Problema 1

$$f(t) = \begin{cases} \frac{1}{2}t & 0 < t \leq 2 \\ 1 & 2 < t \leq 4 \\ -\frac{1}{2}(t-6) & 4 < t \leq 6 \\ f(t+6) & \text{otro caso} \end{cases}$$

Con $T = 6$

$$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{6} = \frac{\pi}{3}$$

Ya que $f(t)$ es una función par $b_n = 0$

Calculando a_n

$$a_n = \frac{2}{T}\int_{t_0}^{t_0+T} f(t)\cos(n\omega_0 t)\,dt$$; pero por simetría podemos calcularla así:

$$= \frac{4}{6}\left\{\frac{1}{2}\int_0^2 t\cos\left(n\frac{\pi}{3}t\right)dt + \int_2^3 \cos\left(n\frac{\pi}{3}t\right)dt\right\}$$

$$= \frac{2}{3}\left\{\frac{1}{2}\int_0^2 t\cos\left(\frac{n\pi t}{3}\right)dt + \int_2^3 \cos\left(n\frac{\pi t}{3}\right)dt\right\}$$

$$= \frac{2}{3}\left\{\frac{1}{2}\left[\frac{3t}{n\pi}\operatorname{sen}\left(\frac{n\pi t}{3}\right) + \left(\frac{3}{n\pi}\right)^2\cos\left(\frac{n\pi t}{3}\right)\right]_0^2 + \left.\frac{3}{n\pi}\operatorname{sen}\left(\frac{n\pi t}{3}\right)\right|_2^3\right\}$$

$$= \frac{2}{3}\left\{\frac{1}{2}\left[\frac{6}{n\pi}\operatorname{sen}\left(\frac{2n\pi}{3}\right) + \frac{9}{n^2\pi^2}\cos\left(\frac{2n\pi}{3}\right) - \frac{9}{n^2\pi^2}\cos(0)\right] + \cancel{\frac{3}{n\pi}\operatorname{sen}\left(\frac{3n\pi}{3}\right)}^{0} - \frac{3}{n\pi}\operatorname{sen}\left(\frac{2n\pi}{3}\right)\right\}$$

$$= \frac{2}{3}\left\{\frac{1}{2}\left[\frac{6}{n\pi}\operatorname{sen}\left(\frac{2n\pi}{3}\right) + \frac{9}{n^2\pi^2}\cos\left(\frac{2n\pi}{3}\right) - \frac{9}{n^2\pi^2}\right] - \frac{3}{n\pi}\operatorname{sen}\left(\frac{2\pi n}{3}\right)\right\}$$

$$= \frac{2}{3}\left\{\cancel{\frac{3\operatorname{sen}\left(\frac{2n\pi}{3}\right)}{n\pi}} + \frac{9\left(\cos\left(\frac{2n\pi}{3}\right)-1\right)}{2n^2\pi^2} - \cancel{\frac{3\operatorname{sen}\left(\frac{2\pi n}{3}\right)}{n\pi}}\right\}$$

$$= \frac{3\left(\cos\left(\frac{2n\pi}{3}\right)-1\right)}{n^2\pi^2} \quad \forall n \neq 0$$

Calculando a_0

$$a_0 = \frac{2}{T}\int_0^{T/2} f(t)\,dt \leftarrow \text{Por simetria}$$

$$a_0 = \frac{2}{6}\int_0^{3} f(t)\,dt = \frac{1}{3}\left\{\int_0^2 \frac{1}{2}t\,dt + \int_2^3 1\,dt\right\}$$

$$= \frac{1}{3}\left\{\left.\frac{t^2}{4}\right|_0^2 + \left.t\right|_2^3\right\} = \frac{1}{3}\left\{\frac{4}{4} - 0 + 3 - 2\right\}$$

$$= \frac{1}{3}\left\{1 - 0 + 3 - 2\right\} = \frac{1}{3}(2) = \frac{2}{3}$$

Por lo tanto

$$F(t) = \frac{2}{3} + \sum_{n=1}^{\infty} \frac{3\left(\cos\left(\frac{2n\pi}{3}\right) - 1\right)}{n^2\pi^2}\cos\left(\frac{n\pi}{3}t\right)$$

Problema (2)

La S.E. está dado por

$$C_n = \frac{1}{2}(a_n - i b_n) \quad \text{pero } b_n = 0$$

Por lo tanto

$$C_n = \frac{1}{2}\left(\frac{3\left(\cos\left(\frac{2n\pi}{3}\right) - 1\right)}{n^2\pi^2}\right) \quad \forall n \neq 0$$

y $C_0 = a_0 = \frac{2}{3}$

Entonces

$$F(t) = \frac{2}{3} + \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{3\left(\cos\left(\frac{2\pi n}{3}\right) - 1\right)}{2n^2\pi^2} e^{\frac{in\pi t}{3}}$$

Problema (3.)

$$\mathcal{F}\{f(t)\} = 2\pi \sum_{n=-\infty}^{\infty} C_n \delta(\omega - n\omega_0) \quad \text{Sea } T=1 \,; \; \omega_0 = 2\pi$$

$$C_n = \frac{1}{T}\int_{-T/2}^{T/2} f(t) e^{-in\omega_0 t} dt = \int_{-1/2}^{1/2} f(t) e^{-2in\pi t} dt = \int_{0^-}^{0^+} \delta(t) e^{-2in\pi t} dt$$

Por la propiedad de muestreo de $\delta(t)$

$$C_n = e^0 = 1$$

Por lo tanto:

$$\mathcal{F}\{f(t)\} = 2\pi \sum_{n=-\infty}^{\infty} \delta(\omega - 2\pi n)$$

Problema 4

$$? \leftrightarrow \underbrace{6\delta(3\omega-10)\cos(15\omega)}_{\mathrm{I}} + \underbrace{\frac{1}{3-i\omega\omega^2}}_{\mathrm{II}} + \underbrace{e^{4i\omega}(\omega-1)}_{\mathrm{III}}$$

Para I

Partiendo de

$$\delta(t) \leftrightarrow 1$$

$$1 \leftrightarrow 2\pi\delta(-\omega)$$

$$\frac{1}{2\pi} \leftrightarrow \delta(-\omega)$$

$$\frac{1}{2\pi} \leftrightarrow \delta(\omega) \leftarrow \text{func. par}$$

$$\frac{1}{2\pi} e^{10it} \leftrightarrow \delta(\omega-10)$$

$$\frac{1}{2\pi} e^{\frac{-10it}{3}} \leftrightarrow 3\delta(3\omega-10)$$

$$\frac{1}{\pi} e^{\frac{10it}{3}} \leftrightarrow 6\delta(3\omega-10)$$

$$\frac{1}{2\pi}\left[e^{\frac{i10}{3}(t+15)} + e^{\frac{i10}{3}(t-15)}\right] \leftrightarrow 6\delta(3\omega-10)\cos(15\omega)$$

Para II

Partiendo de

$$e^{-at}u(t) \leftrightarrow \frac{1}{a+i\omega}$$

$$e^{-3t}u(t) \leftrightarrow \frac{1}{3+i\omega}$$

$$\frac{d^2}{dt}\left(e^{-3t}u(t)\right) \leftrightarrow \frac{(i\omega)^2}{3+i\omega}$$

$$-\frac{d^2}{d\omega}\left(e^{3t}u(-t)\right) \leftrightarrow \frac{\omega^2}{3-i\omega^2}$$

Para III

Partiendo de

$$\delta(t) \leftrightarrow 1$$

$$\frac{d}{dt}\delta(t) \leftrightarrow i\omega$$

$$-i\frac{d}{d\omega}\delta(t)e^{it} \leftrightarrow (\omega-1)$$

$$-i\frac{d}{d\omega}\delta(t+4)e^{i(t+4)} \leftrightarrow (\omega-1)e^{4i\omega}$$

Finalmente

$$\frac{1}{2\pi}\left[e^{\frac{10i}{3}(t+15)} + e^{\frac{10i}{3}(t-15)}\right] - \frac{d^2}{d\omega}\left(e^{3t}u(t)\right) - i\frac{d}{d\omega}\delta(t+4)e^{i(t+4)}$$