30-4-2021

Problemario 1

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Problema 1 Encontrar la STF en (0, 1) de:

a) e^{-t} $\quad T = 1 \quad \omega_0 = 2\pi$

$$a_0 = \frac{1}{1}\int_0^1 e^{-t}dt = -e^{-t}\Big|_0^1 = -\frac{1}{e} + 1 = \frac{e-1}{e} = 1 - e^{-t}$$

$$a_n = 2\int_0^1 e^{-t}\cos(2nt\pi)dt \qquad \boxed{= \frac{e^{ax}(a\cos bx + b\,\text{sen}\, bx)}{a^2+b^2}}$$

$$= 2\left[\frac{e^{-t}(-\cos 2n\pi t + \cancel{2n\pi\,\text{sen}\, 2n\pi t})}{1+4n^2\pi^2}\right]_0^1 \qquad a = -1,\ b = 2n\pi$$

$$= 2\left\{\frac{e^{-1}(-1) - (-1)}{1+4n^2\pi^2}\right\} = \frac{2(1-e^{-1})}{1+4n^2\pi^2}$$

$$b_n = 2\int_0^1 e^{-t}\,\text{sen}(2n\pi t)dt \qquad \boxed{= \frac{e^{ax}(a\,\text{sen}\, bx - b\cos bx)}{a^2+b^2}}$$

$$= 2\left[\frac{e^{-t}(-\cancel{\text{sen}\, 2n\pi t}^{\,0} - 2n\pi\cos 2n\pi t)}{1+4n^2\pi^2}\right]_0^1 \qquad a = -1,\ b = 2n\pi$$

$$= 2\left[\frac{-2n\pi e^{-1} + 2n\pi}{1+4n^2\pi^2}\right] = \frac{4n\pi(1-e^{-1})}{1+4n^2\pi^2}$$

$$f(t) = \frac{e-1}{e} + 2\sum_{n=1}^{\infty}\left(\frac{1-e^{-1}}{1+4n^2\pi^2}\right)\left(\cos(2n\pi t) + 2n\pi\,\text{sen}(2n\pi t)\right)$$

b) $t^2 \quad 0<t<1 \quad T=1 \quad \omega_0 = 2\pi$

$a_0 = \int_0^1 t^2 dt = t^3/3 \Big|_0^1 = 1/3$

$a_n = 2\int_0^1 t^2 \cos(2n\pi t)dt \qquad \boxed{= \frac{2t}{a^2}\cos ax + \left(\frac{t^2}{a} - \frac{2}{a^3}\right)\operatorname{sen} at}$

$= 2\left[\frac{2t}{(2n\pi)^2}\cos(2n\pi t) + \left(\frac{t^2}{2n\pi} - \frac{2}{8n^3\pi^3}\right)\cancel{\operatorname{sen}(2n\pi t)}^{0}\right]_0^1$

$= 2\left[\frac{1}{2n^2\pi^2}\right] = \frac{1}{n^2\pi^2}$

$b_n = 2\int_0^1 t^2 \operatorname{sen}(2n\pi t)dt \qquad \boxed{= \frac{2t}{a^2}\operatorname{sen} ax + \left(\frac{2}{a^3} - \frac{t^2}{a}\right)\cos ax}$

$= 2\left[\cancel{\frac{2t\operatorname{sen}(2n\pi t)}{4n^2\pi^2}}^{0} + \left(\frac{2}{8n^3\pi^3} - \frac{t^2}{2n\pi}\right)\cos \cancel{2n\pi t}^{1}\right]_0^1$

$= 2\left[\cancel{\frac{2}{8n^3\pi^3}} - \frac{1}{2n\pi} - \cancel{\frac{2}{8n^3\pi^3}}\right] = -\frac{1}{n\pi}$

$$t^2 = \frac{1}{3} + \sum_{n=1}^{\infty} \frac{\cos 2n\pi t}{n^2\pi^2} - \frac{\operatorname{sen} 2n\pi t}{n\pi}$$

c) $2t \qquad a_0 = \int_0^1 2t\,dt = t^2 \Big|_0^1 = 1$

$a_n = 2\cdot 2\int_0^1 t\cos(2n\pi t)dt = \cancel{4}\left[\frac{\cos(2n\pi t)}{\cancel{4}n^2\pi^2} + \frac{t\operatorname{sen}\cancel{2n\pi t}^{0}}{2n\pi}\right]_0^1$

$a_n = 0$

$b_n = 1\int_0^1 t\operatorname{sen}(2n\pi t)dt = \left(\cancel{\frac{\operatorname{sen} 2n\pi t}{4n^2\pi^2}}^{0} - \frac{t\cos 2n\pi t}{2n\pi}\right)\Big|_0^1 \cdot 4$

$b_n = 2(-1/n\pi)$

$$2t = 1 - 2\sum_{n=1}^{\infty} \operatorname{sen}(2n\pi t)/n\pi$$

Problema 2

a) $f(t)$ es par $\Rightarrow$ $b_n = 0$ $\boxed{T = 2\pi \quad \omega_0 = 1}$

$$a_0 = \frac{A}{2\pi}\int_{-\pi/2}^{\pi/2} \cos t\,dt = \frac{A}{2\pi}\left[\operatorname{sen}\frac{\pi}{2} - \operatorname{sen}-\frac{\pi}{2}\right] = A/\pi$$

$$a_n = A\int_{-\pi/2}^{\pi/2} \cos t \cos nt\,dt = \boxed{\text{De Evidencia 1.1}} \qquad a_1 = \frac{1}{2}$$

$$f(t) = \left\{\frac{1}{\pi} + \frac{\cos t}{2} + \frac{1}{2\pi}\sum_{n=2}^{\infty}\left[\frac{\operatorname{sen}(n+1)\pi/2 - \operatorname{sen}-(n+1)\pi/2}{n+1} + \frac{\operatorname{sen}\frac{(n-1)\pi}{2} - \operatorname{sen}\frac{-(n-1)\pi}{2}}{n-1}\right]\right.$$

$$\left.\cos nt\right\} A$$

b) $f(t)$ es par $A(1+t)$ en $-\pi<t<0$; $A(1-t)$ $0<t<\pi$

$$a_0 = \frac{A}{2\pi}\int_{-\pi}^{0}(A+t)\,dt + \frac{A}{2\pi}\int_{0}^{\pi}(A-t)\,dt = \frac{A}{2\pi}\left(At + \frac{t^2}{2}\right)\Big|_{-\pi}^{0} + \frac{A}{2\pi}\left(At - \frac{t^2}{2}\right)\Big|_{0}^{\pi}$$

$$= \frac{-A}{2\pi}\left(-\cancel{\pi}A + \frac{\pi^2}{2}\right) + \frac{A}{2\pi}\left(\pi A - \frac{\pi^2}{2}\right) = \frac{A}{2}\left(A - \frac{\pi}{2} + A - \frac{\pi}{2}\right)$$

$$= (2A - \pi)/2$$

$$a_n = \frac{A}{\pi}\int_{-\pi}^{0}(1+t)\cos nt\,dt + \frac{A}{\pi}\int_{0}^{\pi}(1-t)\cos nt\,dt$$

$$= \frac{A}{\pi}\left[\cancel{\frac{\operatorname{sen}(nt)}{n}}^{0} + \frac{\cos nt}{n^2} + \cancel{\frac{t\operatorname{sen}(nt)}{n}}^{0}\right]_{-\pi}^{0} + \frac{A}{\pi}\left[\cancel{\frac{\operatorname{sen}(nt)}{n}}^{0} - \frac{\cos nt}{n^2} - \cancel{\frac{t\operatorname{sen}(nt)}{n}}^{0}\right]_{0}^{\pi}$$

$$= \frac{A}{\pi}\left(\frac{1}{n^2} - \cancel{\frac{\cos -\pi n}{n^2}}\right) + \frac{A}{\pi}\left(-\cancel{\frac{\cos n\pi}{n^2}} + \frac{1}{n^2}\right) = \frac{2}{n^2\pi}(1 - \cos n\pi)$$

$$f(t) = \frac{2A-\pi}{2} + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1-\cos n\pi}{n^2}\cos nt$$

c) $P(t)$ es impar

$$b_n = \frac{\cancel{2}A}{\cancel{2}\pi}\int_{-\pi}^{0} \text{sen}\, nt\, dt + \frac{\cancel{2}A}{\cancel{2}\pi}\int_{0}^{-\pi} -\text{sen}\, nt\, dt = \frac{A}{\pi}\left(-\frac{\cos nt}{n}\right)\Big|_{-\pi}^{0} + \frac{A}{\pi}\left[\frac{\cos nt}{n}\right]_{0}^{\pi}$$

$$= \frac{A}{\pi}\left[-\frac{1}{n} + \frac{\cos - n\pi}{n} + \frac{\cos n\pi}{n} - \frac{1}{n}\right] = \frac{2A}{\pi n}\left(-1 + \cos n\pi\right)$$

$$P(t) = \frac{2A}{\pi}\sum_{n=1}^{\infty}\frac{\cos(n\pi) - 1}{n}\cos nt$$

d) $P(t)$ es impar

$$b_n = \frac{A}{\pi}\int_{-\pi/2}^{\pi/2} \text{sen}\, 2t\, \text{sen}\, nt = \frac{A}{2\pi}\int_{-\pi/2}^{\pi/2}\left[\cos(2+n)t + \cos(2-n)t\right]dt$$

$$= \frac{A}{2\pi}\left[\frac{-\text{sen}(2+n)t}{2+n} + \frac{\text{sen}(2-n)t}{2-n}\right]_{-\pi/2}^{\pi/2}$$

$$= \frac{A}{2\pi}\left[\frac{-\text{sen}(2+n)\pi/2}{2+n} + \frac{\text{sen}(2-n)\pi/2}{2-n} + \frac{\text{sen}(2+n)\pi/2}{2+n} - \frac{\text{sen}(2-n)\pi/2}{2-n}\right]$$

$$= \frac{A}{\pi}\left(-\frac{\text{sen}(2+n)\pi/2}{2+n} + \frac{\text{sen}(2-n)\pi/2}{2-n}\right)$$

$$= \frac{A}{\pi}\left(+\frac{\text{sen}\, n\pi/2}{2+n} + \frac{\text{sen}\, n\pi/2}{2-n}\right) = \frac{A\,\text{sen}\, n\pi/2}{\pi}\left(\frac{1}{2+n} + \frac{1}{2-n}\right)$$

$$b_1 = \frac{A}{\pi}\left(\frac{1}{3} + 1\right) = \frac{4A}{3\pi} \qquad b_2 = \frac{0}{0} \to \frac{\cancel{\pi}}{2}\frac{A\cos(\pi)}{-\cancel{\pi}} = \frac{1}{2}$$

Problema 3

a) $f(t) = \begin{cases} \frac{A}{2}(x+1) & -1<t<0 \\ \frac{A}{2}(x-1) & 0<t<1 \end{cases}$ $\qquad f(t)$ es impar

$(-1, 0)$ $\quad m = \frac{-A/2}{-1} = \frac{A}{2}$ $\qquad (0, -\frac{A}{2})$ $\quad m = \frac{A}{2}$

$(0, \frac{A}{2})$ $\qquad\qquad (1, 0)$

$y = \frac{A}{2}(x+1)$ $\qquad T = 2$, $\omega_0 = \pi$ $\qquad y = \frac{A}{2}(x-1)$

$$b_n = \frac{4}{2}\int_0^1 \frac{A}{2}(t-1)\,\text{sen}(n\pi t)\,dt = A\left[\cancel{\frac{\text{sen}\, n\pi t}{(n\pi)^2}}^{0} - \frac{t\cos n\pi t}{n\pi} + \frac{\cos n\pi t}{n\pi}\right]$$

$$= A\left(\frac{\cos n\pi}{n\pi} - \frac{1}{n\pi}\right) = \frac{A}{n\pi}(\cos n\pi - 1)$$

$$f(t) = \frac{A}{\pi}\sum_{n=1}^{\infty}\left(\frac{\cos n\pi - 1}{n}\right)\text{sen}(n\pi t)$$

b) $f(t) = \begin{cases} A\,\text{sen}\,t & 0<t<2\pi \\ 0 & \text{en otro caso} \end{cases}$ $\qquad T = 4\pi \qquad \omega_0 = 1/2$

$$a_n = \frac{A}{2\pi}\int_0^{2\pi}\text{sen}\,t\cos\left(\frac{nt}{2}\right)dt = \frac{A}{2\pi}\int_0^{2\pi}\text{sen}\,t\left(1-\frac{n}{2}\right) + \text{sen}\left(t\left(1+\frac{n}{2}\right)\right)dt$$

$$= \frac{A}{4\pi}\left[\frac{-\cos t(1-\frac{n}{2})}{1-\frac{n}{2}} - \frac{\cos t(1+\frac{n}{2})}{1+\frac{n}{2}}\right]_0^{2\pi}$$

$$= \frac{-A2}{4\pi}\left(\frac{\cos(2-n)\pi}{2-n} + \frac{\cos(2+n)\pi}{2+n} - \frac{1}{2-n} - \frac{1}{2+n}\right)$$

$$= -\frac{A}{2\pi}\left(\frac{\cos(2-n)\pi - 1}{2-n} + \frac{\cos(2+n)\pi - 1}{2+n}\right) \quad \begin{cases} a_2 = 0 \\ a_0 = 0 \end{cases}$$

$$b_n = \frac{1}{4\pi}\int_0^{2\pi} \cos t\left(1-\frac{n}{2}\right) \operatorname{sen} \frac{n}{2} t \, dt = \frac{1}{4\pi}\int_0^{2\pi} \cos t\left(\left(1-\frac{n}{2}\right)t\right) - \cos t\left(1+\frac{n}{2}\right) dt$$

$$= \frac{A}{4\pi}\left(\frac{\operatorname{sen}(2\pi - \pi n)}{\frac{2-n}{2}} - \frac{\operatorname{sen}(2\pi + \pi n)}{\frac{2+\pi}{2}}\right) = \frac{A}{2\pi}\left(0 \text{ si } n \neq 2\right)$$

$$b_2 = \frac{A}{2} \qquad f(t) = \frac{A}{2}\operatorname{sen} t + \frac{4A}{3\pi}\cos\frac{t}{2} + \sum_{n=3}^{\infty} \frac{2A[(-1)^n - 1]}{\pi(n^2 - 4)} \cos\frac{nt}{2}$$

c) $T = 3$ $\quad \omega_0 = 2\pi/3 \quad$ $f(t)$ es impar

$$b_n = \frac{4}{3}\int_0^{3/2} f(t)\operatorname{sen}\frac{2\pi}{3}t\,dt = \frac{4A}{3}\int_0^1 t\operatorname{sen}\frac{n2\pi}{3}t - \operatorname{sen}\frac{2n\pi t}{3}\,dt$$

$$= \frac{4A}{3}\left[\frac{3t\cos\left(\frac{2n\pi t}{3}\right)}{2n\pi} + \frac{9\operatorname{sen}(2n\pi t)/3}{4n^2\pi^2} + \frac{3\cos(n2\pi t/3)}{2n\pi}\right]_0^1$$

$$= \frac{4A}{3}\left[\frac{9\operatorname{sen}(2n\pi/3)}{4\pi^2} - \frac{3}{2\pi}\right] = \frac{3A\operatorname{sen}(2n\pi/3)}{n^2\pi^2} - \frac{2A}{n\pi}$$

$$= \frac{3A\operatorname{sen}(2n\pi/3)}{\pi^2 n^2} - \frac{2A}{n\pi}$$

$$f(t) = \sum_{n=1}^{\infty}\left(\frac{3A\operatorname{sen}(2n\pi/3)}{\pi^2 n^2} - \frac{2A}{n\pi}\right)\operatorname{sen}\left(\frac{2n\pi t}{3}\right)$$

d) $f(t) = A\cos t \qquad T = 2\pi \qquad \omega_0 = 1 \qquad b_n = 0$

$$a_n = \frac{2}{\pi}\int_0^{\pi/2} \frac{1}{2}\cos t(1-n) + \cos t(1+n)\, dt$$

$$= \frac{1}{\pi}\left[\frac{\operatorname{sen} t \cos tn - \cos t \operatorname{sen} tn}{1-n} + \frac{\operatorname{sen} t \cos nt + \cos t \operatorname{sen} tn}{1+n}\right]_0^{\pi/2}$$

$$= \frac{1}{\pi}\left(\frac{\operatorname{sen}\pi/2 \cos n\pi/2}{1-n} + \frac{\operatorname{sen}\pi/2 \cos n\pi/2}{1+n}\right)$$

$$= \frac{2}{\pi}\left(\frac{\cos n\pi/2}{1-n^2}\right) \qquad a_0 = \frac{2}{\pi}$$

$$a_1 = \frac{1}{\pi}\lim_{n\to 1} \frac{2\cos n\pi/2}{1-n^2} = \frac{0}{0} \Rightarrow \frac{1}{\pi}\left(\frac{\pi}{2}\right) = \frac{1}{2}$$

$$f(t) = \frac{2}{\pi} + \frac{1}{2}\cos\left(t - \frac{\pi}{2}\right) + \sum_{n=1}^{\infty} \frac{1}{\pi}\left(\frac{2\cos n\pi/2}{1-n^2}\right)\cos n(t - \pi/2)$$

e) $f(t) = \begin{cases} A & -\frac{1}{2} < t < \frac{1}{2} \\ -2A\left(t - \frac{1}{2}\right) + A & \frac{1}{2} < t < 1 \\ 0 & 1 < t < 2 \end{cases} \qquad T = 3 \qquad \omega_0 = 2\pi/3$

par $\Rightarrow b_n = 0$

$$a_n = \frac{4}{3}\int_0^1 -2A(t-2)\cos\left(\frac{2n\pi t}{3}\right) dt = -\frac{8A}{3}\left[\frac{-6\operatorname{sen}(2n\pi t/3)}{2n\pi} + \frac{3t\operatorname{sen}(2n\pi t/3)}{2n\pi} + \frac{9\cos(2n\pi t/2)}{4\pi^2 n^2}\right]_0^1$$

$$= -\frac{8A}{3}\left[\frac{-3\operatorname{sen}(2\pi/3)}{2n\pi} + \frac{4\cos(2n\pi/3)}{4\pi^2 n^2} - \frac{9}{4\pi^2 n^2}\right]$$

$$= -\frac{8A}{4\pi^2 n^2}\left(2n\pi\operatorname{sen}(2n\pi/3) - 3\cos(2n\pi/3) + 3\right)$$

$$= -\frac{2A}{n^2\pi^2}\left(2n\pi\operatorname{sen}\left(\frac{2n\pi}{3}\right) - 3\cos\left(\frac{2n\pi}{3}\right) + 3\right)$$

$a_0 = 2A \qquad P(t) = 2A + \sum_{n=1}^{\infty} -2A\left(\frac{2n\pi \operatorname{sen}(2n\pi/3) - 3\cos(2n\pi/3) + 3}{n^2\pi^2}\right) \operatorname{sen}$

Problema 4

a) $P(t) = \begin{cases} A & -\frac{1}{2} < t < \frac{1}{2} \\ 0 & \frac{1}{2} < t < 3 \end{cases} \qquad T = \frac{7}{2} \quad \omega_0 = \frac{4\pi}{7}$

$$C_n = \frac{1}{T}\int_{-1/2}^{3} f(t) e^{-n\frac{4\pi}{7}t}\,dt = \frac{2}{7}\int_{-1/2}^{1/2} A e^{-in\frac{4\pi}{7}t}\,dt + 0$$

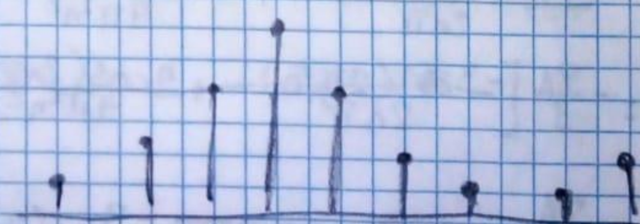
$$= \frac{2A}{7}\int_{-1/2}^{1/2} e^{-in\frac{4\pi}{7}t}\,dt = \frac{2}{7}A e^{-in\frac{4\pi}{7}} \Big/ -\frac{in4\pi}{7} \Bigg|_{-1/2}^{1/2} = \frac{2}{7}\frac{7e^{-in\frac{4\pi}{7}t}}{-in4\pi}\left(\frac{-1}{-1}\right)$$

$$= \frac{A}{n\pi}\left(\frac{e^{in\frac{2\pi}{7}} - e^{-in\frac{2\pi}{7}}}{2i}\right) = \frac{A}{n\pi}\operatorname{sen}\left(\frac{n2\pi}{7}\right)$$

$$C_0 = \lim_{n\to 0}\frac{A\operatorname{sen}\left(\frac{2\pi n}{7}\right)}{\pi n \pi} = \frac{A}{\pi}\frac{2\pi}{7}\cos\left(\frac{2\pi n}{7}\right) = \frac{A}{\pi}\frac{2\pi}{7} = \frac{2A}{7}$$

$$f(t) = \frac{2A}{7} + \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{A}{n\pi}\operatorname{sen}\left(\frac{2n\pi}{7}\right) e^{\frac{in4\pi t}{7}}$$

n	$n\omega_0$	$\lvert C_n \rvert$	θ_n
0	0	0.29A	0
1	$8\pi/7$	0.25A	0
2	$12\pi/7$	0.15A	0
3	$16\pi/7$	0.05A	0
4		0.03A	0
5	$20\pi/7$	0.06A	0



Espectro de Frecuencia

b) $f(t) = \begin{cases} At + A & -1 < t < 0 \\ -At + A & 0 < t < 1 \\ 0 & 1 < t < 3 \end{cases}$ $\quad T = 4$ $\quad \omega_0 = \pi/2$

$$C_n = \frac{1}{4}\int_{-1}^{0} A(t+1)e^{-in\frac{\pi}{2}t}dt + \frac{1}{4}\int_{0}^{1} A(-t+1)e^{-in\frac{\pi}{2}t}dt$$

$$= \frac{A}{4}\left[\frac{2te^{-\frac{in\pi}{2}t}}{-in\pi} - \frac{4e^{-\frac{in\pi}{2}t}}{(-in\pi)^2} + \frac{e^{-\frac{2n\pi}{2}t}}{-in\pi}\right]_{-1}^{0} + \left[\frac{-2te^{-\frac{in\pi}{2}t}}{-in\pi} + \frac{4e^{-\frac{in\pi}{2}t}}{(-in\pi)^2} + \frac{e^{-\frac{in\pi}{2}t}}{in\pi}\right.$$

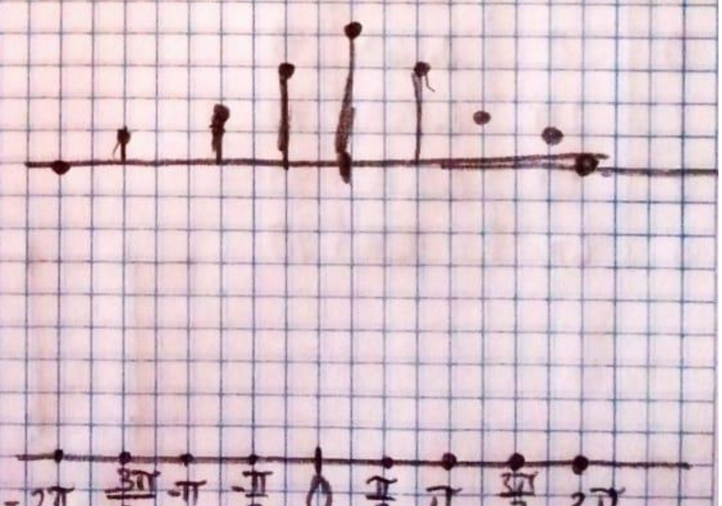
$$= \frac{A}{4}\left(\frac{4}{n^2\pi^2} - \frac{1}{in\pi} - \frac{2e^{\frac{in\pi}{2}t}}{in\pi} - \frac{4e^{\frac{in\pi}{2}}}{n^2\pi^2} + \frac{e^{\frac{in\pi}{2}}}{in\pi} + \frac{2e^{-\frac{in\pi}{2}}}{in\pi} - \frac{4e^{-\frac{in\pi}{2}}}{in\pi} - \frac{e^{-\frac{in\pi}{2}}}{in\pi}\right.$$

$$= \frac{2A}{n^2\pi} - \frac{A2e^{\frac{in\pi}{2}}}{2n^2\pi^2} - \frac{A2e^{-\frac{in\pi}{2}}}{2n^2\pi^2} - \frac{Ae^{\frac{in\pi}{2}}}{4in\pi} + \frac{Ae^{-\frac{in\pi}{2}}}{4in\pi}$$

$$= \frac{2A}{n^2\pi^2} - \frac{2A}{n^2\pi^2}\cos\left(\frac{n\pi}{2}\right) + \frac{A}{n\pi}\operatorname{sen}\left(\frac{n\pi}{2}\right)$$

$$C_0 = \lim_{n\to 0} \frac{4A\operatorname{sen}^2\left(\frac{\pi n}{2}\right)}{\pi^2 n^2} = \frac{A}{4} \qquad f(t) = \frac{A}{4} + \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{4A\operatorname{sen}^2\left(\frac{\pi n}{4}\right)}{\pi^2 n^2}$$

n	$n\omega_0$	$\lvert C_n \rvert$	θ_n
0	0	0.25A	0
1	$\pi/2$	0.2A	0
2	π	0.1A	0
3	$3\pi/2$	0.02A	0
4	2π	0	0
5	$5\pi/2$	0.001A	0



$$c)\ f(t)=\begin{cases}-A(t-1) & 0<t<1\\ f(t+T) & \text{en otro caso}\end{cases}\qquad T=1\qquad \omega_0=2\pi$$

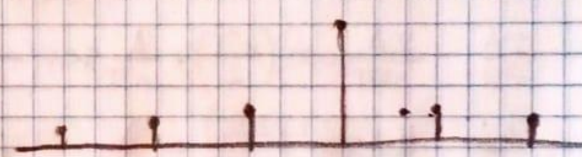
$$C_n=\int_0^1 -A(t-1)e^{-in2\pi t}\,dt=\frac{A}{2\pi n^2}\left[e^{-2\pi i n}(2\pi i n+1)-1\right]+\frac{iA}{2\pi n}\left(e^{-2\pi i n}-1\right)$$

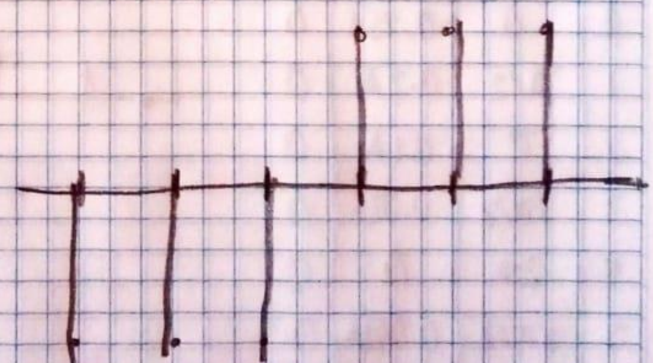
$$=-\frac{A}{(2\pi n)^2}\left[e^{-2\pi i n}-1+2\pi i n\right]=\frac{A}{(2\pi n)^2}\left[\cos(-2\pi n)+i\,\mathrm{sen}(-2n\pi)-1+2\pi i n\right]$$

$$=\frac{A}{2n\pi}$$

$$C_0=A\left[\int_0^1(t-1)\,dt\right]=\frac{A}{2}\qquad f(t)=\frac{A}{2}-\frac{Ai}{2\pi}\sum_{\substack{n=-\infty\\ n\neq 0}}^{\infty}\frac{e^{2nit\pi}}{n}$$

n	$n\omega_0$	$\lvert C_n\rvert$	θ
-3	-6π	0.053A	-90
-2	-4π	0.08A	-90
-1	-2π	0.5A	-90
0	0	0.16A	90
1	2π	0.08A	90
2	4π	0.08A	90
3	6π	0.053A	90





Problema 5 $T=4 \quad \omega_0 = \frac{\pi}{2}$

a) $P(t)$ es par, $b_n = 0$

$$a_0 = \frac{2}{4}\int_0^{3/2}(4-3t)\,dt + \frac{2}{4}\int_{3/2}^{2}(t-2)\,dt = \frac{1}{2}\left[4t - \frac{3t^2}{2}\right]_0^{3/2} + \left[\frac{t^2}{2} - 2t\right]_{3/2}^{2}$$

$$= \frac{1}{2}\left(\frac{21}{8}\right) + \frac{1}{2}\left(-2 + \frac{15}{8}\right) = \frac{5}{4}$$

$$a_n = \int_0^{3/2}(4-3t)\cos n\pi t/2\,dt + \int_{3/2}^{2}(t-2)\cos n\pi t/2\,dt$$

$$= \left[\frac{4\,\text{sen}\,n\pi t/2}{n\pi/2} - 3\left(\frac{\cos n\pi t/2}{(n\pi/2)^2} + \frac{t\,\text{sen}\,n\pi t/2}{n\pi/2}\right)\right]_0^{3/2} +$$

$$\left[\frac{\cos n\pi t/2}{(n\pi/2)^2} + \frac{t\,\text{sen}\,n\pi t/2}{n\pi/2} - 2\,\frac{\text{sen}\,n\pi t/2}{n\pi/2}\right]_{3/2}^{2}$$

$$= \frac{4\,\text{sen}\,3n\pi/4}{n\pi/2} - 3\left(\frac{\cos 3n\pi/4}{(n\pi/2)^2} + \frac{3\,\text{sen}\,3n\pi/4}{2n\pi/2}\right) + \frac{3}{(n\pi/2)^2}$$

$$+ \frac{\cos n\pi}{(n\pi/2)^2} - \frac{\cos 3n\pi/4}{n\pi/2} + \frac{2\,\text{sen}\,3n\pi/4}{n\pi/2} - \frac{3\,\text{sen}\,3n\pi/4}{2n\pi/2}$$

b) $A\cos t \quad -\frac{\pi}{2} < t < \frac{\pi}{2}$

$$F(t) = \int_{-\pi/2}^{\pi/2} A\cos t\, e^{-i\omega_0 t}\,dt = A\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos t(\cos\omega t - i\,\text{sen}\,\omega t)\,dt$$

$$= A\left[\int_{-\pi/2}^{\pi/2}\cos t\cos\omega_0 t\,dt - i\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos t\,\text{sen}\,\omega_0 t\,dt\right]$$

1] $\frac{1}{2}\left[\int\cos(t-\omega t) + \int\cos(t+\omega t)\,dt\right] = \frac{1}{2}\left[\frac{\text{sen}(t-\omega t)}{1-\omega} + \frac{\text{sen}(t+\omega_0 t)}{1+\omega}\right]$

$$\frac{1}{2}\left[\cancel{\frac{(1+\omega_0)\,\text{sen}(t-\omega_0 t) + (1-\omega_0)\,\text{sen}(t+\omega_0 t)}{(1-\omega_0)^2}}\right]$$

$$= \frac{1}{2(1-w^2)}\left[(1+w)[\text{sen}t\cos wt - \cos t\,\text{sen}\,wt] + (1-w_0)(\text{sen}t\cos t + \cos t\,\text{sen}\,wt)\right]$$

$$\int_{-\pi/2}^{\pi/2} \cos t \cos wt = \frac{1}{2(1-w^2)}\Big[(1+w)\cos \pi w/2 + (1-w)\cos \pi w/2 + (1+w)$$

$$\cos w\pi/2 + (1-w)\cos \pi w/2\Big]$$

$$= \frac{1}{2(1-w^2)}\left[\cos \frac{\pi w}{2}\right] = \frac{2\cos \pi w/2}{1-w^2}$$

$b_n = 0$

c) $P(t) = \begin{cases} A & -6 < t < -3 \\ A & 3 < t < 6 \\ 0 & \text{en otro caso} \end{cases}$ $\quad T = 12 \quad w_0 = \pi/6$

$$a_0 = \frac{1}{12}\int_3^6 A\,dt = \frac{2A}{12}[6-3] = 2\frac{3A}{12} = \frac{2A}{4} = \frac{A}{2}$$

$$a_n = \frac{2}{6}\int_3^6 A\cos n\pi t/6\;dt = \frac{A}{3}\left[\frac{\text{sen}\,n\pi t/6}{n\pi/6}\right]_3^6 = \frac{A}{3}\left(-\frac{\text{sen}\,n\pi/2}{n\pi/6}\right)$$

$b_n = 0$

$$= -\frac{2A}{n\pi}\text{sen}\left(\frac{n\pi}{2}\right)$$

d) $A = 1$ $\quad P(t) = \begin{cases} t-6 & 6 < t < 7 \\ 0 & \text{en otro caso} \end{cases}$ $\quad T = 20 \quad w_0 = \pi/10$

$$a_0 = \frac{4}{10}\int_6^7 (t-6)\,dt = \frac{4}{10}\left(\frac{1}{2}\right) = \frac{2}{10} = \frac{1}{5}$$

$$a_n = \frac{4}{5}\int_6^7 (t-6)\cos n\pi t/10\;dt = \frac{4}{5}\left[\frac{\text{sen}\,n\pi t/10}{(n\pi/10)^2} + \frac{t\cos n\pi t/10}{n\pi/10} - \frac{6\,\text{sen}\,n\pi t/10}{n\pi/10}\right]_6^7$$

$$= \frac{8}{n\pi}\left[\frac{\text{sen}\,7n\pi/10 + 7\cos 7n\pi/10}{n\pi/10} - 6\cos 7n\pi/10 - \frac{\text{sen}\,6n\pi t/10}{n\pi/10}\right]$$

$b_n = 0$

⑥ a

$$f(t) = \frac{1}{2\pi}\int_{-w}^{w} A e^{-wt} e^{iwt}\, dw = \frac{A}{2\pi}\int_{-w}^{w} e^{wt(i-1)}\, dw$$

$$= \frac{A}{2\pi}\left[\frac{e^{wt(i-1)}}{(i-1)t}\right]_{-w}^{w} = \frac{A}{2\pi(i-1)t}\left(e^{wt(i-1)} - e^{-w(i-1)}\right)$$

$$= \frac{A e^{wt(i-1)}}{2\pi(i-1)t}\left(e^{w} - e^{-w}\right)$$

ⓑ

$$f(t) = \frac{1}{2}\int_{-w}^{0} e^{\pi/2} e^{iwt}\, dw + \frac{1}{2}\int_{0}^{w} e^{-\pi/2} e^{iwt}\, dw = \frac{e^{\frac{\pi}{2}}}{2}\left[\frac{e^{iwt}}{it}\right]_{-w}^{0} + \frac{e^{-\frac{\pi}{2}}}{2}\left[\frac{e^{iwt}}{it}\right]_{0}^{w}$$

$$= \frac{e^{\pi/2}}{2it}\left(1 - e^{-iwt}\right) + \frac{e^{-\pi/2}}{2it}\left(e^{iwt} - 1\right)$$

ⓒ

$$f(t) = \frac{1}{\pi}\int_{0}^{1} A(1-w) e^{iwt}\, dw = \frac{A}{\pi}\left[\frac{e^{iwt}}{it}\right]_{0}^{1} - \frac{1}{\pi}\int_{0}^{1} w e^{iwt}\, dw$$

$$= \frac{A}{\pi it}\left(e^{it} - 1\right) - \frac{1}{\pi}\left[\frac{e^{iwt}}{it}\left(w - \frac{1}{it}\right)\right]_{0}^{1}$$

$$= \frac{1}{\pi it}\left\{A(e^{it}-1) - \left[e^{it}\left(1 - \frac{1}{it}\right) + \frac{1}{it}\right]\right\}$$

ⓔ

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(t) e^{iwt}\, dw = \frac{8}{2\pi} e^{iwt}\Big|_{+1} + \frac{8}{2\pi} e^{iwt}\Big|_{-1}$$

$$= \frac{4}{\pi}\left(e^{it} + e^{-it}\right)$$

7

a) $\int_{-\infty}^{\infty} \delta(t-5) \operatorname{sen} 2t \, dt = \operatorname{sen}(2t) \Big|_{5} = \operatorname{sen}(10)$

b) $\int_{-\infty}^{\infty} \delta(2-t)(t^5-3)\, dt = t^5-3 \Big|_{2} = 32-3 = 29$

c) $\int_{+1}^{x} e^{-x^2} \delta(x)\, dt = 0 \quad si \ x>0$

d) $\int_{-\infty}^{\infty} \delta(t-2) \cos[\pi(t-3)] = \cos[\pi(t-3)] \Big|_{2} = \cos -\pi = -1$

e) $\int_{-\infty}^{\infty} \delta(t+2) e^{-2t}\, dt = e^{-2t} \Big|_{-2} = e^{4}$

f) $\int_{-\infty}^{\infty} e^{\cos t} \delta(t-\pi)\, dt = e^{\cos t} \Big|_{\pi} = e^{1} = e$

g) $\int_{1}^{100} \log_{10}(t)\, \delta(t-10)\, dt = \log_{10} t \Big|_{10} = \log_{10} 10 = 1$

(8)

a) $f(t) \rightarrow f(-t) \rightarrow f(2-t)$

$F(w) \quad F(-w) \quad F(-w)e^{2iw}$

b) $f(t) \rightarrow f(t-3) \rightarrow f((t-3)-3)$

$F(w) \quad F(w)e^{-3iw} \quad F(w)e^{-3iw}e^{-3iw}$

c) $f(t) \rightarrow \frac{df(t)}{dt} \rightarrow \left(\frac{df(t)}{dt}\right) \operatorname{sen} t$

$F(w) \quad jwF(w) \quad \frac{1}{2}\left[F(w)-F(w)\right]$

d)
$$\begin{array}{ccccc} f(t) & \rightarrow & f(-2t) & & df(-2t)/dt \\ F(w) & & \frac{1}{2}F\left(\frac{-w}{2}\right) & \rightarrow & jw\,F(-w/2)/2 \end{array}$$

---

e)
$$\begin{array}{ccccccc} f(t) & \rightarrow & f(3t) & \rightarrow & -itf(3t) & \rightarrow & tf(3t) \\ F(w) & & \frac{1}{3}F\left(\frac{w}{3}\right) & & \frac{1}{3}\frac{dF(w/3)}{dw} & & \frac{i}{3}\frac{dF(w/3)}{dw} \end{array}$$

f)
$$\begin{array}{ccccc} f(t)-f(t) & \rightarrow & -itf(t)-5f(t) & \rightarrow & (t-5)f(t) \\ F(w)-F(w) & & \frac{dF(w)}{dw}-5F(w) & & i\frac{dF(w)}{dw}-5F(w) \end{array}$$

g)
$$\begin{array}{ccccc} f(t)-3f(t) & \rightarrow & f(-3t)-3f(-3t) & \rightarrow & -itf(-3t)-3f(-3t) \\ F(w)-3F(w) & & \frac{1}{3}\left(F\left(\frac{w}{-3}\right)-3F\left(\frac{w}{-3}\right)\right) & & \frac{dF(w/3)}{3dw}-F\left(\frac{w}{-3}\right) \end{array}$$

$$\begin{array}{cc} \rightarrow & (t-3)f(-3t) \\ & \frac{dF(w/-3)}{3\,dw}-F\left(\frac{w}{-3}\right) \end{array}$$

h)
$$\begin{array}{ccccccc} f(t) & \rightarrow & df(t)/dt & \rightarrow & -itdf(w)/dt & \rightarrow & t\,df(t)/dt \\ F(w) & & iwF(w) & & iw\,dF(w)/dw & & -w\,dF(w)/dw \end{array}$$

i)
$$\begin{array}{ccccc} f(t) & \rightarrow & f(-t) & \rightarrow & f(6-t) \\ F(w) & & F(w) & & F(-w)e^{i6w} \end{array}$$

a) $\delta(t) \leftrightarrow 1$

$5\delta(t) \leftrightarrow 5$

$5\delta(t-1) \leftrightarrow 5e^{-i\omega}$

b) $1 \leftrightarrow \delta(-t)\, 2\pi$

$\frac{16}{2\pi} \leftrightarrow 16\delta(t)$

$\frac{8}{\pi} \leftrightarrow 16\delta(t)$

$\frac{8}{\pi}\cos(t) \leftrightarrow 8\delta(t+1) + 8\delta(t-1)$

c) $1 \leftrightarrow \delta(t)\, 2\pi$

$-it \leftrightarrow \frac{d\delta(t)}{dt}\, 2\pi$

$t \leftrightarrow 2\pi i\, d\delta(t)/dt$

d) $t \leftrightarrow 2\pi i\, d\delta(t)/dt$

$-it^2 \leftrightarrow 2\pi i\, d^2\delta(t)/dt^2$

$t^2 \leftrightarrow 2\pi\, d^2\delta(t)/dt^2$

e) $2C_2(t) \leftrightarrow 4\,Sa(\omega)$

$2C_2(t)\cos 1000t \leftrightarrow 2[Sa(\omega+1000) - Sa(\omega-$ [illegible] $]$

f) $\cos 1000t \leftrightarrow [\delta(\omega+1000) - \delta(\omega-1000)]\pi$

10

a) $\text{sgn}(t) \leftrightarrow \frac{2}{jw}$

$\frac{2}{it} \leftrightarrow 2\pi\,\text{sgn}(-w)$

$\frac{2i}{4t} \leftrightarrow \frac{2\pi}{4}\text{sgn}(4w)$

$\frac{i}{2t}e^{2it} \leftrightarrow \frac{\pi}{2}\text{sgn}(4w-2)$

$\frac{3i}{\pi t}e^{2it} \leftrightarrow 3\,\text{sgn}(4w-2)$

---

b) $C_2(t) \leftrightarrow 2\,Sa(w)$

$C_2\left(\frac{2}{3}t\right) \leftrightarrow 3\,Sa\left(\frac{3w}{2}\right)$

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c) $2C_2(t) \leftrightarrow 4\,Sa(w)$

$2C_2(t)\cos 250t \leftrightarrow 2\left[Sa(w+250) + Sa(w-250)\right]$

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d) $u(t) \leftrightarrow \pi\delta(w) + \frac{1}{jw}$

$u(10t) \leftrightarrow \frac{\pi}{10}\delta\left(\frac{w}{10}\right) + \frac{1}{10iw}$

$u(10t-1) \leftrightarrow \frac{e^{-iw}}{10}\left[\pi\delta\left(\frac{w}{10}\right) + \frac{1}{iw}\right]$

f) $C_8(t) \leftrightarrow 8\,Sa(4w)$

$C_8(t)e^{+2it} \leftrightarrow Sa(4w-2)$

$\frac{4}{\pi}C_8(t)e^{2it} \leftrightarrow \frac{4}{\pi}Sa(4w-2)$

h) $C_{\frac{4}{3}}(t) \leftrightarrow \frac{4}{3}Sa\left(\frac{2w}{3}\right)$

$C_{\frac{4}{3}}(t+6) \leftrightarrow \frac{4}{3}Sa\left(\frac{2w}{3}\right)e^{6iw}$

---

j) $\delta(t) \leftrightarrow 1$

$1 \leftrightarrow 2\pi\delta(-t)$

$2\cos 500t \leftrightarrow \pi\left[\delta(500-t)+\delta(-500-t)\right]$

$\pi\left[\delta(500-t)+\delta(-500-t)\right] \leftrightarrow 4\pi\cos -500t$

$\frac{1}{2}\left[\delta(500-t)+\delta(-500-t)\right] \leftrightarrow 2\cos 500t$

$\frac{1}{2}\left[\delta(500+t)+\delta(t-500)\right] \leftrightarrow 2\cos 500t$

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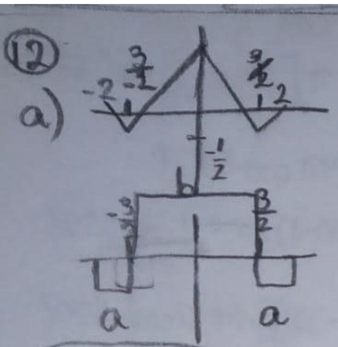
m) $\text{sgn}(t) \leftrightarrow \frac{2}{iw}$

$\frac{i}{2}\text{sgn}(t) \leftrightarrow \frac{1}{w}$

(12)

a)



$$a = \frac{-\frac{1}{2} - 0}{-\frac{3}{2} + 2} = -1$$

$$b = \frac{4+1}{0 + \frac{3}{2}} = \frac{10}{3}$$

$$f''(t) = -\delta(t+2) + \frac{13}{3}\delta\left(t+\frac{3}{2}\right) - \frac{13}{3}\delta\left(t-\frac{3}{2}\right) + \delta(t-2)$$

$$F(w) = \frac{1}{(iw)^2}\left[-e^{2iw} + \frac{13}{3}e^{\frac{3}{2}iw} - \frac{13}{3}e^{-\frac{3}{2}iw} + e^{-2iw}\right]$$

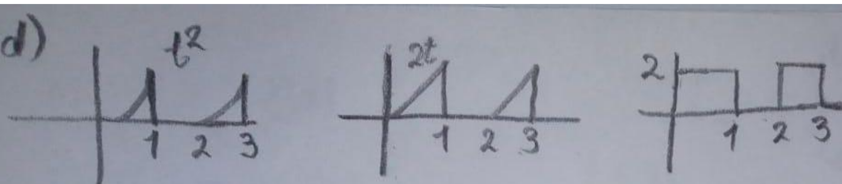
c) $f'(t) = A\delta(t+6) - A\delta(t+4) + A\delta(t-4) - A\delta(t-6)$

$$F(w) = \frac{A}{iw}\left[e^{6iw} - e^{4iw} + e^{-4iw} - e^{-6iw}\right]$$

f) $f'(t) = \delta(t) + \delta(t-1) + \delta(t-2) - \delta(t-3) - \delta(t-4) - \delta(t-5)$

$$F(w) = \frac{1}{iw}\left[1 + e^{-iw} + e^{-2iw} - e^{-3iw} - e^{-4iw} - e^{-5iw}\right]$$

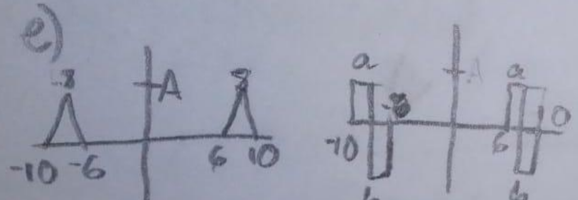
d)



$$f'''(t) = 2\delta(t) - 2\delta(t-1) + 2\delta(t-2) - 2\delta(t-3)$$

$$F(w) = \frac{2}{(iw)^3}\left[1 - e^{-iw} + e^{-2iw} - e^{-3iw}\right]$$

e)



$$a = \frac{A-0}{-8+10} = \frac{A}{2}$$

$$b = \frac{0-A}{-6+8} = -\frac{A}{2}$$

$$f''(t) = \frac{A}{2}\delta(t+10) - A\delta(t+8) + \frac{A}{2}\delta(t+6) + \frac{A}{2}\delta(t-6) - A\delta(t-8) + \frac{A}{2}\delta(t-10)$$

$$F(w) = \frac{A}{(iw)^2}\left[\frac{e^{10iw}}{2} - e^{8iw} + \frac{e^{6iw}}{2} + \frac{e^{-6iw}}{2} - e^{-8iw} + \frac{e^{-10iw}}{2}\right]$$

a) $f(t) = A C_{\frac{2\pi}{5}}(t) \cos(20t) \leftrightarrow A \frac{\pi}{5}\left[Sa\left(\frac{\pi(\omega+20)}{5}\right) + Sa\left(\frac{\pi(\omega-20)}{5}\right)\right]$

b) $f(t) = A C_{\frac{9\pi}{20}}(t) \cos(20t) \leftrightarrow A \frac{9\pi}{40}\left[Sa\left(\frac{9\pi(\omega+20)}{40}\right) + Sa\left(\frac{9\pi}{40}(\omega-20)\right)\right]$

c) $A g(t) \cos 200\pi t$

$T = 4 \quad \omega_0 = \frac{\pi}{2} \qquad G(\omega) = 2\pi \sum_{-\infty}^{\infty} C_n \delta(\omega - n\omega_0)$ $\pi/2$

$C_n = \frac{1}{4}\int_{-1}^{1} e^{-\frac{in\pi t}{2}} dt = \frac{-1}{2in\pi} e^{-\frac{in\pi}{2}t}\Big|_{-1}^{1} = \frac{1}{n\pi}\left(e^{\frac{-in\pi}{2}} - 1\right)$

$A g(t) \cos 200\pi t \leftrightarrow \frac{A}{n\pi} \sum_{-\infty}^{\infty} \left(e^{-\frac{in\pi}{2}} - 1\right)\left[\delta\left(\omega - \frac{n\pi}{2} + 200\pi\right) + \delta\left(\omega - \frac{n\pi}{2} - 200\pi\right)\right]$