Understanding the Tails of the Return Distribution | May 2009

Modeling Financial Time Series

Traditional models in finance rely heavily on the use of the normal (Gaussian) distribution. For example, Value at Risk (VaR) measures are often calculated under the assumption that the underlying return series are normally distributed. Implied volatilities are calculated from option prices based on geometric Brownian motion, which is another manifestation of the normal distribution. The implication of normality is that abrupt changes in asset prices have a very low probability. Figure 1 shows a price series of a large pharmaceutical company and a simulated series that assumes normality (using the same mean and standard deviation). Could you tell by looking at chart which one is which?

While the two price series may look similar, the answer is clearly revealed when we plot the time series of actual and simulated returns in Figure 2. Both series have the same variance, but the simulated Brownian motion series achieves it by generating returns which always have roughly the same amplitude. The actual return series of Merck are more widely dispersed in their amplitude and manifest a few large peaks corresponding to jumps in the price. The largest price drop at the end of September 2004 corresponds to the period when the company was forced to withdraw its blockbuster arthritis drug Vioxx from the market on concerns over its side effects.

Figure 1: Actual and simulated share price series

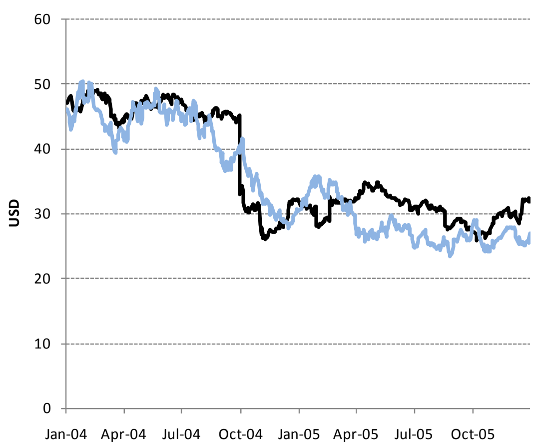
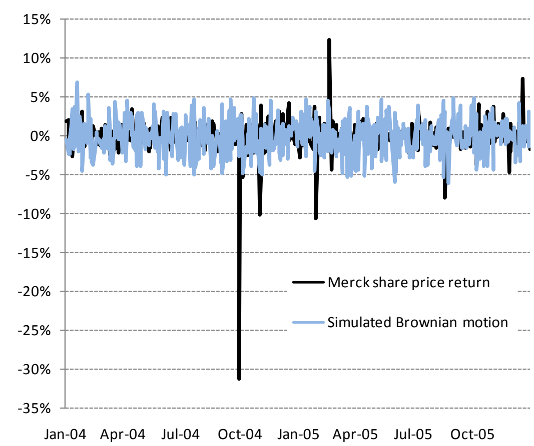


Figure 2: Actual and simulated return series



The comparison of the simulated Gaussian return series and the empirical series illustrates that the distribution of price changes often has fat tails relative to the normal distribution. This means that large negative and positive returns have a higher empirical probability of occurring than the normal model would postulate. This same phenomenon is observed at the level of factor and index returns, as well as for individual securities. Figure 3 shows the estimate of the distribution of the daily returns to the momentum factor in the Barra Global Equity Model (GEM 2) over the course of 2007 and compares it to a normal distribution with the same mean and variance. We observe that while the core of the empirical distribution is more condensed, both tails are fatter. In particular, we can see that large negative daily returns occur far more often than the normal distribution suggests. The largest negative daily return to the momentum factor occurred during the August quant crisis—the daily factor return of -0.7% corresponds to a Z-score of -5.5 (or a probability of occurring approximately once in every two hundred thousand years under the normal distribution).

Figure 3: Distribution of daily momentum factor returns in GEM 2 versus the normal distribution

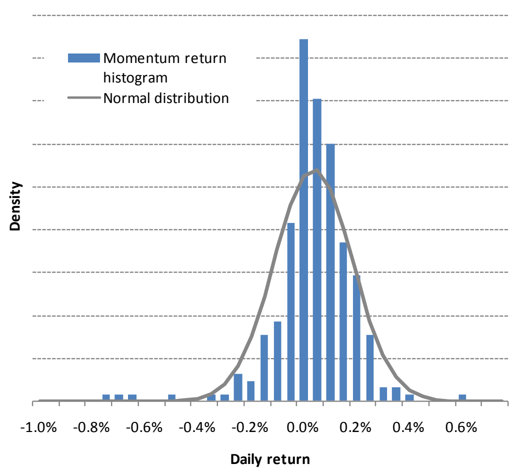
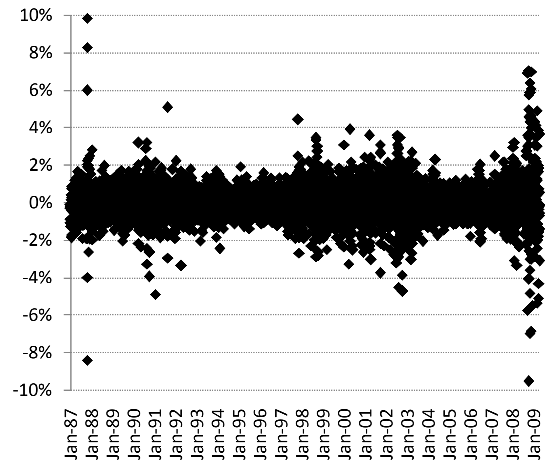


Figure 4: One-day losses in MSCI World Index



Large losses may occur due to a general rise in volatility, or they can occur as isolated events (jumps), or as a combination of the two. In general, financial time series are characterized by both changing volatility dynamics and a non-trivial probability of returns of several standard deviations in magnitude. Figure 4 illustrates this using the example of losses (returns with the sign changed, so negative returns take positive values in the chart) in the MSCI World Index. Graphically, we can think of volatility as a thickness of the cloud around zero and isolated extreme gains and losses as the points that are outside that cloud. The crash of 1987 was characterized mainly by a few extreme events. Then, through the early 1990s, volatility declined—the cloud shrunk. The events surrounding the Asian and Russian crises of 1997-1998 as well as the technology bubble can be seen mostly as an increase in volatility with few true extremes. Through 2003-2007, volatility declined again with almost no extreme returns. The latest volatile episode is clearly a mixture of a substantial rise in the general level of volatility—the cloud widens aggressively—as well as some well pronounced extremes.

The discussion above highlights the fact that investors cannot rely on a single risk measure. If returns followed a normal distribution, then volatility would be a complete measure of risk. However, we see with examples of asset, factor and index-level returns that the assumption of normality does not capture the empirical properties of returns, so volatility alone does not capture all aspects of portfolio risk. The number of empirically-observed extreme events far outnumbers the number of events forecast by the normal distribution.

Extreme Value Theory and the Estimation of Tail Risk

Non-Gaussian distributions are not unique to financial markets. Researchers in diverse fields, such as hydrology and structural engineering, have also modeled non-Gaussian distributions using *extreme value theory*, which describes the probability of extreme events. A direct approach to estimating the tail of the distribution is a semi-parametric procedure known as “peaks over thresholds.” This method takes the core of the distribution from historical data and fits the tails parametrically—illustrated in Figure 5, taken from Goldberg, Miller and Weinstein (2008). Under mild assumptions, given independent identically distributed data, the distribution of events beyond a certain threshold tends to follow a generalized Pareto distribution, as the threshold becomes large. The formula for the generalized Pareto distribution is

$$G_{\alpha,\beta}(x) = \begin{cases} 1-\left(1+\dfrac{x}{\alpha\beta}\right)^{-\alpha} & \text{if } 1/\alpha \neq 0, \\ 1-e^{-x/\beta} & \text{if } 1/\alpha = 0, \end{cases}$$

where

$$x \in \begin{cases} [0,\infty) & \text{if } 1/\alpha \geq 0, \\ [0,-\alpha\beta) & \text{if } 1/\alpha < 0. \end{cases}$$

A Pareto distribution is specified by a scale parameter $\beta > 0$ that adjusts the scale of the excesses, and a tail index α that can be positive, negative or infinite, but not zero. The value of $\alpha = \infty$, which is a limiting case of the distribution for a large, positive α ,corresponds to a diverse collection of initial distributions, including the normal. Positive α indicates that the initial distribution follows a power law, while negative α corresponds to an initial distribution with a finite upper limit. As a consequence, this approach can be used to estimate both heavy- and light-tailed distributions. This is illustrated in Figure 6, where the dotted line corresponds to $\alpha \to \infty$.

Figure 5: Modeling loss innovation to S&P 500 (1980-2005)

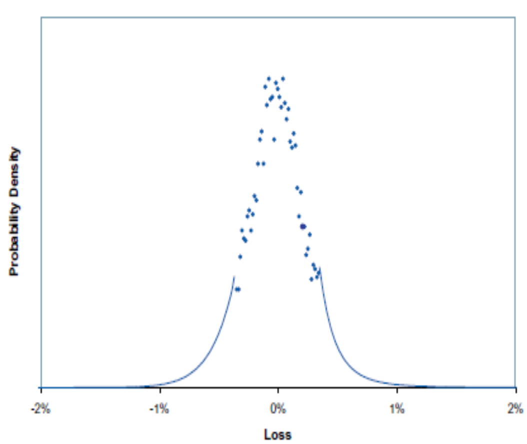
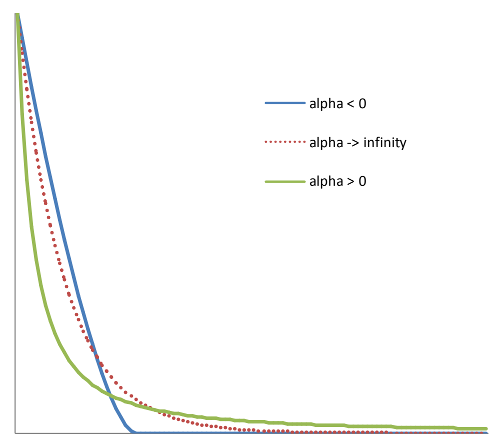


Figure 6: The probability density function of the Generalized Pareto Distribution for various values of alpha



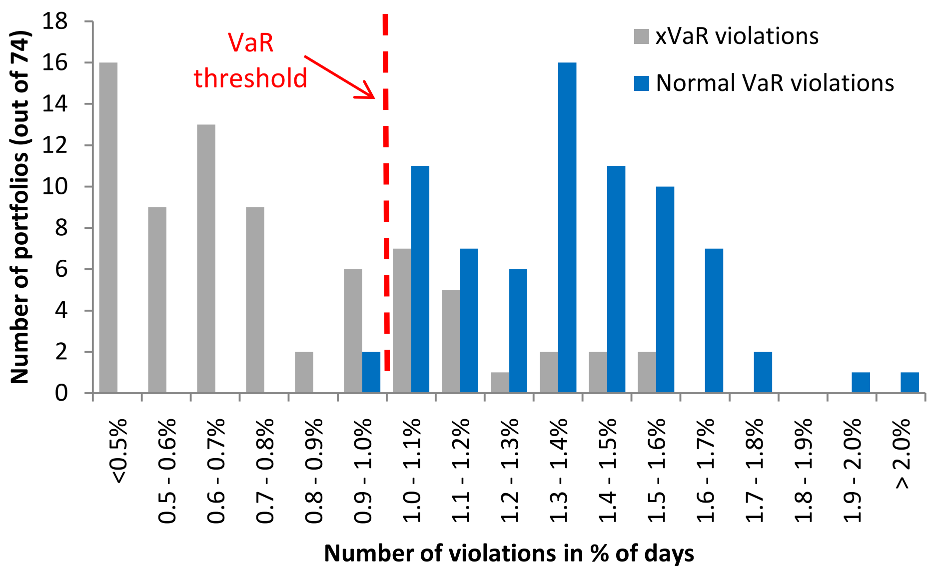
Accounting for Extremes: A Practical Example

Extreme value theory is a broader statistical theory than that specified by a normal distribution, while still covering normal as a special case. Let us consider a practical example, further described in Goldberg, et al (2008), of how extreme value theory can provide a better reflection of the tail risk. We shall compare the relative robustness of the traditional VaR and Barra Extreme VaR (xVaR) for a variety of portfolios composed of US equities. Barra Extreme Risk offers a robust method for estimating tail risk by relying on a long history of factor returns from Barra equity models. Daily returns are taken from December 1996 to October 2007, a period that covers major events, including the Asian crisis, LTCM, the Tech Bubble, September 11, and the Quant Meltdown in August 2007. VaR figures are generated using two methods: the traditional way, in which returns are assumed to be normally distributed and exponentially-weighted across time, as well as using the Barra Extreme Risk methodology. We choose a confidence level of

99%, and a time horizon of one day, which implies that the resultant VaR forecasts should represent the maximum daily loss that would be incurred with 99% probability.

In order to compare the two measures over a variety of different portfolios, a collection of 74 factor-tilted portfolios are created. It is important to note that the VaR numbers generated here are forecast values. Figure 7 shows a histogram of portfolios by percentage of VaR violations. The horizontal axis shows the percentage of days in which actual losses are greater than the VaR, while the vertical axis displays the number of portfolios (out of the 74 in our sample) within each interval. Ideally, all portfolios should be to the left of the broken line. This is true for the majority of the portfolios when using xVaR, but does not hold in the case of the traditional VaR. While about 75% of the portfolios meet this criterion under xVaR, only 3% do so in the case of the traditional VaR. This evidence demonstrates the relative robustness of Barra Extreme Risk methodology in determining tail risk.

Figure 7: Histogram of portfolios by percentage of VaR violations



Conclusion

If returns followed a normal distribution, then volatility would be a complete measure of risk. In this paper, we used examples of asset, factor and index returns, where the assumption of normality does not capture the empirical properties of returns, and volatility alone is not a reliable measure of portfolio risk. We also outlined how extreme value theory can help to more accurately model the tails of the return distribution. Finally, we illustrate how using extreme value theory can improve estimates of VaR for a collection of factor-tilted portfolios using data from 1996 to 2007.

This is the second in a series of three bulletins outlining the use of extreme value theory in risk management and portfolio construction. The first bulletin *Managing Risk Beyond the Normal Distribution* (May 7, 2009) is available from www.mscibarra.com, and the third bulletin, *Shortfall in Portfolio Construction,* will be released in early June 2009.

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Goldberg, L., G. Miller and J. Weinstein (2008) Beyond Value at Risk: Forecasting Portfolio Loss at Multiple Horizons, *Journal of Investment Management*, 6(2), pp. 73-98.

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