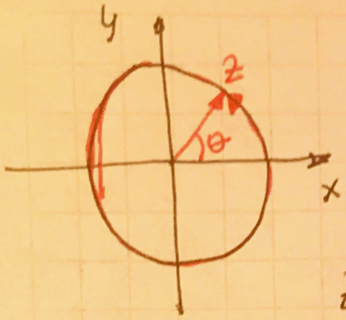
(d) $I = \int_C x\,dz$ SOBRE EL CÍRCULO $|z|=1$ DESCRITO EN LA DIRECCIÓN CONTRARIA A LAS MANECILLAS DEL RELOJ. (91-I)



$$z = e^{it},\ 0 \le t \le 2\pi$$

$$x(t) = \cos t,\ y(t) = \operatorname{sen} t$$

$$dz = (-\operatorname{sen} t + i\cos t)\,dt$$

$$z(t) = x(t) + i\operatorname{sen}(t) = \cos t + i\operatorname{sen} t \Rightarrow z'(t) = -\operatorname{sen} t + i\cos t.$$

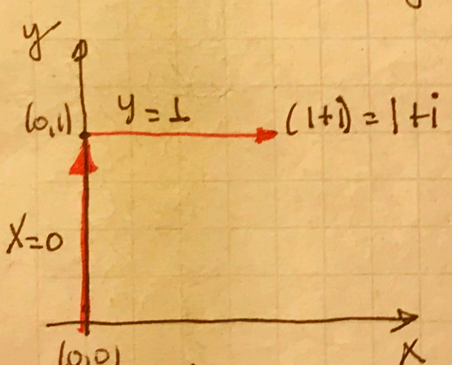
$$I = \int_C x\,dz = \int_0^{2\pi} \cos t\left[-\operatorname{sen} t + i\cos t\right] dt = -\int_0^{2\pi} \cos t \operatorname{sen} t\,dt + i\int_0^{2\pi} \cos^2 t\,dt$$

$$= \frac{\cos^2 t}{2}\Big|_0^{2\pi} + i\left(\frac{t}{2} + \frac{1}{4}\operatorname{sen} 2t\right)\Big|_0^{2\pi} = \pi i$$

$$\int \cos^2 u\,du = \frac{u}{2} + \frac{\operatorname{sen} 2u}{4} = \frac{1}{2}\left[u + \operatorname{sen} u \cos u\right]$$

$$\therefore I = \pi i$$

(e) $I = \int_0^{1+i} (x+y-ix^2)\,dz$ SOBRE LA TRAYECTORIA A LO LARGO DE LAS LÍNEAS $x=0$ y $y=1$.



(1) $x(t) = 0,\ y(t) = t \Rightarrow z(t) = x(t) + iy(t)$

(*) $z(t) = 0 + it = it \Rightarrow z'(t) = i;\ dz = i\,dt$

(2) $x(t) = t;\ y(t) = 1;\ z(t) = x(t) + iy(t)$

$z(t) = t + i \Rightarrow z'(t) = 1;\ dz = dt$

$$I = \int_0^{1+i} (x+y-ix^2)\,dz = \int_0^1 (0 + t - i(0))\,i\,dt + \int_0^1 (t + 1 - it^2)\,dt$$

$$= \int_0^1 ti\,dt + \int_0^1 (t+1-it^2)\,dt = i\frac{t^2}{2}\Big|_0^1 + \frac{t^2}{2} + t - i\frac{t^3}{3}\Big|_0^1$$

$$= \frac{i}{2} + \frac{1}{2} + 1 - i\frac{1}{3} = \frac{i}{2} - i\frac{1}{3} + \frac{1}{2} + 1 = \frac{3}{2} + \frac{i}{6} \qquad \therefore I = \frac{3}{2} + \frac{i}{6}$$