COMP5421 Homework4 Report

Problem 1. Theory

Q1.1. We know that in the given case, both coordinates in homogenous are $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$.

Because $X_m^T F X_m = 0$

we have $[0, 0, 1] \begin{bmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = F_{33} = 0$

So $F_{33} = 0$

Q1.2 the translation matrix $t = \begin{bmatrix} t_1 \\ 0 \\ 0 \end{bmatrix}$

no rotation so $R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$

$E = R t_x$

we know $t_x = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_1 \\ 0 & t_1 & 0 \end{bmatrix}$

So $E = I \times t_x = t_x = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_1 \\ 0 & t_1 & 0 \end{bmatrix}$

so $l_1 = E \tilde{x}_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_1 \\ 0 & t_1 & 0 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -t_1 \\ b_2 t_1 \end{bmatrix}$

$l_2 = E \tilde{x}_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_1 \\ 0 & t_1 & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -t_1 \\ a_2 t_1 \end{bmatrix}$

write in the common form: $l_1 \quad -t_1 y + b_2 t_1 = 0$

$l_2 \quad -t_1 y + a_2 t_1 = 0$

we could easily see that they are all parallel to x axies since they don't have x elements.

Q1.3 suppose we have $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$ as 3D points and $\begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix}$ $\begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix}$ as points on 2D respect to 2 cameras.

So $\begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} = K(R_1 \begin{bmatrix} a \\ b \\ c \end{bmatrix} + t_1)$

inverse it $\begin{bmatrix} a \\ b \\ c \end{bmatrix} = R_1^{-1} (K^{-1} \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} - t_1)$

$= R_1^{T} K^{-1} \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} - R_1^{T} t_1$

$\begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} = K(R_2 \begin{bmatrix} a \\ b \\ c \end{bmatrix} + t_2)$ plug the previous equation in.

$$= K R_2 R_1^T K^{-1} \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} - K R_2 R_1^T t_1 + K t_2$$

So $R_{rel} = K R_2 R_1^T K^{-1}$

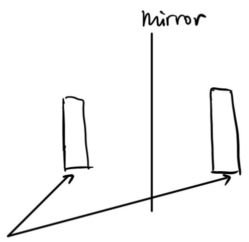
$t_{rel} = -K R_2 R_1^T t_1 + K t_2$

$E = t_{rel} \times R_{rel}$

$F = (K^{-1})^T E K^{-1}$

$= (K^{-1})^T (t_{rel} \times R_{rel}) K^{-1}$

Q1.4



we could see that, when the camera view this two objects. the transformation between them is actually just translation.

So $R = I$ $\quad t = \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}$

From 1.3, we could obtain

$$F = (K^{-1})^T (t_{rel} \times R_{rel}) K^{-1}$$

$$= (K^{-1})^T \begin{bmatrix} 0 & t_z & -t_y \\ -t_z & 0 & t_x \\ t_y & -t_x & 0 \end{bmatrix} K^{-1}$$

if we transpose F, we get

$$F^T = (K^{-1})^T \begin{bmatrix} 0 & -t_z & t_y \\ t_z & 0 & -t_x \\ -t_y & t_x & 0 \end{bmatrix} K^{-1}$$

we see that this is equivalent to $-F$

$$F^T = -F$$

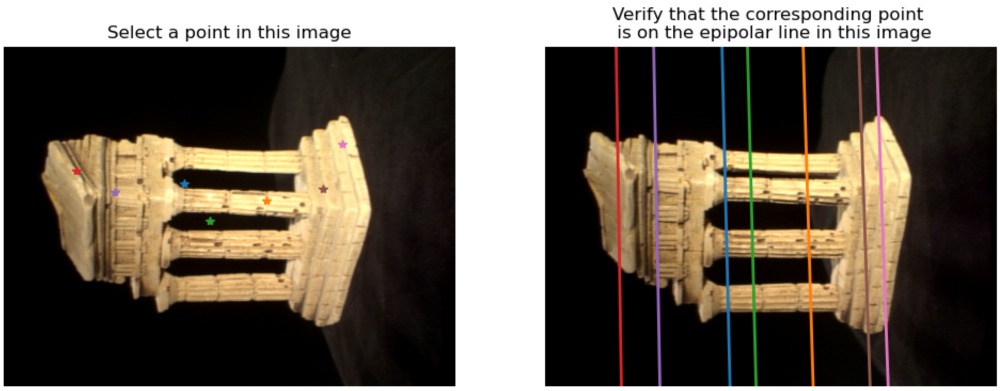
So F is skew-symmetric.

Problem 2.1. Eight-Point Algorithm

The matrix F is:

```
[[ 9.80213865e-10 -1.32271663e-07  1.12586847e-03]
 [-5.72416248e-08  2.97011941e-09 -1.17899320e-05]
 [-1.08270296e-03  3.05098538e-05 -4.46974798e-03]]
```

And the image view is:



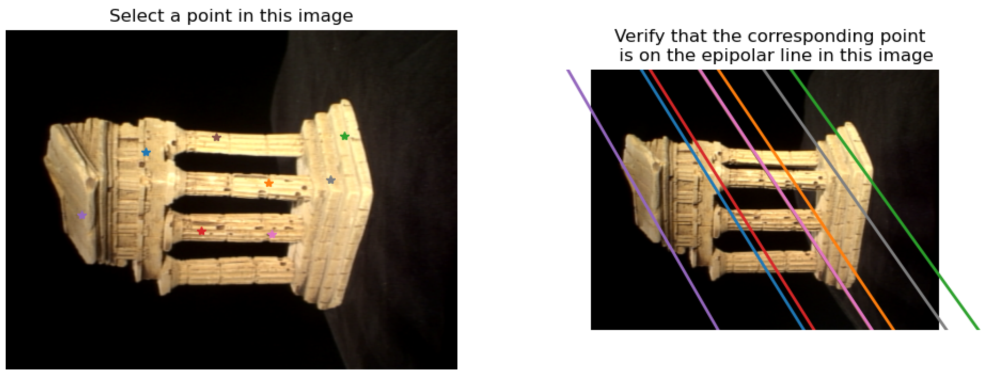
Problem 2.2 Seven-Point Algorithm.

We randomly picked the seven points from the database and compute the fundamental matrix.

We tried several times and find that once we could get result like this:

```
[[ 9.05782568e-08  2.10609816e-08  9.84762396e-04]
 [-3.91155235e-07  1.71458944e-07 -5.79509543e-04]
 [-9.75321578e-04  5.31396432e-04  9.02122037e-03]]
```

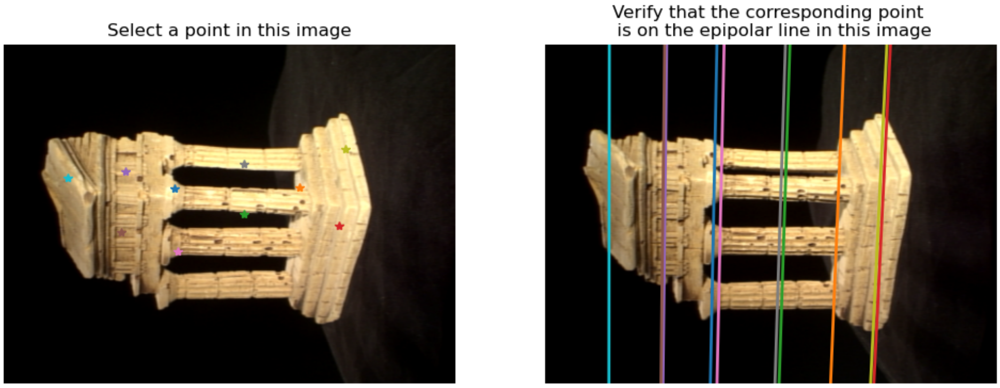
And the visualization is:



First, it is purely random and largely depends on our chosen data points. Sometimes we got some very bad result.

Second, this is still not as good as the previous 8-point algorithm.

> *Below is another try for the algorithm and we get a good result. But since it is purely random I could not recurrent it.*



Problem 3.1.

The essential matrix is

```
[[ 2.26587821e-03 -3.06867395e-01  1.66257398e+00]
 [-1.32799331e-01  6.91553934e-03 -4.32775554e-02]
 [-1.66717617e+00 -1.33444257e-02 -6.72047195e-04]]
```

Problem 3.2. Triangulate

The matrix of M2 is attached below for reference:

```
[[ 0.99942697  0.03331533  0.00598477 -0.02599827]
 [-0.03372859  0.96531605  0.25889634 -1.        ]
 [ 0.00284802 -0.25894984  0.96588657  0.07961991]]
```

Q3.2 we need to construct matrix A

C is a 3x4 matrix

$C_1 P_i = \tilde{x}_1$ $C_2 P_i = \tilde{x}_2$

So $\begin{bmatrix} C_{11} \\ C_{12} \\ C_{13} \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ 1 \end{bmatrix} = \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix}$ $\begin{bmatrix} C_{21} \\ C_{22} \\ C_{23} \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ 1 \end{bmatrix} = \begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix}$

So $C_{11} P = x_1$ $C_{21} P = x_2$

$C_{12} P = y_1$ $C_{22} P = y_2$

$C_{13} P = 1$ $C_{23} P = 1$

View x_1, y_1, x_2, y_2 as $x_1 * 1$, $y_1 * 1$

and substitute 1 with $C_{13} P$ / $C_{23} P$ respectively

get $C_{11} P = x_1 C_{13} P$ $C_{21} P = x_2 C_{23} P$

$C_{12} P = y_1 C_{13} P$ $C_{22} P = y_2 C_{23} P$

So rewrite as A

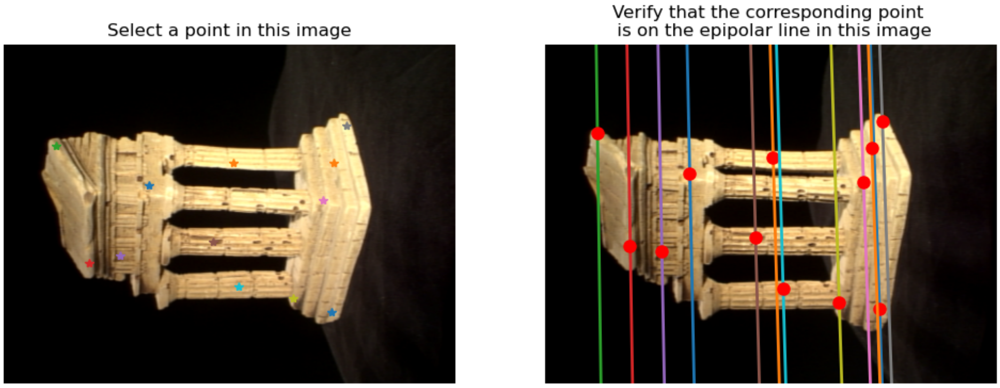
$$A = \begin{bmatrix} x_1 C_{13} - C_{11} \\ y_1 C_{13} - C_{12} \\ x_2 C_{23} - C_{21} \\ y_2 C_{23} - C_{22} \end{bmatrix}$$

Problem 4.1. Epipolar Correspondence

We finally choose window size 4 which means we will generate a 9*9 matrix each time.

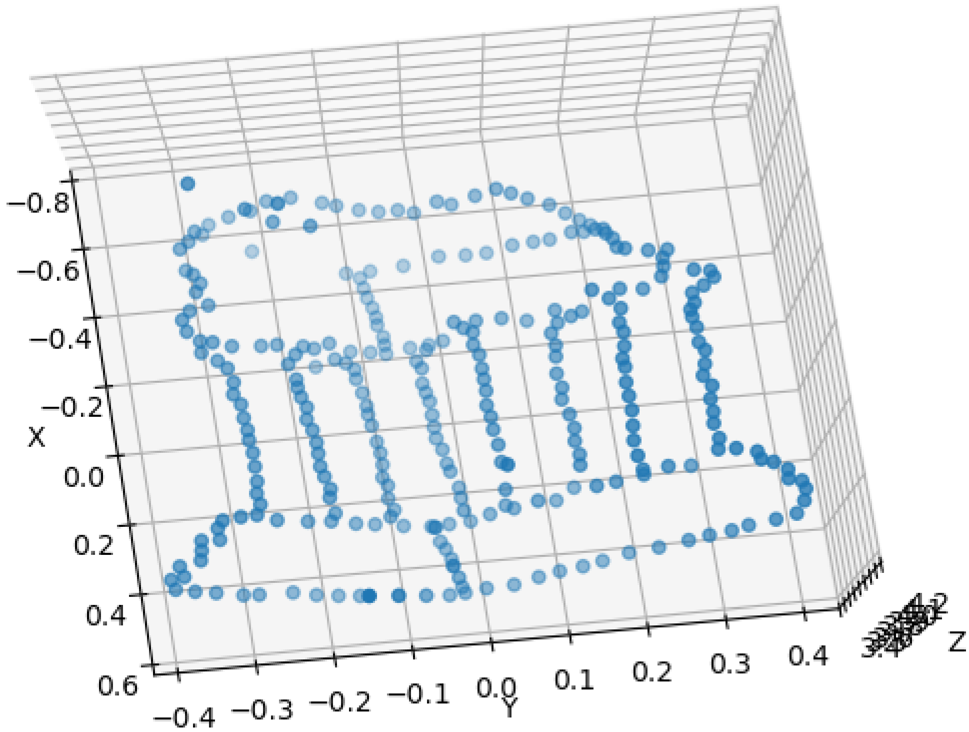
We iterate from y - 25 to y + 25 and find the one with minimum error.

The results are shown below:

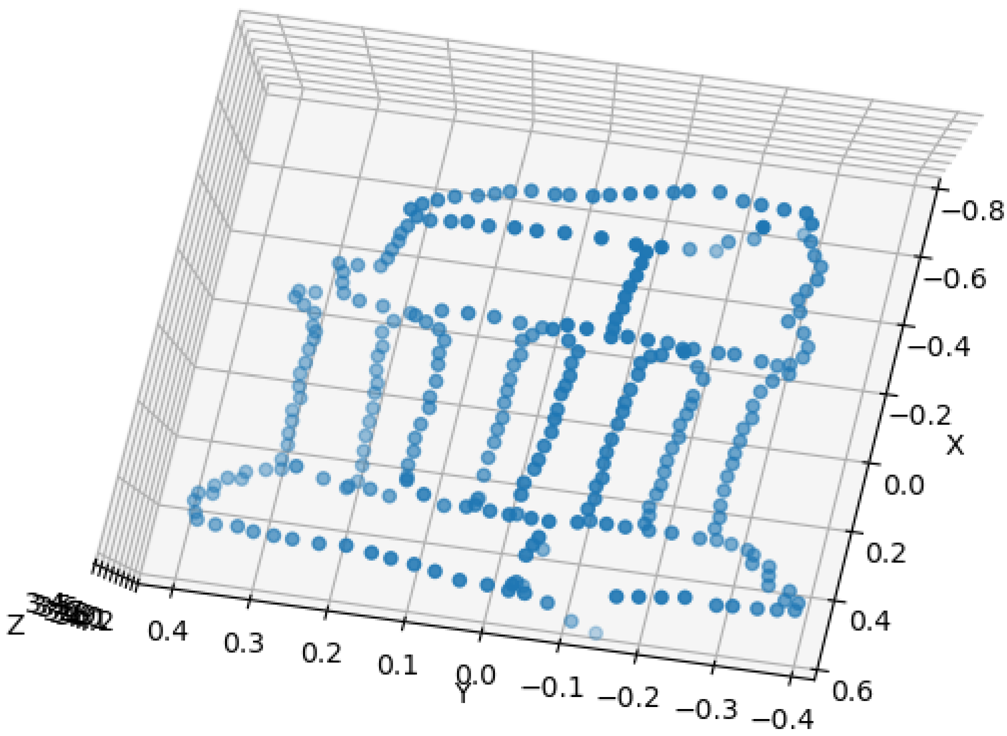


We could see that the results are quiet good.

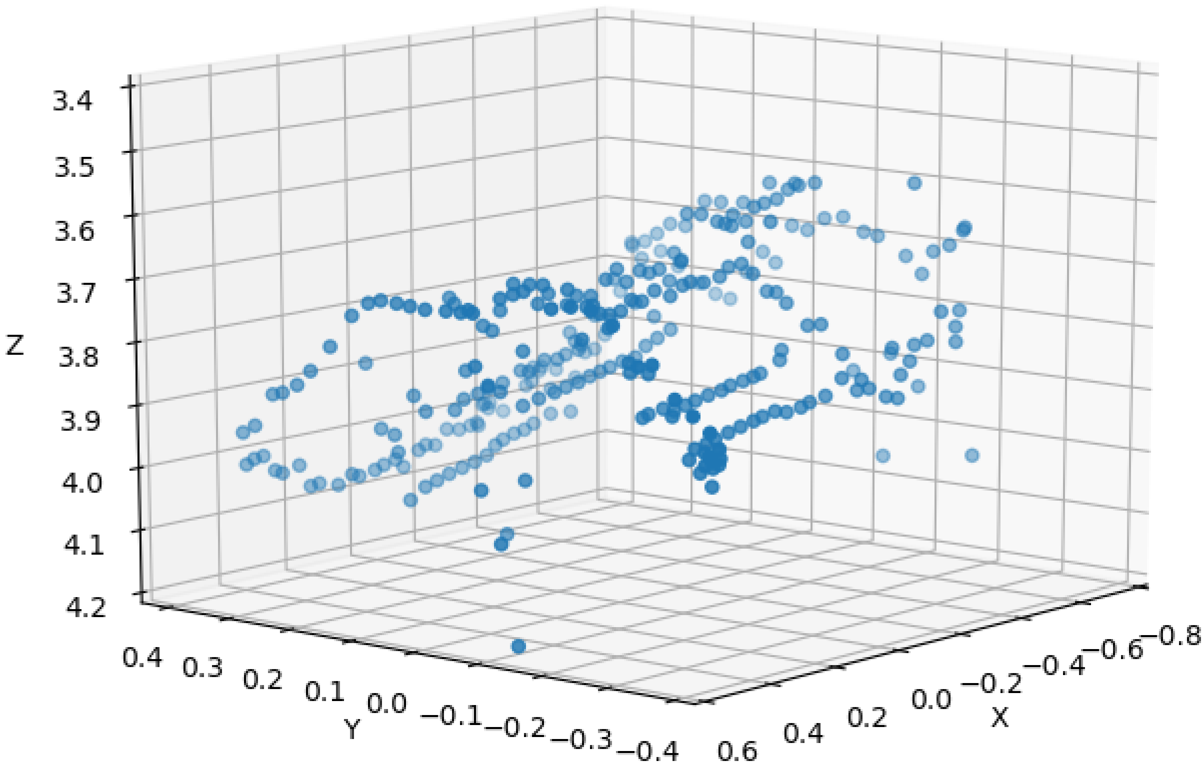
Problem 4.2. 3D Visualization

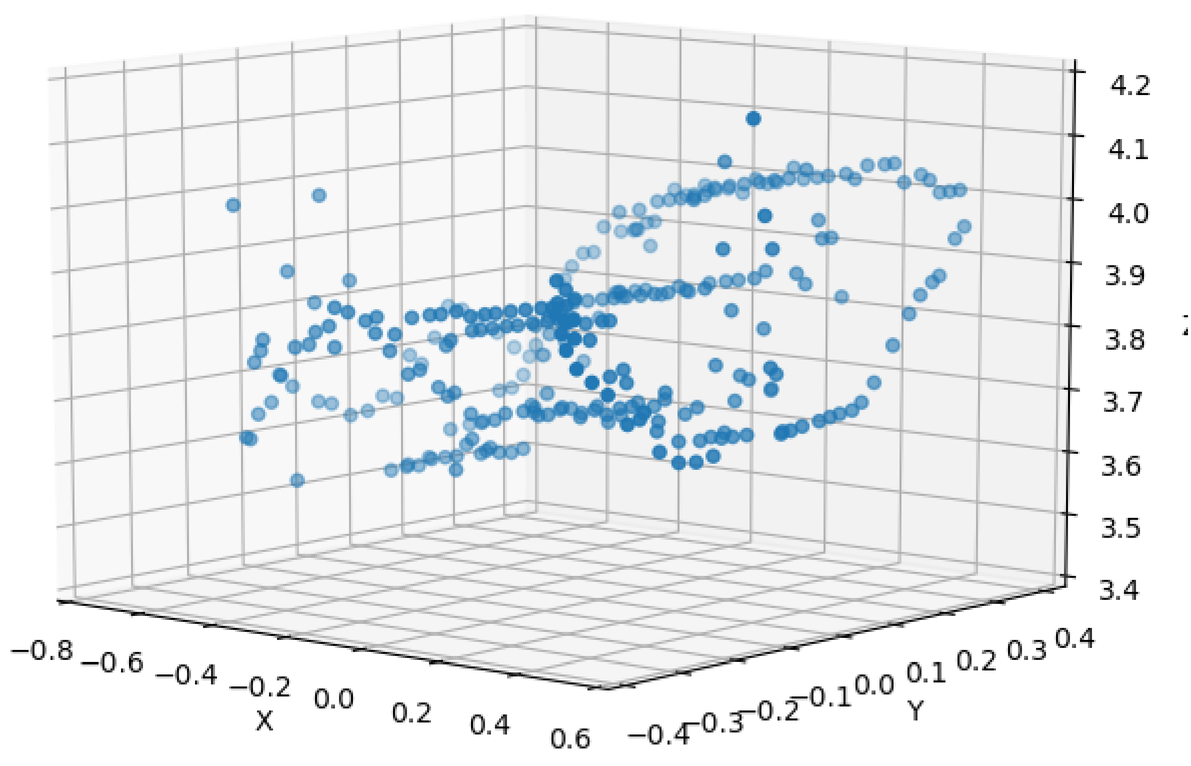


−0.8
−0.6
−0.4
−0.2
X
0.0
0.2
0.4
0.6
−0.4 −0.3 −0.2 −0.1 0.0 0.1 0.2 0.3 0.4
Y
Z



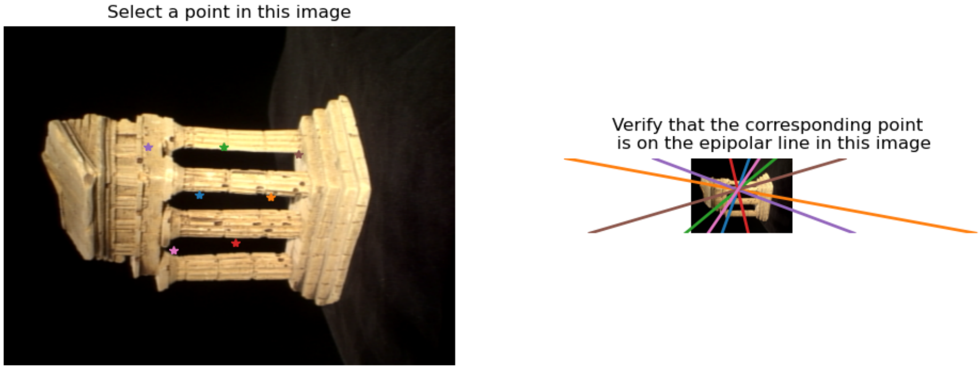
−0.8
−0.6
−0.4
−0.2
X
0.0
0.2
0.4
0.6
0.4 0.3 0.2 0.1 0.0 −0.1 −0.2 −0.3 −0.4
Y
Z


Z
3.4
3.5
3.6
3.7
3.8
3.9
4.0
4.1
4.2
Y
0.4 0.3 0.2 0.1 0.0 −0.1 −0.2 −0.3 −0.4
X
0.6 0.4 0.2 0.0 −0.2 −0.4 −0.6 −0.8


4.2
4.1
4.0
3.9
3.8
3.7
3.6
3.5
3.4
X
−0.8 −0.6 −0.4 −0.2 0.0 0.2 0.4 0.6
Y
−0.4 −0.3 −0.2 −0.1 0.0 0.1 0.2 0.3 0.4

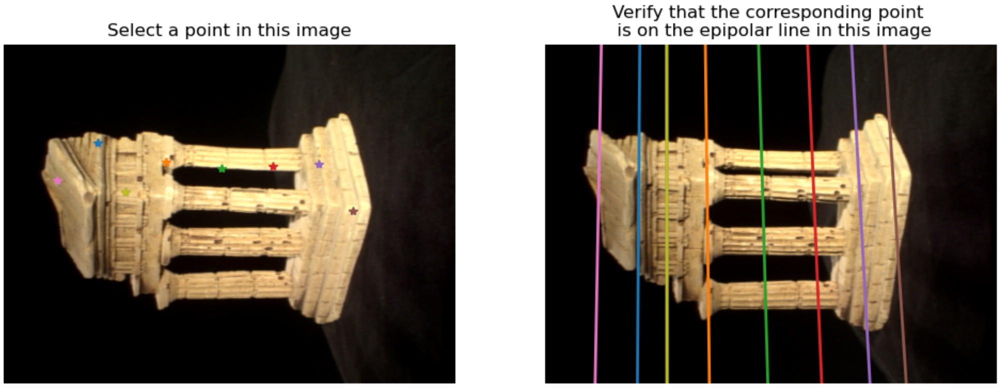
Problem 5.1. RANSAC

If we just use the noisy data without the RANSAC technique, we get the result as below:



We could see that this is pretty bad.

After applying the RANSAC method, we got:



Which is quiet good.

The error matrix we used is based on the feature of fundamental matrix. which is:

> *x2.T @ F @ x1 = 0*

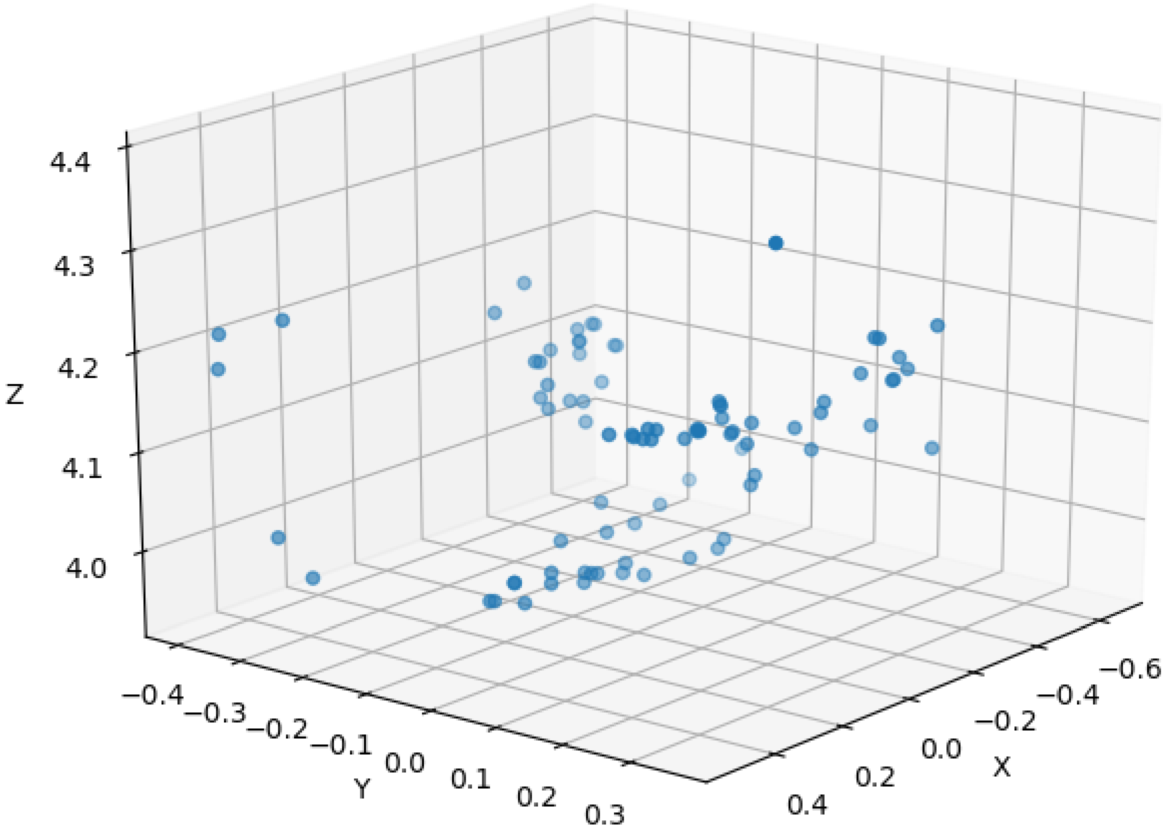
So we define err = abs(x2.T @ F @ x1)

The threshold is 0.001 and we believe it is small enough.

So if the error is less than the threshold, we view it as inlier. After 100 iterations, we found the one with most inliers and use them to compute the F using eight point algorithm.

Problem 5.3. Bundle Adjustment

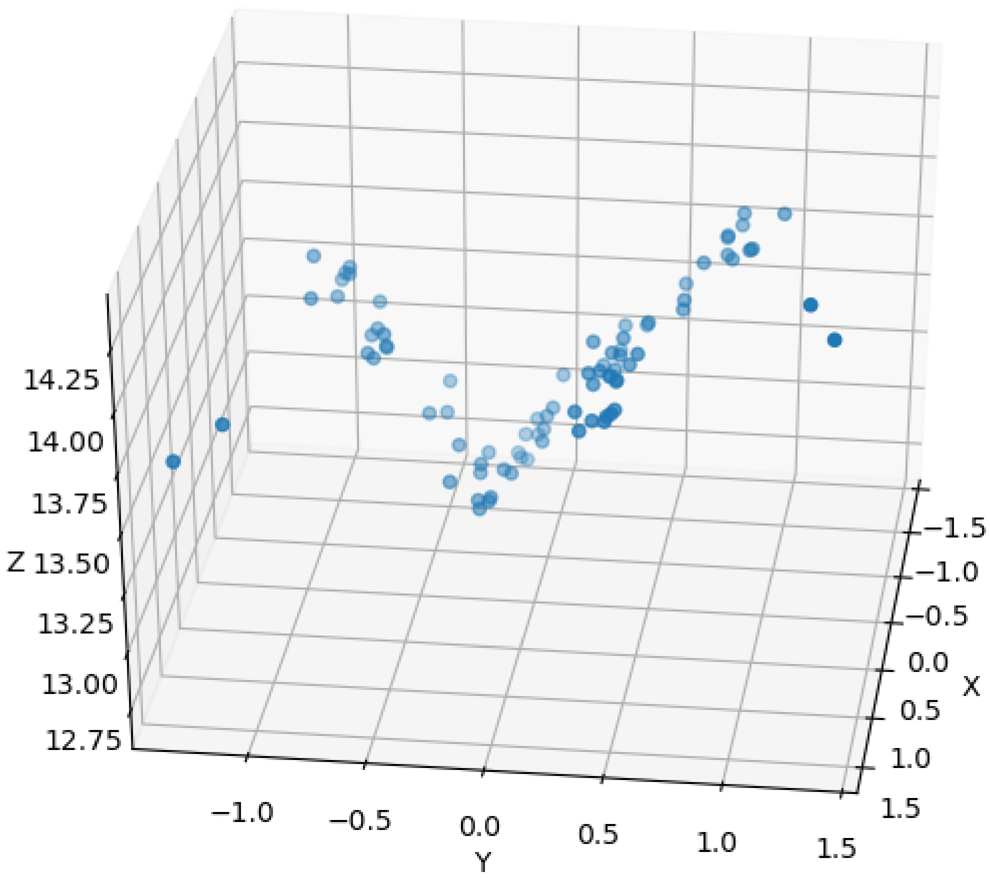
Without the bundle adjustment, we got the plotted figure as below:



And the loss we get is:

```
67.45223569152305
```

With the bundle adjustment, we got the plotted figure as below:



And the loss we get is:

```
7.2708659198453365
```

Which is much less.