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Evidencia 1.3

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$$a_n = \frac{2}{T}\int_{t_0}^{t_0+T} f(t)\cos(n\omega_0 t)\,dt \qquad b_n = \frac{2}{T}\int_{t_0}^{t_0+T} f(t)\,\text{sen}(n\omega_0 t)\,dt$$

$$a_0 = \frac{1}{T}\int_{t_0}^{t_0+T} f(t)\,dt$$

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$$f(t) = \begin{cases} \cos t & -\frac{\pi}{2} < t < \frac{\pi}{2} \\ 0 & \text{en otro caso} \end{cases}$$

STF de $-\pi < t < \pi$ es: $a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + b_n \,\text{sen}\, \omega_0 t n$, $\omega_0 = \frac{2\pi}{2\pi}$

$$a_0 = \frac{1}{T}\int_{-\pi}^{\pi} f(t)\,dt = 0 + \int_{-\pi/2}^{\pi/2} \frac{\cos t\,dt}{2\pi} + 0 = \frac{\text{sen}\,\frac{\pi}{2} - \text{sen}\,-\frac{\pi}{2}}{2\pi} = 2/2\pi = \underline{\underline{1/\pi}}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\cos n\omega_0 t\,dt = \frac{1}{\pi}\int_{-\pi/2}^{\pi/2} \cos t \cos n\omega_0 t\,dt = \frac{1}{2\pi}\int_{-\pi/2}^{\pi/2} \cos(n\omega_0+1)t + \cos(n\omega_0-1)t\,dt$$

$$= \frac{1}{2\pi}\left[\frac{\text{sen}(n\omega_0+1)t}{n\omega_0+1} + \frac{\text{sen}(n\omega_0-1)t}{n\omega_0-1}\right]_{-\pi/2}^{\pi/2} = \frac{1}{2\pi}\left[\frac{\text{sen}\,(n+1)t}{n+1} + \frac{\text{sen}\,(n-1)t}{n-1}\right]_{-\pi/2}^{\pi/2}$$

$\in \mathbb{Z}$ $\quad \in \mathbb{Z}$

$$= \frac{1}{2\pi}\left[\frac{\text{sen}\,\frac{\pi}{2}(n+1)}{n+1} + \frac{\text{sen}\,\frac{\pi}{2}(n-1)}{n-1} - \frac{\text{sen}\,-\frac{\pi}{2}(n+1)}{n+1} - \frac{\text{sen}\,-\frac{\pi}{2}(n-1)}{n-1}\right] \quad \text{sen}\,-a = -\text{sen}\,a$$

$$= \frac{1}{2\pi}\left[\frac{1}{n+1}\left(\text{sen}\left(\frac{(n+1)\pi}{2}\right) - \text{sen}\left(\frac{-(n+1)\pi}{2}\right)\right) + \frac{1}{n-1}\left(\text{sen}\left(\frac{(n-1)\pi}{2}\right) - \text{sen}\left(\frac{-(n-1)\pi}{2}\right)\right)\right]$$

Se indetermina

$b_n = 0$, $f(t)$ es par

$$\lim_{n\to 1} a_n = \frac{1}{2\pi}\left\{\cancel{\frac{1}{2}[0-0]} + \frac{1}{0}[0-0]\right\} = \frac{0}{0} \quad \text{por L'Hopital para (2)}$$

$$= \frac{1}{2\pi}\left\{\frac{\frac{2\pi}{2}\cos\frac{(n-1)}{2}\pi - \frac{-2\pi}{2}\cos\frac{-(n-1)}{2}\pi}{1}\right\} = \frac{1}{2}\left(\frac{1}{2}+\frac{1}{2}\right) = \frac{1}{2}$$

$$f(t) = \frac{1}{\pi} + \frac{\cos t}{2} + \frac{1}{2\pi}\sum_{n=2}^{\infty}\left\{\frac{\text{sen}\,\frac{(n+1)\pi}{2} - \text{sen}\,\frac{-(n+1)\pi}{2}}{n+1} + \frac{\text{sen}\,\frac{(n-1)\pi}{2} - \text{sen}\,\frac{-(n-1)\pi}{2}}{n-1}\right\}\cos nt$$

Con n = 1000