HOMEWORK 4

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Solution 1

Solution 1.1: Strategy 1

If we $\hat{x} \in argmax_x \theta_x$, and if let j be the x at which the largest probability is taken, then $E[1[\hat{x} = j]] = \theta_j$. Therefore, $E[1[\hat{x} \neq j]] = 1 - \theta_j$. By changing this notation j to x, we get the following equation.

$$E[1[\hat{x} \neq x]] = 1 - \theta_x$$

Solution 1.2: Strategy 2

Suppose $\hat{x}$ is generated from a multinomial distribution. $\hat{x} = x$ because the probability of x and $\hat{x}$ occurring is θ_x, which is θ_x^2 since the two events are independent. Therefore, we get below:

$$E[1[\hat{x} \neq x]] = 1 - \theta_x^2$$

Solution 2

The probability that $x = i$ occurs can be expressed as θ_i, and the probability that $\hat{x} = j$ occurs can be expressed as θ_j. Therefore, we can write $c_{i,j} = \theta_i\theta_i$.
Under the knowledge of θ, to minimize the expected value of $c_{i,j}$, it is better to set $\hat{x}$ to be the index representing the lowest probability among θ vector. Because θ_i is given and what we can not control, θ_i is treated as a constant. As a result, to minimize $c_{i,j} = \theta_i\theta_i$, it is a necessary and sufficient condition to minimize θ_j.

Solution 3

Solution 3.1

Let the mean of the distribution of Y_t be μ and the variance be σ^2. In the setting of this question, since we know these two variables, the optimal strategy is to set $x_t = \mu$ at any time. It is because $E[y_t] = \mu$, meaning that the value of y_t that is most likely to appear is μ.
At this time, the expected payment for the Tth time is as follows:

$$\begin{aligned} E[T(x_t - y_t)^2] &= T(\mu^2 - 2\mu E[y_t] + E[y_t^2)]) = T(\mu^2 - 2\mu^2 + \sigma^2 + \mu^2) \\ &= T\sigma^2 \end{aligned}$$

During the development of the above equation, I used the following:

$$\begin{aligned} Var(y_t) = \sigma^2 &= E[y_t^2] - (E[y_t])^2 = E[y_t^2] - \mu^2 \\ \therefore E[y_t^2] &= \sigma^2 + \mu^2 \end{aligned}$$

Solution 3.2

Based on previous problem, we can set the "payment" function below:

$$argmin_{x_t} Payment = T(E[x_t]^2 - 2E[x_t]E[y_t] + E[y_t^2)]) \tag{1}$$

At the end of the mth enforcement, calculate the following with the realized values.

$$E[x_t] = \frac{1}{m}\sum_{i=1}^{m} x_i = \bar{x}$$
$$E[x_t]^2 = \frac{1}{m}\sum_{i=1}^{m}(x_i - \bar{x})^2 + \bar{x}^2 = Var(x) + \bar{x}^2$$
$$E[y_t] = \frac{1}{m}\sum_{i=1}^{m} y_i = \bar{y}$$
$$E[y_t]^2 = \frac{1}{m}\sum_{i=1}^{m}(y_i - \bar{y})^2 + \bar{y}^2 = Var(y) + \bar{y}^2$$

Then, substitute the above four equations into equation (1).

$$\begin{aligned} argmin_{x_t} Payment &= T(\frac{1}{m}\sum_{i=1}^{m}(x_i - \bar{x})^2 + \bar{x}^2 + \frac{1}{m}\sum_{i=1}^{m}(y_i - \bar{y})^2 + \bar{y}^2 - 2\bar{x}\bar{y}) \\ &= T(Var(x) + Var(y) + (\bar{x} - \bar{y})^2) \end{aligned} \tag{2}$$

Calculate equation (2) above for the $m+1$th time as well, and if equation (2), or the benchmark, is lower than the mth time, we can conclude that the strategy we are currently using is correct. In other words, equation (2) is the benchmark.

Solution 4

Solution 4.1

From the setting of this question, $N = 30$, $K_L = 3$, and $\alpha = 0.5$. From the setup of this question, $N = 30$, $K_L = 3$, and $\alpha = 0.5$. Using these, calculate the prior probabilities.

$$\begin{aligned} \hat{p}_\alpha(y = e) &= \frac{\sum_{i=1}^{N} 1[y^{(i)} = e] + \alpha}{N + K_L\alpha} \\ &= \frac{10 + 0.5}{30 + 3 * 0.5} = \frac{3}{10} = 0.33 \end{aligned}$$

Similarly, the two prior probabilities of ta are calculated and the answers are as follows.

$$\hat{p}_\alpha(y = e) = \hat{p}_\alpha(y = j) = \hat{p}_\alpha(y = s) = \frac{3}{10} = 0.33$$

Solution 4.2

From the given conditions, calculate the following using $p(y = e) = \frac{1}{3}$.

$$\theta_{i,e} = \hat{p}(c_i|y = e) = \frac{\hat{p}(c_i \cap y = e)}{\hat{p}(y = e)} = \frac{10c_i}{3M_e}$$

Here, M_e represents the total number of characters in the data labeled $y = e$. Then, compute the θ for each i. θ_e is below:

Table 1: θ_e

character	probability
a	0.06
b	0.01
c	0.02
d	0.02
e	0.11
f	0.02
g	0.02
h	0.05
i	0.05
j	0.0
k	0.0
l	0.03
m	0.02
n	0.06
o	0.07
p	0.02
q	0.0
r	0.05
s	0.07
t	0.08
u	0.03
v	0.01
w	0.02
x	0.0
y	0.01
z	0.0
(space)	0.18

Solution 4.3

Table 2: θ_j

character	probability
a	0.13
b	0.01
c	0.01
d	0.02
e	0.06
f	0.0
g	0.01
h	0.03
i	0.1
j	0.0
k	0.06
l	0.0
m	0.04
n	0.06
o	0.09
p	0.0
q	0.0
r	0.04
s	0.04
t	0.06
u	0.07
v	0.0
w	0.02
x	0.0
y	0.01
z	0.01
(space)	0.12

Table 3: θ_s

character	probability
a	0.1
b	0.01
c	0.04
d	0.04
e	0.11
f	0.01
g	0.01
h	0.0
i	0.05
j	0.01
k	0.0
l	0.05
m	0.03
n	0.05
o	0.07
p	0.02
q	0.01
r	0.06
s	0.07
t	0.04
u	0.03
v	0.01
w	0.0
x	0.0
y	0.01
z	0.0
(space)	0.17

Solution 4.4

Table 4: x of e10.txt

character	probability
a	164
b	32
c	53
d	57
e	311
f	55
g	51
h	140
i	140
j	3
k	6
l	85
m	64
n	139
o	182
p	53
q	3
r	141
s	186
t	225
u	65
v	31
w	47
x	4
y	38
z	2
(space)	498

Solution 4.5

Taking the logarithm of the equation, we get $\log \hat{p}(x|y) = \sum_{i=1}^{d} x_i \theta_{i,y}$. Based on previous problems, $\log \hat{p}(x|y = e) = -7840.77$, $\log \hat{p}(x|y = j) = -8727.17$, and $\log \hat{p}(x|y = s) = -8478.20$.

Solution 4.6

Taking the logarithm of the Bayes law, we get $\log \hat{p}(y|x) = \log \hat{p}(x|y) + \log \hat{p}(y)$. Then, based on previous problems, $\log \hat{p}(y = e|x) = -7841.87$, $\log \hat{p}(y = j|x) = -8728.27$, and $\log \hat{p}(y = s|x) = -8479.30$.

Solution 4.7

The answer table is below.

Table 5: Classification among English, Spanish and Japanese

	English	Spanish	Japanese
English	10	0	0
Spanish	0	10	0
Japanese	0	0	10

Solution 4.8

Shuffling the order of the letters does not affect the results of the estimation using this Naive Bayes classifier's prediction. This is because the order information has been omitted when creating the bag-of-words for each training and validation data, i.e. mathematically, when calculating the likelihood.

Solution 5

Solution 5.1

Set z as $z = W_2\sigma(W_1 x)$ and $\delta_{i,j} = 1(\text{when } i = j)$, 0 otherwise.

$$\begin{aligned}
\frac{\partial L}{\partial x} &= \frac{\partial L}{\partial \hat{y}} \frac{\partial g(z)}{\partial z} \frac{\partial z}{\partial \sigma(W_1 x)} \frac{\partial \sigma(W_1 x)}{\partial (W_1 x)} \frac{\partial W_1 x}{\partial x} \\
&= -\sum_{i=1}^{k} \frac{y}{\hat{y}} y_i(\delta_{i,j} - y_j) W_2 \sigma(W_1 x)(1 - \sigma(W_1 x)) W_1 \\
&= (\hat{y} - y) W_2 \sigma(W_1 x)(1 - \sigma(W_1 x)) W_1
\end{aligned}$$

Solution 5.2

Figure 1: Learning curve by my model

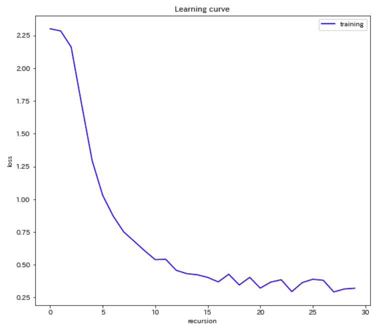
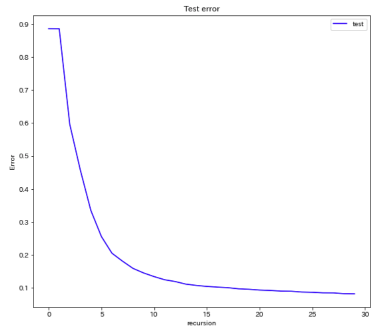


Figure 2: Test error by my model



Solution 5.3

Figure 3: Learning curve by pytorch model

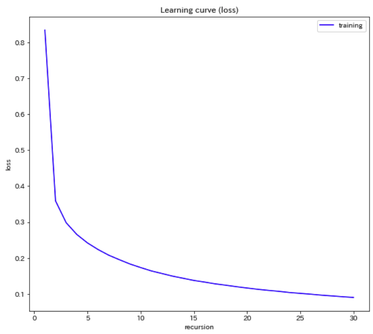
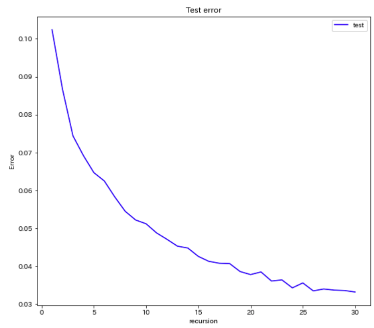


Figure 4: Test error by pytorch model



Solution 5.4.a

Figure 5: Learning curve by pytorch model

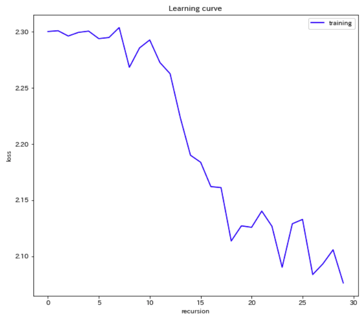
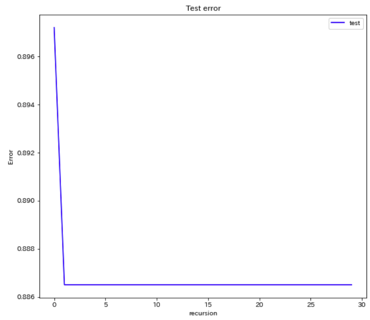


Figure 6: Test error by pytorch model



Solution 5.4.b

Figure 7: Learning curve by pytorch model

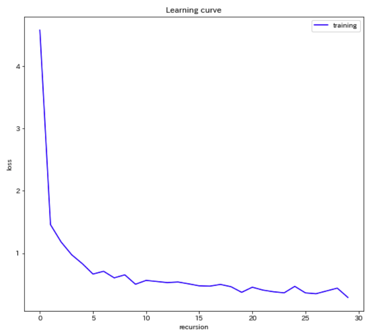


Figure 8: Test error by pytorch model