Model Insight

Barra US Total Market Equity Model for Long-Term Investors

Empirical Notes

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1. Introduction

This paper provides the model methodology review and empirical results for the new Barra US Total Market Equity Model for Long-Term Investors.[1] The model introduces the latest innovations from MSCI for building multi-factor equity models: *Systematic Equity Strategies (SES)* and the alignment of the model's factor structure with the investment horizon.

The Systematic Equity Strategies, when represented as factors, significantly enhance the information content of the model. The model includes five new SES factors: *Management Quality*, *Profitability, Prospect, Long-Term Reversal,* and *Earnings Quality*. In addition, significant data and methodological enhancements have been made to many existing factors such as *Value, Earnings Yield, Dividend Yield, Beta, Residual Volatility*, and *Liquidity*. The enriched content improves model performance across multiple use cases, such as Portfolio Construction, Hedging Market Risk, and Risk Monitoring.

The alignment of the factor structure with the investment horizon is one of the key innovations of the new Barra US Total Market Equity Models. The suite of new Barra US Total Market Equity Models consists of:

- Barra US Total Market Equity Model for Long-Term Investors
- Barra US Total Market Equity Model for Medium-Term Investors
- Barra US Total Market Equity Trading Model

The Long-Term Model is comprised of stable factors. These factors tend to be the focus of long-term investors with low portfolio turnover[2]. Compared to the Long-Term Model, the Medium-Term and the Trading models include additional factors that are relevant at shorter investment horizons but are considered to be "too fast", i.e. exposures change too much and too quickly, for long-term investors. The Long-Term Model helps achieve risk-reduction while keeping portfolio turnover and transaction costs at low levels.

The main characteristics of the new Barra US Long-Term Equity Model are:

- Alignment of the factor structure and long-term investment horizon for reduction of risk at low portfolio turnover levels.
- Style factors that reflect the latest research on Systematic Equity Strategies to capture new sources of investment risk.
- Historical point-in-time fundamental data updated daily for more realistic backtests.
- Enhanced beta estimation, with Bayesian shrinkage to industry betas to increase accuracy.
- Volatility Regime Adjustment methodology designed to calibrate factor volatilities and specific risk forecasts to current market volatility levels[3].
- Optimization Bias Adjustment methodology designed to improve risk forecasts of the optimized portfolios by reducing the effects of sampling error on the factor covariance matrix[4].

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[1] We refer to this model as the Barra US Long-Term Equity Model in this document. The acronym for the Barra US Total Market Equity Model for Long-Term Investors is USSLOW.

[2] According to our estimates, the average annualized turnover of an active US large-cap long-only mutual fund has been about 45% over the 2009-2014 period. We think of low-turnover portfolios as having turnover less than or equal to this average.

[3] Applied to the Responsive variant of the model only.

[4] Applied to the Responsive variant of the model only.

- Separation of market and industry effects through a country factor to better capture correlations among industries.
- Robust specific risk model based on daily asset-level returns, incorporating Volatility Regime Adjustment[5] and Bayesian Adjustment techniques for greater forecasting accuracy.
- Daily update for all versions of the model. Deep daily model history goes back to July 1995.
- Sixty industry factors based on the Global Industry Classification Standard (GICS®).

The Barra US Long-Term Equity Model is offered in Stable and Responsive variants. Both models have identical factor structures but differ in the responsiveness of risk forecasts.

In this paper, we discuss the new Barra US Long-Term Equity Model's factor structure, explanatory power, and performance, with a side-by-side comparison of its forecasting accuracy and backtesting performance versus its predecessors[6].

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[5] Applied to the Responsive variant of the model only

[6] The predecessors of the Barra US Long-Term Equity model are the Barra US Equity Models USE4 and USE3.

2. Methodology Highlights

2.1. Factor Structure and Investment Horizon

The new suite of Barra US Total Market Equity Models includes Long-Term, Medium-Term, and Trading Models. Each model is built on a factor set that is appropriate for a specific investment horizon. We employ factor exposure stability as an objective criterion to assess the relevance of a factor for a given time-frame.

We can think of the factor stability as the rate of information decay, which depends on both (i) the frequency of data updates and (ii) the magnitude of the changes between the updates. A high frequency of data updates and large changes in data between the updates tends to lead to faster information decay and less stability in factor exposures. We use cross-sectional correlation of monthly factor exposures for measuring factor stability.

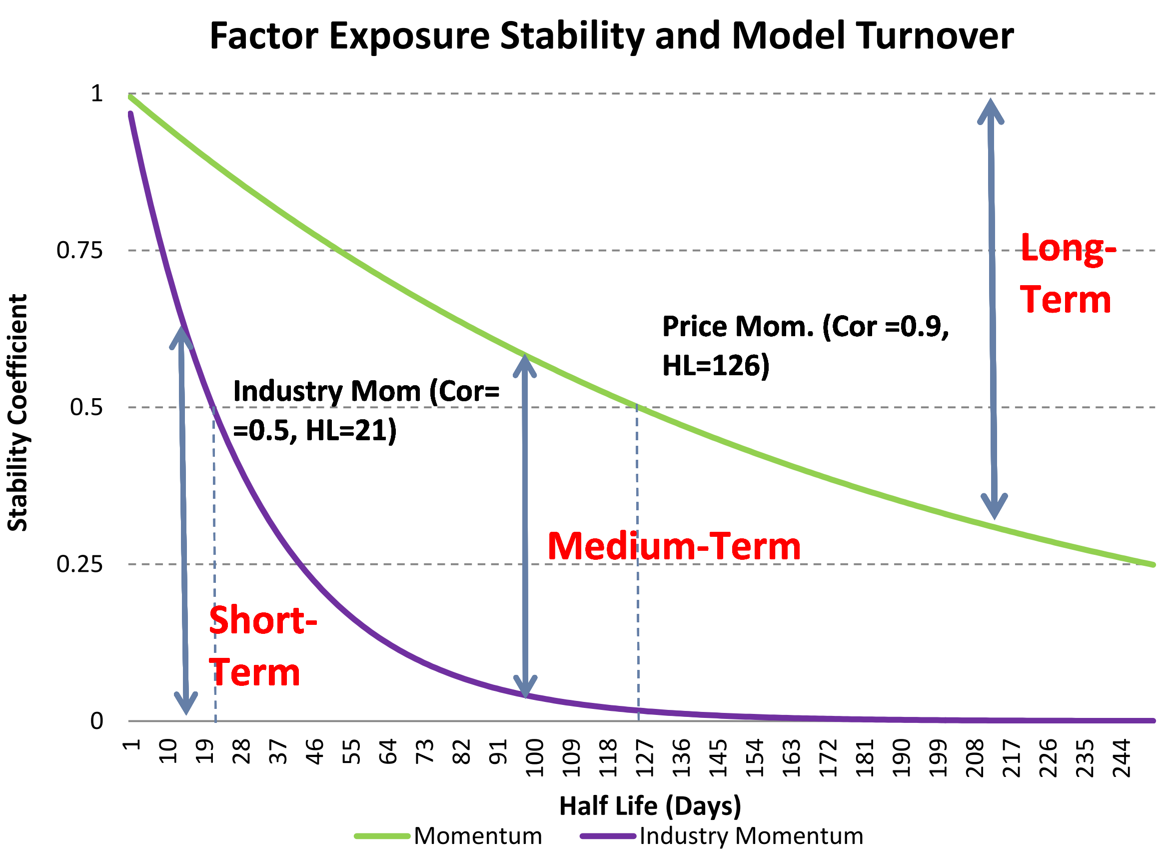
Factor stability plays an important role in portfolio management because there is a direct link between factor stability and portfolio turnover. For faster factors, the underlying information is incorporated more frequently and changes are larger in magnitude, implying that portfolio positions need to be rebalanced more frequently and more substantially to optimally reflect the updates in factor exposures. This leads to higher portfolio turnover. While high turnover may be required to pursue short-term strategies, it may be detrimental to the performance of long-term investors due to transaction costs. Therefore, it is important to align the factor structure of the model with the investment horizon.

The Barra US Long-Term Equity Model incorporates stable risk factors that are effective in capturing long-term stock volatilities and correlations. The Medium-Term and Trading models include faster factors that are relevant for their respective investment horizons.

In **Figure 2.1**, we illustrate how stability can be categorized into the three horizons offered with the Barra US Total Market Equity Model. Assuming the factor exposure correlation decays exponentially, the 0.90 monthly stability coefficient corresponds to the green line, and is equivalent to a 6-month half-life. Factors that are more stable will have decay curves above the green line, and are included in the Barra US Long-Term Equity Model.

Likewise, the purple line represents a monthly stability of 0.50 or a half-life of 21-days. The Barra US Total Market Equity Model for Medium-Term Investors includes all factors with exposure stability above the purple line, and the factors that are added to the Barra US Total Market Equity Trading Model (shown as Short-Term) fall below the purple line.

Figure 2.1: Factor Exposure Stability and Model Turnover



In **Table 2.1**, we rank factors by their stability and group them by model. The Barra US Total Market Trading Model includes all the factors; the Barra US Total Market Equity Model for Medium-Term Investors includes the low and medium-turnover factors (top two panels), and the Barra US Long-Term Equity Model includes only the most stable factors in the top panel.

Table 2.1: Barra US Total Market Equity Model factors by investment horizon

Horizon	Factor	Description
Long-Horizon & Low Turnover Factors	Size	Log of Market Capitalization
	Dividend Yield	Historical and predicted dividend yield
	Liquidity	Composite of share turnover, Amihud and Pastor-Stambaugh measures
	Management Quality	Composite of asset growth, capital expenditure growth, and net issuance growth
	Profitability	Composite of gross profitability, gross margin, ROE, ROA, and asset turnover
	Mid Capitalization	Mid-capitalization effect
	Prospect	Composite of long-term stock skewness and recent drawdown
	Value	Composite of book-to-price, sales-to-price, cash flow-to-price and fundamental value
	Growth	Composite of earnings and sales growth measures
	Leverage	Composite of book and market leverage and debt-to–assets ratio
	Long-Term Reversal	5-year reversal excluding 1-year momentum
	Beta	Historical beta with Bayesian shrinkage
	Earnings Yield	Forward and trailing earnings-to-price, EBITDA/EV
	Earnings Quality	Composite of accruals, estimate dispersion, variability in sales, earnings, and cash-flows
	Residual Volatility	Composite of option implied volatility and CAPM idiosyncratic volatility
	Momentum	Stock momentum
Mid-Horizon & Medium Turnover Factors	Analyst Sentiment	Analyst estimate revisions and up-down ratio
	Short Interest	Short interest as percent of total available to short
	Downside Risk	Co-movement of stock returns conditional on market and own performance
	Regional Momentum	Mid-term momentum in stock-regional performance
	Industry Momentum	Mid-term momentum in GICS® sub-industries
Short-Horizon & High Turnover Factors	Volatility Skew	Skew in implied volatilities
	News Sentiment	News related sentiment
	Seasonality	Time-of-the-year seasonality effect
	Short-term Reversal	Reversal in stock returns over short-horizon
	1-Day Reversal	Reversal in stock returns over 1-day

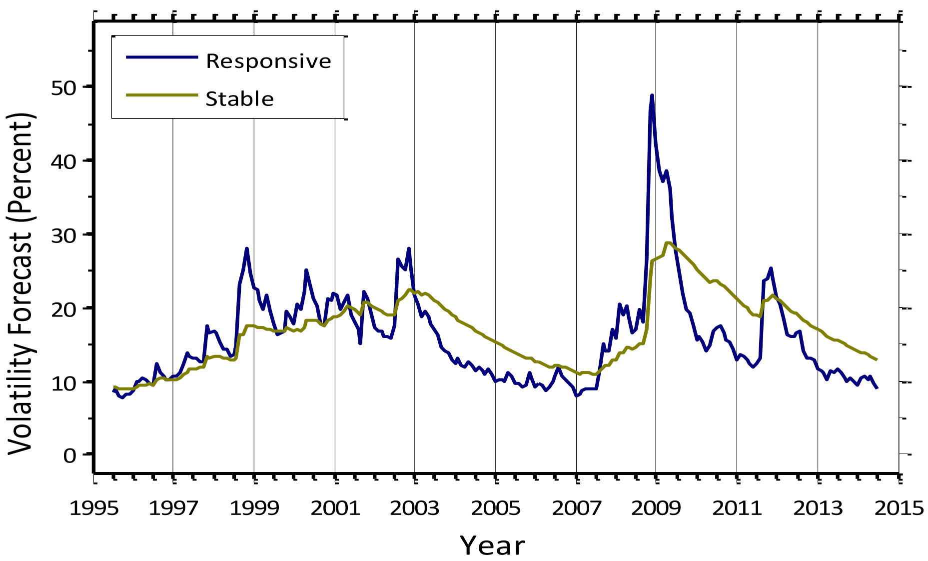
Aligning the factor structure with the investment horizon, the Barra US Long-Term Equity Model is better suited for typical use cases of long-term investors. For risk monitoring, removing fast factors prevents the model from unduly reflecting risk properties that are expected to disappear over the long-term investment horizon. For constructing portfolios, limiting the model to stable factors reduces the portfolio turnover.

2.2. Model Responsiveness

The Barra US Total Market Equity Model for Long-Term Investors comes in two variants: the Stable[7] and the Responsive[8]. Both variants use the same factor structure and factor returns, but differ in risk estimation. The Responsive variant, with shorter half-lives for variance and correlation estimators, adapts faster to changes in volatility. The Stable variant has longer half-lives and is designed to be less responsive. Additionally, the Responsive variant is leveraging the Volatility Regime Adjustment, which calibrates factor volatilities to current levels resulting in faster response to recent market trends.

In **Figure 2.2**, we plot the time series of volatility forecasts for the market portfolio described in **Section 3.1** for the Stable variant and the Responsive variant. Note that the Responsive variant adapts much faster to market shocks.

Figure 2.2: Market portfolio risk forecasts for Stable and Responsive variants available for the Barra US Long-Term Equity Model



2.3. Systematic Equity Strategies as Risk Factors

The concept of Systematic Equity Strategies was introduced and discussed by Bayraktar, Radchenko, Winkelmann, and Zangari (2013) and is implemented in the recently introduced Barra equity models.[9] The new Barra US Long-Term Equity Model includes these strategies as style risk factors. The following Systematic Equity Strategy factors are incorporated:

- Value
- Earnings Yield
- Dividend Yield

[7] Similar to the Long version of previous Barra US Equity Models

[8] Similar to the Short version of previous Barra US Equity Models

[9] Systematic Equity Strategies were introduced in the Barra Japan Equity Model (JPE4), the Barra Korea Equity Model (KRE3), the Barra US Sector Equity Models (USSM1), the Barra US Small Cap Equity Model, and the Barra Emerging Market Equity Model (EMM1).

- Profitability
- Earnings Quality
- Management Quality
- Momentum
- Long-Term Reversal
- Prospect[10]

Value, Earnings Yield, Dividend Yield, and Momentum are available in the predecessor model, USE4, but Profitability, Earnings Quality, Management Quality, Long-Term Reversal and Prospect are new additions. These factors are also commonly employed by investment practitioners as either factors in the quantitative process, or as screens for fundamental managers.

The Barra US Long-Term Equity Model allows investors to measure their exposure to popular but potentially crowded investment strategies. Furthermore, asset managers can attribute realized risk and returns to these factors and obtain more meaningful insights into drivers of their investment strategies.

The empirical analysis in this paper supports our intuition that the inclusion of these Systematic Equity Strategy factors in a risk model can lead to more accurate risk forecasts and enhanced portfolio performance, particularly for portfolios that are based on a systematic investment approach[11].

2.4. Volatility Regime Adjustment

A major source of bias in a risk model is the change in the level of volatility, a characteristic known as *non-stationarity*. Since risk models look backward to make predictions about the future, they tend to underpredict risk in times of rising volatility, and overpredict risk in times of falling volatility.

The Volatility Regime Adjustment is used for adjusting factor volatilities. It relies on a cross-sectional bias statistic, which may be interpreted as an *instantaneous* measure of risk model bias. By taking a weighted average of this measure over a suitable interval, the bias can be significantly reduced.

Just as factor volatilities are not stable across time, the same holds true for specific risk. We therefore also apply a Volatility Regime Adjustment to the specific risk model.

For more technical details on Volatility Regime Adjustment, see **Appendix A: Volatility-Regime Adjustment**.

2.5. Optimization Bias Adjustment

Another significant bias exhibited by risk models is the tendency to underpredict the risk of optimized portfolios, as demonstrated empirically by Muller (1993). More recently, Shepard (2009) derived an analytic result for the magnitude of the bias, showing that the under-forecasting becomes increasingly severe as the number of factors grows relative to the number of time periods used to estimate the factor covariance matrix. The basic source of this bias is sampling error. Specifically, spurious correlations may cause certain stocks to appear as good hedges in-sample, while these hedges fail to perform as effectively out-of-sample.

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[10] We refer readers to Bayraktar, Mashtaler, Meng and Radchenko (2013) for a more detailed discussion of Prospect factor, economic intuition behind the factor, and its importance from the perspective of risk.

[11] See section 5

We identify portfolios that capture biases and correct them directly within the factor covariance matrix. As shown by Menchero, Wang, and Orr (2011), the *eigenfactors* of the sample covariance matrix are systematically biased. Specifically, the sample covariance matrix tends to underpredict the risk of low-volatility eigenfactors, while overpredicting the risk of high-volatility eigenfactors.

We estimate the biases via Monte Carlo simulation, and then adjust the eigenvalues of the estimated covariance matrix to correct for these biases. This procedure helps improve factor risk forecasts for optimized portfolios. Furthermore, it builds the corrections directly into the factor covariance matrix, while fully preserving the meaning and intuition of the pure factors. Lee, Stefek, Xu and Yao (2011) demonstrate the effectiveness of this approach by backtesting active portfolios.

For a more technical discussion on Optimization Bias Adjustment, see **Appendix B: Optimization Bias Adjustment**.

2.6. Specific Risk Model with Bayesian Shrinkage

The specific risk model builds upon methodological advances introduced with the latest generation of Barra equity models[12]. All subsequent models utilize daily observations to provide timely estimates of specific risk directly from the time series of specific returns. A significant benefit to this approach is that specific risk is estimated individually for every stock, thus reflecting the idiosyncratic nature of this risk source.

A potential shortcoming of a pure time-series approach is that specific volatilities may not fully persist out-of-sample. In fact, there is a tendency for time-series volatility forecasts to overpredict the specific risk of high-volatility stocks, and underpredict the risk of low-volatility stocks, as shown in Menchero, Orr, and Wang (2011).

To reduce these biases, we introduce a Bayesian shrinkage technique. Stocks are segmented into deciles based on their market capitalization. Within each market capitalization decile, the mean and standard deviation of the specific risk forecasts are computed. The volatility forecast is then scaled towards the mean within each size decile. The scaling magnitude depends on the standard deviation of the specific risk forecasts.

For technical details on Bayesian Shrinkage, see **Appendix C: Specific Risk Bayesian Shrinkage**.

[12] The Barra US Equity Model (USE4) was the first model to introduce this methodology.

3. Factor Structure Overview

3.1. Estimation Universe

The coverage universe is the set of all securities for which the model provides risk forecasts. By contrast, the estimation universe is the subset of stocks used to actually estimate the model. Judicious selection of the estimation universe is an important part of building a sound risk model. The estimation universe must be broad enough to accurately represent the investment opportunity set of investors, without being so broad as to include illiquid stocks that may introduce spurious return relationships into the model. Furthermore, the estimation universe must be sufficiently stable to ensure that factor exposures are well behaved across time. Representation, liquidity, and stability, therefore, are the three primary issues to address when selecting a risk model estimation universe.

A well-constructed equity index must address these very same issues, and therefore serves as an excellent basis for the estimation universe. The Barra US Long-Term Equity Model estimation universe utilizes the MSCI USA Investable Markets Index (USA IMI), which aims to reflect the full breadth of investment opportunities within the US market by targeting 99 percent of the float-adjusted market capitalization. The MSCI index construction methodology applies innovative rules designed to achieve index stability, while reflecting the evolving equity markets in a timely fashion. Moreover, liquidity screening rules are applied to ensure that only investable stocks that meet the index methodological requirements are included for index membership.

3.2. Country Factor

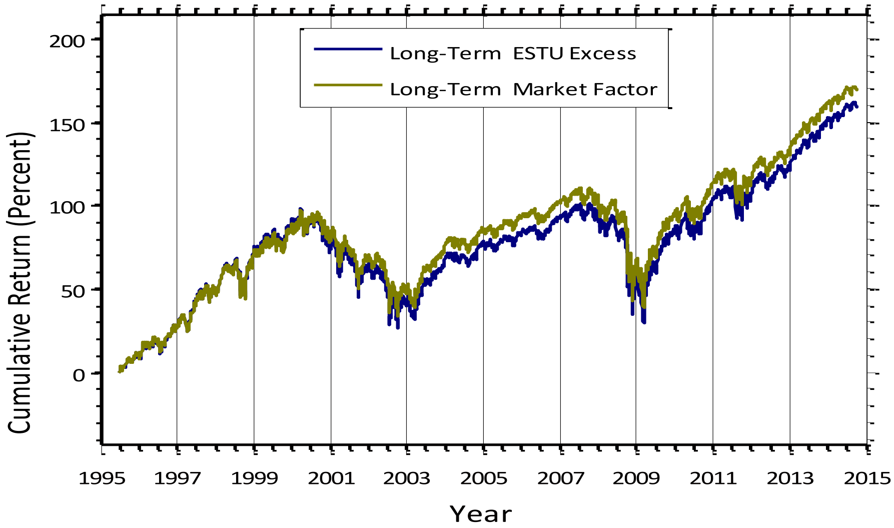
An important innovation included in this model is to explicitly include a Country factor, which is analogous to the World factor introduced and described by Menchero, Morozov, and Shepard (2008, 2010). One significant benefit of the Country factor is the insight and intuition it affords to portfolio managers. Menchero, Orr, and Wang (2011) show that the Country factor portfolio can be interpreted as the capitalization-weighted market portfolio and it can disentangle the pure industry effect from the overall market effect. The Country factor can thus provide a more intuitive interpretation of the industry factors.

Without the Country factor, industry factors represent portfolios that are 100 percent net long the particular industry, with zero net weight in every other industry. With the Country factor, by contrast, industry factors represent *dollar-neutral* portfolios that are 100 percent long the industry and 100 percent short the Country factor; that is, industry performance is measured net of the market.

Dollar-neutral industry factor portfolios are important for attribution. For instance, suppose that a portfolio manager overweights an industry that *underperforms* the market, but the industry nonetheless has a *positive* return. Clearly, overweighting an underperforming industry *detracts* from performance. If the industry factors are represented by net-long portfolios, however, an attribution analysis would show that overweighting the underperforming industry contributed *positively* to performance. This non-intuitive result is resolved by introducing the Country factor. Including the Country factor also has benefits in risk attribution, which are fully discussed in Davis and Menchero (2011).

Another benefit of the Country factor is improvement in risk forecasting. Intuitively and empirically, we know that industries tend to become more correlated in times of financial crisis. As shown in Menchero, Orr, and Wang (2011), the Country factor is able to capture these changes in industry correlation in a more timely fashion. The underlying mechanism for this effect is that net-long industry portfolios have common exposure to the Country factor, and when the volatility of the Country factor rises during times of market stress, it explains the increased correlations for the industries.

Figure 3.1: Cumulative Returns of the Country factor and the estimation universe (ESTU)



3.3. Industry Factors

Industries are important variables for explaining the sources of equity return co-movement. We construct the industry factor structure of the model using Global Industry Classification Standard (GICS®) as an input. The GICS scheme is hierarchical, with 10 sectors at the top, 24 industry groups at the next level, followed with increasing granularity at the industry and sub-industry levels. GICS applies a consistent global methodology to classify stocks based on evaluation of each firm's business model and economic operating environment. The model follows the same industry structure as the Barra US Equity Model (USE4), described by Menchero, Orr, and Wang (2011). For further details on the industry factors, refer to **Appendix J: Industry Factor Characteristics.**

3.4. Style Factors

Investment style represents another major source of systematic risk for equity portfolios. Style factors are constructed from financially intuitive stock attributes called *descriptors*, which serve as effective predictors of equity return covariance. **Appendix E: Descriptors** summarizes the descriptor definitions for each style factor. To facilitate comparison across style factors, the factors are standardized to have a capitalization-weighted mean of zero and an equal-weighted standard deviation of one. The capitalization-weighted estimation universe, therefore, is *style neutral*.

The following enhancements have been made to the factor structure in the new Barra US Long-Term Equity Model as compared to the Barra US Equity Model (USE4):

1. Addition of five Systematic Equity Strategy factors, as listed in Section 2.3
2. Removal of Non-Linear Beta to improve factor structure interpretation

A summary of all style factors is as follows (listed alphabetically):

- *Beta* - Explains common variation in stock returns due to different stock sensitivities to market or systematic risk that cannot be explained by the US Country factor.
- *Dividend Yield* - Captures differences in stock returns attributable to stock's historical and predicted dividend-to-price ratios.
- *Earnings Quality* - Explains stock return differences due to the uncertainty around company operating fundamentals (sales, earnings, cash flows) and the accrual components of their earnings.
- *Earnings Yield* - Describes stock return differences due to various ratios of the company's earnings relative to its price.
- *Growth* - Differentiates stocks based on their prospects for sales or earnings growth. This factor contains forward-looking long-term analyst predicted earnings growth descriptor and historical descriptors for sales and earnings growth over the trailing five years.
- *Leverage* - Captures common variation in stock returns due to differences in the level of company leverage.
- *Liquidity* - Captures common variations in stock returns due to the amount of relative trading and differences in the impact of trading on stock returns.
- *Long Term Reversal* - Explains common variation in returns related to a long-term (five years ex. recent thirteen months) stock price behavior.
- *Management Quality* – A combination of asset, investment, net issuance growth measures that captures common variation in stock returns of companies experiencing rapid growth or contraction of assets.
- *Mid Capitalization* - Captures deviations from linearity in the relationship between returns and log of market capitalization (Size factor). This factor measures the returns of mid-capitalization stocks relative to large- and small-cap stocks.
- *Momentum* - Explains common variation in stock returns related to recent (twelve months) stock price behavior.
- *Profitability* – A combination of profitability measures that characterizes efficiency of a firm's operations and total activities.
- *Prospect* - Explains common variation in stock returns that have exhibited a lottery-like behavior identified through a combination of stock return skewness over a long horizon and drawdown in returns over the recent period.
- *Residual Volatility* - Captures relative volatility in stock returns that is not explained by differences in stock sensitivities to market returns (Country and Beta factors).
- *Size* - Captures differences in stock returns and risk due to differences in the market capitalization of companies.
- *Value* - Captures the extent to which a company is overpriced or underpriced using a combination of several relative valuation metrics and one structural valuation factor.

For style factor descriptions, see **Appendix D: Factor Descriptions**. For descriptor definitions by style factor, see **Appendix E: Descriptors**.

4. Model Characteristics and Properties

One requirement of a high-quality factor structure is that factor returns be statistically significant. This helps prevent weak or noisy factors from finding their way into the model. We measure statistical significance by the *t*-statistic of the factor return. Assuming normality, absolute *t*-statistics greater than 2 are considered significant at the 95-percent confidence level. In other words, even if the factor had no explanatory power (that is, the factor is pure noise), then there is a chance that we would still observe a t-statistic with a value above two about five percent of the time.

In **Table 4.1**, we report summary statistics for the model style factors, along with the Factor Stability Coefficient and the Variance Inflation Factor (see Menchero, Orr, and Wang 2011). The Factor Stability Coefficient is computed as the cross-sectional correlation of factor exposures from one day to the next. Variance Inflation Factor (VIF) indicates the degree of collinearity among the factors. It concretely measures the extent a factor can be explained by the remaining factors. Excessive collinearity can lead to increased estimation error in the factor returns and non-intuitive correlations among factors.

Table 4.1: Style factor summary statistics computed using daily cross-sectional regressions (June 30, 1995 – September 30, 2014)

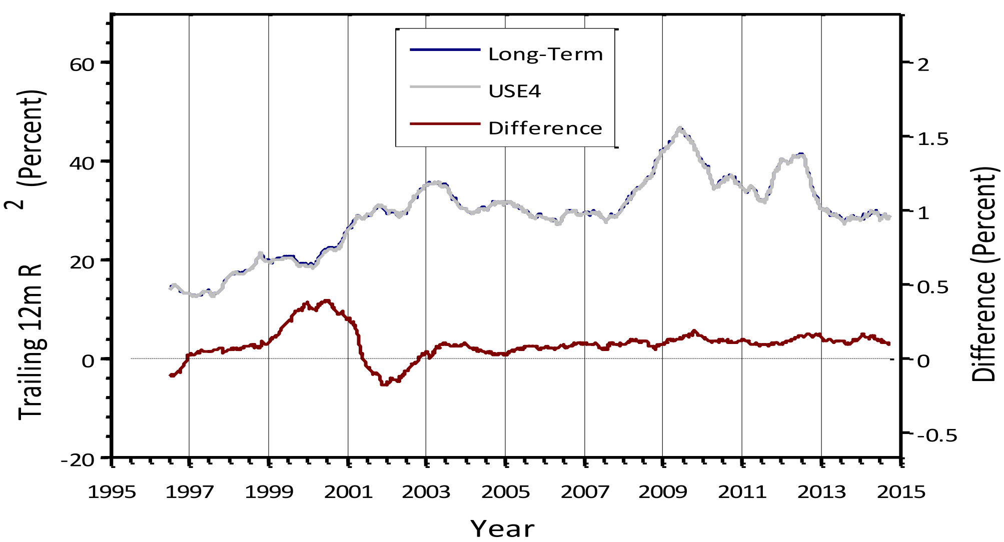
Style Factor	Average Absolute T-Stat	Percent Observ. |t|>2	Variance Inflation Factor	Annual Factor Return	Annual Factor Volatility	Factor IR	Correl. with ESTU	Factor Stability Coeff	Aug 2007 Drawdown
Country	14.30	89.66		8.81	19.70	0.45	1.00		-0.73
Beta	4.54	70.04	3.36	0.30	8.53	0.03	0.84	0.9554	1.71
Dividend Yield	1.19	17.83	2.39	0.29	1.84	0.15	0.09	0.9878	6.56
Earnings Quality	1.24	19.15	2.23	1.60	2.00	0.80	-0.05	0.9328	-2.63
Earnings Yield	1.41	25.60	2.17	2.47	2.27	1.09	-0.01	0.9541	-11.18
Growth	1.28	20.53	2.27	0.47	2.02	0.23	0.06	0.9721	2.95
Leverage	1.29	21.04	1.77	-0.31	1.84	-0.17	0.00	0.9880	-3.15
Liquidity	1.48	27.11	1.68	-0.02	2.12	-0.01	0.15	0.9916	-3.52
Long-Term Reversal	1.26	20.16	1.67	0.64	1.65	0.39	-0.05	0.9574	-3.20
Management Quality	1.05	12.87	1.58	1.36	1.26	1.08	0.01	0.9736	-11.62
Mid Capitalization	1.95	41.02	1.38	-0.58	3.01	-0.19	0.09	0.9839	-1.02
Momentum	2.57	51.66	1.93	2.26	4.24	0.53	0.05	0.8894	-8.37
Profitability	1.28	20.96	2.72	2.33	1.94	1.20	-0.10	0.9841	-6.82
Prospect	1.07	13.31	1.20	1.18	1.34	0.88	0.21	0.9650	3.51
Residual Volatility	2.28	46.73	2.04	-0.84	3.73	-0.22	0.44	0.9325	-2.33
Size	3.11	59.81	1.92	-2.10	3.39	-0.62	-0.04	0.9955	0.98
Value	1.29	21.21	2.28	1.38	2.09	0.66	0.07	0.9738	-10.71

4.1. Explanatory Power

The explanatory power of the factors, as measured by adjusted *R*-squared, is a key measure of model quality. However, the value of adjusted *R*-squared can be significantly impacted by the regression weighting scheme, the estimation universe, and the time period under consideration. Caution is required when comparing adjusted *R*-squared values across different models. Nevertheless, if each of these variables is carefully controlled, a meaningful comparison between models is possible.

In **Figure 4.1**, we report the trailing 252-day adjusted *R*-squared for the Barra US Long-Term Equity Model and its predecessor. The estimation universe and regression weighting scheme (square root of market capitalization) were identical for the two sets of regressions. The improvement in adjusted R-squared is significant when using the latest model.

Figure 4.1: Trailing 252-day adjusted *R*-squared for the Barra US Long-Term Equity Model and its predecessor



4.2. Cross-Sectional Dispersion

Cross-sectional dispersion can be measured in two ways; first is by cross-sectional volatility (CSV), which measures the dispersion relative to the *mean* return, and second is by root mean square (RMS) return, which measures the dispersion relative to *zero* return. The main difference between the two is that the Country factor makes no contribution to CSV, whereas it does contribute to RMS levels. As discussed by Menchero and Morozov (2011), the RMS return can be decomposed and attributed to individual factors or groups of factors.

In **Figure 4.2**, we show the net root mean square contributions from factors and stock-specific sources with a trailing 252-day total RMS return. In **Figure 4.3**, we further decompose the factor RMS into Country factor, industry, and style components.

Figure 4.2: Total daily cross-sectional dispersion as measured by root mean square (RMS) return

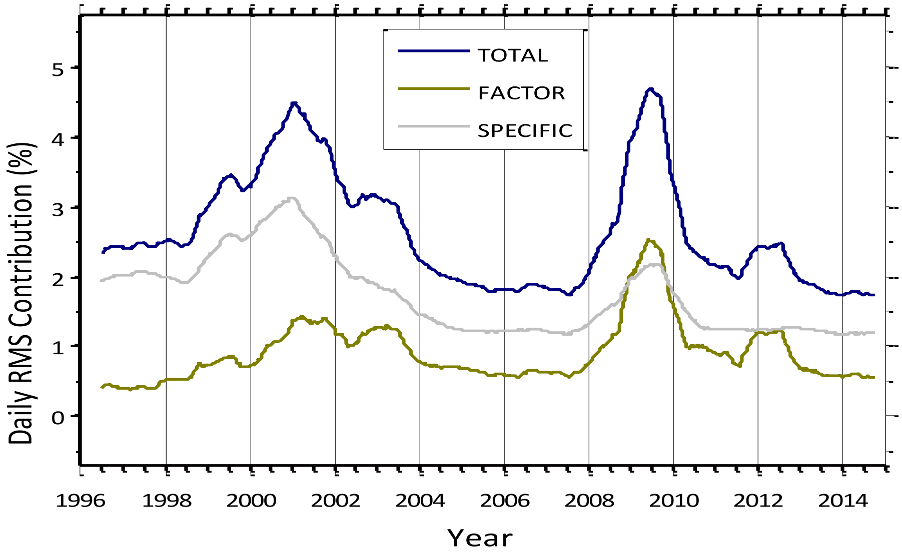
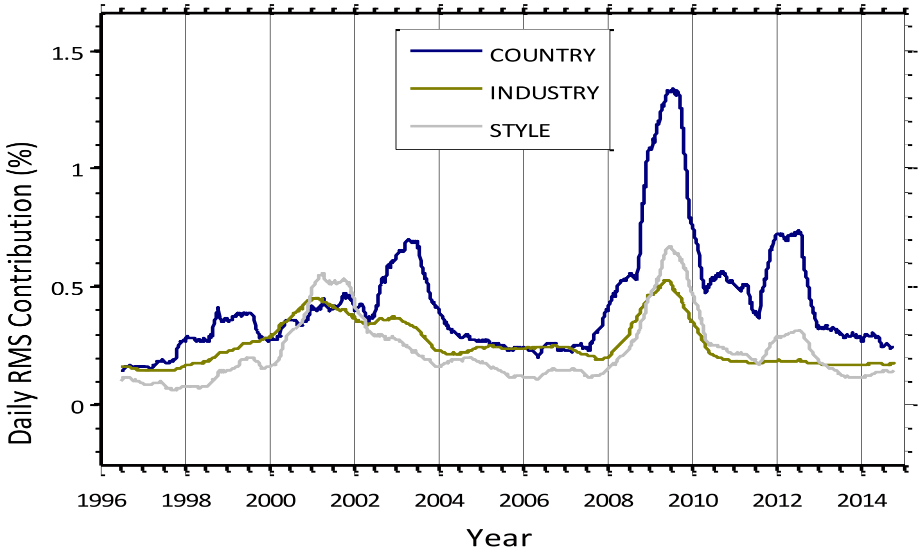


Figure 4.3: Contributions to daily root mean square (RMS) return from the Country factor, industries, and styles



4.3. Specific Risk

In **Figure 4.4**, we plot a sample histogram of specific risk forecasts. In **Figure 4.5**, we plot the 5th-percentile, mean, and 95th-percentile values for the specific risk distribution across time.

Figure 4.4: Histogram of responsive specific risk forecasts as of September 30, 2014

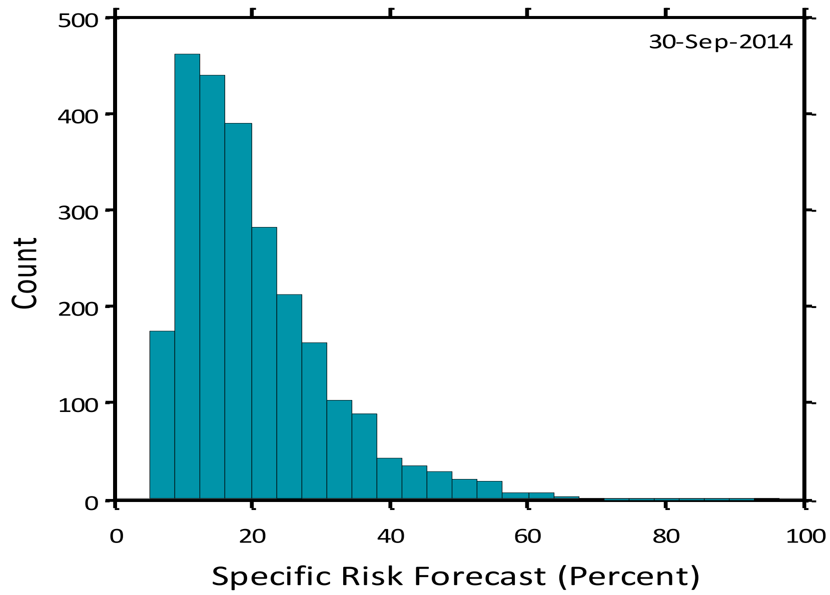
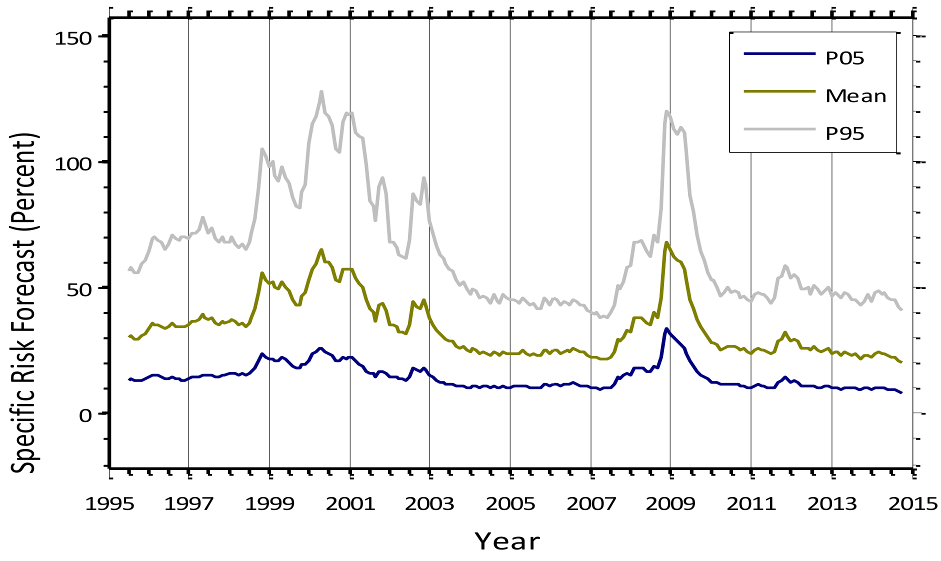


Figure 4.5: Specific risk levels versus time for Responsive model



5. Risk Forecasting Accuracy

5.1. Risk Forecasting Accuracy

In this section we compare the risk forecasting accuracy of the Barra US Long-Term Equity Model and its predecessor, the Barra US Equity Model (USE4). Our methodology for evaluating and comparing the accuracy of forecasts is based on the Q-statistic and the bias statistic. We use the Q-statistic to quantify the differences between models, and the bias statistic to build intuition about the periods when a model underforecasts or overforecasts risk. For a more technical discussion of measures of bias, see **Appendix G: Review of Bias Statistics** and Patton (2011).

The bias statistic is an out-of-sample measure that represents the ratio of realized risk to predicted risk. The bias statistic for perfect risk forecasts is one. By plotting the mean rolling-window bias statistic across time for a collection of portfolios in **Figures 5.1 to 5.7,** we can visualize the magnitude of the average biases and judge if they are persistent or regime-dependent.

One shortcoming of the bias statistic is that over a long period, we may have sub-periods of overforecasting and underforecasting, yet obtain a bias statistic close to one over the entire period. Forecasting errors can cancel out over the long term, even though the accuracy may be poor over sub-periods. For this reason, we focus on the mean Q-statistic. The Q-statistic provides a measure of the forecast error and grows with the error size. The mean Q-statistic is not prone to the error cancellation and is minimized by having the exact forecast for every portfolio for every time period. This gives us a tool to measure the improvements between models on the same set of portfolios. The more accurate model will have a lower average Q-statistic.

The following tables summarize our findings for the test cases presented from **Figures 5.1 to 5.7**. It is clear that the new Barra US Long-Term Equity Model provides more timely and accurate risk forecasts than the previous model[13].

The forecasting accuracy test is most relevant for the Responsive variant of the model, as its parameters are adjusted to provide the most accurate risk forecasts for the monthly horizon. While we include the results for the Stable version for completeness, note that the Stable version targets portfolio construction and risk-budgeting use-cases where more stable forecasts go beyond the one month prediction horizon.

Table 5.1: Bias statistic and average Q-statistic for the Barra US Long-Term Equity Model (Responsive variant) and the Barra US Equity Model Short-Horizon (USE4S)

		USE4S		Responsive		
Portfolio Type	**Figure**	**Bias**	**Q**	**Bias**	**Q**	**Q Diff**
Factor returns	5.1	1.03		1.05		
Specific returns	5.2	1.03		1.02		
Market (ESTU)	5.3	0.94	1.9679	0.95	1.9419	-0.0260
Random active	5.4	1.04	2.3323	1.03	2.3293	-0.0030
Factor-tilt (long)	5.5	1.02	2.2716	1.01	2.2421	-0.0294
Factor-tilt (active)	5.6	1.08	2.3562	1.08	2.3435	-0.0127
Optimized styles	5.7	1.07	2.3728	1.06	2.3660	-0.0068

[13] A comparison with USE3 is presented in Appendix K.

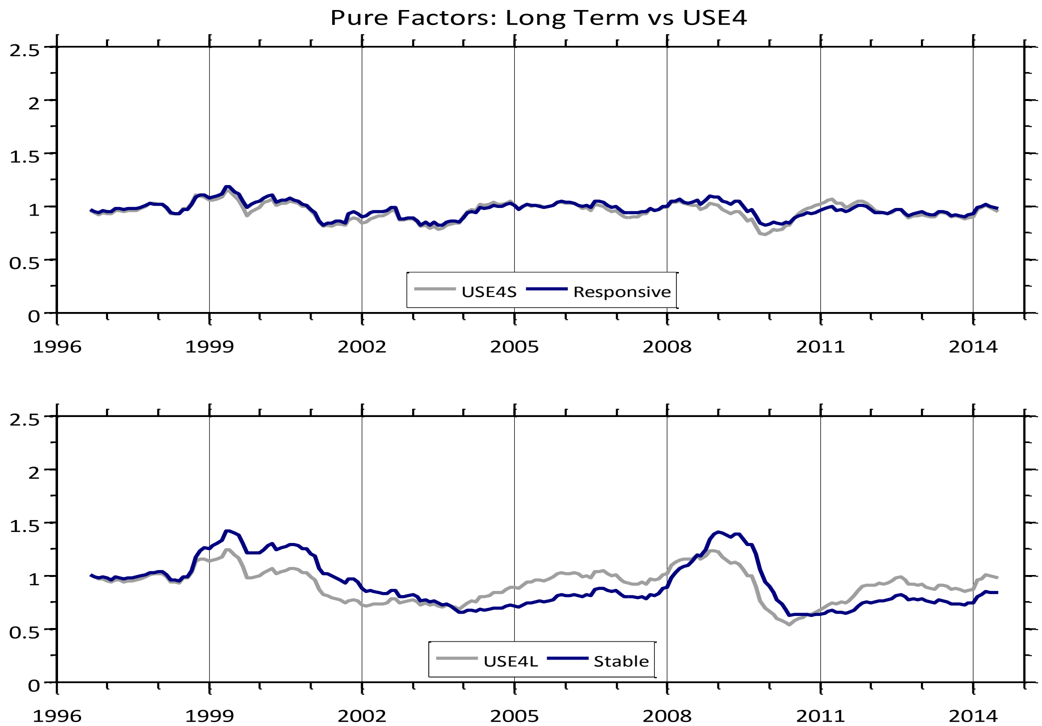
Table 5.2: Bias statistic and average Q-statistic for the Barra US Long-Term Equity Model (Stable variant) and the Barra US Equity Model Long-Horizon (USE4L)

	Figure	USE4L		Stable		
		Bias	Q	Bias	Q	Q Diff
Factor returns	5.1	1.01		1.03		
Specific returns	5.2	1.02		0.99		
Market	5.3	0.92	2.0165	0.94	2.0796	0.0631
Random active	5.4	1.02	2.3586	1.00	2.3881	0.0295
Factor-tilt (long)	5.5	1.00	2.3098	0.99	2.3555	0.0457
Factor-tilt (active)	5.6	1.06	2.3777	1.05	2.4292	0.0515
Optimized styles	5.7	1.06	2.3991	1.06	2.4290	0.0299

In the following figures, we compare the Barra US Long-Term Equity Model (Stable and Responsive variants) against its predecessor model, USE4 Long (L) and Short (S) horizons.

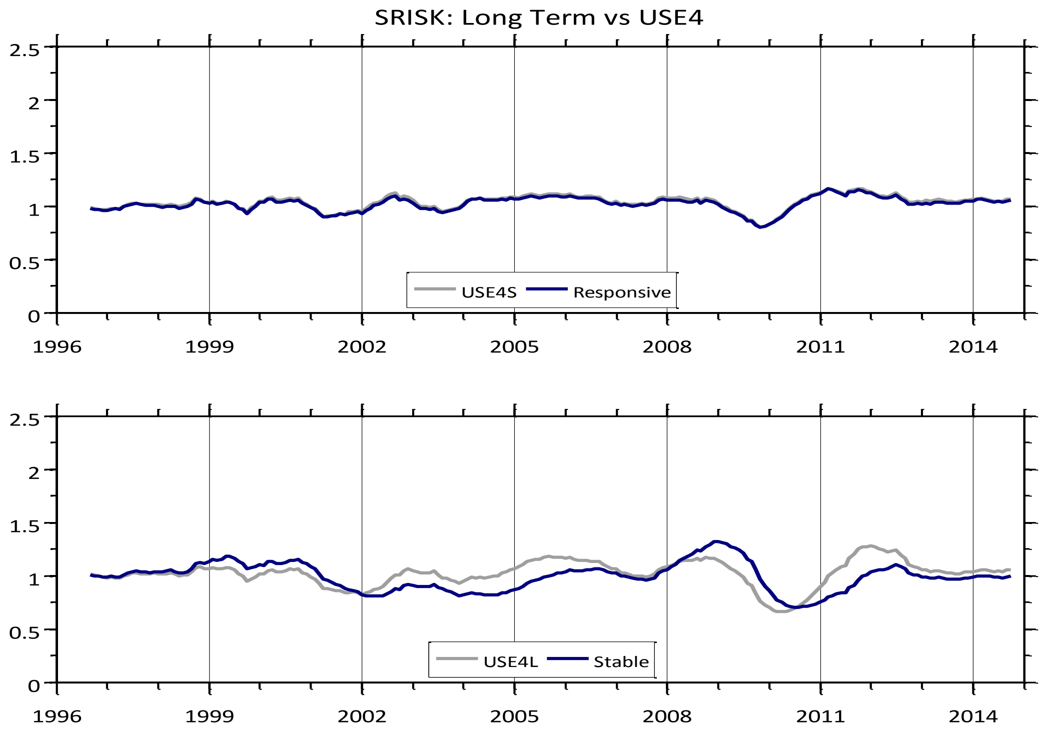
In **Figure 5.1**, we plot the rolling window bias statistics for the pure factors. Throughout this paper, we use rolling windows of 12 months for all models.

Figure 5.1: Rolling bias statistics for factor volatility forecast



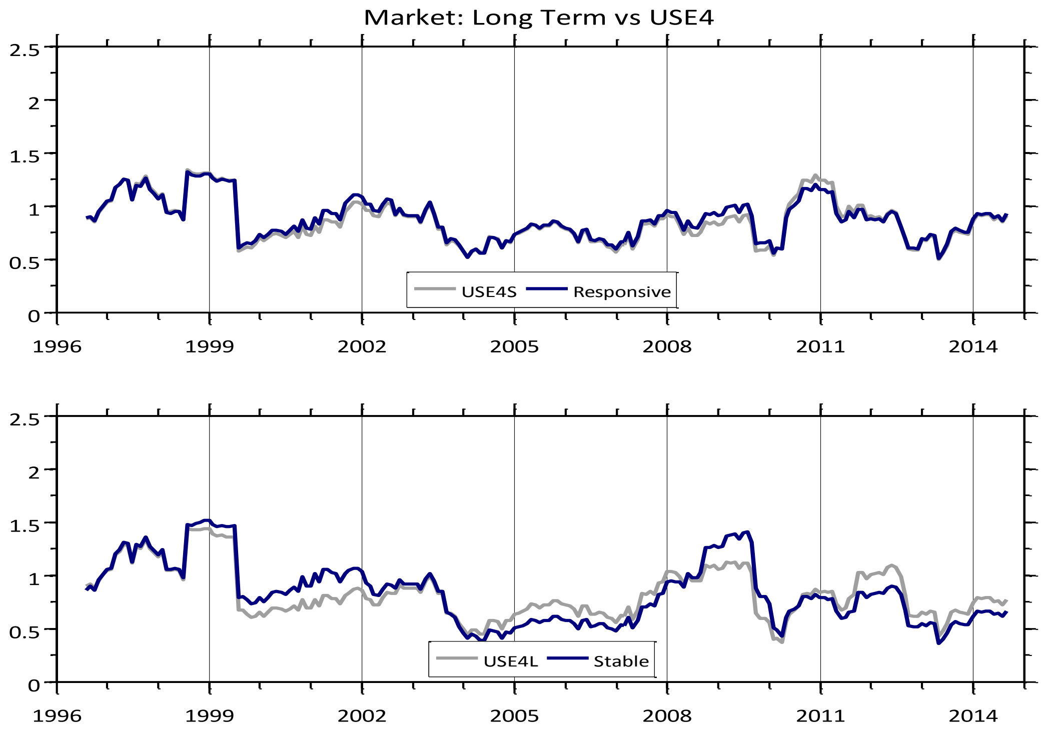
In **Figure 5.2***,* we plot the rolling average cross-sectional bias statistics for the specific risk forecasts for all stocks in the estimation universe. The cross-sectional bias statistic is capitalization-weighted.

Figure 5.2: Rolling bias statistics for specific risk forecast



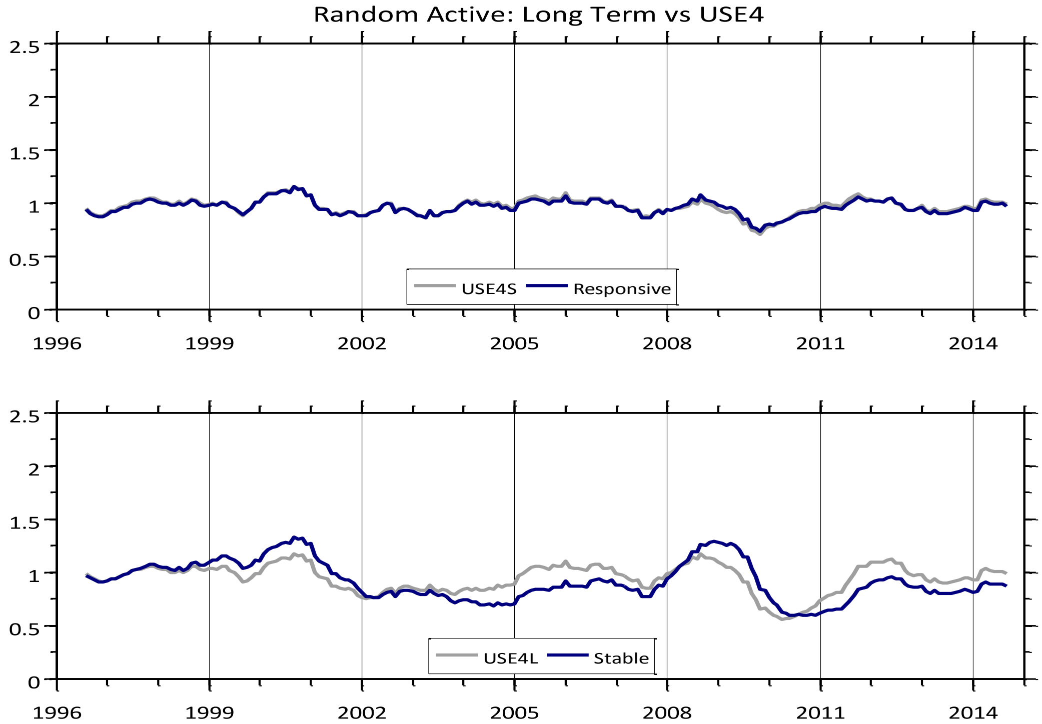
In **Figure 5.3**, we plot the rolling window bias statistics for the cap-weighted portfolio of the stocks present in estimation universes of the two models.

Figure 5.3: Comparison of bias statistics for the cap-weighted estimation universe portfolio.



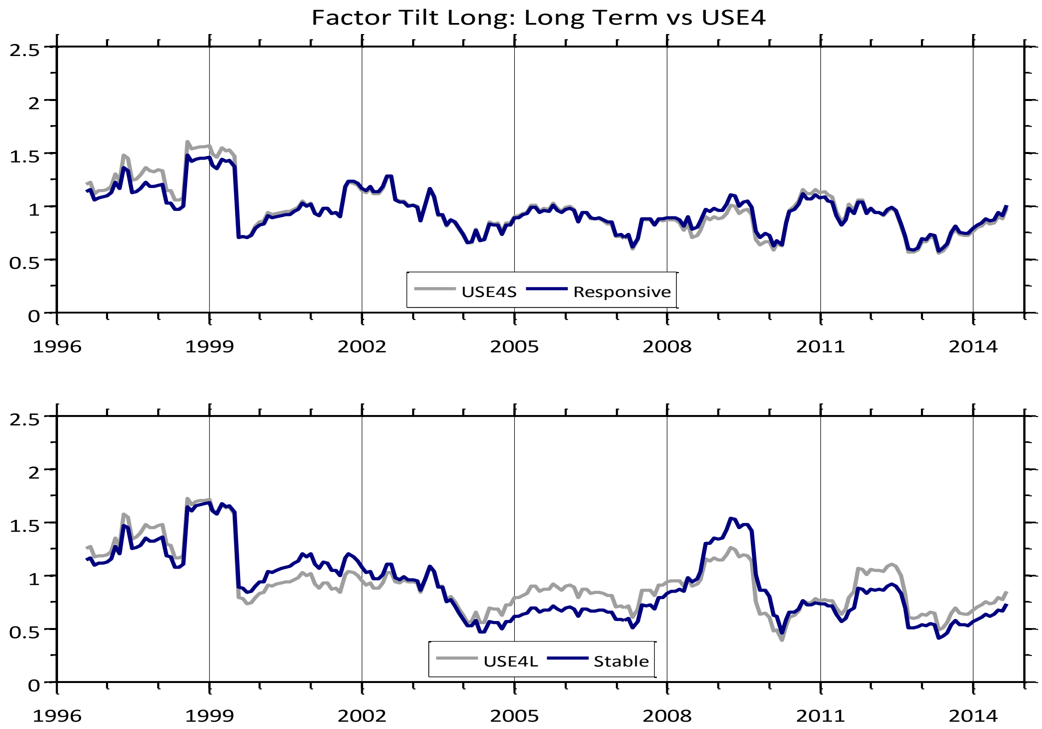
In **Figure 5.4**, we plot the rolling window bias statistics for the active risk for 100 random portfolios. The portfolios are constructed by going long 100 randomly selected stocks and weighted by their market capitalization. The cap-weighted estimation universe portfolio in Figure 5.3 is used as the benchmark. To reduce turnover, the list of stocks used to construct the random portfolios is fixed unless a stock drops out of the estimation universe, in which case it is replaced by another randomly selected stock.

Figure 5.4: Comparison of bias statistics for 100 random active portfolios



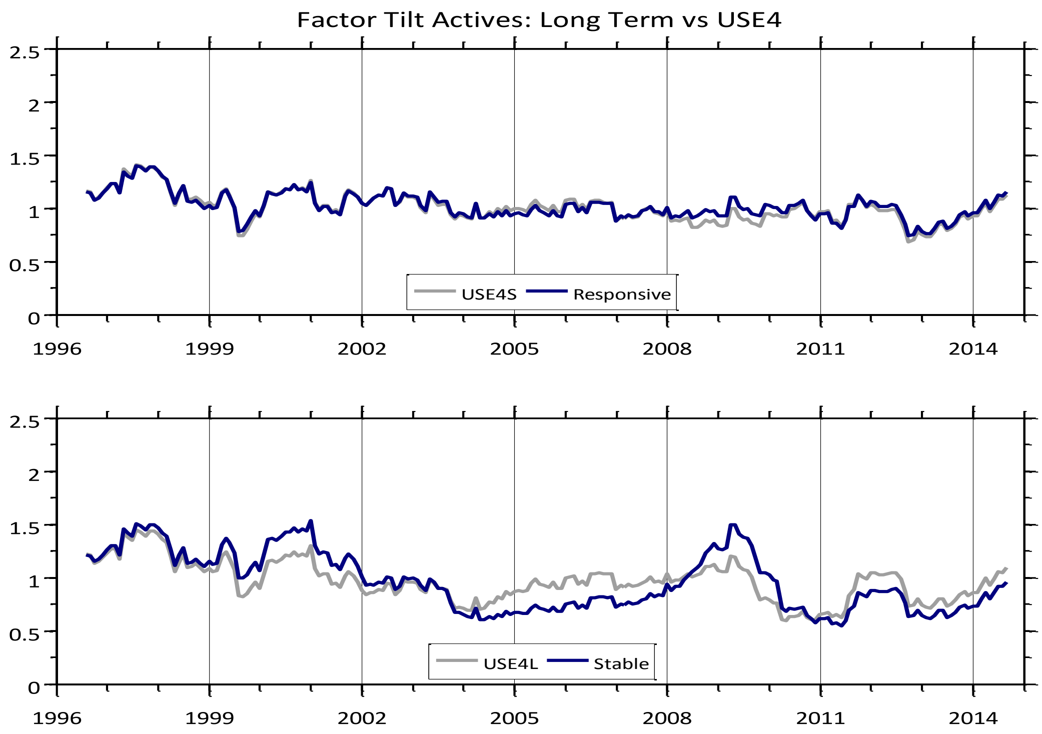
In **Figure 5.5**, we plot rolling bias statistics for long-only factor-tilt portfolios. The cap-weighted portfolios were constructed for each industry and the top and bottom quintiles of each style factor.

Figure 5.5: Comparison of bias statistics for long-only factor-tilt portfolios



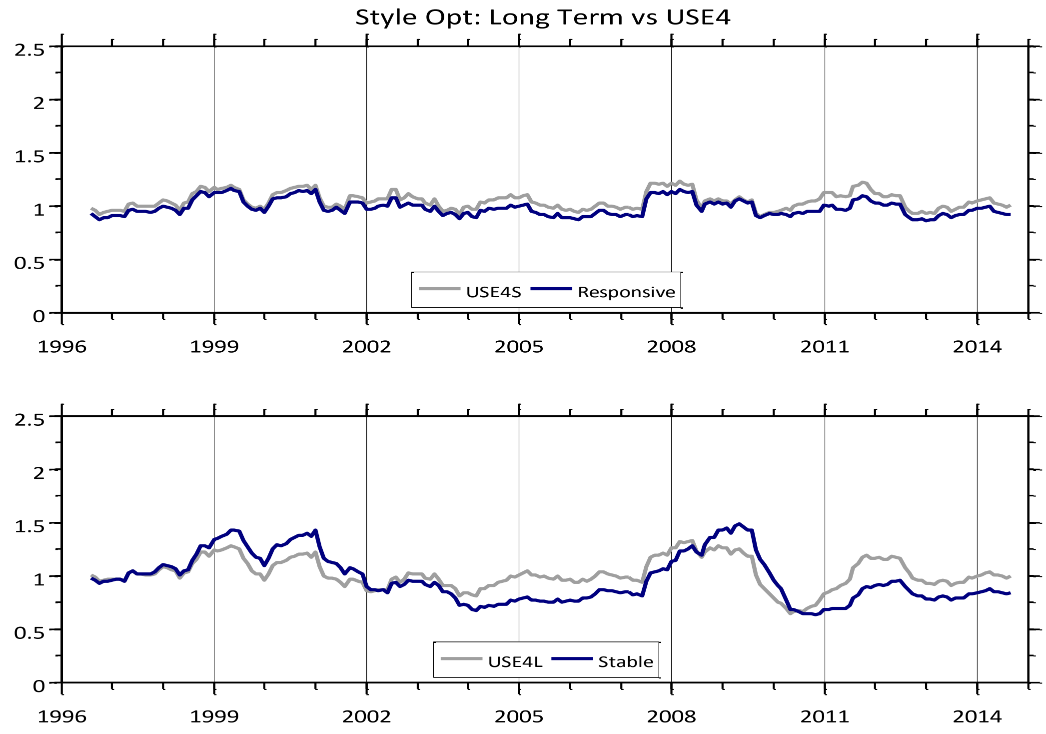
In **Figure 5.6**, we plot rolling-window bias statistics for the active factor-tilt portfolios. The portfolios were constructed by going long the factor-tilt portfolios of Figure 5.5 and shorting the estimation universe portfolio.

Figure 5.6: Comparison of bias statistics for active factor-tilt portfolios



In **Figure 5.7**, we plot the rolling bias statistics for optimized style-tilt portfolios. These portfolios are constructed by using style factors from both models as “alpha signals” and forming the unit-alpha, minimum volatility portfolios for 20 draws of 100 random stocks. No other constraints are imposed in the portfolio construction.

Figure 5.7: Rolling bias statistics of optimized style-tilt portfolios



5.2. Beta Forecasting Accuracy

In this section, we compare the forecasting accuracy of fundamental betas introduced by Rosenberg (1985) for the Barra US Long-Term Equity Model and its predecessor model, USE4. Our methodology for evaluating and comparing the accuracy of beta forecasts is based on cross-sectional volatility of the residual returns based on a one-factor market model. We obtain these residuals for each stock by subtracting the estimation universe return multiplied by the stock’s predicted beta from the stock’s excess returns. As Menchero, Nagy, and Singh (2014) demonstrated, cross-sectional residual volatility can be directly linked to the estimation error of betas. Hence, a model that produces less residual volatility provides more accurate beta forecasts. For the sake of comparability over time, we report the residual variance relative to the cross sectional variance of asset returns. As an analogy to the concept of regression R-squared, it can be understood as the portion of cross-sectional variance that cannot be explained by the market component, using the fundamental beta forecast.

In **Figures 5.8 and 5.9**, we report the trailing 12-month of residual return variance in relation to total cross-sectional stock return variance for the Responsive variant of the Barra US Long-Term Equity Model and USE4. Positive differences indicate that the new model provides more accurate beta forecasts.

Figure 5.8: Trailing 12-month cross-sectional residual variance relative to total variance for the Responsive variant of the Barra US Long-Term Equity Model and USE4 Short-Horizon

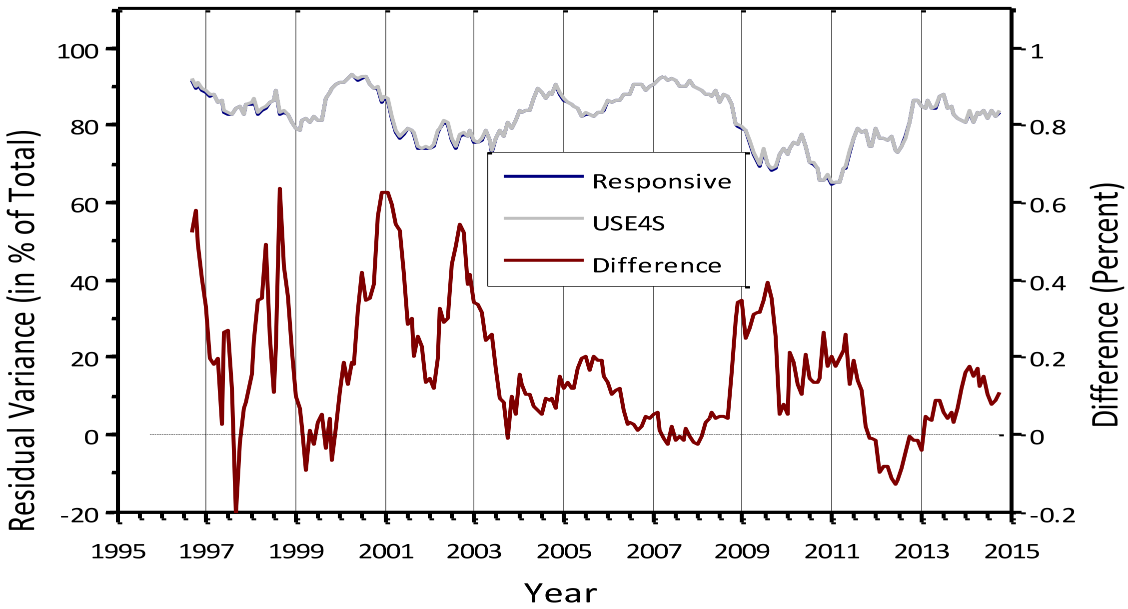
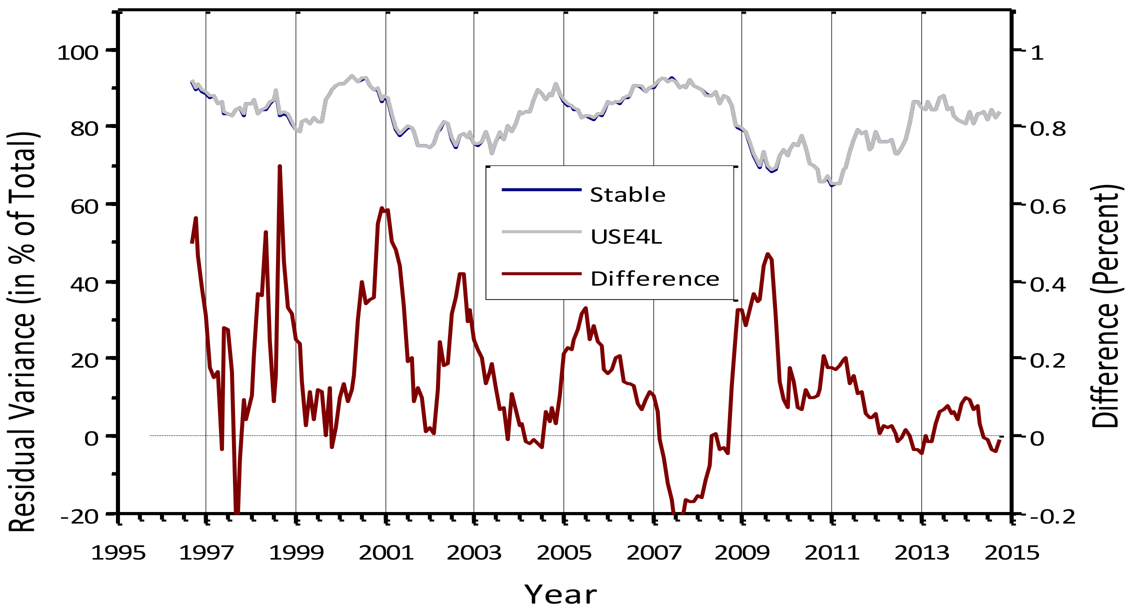


Figure 5.9: Trailing 12-month cross-sectional residual variance relative to total variance for the Stable variant of the Barra US Long-Term Equity Model and USE4 Long-Horizon



In **Table 5.3**, we report the average of the ratio of residual return variance to total cross-sectional stock return variance for the Barra US Long-Term Equity Model and its predecessor by capitalization deciles[14]. Positive differences between the USE4 Short (S) and Long (L) horizons, and the Barra US Long-Term Equity Model (Responsive and Stable variants) indicate that the new model provides more accurate beta forecasts across all market capitalization buckets.

Table 5.3: Average of the ratio of cross-sectional residual variance and total variance for Barra US Long-Term Equity Model and USE4

Decile	USE4S	Responsive	Difference	USE4L	Stable	Difference
Small	88.46%	88.31%	0.15%	88.45%	88.32%	0.13%
2	87.95%	87.92%	0.02%	87.97%	87.99%	-0.02%
3	86.41%	86.30%	0.11%	86.42%	86.34%	0.09%
4	85.85%	85.78%	0.07%	85.86%	85.83%	0.03%
5	85.00%	84.91%	0.08%	85.04%	85.01%	0.04%
6	84.55%	84.44%	0.12%	84.60%	84.53%	0.07%
7	82.78%	82.66%	0.12%	82.82%	82.72%	0.09%
8	81.85%	81.69%	0.16%	81.88%	81.75%	0.13%
9	80.40%	80.23%	0.17%	80.44%	80.30%	0.14%
Large	77.70%	77.43%	0.27%	77.73%	77.47%	0.26%

[14] A comparison with USE3 is presented in Appendix K.

6. Backtesting Results

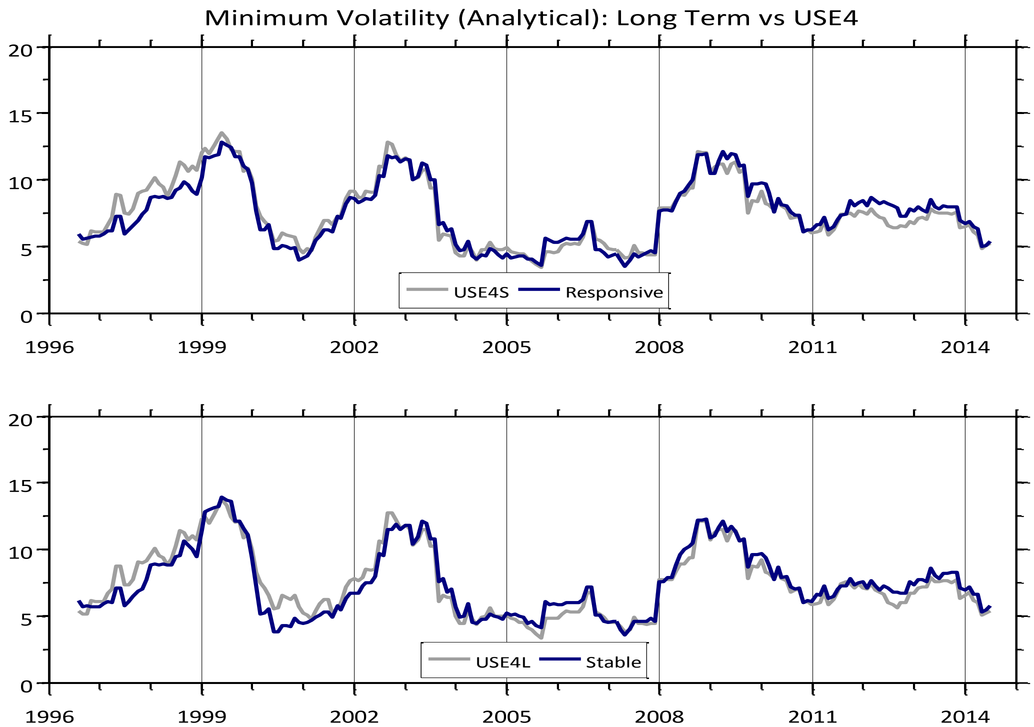
In this section, we test the portfolio construction processes for three kinds of long-only portfolios: minimum risk, benchmark tracking, and active portfolios. We compare the Barra US Long-Term Equity Model (Responsive and Stable variants) with the USE4 Short (S) and Long (L) horizons[15].

6.1. Minimum Risk Portfolio

In this set of backtests, we construct long-only minimum-risk portfolios with monthly rebalancing frequency. The investment universe is the intersection of the new and previous estimation universes. The backtest period is July 1995 through September 2014. We constrain monthly portfolio turnover at 2 percent, and asset weights at 3 percent. We also provide the results for the unconstrained long/short case.

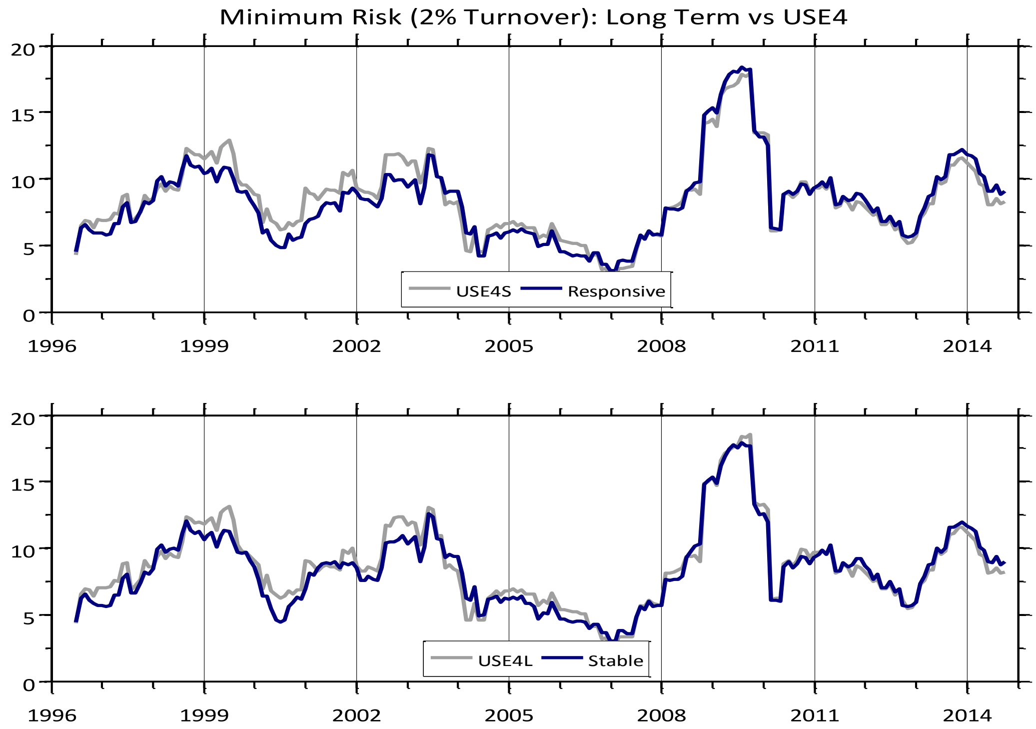
In **Figure 6.1**, we show the rolling 12-month volatility of the unconstrained minimum-risk portfolio. In **Figure 6.2**, we depict the realized volatility of a long-only minimum-risk portfolio with 2% turnover and asset weight constraint of 3%.

Figure 6.1: Realized volatility of analytical minimum risk portfolios estimated using 12-month rolling window



[15] A comparison with USE3 is presented in Appendix K.

Figure 6.2: Realized volatility of long-only minimum risk portfolios estimated using 12-month rolling window



In **Table 6.1**, we summarize the realized volatility and bias statistics for minimum risk for monthly horizon for the sample period July 1995 through September 2014. We compare returns (Ret), volatility (Vol), Information Ratio (IR), and Turnover (Turn).

Table 6.1: Summary of minimum-risk portfolio statistics

	USE4S				Responsive			
	Ret	**Vol**	**IR**	**Turn**	**Ret**	**Vol**	**IR**	**Turn**
Min-volatility	9.83%	8.14%	1.21	193%	10.20%	8.06%	1.26	169%
Min-volatility (long-only,2% monthly turnover)	8.01%	9.40%	0.85	29%	8.30%	9.22%	0.90	29%
Min-volatility (long-only, 4% monthly turnover)	8.03%	9.23%	0.87	52%	8.08%	8.99%	0.90	53%

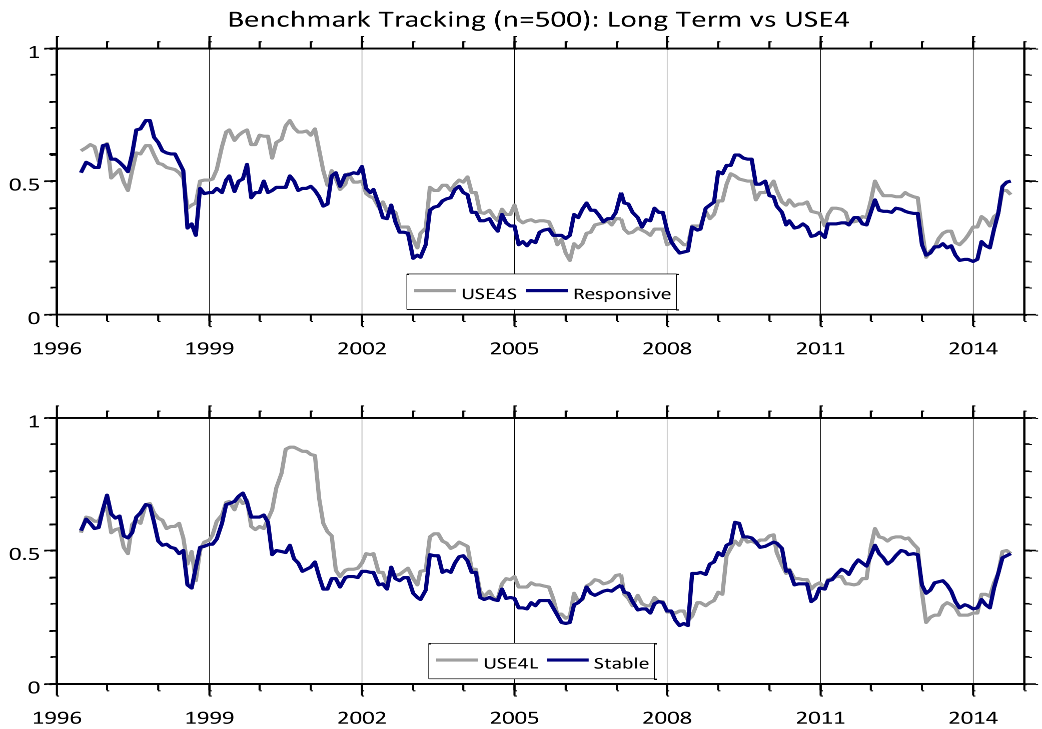
	USE4L				Stable			
	Ret	**Vol**	**IR**	**Turn**	**Ret**	**Vol**	**IR**	**Turn**
Min-volatility	9.95%	8.15%	1.22	185%	10.63%	8.11%	1.31	156%
Min-volatility (long-only, 2% monthly turnover)	8.16%	9.59%	0.85	29%	8.64%	9.28%	0.93	29%
Min-volatility (long-only, 4% monthly turnover)	8.34%	9.46%	0.88	52%	8.43%	9.16%	0.92	53%

6.2. Index Tracking

In this section, we provide results for a long-only backtest for monthly rebalanced portfolios tracking the intersection of estimation universes. The turnover is set to the maximum of 4 percent and the active asset weight is capped at 2 percent. The maximum number of allowed stocks is constrained to 500/250. The sample period is July 1995 through September 2014.

In **Figure 6.3**, we show the rolling 12-month volatility of the index-tracking portfolio.

Figure 6.3: Realized volatility of index-tracking portfolios estimated using 12-month rolling window



Tables 6.2 Provides the tracking error for the portfolios in **Figure 6.3**.

Table 6.2: Summary of index tracking portfolios over entire sample period from July 1994 through July 2013

	USE4S	Responsive
Benchmark (250)	0.84%	0.78%
Benchmark (500)	0.48%	0.45%

	USE4L	Stable
Benchmark (250)	0.86%	0.82%
Benchmark (500)	0.50%	0.47%

6.3. Optimized Style-Tilt Portfolios

In this section, we construct optimized style-tilt portfolios that are long-short portfolios minimizing volatility with unit exposure to a certain style characteristic. As the style exposure is fixed to be one in all cases, the comparison focuses on the realized volatility of these portfolios. Lower realized volatility indicates that a model is better capable in hedging style risk in portfolio construction.

The following tables exhibit the average realized volatility of optimized style-tilt portfolios categorized by styles in the Barra US Long-Term Equity model (Responsive variant)only, its predecessor model (USE4) only, and a combination of styles in both models.

Table 6.3: Summary of realized volatility of optimized style-tilt portfolios over entire sample period from July 1995 through September 2014

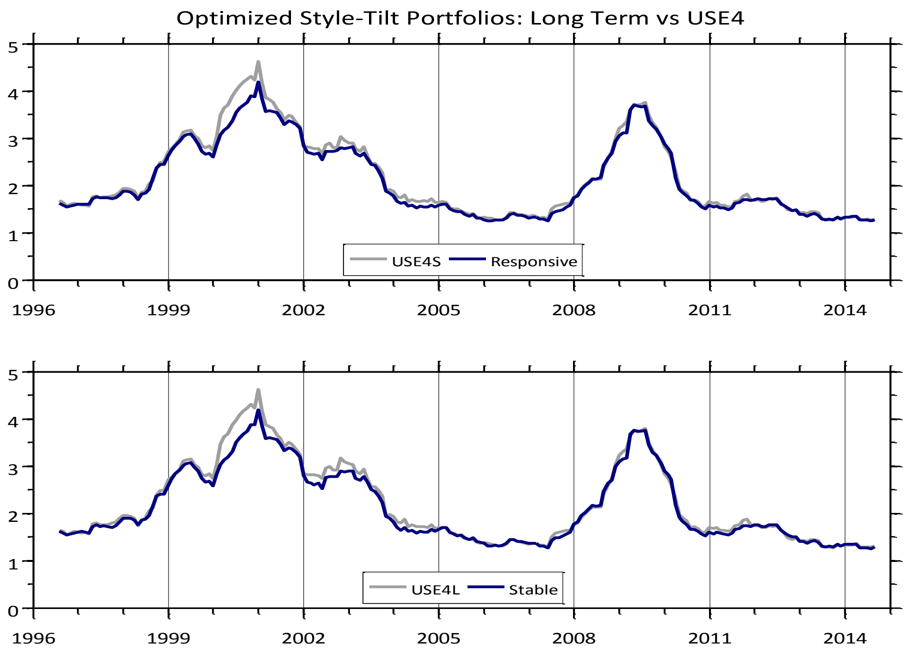
	USE4S	Responsive	Difference
USE4 Styles	2.72%	2.67%	-0.05%
Long-Term Styles	2.39%	2.25%	-0.14%
All Styles	2.53%	2.43%	-0.11%

Table 6.4: Summary of realized volatility of optimized style-tilt portfolios over entire sample period from July 1995 through September 2014

	USE4L	Stable	Difference
USE4 Styles	2.75%	2.67%	-0.08%
Long-Term Styles	2.41%	2.30%	-0.11%
All Styles	2.56%	2.46%	-0.11%

In **Figure 6.4**, we plot the average 12-month rolling volatility of all optimized style-tilt portfolios.

Figure 6.4: Average realized volatility of all optimized style-tilt portfolios estimated using a 12-month rolling window



7. Conclusion

This paper describes the Barra US Total Market Equity Model for Long-Term Investors. The model builds on the following key innovations:

- Factor structure and investment horizon alignment
- Systematic Equity Strategy factors
- Enhanced informational content and descriptor set

The Barra US Long-Term Model Equity model includes stable factors relevant to long-term investors. Our tests show that the increased factor exposure stability allows the construction of portfolios with lower realized volatilities at lower portfolio turnover compared to the predecessor models.

The model includes five new Systematic Equity Strategy factors: *Management Quality*, *Profitability, Prospect, Long-Term Reversal,* and *Earnings Quality*. These factors are commonly employed by investment practitioners as either factors in the quantitative process, or as screens for fundamental managers. The enhanced factor structure allows investors to attribute realized risk and returns to these factors and obtain more meaningful insights into drivers of their investment strategies. It also leads to more accurate risk forecasts, particularly for portfolios that are based on a systematic investment approach.

The inclusion of the new factors and the enhancements to existing factors significantly increases the number of underlying descriptors used in the model. The new descriptors leverage both existing and new data sources. These modifications enhance the informational content of the model.

This paper provides an empirical analysis of the Barra US Long-Term Equity Model. We offer a detailed presentation of the style factors, and key metrics that are reported at the individual factor level, including statistical significance, performance, volatility, and correlation.

We compare the explanatory power of this model versus its predecessors, USE3 and USE4[16], and decompose cross-sectional dispersion into contributions from factors and stock-specific sources. Further, we decompose the factor contribution into country, industry, and style components.

Finally, we show that the model improves the risk forecasting accuracy of portfolios and leads to better estimates of market betas. Minimum volatility portfolios and index tracking portfolios illustrate the benefits of the model in terms of lower realized volatility and turnover.

[16] A comparison with USE3 is presented in **Appendix K**

Appendix A: Volatility Regime Adjustment

Let f_{kt} be the return to factor *k* on day t , and let σ_{kt} be the one-day volatility forecast for the factor at the start of the day. The standardized return of the factor is given by the ratio f_{kt}/σ_{kt}, and should have standard deviation close to 1 if the risk forecasts are accurate. Normally, as described in **Appendix G: Review of Bias Statistics**, we compute the *time-series* standard deviation to investigate whether an individual factor is unbiased across time.

Alternatively, we can compute the *cross-sectional* standard deviation to investigate whether the factor volatility forecasts are collectively unbiased at a given point in time. We define the factor cross-sectional bias statistic B_t^F on day *t* as

$$B_t^F = \sqrt{\frac{1}{K}\sum_k \left(\frac{f_{kt}}{\sigma_{kt}}\right)^2} \qquad \text{(A1)}$$

where *K* is the total number of factors. This quantity represents an instantaneous measure of factor risk bias. For instance, if the risk forecasts were too small on a particular day, then $B_t^F > 1$. By observing the cross-sectional bias statistics over time, we can determine the extent to which volatility forecasts should be adjusted to remove these biases.

We define the *factor volatility multiplier* λ_F as an exponentially weighted average

$$\lambda_F = \sqrt{\sum_t (B_t^F)^2 w_t} \qquad \text{(A2)}$$

where w_t is an exponential weight with Volatility Regime Adjustment half-life τ_{VRA}^F. This parameter serves as the primary determinant of model responsiveness for factor risk. The Volatility Regime Adjustment forecasts are given by

$$\tilde{\sigma}_k = \lambda_F \sigma_k \qquad \text{(A3)}$$

This is equivalent to multiplying the entire factor covariance matrix by a single number, λ_F^2 . As a result, the Volatility Regime Adjustment has no effect on factor correlations.

Appendix B: Optimization Bias Adjustment

Let F_0 denote the $K\mathrm{x}K$ sample factor correlation matrix (FCM),

$$F_0 = cor(f, f) \tag{B1}$$

where f is the $K\mathrm{x}T$ matrix of realized factor returns, K is the number of factors and T is the number of periods. More detail on how to estimate F_0 is provided in **Appendix I: Covariance Matrix Estimation**.

The sample FCM can be expressed in diagonal form as

$$D_0 = U_0' F_0 \; U_0 \tag{B2}$$

where U_0 is the $K\mathrm{x}K$ rotation matrix whose columns are given by the eigenvectors of F_0 . The j^{th} element of the k^{th} column of U_0 gives the weight of pure factor j in eigenfactor k . The predicted eigenvalues of the eigenfactors are given by the diagonal elements of D_0. The fact that D_0 is diagonal indicates that the eigenfactors are mutually uncorrelated.

Although the true FCM is unobservable, we suppose for simulation purposes that the sample FCM F_0 governs the "true" return-generating process. We generate a set of factor returns for simulation m as

$$f_m = U_0 \; b_m \tag{B3}$$

where b_m is a $K\mathrm{x}T$ matrix of simulated eigenfactor returns. The elements of row k of b_m are drawn from a random normal distribution with mean zero and eigenvalues given by the diagonal element $D_0(k)$ of matrix D_0. It can be easily verified that the simulated returns in Equation B3 have a true FCM given by F_0. Due to sampling error, however, the *estimated* FCM

$$F_m = cor(f_m, f_m) \tag{B4}$$

will differ from the true FCM F_0 . Nevertheless, F_m is unbiased in the sense that $\mathrm{E}[F_m] = F_0$. We diagonalize the simulated FCM

$$D_m = U_m' F_m \, U_m \tag{B5}$$

where U_m denotes the simulated eigenfactors with estimated eignevalues given by the diagonal elements of D_m, i.e. $D_m(k)$.

Since we know the true distribution that governs the simulated factor returns, we can compute the true FCM of the simulated eigenfactors,

$$\widetilde{D}_m = U_m' F_m \, U_m \tag{B6}$$

Note that since U_m is not composed of the "true" eigenfactors, the matrix $\widetilde{D}_m$ is not diagonal. Nevertheless, our current focus is on the diagonal elements of the matrix. We compute the *simulated* eigenvalue biases according to

$$\nu^2(k) = \frac{1}{M} \sum_m \frac{\widetilde{D}_m(k)}{D_m(k)} \tag{B7}$$

where M is the total number of simulations. The simulated eigenvalue bias is computed daily, and the average over the entire sample period.

We now assume that the sample FCM F_0 , which uses the same correlation estimator as the simulated FCM F_m , also suffers from the same biases. Let $\widetilde{D}_0$ denote the diagonal FCM whose eigenvalues have been adjusted

$$\widetilde{D}_0 = v^2 D_0 \tag{B8}$$

where v^2 is a diagonal matrix whose elements are given by $v^2(k)$. The FCM in Equation D9 is now rotated from the diagonal basis to the pure factor basis using the sample eigenfactors. That is,

$$\tilde{F}_0 = U_0 \widetilde{D}_0 U_0' \tag{B9}$$

Where $\tilde{F}_0$ denotes the eigen-adjusted factor correlation matrix.

For further details, refer to Menchero, Wang, and Orr (2011).

Appendix C: Specific Risk Bayesian Shrinkage

One potential problem with using a pure time-series approach is that specific volatilities may not fully persist out-of-sample. In particular, stocks with either extremely low or extremely high specific volatility forecasts tend to revert to the mean.

To remove this bias, we shrink our estimates toward the cap-weighted mean specific volatility for the size decile s_n to which the stock belongs. More precisely, the shrunk estimate σ_n^{SH} is given by

$$\sigma_n^{SH} = v_n \bar{\sigma}(s_n) + (1 - v_n)\, \hat{\sigma}_n \tag{C1}$$

where $\hat{\sigma}_n$ is the original forecast and v_n is the shrinkage intensity that determines the weight given to the Bayesian prior, also known as the shrinkage target,

$$\bar{\sigma}(s_n) = \sum_{n \in S_n} w_n\, \hat{\sigma}_n \tag{C2}$$

where w_n is the capitalization weight of stock n with respect to the size decile. The shrinkage intensity is given by

$$v_n = \frac{q|\hat{\sigma}_n - \bar{\sigma}(s_n)|}{\Delta_\sigma(s_n) + q|\hat{\sigma}_n - \bar{\sigma}(s_n)|} \tag{C3}$$

where q is an empirically determined shrinkage parameter and

$$\Delta_\sigma(s_n) = \sqrt{\frac{1}{N(s_n)} \sum_{n \in S_n} (\hat{\sigma}_n - \bar{\sigma}(s_n))^2} \tag{C4}$$

is the standard deviation of specific risk forecasts within the size decile. The intuition behind this approach is straightforward: the more $\hat{\sigma}_n$ deviates from the mean, the greater the weight we assign to the Bayesian prior $\bar{\sigma}(s_n)$.

Appendix D: Factor Descriptions

This appendix provides the description, motivation, and summary of factor risk and return performance for all factors in the Barra US Total Market Equity Model for Long-Term Investors. More details on descriptors for each factor are provided in Appendix E.

For each factor, there are four plot diagrams that summarize factor risk and return performance in the univariate setting (agnostic to other risk model factors) and multivariate setting (in the presence of all factors) in the Barra US Total Market Equity Model for Long-Term Investors. The four factor plots are constructed as follows:

1. The **Factor Ventile** plot illustrates the performance of different stock segments grouped by their factor exposure. To construct Factor Ventile portfolios, we do the following:
    a. At each month-end, rank all the stocks in the estimation universe from smallest to largest, according to their factor exposure. Ranking stocks at each month-end corresponds to the monthly rebalancing of factor ventile portfolios.
    b. Split stocks into 20 ventile portfolios so that each ventile portfolio contains 5% of stocks in the estimation universe. Ventile portfolio 1 has 5% of stocks with the lowest factor exposure, while ventile portfolio 20 has 5% of stocks with the highest factor exposure.
    c. Compute the excess of market equal-weighted performance of all stocks in ventile portfolios. When computing market returns, we use the equal-weighted performance of all stocks in the estimation universe.
    d. Plot the average excess of market monthly returns for each ventile portfolios.
2. The **Monthly Performance** plot illustrates historical univariate and multivariate factor performance using the monthly rebalance. Multivariate monthly performance is computed using monthly cross-sectional regressions in the Barra US Total Market Equity Model for Long-Term Investors and measures factor performance orthogonal to all factors in the risk model. Univariate monthly performance is computed as the return to a portfolio that goes long the top 2 ventile portfolios and goes short the bottom 2 ventile portfolios that are rebalanced monthly. Unlike factor portfolios in a multivariate setting, factor portfolios in a univariate setting may have exposure to other factors in the risk model.
3. The **Daily Performance** plot illustrates: (i) historical multivariate performance using daily rebalancing, denoted “Ret”, (ii) factor drawdown, denoted ‘DD’, and (iii) factor standardized weekly returns, denoted “Z-score”. Multivariate daily performance is computed using daily cross-sectional regressions in the Barra US Long-Term Equity Model.
4. The **Annualized Volatility** plot illustrates predicted annualized factor volatility using the Barra US Long-Term Equity Model.

Table D.1: Factors and descriptors in the model categorized by similar styles—for descriptor information, refer to Appendix E

Style Factors (16)						
Size	**Value**	**Momentum**	**Quality**	**Volatility**	**Liquidity**	**Growth**
• Size • Mid Capitalization	• Value • Earnings Yield • Dividend Yield	• Momentum • Long-Term Reversal • Prospect	• Leverage • Earnings Quality • Profitability • Management Quality	• Beta • Residual Volatility	• Liquidity	• Growth

Descriptors by Style Factors (46)													
Size		**Value**		**Momentum**		**Leverage**		**Beta**		**Liquidity**		**Growth**	
LNCAP	Natural log of Market Cap	BTOP	Book-to-Price	RSTR	Relative Strength	MLEV	Market Leverage	HBETA	Historical Beta	STOM	Share Turnover, 1month	EGIBS	Earnings per Share Growth
Mid Capitalization		STOP	Sales-to-Price	**Long-Term Reversal**		BLEV	Book Leverage	**Residual Volatility**		STOQ	Average Share Turnover, 3month	EGRO	Trailing Earnings Growth
SIZENL	Cube of Size	CFTOP	Cash-Flow-to-Price	LTRSTR	Relative Strength	DTOA	Debt to Assets	HSIGMA	Historical Sigma	STOA	Average Share Turnover, 12month	SGRO	Sales Growth
		SVAL	Structural Valuation	LTHALPHA	Historical Alpha	**Earnings Quality**		IVOLC1	Implied Volatility 1 month Call	LIQMA	Modified Amihud Measure		
		Earnings Yield		**Prospect**		ABS	Accruals (Balance Sheet)	IVOLC3	Implied Volatility 3 month Call	LIQPS	Pastor-Stambaugh Measure		
		ETOP	Trailing Earnings-to-Price	SKEW	Skewness	ACF	Accruals (Cash Flow)	IVOLP1	Implied Volatility 1 month Put				
		EPIBS	Analyst-Predicted Earnings-to-Price	MAD	Maximum Drawdown	VSAL	Variability in Sales	IVOLP3	Implied Volatility 3 month Put				
		EM	Enterprise Multiple			VERN	Variability in Earnings						
		Dividend Yield				VFLO	Variability in Cash-Flows						
		DTOP	Dividend-to-Price			SPIBS	Variability of Analyst EPS estimates to price						
		DPIBS	Predicted Dividend-to-Price			**Profitability**							
						ROA	Return on Assets						
						ROE	Return on Equity						
						GP	Gross Profitability						
						GM	Gross Margin						
						ATO	Asset Turnover						
						Management Quality							
						AGRO	Asset Growth						
						IGRO	Issuance Growth						
						CX	Capital Expenditure						
						CXGRO	Capital Expenditure Growth						

Beta

Description Explains common variation in stock returns due to different stock sensitivities to market or systematic risk that cannot be explained by the US Country factor.

Motivation The Capital Asset Pricing Model (CAPM), one of the workhorses of finance theory, describes the relationship between risk and expected return. One of the implications of the model is that one of the key determinants of an investor's required rate of return and stock risk is stock beta.

Start Date 30 June 1995

Frequency Daily

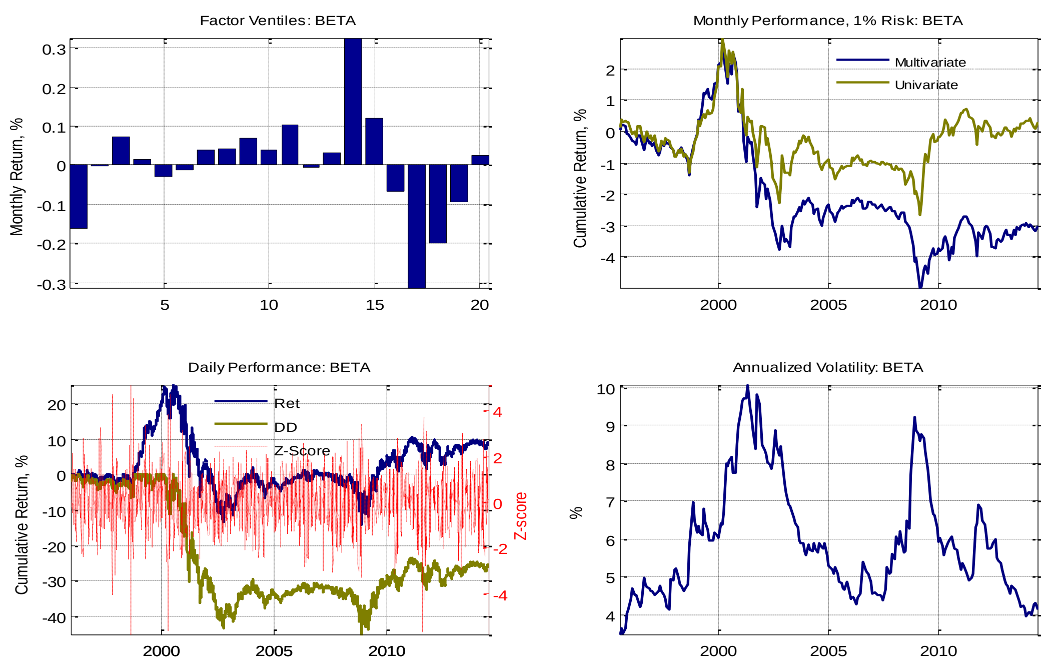
Exposure Interpretation A positive exposure indicates a high beta stock.

A negative exposure indicates a low beta stock.

Descriptors

- Historical beta

Figure D.1: Factor ventiles, monthly performance, daily performance and predicted annualized volatility for the Beta factor



Dividend Yield

Description

Captures differences in stock returns attributable to stock's historical and predicted dividend-to-price ratios.

This factor is based on Systematic Equity Strategies.

Motivation

Dividend is one of the central inputs in Gordon Growth Model for valuing a company's stock price. Rather than building a dividend-based structural model for valuation, we apply a relative valuation approach and use the Dividend Yield factor to capture common variation and risk differences between dividend paying companies.

Start Date

30 June 1995

Frequency

Daily

Exposure Interpretation

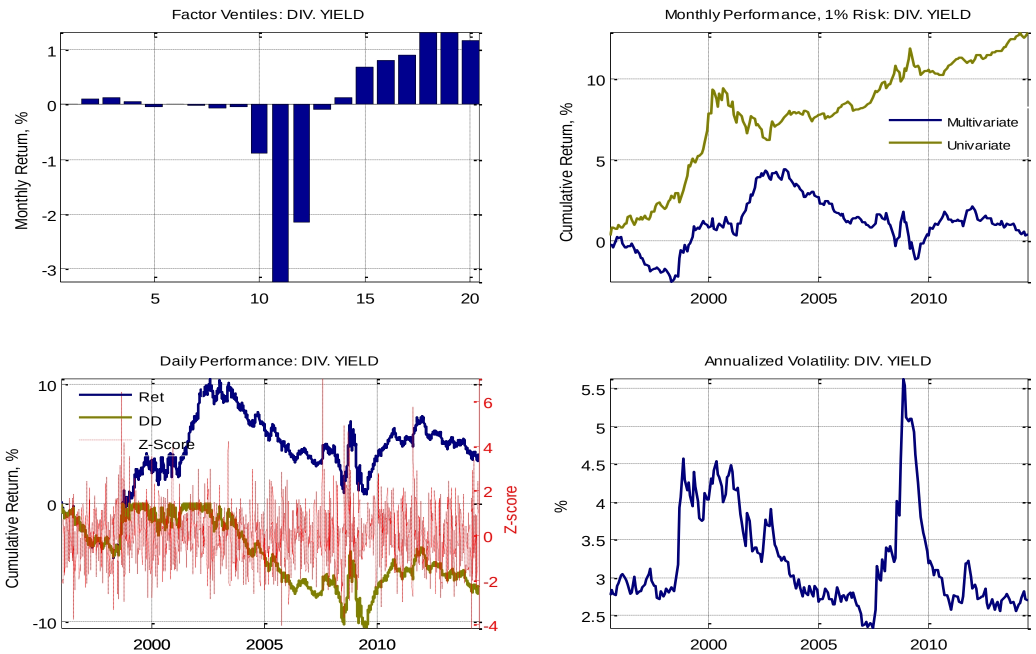
A positive exposure indicates a high historical/predicted dividend yield.

A negative exposure indicates a low historical/predicted dividend yield.

Descriptors

- Historical dividend-to-price ratio
- Analyst-predicted dividend-to-price ratio

Figure D.2: Factor ventiles, monthly performance, daily performance and predicted annualized volatility for the Dividend Yield factor



Earnings Quality

Description Explains stock return differences due to the uncertainty around company operating fundamentals (sales, earnings, cash flows) and the accrual components of their earnings.

This factor is based on Systematic Equity Strategies.

Motivation In an influential paper, Sloan R. (1996) illustrates the importance of distinguishing persistent and non-persistent (accruals) components of company earnings in valuing companies. The accrual components of earnings involve significant management discretion and are more prone to manipulations. Typically, earnings growth driven by a large accrual component is seen as less “sustainable”, hence of “low quality”. Along with the measures of less persistent earnings component (accruals), we augment the Earnings Quality factor with the dispersion of historical earnings, cash flows, and sales, and the variability in analyst earnings estimates. Our motivation is that companies with higher earnings quality have less uncertainty around their fundamentals. [17]

Start Date 30 June 1995

Frequency Daily

Exposure Interpretation A positive exposure indicates a low accruals and low uncertainty around firm fundamentals.
A negative exposure indicates a high accruals and high uncertainty around firm fundamentals.

Descriptors

- Accruals using balance sheet statement
- Accruals using cash-flow statement
- Variability in sales
- Variability in earnings
- Variability in cash-flows
- Variability in Analyst EPS Estimates to Price

[17] For further reading, refer to:

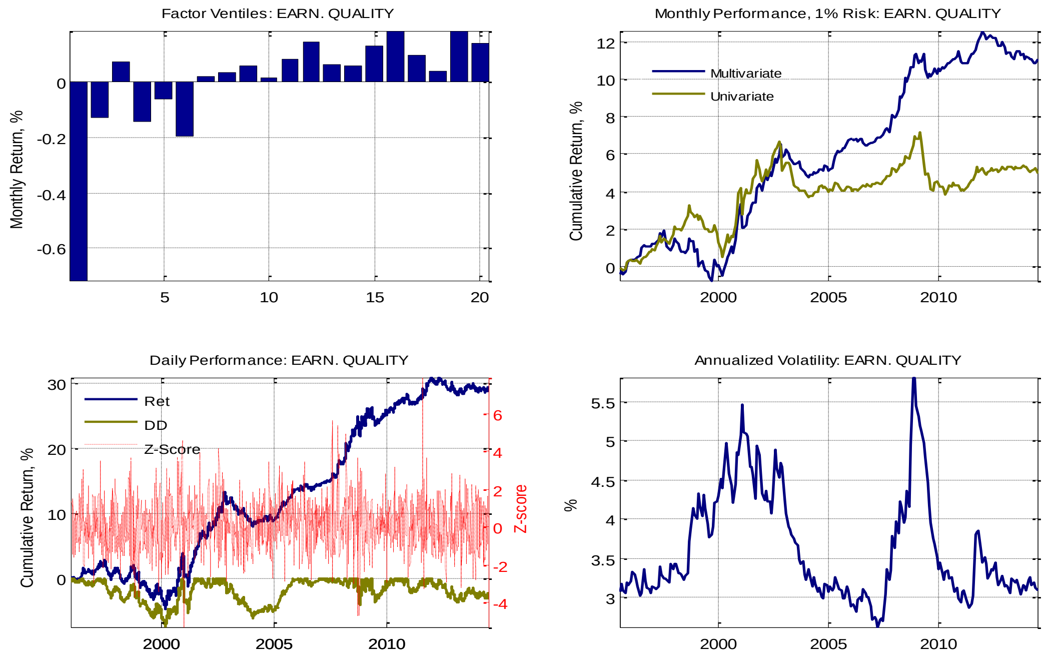
Sloan R., 1996, Do Stock Prices Fully Reflect Information in Accruals and Cash Flows about Future Earnings? The Accounting Review

Diether K.B., C.J. Malloy, 2002, Differences of Opinion and the Cross Section of Sock Returns, The Journal of Finance

Francis J., R. LaFond, P. Olsson, K. Schipper, 2004, Cost of Equity and Earnings Attributes, The Accounting Review

Huang, A.G., 2009, The cross section of cashflow volatility and expected stock returns. Journal of Empirical Finance

Figure D.3: Factor ventiles, monthly performance, daily performance and predicted annualized volatility for the Earnings Quality factor



Earnings Yield

Description Describes stock return differences due to various ratios of the company's earnings relative to its price.

This factor is based on Systematic Equity Strategies.

Motivation The Earnings Yield factor is one of the relative valuation multiples popular in the finance industry. Price multiples characterize a stock's relative "market" valuation and differ from multiples scaled by other metrics (such as assets, sales, or book value.

Most company valuations in the finance industry are relative valuations based on some company multiples and comparables. It is pointed out in academic literature that almost 85% of equity research reports and more than 50% of all acquisitions are based upon a company multiple.

Start Date 30 June 1995

Frequency Daily

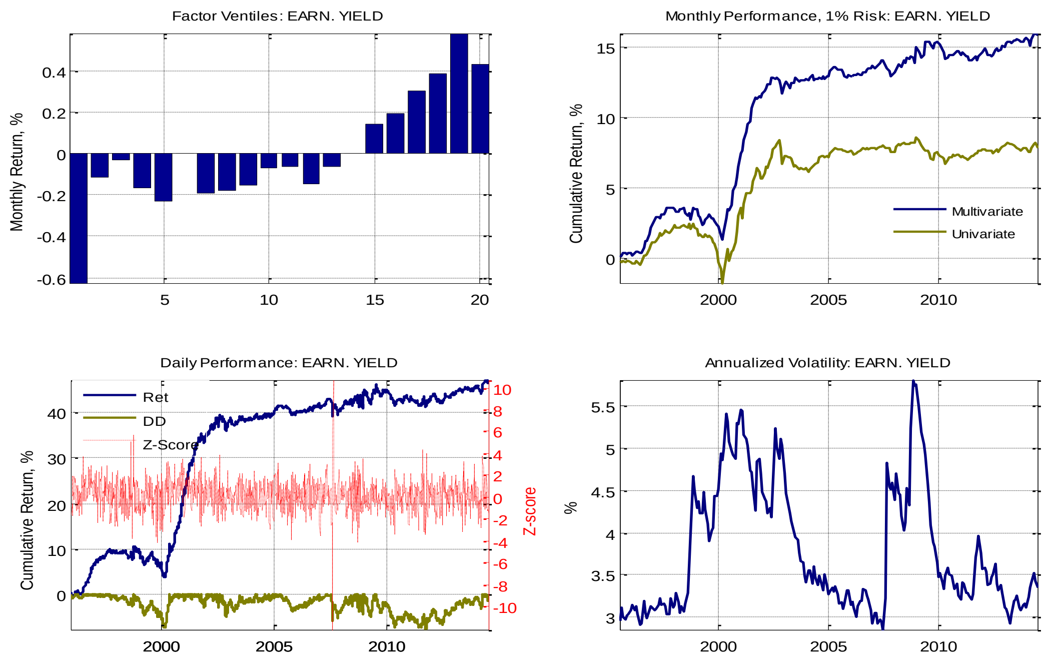
Exposure Interpretation A positive exposure indicates a high historical/predicted earnings yield ('cheap' stocks)

A negative exposure indicates a low historical/predicted earnings yield ('expensive stocks)

Descriptors

- Enterprise Multiple (EBITDA to EV)
- Earnings-to-price ratio
- Analyst-predicted earnings-to-price ratio

Figure D.4: Factor ventiles, monthly performance, daily performance and predicted annualized volatility for the Earning Yield factor



Growth

Description

Measures company growth prospects using historical sales growth and historical and predicted earnings growth.

This factor is based on Systematic Equity Strategies.

Motivation

Growth is one of the factors that determines the future cash flow and dividends paid out to investors and, thus, future stock prices. The Gordon Growth Model, which is an example of the dividend discount model (DDM), for valuing a company's stock price is based on the net present value of future dividends. The model predicts the relationship between stock price, future dividends, cost of capital and the dividend growth rate. We use the Growth factor in the risk model as a proxy for future dividend growth rate.

Start Date

30 June 1995

Frequency

Daily

Exposure Interpretation

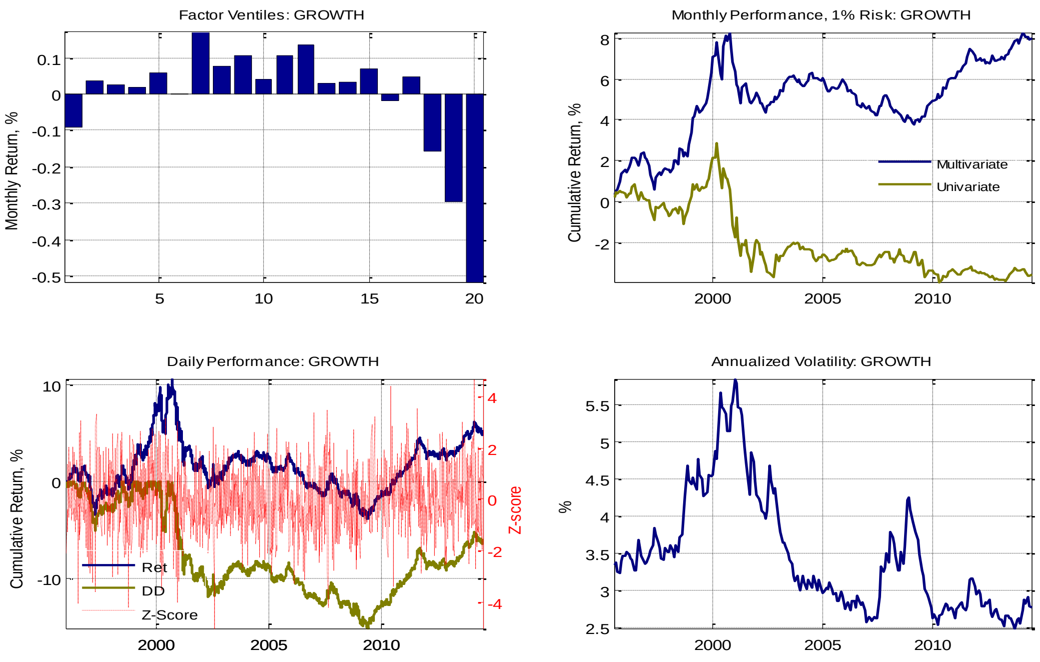
A positive exposure indicates a high historical/predicted growth.

A negative exposure indicates a low historical/predicted growth.

Descriptors

- Long term analyst-predicted earnings per share growth
- Historical earnings per share growth rate
- Historical sales per share growth rate

Figure D.5: Factor ventiles, monthly performance, daily performance and predicted annualized volatility for the Growth factor



Leverage

Description Captures common variation in stock returns due to differences in the level of company leverage.

Motivation We view highly-leveraged firms as riskier because they cannot change their production easily. In particular, they cannot scale down production during recessionary periods without increasing the probability of default. Highly-leveraged firms are saddled with too much capital during periods of low productivity and, relative to the low-leveraged firms, are more sensitive to interest rate shocks. [18]

Type Fundamental

Start Date 30 June 1995

Frequency Daily

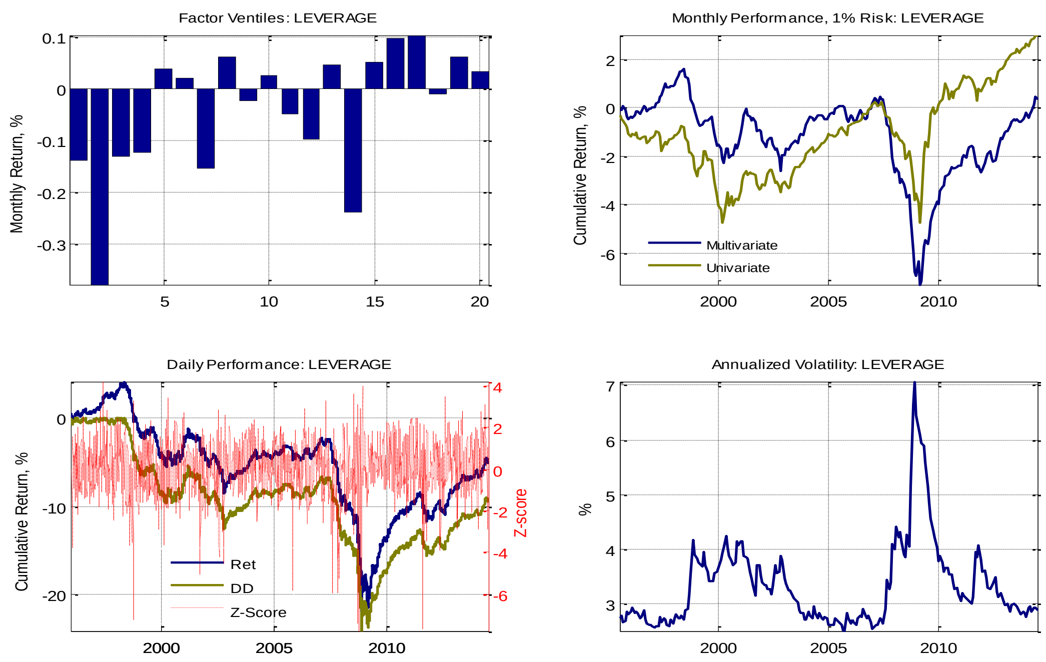
Exposure Interpretation A positive exposure indicates a high leverage.

A negative exposure indicates a low leverage.

Descriptors

- Market leverage
- Book leverage
- Debt-to-assets ratio

Figure D.6: Factor ventiles, monthly performance, daily performance and predicted annualized volatility for the Leverage factor



[18] For further reading, refer to: Bhandari, 1988. Debt/Equity Ratio and Expected Common Stock Returns: Empirical Evidence, Journal of Fianance

Liquidity

Description Captures common variations in stock returns due to the amount of relative trading and differences in the impact of trading on stock returns.

Motivation The ability of an investor to convert stock holdings into cash determines an illiquidity premium, that is, excess returns that investors require for holding difficult-to-sell illiquid stocks. The risk of holding illiquid stocks is that an investor may not be able to sell her holdings without incurring significant losses when she needs to raise cash. While volatility of highly-liquid stocks is primarily driven by changes in company fundamentals, volatility of illiquid stocks may be driven by company fundamentals as well as the investor's needs to raise cash by liquidating illiquid positions. [19]

Start Date 30 June 1995

Frequency Daily

Exposure Interpretation A positive exposure indicates a high liquidity.

A negative exposure indicates a low liquidity.

Descriptors

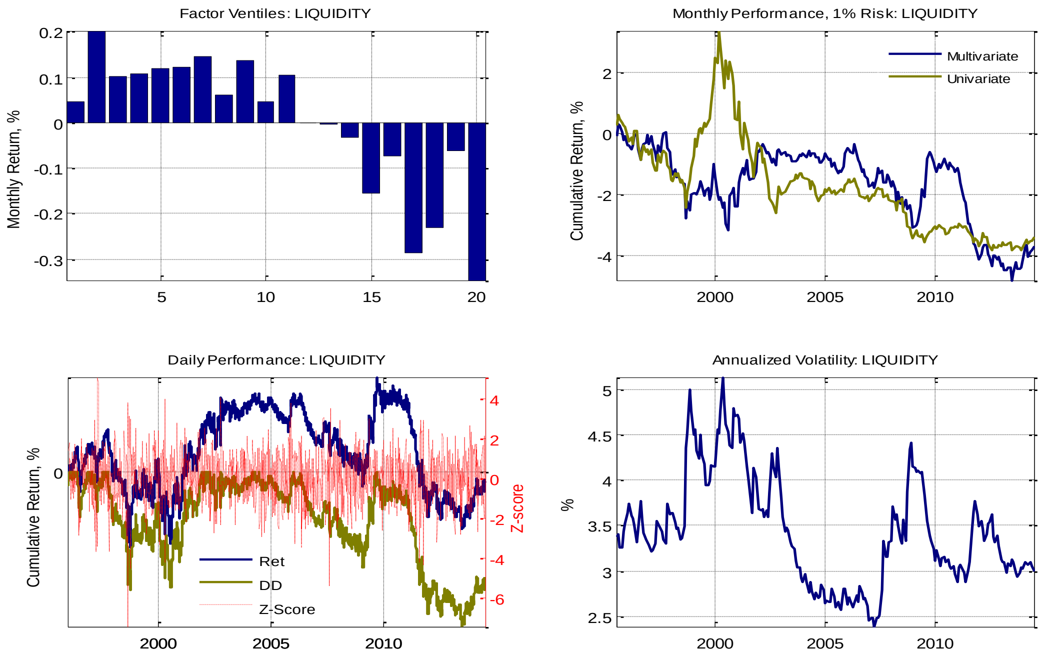
- Monthly share turnover
- Quarterly share turnover
- Annual share turnover
- Modified Amihud illiquidity measure
- Pastor-Stambaugh illiquidity measure

[19] For further reading, refer to:

Amihud, 2002. Illiquidity and stock returns: Cross-section and time series effects. Journal of Financial Markets 5, 31-56

Pastor and Stambaugh, 2003. Liquidity risk and expected stock returns. Journal of Political Economy, 111(3), 642-685

Figure D.7: Factor ventiles, monthly performance, daily performance and predicted annualized volatility for the Liquidity factor



Long-Term Reversal

Description Explains common variation in returns related to a long-term (five years ex. recent thirteen months) stock price behavior.

This factor is based on Systematic Equity Strategies.

Motivation The early evidence for long-term reversal phenomena in stock returns goes back to De Bondt and Thaler (1985). Our own research illustrates that actively managed US mutual funds have significant exposure to this factor. [20]

Start Date 30 June 1995

Frequency Daily

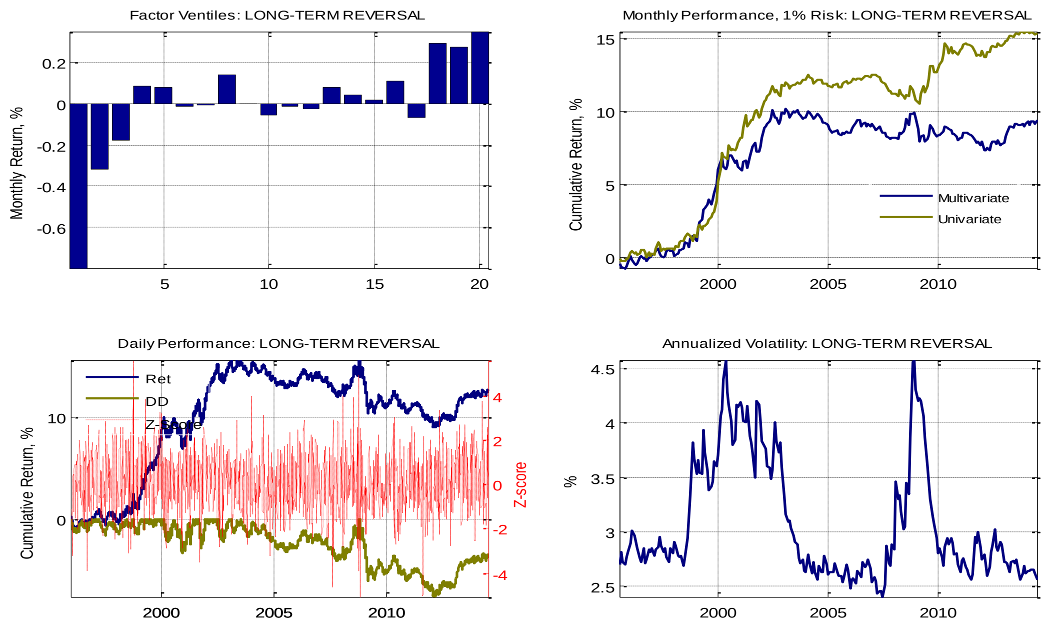
Exposure Interpretation A positive exposure indicates a low long-term momentum (poor long-term performance ex. recent performance).

A negative exposure indicates a high long-term momentum (good long-term performance ex. recent performance).

Descriptors

- Long-term relative strength
- Long-term historical alpha

Figure D.8: Factor ventiles, monthly performance, daily performance and predicted annualized volatility for the Long-Term Reversal factor



[20] For further reading, refer to:

De Bondt and Thaler, 1985. Does the Stock Market Overreact? Journal of Finance, Volume 40, Issue 3

Management Quality

Description A combination of asset, investment, net issuance growth measures that captures common variation in stock returns of companies experiencing rapid growth or contraction of assets.

This factor is based on Systematic Equity Strategies.

Motivation There is evidence that companies with corporate events associated with asset expansion (that is, acquisitions, public equity offering, etc.) tend to experience lower returns than companies with corporate events associated with asset contraction (that is, spinoff, share repurchase, etc.). These findings suggest that investors may have a bias in the capitalization of company asset investments and disinvestment decisions. Related to these findings, there is evidence that management tends to issue or repurchase shares when the company is overvalued or undervalued, and investors underreact to that information. Also, high capital expenditures and asset growth are associated with the phenomenon of "empire building" that has a negative impact on company future performance. [21]

Start Date 30 June 1995

Frequency Daily

Exposure Interpretation A positive exposure indicates low asset and capital expenditure growth and low net equity issuance.

A negative exposure indicates high asset and capital expenditure growth and high net equity issuance.

Descriptors

- Asset growth
- Issuance growth
- Capital expenditure growth
- Capital expenditure

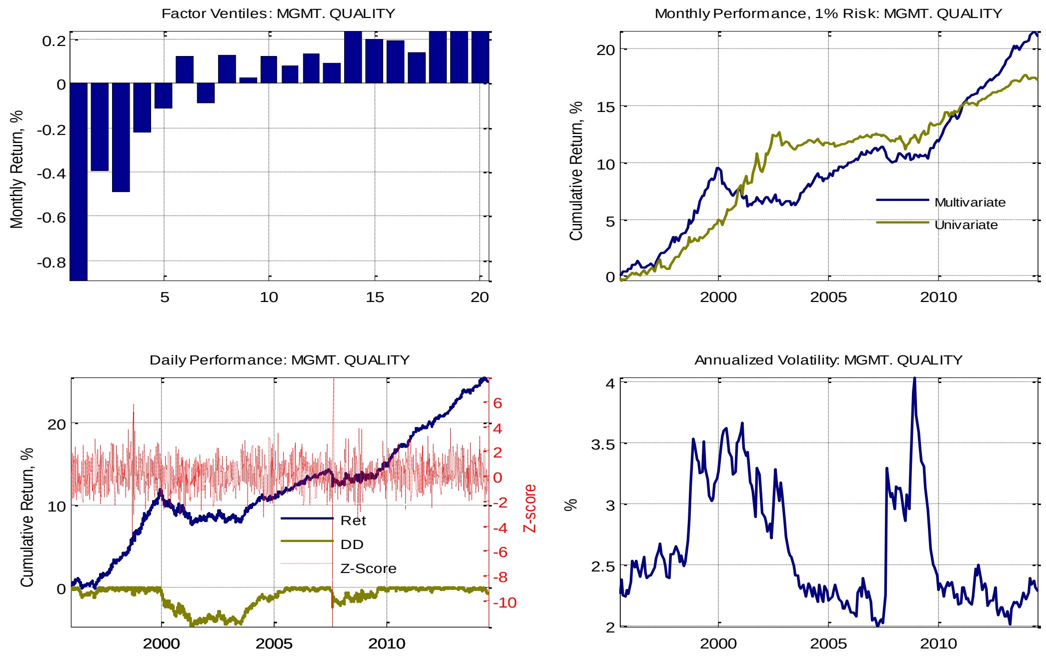
[21] For further reading, refer to:

Cooper, M. J., H. Gulen, M. J. Schill, 2008, Asset Growth and the Cross-Section of Stock Returns, The Journal of Finance

Abarbanell J. S., B. J. Bushee, 1998, Abnormal Returns to a Fundamental Analysis Strategy, The Accounting Review

Pontiff J., A. Woodgate, 2008, Share Issuance and Cross-sectional Returns, The Journal of Finance

Figure D.9: Factor ventiles, monthly performance, daily performance and predicted annualized volatility for the Management Quality factor



Mid Capitalization

Description Captures deviations from linearity in the relationship between returns and the logarithm of market capitalization (Size factor). This factor explains differences in risk and return for mid-capitalization stocks from small-cap and large-cap stocks.

Motivation A closer look at the relationship between company stock returns or risk and the company log of market capitalization reveals deviations from a linear relationship. In particular, a change in expected returns and risk tends to increase more rapidly than implied by a linear model as we move from large-capitalization companies to small-capitalization companies. To capture this non-linear relationship in a linear factor framework, we introduced the Mid Capitalization factor.

Start Date 30 June 1995

Frequency Daily

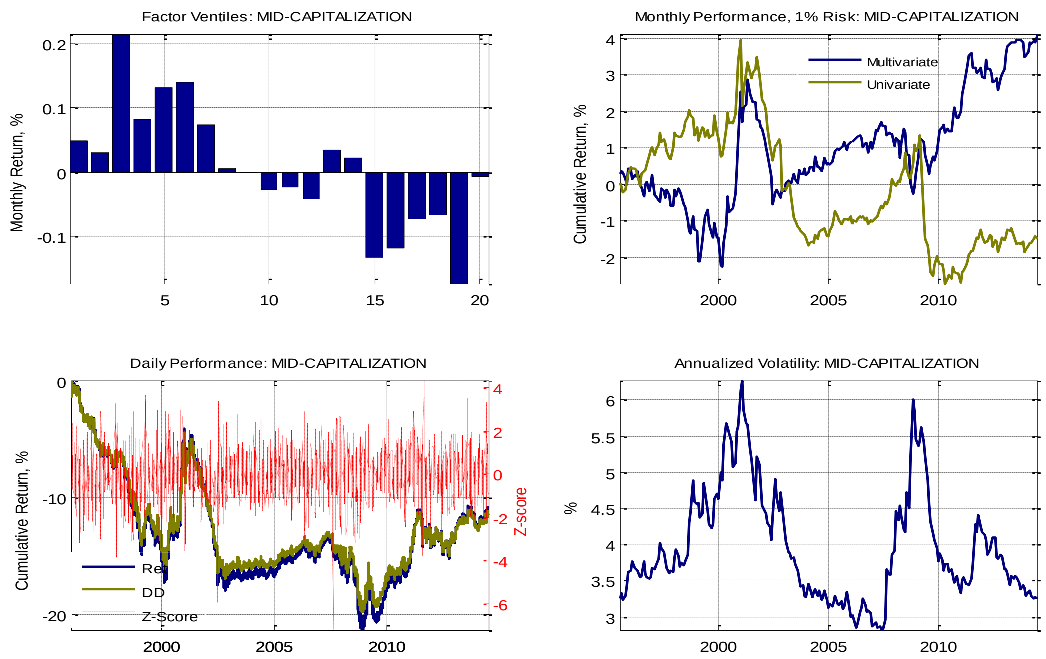
Exposure Interpretation A positive exposure indicates mid capitalization.

A negative exposure indicates the large and small capitalization.

Descriptors

- Cube of Size exposure

Figure D.10: Factor ventiles, monthly performance, daily performance and predicted annualized volatility for the Mid Capitalization factor



Momentum

Description Explains common variation in stock returns related to recent (twelve months) stock price behavior.

This factor is based on Systematic Equity Strategies.

Motivation The importance of the Momentum factor in explaining stock return differences is well established in academia. A variant of the Momentum factor, called Success, was introduced in a previous Barra US equity model developed in the 1980s. The Momentum factor is one of the factors often added to the popular Fama-French Three-Factor Model. The Momentum factor phenomenon spurred a number of often-opposing models trying to explain common variation in stock returns and risk.

Start Date 30 June 1995

Frequency Daily

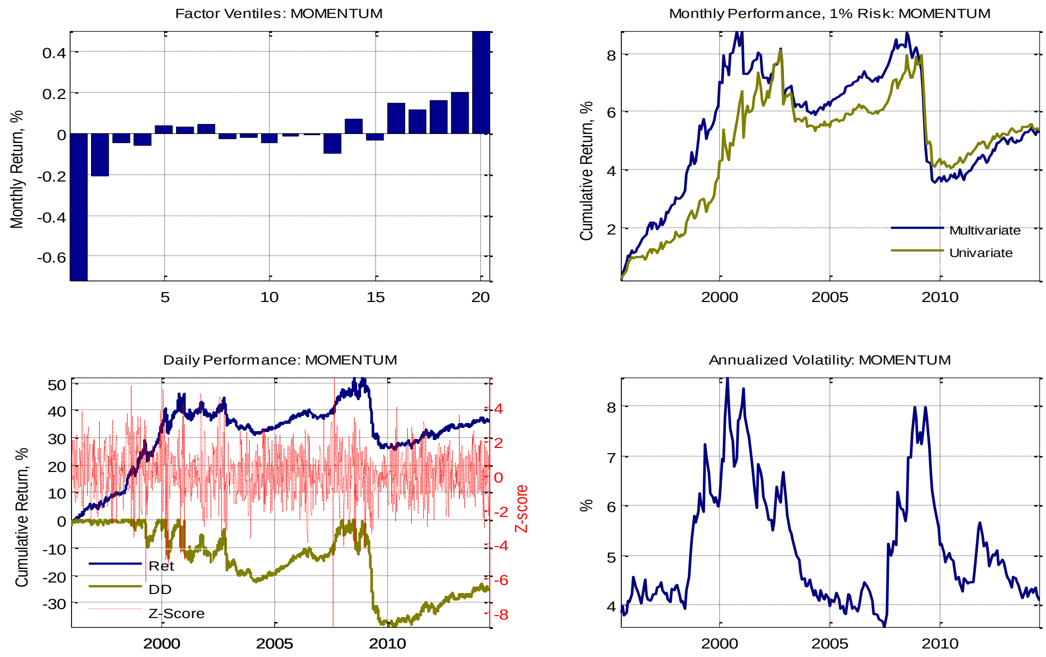
Exposure Interpretation A positive exposure indicates a high medium-term momentum (good recent performance).

A negative exposure indicates a low medium-term momentum (poor recent performance).

Descriptors

- Relative strength

Figure D.11: Factor ventiles, monthly performance, daily performance and predicted annualized volatility for the Momentum factor



Profitability

Description A combination of profitability measures that characterizes efficiency of a firm's operations and total activities.

This factor is based on Systematic Equity Strategies.

Motivation From an academic point of view, the importance of profitability may be demonstrated by using the dividend discount model (DDM). Under some simplifying assumptions, the dividend discount model implies that higher expected future earnings imply a higher expected stock return. Following recent academic research, we use profitability measures as a proxy for future expected earnings. [22]

Start Date 30 June 1995

Frequency Daily

Exposure Interpretation A positive exposure indicates a high profitability and operating efficiency.

A negative exposure indicates a low profitability and operating efficiency.

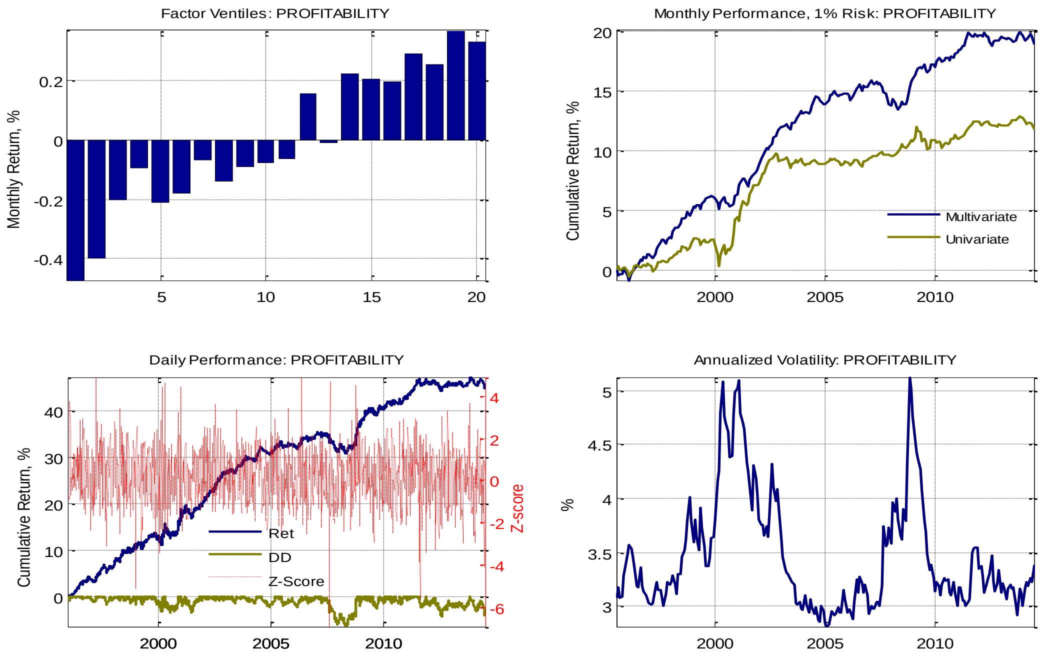
Descriptors

- Return on assets
- Return on equity
- Gross profitability
- Gross margin
- Asset turnover

[22] For further reading, refer to:

Novy-Marx, 2013. The other side of value: The gross profitability premium. Journal of Financial Economics 108(1), 1-28

Figure D.12: Factor ventiles, monthly performance, daily performance and predicted annualized volatility for the Profitability factor



Prospect

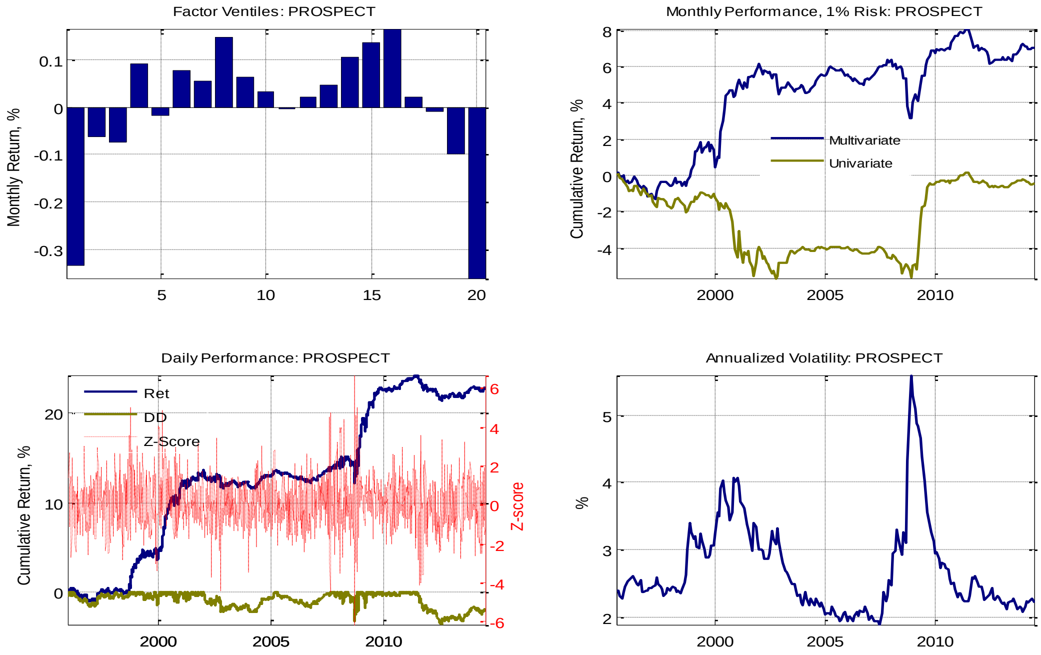
Description	Explains common variation in stock returns that have exhibited a lottery-like behavior identified through a combination of stock return skewness over a long horizon and drawdown in returns over the recent period. This factor is based on Systematic Equity Strategies.
Motivation	The Prospect factor is motivated by the cumulative prospect theory that implies that a security's own skewness may be prices. In particular, companies with positively-skewed returns may be overpriced and have low excess stock returns relative to stocks with negatively-skewed returns. [23] Stocks with positive skewness means its historical return distribution (5 year daily) have a longer right tail, exhibit strong lottery like behaviors, tend to be over-bought by investors, and experience a lower than average realized returns
Start Date	30 June 1995
Frequency	Daily
Exposure Interpretation	A positive exposure indicates a left skew in returns (large negative returns) and large drawdowns in recent performance. A negative exposure indicates a right skew in returns (large positive returns) and low drawdowns in recent performance.
Descriptors	• Skewness • Maximum drawdown

[23] For further reading, refer to:

Barberis and Huang, 2008. Stock as lotteries: The implications of probability weighting for security prices. AER, 98(5), 2066-2100

Boyer, Mitton, Vorkink, 2010. Expected idiosyncratic skewness. RFS, 23(1), 169-202

Figure D.13: Factor ventiles, monthly performance, daily performance and predicted annualized volatility for the Prospect factor



Residual Volatility

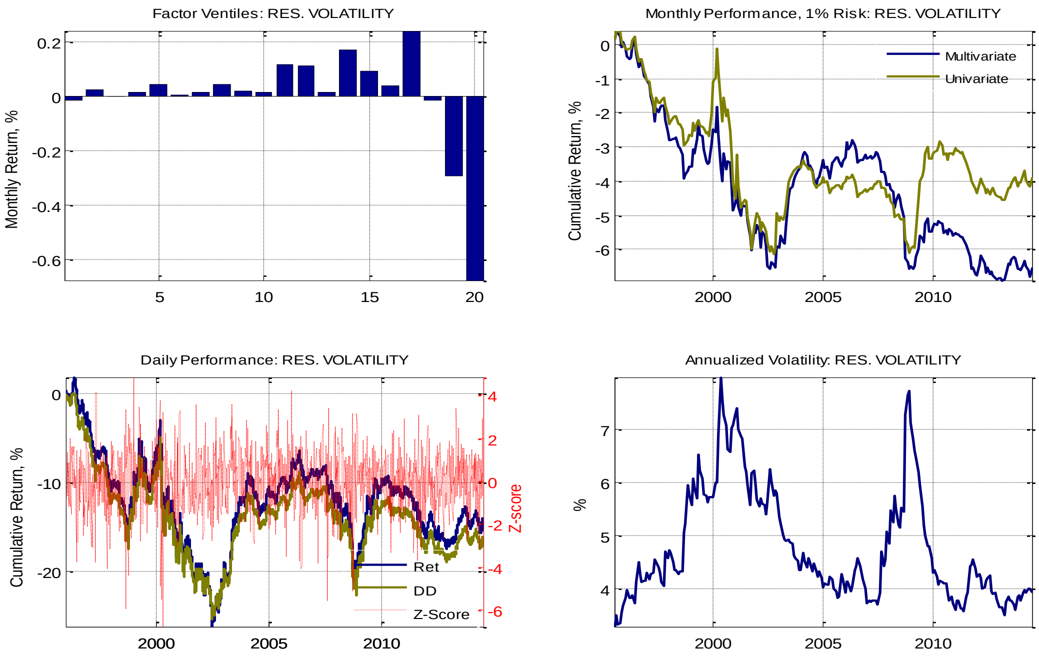
Description	Captures relative volatility in stock returns that is not explained by differences in stock sensitivities to market returns (Country and Beta factors).
Motivation	There is a persuasive evidence that stocks with high residual (idiosyncratic) volatility relative to the Capital Asset Pricing Model (CAPM) or Fama-French Three-Factor Model have unexpectedly low average returns. We include the Residual Volatility factor to capture this pervasive phenomenon. [24]
Start Date	30 June 1995
Frequency	Daily
Exposure Interpretation	A positive exposure indicates a high residual volatility. A negative exposure indicates a low residual volatility.
Descriptors	• Historical sigma • Volatility implied by call options • Volatility implied by put options

[24] For further reading, refer to:

Ang, Hodrick, Xing, Zhang, 2006. The cross-section of volatility and expected returns. JF 61, 259-299

Bali, Cakici, 2008. Idiosyncratic volatility and the cross section of expected returns. JFQA 43(1), 29-58

Figure D.14: Factor ventiles, monthly performance, daily performance and predicted annualized volatility for the Residual Volatility factor



Size

Description Captures differences in stock returns and risk due to differences in of the market capitalization of companies.

Motivation The importance of the Size factor in predicting the cross-section of stock returns has a long history in Barra modelling and academic literature. Also, the Size factor is one of the factors in Fama-French Three-Factor Model. There is consensus that there are significant differences in the behavior of risk and returns of large-capitalization and small-capitalization companies. Historically, small-capitalization companies earned higher returns realizing a higher volatility.

Start Date 30 June 1995

Frequency Daily

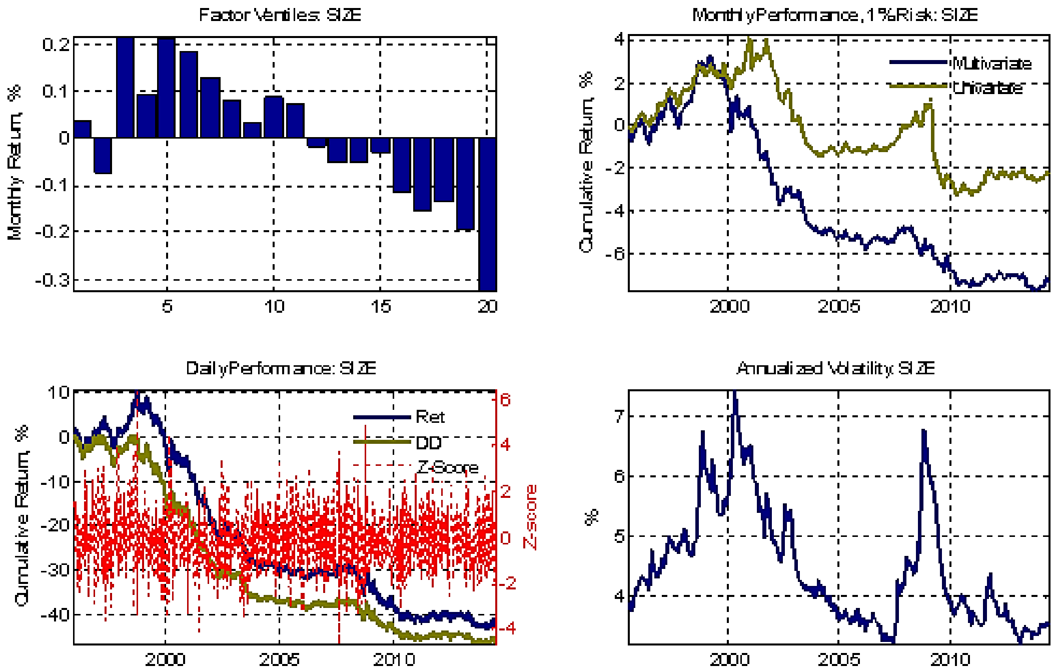
Exposure Interpretation A positive exposure indicates large capitalization.

A negative exposure indicates small capitalization.

Descriptors

- Logarithm of market capitalization

Figure D.15: Factor ventiles, monthly performance, daily performance and predicted annualized volatility for the Size factor



Value

Description	Captures the extent to which a company is overpriced or underpriced using a combination of several relative valuation metrics and one structural valuation factor. This factor is based on Systematic Equity Strategies.
Motivation	The Value factor is a combination of relative valuation multiples popular in the finance industry. The role of the Value factor in explaining stock return and risk differences has also a long history in Barra modelling and academic literature. Barra had the Value factor in its first US equity model developed in the 1970s. Also, the Value factor is one of the factors in the Fama-French Three-Factor Model. Historically, high-value companies earned higher returns and experienced higher risk relative to low-value companies.[25]
Start Date	30 June 1995
Frequency	Daily
Exposure Interpretation	A positive exposure indicates a high value (“cheap”). A negative exposure indicates a low value (“expensive”).
Descriptors	• Book-to-price ratio • Sales-to-price ratio • Cash-flow to price ratio • Structural valuation factor

[25] For further reading, refer to:

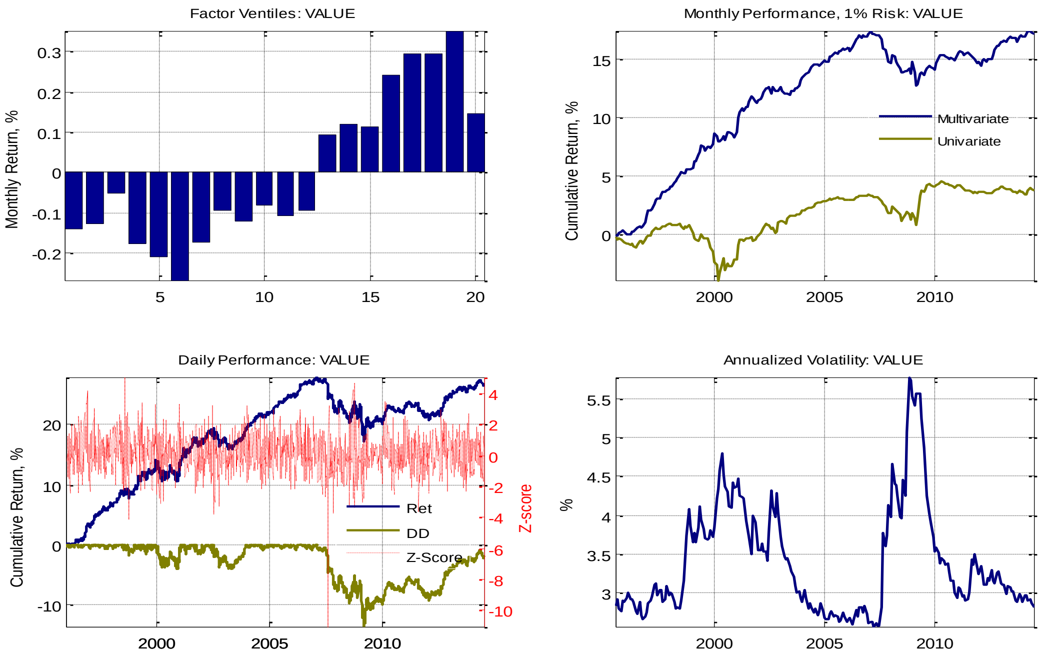
Rosenberg B., K. Reid, R. Lanstein, 1985, Persuasive Evidence of Market Inefficiency

Barbee, W. C., Mukherji, S., Raines, G. A., 1996, Do the sales-to-price and debt-equity ratios explain stock returns better than the book-to-market value of equity ratio and firm size? Financial Analyst Journal

Desai H., Rajgopal S., Venkatachalam M., Value-Glamour and Accruals Mispricing: One anomaly or Two?

Lyle and Wang, 2013. The Cross section of expected holding period returns and their dynamics: A present value approach. JFE, forthcoming

Figure D.16: Factor ventiles, monthly performance, daily performance and predicted annualized volatility for the Value factor



Appendix E: Descriptors

The 16 style factors of the model comprise a total of 48 descriptors. This document defines these descriptors in the style factors. The descriptors are listed under the style factors to which they belong. The factors are listed alphabetically.

Style: **Beta**

Components: HBETA <u>Historical Beta (β)</u>

Computed as the weighted average of (i) the slope coefficient in a time-series regression of excess *stock* return, $\boldsymbol{r_t - r_{ft}}$, against the cap-weighted excess returns of the estimation universe $\boldsymbol{R_t}$,

$$r_t - r_{ft} = \alpha + \beta_s R_t + e_t$$

and (ii) the slope coefficient in a time-series regression of excess cap-weighted industry returns, $\boldsymbol{r_{ind,t} - r_{ft}}$, against the cap-weighted excess returns of the estimation universe $\boldsymbol{R_t}$,

$$r_{ind,t} - r_{ft} = \alpha + \beta_{ind} R_t + u_t$$

The regression coefficients for stock and industry beta regressions are estimated over the trailing 252 trading days of returns with a half-life of 63 trading days.

The Beta β is computed as,

$$\beta = (1 - w)\beta_s + w\beta_{ind}$$

$$w = \frac{\sigma(\beta_s)}{\sigma(\beta_s) + \tau\sigma(\beta_{ind})}$$

where the variance terms $\sigma(\beta_s)$and $\sigma(\beta_{ind})$come from time-series regressions, τ is a calibrated parameter.

Style: **Dividend Yield**

Components: DTOP <u>Historical Dividend-to-Price Ratio</u>

Computed by dividing the trailing 12-month dividend per share by the current price.

DPIBS <u>Analyst-Predicted Dividend-to-Price Ratio</u>

Computed by dividing the 12-month forward-looking dividend per share (DPS) by the current price. Forward-looking DPS are defined as a weighted average between the average analyst-predicted DPS for the current and next fiscal years.

Style: **Earnings Quality**

Components:

ABS — Accruals using Balance Sheet Statement

Computed as the change in current assets net of cash, less change in current liabilities net of short-term debt, less depreciation, and standardized by total assets.

$$ABS = [(\Delta CA - \Delta Cash) - (\Delta CL - \Delta STD) - Dep]/TA$$

where **CA** – Current Assets, **CL** – Current Liabilities, **STD** – Short-Term Debt, **TA** – Total Assets, **Dep** – Depreciation.

Note: The factor goes long companies with *low* accruals.

ACF — Accruals using Cash-Flow Statement

Computed as the change in accounts receivable and inventories, less changes in accounts payable, accrued taxes, and other current assets/liabilities, less depreciation, and standardized by total assets.

$$ACF = (\Delta AR + \Delta Inv - \Delta AP - \Delta AT - \Delta OC - Dep)/TA$$

where **AR** – accounts receivable, **Inv** – inventories, **AP** – accounts payable, **AT** – accrued taxes, **OC** – other current assets and liabilities, **Dep** – depreciation, **TA** – total assets.

Note: The factor goes long companies with *low* accruals.

VSAL — Variability in Sales

Computed as the standard deviation of company reported quarterly sales over the last five fiscal years.

Note: The factor goes long companies with *low* variability of sales.

VERN — Variability in Earnings

Computed as the standard deviation of company reported quarterly earnings standardized by sales over the last five fiscal years.

Note: The factor goes long companies with *low* variability of earnings.

VFLO — Variability in Cash-Flows

Computed as the standard deviation of company quarterly cash flows standardized by sales over the last five fiscal years.

Note: The factor goes long companies with *low* variability of cash flows.

SPIBS — Variability of Analyst EPS Estimates to Price

Computed by dividing the standard deviation of 12-month forward-looking earnings per share (EPS) estimates by the current price.
Note: The factor goes long companies with *low* variability of analyst EPS estimates to price.

Style: **Earnings Yield**

Components:

EM — Enterprise Multiple (EBITDA to EV)

Computed by dividing the trailing 12-month earnings before interest, taxes, depreciation and amortization (EBITDA) to enterprise value (EV).

ETOP — Trailing Earnings-to-Price Ratio

Computed by dividing the trailing 12-month earnings by the current market capitalization. Trailing earnings are defined as the last reported fiscal-year earnings plus the difference between the current interim figure and the comparative interim figure from the previous year.

EPIBS — Analyst-Predicted Earnings-to-Price Ratio

Computed by dividing the 12-month forward-looking earnings by the current market capitalization. Forward-looking earnings are defined as a weighted average between the average analyst-predicted earnings for the current and next fiscal years.

Style: **Growth**

Components:

EGIBS — Long Term Analyst-Predicted Earnings per Share Growth

Long-term (3-5 years) earnings growth forecasted by analysts.

EGRO — Historical Earnings per Share Growth Rate

Annual reported earnings per share are regressed against time over the past five fiscal years. The slope coefficient is then divided by the average annual earnings per share to obtain the earnings growth.

SGRO — Historical Sales per Share Growth Rate

Annual reported sales per share are regressed against time over the past five fiscal years. The slope coefficient is then divided by the average annual earnings per share to obtain the earnings growth.

Style: **Leverage**

Components:

MLEV — Market Leverage

Computed as,

$$MLEV = \frac{ME + PE + LD}{ME}$$

where **ME** is the market value of common equity on the last trading day, **PE** is most recent book value of preferred equity, and **LD** is the most recent book value of long-term debt.

BLEV Book Leverage

Computed as,

$$BLEV = \frac{BE + PE + LD}{BE}$$

where **BE** is the most recent book value of common equity, **PE** is the most recent book value of preferred equity, and **LD** is the most recent book value of long-term debt.

DTOA Debt-to-Assets Ratio

Computed as,

$$DTOA = \frac{TD}{TA}$$

where **TD** is the book value of total debt (long-term debt and current liabilities) and **TA** is the most recent book value of total assets.

Style: **Liquidity**

Components: STOM Monthly Share Turnover

Computed as the log of the share turnover over the previous month,

$$STOM = ln\left(\frac{V}{S}\right)$$

where **V** is the trading volume for the month and **S** is the number of shares outstanding.

STOQ Quarterly Share Turnover

Let $STOM_t$ be the share turnover for month t. The quarterly share turnover is defined by,

$$STOQ = ln\left[\frac{1}{T}\sum_{\tau=1}^{T} \exp(STOM_\tau)\right]$$

where T = 3 months.

STOA Annual Share Turnover

Let $STOM_t$ be the share turnover for month t. The annual share turnover is defined by,

$$STOA = ln\left[\frac{1}{T}\sum_{\tau=1}^{T} \exp(STOM_\tau)\right]$$

where T = 12 months.

LIQMA

Modified Amihud Illiquidity Measure

Computed as,

$$LIQMA = \frac{1}{T}\sum_{t=1}^{T}\frac{|r_t|}{V_t/ME_t}$$

where $\boldsymbol{r_t}$ is the stock return at day t, $\boldsymbol{V_t}$ is the traded dollar volume, $\boldsymbol{ME_t}$ is the market value of common equity.

Note: the factor goes long companies with the low estimated price impact, i.e. liquid companies.

LIQPS

Pastor-Stambaugh Illiquidity Measure

The factor construction follows Pastor and Stambaugh (2003) paper.

Step 1. Estimate the liquidity measure for stock i in month t, $\boldsymbol{\gamma_{it}}$ using daily observations d=1,2,…,D from month t and using the following time-series regression:

$$r_{i,d+1,t}^{e} = \theta_{i,t} + \varphi_{i,t} r_{i,d,t} + \gamma_{it} sign\left(r_{i,d,t}^{e}\right) v_{i,d,t} + e_{i,d+1,t}$$

where $r_{i,d,t}$ is the stock return of stock i on day d in month t , $r_{i,d,t}^{e} \ = r_{i,d,t} - r_{m,d,t}$, $r_{m,d,t}$ is the return of cap-weighted estimation universe, $v_{i,d,t}$ is the dollar volume of stock i on day d in month t.

Step 2. Construct the aggregate measure of market liquidity and compute the innovations in aggregate market liquidity

$$\gamma_t = \frac{1}{N}\sum_{i=1}^{N}\gamma_{it}$$

$$\Delta\gamma_t = a + b\Delta\gamma_{t-1} + d\left(\frac{m_{t-1}}{m_1}\right)\gamma_{t-1} + L_t$$

where N is the number of stocks in the estimation universe, m_{t-1} is the total dollar value of the estimation universe stocks at the end of month t-1, m_1 is the total dollar value of the estimation universe stocks at the beginning of the model history. L_t is the measure of unexpected changes in the aggregate liquidity level.

Step 3. Estimate stock liquidity beta, $\beta_{i,L}$, using a time-series regression

$$u_{it} = c_i + \beta_{i,L} L_t + e$$

where $\boldsymbol{u_{it}}$ is the stock specific return from a three factor model with Size, Value, and Momentum factors.

Note: The factor goes long companies with low sensitivity to changes in aggregate liquidity, i.e. liquid companies.

Style: **Long-Term Reversal**

Components: LTRSTR <u>Long-Term Relative Strength</u>

The long term relative strength signal for day t is computed as the sum of excess log returns over the trailing T=1008 trading days (4 years),

$$RS(t) = \sum_{\tau=1}^{T} w_\tau [\ln(1 + r_{t-\tau-273}) - \ln(1 + r_{ft-\tau-273})]$$

where $\boldsymbol{r_t}$ is the return on day t, $\boldsymbol{r_{ft}}$ is the risk-free return, and $\boldsymbol{w_\tau}$ is an exponential weight with a half-life of 504 trading days.

LTHALPHA <u>Long-Term Historical Alpha</u>

Computed as the intercept coefficient in a time-series regression of excess monthly stock return, $\boldsymbol{r_t - r_{ft}}$ against the cap-weighted excess returns of the estimation universe $\boldsymbol{R_t}$,

$$r_t - r_{ft} = \alpha + \beta_s R_t + e_t$$

The regression coefficients are estimated over the trailing 48 monthly returns with 13 months lag.

Style: **Management Quality**

Components: AGRO <u>Asset Growth</u>

Annual reported company assets are regressed against time over the past five fiscal years. The slope coefficient is then divided by the average annual assets to obtain the asset growth.

Note: The factor goes long companies with *low* asset growth.

IGRO <u>Issuance Growth</u>

Annual reported company number of shares outstanding regressed against time over the past five fiscal years. The slope coefficient is then divided by the average annual number of shares outstanding.

Note: The factor goes long companies with *low* net issuance growth.

CXGRO <u>Capital Expenditure Growth</u>

Annual reported company capital expenditures are regressed against time over the past five fiscal years. The slope coefficient is then divided by the average annual capital expenditures to obtain the capital expenditures growth.

Note: The factor goes long companies with *low* capital expenditures growth.

CX Capital Expenditure

The most recent capital expenditures are scaled by the average of capital expenditures over the last five fiscal years.

Note: The factor goes long companies with *low* capital expenditures.

Style: **Mid Capitalization**

Components: SIZENL Cube of the Size Exposure

First, the standardized Size exposure (log of market capitalization) is cubed. The resulting factor is then orthogonalized to the Size factor on a regression-weighted basis. Finally, the factor is winsorized and standardized.

Style: **Momentum**

Components: RSTR Relative Strength

The relative strength signal for day t is computed as the sum of excess log returns over the trailing T=504 trading days,

$$RS(t) = \sum_{\tau=1}^{T} w_\tau [\ln(1 + r_{t-\tau-21}) - \ln(1 + r_{ft-\tau-21})]$$

where $\boldsymbol{r_t}$ is the return on day t, $\boldsymbol{r_{ft}}$ is the risk-free return, and $\boldsymbol{w_\tau}$ is an exponential weight with a half-life of 126 trading days.

Style: **Profitability**

Components: ROA Return on Assets

Computed as,

$$ROA = \frac{Earnings}{TA}$$

where **Earnings** are most recently reported company earnings (operating income before depreciation, interest and taxes), **TA** is the most recently reported company total assets.

ROE Return on Equity

Computed as,

$$ROE = \frac{Earnings\ for\ Common}{BE}$$

where **Earnings for Common** are the most recently available earnings for common equity, **BE** is the most recently reported book value of common equity.

GP

Gross Profitability

Computed as,

$$GP = \frac{Sales - COGS}{TA}$$

where, **GP** is gross profitability, **SALES** is the most recently reported company sales, COGS is the most recently reported cost of goods sold, **TA** is the most recently reported company total assets.

GM

Gross Margin

Computed as,

$$GM = \frac{Sales - COGS}{Sales}$$

where **GM** is gross margin, **SALES** is the most recently reported company sales, **COGS** is the most recently reported cost of goods sold.

ATO

Asset Turnover

Computed as,

$$ATO = \frac{Sales}{TA}$$

where **SALES** is the most recently reported company sales, **TA** is the most recently reported company total assets.

Style: **Prospect**

Components: SKEW

Skewness

Computed as the skewness of monthly stock specific returns from a model with industries, Size, Value, Momentum, Leverage, and Liquidity. The period used in estimation is five years.

MAD

Maximum Drawdown

Computed as the maximum price drawdown from peak to trough that occurred within a window of last 300 days. This measure represents the maximum investors could lose if they purchased and sold a security within the window.

Style: **Residual Volatility**

Components: HSIGMA

Historical Sigma (σ)

Computed as the volatility of CAPM residual returns,

$$\sigma = std(e_t)$$

The volatility is estimated over the trailing 252 trading days of returns with a half-life of 63 trading days. The Residual volatility factor is orthogonalized to Beta and Size factors to reduce collinearity.

IVOLC1 Volatility Implied by One-Month Call Option

Implied volatility corresponding to one month at-the-money call option.

IVOLC3 Volatility Implied by Three-Month Call Option

Implied volatility corresponding to three month at-the-money call option.

IVOLP1 Volatility Implied by One-Month Put Optio

Implied volatility corresponding to one month at-the-money put option.

IVOLP3 Volatility Implied by Three-Month Put Option

Implied volatility corresponding to three month at-the-money put option.

Style: **Size**

Components: LNCAP Logarithm of Market Capitalization

Computed as the natural logarithm of the market capitalization of the firm.

Style: **Value**

Components: BTOP Book-to-Price Ratio

Last reported book value of common equity divided by current market capitalization.

STOP Sales-to-Price Ratio

Last reported company sales divided by current market capitalization.

CFTOP Cash-Flow to Price Ratio

Most recently available company cash flows divided by current market capitalization.

SVAL Structural Valuation Factor

The structural valuation factor is constructed following the structural model in Lyle and Wang (2013). They assume that:

- Log quarterly returns and expected log quarterly ROE follow AR(1) process, with AR(1) parameters k and ω respectively, and a common long-run mean μ
- The model structural parameters driving stock return dynamics are industry specific and may be time-varying.

The model estimation consists of a several steps.

Step 1, estimate the following industry wide pooled quarterly regression:

$$r_{it} = \beta_0 + \beta_1 bm_{it} + \beta_2 roe_{it} + e_{it}$$

where $\boldsymbol{r_{it}}$ is the quarterly return of stock i, $bm = \ln\left(\frac{BE}{ME}\right)$, **BE** is the stock book value of common equity, **ME** is the stock market value of common equity, $roe = ln\left(1 + \frac{E}{BE}\right)$, where **E** is the company earnings. The estimated regression parameters are industry specific. We use an extending window in the pooled regression.

Step 2, compute the expected stock returns as,

$$E[\log(r_{it+1})] = \mu_i + \frac{1}{\alpha_2}[bm_{it} + \alpha_1(roe_{it} - \mu_i)]$$

where $\beta_1 = \frac{1}{\alpha_2}, \beta_2 = \omega_i \frac{\alpha_2}{\alpha_1}, \mu = \frac{\beta_0}{(1-\beta_1)}, \omega = \frac{\beta_2/\beta_1}{1+(\beta_2/\beta_1)k_1}, k_1 = 0.98$

Notice that for stock i, we use β regression parameters estimated for the industry to which stock i belongs. We transform the expected value for log stock returns into the expected value for stock returns to obtain our descriptor value. Notice that this is a simplified formula of Lyle and Wang (2013) general formula to compute the average expected returns over time T, $\frac{1}{T}\sum_{j=1}^{T} E\left[\log\left(r_{it+j}\right)\right]$.

Appendix F: Decomposing RMS Returns

We decompose excess stock returns r_n into a systematic component, due to factors, and a stock-specific component u_n. The factor returns f_k are estimated each period by cross-sectional regression

$$r_n = \sum_k X_{nk} f_k + u_n \ , \quad \text{(F1)}$$

where X_{nk} is the exposure of stock n to factor k. The specific returns are assumed to be uncorrelated with one another as well as to the other factors.

The total *R*-squared of a regression measures the cross-sectional variation explained by the factors,

$$R_T^2 = 1 - \frac{\sum_n v_n u_n^2}{\sum_n v_n r_n^2} \ , \quad \text{(F2)}$$

where v_n is the regression weight of stock n (proportional to square-root of market capitalization). The root mean square (RMS) return, computed as

$$RMS = \sqrt{\sum_n v_n r_n^2} \ , \quad \text{(F3)}$$

measures the cross-sectional dispersion from zero return. As described by Menchero and Morozov (2011), the RMS return can be exactly decomposed into the return sources of Equation B1 using a cross-sectional version of the *x-sigma-rho* formula,

$$RMS = \sum_k f_k \sigma(X_k) \rho(X_k, r) + \sigma(u) \rho(u, r) \ , \quad \text{(F4)}$$

where $\sigma(X_k)$ is the RMS dispersion of factor k, and $\rho(X_k, r)$ is the cross-sectional correlation between factor k and the asset returns. The last term in Equation E4 represents the contribution to RMS coming from stock-specific sources.

Appendix G: Review of Bias Statistics

G.1. Single-Window Bias Statistics

A commonly used measure for a risk model's accuracy is the bias statistic. Conceptually, the bias statistic represents the ratio of realized risk to forecast risk.

Let R_{nt} be the return to portfolio n over period t, and let σ_{nt} be the beginning-of-period volatility forecast. Assuming perfect forecasts, the *standardized* return,

$$b_{nt} = \frac{R_{nt}}{\sigma_{nt}}, \tag{G1}$$

has an expected standard deviation of 1. The bias statistic for portfolio n is the *realized* standard deviation of standardized returns,

$$B_n = \sqrt{\frac{1}{T-1}\sum_{t=1}^{T}\left(b_{nt} - \bar{b}_n\right)^2}, \tag{G2}$$

where T is the number of periods in the observation window.

Assuming normally distributed returns and perfect risk forecasts, for sufficiently large T the bias statistic B_n is approximately normally distributed about 1, and roughly 95 percent of the observations fall within the confidence interval,

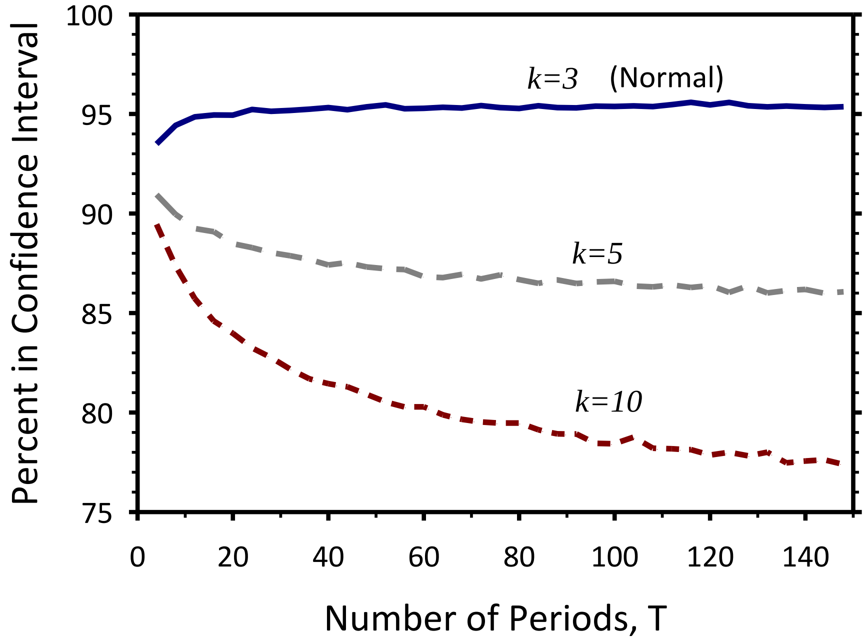
$$B_n \in \left[1-\sqrt{2/T}, 1+\sqrt{2/T}\right]. \tag{G3}$$

If B_n falls outside this interval, we reject the null hypothesis that the risk forecast is accurate.

If returns are not normally distributed, however, then fewer than 95 percent of the observations will fall within the confidence interval, even for perfect risk forecasts. In **Figure F.1**, we show simulated results for the percentage of observations actually falling within this interval, plotted versus observation window length T, for several values of kurtosis k.

For the normal case (kurtosis $k = 3$), except for the smallest values of T, the confidence interval indeed captures about 95 percent of the observations. As the kurtosis increases, however, the percentage falling within the interval drops significantly. For instance, at a kurtosis level of 5, only 86 percent of bias statistics fall inside the confidence interval for an observation window of 120 periods.

Figure G.1: Percent of observations falling within the confidence interval $1 \pm \sqrt{2/T}$, where T is the number of periods in the observation window. Results were simulated using a normal distribution $k=3$, and using a *t*-distribution with kurtosis values $k=5$ and $k=10$.



The standard deviations were equal to 1 in all cases. For the normal distribution, the percentage of observations inside the confidence interval quickly approaches 95 percent. As kurtosis is increased, however, the proportion within the confidence interval declines considerably.

G.2. Rolling-Window Bias Statistics

The purpose of bias-statistic testing is to assess the accuracy of risk forecasts, typically over a long sample period. One possibility is to select the entire sample period as a single window, and to compute the bias statistic as in Equation F2. This would be a good approach if financial data were stationary, as sampling error is reduced by increasing the length of the window. In reality, however, financial data are not stationary. It is possible to significantly overpredict risk for some years, and underpredict it for others, while ending up with a bias statistic close to 1.

Often, a more relevant question is to study the accuracy of risk forecasts over a window of k observations. For this purpose, we define the rolling window bias statistic for portfolio n,

$$B_n^\tau = \sqrt{\frac{1}{k}\sum_{t=\tau-k+1}^{\tau}\left(b_{nt} - \bar{b}_n\right)^2}, \tag{G4}$$

where τ denotes the last observation of the window. The windows are rolled forward one observation at a time until reaching the end of the sample period. If T is the number of observations in the sample period, then each portfolio will have $T-k+1$ (overlapping) k-observation windows.

It is useful to consider, for a collection of N portfolios, the mean of the rolling window bias statistics,

$$\bar{B}^\tau = \frac{1}{N}\sum_n B_n^\tau . \tag{G5}$$

We also define $B^{\tau}(5\%)$ and $B^{\tau}(95\%)$ to be the 5-percentile and 95-percentile values for the rolling window bias statistics at a given point in time.

G.3. Q-Statistic

The Q-statistic is defined as $Q_{nt} = b_{nt}^2 - \ln b_{nt}^2$, where b_{nt} is a standardized return introduced in (G1). The Q-statistic penalizes both under and over forecast and is not prone to "error cancellation" when averaged across time and/or test portfolios. For averaging, we define the mean of Q-statistic as follows:

$$\overline{Q} = \sum_{n=1}^{N} \sum_{t=1}^{T} Q_{nt} \tag{G6}$$

where N is a number of portfolios and T is a sample size. Further information on Q-statistic can be found in Patton (2011).

Appendix H: Historical Beta Estimation

To make estimation of stock betas more robust, we estimate stock betas with respect to the model estimation universe using (i) Bayesian shrinkage of stock beta estimates to the estimates of industry betas with respect to market (ii) shrinking Bayesian estimates of beta to the value of one (Vasicek shrinkage). The amount of Bayesian shrinkage of individual stock betas to the industry betas depends on how accurately we estimate stock betas relative to the industry betas. If the individual stock betas are estimated accurately, the amount of shrinkage to the industry beta is minimal. The amount of shrinkage of Bayesian estimates of beta to one depends on the degree of cross-sectional dispersion of betas. For periods when the cross-sectional dispersion of betas is high, the amount of shrinkage to one is minimal.

Technically, estimation of betas is done in four steps:

__Step 1:__ Estimate the beta of each industry with respect to the market, the so-called industry betas. This is done by running a time-series regression of industry returns on the market return using a 252-day estimation window and 63-day half-life:

$$r_{industry,i,t} = \alpha + \beta_{industry,i,t} r_{mt} + e_{industry,i,t} \tag{H1}$$

where $r_{industry,i,t}$ represents the industry return in period t, r_{mt} represents the market return, and $\beta_{industry,i,t}$ is the industry beta estimate at period t. For each industry beta, we compute the standard error around the estimate, $SE(\beta_{industry,i,t})$, which gives us an indication of how accurately we estimate industry betas.

__Step 2:__ Estimate the beta of each individual stock with respect to the market. Similar to the estimation of industry betas, we use a time series regression with a 252-day estimation window and a 63-day half-life:

$$r_{i,t} = \alpha + \beta_{i,t} r_{mt} + e_{ry,i,t} \tag{H2}$$

where $r_{i,t}$ represents stock i return in period t and $\beta_{i,t}$ is the individual stock beta estimate at period t. For each stock beta, we compute the standard error around the estimate, $SE(\beta_{i,t})$.

__Step 3:__ Compute the Bayesian estimates of betas by shrinking the estimates of individual stock betas from Step 2 with estimates of industry betas from Step 1. This is done using a Bayesian updating (shrinkage) formula:

$$\beta_{i,Bayes,t} = \left(\frac{1}{SE(\beta_{i,t})} + \frac{1}{\tau SE(\beta_{industry,i,t})}\right)^{-1} \left(\frac{\beta_{i,t}}{SE(\beta_{i,t})} + \frac{\beta_{industry,i,t}}{\tau SE(\beta_{industry,i,t})}\right) \tag{H3}$$

where τ is the calibrated parameter for scaling up the standard error estimates on industry betas. The choice of τ determines the degree of shrinkage. The higher the value of τ, the smaller is the amount of shrinkage to industry beta.

Looking at the formula, one can see that the Bayesian estimate of beta is a weighted average of the individual stock beta and the industry beta. The amount of shrinkage of individual stock betas to industry betas (or the weight that we put on the industry beta) depends on how large the stock beta standard error, $SE(\beta_{i,t})$, is with respect to industry beta standard error, $SE(\beta_{industry,i,t})$. If $SE(\beta_{i,t})$ is relatively small, we put a large weight on the stock estimate of beta. If $SE(\beta_{i,t})$ is relatively large which may happen when stock has a short history or sparse returns, we put a large weight on the industry beta estimate.

Step 4: Calculate the final estimate of individual stock beta by shrink Bayesian estimates of beta to value of one using Vasicek shrinkage:

$$\beta_{i,final,t} = w_{i,t}\beta_{i,Bayes,t} + \left(1 - w_{i,t}\right)1$$

$$w_{i,t} = \frac{VAR(\beta_t)_{CS}}{VAR\left(\beta_{i,t}\right) + VAR(\beta_t)_{CS}} \tag{H4}$$

where $VAR(\beta_t)_{CS}$ is the cross sectional variance of the estimated Bayesian betas in Step 3.

Prior to this step, the stock beta estimates should be not "too far away" from one. To make this notion concrete, we follow Vasicek's suggestion and look at the cross-sectional dispersion of betas as a measure of distance of how far away betas are likely to be from one. If the cross-section dispersion of betas is low for a given time period, then we know that beta estimates that are much different from one are likely to be outliers caused by estimation noise. These beta estimates will be shrunk to one.

Appendix I: Covariance Matrix Estimation

Estimation of the model's factor covariance matrix follows a multi-step process. The first step is to compute the factor correlation matrix from the set of daily factor returns. We employ exponentially weighted averages, characterized by the factor correlation half-life parameter τ_{ρ}^{F}. This approach gives more weight to recent observations and is an effective method for dealing with data non-stationarity.

For the Barra US Total Market Equity Model for Long-Term investors, the prediction horizon is one month. The factor correlation matrix, however, is estimated from daily factor returns. We must therefore account for the possibility of serial correlation in factor returns, as these may affect risk forecasts over a longer horizon.

We employ the Newey-West methodology (1987) to account for serial-correlation effects. A key parameter in this approach is the number of lags L_{ρ}^{F} over which the serial correlation is deemed important. For instance, $L_{\rho}^{F} = 2$ implies that the return of any factor may be correlated with the return of any other factor within a two-day time span.

Another complication in estimating the factor correlation matrix arises from the case of missing factor returns. In models, missing factor returns can arise from using time series of differing lengths. For instance, an industry factor may appear in the model only after the start date of the cross-sectional regressions. The industry factors proxy early history using the returns of the parent industry. For style factors, we use the Expectation Maximization (EM) algorithm of Dempster (1977) to estimate the correlation matrix for the case of missing factor returns. This method employs an iterative procedure to estimate the correlation matrix. The EM algorithm also refines the correlation forecasts as new information flows into the model.

With the correlation matrix thus computed, the next step is to calculate the factor volatilities. We use exponentially weighted averages, with half-life parameter τ_{σ}^{F}. In estimating monthly factor volatilities, we also scale daily volatility by a ratio of volatility estimates calculated using overlapping monthly returns and daily returns.

Next, we construct the initial covariance matrix by combining the factor volatilities and correlations. That is, the covariance between factors i and j is given by

$$F_{ij}^{0} = \rho_{ij}\,\sigma_i \sigma_j \tag{I1}$$

where σ_i and σ_j are the factor volatilities and ρ_{ij} is their respective correlation.

Appendix J: Industry Factor Characteristics

Table J.1: Barra US Long-Term Equity Model industry factors. Weights were determined within the estimation universe using market capitalization. Averages were computed over the sample period from June 30, 1995 to Sep 30, 2014

Sector	Industry Factor Name	Average Weight	30-Sep-14
Energy	Oil and Gas Drilling	0.48%	0.36%
Energy	Oil and Gas Exploration and Production	2.38%	3.60%
Energy	Oil Gas and Consumable Fuels	3.13%	2.89%
Energy	Oil and Gas Equipment and Services	1.11%	1.19%
Materials	Chemicals	1.43%	1.51%
Materials	Construction Materials	0.09%	0.07%
Materials	Containers and Packaging	0.30%	0.27%
Materials	Aluminum Steel	0.36%	0.26%
Materials	Paper and Forest Products	0.31%	0.14%
Materials	Precious Metals Gold Mining	0.48%	0.36%
Materials	Specialty Chemicals	0.75%	0.79%
Industrials	Aerospace and Defense	1.51%	1.62%
Industrials	Airlines	0.24%	0.45%
Industrials	Building Products	0.23%	0.19%
Industrials	Commercial and Professional Services	2.33%	1.71%
Industrials	Construction and Farm Machinery	0.50%	0.45%
Industrials	Construction and Engineering	0.24%	0.38%
Industrials	Electrical Equipment	1.01%	1.09%
Industrials	Industrial Conglomerates	1.02%	0.62%
Industrials	Industrial Machinery	1.20%	1.23%
Industrials	Road and Rail	0.95%	1.43%
Industrials	Transportation Air Freight and Marine	0.86%	0.95%
Industrials	Trading Companies and Distributors	0.59%	0.63%

Sector	Industry Factor Name	Average Weight	30-Sep-14
Consumer Discretionary	Apparel and Textiles	0.90%	0.62%
Consumer Discretionary	Automobiles and Components	0.75%	1.01%
Consumer Discretionary	Distributors Multiline Retail	1.19%	0.63%
Consumer Discretionary	Homebuilding	0.21%	0.20%
Consumer Discretionary	Household Durables (non-Homebuilding)	0.54%	0.40%
Consumer Discretionary	Specialty Retail	1.64%	1.75%
Consumer Discretionary	Leisure Products Textiles Apparel and Luxury	0.87%	1.08%
Consumer Discretionary	Hotels Leisure and Consumer Services	1.06%	1.17%
Consumer Discretionary	Media	4.87%	4.48%
Consumer Discretionary	Internet and Catalog Retail	0.98%	1.88%
Consumer Discretionary	Restaurants	0.98%	1.36%
Consumer Discretionary	Specialty Stores	0.48%	0.58%
Consumer Staples	Beverages Tobacco	2.87%	3.00%
Consumer Staples	Food Products	1.98%	1.51%
Consumer Staples	Food and Staples Retailing	1.66%	1.69%
Consumer Staples	Household and Personal Products	2.14%	2.18%
Health Care	Biotechnology Life Sciences	1.93%	3.47%
Health Care	Health Care Providers (non-HMO)	1.30%	1.41%
Health Care	Health Care Equipment and Technology	2.05%	2.35%
Health Care	Managed Health Care	0.78%	0.91%
Health Care	Pharmaceuticals	5.84%	4.37%
Financials	Diversified Financials	6.68%	5.92%
Financials	Banks	5.59%	4.06%
Financials	Insurance Brokers and Reinsurance	3.22%	2.24%
Financials	Life Health and Multi-line Insurance	1.20%	0.72%
Financials	Real Estate	2.32%	3.23%
Information Technology	Communications Equipment	2.82%	1.34%
Information Technology	Computers Electronics	3.86%	4.24%
Information Technology	Internet Software and IT Services	3.27%	8.12%
Information Technology	Semiconductors	2.54%	2.02%
Information Technology	Semiconductor Equipment	0.41%	0.32%
Information Technology	Software	4.35%	4.72%
Telecom	Diversified Telecommunication Services	3.02%	1.15%
Telecom	Wireless Telecommunication Services	1.07%	1.09%
Utilities	Electric Utilities	1.96%	1.42%
Utilities	Gas Utilities	0.44%	0.32%
Utilities	Multi-Utilities Water Utilities Power	0.74%	0.84%

In **Table J.2**, we report the underlying GICS codes that map to each of the industry factors. This table helps illustrate the customization that takes place within each sector.

Table J.2: Mapping of Barra US Long-Term Equity Model industry factors to GICS® codes.

	Industry Factor Name	GICS Codes
1	Oil and Gas Drilling	10101010
2	Oil and Gas Equipment and Services	10101020
3	Oil Gas and Consumable Fuels	10102010, 10102030, 10102040, 10102050
4	Oil and Gas Exploration and Production	10102020
5	Chemicals	15101010, 15101020, 15101030, 15101040
6	Specialty Chemicals	15101050
7	Construction Materials	151020
8	Containers and Packaging	151030
9	Aluminum Steel	15104010, 15104050
10	Precious Metals Gold Mining	15104020, 15104030, 15104040
11	Paper and Forest Products	151050
12	Aerospace and Defense	201010
13	Building Products	201020
14	Construction and Engineering	201030
15	Electrical Equipment	201040
16	Industrial Conglomerates	201050
17	Construction and Farm Machinery	20106010
18	Industrial Machinery	20106020
19	Trading Companies and Distributors	201070
20	Commercial and Professional Services	2020
21	Transportation Air Freight and Marine	203010, 203030, 203050
22	Airlines	203020
23	Road and Rail	203040
24	Automobiles and Components	2510
25	Household Durables (non-Homebuilding)	25201010, 25201020, 25201040, 25201050
26	Homebuilding	25201030
27	Leisure Products Textiles Apparel and Luxury	252020, 252030
28	Hotels Leisure and Consumer Services	25301010, 25301020, 25301030, 253020
29	Restaurants	25301040
30	Media	2540
31	Distributors Multiline Retail	255010, 255030
32	Internet and Catalog Retail	255020
33	Apparel and Textiles	25504010
34	Specialty Retail	25504020, 25504030, 25504050, 25504060
35	Specialty Stores	25504040

	Industry Factor Name	GICS Codes
36	Food and Staples Retailing	3010
37	Beverages Tobacco	302010, 302030
38	Food Products	302020
39	Household and Personal Products	3030
40	Health Care Equipment and Technology	351010, 351030
41	Health Care Providers (non-HMO)	35102010, 35102015, 35102020
42	Managed Health Care	35102030
43	Biotechnology Life Sciences	352010, 352030
44	Pharmaceuticals	352020
45	Banks	4010
46	Diversified Financials	4020
47	Insurance Brokers and Reinsurance	40301010, 40301040, 40301050
48	Life Health and Multi-line Insurance	40301020, 40301030
49	Real Estate	4040
50	Internet Software and IT Services	451010, 45102010, 45102020
51	Software	451030
52	Communications Equipment	452010
53	Computers Electronics	452020, 452030, 452040
54	Semiconductor Equipment	45205010, 45301010
55	Semiconductors	45205020, 45301020
56	Diversified Telecommunication Services	501010
57	Wireless Telecommunication Services	501020
58	Electric Utilities	551010
59	Gas Utilities	551020
60	Multi-Utilities Water Utilities Power	551030, 551040, 551050

In **Table J.3**, we report the largest firm within each industry, as well as the total market capitalization as of September 30, 2014.

Table J.3: Largest stock within each industry as of September 30, 2014. Market capitalizations are reported in billions of US dollars

Industry Factor Name	Largest Stock	Mkt. Cap ($bln)
Oil and Gas Drilling	HELMERICH & PAYNE INC	10.6
Oil and Gas Exploration and Production	CONOCOPHILLIPS	93.9
Oil Gas and Consumable Fuels	EXXON MOBIL CORP	403.9
Oil and Gas Equipment and Services	SCHLUMBERGER LTD	132.3
Chemicals	E I DU PONT DE NEMOURS & CO	65.9
Construction Materials	MARTIN MARIETTA MATLS INC	8.6
Containers and Packaging	BALL CORP	8.8
Aluminum Steel	ALCOA INC	18.9
Paper and Forest Products	INTL PAPER CO	20.7
Precious Metals Gold Mining	FREEPORT-MCMORAN	33.9
Specialty Chemicals	ECOLAB INC	34.5
Aerospace and Defense	UNITED TECHNOLOGIES CORP	96.8
Airlines	DELTA AIR LINES INC DEL	30.7
Building Products	MASCO CORP	8.5
Commercial and Professional Services	WASTE MGMT INC DEL	22.1
Construction and Farm Machinery	CATERPILLAR INC DEL	61.8
Construction and Engineering	FLUOR CORP NEW	10.6
Electrical Equipment	EMERSON ELEC CO	43.9
Industrial Conglomerates	GENERAL ELECTRIC CO	256.9
Industrial Machinery	ILLINOIS TOOL WKS INC	34.8
Road and Rail	UNION PACIFIC CORP	98.3
Transportation Air Freight and Marine	UNITED PARCEL SERVICE INC	90.0
Trading Companies and Distributors	GRAINGER W W INC	17.2
Apparel and Textiles	TJX COS INC NEW	41.4
Automobiles and Components	FORD MTR CO DEL	58.5
Distributors Multiline Retail	TARGET CORP	39.7
Homebuilding	LENNAR CORP	7.7
Household Durables (non-Homebuilding)	WHIRLPOOL CORP	11.3
Specialty Retail	HOME DEPOT INC	125.5
Leisure Products Textiles Apparel and Luxury	NIKE INC	78.4
Hotels Leisure and Consumer Services	LAS VEGAS SANDS CORP	50.3
Media	WALT DISNEY CO	154.2
Internet and Catalog Retail	AMAZON COM INC	148.4
Restaurants	MCDONALDS CORP	93.7
Specialty Stores	TIFFANY & CO NEW	12.4

Industry Factor Name	Largest Stock	Mkt. Cap ($bln)
Beverages Tobacco	COCA COLA CO	187.5
Food Products	MONDELEZ INTERNATIONAL INC	58.0
Food and Staples Retailing	WAL MART STORES INC	246.8
Household and Personal Products	PROCTER & GAMBLE CO	226.6
Biotechnology Life Sciences	GILEAD SCIENCES INC	163.5
Health Care Providers (non-HMO)	EXPRESS SCRIPTS INC	54.8
Health Care Equipment and Technology	ABBOTT LABS	62.5
Managed Health Care	UNITEDHEALTH GROUP INC	84.5
Pharmaceuticals	JOHNSON & JOHNSON	301.6
Diversified Financials	BERKSHIRE HATHAWAY [B]	336.1
Banks	WELLS FARGO & CO NEW	273.2
Insurance Brokers and Reinsurance	ACE LTD	35.4
Life Health and Multi-line Insurance	AMERICAN INTL GROUP INC	78.1
Real Estate	SIMON PPTY GROUP INC NEW	51.1
Communications Equipment	CISCO SYS INC	128.9
Computers Electronics	APPLE INC	607.5
Internet Software and IT Services	GOOGLE INC [A]	393.2
Semiconductors	INTEL CORP	173.3
Semiconductor Equipment	APPLIED MATLS INC	26.3
Software	MICROSOFT CORP	383.0
Diversified Telecommunication Services	VERIZON COMMUNICATIONS INC	207.0
Wireless Telecommunication Services	SPRINT CORP	25.0
Electric Utilities	DUKE ENERGY CORP NEW	52.9
Gas Utilities	AGL RESOURCES INC	6.1
Multi-Utilities Water Utilities Power	DOMINION RES INC VA NEW	40.2

In **Table J.4** we report mean absolute *t*-statistics for the Barra US Long-Term Equity Model industry factors, as well as the percentage of observations with $|t|>2$. We also report the returns, volatilities, and Information Ratios (IR) for the factors, and the correlations of the daily factor returns with the estimation universe.

Table J.4: Industry factor summary statistics computed using daily cross-sectional regressions. The sample period is from June 30, 1995 to Sep 30, 2014 (5101 days)

Factor Name	Average Absolute t-stat	Percent Observ. |t|>2	Annual Factor Return	Annual Factor Volatility	Factor Sharpe Ratio	Correl. with ESTU
Oil and Gas Drilling	2.63	51.33	6.15	30.37	0.20	-0.01
Oil and Gas Exploration and Production	3.20	57.44	5.14	21.10	0.24	0.00
Oil Gas and Consumable Fuels	2.11	41.12	4.62	16.84	0.27	-0.02
Oil and Gas Equipment and Services	2.92	55.42	3.97	25.93	0.15	-0.01
Chemicals	1.45	25.71	1.35	14.83	0.09	0.00
Construction Materials	0.86	7.94	0.89	20.72	0.04	-0.01
Containers and Packaging	0.98	11.02	-2.99	14.81	-0.20	-0.01
Aluminum Steel	1.70	31.28	-6.22	20.31	-0.31	0.07
Paper and Forest Products	1.24	18.67	-2.94	19.87	-0.15	0.03
Precious Metals Gold Mining	2.08	41.41	3.33	25.73	0.13	-0.06
Specialty Chemicals	0.96	9.90	-3.53	10.61	-0.33	0.10
Aerospace and Defense	1.49	26.45	1.67	13.83	0.12	-0.07
Airlines	2.08	40.73	-1.66	31.78	-0.05	-0.03
Building Products	0.91	9.06	-4.93	15.12	-0.33	0.04
Commercial and Professional Services	1.13	15.35	-4.40	6.10	-0.72	0.00
Construction and Farm Machinery	1.32	21.29	0.53	16.04	0.03	0.06
Construction and Engineering	0.99	11.68	-3.72	15.82	-0.24	0.01
Electrical Equipment	1.05	13.29	-1.67	10.52	-0.16	0.10
Industrial Conglomerates	0.91	8.87	-3.24	25.03	-0.13	-0.01
Industrial Machinery	1.15	16.05	-1.42	9.50	-0.15	0.13
Road and Rail	1.47	25.38	2.30	13.90	0.17	-0.01
Transportation Air Freight and Marine	1.11	15.33	-2.45	13.39	-0.18	0.02
Trading Companies and Distributors	0.87	7.43	-5.96	11.94	-0.50	0.05
Apparel and Textiles	1.92	37.34	-1.78	17.75	-0.10	-0.01
Automobiles and Components	1.46	25.23	-10.89	16.27	-0.67	0.03
Distributors Multiline Retail	1.74	33.44	-3.14	15.99	-0.20	-0.02
Homebuilding	1.95	35.49	3.80	26.23	0.14	0.01
Household Durables (non Homebuilding)	1.00	11.29	-5.96	11.77	-0.51	0.02
Specialty Retail	1.68	32.08	1.32	16.64	0.08	0.00

Leisure Products Textiles Apparel and Luxury	1.39	23.93	-7.11	11.32	-0.63	-0.01
Hotels Leisure and Consumer Services	1.55	27.36	-3.15	12.33	-0.26	-0.03
Media	1.73	33.57	1.30	8.82	0.15	0.00
Internet and Catalog Retail	1.45	24.45	3.66	21.83	0.17	0.00
Restaurants	1.45	25.01	-0.69	12.44	-0.06	-0.05
Specialty Stores	1.30	21.44	-6.76	13.48	-0.50	0.00

Factor Name	Average Absolute t-stat	Percent Observ. |t|>2	Annual Factor Return	Annual Factor Volatility	Factor Sharpe Ratio	Correl. with ESTU
Beverages Tobacco	1.29	21.25	0.15	11.37	0.01	-0.14
Food Products	1.30	21.93	-1.79	9.07	-0.20	-0.15
Food and Staples Retailing	1.32	22.32	-5.72	10.93	-0.52	-0.12
Household and Personal Products	1.26	19.97	-1.80	11.67	-0.15	-0.09
Biotechnology Life Sciences	2.45	46.89	16.77	16.67	1.01	-0.07
Health Care Providers (non HMO)	1.59	29.67	-1.48	11.71	-0.13	-0.12
Health Care Equipment and Technology	1.53	28.20	0.80	9.02	0.09	-0.13
Managed Health Care	1.87	34.52	1.84	21.56	0.09	-0.09
Pharmaceuticals	1.99	39.61	4.37	11.69	0.37	-0.12
Diversified Financials	2.28	46.09	3.75	12.16	0.31	0.15
Banks	2.41	46.28	-2.39	13.01	-0.18	0.07
Insurance Brokers and Reinsurance	1.46	25.54	-0.28	10.33	-0.03	-0.04
Life Health and Multi-line Insurance	1.28	19.43	-4.64	18.43	-0.25	0.13
Real Estate	2.01	35.34	1.07	12.98	0.08	0.05
Communications Equipment	2.04	40.75	3.27	13.87	0.24	0.05
Computers Electronics	2.09	42.25	0.09	10.95	0.01	0.05
Internet Software and IT Services	1.66	32.31	0.64	10.94	0.06	0.04
Semiconductors	3.14	58.49	5.39	20.73	0.26	0.06
Semiconductor Equipment	2.25	43.80	3.06	26.59	0.12	0.07
Software	2.03	41.26	2.39	11.16	0.21	0.05
Diversified Telecommunication Services	1.66	30.95	-3.69	14.50	-0.25	-0.06
Wireless Telecommunication Services	1.60	28.72	3.41	19.56	0.17	-0.05
Electric Utilities	1.73	33.81	-1.63	13.30	-0.12	-0.12
Gas Utilities	0.88	7.96	0.20	11.67	0.02	-0.03
Multi-Utilities Water Utilities Power	1.19	18.44	-5.78	16.49	-0.35	-0.09
Average	1.63	28.48		15.66		

In the following figures, we plot the cumulative factor returns for the industry factors.

Figure J.1: Cumulative returns of the Oil and Gas Drilling factor, and the Oil & Gas Exploration and Production factor

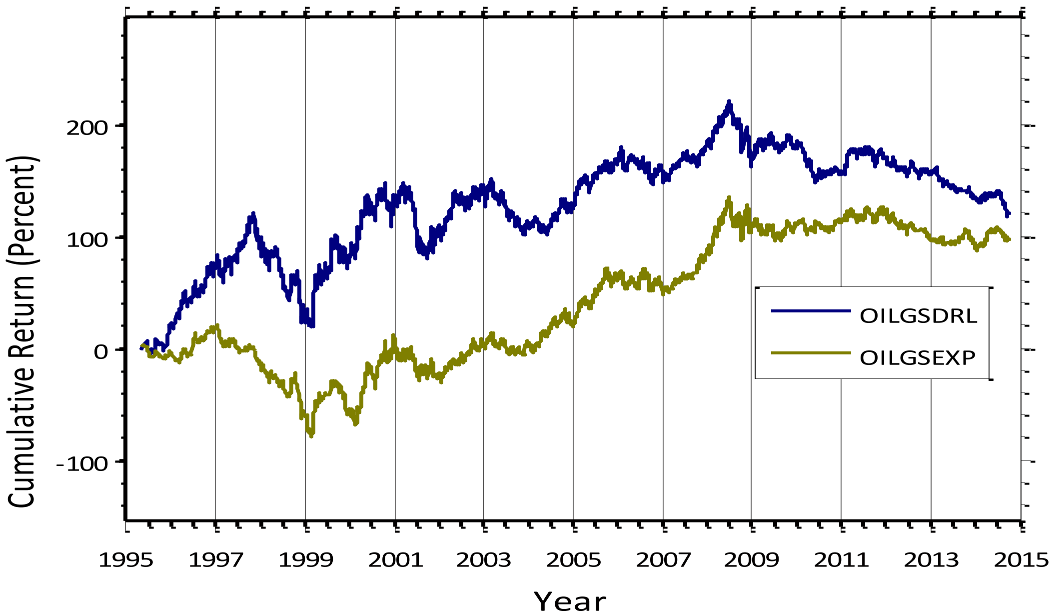


Figure J.2: Cumulative returns of the Oil Gas and Consumable Fuels factor, and the Oil and Gas Equipment and Services factor

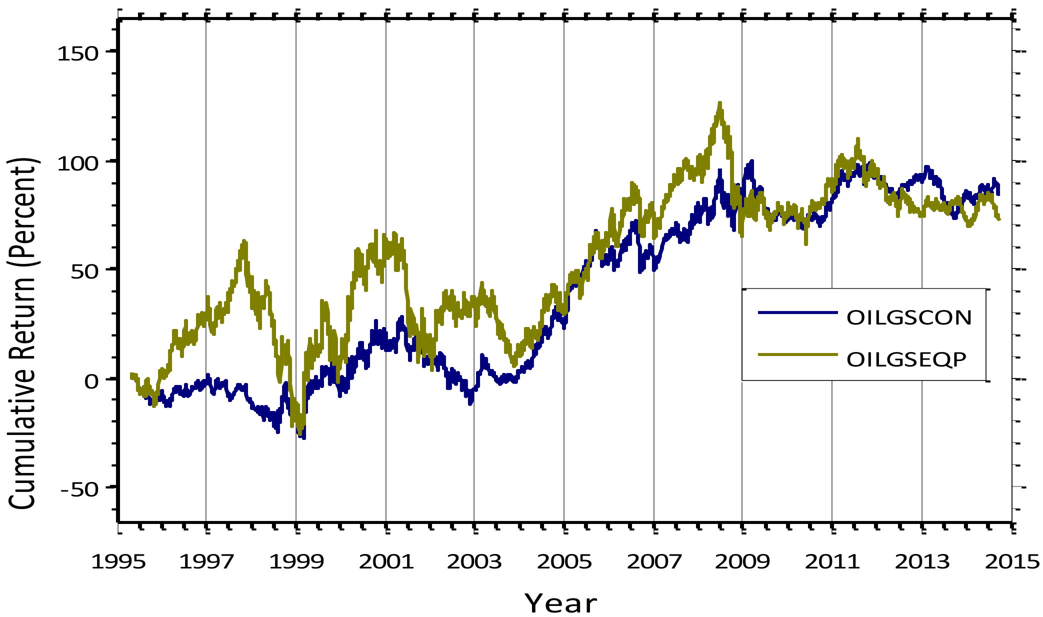


Figure J.3: Cumulative returns of the Chemicals factor, the Specialty Chemicals factor, and the Construction Materials factor

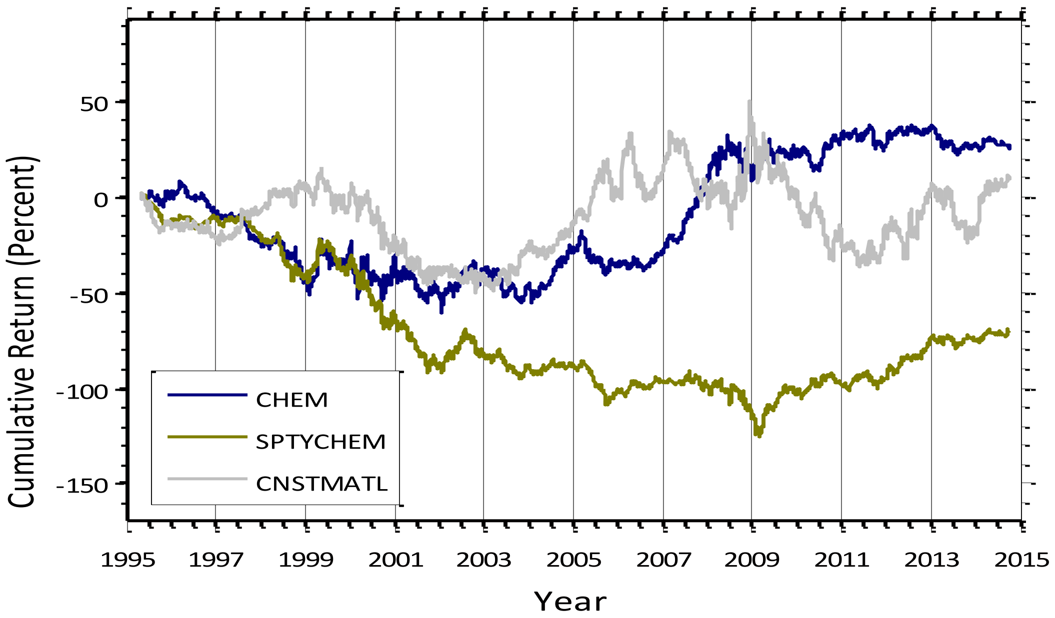


Figure J.4: Cumulative returns of the Containers and Packaging factor, and the Paper and Forest Products factor

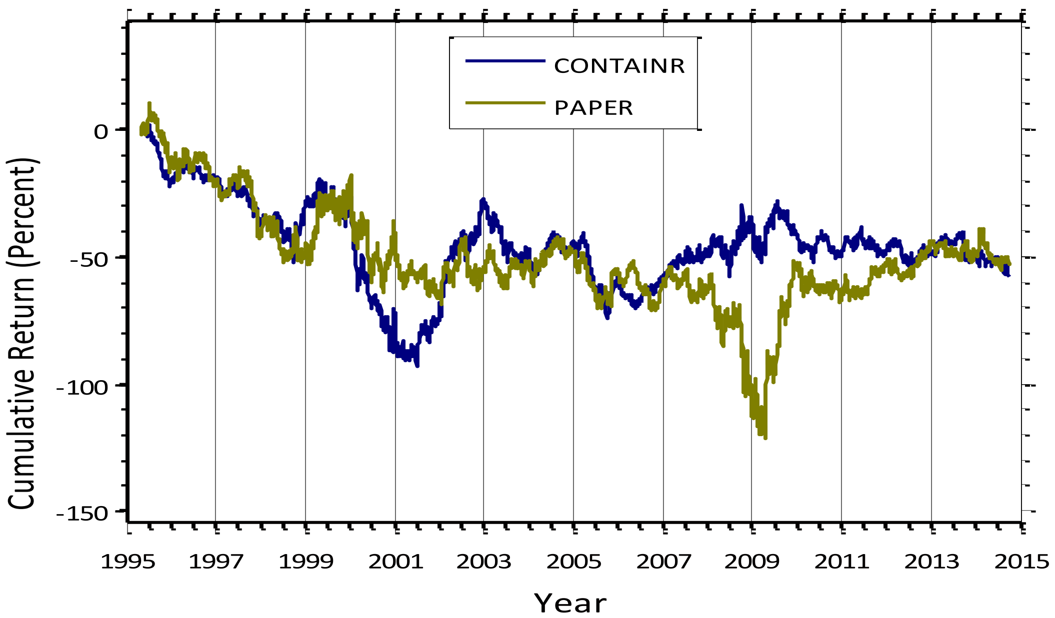


Figure J.5: Cumulative returns of the Aluminum Steel factor, and the Precious Metals Gold Mining factor

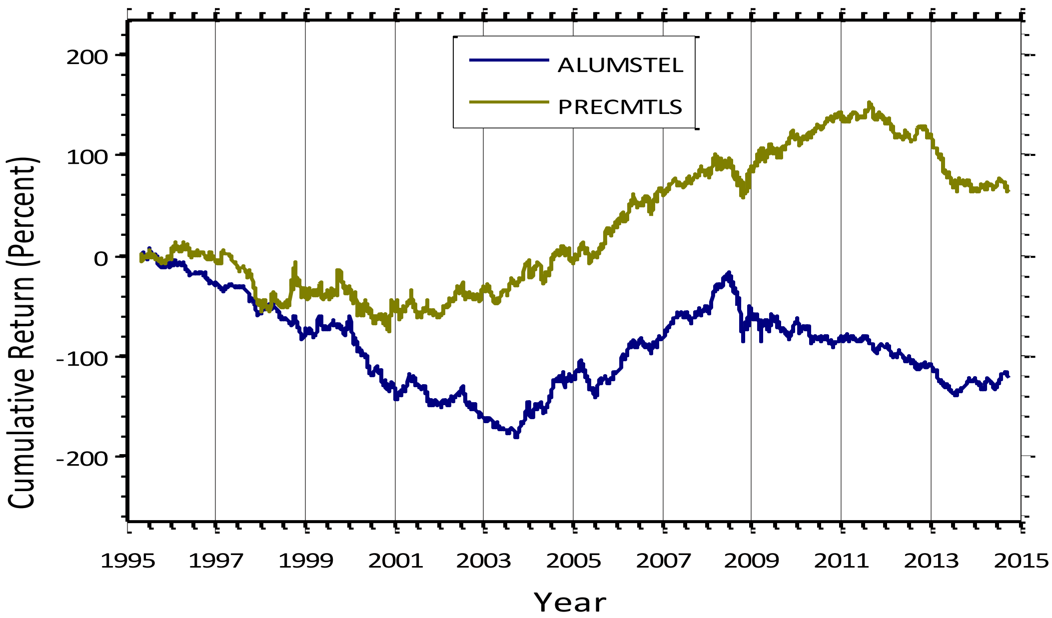


Figure J.6: Cumulative returns of the Aerospace and Defense factor, and the Industrial Conglomerates factor

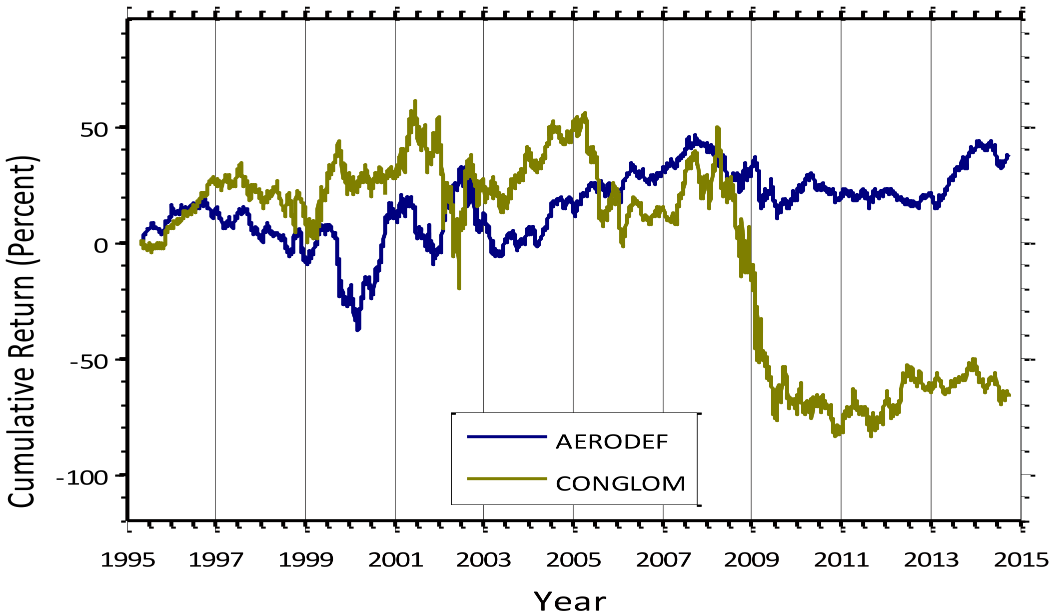


Figure J.7: Cumulative returns of the Construction and Engineering factor, and the Building Products factor

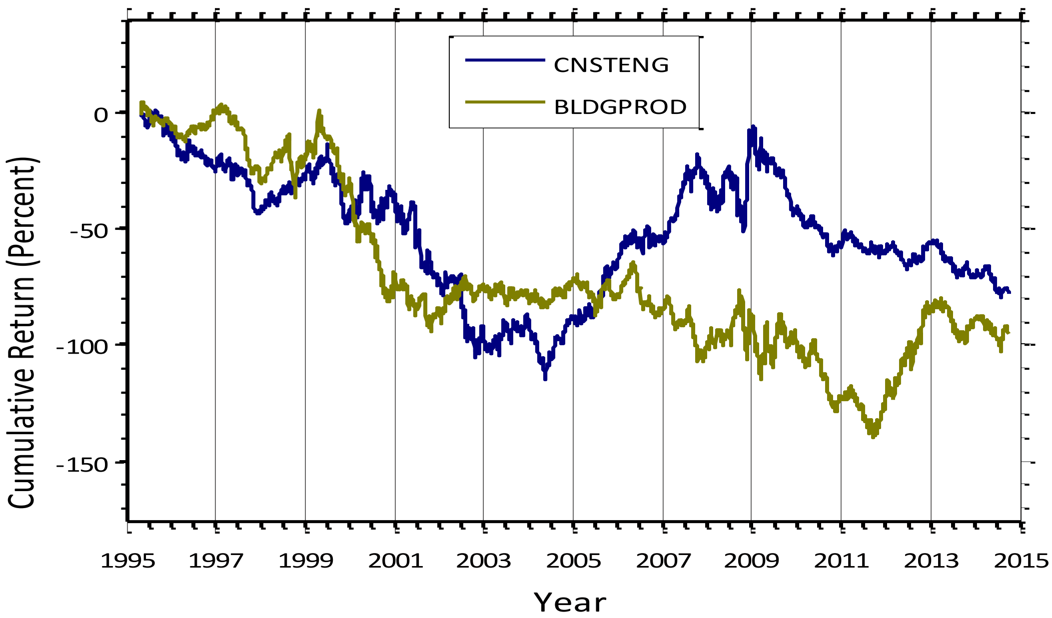


Figure J.8: Cumulative returns of the Electrical Equipment factor, the Construction and Farm Machinery factor, and the Industrial Machinery factor

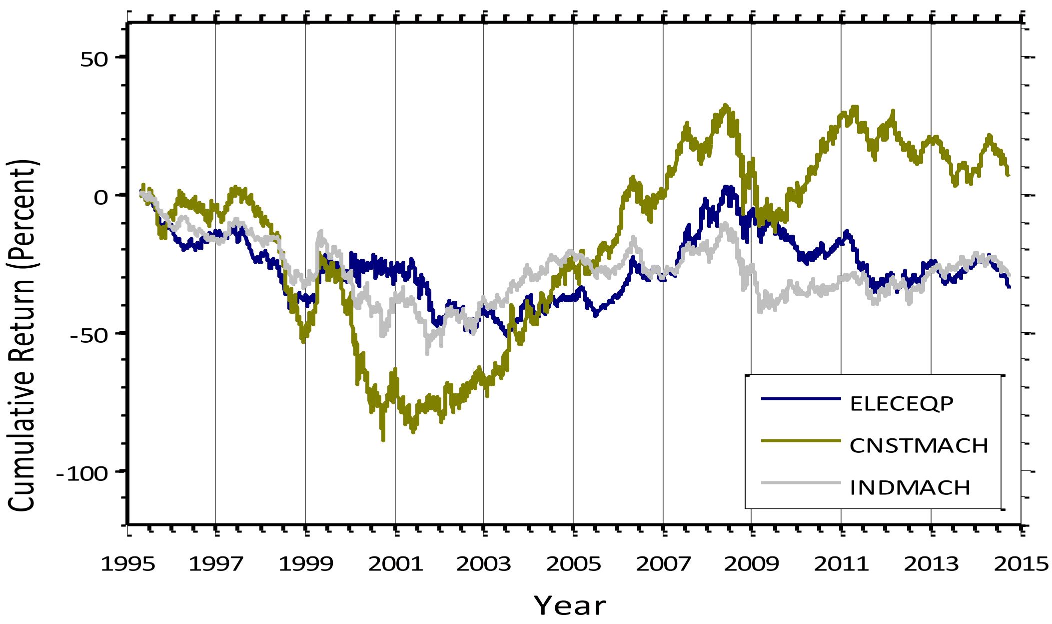


Figure J.9: Cumulative returns of the Trading Companies and Distributors factor, and the Commercial and Professional Services factor

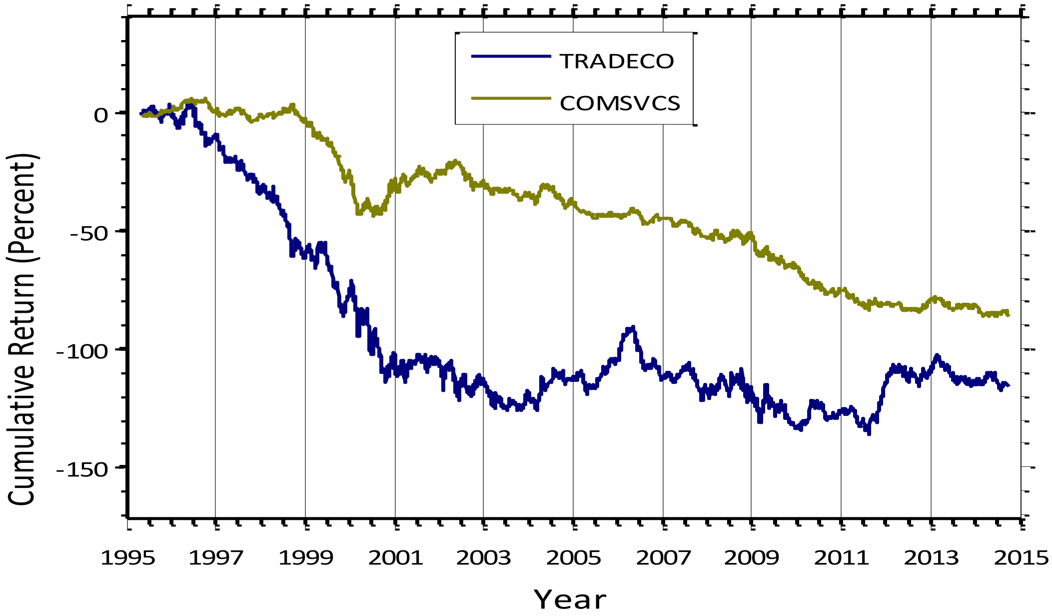


Figure J.10: Cumulative returns of the Transportation Air Freight and Marine factor, the Airlines factor, and the Road and Rail factor

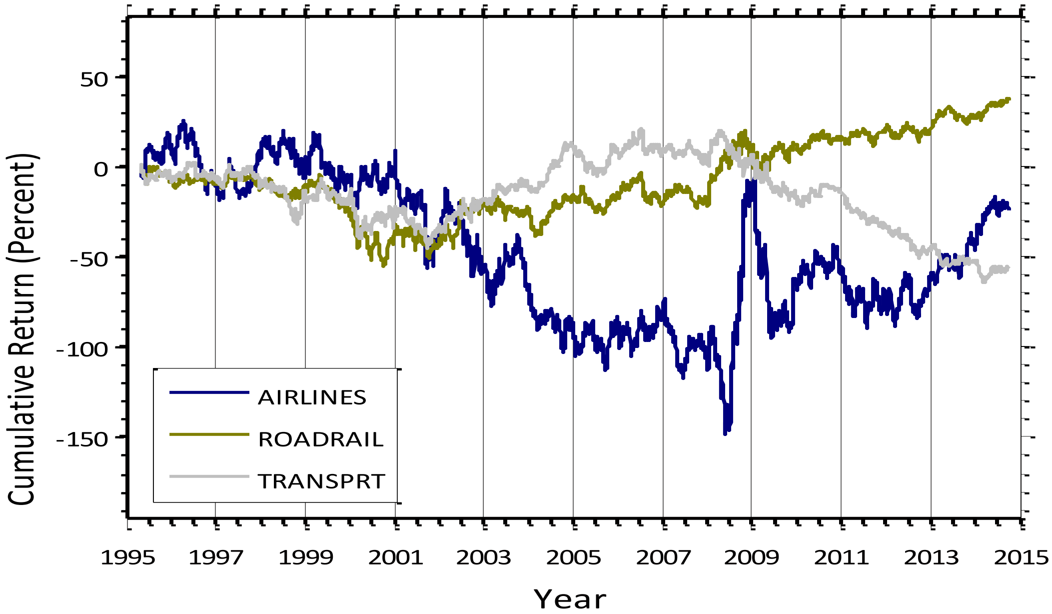


Figure J.11: Cumulative returns of the Automobiles and Components factor, the Household Durables (non-Homebuilding) factor, and the Homebuilding factor

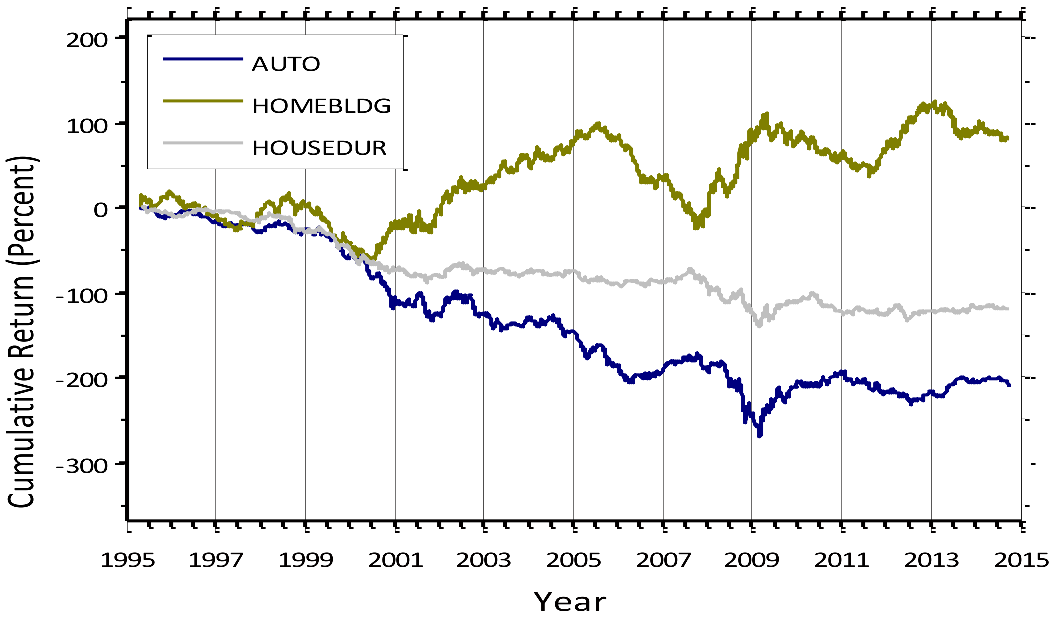


Figure J.12: Cumulative returns of the Leisure Products Textiles Apparel and Luxury factor, and the Leisure Products Textiles Apparel and Luxury factor

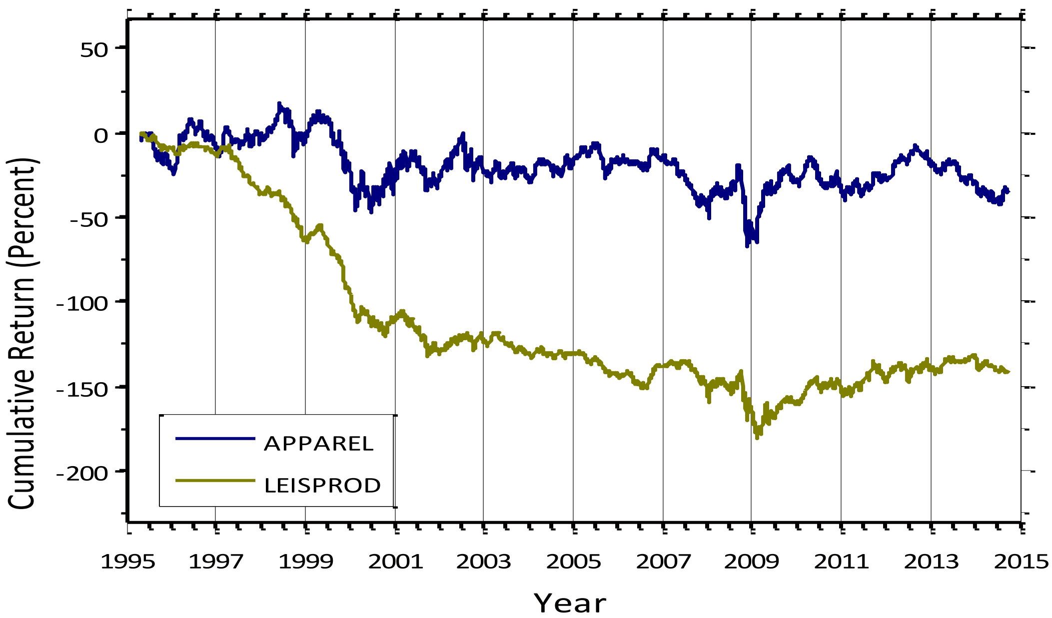


Figure J.13: Cumulative returns of the Distributors Multiline Retail factor, the Media factor, and the Internet and Catalog Retail factor

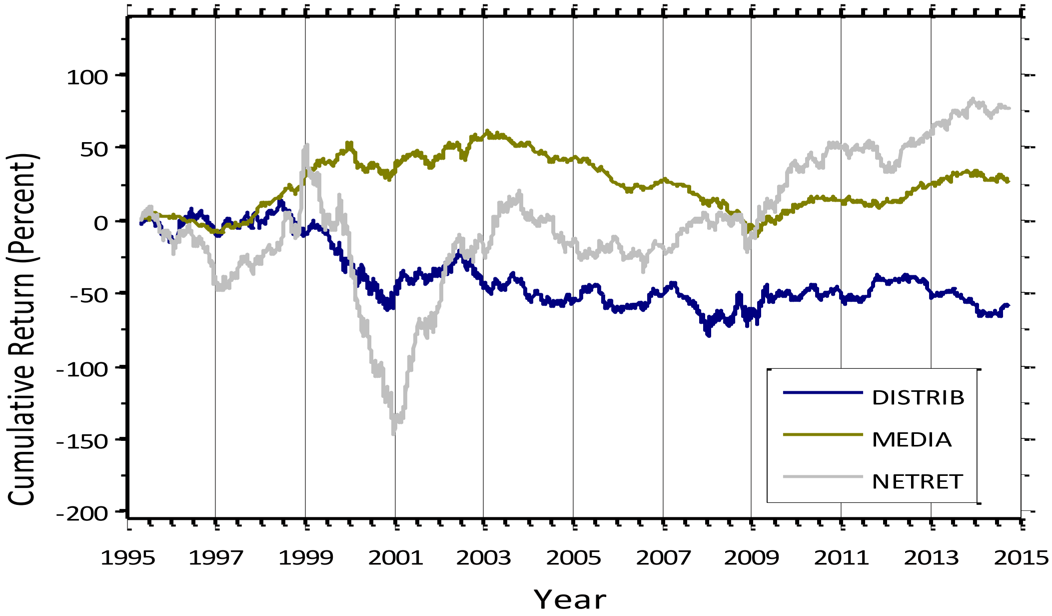


Figure J.14: Cumulative returns of the Specialty Retail factor, and the Specialty Stores factor

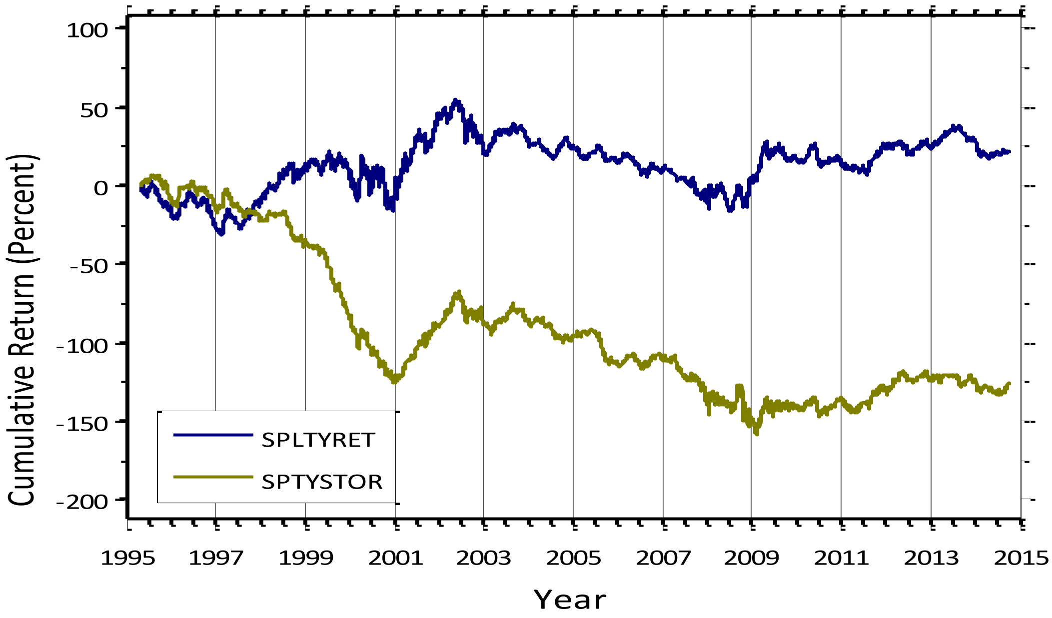


Figure J.15: Cumulative returns of the Hotels Leisure and the Consumer Services factor, and the Restaurants factor

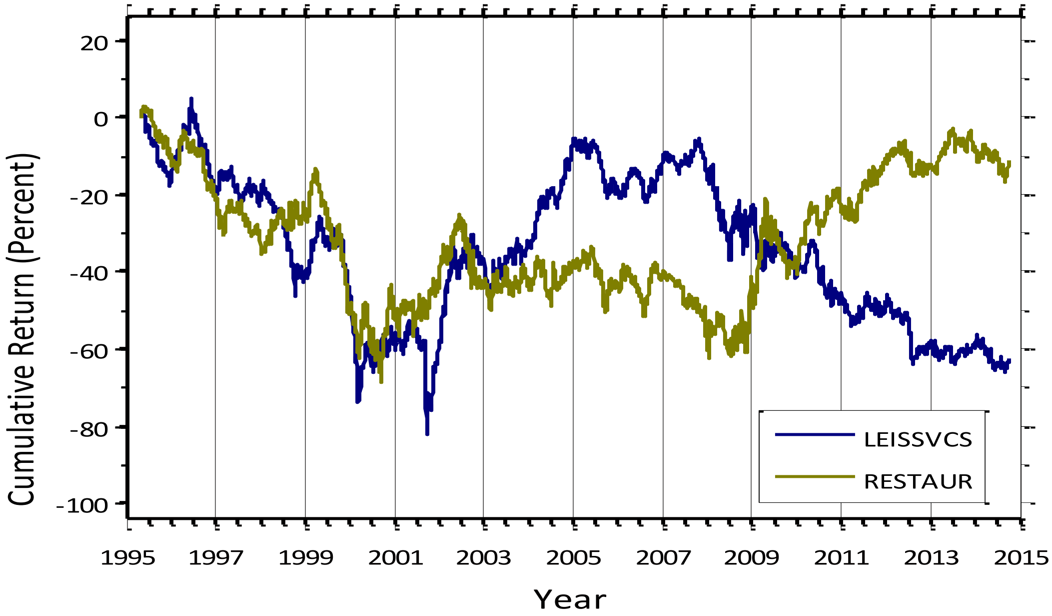


Figure J.16: Cumulative returns of the Beverages Tobacco factor, and the Food and Staples Retailing factor

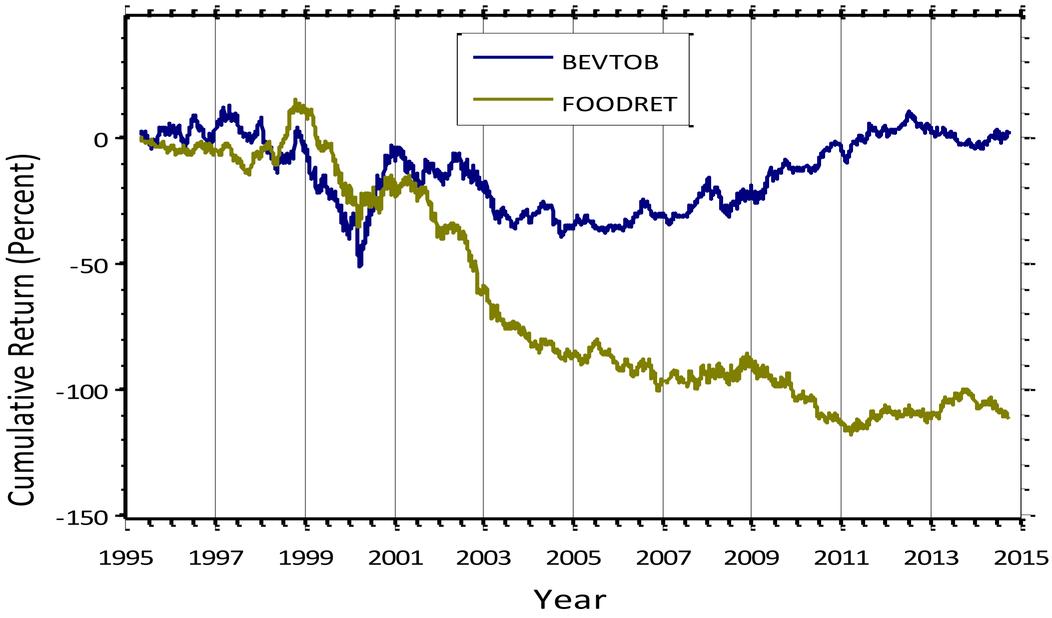


Figure J.17: Cumulative returns of the Food Products factor, and the Household and Personal Products factor

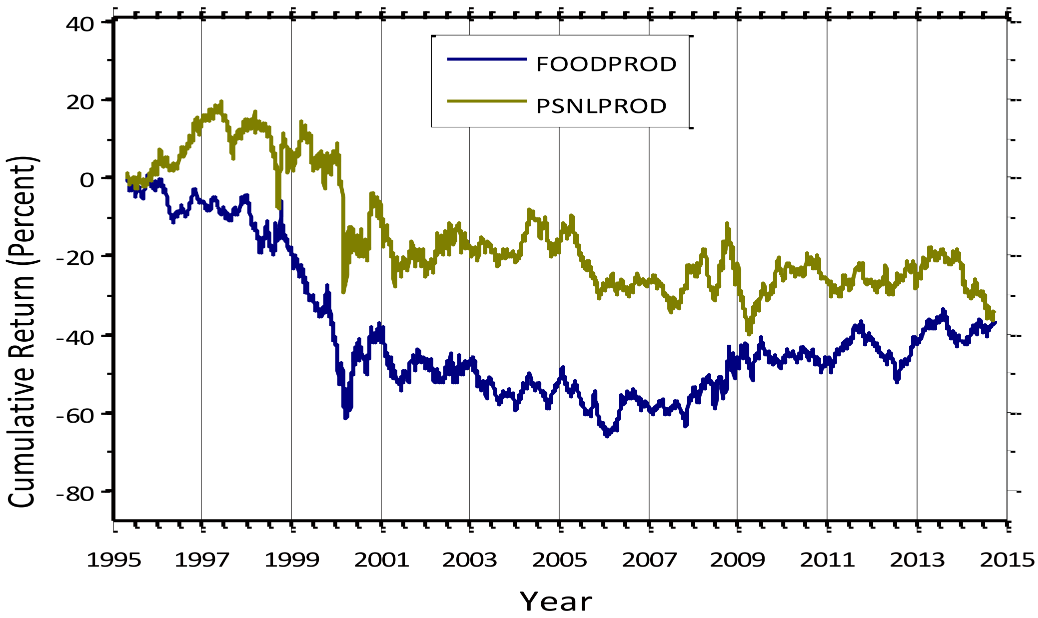


Figure J.18: Cumulative returns of the Biotechnology Life Sciences factor, the Health Care Equipment and Technology factor, and the Pharmaceuticals factor

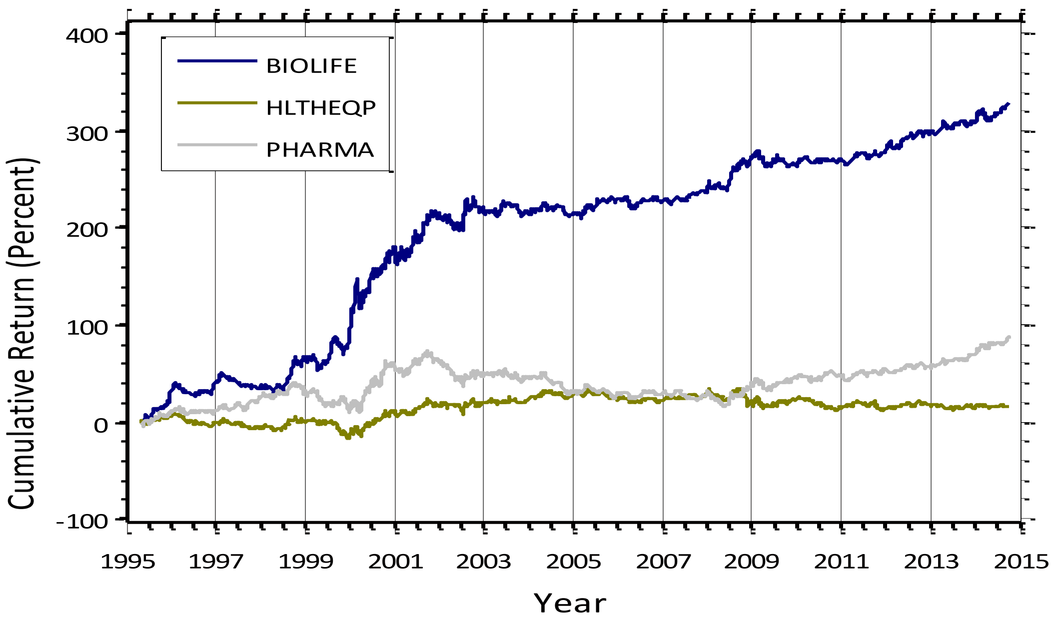


Figure J.19: Cumulative returns of the Health Care Providers (non-HMO) factor, and the Managed Health Care factor

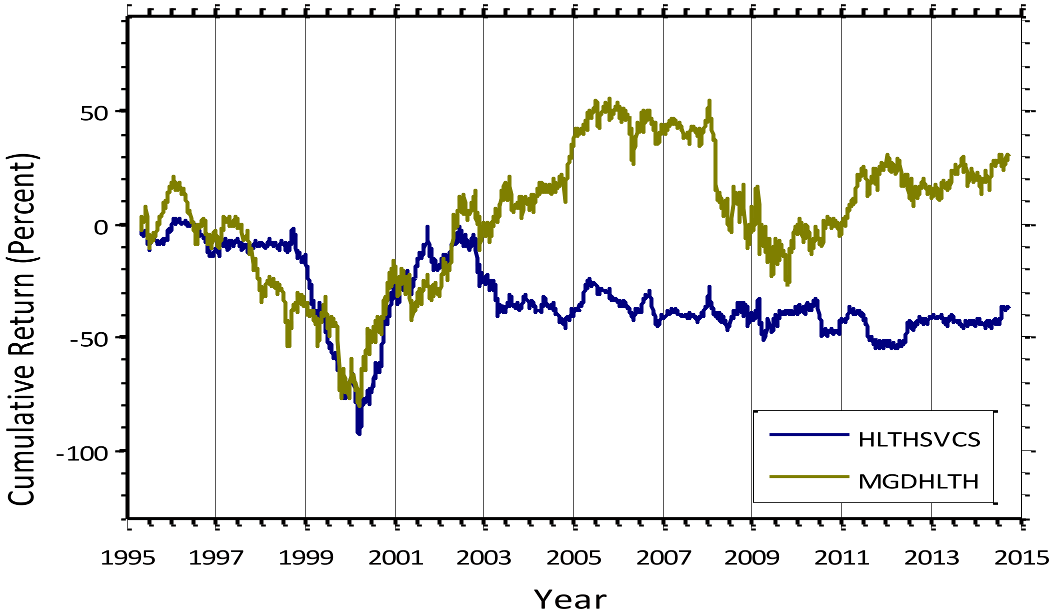


Figure J.20: Cumulative returns of the Banks factor, the Diversified Financials factor, and the Insurance Brokers and Reinsurance factor

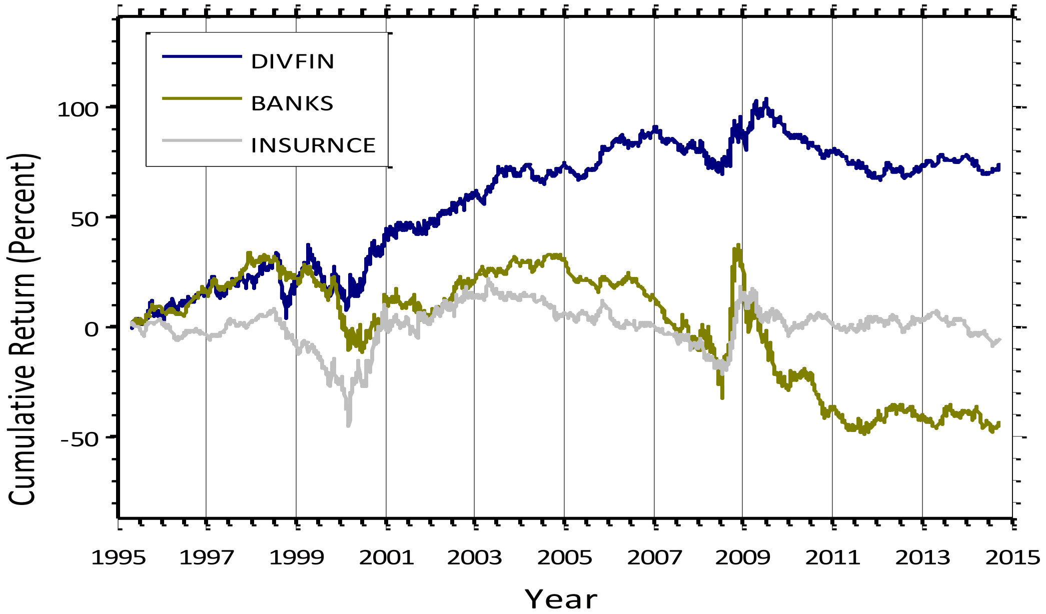


Figure J.21: Cumulative returns of the Life Health and Multi-line Insurance factor, and the Real Estate factor

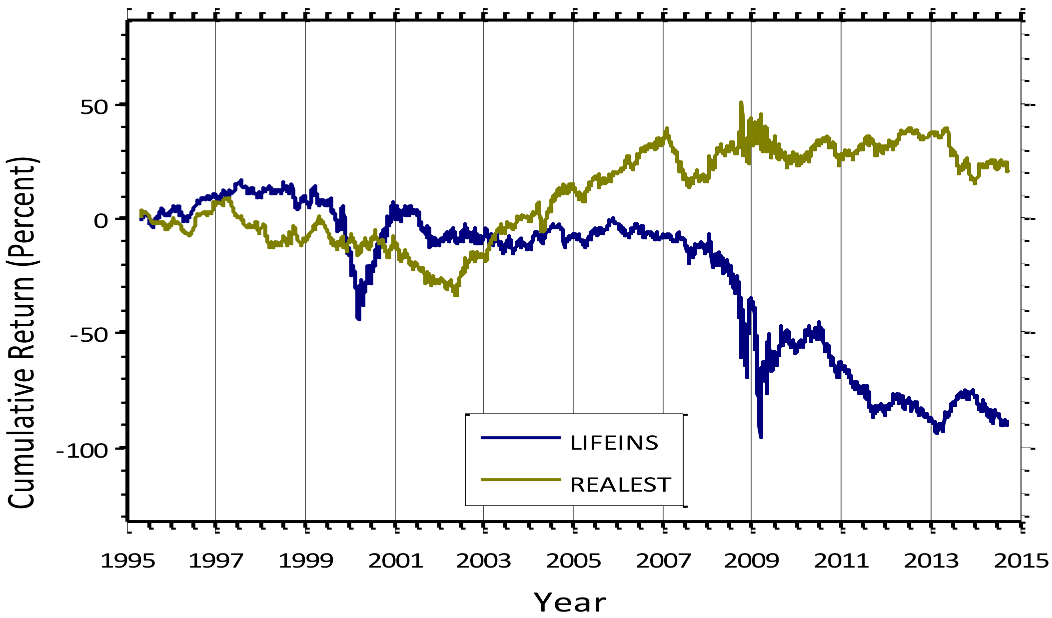


Figure J.22: Cumulative returns of the Communications Equipment factor, and the Computers Electronics factor

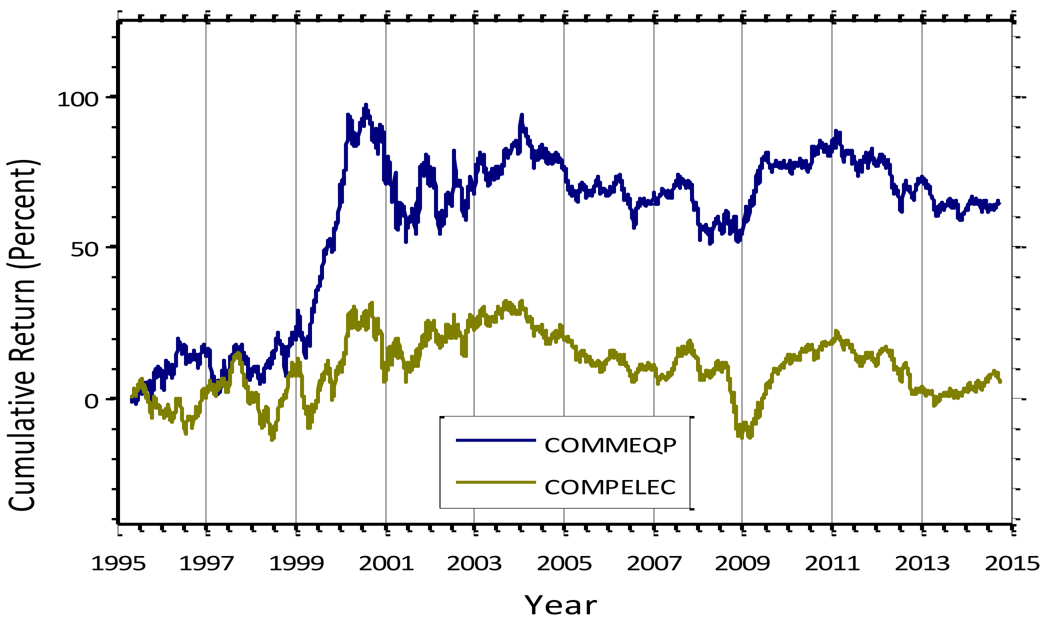


Figure J.23: Cumulative returns of the Internet Software and the IT Services factor, and the Software factor

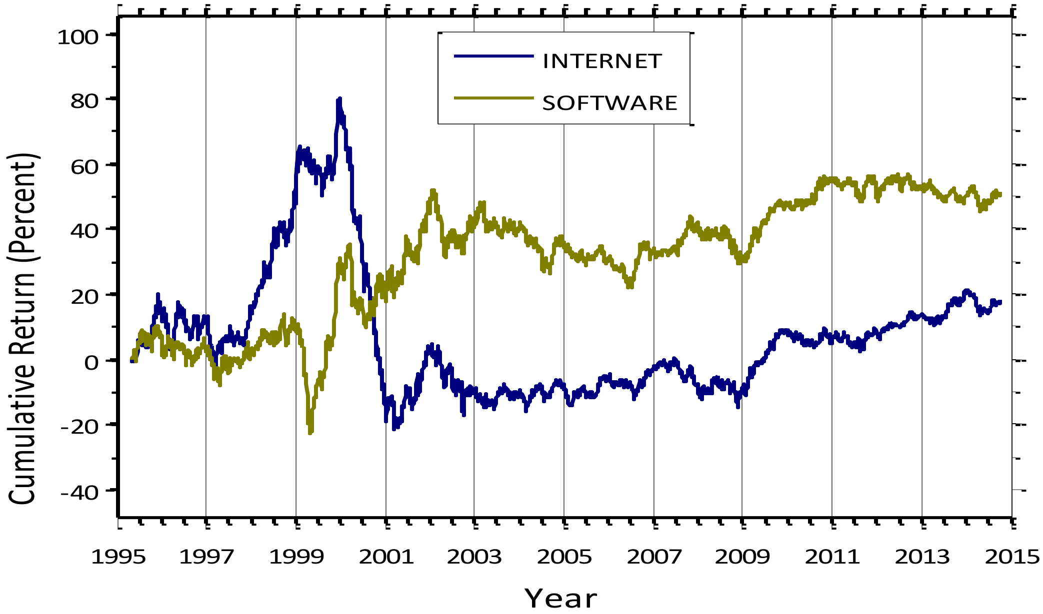


Figure J.24: Cumulative returns of the Semiconductor Equipment factor, and the Semiconductors factor

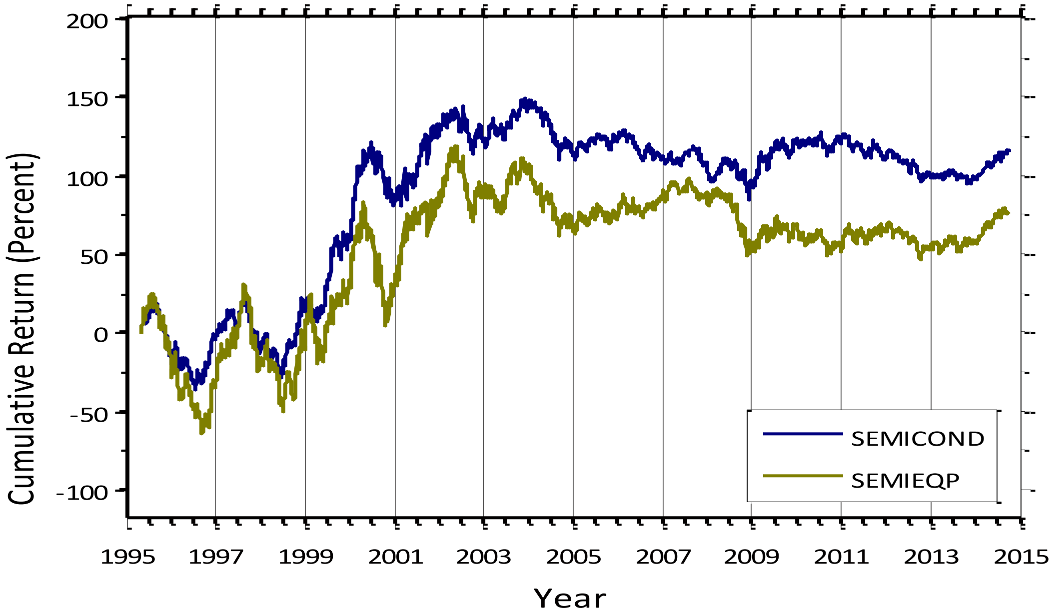


Figure J.25: Cumulative returns of the Diversified Telecommunication Services factor, and the Wireless Telecommunication Services factor

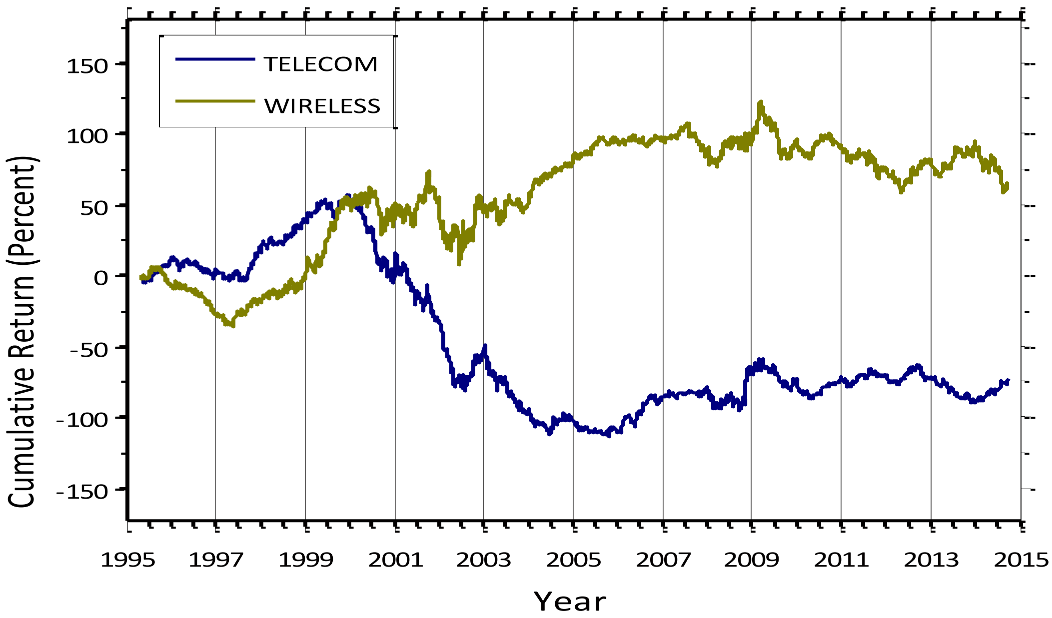
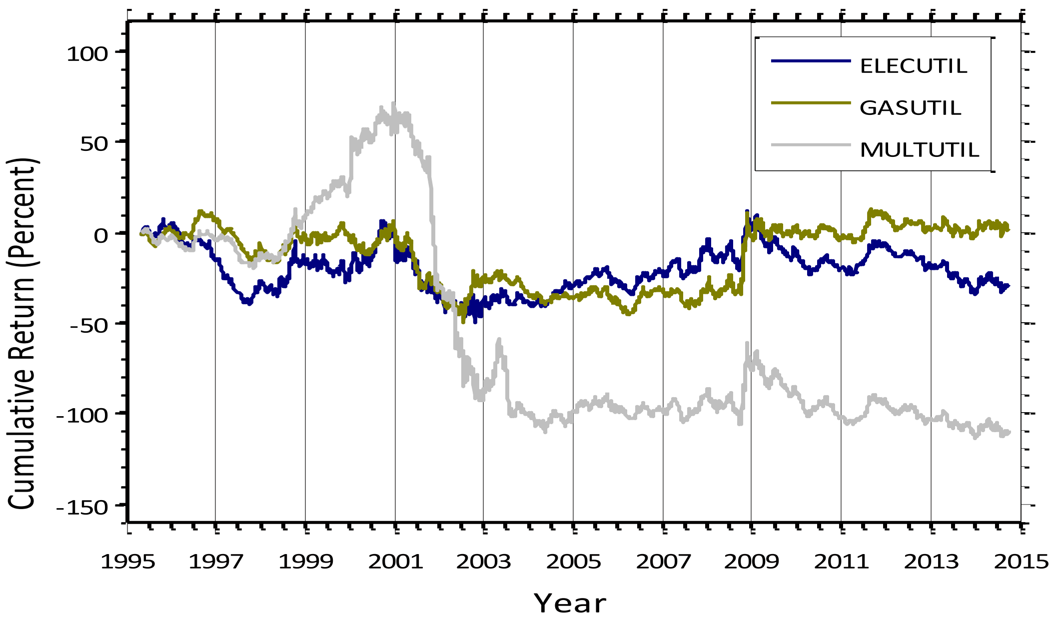


Figure J.26: Cumulative returns of the Electric Utilities factor, the Gas Utilities factor, and the Multi-Utilities Water Utilities Power factor



Appendix K: Comparison with Barra US Equity Model (USE3)

In this section, we compare the Barra US Long-Term Equity Model (Responsive and Stable variants) with USE3 Short (S) and Long (L) horizons.

Table K.1: Bias statistic and average Q-statistic for the Responsive Long-term model and USE3 Short-Horizon model

Portfolio Type	Figure	USE3S		Responsive		Q Diff
		Bias	Q	Bias	Q	
Factor Returns	K.1	0.97		1.05		
Specific Returns	K.2	0.96		1.02		
Market	K.3	0.93	1.9959	0.95	1.9419	-0.0540
Random Active	K.4	0.97	2.3511	1.03	2.3293	-0.0218
Factor-Tilt (Long)	K.4	0.99	2.2826	1.01	2.2421	-0.0405
Factor-Tilt (Active)	K.6	1.08	2.3743	1.08	2.3435	-0.0308
Optimized Styles	K.7	1.03	2.3630	1.06	2.3660	0.0030

Table K.2: Bias statistic and average Q-statistic for the Stable Long-term model and USE3 Long-Horizon model

Portfolio Type	Figure	USE3L		Stable		Q Diff
		Bias	Q	Bias	Q	
Factor Returns	K.1	0.96		1.03		
Specific Returns	K.2	0.96		0.99		
Market	K.3	0.93	2.0979	0.94	2.0796	-0.0183
Random Active	K.4	0.96	2.3783	1.00	2.3881	0.0098
Factor-Tilt (Long)	K.4	0.98	2.3617	0.99	2.3555	-0.0062
Factor-Tilt (Active)	K.6	1.07	2.4356	1.05	2.4292	-0.0064
Optimized Styles	K.7	1.02	2.3783	1.06	2.4290	0.0507

Table K.3: Average of the ratio of cross-sectional residual variance and total variance for Barra US Long-Term Equity Model and USE3

Decile	USE3S	Responsive	Difference	USE3L	Stable	Difference
Small	88.46%	88.31%	0.15%	88.42%	88.32%	0.09%
2	88.03%	87.92%	0.11%	88.03%	87.99%	0.04%
3	86.48%	86.30%	0.18%	86.46%	86.34%	0.12%
4	86.11%	85.78%	0.33%	86.05%	85.83%	0.22%
5	85.15%	84.91%	0.23%	85.20%	85.01%	0.20%
6	84.62%	84.44%	0.18%	84.69%	84.53%	0.16%
7	82.88%	82.66%	0.23%	82.93%	82.72%	0.21%
8	82.00%	81.69%	0.31%	82.09%	81.75%	0.33%
9	80.65%	80.23%	0.42%	80.74%	80.30%	0.44%
Large	77.66%	77.43%	0.23%	77.67%	77.47%	0.20%

Table K.4: Summary of minimum-risk portfolio statistics for the sample period July 1995 through September 2014. We compare returns (Ret), volatility (Vol), Information Ratio (IR), and Turnover (Turn).

	USE3S				Responsive			
	Ret	Vol	IR	Turn	Ret	Vol	IR	Turn
Min-volatility	9.71%	8.82%	1.10	181%	10.20%	8.06%	1.26	169%
Min-volatility (long-only, 2% monthly turnover)	8.18%	9.90%	0.83	29%	8.30%	9.22%	0.90	29%
Min-volatility (long-only, 4% monthly turnover)	8.78%	9.90%	0.89	53%	8.08%	8.99%	0.90	53%

	USE3L				Stable			
	Ret	Vol	IR	Turn	Ret	Vol	IR	Turn
Min-volatility	9.57%	8.91%	1.07	166%	10.63%	8.11%	1.31	156%
Min-volatility (long-only, 2% monthly turnover)	8.80%	9.87%	0.89	29%	8.64%	9.28%	0.93	29%
Min-volatility (long-only, 4% monthly turnover)	9.14%	9.81%	0.93	53%	8.43%	9.16%	0.92	53%

The following tables exhibit the average realized volatility of optimized style-tilt portfolios categorized by styles in the Barra US Long-Term Equity model (Responsive variant) only, two predecessor models (USE3 and USE4) only, and a combination of styles in all models.

Table K.5: Summary of realized volatility of optimized style-tilt portfolios.

	USE3S	Responsive	Difference
USE3 Styles	2.63%	2.44%	-0.19%
Long-Term Styles	2.36%	2.25%	-0.11%
All Styles	2.48%	2.33%	-0.15%

	USE3L	Stable	Difference
USE3 Styles	2.61%	2.46%	-0.16%
Long-Term Styles	2.37%	2.30%	-0.07%
All Styles	2.47%	2.37%	-0.11%

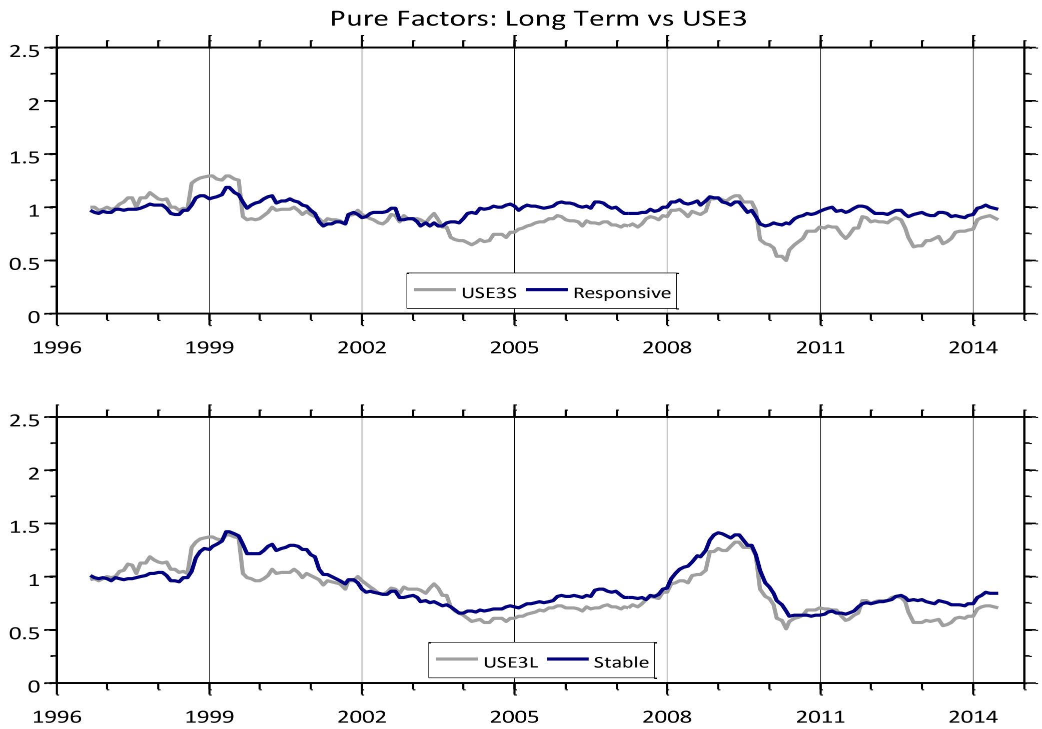
Table K.6: Summary of index tracking portfolios over entire sample period from July 1994 through July 2013

	USE3S	Responsive
Benchmark (250)	0.90%	0.78%
Benchmark (500)	0.48%	0.45%

	USE3L	Stable
Benchmark (250)	0.90%	0.82%
Benchmark (500)	0.47%	0.47%

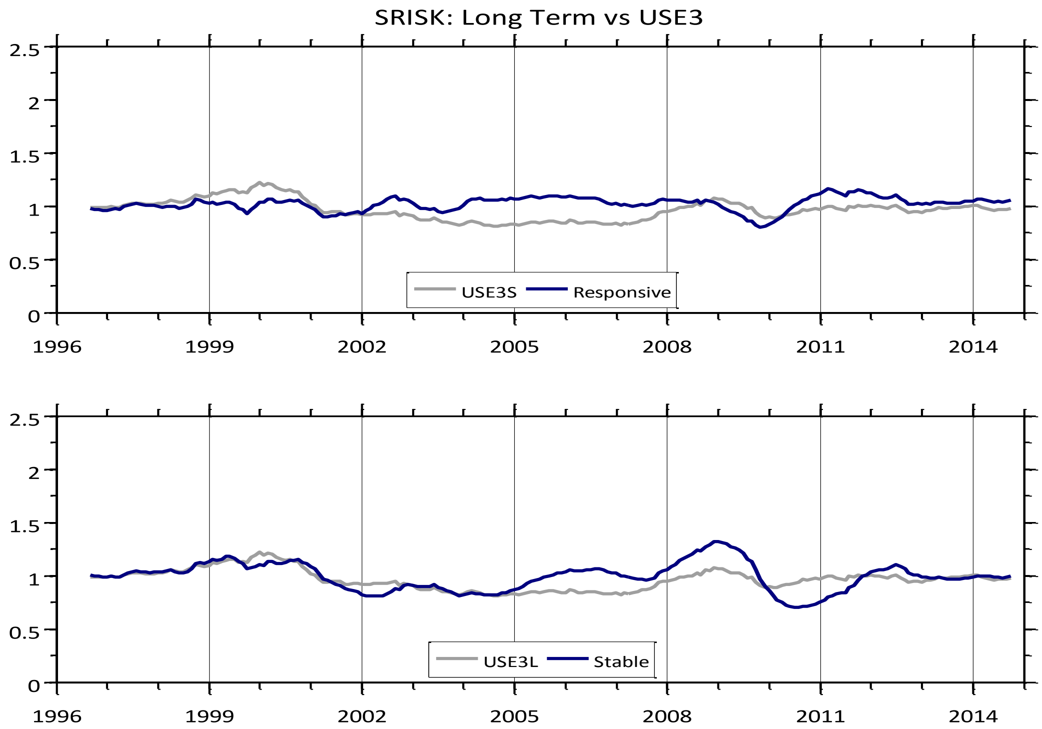
In **Figure K.1,** we plot the rolling window bias statistics for the pure factors. Throughout this paper, we use rolling windows of 12 months for all models.

Figure K.1: Rolling bias statistics for factor volatility forecast



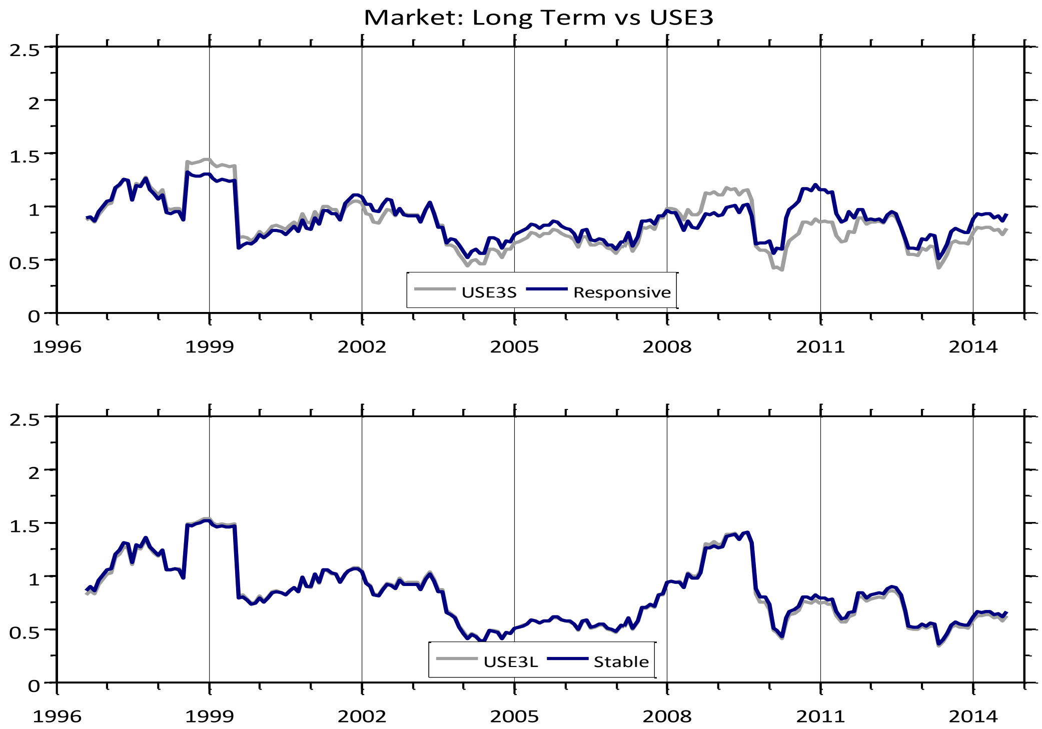
In **Figure K.2,** we plot the rolling average cross-sectional bias statistics for the specific risk forecasts for all stocks in the estimation universe. The cross-sectional bias statistic is cap-weighted.

Figure K.2: Rolling bias statistics for specific risk forecast



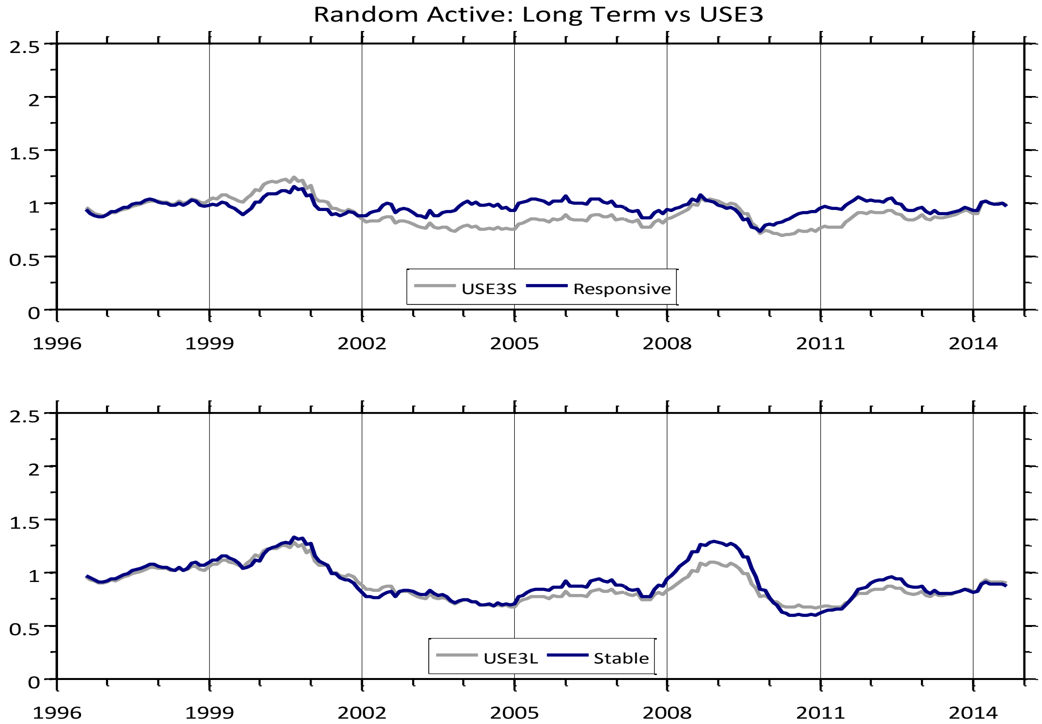
In **Figure K.3,** we plot the rolling window bias statistics for the cap-weighted portfolio of the stocks present in estimation universes of the two models.

Figure K.3: Comparison of bias statistics for the cap-weighted estimation universe portfolio



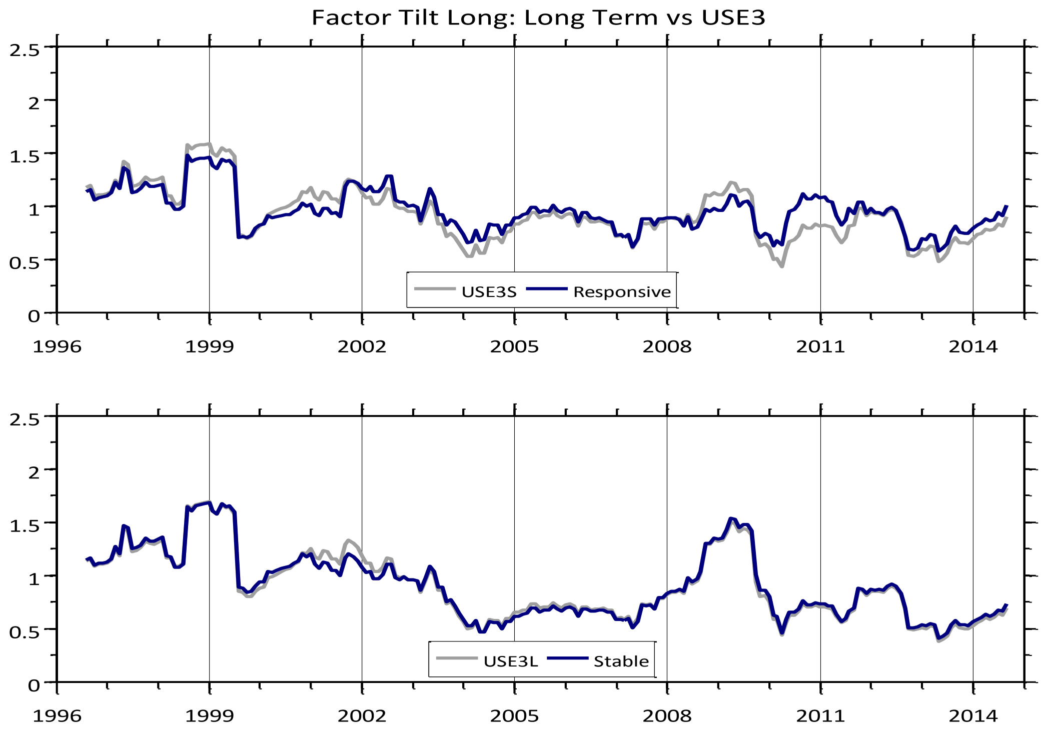
In **Figure K.4,** we plot the rolling window bias statistics for the active risk for 100 random portfolios. The portfolios are constructed by going long 100 randomly selected stocks and weighted by their market capitalization. The cap-weighted estimation universe portfolio in Figure 5.3 is used as the benchmark. To reduce turnover, the list of stocks used to construct the random portfolios is fixed unless a stock drops out of the estimation universe, in which case it is replaced by another randomly selected stock.

Figure K.4: Comparison of bias statistics for 100 random active portfolios



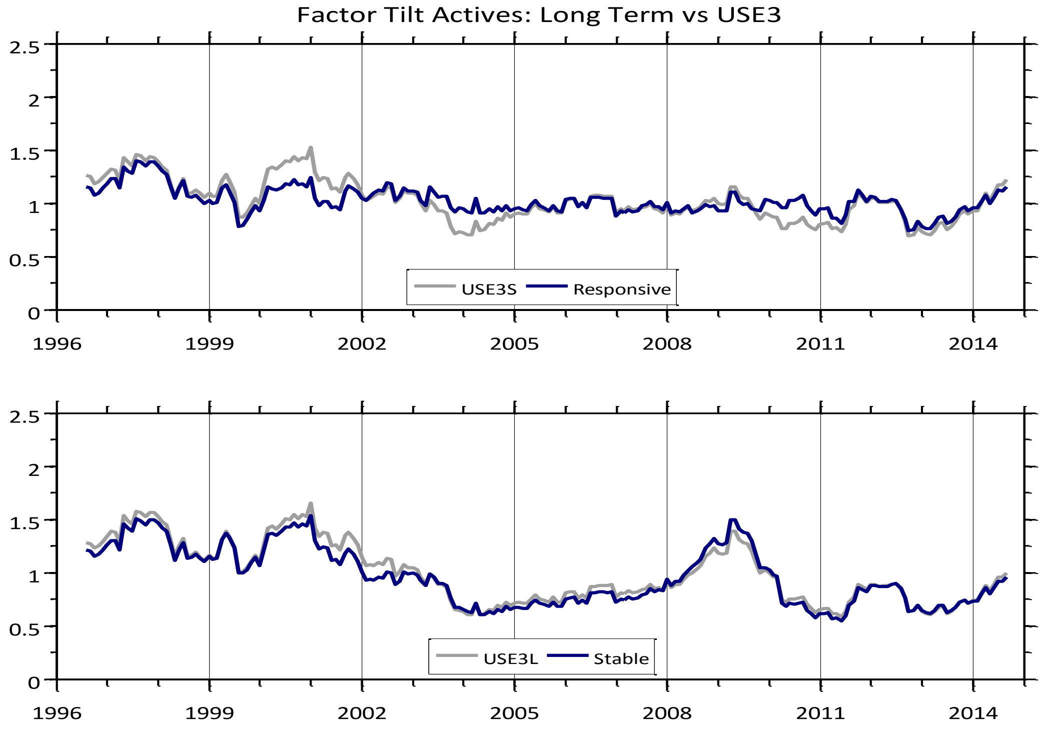
In **Figure K.5,** we plot rolling bias statistics for long-only factor-tilt portfolios. The cap-weighted portfolios were constructed for each industry and the top and bottom quintiles of each style factor.

Figure K.5: Comparison of bias statistics for long-only factor-tilt portfolios



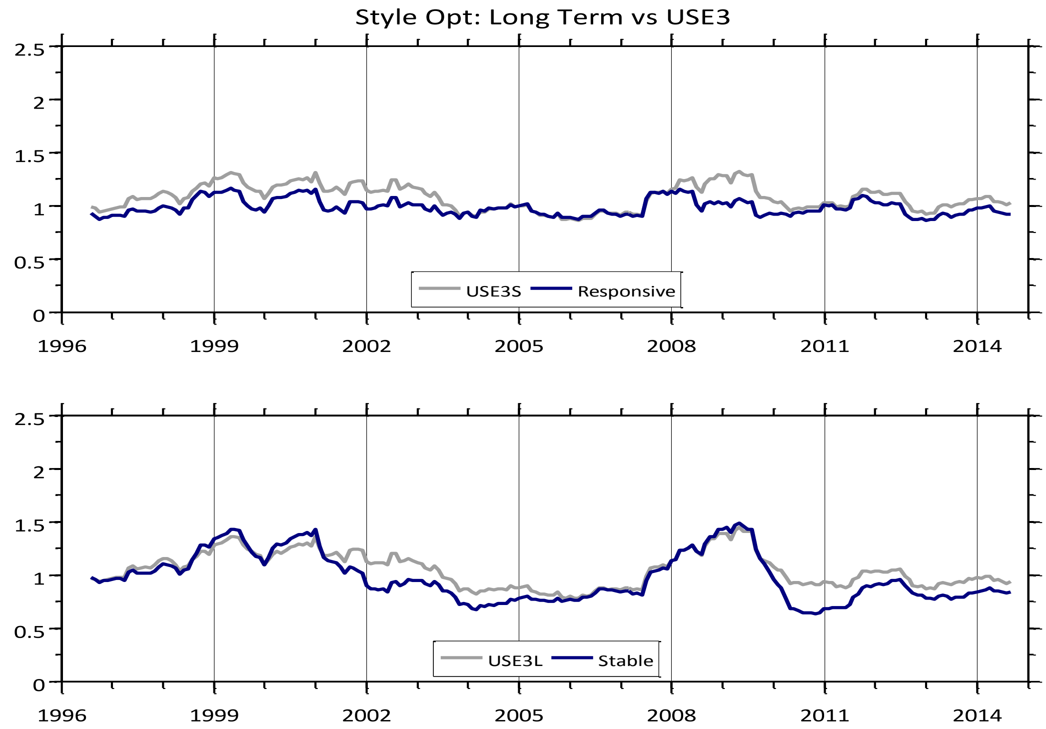
In **Figure K.6,** we plot rolling-window bias statistics for the active factor-tilt portfolios. The portfolios were constructed by going long the factor-tilt portfolios of Figure 5.5 and shorting the estimation universe portfolio.

Figure K.6: Comparison of bias statistics for active factor-tilt portfolios



In **Figure K.7,** we plot the rolling bias statistics for optimized style-tilt portfolios. These portfolios are constructed by using style factors from both models as “alpha signals” and forming the unit-alpha, minimum volatility portfolios for 20 draws of 100 random stocks. No other constraints are imposed in the portfolio construction.

Figure K.7: Rolling bias statistics of optimized style-tilt portfolios



In **Figure K.8 and Figure K.9,** we report the trailing 12-month of residual return variance in relation to total cross-sectional stock return variance for the Responsive variant of the Barra US Long-Term Equity Model and the USE3 model. Positive differences between the predecessor model and the Long-Term model indicate that the new model provides more accurate beta forecasts.

Figure K.8: Trailing 12-month cross-sectional residual variance relative to total variance for the Responsive variant of the Barra US Long-Term Equity Model and USE3 Short-Horizon.

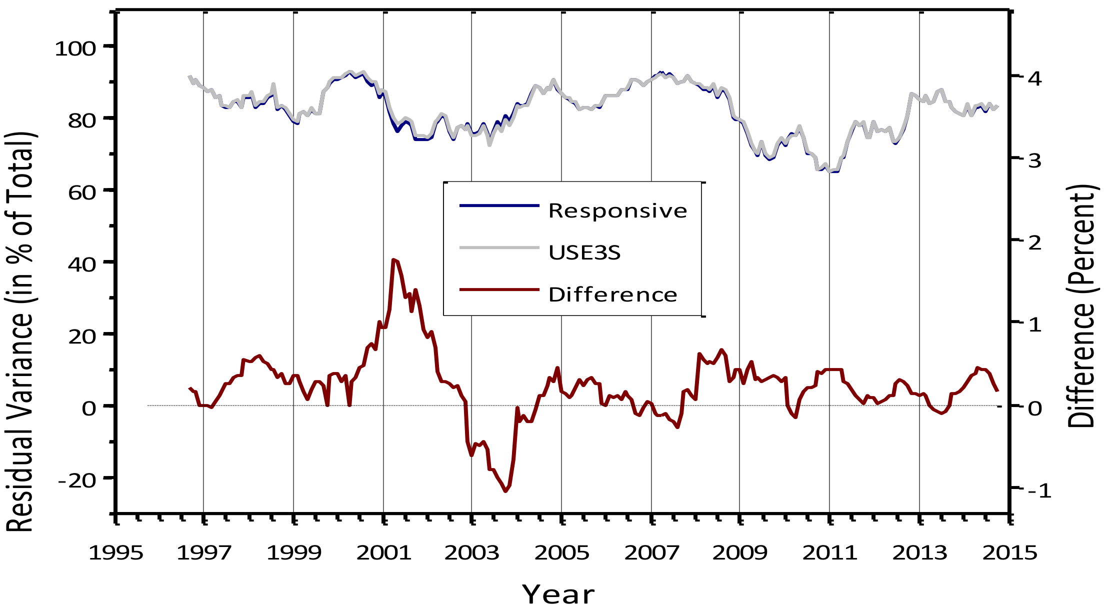
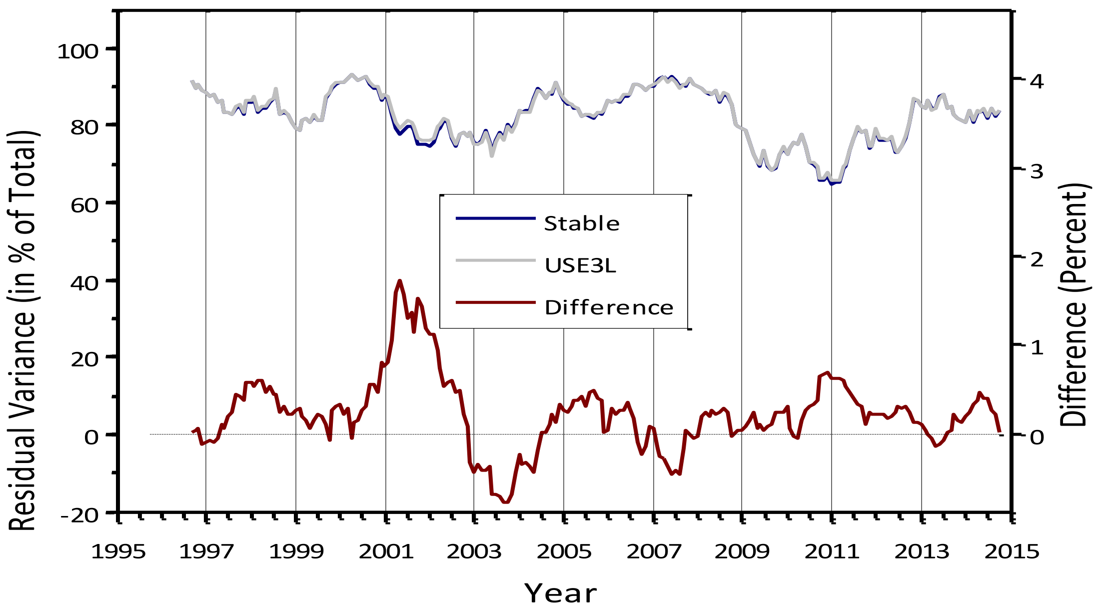


Figure K.9: Trailing 12-month cross-sectional residual variance relative to total variance for the Stable variant of the Barra US Long-Term Equity Model and USE3 Long-Horizon



In **Figure K.10**, we show the rolling 12-month volatility of the unconstrained minimum-risk portfolio. In **Figure K.11**, we depict the realized volatility of a long-only minimum-risk portfolio with 2% turnover and asset weight constraint of 3%.

Figure K.10: Realized volatility of analytical minimum risk portfolios estimated using 12-month rolling window

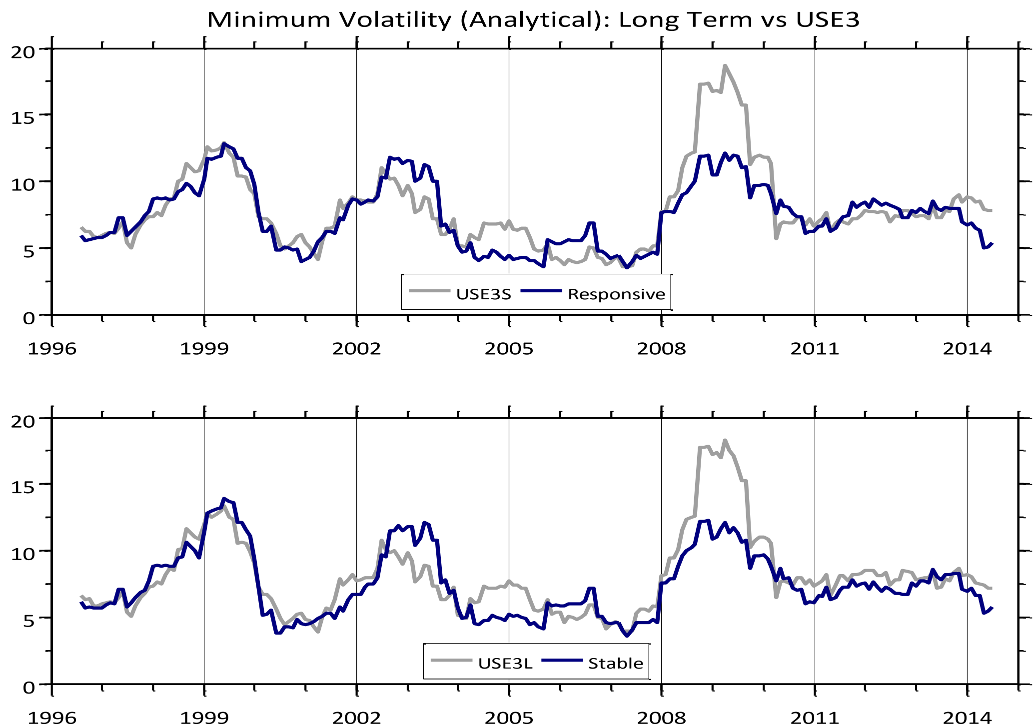
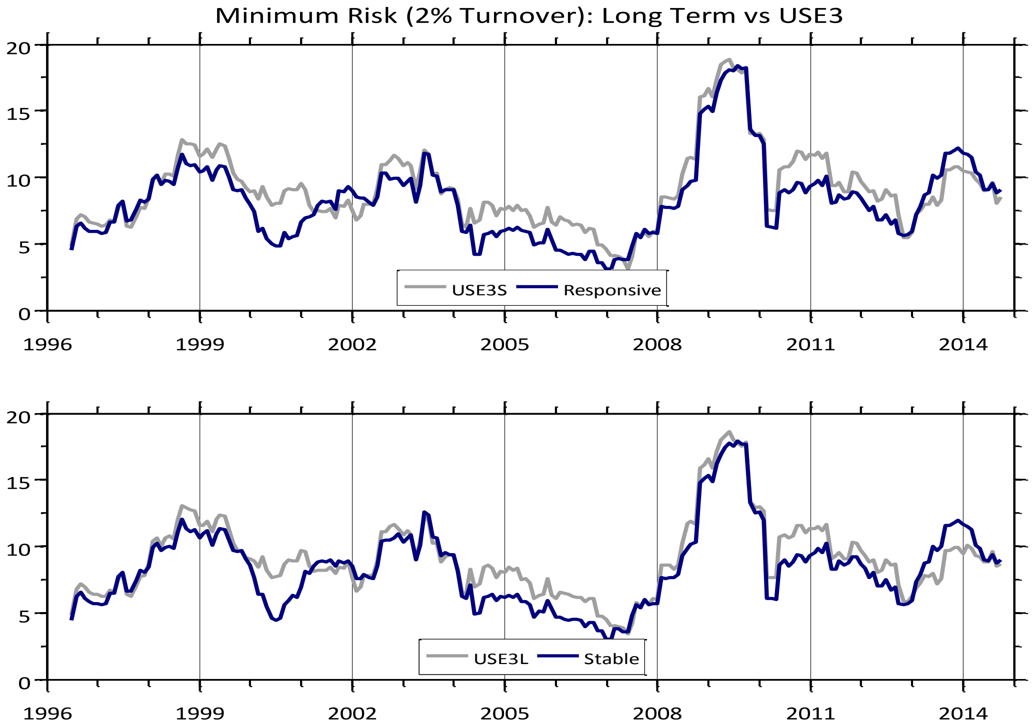
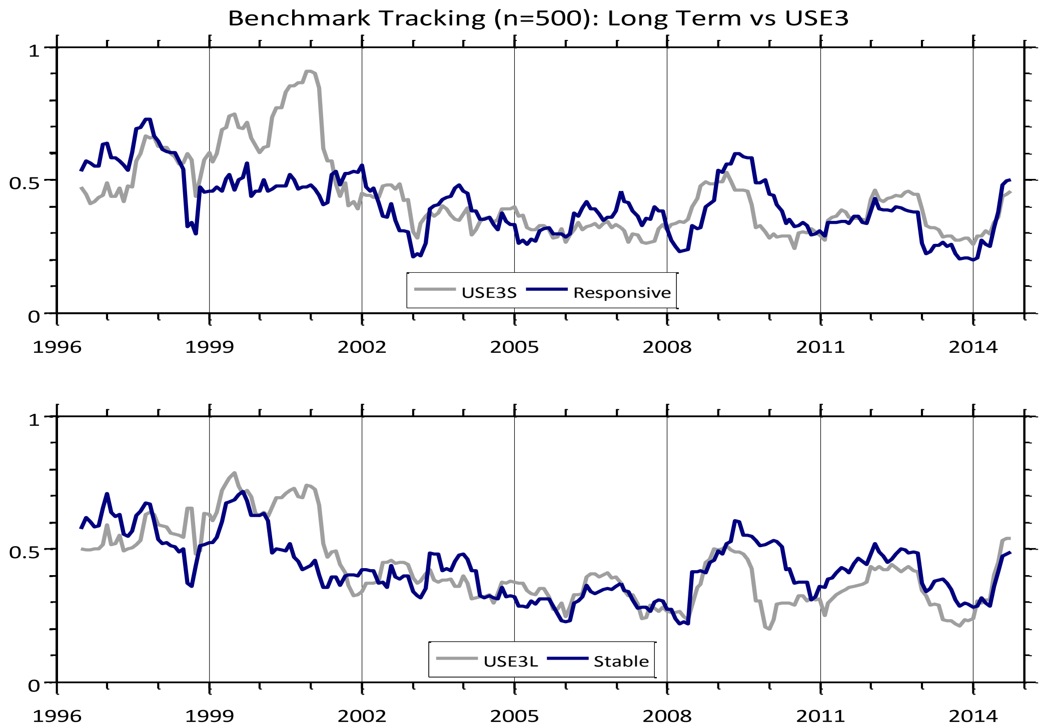


Figure K.11: Realized volatility of long-only minimum risk portfolios estimated using 12-month rolling window



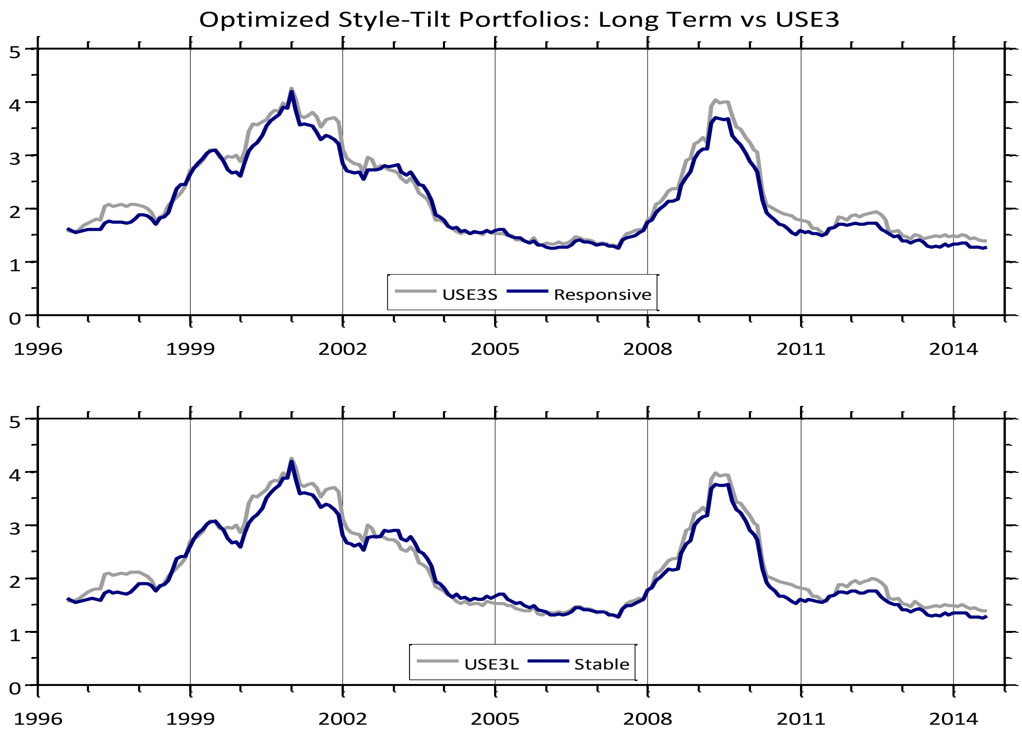
In **Figure K.12**, we show the rolling 12-month volatility of the index-tracking portfolio that is constructed using the Long-Term model and USE3.

Figure K.12: Realized volatility of index-tracking portfolios estimated using 12-month rolling window



In **Figure K.13**, we plot the average 12-month rolling volatility of all optimized style-tilt portfolios.

Figure K.13: Average realized volatility of all optimized style-tilt portfolios estimated using a 12-month rolling window



Appendix I: Model Estimation Parameters

Table I1 contains factor covariance model estimation parameters for the Barra US Total Market Equity Model for Long-Term Investors. The parameters are described in *USE4 Methodology Notes*.

Table I1: Factor covariance matrix parameters. All values are reported in trading days.

	Stable	Responsive
Factor Volatility Half-Life	252	84
Factor Serial Correlation Adjustment		
Half-Life	756	504
Newey-West Lags	10	10
Factor Correlation Matrix		
Half-Life	756	504
Newey-West Lags	4	4
Factor Volatility Regime Adjustment Half-Life	N/A	21
Eigenfactor Correction	N/A	On Correlation Matrix

Table I2 contains specific risk model estimation parameters for the Barra US Total Market Equity Model for Long-Term Investors. The parameters are described in *USE4 Methodology Notes*.

Table I2: Specific risk parameters. Except for the dimensionless shrinkage parameter q, all values are reported in trading days.

	Stable	Responsive
Specific Volatility Half-Life	252	84
Specific Serial Correlation Adjustment		
Half-Life	252	252
Newey-West Lags	5	5
Specific Volatility Regime Adjustment Half-Life	N/A	42
Bayesian Shrinkage Parameter	0.05	0.05

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[1] *As of June 30, 2014, as reported on September 30 2014 by eVestment, Morningstar and Bloomberg* *Nov 2014*