Average Absolute <i>t</i> -stat	Percent Observ. $ t > 2$	Annual. Factor Return	Annual. Factor Volatility	Factor Sharpe Ratio	Correl. with ESTU
Austria	12/30/1994	1.40	23.1%	0.93%	11.05%	0.08	-0.220
Belgium	12/30/1994	1.60	29.0%	0.35%	7.91%	0.04	-0.138
Denmark	12/30/1994	1.86	39.1%	0.89%	10.51%	0.08	-0.186
Finland	12/30/1994	1.97	42.1%	3.34%	11.87%	0.28	0.023
France	12/30/1994	2.43	48.1%	0.27%	5.12%	0.05	0.115
Germany	12/30/1994	2.57	48.6%	-1.05%	6.73%	-0.16	0.037
Greece	12/30/1994	3.75	60.5%	-2.38%	25.18%	-0.09	-0.181
Ireland	12/30/1994	1.70	37.0%	2.60%	12.01%	0.22	-0.191
Italy	12/30/1994	3.15	54.7%	-2.85%	10.33%	-0.28	0.051
Netherlands	12/30/1994	1.58	30.0%	-1.54%	6.22%	-0.25	0.149
Norway	12/30/1994	2.03	44.2%	0.99%	12.55%	0.08	-0.121
Portugal	12/30/1994	1.76	36.3%	-1.73%	11.24%	-0.15	-0.192
Spain	12/30/1994	2.58	51.6%	0.33%	8.83%	0.04	0.075
Sweden	12/30/1994	2.50	51.9%	3.12%	10.17%	0.31	0.004
Switzerland	12/30/1994	1.99	38.8%	0.75%	6.14%	0.12	0.048
UK	12/30/1994	3.93	66.4%	0.67%	4.79%	0.14	-0.069
Average		2.30	43.8%		10.04%		

Table 4.1 (B)

Country factor summary statistics for emerging markets. The two columns pertaining to *t*-statistics were computed using monthly cross-sectional regressions. The last four columns were computed based on daily factor returns. The sample period is from 30-Dec-1994 to 30-Nov-2012 (215 months of returns).

Factor name	Factor start date	Average Absolute <i>t</i> -stat	Percent Observ. $ t > 2$	Annual. Factor Return	Annual. Factor Volatility	Factor Sharpe Ratio	Correl. with ESTU
Czech Republic	12/31/1996	1.75	33.8%	-1.20%	17.50%	-0.07	-0.184
Hungary	12/31/1996	1.64	28.2%	-4.83%	21.01%	-0.23	-0.105
Poland	12/31/1996	2.60	52.8%	-1.66%	28.36%	-0.06	-0.186
Russia	12/31/1996	5.74	78.1%	4.20%	39.44%	0.11	-0.064
Turkey	12/31/1996	5.99	77.1%	5.43%	37.07%	0.15	-0.154
Average		3.55	54.0%		28.67%		

Table 4.1 (C)

Country factor summary statistics for markets outside AC Europe IMI. The two columns pertaining to *t*-statistics were computed using monthly cross-sectional regressions. The last four columns were computed based on daily factor returns. The sample period is from 30-Dec-1994 to 30-Nov-2012 (215 months of returns).

Factor name	Factor start date	Average Absolute <i>t</i> -stat	Percent Observ. $ t > 2$	Annual. Factor Return	Annual. Factor Volatility	Factor Sharpe Ratio	Correl. with ESTU
Bosnia Herzegovina	4/29/2011	0.75	11.5%	5.31%	16.51%	0.32	-0.462
Bulgaria	1/31/2006	1.88	15.3%	-8.90%	23.36%	-0.38	-0.487
Croatia	11/30/2001	1.56	27.2%	3.87%	20.23%	0.19	-0.352
Cyprus	1/31/2003	1.01	10.8%	-7.11%	22.91%	-0.31	-0.380
Estonia	12/31/2002	1.24	19.2%	8.60%	19.17%	0.45	-0.342
Iceland	9/28/2001	1.32	23.4%	-4.43%	21.43%	-0.21	-0.420
Kazakhstan	7/31/2006	2.16	47.0%	4.45%	38.17%	0.12	-0.228
Latvia	11/29/2002	0.89	7.9%	2.87%	24.69%	0.12	-0.442
Lithuania	11/29/2002	1.50	23.6%	6.84%	20.64%	0.33	-0.387
Romania	11/29/2002	1.80	33.1%	1.36%	25.36%	0.05	-0.320
Serbia	10/30/2009	1.15	8.9%	-14.59%	26.00%	-0.56	-0.607
Slovenia	8/31/2000	1.07	16.2%	1.25%	18.85%	0.07	-0.343
Ukraine	12/29/2006	2.72	62.3%	-21.28%	30.71%	-0.69	-0.358
Average		1.47	23.6%		23.69%		

Figure 4.1
 $|t|$ values for Italy, Spain, Greece and Portugal country factors. Lines were smoothed using 150 days moving averages.

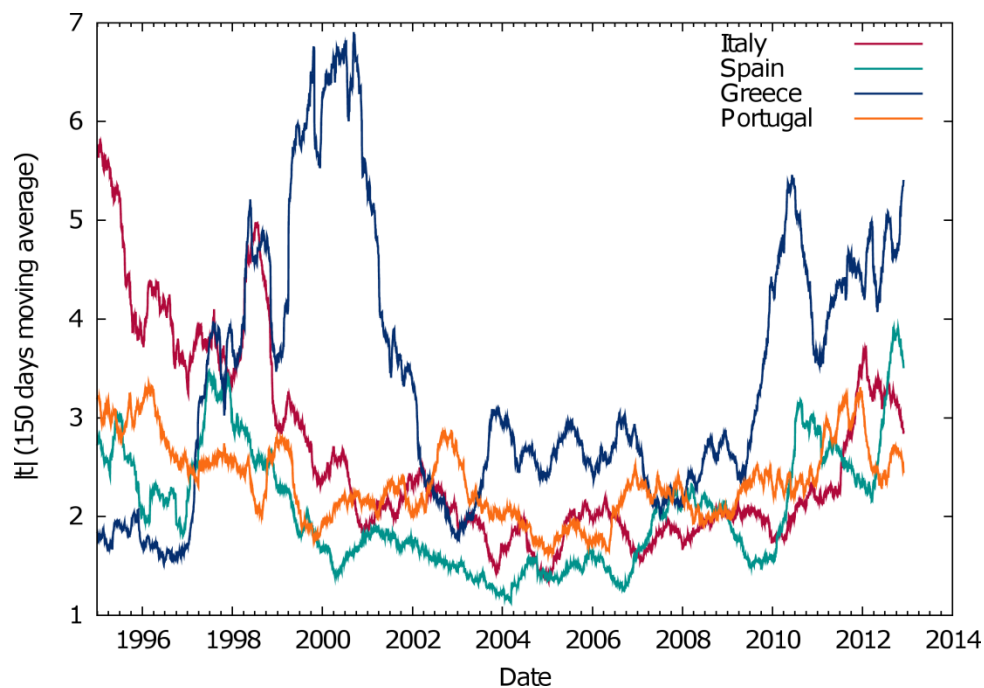


Figure 4.1 uses the Italy, Spain, Greece and Portugal country factors as an example to illustrate the gradually increasing market integration within Europe from 2000-2008, which is followed by a period when country factors are again increasingly important. Before 2000, country factors had high $|t|$ values which then gradually decreased indicating that individual countries became less distinguishable from the European regional average. That trend was reversed in 2008 reflecting the increasing importance of country-specific effects within Europe.

4.2. Industry Factors

In **Table 4.2** we report mean absolute t -statistics for the EUE4 Market factor and industry factors, as well as the percentage of observations with $|t| > 2$. Again, the t -statistics reported in Table 4.2 were computed using *monthly* cross-sectional regressions, even though we run daily cross-sectional regressions for purposes of constructing the factor covariance matrix.

From Table 4.2, we see that the Market factor was by far the strongest factor during the sample period. On average, it had an absolute t -statistic of 14.42, and was significant about 90 percent of the months. Of the industry factors, Metals & Mining had the highest statistical significance. Across all industries, the t -statistics were significant in 39.1 percent of the observations. This is higher than the mean statistical significance of the country factors (37.6 percent), and reflects the strength of the industry factors.

Table 4.2

Industry factor summary statistics. The first two columns pertain to *t*-statistics, and were computed using monthly cross-sectional regressions. The last four columns were computed based on daily factor returns. The sample period is from 30-Dec-1994 to 30-Nov-2012 (215 months of returns).

Factor name	Average Absolute <i>t</i> -stat	Percent Observ. $ t > 2$	Annual. Factor Return	Annual. Factor Volatility	Factor Sharpe Ratio	Correl. with ESTU
Market factor	14.42	89.5%	4.63%	17.53%	0.26	0.998
Energy Equipment & Services	1.83	38.4%	3.36%	13.45%	0.25	0.043
Oil, Gas and Consumable Fuels	2.55	50.0%	0.72%	10.38%	0.07	-0.034
Other Materials	1.90	37.5%	-3.35%	5.15%	-0.65	0.052
Metals & Mining	2.77	54.6%	-3.69%	11.40%	-0.32	0.137
Other Capital Goods	1.70	32.9%	-3.11%	5.06%	-0.61	0.084
Construction & Engineering	1.52	30.1%	-2.35%	6.26%	-0.38	0.069
Machinery	1.96	41.7%	-2.03%	6.18%	-0.33	0.105
Commercial & Professional Services	1.32	22.2%	-0.30%	6.12%	-0.05	-0.064
Transportation (non-Airlines)	1.29	21.3%	-1.06%	5.97%	-0.18	-0.067
Airlines	1.76	36.1%	-3.42%	12.01%	-0.29	-0.048
Automobiles & Components	2.18	43.1%	-2.78%	10.82%	-0.26	0.014
Consumer Durables and Apparel	1.84	36.1%	-1.63%	6.68%	-0.24	0.025
Hotels, Restaurants & Leisure	1.50	27.8%	-1.53%	7.03%	-0.22	-0.060
Media	1.97	40.3%	-1.51%	6.66%	-0.23	-0.056
Retailing	1.84	35.6%	-0.45%	6.83%	-0.07	-0.083
Food, Drug & Staples Retailing	1.64	32.9%	-0.01%	8.43%	0.00	-0.124
Food, Beverage and Tobacco	1.78	39.4%	1.70%	5.47%	0.31	-0.188
Household & Personal Products	1.17	19.0%	3.88%	9.23%	0.42	-0.255
Health Care Equipment & Services	1.27	21.3%	3.55%	6.97%	0.51	-0.151
Pharmaceuticals & Life Sciences	2.02	42.6%	4.94%	7.44%	0.66	-0.150
Banks	2.38	45.8%	-2.28%	7.22%	-0.32	0.120
Diversified Financials	1.64	37.0%	-1.93%	4.95%	-0.39	0.214
Insurance	2.21	46.3%	-3.78%	7.19%	-0.53	0.147
Real Estate	1.95	36.1%	-0.91%	7.69%	-0.12	0.002
IT Services and Software	2.11	34.7%	3.03%	9.98%	0.30	0.055
Technology Hardware & Equipment	1.77	37.0%	-1.37%	8.26%	-0.17	0.096
Semiconductors	2.21	46.3%	7.82%	17.70%	0.44	0.156
Telecommunication Services	2.64	54.2%	5.63%	8.41%	0.67	-0.085
Utilities	2.06	43.1%	0.50%	6.78%	0.07	-0.136
Average	2.31	39.1%		8.44%		

Also reported in Table 4.2 are the returns, volatilities, and Sharpe ratios for the industry factors. These quantities were computed using *daily* factor returns and stated on an annualized basis. The Market factor had an annualized return of 4.63 percent and a volatility of 17.53 percent, leading to a Sharpe ratio of 0.26 over the sample period. The best-performing industries tended to be concentrated in the

Telecom and Health Care sectors, whereas the worst-performing industries tended to be in the Industrials and Financials sectors.

Table 4.2 also reports the correlations of the daily factor returns with the estimation universe. Particularly noteworthy is the 99.8 percent correlation between the Market factor and the estimation universe, indicating the essential equivalence of the two. By contrast, most industry factors, being dollar-neutral portfolios, had relatively small correlations with the estimation universe. Industry factors within the Consumer Staples, Health Care, and Utilities tended to have negative correlations, whereas factors within Financials and Information Technology were positive. It is important to stress that these correlations represent averages over the entire sample period. Within different sub-periods or market regimes, the correlations may deviate significantly from these reported values.

4.3. Style Factors

In **Table 4.3**, we report summary statistics for the EUE4 style factors, during the sample period. Note that the statistical significance of the style factors, on the whole, was on par with that for the industry factors. As measured by statistical significance, the strongest factors were generally Beta and Momentum, followed by Residual Volatility. In the sample period (30-Dec-1994 to 30-Nov-2012), Momentum, Earnings Yield, and Dividend Yield performed extremely well, while Residual Volatility and Size performed poorly.

Most style factors, being dollar-neutral portfolios, had relatively small correlation with the estimation universe. The glaring exception is the Beta factor, which had a correlation of 0.87 in the sample period. The Residual Volatility and Size factors also have a sizeable positive correlation with the estimation universe. Again, it is important to stress that the correlations reported in Table 4.3 represent averages, and that the actual correlations in different market regimes may deviate from these reported values.

Also reported in Table 4.3 is the factor stability coefficient, described in the *USE4 Methodology Notes*. Briefly, this coefficient is computed as the cross-sectional correlation of factor exposures from one month to the next. Although there is no strict lower limit for what is considered acceptable, a useful rule of thumb is that values below 0.80 are regarded as too unstable for model inclusion, while those above 0.90 are considered sufficiently stable. From Table 4.3, we see that the average factor stability coefficient for style factors was 0.96 or above during both sample periods.

Table 4.3 also reports the Variance Inflation Factor (VIF). As explained in the *USE4 Methodology Notes*, VIF measures the degree of collinearity among the factors. Excessive collinearity can lead to increased estimation error in the factor returns and non-intuitive correlations among factors. Although there exists no strict upper bound, VIF scores above 5 are generally considered problematic. As shown in Table 4.3, all EUE4 style factors were well below this level during the sample period.

Table 4.3

Style factor summary statistics. The first two columns pertain to *t*-statistics, and were computed using monthly cross-sectional regressions. The next four columns were computed based on daily factor returns. The factor stability coefficient and Variance Inflation Factor were computed on monthly data using square root of market-cap weighting. The entire sample period comprises 215 months (30-Dec-1994 to 30-Nov-2012).

Factor name	Average Absolute <i>t</i> -stat	Percent Observ. $ t > 2$	Annual. Factor Return	Annual. Factor Volatility	Factor Sharpe Ratio	Correl. with ESTU	Factor stability coeff.	Variance inflation factor
Beta	3.79	63.9%	-0.18%	5.47%	-0.03	0.87	0.95	2.68
Book-to-Price	1.50	31.5%	0.96%	1.89%	0.51	0.07	0.98	2.00
Dividend Yield	1.60	28.2%	1.52%	1.75%	0.87	-0.01	0.96	1.65
Earnings Yield	1.98	40.3%	2.28%	2.41%	0.95	0.04	0.96	1.96
Growth	1.26	19.0%	0.44%	1.27%	0.35	0.08	0.95	1.22
Leverage	1.41	27.8%	-1.16%	1.59%	-0.73	0.02	0.99	1.58
Liquidity	1.34	23.1%	0.30%	1.54%	0.19	0.10	0.95	1.68
Momentum	3.20	65.3%	4.44%	4.23%	1.05	-0.19	0.89	1.76
Residual Volatility	2.62	57.9%	-2.08%	3.57%	-0.58	0.56	0.96	1.85
Size	2.46	49.5%	-0.78%	2.87%	-0.27	0.56	1.00	4.10
Non-Linear Size	1.46	25.0%	1.35%	1.72%	0.78	0.22	0.99	1.29
Average	2.06	39.2%	0.64%	2.57%	0.28	0.21	0.96	1.98

4.4. Cross-Sectional Regression

EUE4 estimates factor returns by regressing daily local excess returns against the market, industry, country, and style factor exposures. In the predecessor model, EUE3, the factor regression proceeds in two stages: the first stage estimates the returns of all factors with exposures in the core universe, while in the second stage the non-core country returns are estimated. The key idea behind this treatment is to avoid any effect of the non-core universe on market, industry and style factor returns.

The same goal is achieved in EUE4 in a single regression, where the non-core universe assets are down-weighted. If the down-weighting parameter tends to zero then the two approaches give the same results. The reason to choose the single stage version in EUE4 is twofold:

- First, this makes it easier to introduce a more-than-two-level hierarchy in the estimation universe without increasing the number of regression stages. We make use of this feature when we down-weight Frontier Markets with respect to Emerging Markets to minimize the effect of the former on the Eastern European industry factor returns in Eastern European model version.
- Second, this makes it much easier to define further derived models with a home region which contains both core and non-core countries.

To control the influence of outlier returns on the regression results, EUE4 applies a two-step truncation scheme. First, we introduce hard bounds for daily excess returns at -0.8 and 4. Second, after the regression we identify residual return outliers and pulling them back to ± 5 cross-sectional standard deviations. Finally, we re-run the regression after truncating the returns of outlier stocks.

EUE4 (especially the derived model versions) includes some industries that contain few assets at some point in the back-testing period. Factor returns of such thin categories are affected by small-sample biases; they tend to be overestimated because they are derived from a poorly diversified sample. The

same problem also occurs in a category with more assets if the category is dominated by few large cap stocks. To correct for thin industry biases EUE4 adds synthetic assets that act as Bayesian priors. The procedure is very similar to that of EUE3 with two important differences:

- First, in EUE4 we supplement thin industries only; we do not add priors to thin countries.
- Second, the return of the synthetic asset is the average excess return of the pan-European parent industry with respect to the market. If the parent industry is also thin, we introduce a second prior whose return is the average excess return of the GICS sector. In EUE3, a category containing many stocks but dominated by few large cap stocks were treated differently, i.e. a synthetic asset with the equal-weighted average category return was added. Another important difference to note is that in EUE3 derived model versions, the complementary regional industry serves as a prior for thin home-region industries. For example, the Continental Europe Airlines industry is used as a prior in the thin category correction of the UK Airlines industry. If the complementary regional industry is also thin, the regression uses a pan-European sector as a second-stage prior. In contrast, in all the variants of EUE4 the pan-European categories serve as priors.

Synthetic assets are given zero exposures to all factors except their own industry factor. The regression weights of the synthetic assets are proportional to $(n_{eff}^\varphi)^{-1/2}$, n_{eff}^φ being the effective number of assets in the thin industry.

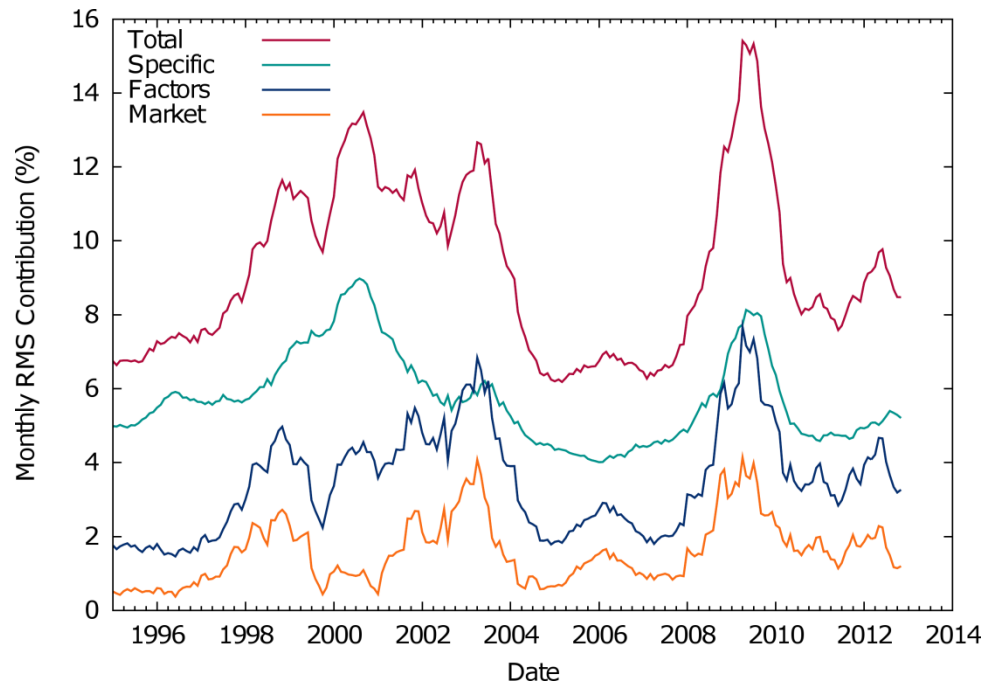
4.5. Cross-Sectional Dispersion

To gain a better insight into the relative importance of various factors and factor groups as a function of time it is informative to study the cross-sectional dispersion of monthly stock returns. As discussed by Menchero and Morozov (2011), dispersion can be measured in one of two ways. The first is by cross-sectional volatility (CSV), which measures the dispersion relative to the *mean* return. The second way is by root mean square (RMS) return, which measures the dispersion relative to *zero* return. The main difference between the two is that the Market factor –since the cross sectional volatility of stocks' exposures to it is identically zero- makes no contribution to CSV, whereas it does contribute to RMS levels.

In **Figure 4.1**, we plot the trailing 12-month total RMS return. The two most prominent features corresponded to the Internet Bubble and the 2008/2009 financial crisis. Note that the Internet Bubble peak was much broader — the buildup and aftermath spanned several years — whereas the financial-crisis peak was relatively short in duration.

Figure 4.1

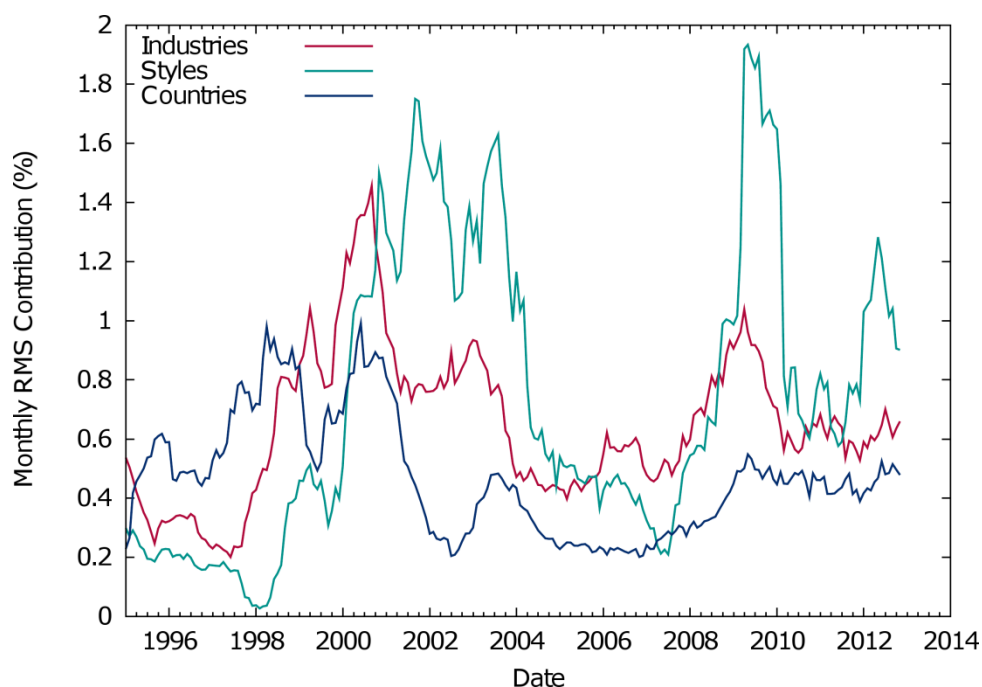
Total monthly cross-sectional dispersion as measured by root mean square (RMS) return. Also displayed are the stock-specific and factor contributions, and the contribution from the Market factor. Lines were smoothed using 12-month moving averages.



As discussed by Menchero and Morozov (2011), and shown in Appendix B, the RMS return can be decomposed and attributed to individual factors or groups of factors. Figure 4.1 shows the net RMS contributions from stock-specific sources, all factors, and the Market factor. The stock-specific contribution dominated throughout the sample period except for two short periods: In 2003 and during the financial crisis, factors became as important as the stock-specific contribution. Note that the contribution of the Market factor became particularly important during late 2008 and early 2009 as stocks moved together in aggregate.

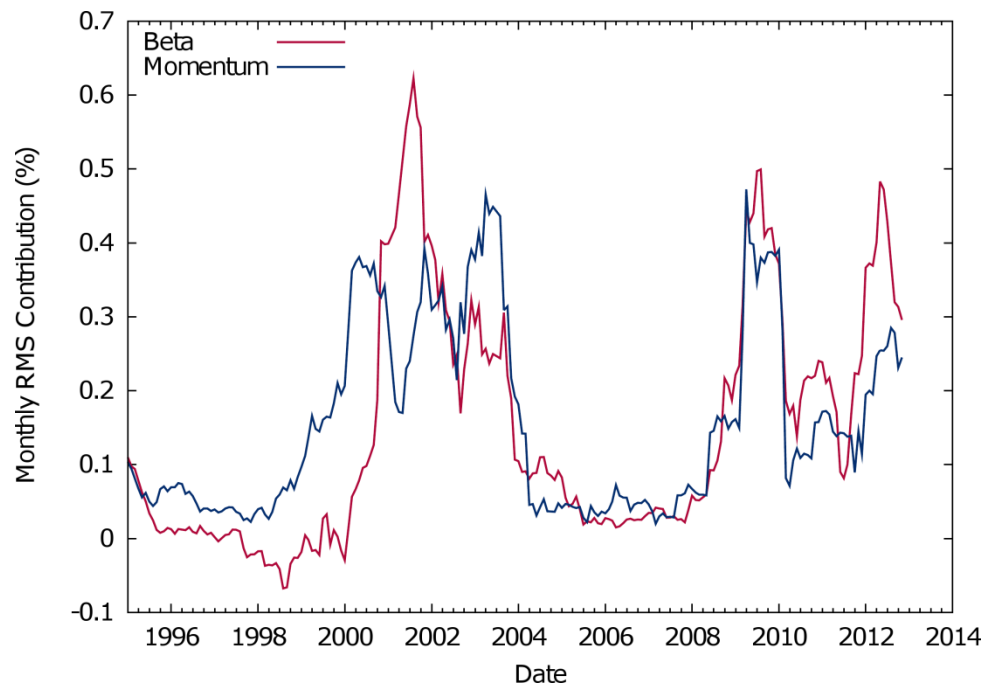
In **Figure 4.2**, we report the factor RMS contributions from countries, industries, and styles. We see that all three sources were of comparable importance in explaining the cross section of RMS returns. However, the relative importance of these factors varied over time. For instance, country factors were the largest contributor to RMS return from 1996-1999, whereas styles dominated from 2001-2004. Industries dominated during a brief period in 2000.

Figure 4.2
Contributions to monthly root mean square (RMS) return from countries, industries, and styles. Lines were smoothed using 12-month moving averages. All three sources are important contributors to cross-sectional dispersion.



In **Figure 4.3** we report RMS contributions from the Beta and Momentum factors. Particularly noteworthy is the large peak attributed to Beta in 2002. The Momentum factor dominated in the late 1990s.

Figure 4.3
Contributions to monthly root mean square (RMS) return from Beta and Momentum factors. Lines were smoothed using 12-month moving averages.



4.6. Factor Covariance Estimation

In line with other Barra models, EUE4 uses exponentially weighted moving average (EWMA) to calculate daily factor covariance matrix forecasts, with a shorter half-life for the estimation of factor variances than for the factor correlation estimates. This treatment is based on the observation that correlations tend to be more stable over time than factor variances. EUE4 is published in 3 variants that use different sets of half-lives, as summarized in Appendix D.

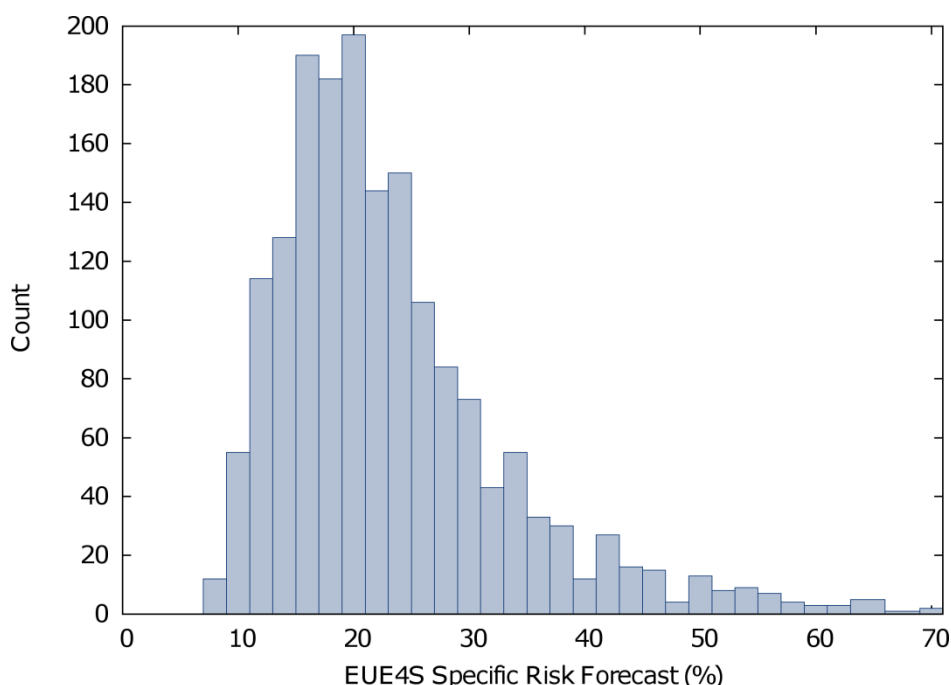
Apart from the slightly different parameters than those used in EUE3, there are few differences in the estimation which are worth to mention.

- When estimating the factor variance, we do not subtract the weighted mean of the factor returns, because users of a risk model generally take the view of zero expected *ex ante* factor return. Second, raw rather than demeaned returns are used in the back-testing procedure. Not subtracting weighted mean aligns model estimation with model performance evaluation.
- The complex factor return outlier filtering of the predecessor model, EUE3, is not used in EUE4. The reason is that the Volatility Regime Adjustment (VRA) achieves the same goal as the effect of an outlier return is gradually suppressed as time proceeds. When combining VRA and EUE3-type factor return filtering, we did not see any significant improvement in the forecast accuracy as compared to the case when VRA is applied alone.

4.7. Specific Risk

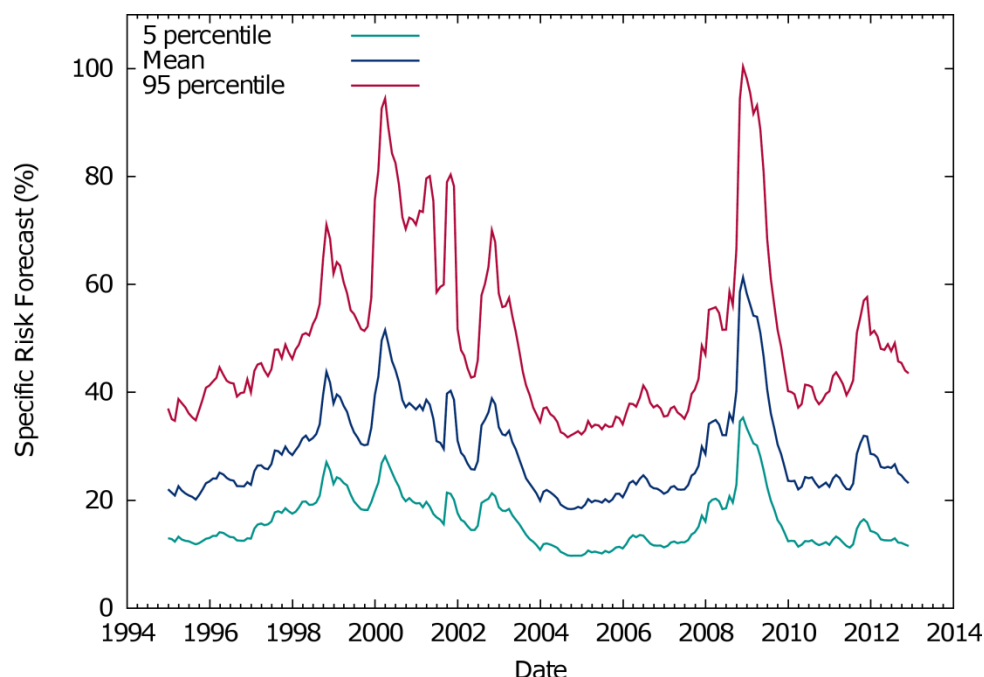
The distribution of specific volatilities is an important characteristic to examine. In **Figure 4.4** we plot the histogram of EUE4S specific risk forecasts for analysis date 30-Nov-2012. Most stocks had specific risk forecasts within the range 15-50 percent, although the most volatile stocks had forecasts exceeding 60 percent. The mean specific risk forecast on 30-Nov-2012 was 23.7 percent.

Figure 4.4
Histogram of EUE4S specific risk forecasts as of 30-Nov-2012.



It is also interesting to study how the distribution of specific risk varied over time. In **Figure 4.5**, we plot the 5-percentile, mean, and 95-percentile values for the specific risk distribution. We see that the 5-percentile specific volatility historically ranged from about 10-30 percent, with the maximum occurring in late 2008. The mean specific risk varied within the range of 20-60 percent. Again, the peak happened in late 2008, although the Internet Bubble period saw comparable levels of specific risk. The 95-percentile specific volatility ranged from a low of 35 percent in 2005, to highs in excess of 90 percent during the Internet Bubble and financial crisis.

Figure 4.5
Specific risk levels versus time for EUE4S.



5. Derived Model Versions

EUE4 is also published in *derived versions*. Derived model variants offer clients the option to select different levels of granularity in the factor structure of EUE4. Users who build diversified European portfolios may be well served by the single set of pan-European industry factors in the EUE4 base version because this structure matches their regional approach to industry allocation. However, some users may prefer a *home-centric* approach to portfolio construction; they may overweight their home market and take small cap exposure in home-market stocks, whilst concentrating only on the large caps in the rest of the region. This approach to portfolio construction treats the home-market portfolio as a separate entity. If a home market is broad enough to support its own set of industry factors, adding granularity to a regional model by splitting the industries can be beneficial for home-centric portfolios. MSCI provides three different versions of EUE4. In addition to the base version, users can select between two derived models which use *dual* market factors and *dual* sets of 29 industry factors; EUE3UK and EUE3EE. All models are published in short-horizon, long-horizon and daily-horizon variants.

Model Version	Market Factors	Industry Factors
EUE4	1 Europe	29 European industries
EUE4UK	1 United Kingdom	29 UK industries
	1 Continental Europe	29 Continental Europe industries
EUE4EE	1 Western Europe	29 Western Europe industries
	1 Eastern Europe	29 Eastern Europe industries

EUE4UK is the derived model with UK market focus. The United Kingdom is the largest market in Europe. As of November 2012 it represented about 27.45% of the total estimation universe cap of EUE4. Even though the UK market exhibits increased convergence with the rest of Western Europe, this convergence is not perfect. Adding separate UK industries can therefore enhance the explanatory power of the model for UK-centric portfolios. The predecessor model, EUE3UK, also used separate UK industry factors. Therefore, users that prefer a deeper level of detail in the UK market can replace EUE3UK with EUE4UK; keeping the familiar dual industry structure, while benefiting from all improvements of the EUE4 model series.

The second derived model, EUE4EE, similarly to its predecessor, EUE3EE, provides an enhanced level of detail for Eastern European stocks. The EUE4 base version explains the industry effects of Eastern European stocks by projecting Western European industry factor returns onto Eastern European stocks. This approach can, however, miss some characteristic differences between the Western European and Eastern European industries. EUE4EE may be considered by clients who build portfolios with significant exposure to Eastern Europe or who aim to analyze market risk in this region in detail. The performance of EUE4EE is equivalent to the EUE4 base version on a Western European universe; therefore there is no need for clients to subscribe the Eastern Europe derived model if they only invest in Western European markets. Diversification within each industry factor also plays a role when deciding which model version is best for a given use case.

Derived versions provide extra granularity within their home region, but this comes at the cost of adding some industries with only few constituents. For example, the UK Airlines industry had just one constituent in November 2012. To avoid over-fitting the returns of individual stocks within such thin industries, the EUE4 regression code applies a Bayesian correction which pulls the thin derived industry factor returns towards the pan-European parent industry.

It is worth to note here that while EUE3 derived model versions were designed to obtain detailed risk forecast of portfolios with a home-market bias, for optimization purposes the clients were referred to the base version. In EUE4, however, the difference in optimization bias between the base and derived model versions is considerably reduced by the Optimization Bias Adjustment, as shown in Section 6.

6. Forecasting Accuracy

6.1. Overview of Testing Methodology

As part of the model development process we subject the EUE4 model to rigorous and extensive testing to ensure its quality. To that end we perform side-by-side comparison for the EUE3 and EUE4 Models based on Q statistics and bias statistics for 580 portfolios, including long only and active portfolios for a testing period of up to 215 months.

In this section, we describe our methodology for evaluating and comparing the accuracy of risk model forecasts. We aim for a systematic and quantitative approach, yet one that is also visually intuitive.

The foundation of our approach rests on the bias statistic, described in Appendix C. Conceptually, the bias statistic is an out-of-sample measure that represents the ratio of realized risk to predicted risk. The ideal bias statistic for perfect risk forecasts should be close to 1. However, even for perfect risk forecasts, the bias statistic will never be *exactly* 1 due to sampling error. Nevertheless, we may define a confidence interval that is expected to contain 95 percent of the observations under the hypothesis of

perfect risk forecasts. If the bias statistic falls outside of the confidence interval, we infer that the risk forecast was not accurate.

When determining the size of the confidence interval, standard practice is to assume that returns are normally distributed. In reality, however, stock returns tend to have fat tails (i.e., positive excess kurtosis). As shown in Appendix C, fewer than 95 percent of the observations are expected to fall within the standard confidence interval when kurtosis is taken into account.

We are interested in testing the full sample period. One potential shortcoming of the bias statistic is that over long windows, we may have sub-periods of overforecasting and underforecasting, yet obtain a bias statistic close to 1 over the entire window. In other words, forecasting errors may cancel out over the long term, even though the risk forecasts may be poor over shorter periods. For a portfolio manager who may be devastated by a single year of poor performance, it is small consolation knowing that a risk forecast is good *on average*.

For this reason, we focus also on 12-*period* rolling windows. By plotting the mean rolling 12-month bias statistic across time for a collection of portfolios, we quickly visualize the magnitude of the average biases and can judge whether they were persistent or regime-dependent.

It is not enough, however, knowing the average bias statistic. We must also understand the extremes. We also compute, therefore, the 5-percentile (P5) and 95-percentile (P95) bias statistics across time. Assuming normally distributed returns and perfect risk forecasts, on average 5 percent of the rolling 12-month bias statistics will fall below 0.66 by pure chance. Therefore, if the P5 bias statistic falls significantly below this level, we infer that we are likely overpredicting the risk of at least some of the portfolios with bias statistics below 0.66. Similarly, if the P95 bias statistic lies well above 1.34, we infer that we are underpredicting the risk of some portfolios with bias statistics above 1.34. It is worth pointing out, however, that if we relax the normality assumption and allow for fat-tailed distributions, then for perfect risk forecasts the P5 bias statistic tends to fall below 0.66, and the P95 value generally lies above 1.34.

Q-statistics

Patton (2011) describes measures of forecast accuracy in terms of “loss functions.” He defines a loss function as “robust” if the ranking of any two volatility forecasts by expected loss is the same whether the ranking is done using the true variance (unobservable) or some unbiased variance proxy (e.g., squared return). One example of a robust loss function is the Q-statistic, defined for portfolio n and time t as

$$Q_{nt} = z_{nt}^2 - \ln z_{nt}^2$$

Patton further shows that the Q-statistic is the unique loss function (up to trivial additive and multiplicative constants) that depends solely on standardized returns (i.e., z-scores). This makes the Q-statistic ideal for evaluating risk model accuracy; because it places every observation on an equal footing (whether the volatility is high or low). Another key property of robust loss functions is that they are minimized in expectation when the predicted volatility equals the true volatility. For other loss functions, this is not true. That is, a biased volatility forecast can minimize the loss function. This would obviously be problematic for risk model calibration purposes. For further details of benefits of the Q-statistic over other loss functions we refer the reader to “*Predicting Risk at Short Horizons*” by Menchero, Morozov, and Pasqua (2012).

It is worth to mention that the Q-statistic may appear be noisy if the number of test portfolios and/or the number of test periods is not sufficiently large. To circumvent that difficulty, one can perform some kind of outlier filtering. Below we report both raw and filtered results. In the filtered version, we omit

the contribution of portfolio n at time t if $Q_{nt} > 11$ (this corresponds to $z_{nt} > 3.7$) symmetrically, i.e. we remove the outliers from both models if the Q -statistic exceeds the threshold. To justify the applicability of this procedure, we put together all the portfolios we considered and looked for the average effect of outlier filtering over the entire testing period (>500 portfolios, 215 months). This sample is of adequate size to obtain a clear picture how the filtering affects the results on average. The results are summarized in **Table 6.1**. The results clearly show that on average both EUE3 and EUE4 benefit from the filtering, even though EUE3 benefits more. In the next section we show the results both without filtering and with the unanticipated noise of outliers filtered out. The raw and filtered results may be considered as complementing each other.

Table 6.1

The effect of outlier filtering on average Q -statistics for EUE3BAS and EUE4BAS Models. The results were computed on the set of all the portfolios described in details in the next section.

Q (EUE3L)	Bias (EUE3L)	Q (EUE4L)	Bias (EUE4L)	Q Diff (bp)	All portfolios
2.4468	1.14	2.3977	1.05	491	w/o outlier filter
2.3427	1.10	2.3219	1.03	207	with outlier filter

Q (EUE3S)	Bias (EUE3S)	Q (EUE4S)	Bias (EUE4S)	Q Diff (bp)	All portfolios
2.5046	1.22	2.3763	1.08	1284	w/o outlier filter
2.3556	1.16	2.2955	1.06	601	with outlier filter

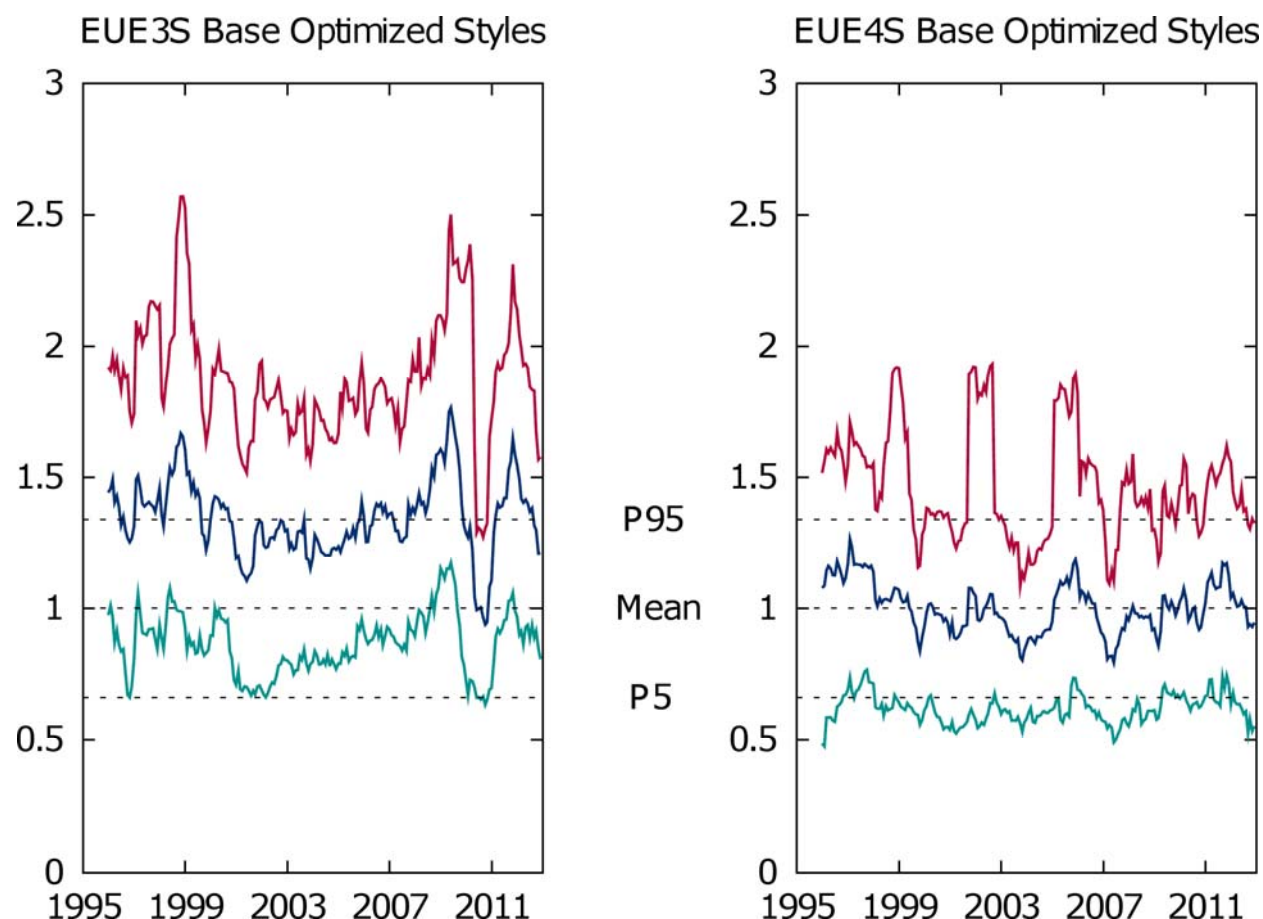
6.2. Backtesting Results

In this section we perform side-by-side comparison for the EUE3 and EUE4 Models. We compare Q statistics and bias statistics for a variety of portfolios using short and long horizon models. The analysis period is 215 months, running from December 1994 through November 2012. Since in the predecessor model, EUE3, there was no daily horizon forecast we compare EUE4D to EUE4S scaled to daily horizon.

In **Figure 6.1** we plot 12-month rolling window bias statistics for optimized style-tilt portfolios. The optimized portfolios were constructed by using EUE4 styles factors as “alpha signals” and then forming the minimum volatility portfolio (with alpha equal 1) for 20 draws of 500 randomly selected stocks using EUE3S BASE and EUE4S BASE models. The mean bias statistics for the EUE3S model were greater than 1 for virtually the entire sample period, indicating underprediction of risk for these optimized portfolios. By contrast, the mean bias statistics for EUE4S were close to 1 on average, indicating that the Optimization Bias Adjustment was effective at reducing the underforecasting biases for these optimized portfolios.

Figure 6.1

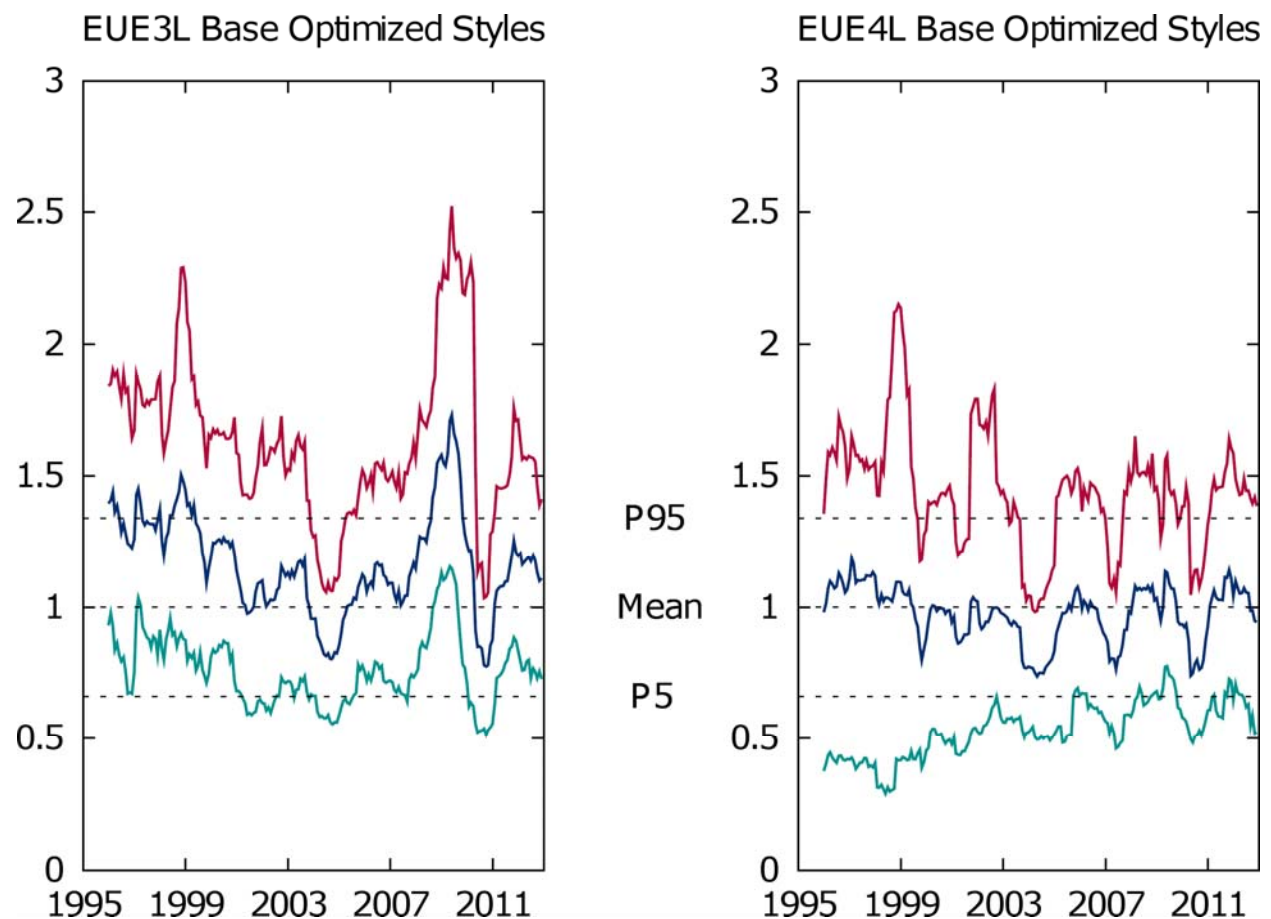
Comparison of EUE3S Model and EUE4S Model for optimized portfolios. The portfolios were constructed by minimizing risk subject to the unit alpha constraint, where the alpha signals were taken from the style factors of each model. Twenty sets of optimizations were performed using 500 randomly selected stocks for each style factor. Each plot reports rolling 12-month values for the mean bias statistic, the 5-percentile and 95-percentile bias statistics. The dashed horizontal lines indicate the ideal positions assuming normally distributed returns and perfect risk forecasts.



In **Figure 6.2** we plot bias statistics for optimized style-tilt portfolios constructed in the same fashion as in Figure 6.1, except now using the long-horizon models. Even though EUE3L clearly has less bias than EUE3S, the bias statistics for the EUE3L Model were shifted upward throughout most of the sample period, indicating underprediction of risk for the optimized portfolios. The mean bias statistics for the EUE4L Model were closer to 1 on average.

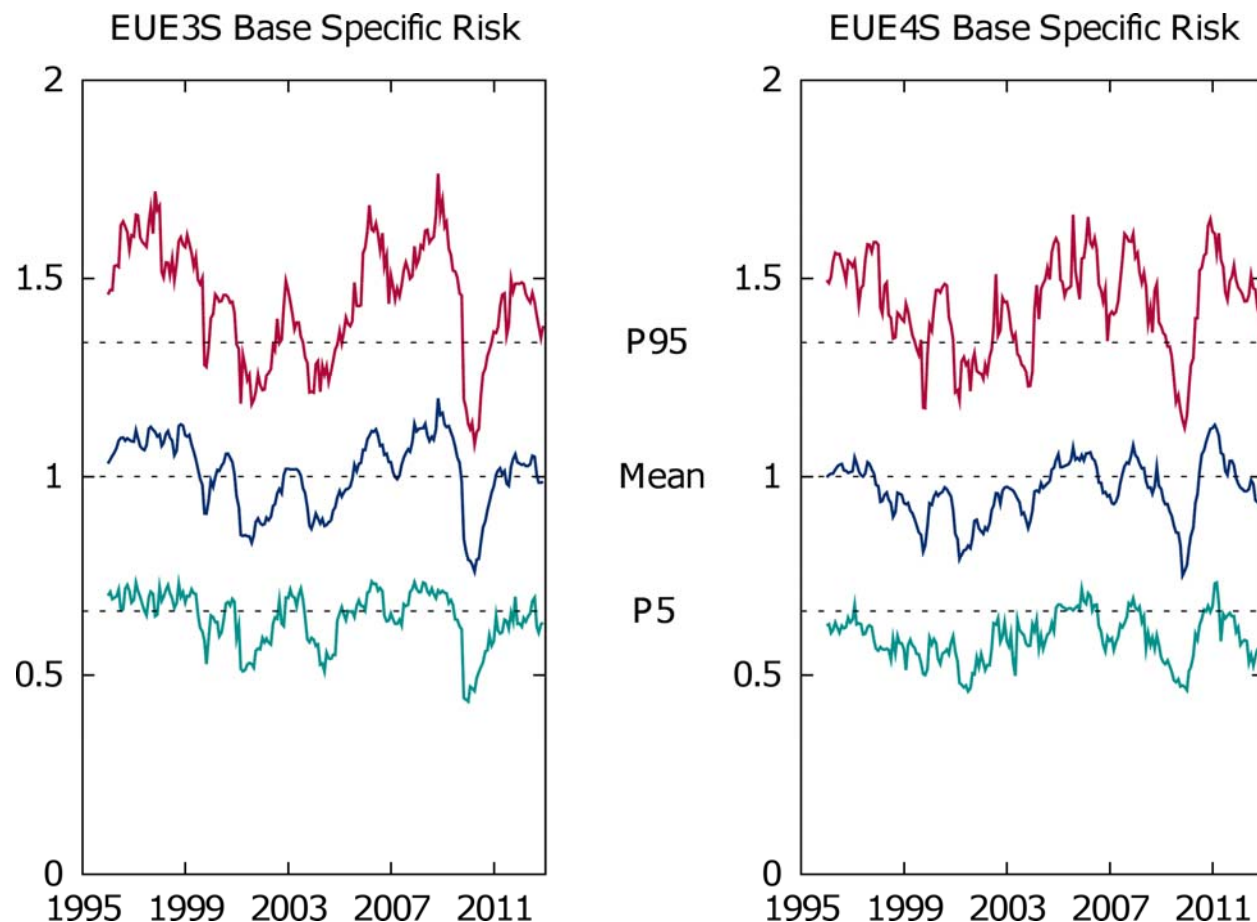
Figure 6.2

Comparison of EUE3L Model and EUE4L Model for optimized portfolios. The portfolios were constructed by minimizing risk subject to the unit alpha constraint, where the alpha signals were taken from the style factors of each model. Twenty sets of optimizations were performed using 500 randomly selected stocks for each style factor. Each plot reports rolling 12-month values for the mean bias statistic, the 5-percentile and 95-percentile bias statistics. The dashed horizontal lines indicate the ideal positions assuming normally distributed returns and perfect risk forecasts.



In **Figure 6.3** we plot cap-weighted bias statistics for the specific returns of all stocks in the intersection of EUE4 and EUE3 estimation universes using the short-horizon models. Both models do an excellent job predicting specific volatility levels. Qualitatively, the main features are similar.

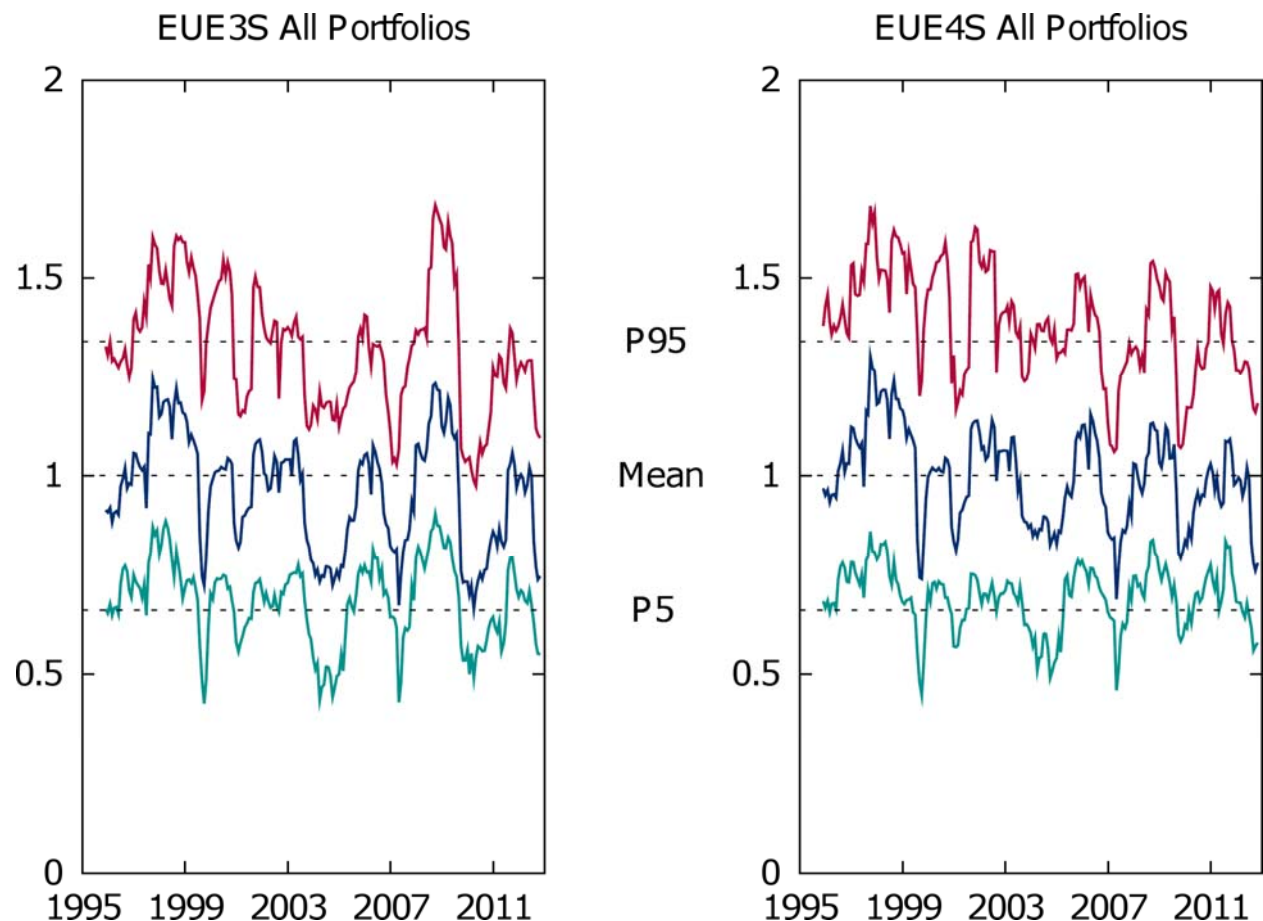
Figure 6.3
Comparison of EUE3S Model and EUE4S Model for specific risk. Results were capitalization weighted. Each plot reports rolling 12-month values for the mean bias statistic, the 5-percentile and 95-percentile bias statistics. The dashed horizontal lines indicate the ideal positions assuming normally distributed returns and perfect risk forecasts.



In **Figure 6.4** we plot summary bias statistics for all the test portfolios considered, for the short horizon models. The test portfolios include random long, random active, factor tilt long, factor tilt active and optimized style portfolios. The mean bias statistics for the EUE4S/L Models were closer to 1 on average than for their EUE3 counterparts.

Figure 6.4

Comparison of EUE3S Model and EUE4S Model for all the portfolios we considered: 100 random portfolios (randomly selected 500 stocks, which were then capitalization weighted), 100 random active portfolios (random long portfolios were run against the estimation universe to form euro-neutral portfolios), factor tilt long portfolios (capitalization weighted country, industry and style-tilt portfolios), factor tilt active portfolios (going long the factor-tilt portfolios and shorting the estimation universe) and optimized style portfolios of Figure 6.1.



In **Table 6.2** we present summary Q-statistic and mean bias statistic numbers for the EUE3SBAS and EUE4LBAS Models for the test portfolios for the period **1994-12-30 to 2012-11-30**. For every class of portfolios, the EUE4BAS Model produced more accurate risk forecasts as measured by the Q statistic. The average outperformance by the Q measure was 351 bps for the short-horizon model and 164 bps for the long-horizon models.

Table 6.2
Summary of mean bias statistics and Q-statistics for EUE3BAS and EUE4BAS Models.
Panel A: outliers filtered

	Q (EUE3L)	Bias (EUE3L)	Q (EUE4L)	Bias (EUE4L)	Q Diff (bp)	Portfolio Type
	2.3615	1.03	2.3379	1.02	236	Pure Factors
	2.2885	1.02	2.2771	0.99	115	Random Active
	2.3641	0.99	2.3530	1.03	110	Factor Tilts Long
	2.3673	1.05	2.3614	1.03	59	Factor Tilts Active
	2.3564	1.21	2.3241	1.04	323	Optimized Styles
	2.5105	1.12	2.4965	1.02	140	Specific Risk
Average	2.3747	1.07	2.3583	1.02	164	

	Q (EUE3S)	Bias (EUE3S)	Q (EUE4S)	Bias (EUE4S)	Q Diff (bp)	Portfolio Type
	2.3249	1.05	2.3058	1.05	190	Pure Factors
	2.2700	1.01	2.2678	1.04	21	Random Active
	2.3204	0.99	2.3166	1.04	38	Factor Tilts Long
	2.3348	1.04	2.3318	1.06	30	Factor Tilts Active
	2.4416	1.33	2.2946	1.07	1470	Optimized Styles
	2.5069	1.09	2.4715	1.02	354	Specific Risk
Average	2.3664	1.08	2.3314	1.05	351	

Panel B: without filtering

	Q (EUE3L)	Bias (EUE3L)	Q (EUE4L)	Bias (EUE4L)	Q Diff (bp)	Portfolio Type
	2.3615	1.05	2.3379	1.03	236	Pure Factors
	2.3587	1.04	2.3355	1.01	232	Random Active
	2.4220	1.01	2.4255	1.05	-35	Factor Tilts Long
	2.4532	1.08	2.4498	1.05	34	Factor Tilts Active
	2.4947	1.25	2.3955	1.03	992	Optimized Styles
	2.5115	1.13	2.5034	1.02	81	Specific Risk
Average	2.4336	1.09	2.4080	1.03	257	

	Q (EUE3S)	Bias (EUE3S)	Q (EUE4S)	Bias (EUE4S)	Q Diff (bp)	Portfolio Type
	2.3249	1.07	2.3058	1.06	190	Pure Factors
	2.3282	1.02	2.3256	1.05	26	Random Active
	2.3756	1.01	2.3950	1.05	-194	Factor Tilts Long
	2.3987	1.06	2.4194	1.07	-207	Factor Tilts Active
	2.6880	1.41	2.3739	1.06	3141	Optimized Styles
	2.5058	1.11	2.4781	1.02	277	Specific Risk
Average	2.4369	1.11	2.3830	1.05	539	

In **Table 6.3** we present summary Q-statistic and mean bias statistic numbers for the EUE3SUK and EUE4LUK Models for the test period **1994-12-30 to 2012-11-30**. For every class of portfolios, the EUE4UK Model produced more accurate risk forecasts as measured by the Q statistic. The average outperformance by the Q measure was 482 bps for the short-horizon model and 259 bps for the long-horizon models.

Table 6.3
Summary of mean bias statistics and Q-statistics for EUE3 DUK and EUE4 DUK Models.
Panel A: outliers filtered

	Q (EUE3L)	Bias (EUE3L)	Q (EUE4L)	Bias (EUE4L)	Q Diff (bp)	Portfolio Type
	2.3497	1.03	2.3327	1.02	170	Pure Factors
	2.3023	1.08	2.2890	0.94	133	Random Active
	2.3600	0.99	2.3470	1.02	130	Factor Tilts Long
	2.3840	1.07	2.3603	1.00	237	Factor Tilts Active
	2.4085	1.27	2.3281	1.04	804	Optimized Styles
	2.5115	1.13	2.5034	1.02	81	Specific Risk
Average	2.3860	1.10	2.3601	1.01	259	

	Q (EUE3S)	Bias (EUE3S)	Q (EUE4S)	Bias (EUE4S)	Q Diff (bp)	Portfolio Type
	2.3107	1.05	2.3030	1.04	77	Pure Factors
	2.2867	1.07	2.2720	0.98	148	Random Active
	2.3162	0.99	2.3147	1.04	14	Factor Tilts Long
	2.3494	1.06	2.3322	1.03	172	Factor Tilts Active
	2.5173	1.38	2.2967	1.06	2206	Optimized Styles
	2.5058	1.11	2.4781	1.02	277	Specific Risk
Average	2.3810	1.11	2.3328	1.03	482	

Panel B: without filtering

	Q (EUE3L)	Bias (EUE3L)	Q (EUE4L)	Bias (EUE4L)	Q Diff (bp)	Portfolio Type
	2.3497	1.03	2.3327	1.02	170	Pure Factors
	2.3870	1.11	2.3453	0.96	416	Random Active
	2.4230	1.02	2.4238	1.03	-8	Factor Tilts Long
	2.4440	1.10	2.4362	1.02	78	Factor Tilts Active
	2.6140	1.34	2.4012	1.03	2128	Optimized Styles
	2.5115	1.13	2.5034	1.02	81	Specific Risk
Average	2.4549	1.12	2.4071	1.01	478	

	Q (EUE3S)	Bias (EUE3S)	Q (EUE4S)	Bias (EUE4S)	Q Diff (bp)	Portfolio Type
	2.3107	1.05	2.3030	1.04	77	Pure Factors
	2.3545	1.09	2.3270	0.99	275	Random Active
	2.3749	1.02	2.3835	1.04	-86	Factor Tilts Long
	2.4264	1.08	2.4374	1.04	-111	Factor Tilts Active
	2.8578	1.51	2.3788	1.06	4791	Optimized Styles
	2.5058	1.11	2.4781	1.02	277	Specific Risk
Average	2.4717	1.14	2.3846	1.03	871	

In **Table 6.4** we present summary Q-statistic and mean bias statistic numbers for the EUE3S DEE and EUE4L DEE Models for the test period **1994-12-30 to 2012-11-30**. For every class of portfolios, the EUE4 DEE Model produced more accurate risk forecasts as measured by the Q statistic. The average outperformance by the Q measure was 377 bps for the short-horizon model and 209 bps for the long-horizon models.

Table 6.4

Summary of mean bias statistics and Q-statistics for EUE3 DEE and EUE4 DEE Models.

Panel A: outliers filtered

	Q (EUE3L)	Bias (EUE3L)	Q (EUE4L)	Bias (EUE4L)	Q Diff (bp)	Portfolio Type
	2.3686	1.03	2.3605	1.00	81	Pure Factors
	2.2884	1.02	2.2791	0.99	93	Random Active
	2.3508	1.00	2.3184	1.01	324	Factor Tilts Long
	2.3755	1.06	2.3654	1.02	101	Factor Tilts Active
	2.3445	1.19	2.2925	1.04	520	Optimized Styles
	2.5105	1.11	2.4969	1.01	136	Specific Risk
Average	2.3731	1.07	2.3521	1.01	209	

	Q (EUE3S)	Bias (EUE3S)	Q (EUE4S)	Bias (EUE4S)	Q Diff (bp)	Portfolio Type
	2.3403	1.05	2.3275	1.03	128	Pure Factors
	2.2733	1.02	2.2699	1.02	35	Random Active
	2.3055	0.99	2.2961	1.03	94	Factor Tilts Long
	2.3425	1.05	2.3410	1.04	15	Factor Tilts Active
	2.4455	1.32	2.2826	1.06	1629	Optimized Styles
	2.5062	1.09	2.4702	1.01	360	Specific Risk
Average	2.3689	1.09	2.3312	1.03	377	

Panel B: without filtering

	Q (EUE3L)	Bias (EUE3L)	Q (EUE4L)	Bias (EUE4L)	Q Diff (bp)	Portfolio Type
	2.3686	1.03	2.3605	1.00	81	Pure Factors
	2.3538	1.04	2.3290	1.01	248	Random Active
	2.4139	1.02	2.4089	1.05	50	Factor Tilts Long
	2.4207	1.06	2.4235	1.03	-28	Factor Tilts Active
	2.4831	1.24	2.3653	1.05	1178	Optimized Styles
	2.5105	1.11	2.4969	1.01	136	Specific Risk
Average	2.4251	1.08	2.3974	1.02	278	

	Q (EUE3S)	Bias (EUE3S)	Q (EUE4S)	Bias (EUE4S)	Q Diff (bp)	Portfolio Type
	2.3403	1.05	2.3275	1.03	128	Pure Factors
	2.3232	1.03	2.3163	1.03	69	Random Active
	2.3669	1.02	2.3659	1.06	9	Factor Tilts Long
	2.4042	1.07	2.4099	1.05	-57	Factor Tilts Active
	2.6972	1.42	2.3590	1.06	3382	Optimized Styles
	2.5062	1.09	2.4702	1.01	360	Specific Risk
Average	2.4397	1.11	2.3748	1.04	648	

In **Tables 6.5, 6.6 and 6.7** we present daily horizon summary Q-statistic and mean bias statistic numbers for the EUE4S and EUE4D BASE/DUK/DEE Models, respectively, for the test period **1994-12-30 to 2012-11-30**. For every class of portfolios, the EUE4D Model produced more accurate daily risk forecasts as measured by the Q statistic. The average outperformance by the Q measure was 508 bps, 477 bps, 492 bps for the BASE/DUK/DEE Models, respectively.

Table 6.5

Summary of daily horizon mean bias statistics and Q-statistics for EUE4S BASE and EUE4D BASE Models, entire sample period (1994-12-30 to 2012-11-30).

Panel A: outliers filtered

	Q (EUE4S)	Bias (EUE4S)	Q (EUE4D)	Bias (EUE4D)	Q Diff (bp)	Portfolio Type
	2.4725	0.92	2.4106	1.00	619	Pure Factors
	2.3109	1.05	2.2928	1.04	182	Random Active
	2.4305	0.93	2.3498	0.97	807	Factor Tilts Long
	2.3543	0.96	2.3119	1.00	425	Factor Tilts Active
	2.3098	0.94	2.2884	0.97	214	Optimized Styles
	2.8605	1.22	2.7806	1.08	799	Specific Risk
Average	2.4564	1.00	2.4057	1.01	508	

Panel B: without filtering

	Q (EUE4S)	Bias (EUE4S)	Q (EUE4D)	Bias (EUE4D)	Q Diff (bp)	Portfolio Type
	2.4725	0.92	2.4106	1.00	619	Pure Factors
	2.3565	1.05	2.3303	0.98	262	Random Active
	2.4874	0.94	2.4071	0.97	803	Factor Tilts Long
	2.4396	0.99	2.3869	1.01	528	Factor Tilts Active
	2.3569	0.92	2.3549	0.95	20	Optimized Styles
	2.8605	1.22	2.7806	1.08	799	Specific Risk
Average	2.4956	1.01	2.4451	1.00	505	

Table 6.6
Summary of daily horizon mean bias statistics and Q-statistics for EUE4S DUK and EUE4D DUK Models,
entire sample period (1994-12-30 to 2012-11-30).
Panel A: outliers filtered

	Q (EUE4S)	Bias (EUE4S)	Q (EUE4D)	Bias (EUE4D)	Q Diff (bp)	Portfolio Type
	2.4744	0.93	2.4212	1.00	532	Pure Factors
	2.3115	0.99	2.2962	0.98	153	Random Active
	2.4506	0.92	2.3780	0.95	726	Factor Tilts Long
	2.3538	0.93	2.3148	0.95	390	Factor Tilts Active
	2.3155	0.94	2.2907	0.96	248	Optimized Styles
	2.8224	1.23	2.7412	1.08	813	Specific Risk
Average	2.4547	0.99	2.4070	0.99	477	

Panel B: without filtering

	Q (EUE4S)	Bias (EUE4S)	Q (EUE4D)	Bias (EUE4D)	Q Diff (bp)	Portfolio Type
	2.4744	0.93	2.4212	1.00	532	Pure Factors
	2.3502	0.99	2.3291	0.98	211	Random Active
	2.5100	0.93	2.4358	0.95	742	Factor Tilts Long
	2.4447	0.95	2.3955	0.97	492	Factor Tilts Active
	2.3597	0.92	2.3507	0.95	89	Optimized Styles
	2.8224	1.23	2.7412	1.08	813	Specific Risk
Average	2.4936	0.99	2.4456	0.99	480	

Table 6.7
Summary of daily horizon mean bias statistics and Q-statistics for EUE4S DEE and EUE4D DEE Models,
entire sample period (1994-12-30 to 2012-11-30).
Panel A: outliers filtered

	Q (EUE4S)	Bias (EUE4S)	Q (EUE4D)	Bias (EUE4D)	Q Diff (bp)	Portfolio Type
	2.4522	0.92	2.3982	0.99	540	Pure Factors
	2.3122	1.03	2.2948	1.03	174	Random Active
	2.4277	0.92	2.3500	0.95	777	Factor Tilts Long
	2.3580	0.93	2.3136	0.96	444	Factor Tilts Active
	2.3068	0.94	2.2850	0.96	218	Optimized Styles
	2.8607	1.22	2.7808	1.08	799	Specific Risk
Average	2.4529	0.99	2.4038	1.00	492	

Panel B: without filtering

	Q (EUE4S)	Bias (EUE4S)	Q (EUE4D)	Bias (EUE4D)	Q Diff (bp)	Portfolio Type
	2.4522	0.92	2.3982	0.99	540	Pure Factors
	2.3553	1.03	2.3308	1.03	245	Random Active
	2.4765	0.91	2.3966	0.95	800	Factor Tilts Long
	2.4381	0.94	2.3843	0.97	538	Factor Tilts Active
	2.3635	0.93	2.3503	0.95	132	Optimized Styles
	2.8607	1.22	2.7808	1.08	799	Specific Risk
Average	2.4910	0.99	2.4402	1.00	509	

In **Tables 6.7 and 6.8** we present comparison between EUE4S/L BASE and derived models based on Q-statistic and mean bias statistic numbers for home region centered and ex-home region centered long and active industry-tilt portfolios. Please note that the active portfolios were constructed as going long the home/ex-home region factors and shorting the entire pan-European estimation universe. The test period was **1994-12-30 to 2012-11-30**.

Table 6.7: Comparison between EUE4S /L BASE and DUK Models based on UK-centered and ex-UK industry-tilt portfolios.

Panel A: outliers filtered

	Q (EUE4L BASE)	Bias (EUE4L BASE)	Q (EUE4L DUK)	Bias (EUE4L DUK)	Q Diff (bp)	Portfolio Type
	2.3101	1.04	2.3062	0.94	40	UK long
	2.3694	1.11	2.3561	0.92	133	UK Active
	2.3511	1.04	2.3416	1.03	95	ex-UK long
	2.3525	1.04	2.3472	1.02	53	ex-UK active
Average	2.3458	1.06	2.3378	0.98	80	

	Q (EUE4S BASE)	Bias (EUE4S BASE)	Q (EUE4S DUK)	Bias (EUE4S DUK)	Q Diff (bp)	Portfolio Type
	2.2975	1.06	2.2826	0.95	150	UK long
	2.3542	1.13	2.3232	0.93	309	UK Active
	2.3196	1.05	2.3118	1.03	78	ex-UK long
	2.3303	1.05	2.3207	1.03	96	ex-UK active
Average	2.3254	1.07	2.3096	0.99	158	

Panel B: without filtering

	Q (EUE4L BASE)	Bias (EUE4L BASE)	Q (EUE4L DUK)	Bias (EUE4L DUK)	Q Diff (bp)	Portfolio Type
	2.4174	1.08	2.3915	0.99	259	UK long
	2.5083	1.18	2.4554	0.99	529	UK Active
	2.4086	1.07	2.4030	1.07	56	ex-UK long
	2.4241	1.06	2.4127	1.06	114	ex-UK active
Average	2.4396	1.10	2.4156	1.03	240	

	Q (EUE4S BASE)	Bias (EUE4S BASE)	Q (EUE4S DUK)	Bias (EUE4S DUK)	Q Diff (bp)	Portfolio Type
	2.4347	1.11	2.3926	0.99	421	UK long
	2.4644	1.20	2.4555	1.00	88	UK Active
	2.4086	1.08	2.4023	1.07	63	ex-UK long
	2.4233	1.09	2.4130	1.07	103	ex-UK active
Average	2.4327	1.12	2.4159	1.03	169	

Table 6.8: Comparison between EUE4S/L BASE and DEE Models based on Eastern European and Western European factor-tilt portfolios.

Panel A: outliers filtered

	Q (EUE4L BASE)	Bias (EUE4L BASE)	Q (EUE4L DEE)	Bias (EUE4L DEE)	Q Diff (bp)	Portfolio Type
	2.3400	1.04	2.3367	0.96	32	East. Europe long
	2.3035	1.01	2.3139	0.94	-104	East. Europe active
	2.3378	1.03	2.3257	1.02	120	West. Europe long
	2.3645	1.08	2.3564	1.07	81	West. Europe active
Average	2.3364	1.04	2.3332	1.00	32	

	Q (EUE4S BASE)	Bias (EUE4S BASE)	Q (EUE4S DEE)	Bias (EUE4S DEE)	Q Diff (bp)	Portfolio Type
	2.3533	1.09	2.3394	1.00	139	East. Europe long
	2.3130	1.02	2.3223	0.95	-94	East. Europe active
	2.3020	1.03	2.3017	1.03	4	West. Europe long
	2.3466	1.09	2.3378	1.08	88	West. Europe active
Average	2.3287	1.06	2.3253	1.02	34	

Panel B: without filtering

	Q (EUE4L BASE)	Bias (EUE4L BASE)	Q (EUE4L DEE)	Bias (EUE4L DEE)	Q Diff (bp)	Portfolio Type
	2.5962	1.18	2.5452	1.09	511	East. Europe long
	2.3741	1.03	2.3763	0.95	-22	East. Europe active
	2.4199	1.08	2.4198	1.08	0	West. Europe long
	2.4269	1.12	2.4237	1.12	32	West. Europe active
Average	2.4543	1.10	2.4413	1.06	130	

	Q (EUE4S BASE)	Bias (EUE4S BASE)	Q (EUE4S DEE)	Bias (EUE4S DEE)	Q Diff (bp)	Portfolio Type
	2.6041	1.19	2.5522	1.09	519	East. Europe long
	2.3827	1.04	2.3841	0.96	-13	East. Europe active
	2.4208	1.08	2.4206	1.08	3	West. Europe long
	2.4301	1.13	2.4281	1.12	20	West. Europe active
Average	2.4595	1.11	2.4462	1.06	132	

While the UK derived model outperforms the Base model version for all the relevant portfolio types, the Eastern European derived model underperforms the Base model in case of forecasting the risk of Eastern European Active portfolios. The origin of this behavior is twofold:

- The Market factor is split into Western and Eastern European Markets, and
- The home region of the DEE model contains Emerging and Frontier Markets (which are down-weighted in the regression), but no Developed Markets.

These two facts together make the DEE home factors and the pan-European Market factor effectively decoupled resulting in noisier forecast for the Eastern European Active portfolios. It is worth mentioning here that the same argument does not hold for the DUK model: UK is the largest market in Europe and UK stocks participate in the regression with their full weight.

7. Conclusion

The new Barra Europe Equity Model (EUE4) is the result of extensive research efforts in combination with client consultations. The EUE4 Model incorporates many methodological innovations and advances designed to address long-standing problems in risk modeling. For instance, the Optimization Bias Adjustment addresses the issue of underestimation of risk for optimized portfolios, and leads to better conditioning of the covariance matrix. The Volatility Regime Adjustment calibrates volatilities to current market levels and represents a key determinant of risk forecasts, especially during times of market turmoil. Another enhancement is the use of a Bayesian adjustment technique which aims to reduce biases in specific risk forecasts.

This document provides a thorough empirical analysis of the EUE4 Model. The factor structure is described in transparency and detail, for countries, industries and styles. The performance of select factors is presented and discussed. Key metrics are reported at the individual factor level, including statistical significance, performance, volatility, and correlation.

In addition, we study the contributions to cross-sectional dispersion from the Market factor, countries, industries, and styles. We find that each category of factors was of comparable importance in explaining the observed cross-sectional dispersion of equity returns.

Lastly, we systematically compare the forecasting accuracy of the EUE4S and EUE4L Models versus their EUE3 counterparts over a roughly 18-year backtesting window. We consider several classes of portfolios, including pure factors, random active portfolios, factor-tilt portfolios (both long-only and euro-neutral), and optimized portfolios. We also compare the accuracy of specific risk forecasts between the two models. For every portfolio type considered, we find that the EUE4S and EUE4L Models provided more accurate risk forecasts than their EUE3 counterparts during the sample period. Since there was no daily horizon version of EUE3, we compared EUE4D to EUE4S scaled to a daily horizon. Our conclusion is that the Volatility Regime Adjusted volatility forecast of the EUE4D model is in fact more accurate than EUE4S on a daily horizon.

Appendix A: Descriptors by Style Factor

Beta

Definition: $1.0 \cdot BETA$

$BETA$ Beta (β)

Computed as the slope coefficient in a time-series regression of excess stock return, $r_t - r_{ft}$, against the cap-weighted excess return of the estimation universe R_t ,

$$r_t - r_{ft} = \alpha + \beta R_t + e_t. \quad (A1)$$

The returns are aggregated over two-day windows to reduce the effects of non-synchronicity. The regression coefficients are estimated over the trailing 252 trading days of returns with a half-life of 63 trading days.

As a methodological improvement, we introduced BETA priors in EUE4 to blend the BETA of recent IPO-s and illiquid assets. The details can be found in Appendix D.

Momentum

Definition: $0.6 \cdot RSTR + 0.4 \cdot HALPHA$

$RSTR$ Relative strength

First, non-lagged relative strength for day τ is computed as the sum of excess log returns over the trailing $T = 504$ trading days,

$$RS(\tau) = \sum_{t=\tau}^{\tau+T} w_t \left[\ln(1 + r_t) - \ln(1 + r_{ft}) \right], \quad (A2)$$

where r_t is the stock return on day t , r_{ft} is the risk-free return, and w_t is an exponential weight with a half-life of 126 trading days.

Relative strength is computed as the equal-weighted average of non-lagged relative strength over the previous 11 to 21 trading days:

$$RSTR = \frac{1}{11} \sum_{\tau=11}^{21} RS(\tau). \quad (A3)$$

This treatment introduces a lag while avoiding undue jumps in relative strength exposures when large returns enter the estimation window.

HALPHA Historical alpha

First, non-lagged values of historical alpha are computed by the time-series regression of Equation A1. Historical alpha is then computed as the equal-weighted average of non-lagged values over the previous 11 to 21 trading days. This treatment introduces a lag while avoiding undue jumps in historical alpha exposures when large returns enter the estimation window.

Size

Definition: $1.0 \cdot LNCAP$

LNCAP Log of market cap

Given by the logarithm of the total market capitalization of the firm.

Earnings Yield

Definition: $0.643 \cdot EPFWD + 0.214 \cdot CETOP + 0.143 \cdot ETOP$

EPFWD Predicted earnings-to-price ratio

Given by the 12-month forward-looking earnings divided by the current market capitalization.

Forward-looking earnings are defined as a weighted average between the average analyst-predicted earnings for the current and next fiscal years.

CETOP Cash earnings-to-price ratio

Given by the trailing 12-month cash earnings divided by current price.

ETOP Trailing earnings-to-price ratio

Given by the trailing 12-month earnings divided by the current market capitalization. Trailing earnings are defined as the last reported fiscal-year earnings plus the difference between current interim figure and the comparative interim figure from the previous year.

Residual Volatility

Definition: $0.60 \cdot DASTD + 0.30 \cdot CMRA + 0.10 \cdot H\SIGMA$

DASTD Daily standard deviation

Computed as the volatility of daily excess returns over the past 252 trading days with a half-life of 42 trading days.

CMRA Cumulative range

This descriptor differentiates stocks that have experienced wide swings over the last 12 months from those that have traded within a narrow range. Let $Z(T)$ be the cumulative excess log return over the past T months, with each month defined as the previous 21 trading days

$$Z(T) = \sum_{\tau=1}^T [\ln(1 + r_{\tau}) - \ln(1 + r_{f\tau})], \quad (A4)$$

where r_{τ} is the stock return for month τ (compounded over 21 days), and $r_{f\tau}$ is the risk-free return.

The cumulative range is given by

$$CMRA = \ln(1 + Z_{\max}) - \ln(1 + Z_{\min}), \quad (A5)$$

where $Z_{\max} = \max\{Z(T)\}$, $Z_{\min} = \min\{Z(T)\}$, and $T = 1, \dots, 12$.

HSIGMA Historical sigma (σ)

Computed as the volatility of residual returns in Equation A1,

$$\sigma = \text{std}(e_t). \quad (A6)$$

The volatility is estimated over the trailing 252 trading days of returns with a half-life of 63 trading days.

Note: The Residual Volatility factor is orthogonalized to Beta to reduce collinearity.

Growth

Definition: $0.70 \cdot EGRLF + 0.15 \cdot EGRO + 0.15 \cdot SGRO$

EGRLF Long-term predicted earnings growth

Long-term (3-5 years) earnings growth forecasted by analysts.

EGRO Earnings growth (trailing five years)

Annual reported earnings per share are regressed against time over the past five fiscal years. The slope coefficient is then divided by the average annual earnings per share to obtain the earnings growth.

SGRO Sales growth (trailing five years)

Annual reported sales per share are regressed against time over the past five fiscal years. The slope coefficient is then divided by the average annual sales per share to obtain the sales growth.

Dividend Yield

Definition: $1.0 \cdot YILD$

YILD Dividend-to-price ratio

Given as the trailing 12-month dividend per share divided by current price.

Book-to-Price

Definition: $1.0 \cdot BTOP$

BTOP Book-to-price ratio

Last reported book value of common equity divided by current market capitalization.

Leverage

Definition: $0.50 \cdot MLEV + 0.10 \cdot DTOA + 0.40 \cdot BLEV$

MLEV Market leverage

Computed as

$$MLEV = \frac{ME + PE + LD}{ME}, \quad (A7)$$

where *ME* is the market value of common equity on the last trading day, *PE* is the most recent book value of preferred equity, and *LD* is the most recent book value of long-term debt.

DTOA Debt-to-assets
Computed as

$$DTOA = \frac{TD}{TA}, \quad (A8)$$

where TD is the book value of total debt (long-term debt and current liabilities), and TA is most recent book value of total assets.

BLEV Book leverage
Computed as

$$BLEV = \frac{BE + PE + LD}{BE}, \quad (A9)$$

where BE is the most recent book value of common equity, PE is the most recent book value of preferred equity, and LD is the most recent book value of long-term debt.

Liquidity

Definition: $0.20 \cdot STOM + 0.35 \cdot STOQ + 0.45 \cdot STOA$

STOM Share turnover, one month
Computed as the log of the share turnover over the previous month,

$$STOM = \ln\left(\frac{V}{S}\right), \quad (A10)$$

where V is the trading volume for the month, and S is the number of shares outstanding.

STOQ Average share turnover, trailing 3 months
Let $STOM_{\tau}$ be the share turnover for month τ . The quarterly share turnover is defined by

$$STOQ = \ln\left[\frac{1}{T} \sum_{\tau=1}^T \exp(STOM_{\tau})\right], \quad (A11)$$

where $T = 3$ months.

STOA Average share turnover, trailing 12 months
Let $STOM_{\tau}$ be the share turnover for month τ . The annual share turnover is defined by

$$STOA = \ln\left[\frac{1}{T} \sum_{\tau=1}^T \exp(STOM_{\tau})\right], \quad (A12)$$

where $T = 12$ months.

Non-linear Size

Definition: $1.0 \cdot NLSIZE$

$NLSIZE$ Cube of Size

First, the standardized Size exposure (i.e., log of market cap) is cubed. The resulting factor is then orthogonalized to the Size factor on a regression-weighted basis. Finally, the factor is winsorized and standardized.

Appendix B: Decomposing RMS Returns

We decompose excess stock returns r_n into a systematic component, due to factors, and a stock-specific component u_n . The factor returns f_k are estimated each period by cross-sectional regression

$$r_n = \sum_k X_{nk} f_k + u_n, \quad (\text{B1})$$

where X_{nk} is the exposure of stock n to factor k . The specific returns are assumed to be uncorrelated with one another as well as to the other factors.

The total R -squared of a regression measures the cross-sectional variation explained by the factors,

$$R_T^2 = 1 - \frac{\sum_n v_n u_n^2}{\sum_n v_n r_n^2}, \quad (\text{B2})$$

where v_n is the regression weight of stock n (proportional to square-root of market capitalization). The root mean square (RMS) return, computed as

$$RMS = \sqrt{\sum_n v_n r_n^2}, \quad (\text{B3})$$

measures the cross-sectional dispersion from zero return. As described by Menchero and Morozov (2011), the RMS return can be exactly decomposed into the return sources of Equation B1 using a cross-sectional version of the x -sigma-rho formula,

$$RMS = \sum_k f_k \sigma(X_k) \rho(X_k, r) + \sigma(u) \rho(u, r), \quad (\text{B4})$$

where $\sigma(X_k)$ is the RMS dispersion of factor k , and $\rho(X_k, r)$ is the cross-sectional correlation between factor k and the asset returns. The last term in Equation B4 represents the contribution to RMS coming from stock-specific sources.

Appendix C: Review of Bias Statistics

A commonly used measure to assess a risk model's accuracy is the bias statistic. Conceptually, the bias statistic represents the ratio of realized risk to forecast risk.

Let R_{nt} be the return to portfolio n over period t , and let σ_{nt} be the beginning-of-period volatility forecast. Assuming perfect forecasts, the *standardized* return,

$$b_{nt} = \frac{R_{nt}}{\sigma_{nt}}, \quad (C1)$$

has an expected standard deviation of 1. The bias statistic for portfolio n is the *realized* standard deviation of standardized returns,

$$B_n = \sqrt{\frac{1}{T-1} \sum_{t=1}^T (b_{nt} - \bar{b}_n)^2}, \quad (C2)$$

where T is the number of periods in the observation window.

Assuming normally distributed returns and perfect risk forecasts, for sufficiently large T the bias statistic B_n is approximately normally distributed about 1, and roughly 95 percent of the observations fall within the confidence interval,

$$B_n \in \left[1 - \sqrt{2/T}, 1 + \sqrt{2/T} \right]. \quad (C3)$$

If B_n falls outside this interval, we reject the null hypothesis that the risk forecast was accurate.

Appendix D: Model Estimation Parameters

Table D1 contains factor covariance model estimation parameters for the EUE4 Model. The parameters are described in *USE4 Methodology Notes*.

Table D1

Factor covariance matrix parameters for the EUE4 model. All values are reported in trading days.

Model	Factor Volatility Half-Life	Newey-West Volatility Lags	Factor Correlation Half-Life	Newey-West Correlation Lags	Factor VRA Half-Life
EUE4D	42	N/A	200	N/A	4
EUE4S	84	10	504	3	21
EUE4L	252	10	504	3	84

Table D2 contains specific risk model estimation parameters for the EUE4 Model. The parameters are described in *USE4 Methodology Notes*.

Table D2

Specific risk parameters for the EUE4 model. Except for the dimensionless shrinkage parameter q , all values are reported in trading days.

Model	Specific Volatility Half-Life	Newey-West Auto-Corr. Lags	Newey-West Auto-Corr. Half-Life	Bayesian Shrinkage Parameter q	Specific VRA Half-Life
EUE4D	42	N/A	N/A	0.1	4
EUE4S	84	5	252	0.15	42
EUE4L	252	5	252	0.15	168

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¹ As of September 30, 2012, as published by eVestment, Lipper and Bloomberg on January 31, 2013