INSTITUTO POLITECNICO NACIONAL

ESCUELA SUPERIOR DE COMPUTO

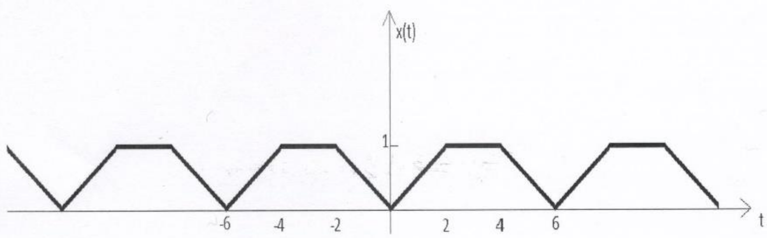
Teoría de Comunicaciones y Señales

1er. Exámen departamental

NOMBRE:____ TIPO: A

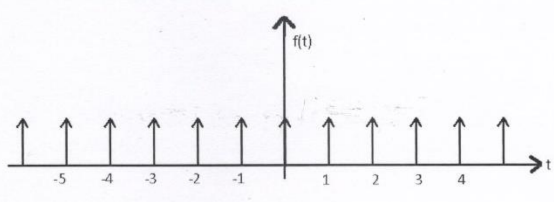
GRUPO:________________________

Problema 1. Encuentre la Serie Trigonométrica de Fourier de x(t)



Problema 2. A partir de la serie obtenida en el problema anterior, encuentra la Serie Exponencial de Fourier.

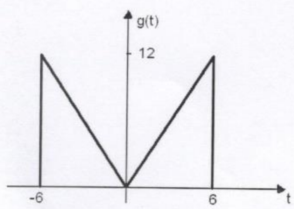
Problema 3. Encuentre la transformada de Fourier de la siguiente función periódica:



Problema 4. Usando propiedades, encuentre la transformada de Fourier de:

$$-6\delta[5t-10]\cdot\cos 15t \quad + \quad \frac{1}{3-jt}\cdot t^2 \quad + \quad e^{j4t}(t-1) \quad \leftrightarrow \quad ?$$

Problema 5. Usando Propiedades de la transformada de Fourier encuentre la transformada de g(t)



1er Departamental

TIPO A

Problema 1

$$x(t)=\begin{cases}\frac{1}{2}t & 0<t\leq 2\\ 1 & 2<t\leq 4\\ -\frac{1}{2}(t-6) & 4<t\leq 6\\ x(t+6) & \text{otro caso}\end{cases}$$

$T=6 \therefore \omega_0=\frac{2\pi}{6}=\frac{\pi}{3}$

como $x(t)$ es par

$\therefore b_n=0$

&

$$a_n=\frac{4}{T}\int_0^{\frac{T}{2}}x(t)\cos n\omega_0 t\,dt \quad ; \quad a_0=\frac{2}{T}\int_0^{\frac{T}{2}}x(t)dt$$

$$a_n=\frac{4}{6}\int_0^3 x(t)\cos\frac{n\pi}{3}t\,dt$$

$$a_n=\frac{\cancel{2}}{3}\int_0^2\frac{1}{\cancel{2}}t\cdot\cos\frac{n\pi}{3}t\,dt+\frac{2}{3}\int_2^3\cos\frac{n\pi}{3}t\,dt$$

$u=t \quad dv=\cos\frac{n\pi}{3}t\,dt$

$du=dt \quad v=\frac{3}{n\pi}\operatorname{sen}\frac{n\pi}{3}t$

$$a_n=\frac{1}{3}\left\{\frac{3t}{n\pi}\operatorname{sen}\frac{n\pi}{3}t\Big|_0^2-\frac{3}{n\pi}\int_0^2\operatorname{sen}\frac{n\pi}{3}t\,dt\right\}$$
$$+\left(\frac{2}{\cancel{3}}\right)\left(\frac{\cancel{3}}{n\pi}\right)\operatorname{sen}\frac{n\pi}{3}t\Big|_2^3$$

$$a_n=\frac{1}{3}\left\{\frac{6}{n\pi}\operatorname{sen}\frac{2}{3}n\pi+\frac{9}{n^2\pi^2}\cos\frac{n\pi}{3}t\Big|_0^2\right.$$
$$+\frac{2}{n\pi}\left(\underbrace{\operatorname{sen} n\pi}_{0}-\operatorname{sen}\frac{2}{3}n\pi\right)$$

$$a_n=\cancel{\frac{2}{n\pi}\operatorname{sen}\frac{2}{3}n\pi}+\frac{3}{n^2\pi^2}\left[\cos\frac{2}{3}n\pi-1\right]$$
$$-\cancel{\frac{2}{n\pi}\operatorname{sen}\frac{2}{3}n\pi} \qquad \forall\, n\neq 0$$

$$a_n=\frac{3}{n^2\pi^2}\left[\cos\frac{2}{3}n\pi-1\right]$$

$$a_0=\frac{2}{6}\int_0^3 x(t)dt$$

$$a_0=\frac{1}{3}\int_0^2\frac{1}{2}t\,dt+\frac{1}{3}\int_2^3 dt$$

$$a_0=\frac{1}{6}\left[\frac{t^2}{2}\right]_0^2+\frac{1}{3}t\Big|_2^3$$

$$a_0=\frac{1}{12}[4-0]+\frac{1}{3}[3-2]$$

$$a_0=\frac{1}{3}+\frac{1}{3}$$

$$a_0=\frac{2}{3}$$

Finalmente:

$$x(t)=\frac{2}{3}+\sum_{n=1}^{\infty}\frac{3}{n^2\pi^2}\left(\cos\frac{2}{3}n\pi-1\right)\cdot\cos\frac{n\pi}{3}t$$

Problema 2

Si $x(t) = \frac{2}{3} + \sum_{n=1}^{\infty} \frac{3}{n^2\pi^2}\left(\cos \frac{2}{3}n\pi - 1\right) \cdot \cos \frac{n\pi}{3} t$

entonces su serie exponencial está dada por:

$C_n = \frac{1}{2}(a_n - i b_n)$, con $b_n = 0$

así $C_n = \frac{1}{2} \cdot \frac{3}{n^2\pi^2}\left(\cos\left(\frac{2}{3}n\pi\right) - 1\right) \quad \forall\, n \neq 0$

$C_0 = a_0 = \frac{2}{3}$

Finalmente:

$$x(t) = \frac{2}{3} + \sum_{n=-\infty}^{\infty} \frac{3}{2n^2\pi^2}\left(\cos \frac{2}{3}n\pi - 1\right) \cdot e^{i\frac{n\pi}{3}t}$$

Problema 3

$$\mathcal{F}\{f(t)\} = 2\pi \sum_{n=-\infty}^{\infty} C_n\, \delta(\omega - n\omega_0)$$

si $T = 1 \therefore \omega_0 = \frac{2\pi}{T} = 2\pi$

& $$C_n = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)\, e^{-jn\omega_0 t}\, dt$$

$$C_n = \int_{-\frac{1}{2}}^{\frac{1}{2}} f(t) \cdot e^{-j2\pi n t}\, dt$$

$$C_n = \int_{0^-}^{0^+} \delta(t)\, e^{-j2\pi n t}\, dt$$

De la Propiedad de muestreo de $\delta(t)$

$$C_n = e^{0} = 1$$

Finalmente:

$$\mathcal{F}\{f(t)\} = 2\pi \sum_{n=-\infty}^{\infty} \delta(\omega - 2\pi n)$$

2.0 pts

Problema 4

$$\underbrace{-6\delta(5t-10)\cdot \cos 15t}_{\text{I}} + \underbrace{\frac{1}{3-jt}t^2}_{\text{II}} + \underbrace{e^{j4t}(t-1)}_{\text{III}} \longleftrightarrow ?$$

SOLUCION:

I Si $\delta(t) \longleftrightarrow 1$

$$\delta(t-10) \longleftrightarrow e^{-j10\omega}$$

$$\delta(5t-10) \longleftrightarrow \frac{1}{|5|} e^{-j\frac{10}{5}\omega}$$

$$\delta(5t-10)\cdot \cos 15t \longleftrightarrow \frac{2}{5}\left[e^{-j2(\omega+15)} + e^{-j2(\omega-15)}\right]$$

$$-6\,\delta(5t-10)\cdot \cos 15t \longleftrightarrow -\frac{12}{5}\left(e^{-j2(\omega+15)} + e^{-j2(\omega-15)}\right]$$

h

II Si $e^{-at}u(t) \longleftrightarrow \frac{1}{a+j\omega}$

$$\frac{1}{a+jt} \longleftrightarrow 2\pi e^{-a(-\omega)} u(-\omega)$$

$$\frac{1}{3+jt} \longleftrightarrow 2\pi e^{3\omega} u(-\omega)$$

$$\frac{1}{3-jt} \longleftrightarrow 2\pi e^{-3\omega} u(\omega)$$

$$\frac{(-jt)^2}{3-jt} \longleftrightarrow 2\pi \frac{d^2}{d^2\omega}\left[e^{-3\omega}u(\omega)\right]$$

$$\frac{t^2}{3-jt} \longleftrightarrow -2\pi \frac{d^2}{d^2\omega}\left[e^{-3\omega}u(\omega)\right]$$

h

III Si $\delta(t) \longleftrightarrow 1$

$$1 \longleftrightarrow 2\pi\delta(-\omega)$$

$$-jt \longleftrightarrow 2\pi \frac{d}{d\omega}\delta(\omega)$$

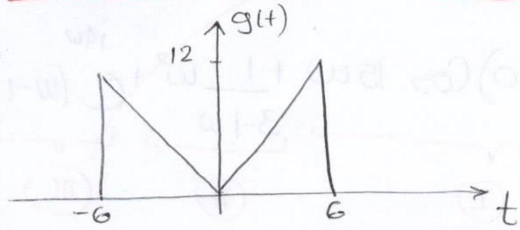
$$t-1 \longleftrightarrow 2\pi j \frac{d}{d\omega}\delta(\omega)\cdot e^{-j\omega}$$

$$e^{j4t}\cdot(t-1) \longleftrightarrow 2\pi j \frac{d}{d\omega}\delta(\omega+4) e^{-j(\omega+4)}$$

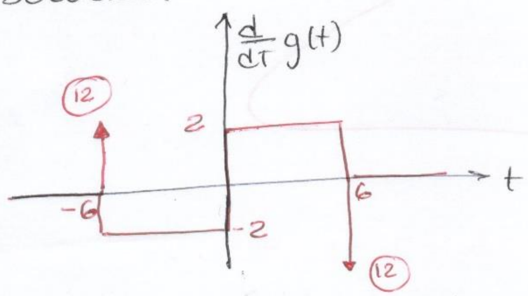
$$e^{j4t}\cdot(t-1) \longleftrightarrow j2\pi e^{-j(\omega+4)} \frac{d}{d\omega}\delta(\omega+4)$$

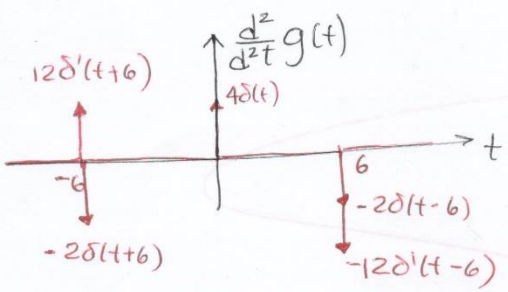
h

Problema 5 (2.0 pts)



SOLUCION:





$$\frac{d^2}{d^2t}g(t) = -2\delta(t+6) + 12\delta'(t+6) + 4\delta(t) - 2\delta(t-6) - 12\delta'(t-6)$$

$$\mathcal{F}\left\{\frac{d^2}{d^2t}g(t)\right\} = -2\mathcal{F}\{\delta(t+6)\} + 12\mathcal{F}\{\delta'(t+6)\} + 4\mathcal{F}\{\delta(t)\} - 2\mathcal{F}\{\delta(t-6)\} - 12\mathcal{F}\{\delta'(t-6)\}$$

$$\mathcal{F}\left\{\frac{d^2}{d^2t}g(t)\right\} = -2e^{i6\omega} + 12j\omega e^{i6\omega} + 4 - 2e^{-i6\omega} - 12j\omega e^{-i6\omega}$$

$$= -2(e^{i6\omega} + e^{-i6\omega}) + 12j\omega(e^{i6\omega} - e^{-i6\omega}) + 4$$

$$= -4\cos 6\omega - 24\omega \operatorname{Sen} 6\omega + 4$$

$$= 4(1 - \cos 6\omega) - 24\omega \operatorname{Sen} 6\omega$$

$$= 8 \cdot \operatorname{sen}^2 3\omega - 24\omega \operatorname{Sen} 6\omega$$

Así

$$\frac{d^2}{d^2t}g(t) \longleftrightarrow 8\operatorname{Sen}^2 3\omega - 24\omega \operatorname{Sen} 6\omega$$

De la Prop. de diferenciación en t

Si $g(t) \longleftrightarrow G(\omega)$

$$\frac{d^2}{d^2t}g(t) \longleftrightarrow (j\omega)^2 G(\omega)$$

Finalmente

$$(j\omega)^2 G(\omega) = 8\operatorname{sen}^2 3\omega - 24\omega \operatorname{Sen} 6\omega$$

$$G(\omega) = \frac{-1}{\omega^2}\left[8\operatorname{sen}^2 3\omega - 24\omega \operatorname{Sen} 6\omega\right]$$

$$G(\omega) = \frac{24}{\omega}\operatorname{Sen} 6\omega - \frac{8}{\omega^2}\operatorname{sen}^2 3\omega$$

$$G(\omega) = 144\, Sa\, 6\omega - 8(9)\frac{\operatorname{sen} 3\omega}{(3)\omega} \cdot \frac{\operatorname{sen} 3\omega}{(3)\omega}$$

$$G(\omega) = 144\, Sa\, 6\omega - 72\, Sa^2\, 3\omega$$