Problemario 1er. Departamental.

Problema 1. Expresar las señales e^{-t}, t^2 y $2t$ como serie trigonométrica de Fourier en el intervalo $(0, 1)$.

① e^{-t}

$$a_0 = \frac{1}{1}\int_0^1 e^{-t}\,dt = -e^{-t}\Big|_0^1 = -e^{-1} + 1$$

$T = 1$

$\omega_0 = \frac{2\pi}{1} = 2\pi$

ILATE

$$a_n = \frac{2}{T}\int_0^1 e^{-t}\cos(2n\pi t)\,dt = (\cos(2n\pi t)e^{-t} + \int_0^1 e^{-t}(2n\pi \operatorname{Sen}(2n\pi t))dt)$$

$$\int u\,dv = uv - \int v\,du$$

$u = \cos(2n\pi t)$

$du = -2n\pi \operatorname{Sen}(2n\pi t)$

$\int dv = \int e^{-t}$

$v = -e^{-t}$

$u = \operatorname{Sen}(2n\pi t)$

$du = \cos(2n\pi t) \cdot 2n\pi t$

$$= 2\left(-\cos(2n\pi t)e^{-t}\Big|_0^1 - 2n\pi\left(-\operatorname{Sen}(2n\pi t)e^{-t}\Big|_0^1 + \int_0^1 e^{-t}(2n\pi\cos(2n\pi t))\,dt\right)\right)$$

$$= -2\cos(2n\pi)e^{-1} + 2 - 4n\pi(-\operatorname{Sen}(2n\pi)e^{-1} + 0 + 2n\pi\int_0^1 \cos(2n\pi t)e^{-t}\,dt$$

$$= -2\cos(2n\pi)e^{-1} + 2 + 4n\pi \operatorname{Sen}(2n\pi)e^{-1} - 8n^2\pi^2\int_0^1 \cos(2n\pi t)e^{-t}\,dt$$

$$(2 + 8n^2\pi^2)\int_0^1 e^{-t}\cos(2n\pi t)\,dt = 4n\pi \operatorname{sen}(2n\pi)e^{-1} - 2\cos(2n\pi)e^{-1} + 2$$

$$2\int_0^1 e^{-t}\cos(2n\pi t)\,dt = \frac{(4n\pi \operatorname{sen}(2n\pi)e^{-1} - 2\cos(2n\pi)e^{-1} + 2)2}{2 + 8n^2\pi^2}$$

$$= \frac{\left(\frac{4n\pi \operatorname{sen}(2n\pi)}{e} - \frac{2\cos(2n\pi)}{e} + 2\right)2}{8n^2\pi^2 + 2} = \frac{2\left(\frac{2e + 4n\pi \operatorname{Sen}(2n\pi) - 2\cos(2n\pi)}{e}\right)}{8n^2\pi^2 + 2}$$

$$= \frac{2(2n\pi \operatorname{sen}(2n\pi) - \cos(2n\pi) + e)}{4e\pi^2 n^2 + e}$$

$$\therefore\ a_n = \frac{2(2n\pi \operatorname{sen}(2n\pi) - \cos(2n\pi) + e)}{4e\pi^2 n^2 + e}$$

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$$b_n=\frac{2}{1}\int_0^1 e^{-t}\operatorname{sen}(2n\pi t)\,dt=2\left(-\operatorname{sen}(2n\pi t)e^{-t}\Big|_0^1+\int_0^1 (2n\pi)\cos(2n\pi t)e^{-t}dt\right)$$

$$\int u\,dv=uv-\int v\,du$$

$u=\operatorname{sen}(2n\pi t)$ $\qquad \int dv=\int e^{-t}dt$ $\qquad u=\cos(2n\pi t)$

$du=2n\pi\cos(2n\pi t)$ $\qquad v=-e^{-t}$ $\qquad du=-2n\pi\operatorname{sen}(2n\pi t)$

$$=-2\operatorname{sen}(2n\pi)e^{-1}+4n\pi\int_0^1\cos(2n\pi t)e^{-t}dt=-2\operatorname{sen}(2n\pi)e^{-1}+4n\pi\left(-\cos(2n\pi t)e^{-t}\Big|_0^1\right.$$

$$\left.-\int_0^1 2n\pi\operatorname{sen}(2n\pi t)e^{-t}dt\right)=-2\operatorname{sen}(2n\pi)e^{-1}-4n\pi\cos(2n\pi)e^{-1}+4n\pi-$$

$$8n^2\pi^2\int_0^1\operatorname{sen}(2n\pi t)e^{-t}dt$$

$$\Rightarrow (2+8n^2\pi^2)\int_0^1 e^{-t}\operatorname{sen}(2n\pi t)dt=4n\pi-2\operatorname{sen}(2n\pi)e^{-1}-4n\pi\cos(2n\pi)e^{-1}$$

$$2\int_0^1 e^{-t}\operatorname{sen}(2n\pi t)dt=\frac{2(4n\pi-2\operatorname{sen}(2n\pi)e^{-1}-4n\pi\cos(2n\pi)e^{-1})}{2+8n^2\pi^2}$$

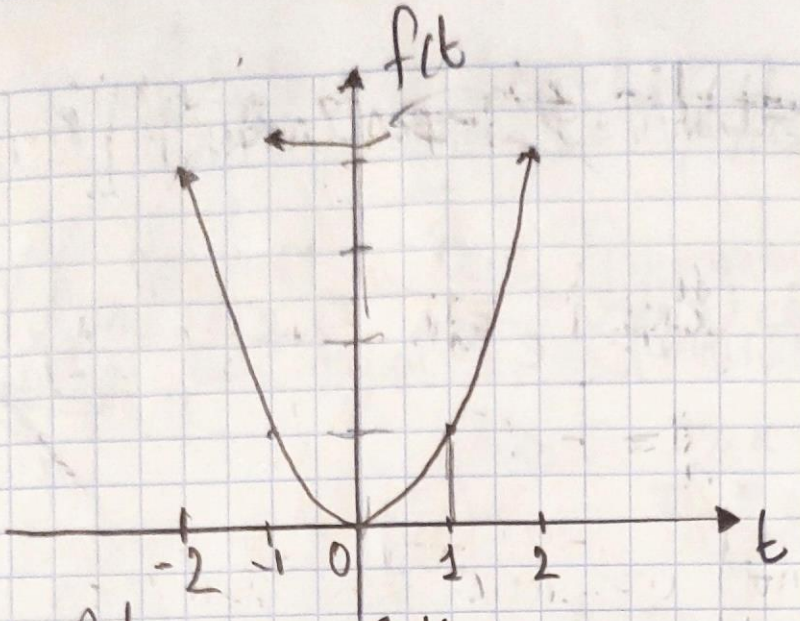
$$b_n=\frac{2\left(2n\pi-\frac{\operatorname{sen}(2n\pi)}{e}-\frac{2n\pi\cos(2n\pi)}{e}\right)}{4n^2\pi^2+1}=\frac{2(2n\pi e-\operatorname{sen}(2n\pi)-2n\pi\cos(2n\pi))}{4en^2\pi^2+e}$$

$$b_n=\frac{2(2n\pi e-\operatorname{sen}(2n\pi)-2n\pi\cos(2n\pi))}{4en^2\pi^2+e}$$

$$f(t)=-e^{-1}+1+\sum_{n=1}^{\infty}\frac{2(2n\pi\operatorname{sen}(2n\pi)-\cos(2n\pi)+e)}{e(4n^2\pi^2+1)}\cdot\cos(2n\pi t)+$$

$$\frac{2(2n\pi e-\operatorname{sen}(2n\pi)-2n\pi\cos(2n\pi))}{e(4n^2\pi^2+1)}\cdot\operatorname{Sen}(2n\pi t)$$

② t^2



$T = 1$

$\omega_0 = 2\pi$

$$a_0 = \frac{1}{1}\int_0^1 t^2\,dt = \frac{t^3}{3}\Big|_0^1 = \frac{1}{3}$$

$$a_n = \frac{2}{1}\int_0^1 t^2 \cos 2\pi n t\,dt = 2\left(\frac{t^2\sin(2n\pi t)}{2n\pi} + \frac{2t\cos(2n\pi t)}{(2n\pi)^2} - \frac{2\sin(2n\pi t)}{(2n\pi)^3}\right)\Big|_0^1$$

$t^2 \;+\; \cos 2n\pi t$

$2t \;-\; \sin 2n\pi t / 2n\pi$

$2 \;+\; -\cos 2n\pi t/(2n\pi)^2$

$0 \;\; -\sin 2n\pi t/(2n\pi)^3$

$$a_n = 2\left(\frac{\sin(2n\pi)}{2n\pi} - 0 + \frac{2\cos(2n\pi)}{(2n\pi)^2} - 0 - \frac{2\sin(2n\pi)}{(2n\pi)^3} - 0\right)$$

$$a_n = \cancel{\frac{\sin(2n\pi)}{n\pi}}^{\to 0} + \frac{\cos(2n\pi)}{n^2\pi^2} - \cancel{\frac{\sin(2n\pi)}{2n^3\pi^3}}^{\to 0}$$

$$a_n = \frac{1}{n^2\pi^2}$$

$$b_n = \frac{2}{1}\int_0^1 t^2 \sin(2n\pi t)\,dt = 2\left(\frac{-t^2\cos(2n\pi t)}{2n\pi} + \frac{2t\sin(2n\pi t)}{(2n\pi)^2} + \frac{2\cos(2n\pi t)}{(2n\pi)^3}\right)\Big|_0^1$$

$t^2 \;+\; \sin(2n\pi t)$

$2t \;-\; -\cos(2n\pi t)/2n\pi$

$2 \;+\; -\sin(2n\pi t)/(2n\pi)^2$

$0 \;\; \cos(2n\pi t)/(2n\pi)^3$

$$b_n = 2\left(\frac{-\cos(2n\pi)}{2\pi n} + 0 + \frac{2\sin(2n\pi)}{(2n\pi)^2} - 0 + \frac{2\cos(2n\pi)}{(2n\pi)^3} - \frac{2}{(2n\pi)^3}\right)$$

$$b_n = \frac{-\cos(2n\pi)}{\pi n} + \cancel{\frac{\sin(2n\pi)}{n^2\pi^2}}^{\to 0} + \frac{\cos(2n\pi)}{2n^3\pi^3} - \frac{1}{2n^3\pi^3}$$

$$b_n = \frac{-1}{\pi n} + \cancel{\frac{1}{2n^3\pi^3}} - \cancel{\frac{1}{2n^3\pi^3}} = \frac{-1}{\pi n}$$

$$f(t) = \frac{1}{3} + \sum_{n=1}^{\infty} \frac{1}{n^2\pi^2} \cdot \cos(2n\pi t) - \frac{1}{\pi n} \cdot \operatorname{Sen}(2n\pi t)$$

3) $2t$ $T=1$ $w_0 = 2\pi$

$$a_0 = \frac{1}{1}\int_0^1 2t\,dt = t^2\Big|_0^1 = 1$$

$$a_0 = 1$$

$$a_n = \frac{2}{1}\int_0^1 2t\cos 2n\pi t\,dt = 4\int_0^1 t\cos 2n\pi t\,dt = 4\left(\frac{t\operatorname{sen} 2n\pi t}{2n\pi} + \frac{\cos 2n\pi t}{(2n\pi)^2}\right)\Big|_0^1$$

$t \quad \cos 2n\pi$

$1 \quad \operatorname{sen} 2n\pi / 2n\pi$

$0 \quad -\cos 2n\pi / (2n\pi)^2$

$$a_n = 4\left(\frac{\cancel{\operatorname{Sen}(2n\pi)}^{0}}{2n\pi} - 0 + \frac{\cos(2n\pi)}{(2n\pi)^2} - \frac{1}{(2n\pi)^2}\right)$$

$$a_n = \frac{4}{4n^2\pi^2} - \frac{4}{4n^2\pi} = 0$$

$$a_n = 0$$

$$b_n = 2\int_0^1 2t\operatorname{sen}(2n\pi t)\,dt = 4\int_0^1 t\operatorname{sen}(2n\pi t)\,dt = 4\left(-\frac{t\cos(2n\pi t)}{2n\pi} + \frac{\operatorname{Sen}(2n\pi t)}{(2n\pi)^2}\right)\Big|_0^1$$

$t \quad \operatorname{sen}(2n\pi t)$

$1 \quad -\cos(2n\pi t)/2n\pi$

$0 \quad -\operatorname{Sen}(2n\pi t)/(2n\pi)^2$

$$b_n = 4\left(-\frac{\cos(2n\pi)}{2n\pi} - 0 + \frac{\operatorname{Sen}(2n\pi)^{0}}{(2n\pi)^2} - 0\right)$$

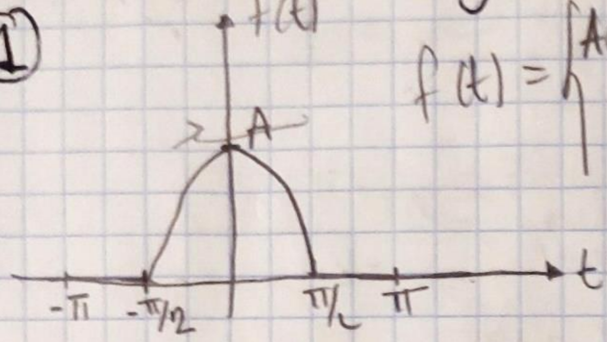
$$b_n = -\frac{4}{2n\pi} = -\frac{2}{n\pi}$$

Norma

$$f(t) = 1 - \sum_{n=1}^{\infty} \frac{2}{n\pi} \cdot \sin(2n\pi t)$$

Problema 2. Encontrar la Serie Trigonometrica de Fourier de cada una de las señales de la figura 1, en el intervalo de $-\pi$ a π.

(1)



$$f(t) = \begin{cases} A\cos t & -\pi/2 < t < \pi/2 \\ 0 & \text{en otro caso} \end{cases}$$

$T = 2\pi$

$w_0 = 1$

Señal Par $b_n = 0$

$$a_0 = \frac{2}{T}\int_0^{T/2} f(t)\,dt$$

$$a_n = \frac{4}{T}\int_0^{T/2} f(t)\cos n w_0 t\,dt$$

$$a_0 = \frac{2}{2\pi}\int_0^{\pi/2} A\cos t\,dt = \frac{A}{\pi}\cdot \sin t\Big|_0^{\pi/2} = \frac{A}{\pi}\left(\sin(\tfrac{\pi}{2}) - 0\right) = \frac{A}{\pi}$$

$a_0 = 0$

$$a_n = \frac{4}{2\pi}\int_0^{\pi/2} A\cos t \cos nt\,dt = \frac{2A}{\pi}\int_0^{\pi/2} \frac{1}{2}\left(\cos(t(1-n)) + \cos(t(1+n))\right)dt$$

$$a_n = \frac{A}{\pi}\int_0^{\pi/2} \cos(t(1-n)) + \cos(t(1+n))\,dt = \frac{A}{\pi}\left[\frac{\sin(t(1-n))}{1-n} + \frac{\sin(t(1+n))}{1+n}\right]_0^{\pi/2}$$

$$a_n = \frac{A}{\pi}\left[\frac{\sin(\frac{\pi}{2}(1-n))}{1-n} + \frac{\sin(\frac{\pi}{2}(1+n))}{1+n} - 0\right]$$

$$a_0 = \frac{A}{\pi}\left[\frac{\sin(\frac{\pi}{2} - \frac{n\pi}{2})}{1-n} + \frac{\sin(\frac{\pi}{2} + \frac{n\pi}{2})}{1+n}\right]$$

$$a_0 = \frac{A}{\pi}\left(\frac{\sin(\frac{\pi}{2})\cos(\frac{n\pi}{2}) + \cancel{\cos(\frac{\pi}{2})\sin(\frac{n\pi}{2})}}{1-n} + \frac{\sin(\frac{\pi}{2})\cos(\frac{n\pi}{2}) + \cancel{\dots}}{1+n}\right]$$

$$a_n = \frac{A}{\pi}\left(\frac{\cos(\frac{n\pi}{2})}{1-n} + \frac{\cos(\frac{n\pi}{2})}{1+n}\right) = \frac{A}{\pi}\left(\frac{\cos(\frac{n\pi}{2}) + n\cos(\frac{n\pi}{2}) + \cos(\frac{n\pi}{2}) - n\cos(\frac{n\pi}{2})}{1^2-n^2}\right) = \frac{A}{\pi}\left(\frac{2\cos(\frac{n\pi}{2})}{1^2-n^2}\right)$$

$$a_n = \frac{2A}{\pi}\left(\frac{\cos(\frac{n\pi}{2})}{1-n^2}\right)$$

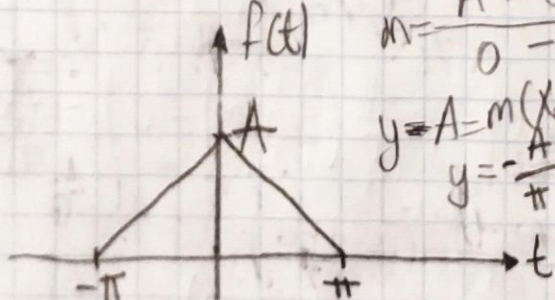
$$a_1 = \lim_{n\to 1}\frac{2A}{\pi}\left(\frac{\cos(n\frac{\pi}{2})}{1-n^2}\right) = \frac{2A}{\pi}\lim_{n\to 1}\frac{\sin(n\frac{\pi}{2})\cdot\frac{\pi}{2}}{-2n}$$

$$a_1 = \frac{\cancel{2}A}{\pi}\,\frac{\sin(\frac{\pi}{2})\cdot\frac{\pi}{2}}{\cancel{2}} = \frac{A\frac{\pi}{2}}{\pi} = \frac{A}{2}$$

$$a_1 = \frac{A}{2}$$

$$f(t) = \frac{1}{\pi} + \frac{A}{2}\cos(t) + \sum_{n=2}^{\infty}\frac{2A}{\pi}\left(\frac{\cos(\frac{n\pi}{2})}{1-n^2}\right)\cos(nt) \quad -\pi<t<\pi$$

②



$m = \frac{A-0}{0-\pi} = -\frac{A}{\pi}$

$T = 2\pi$

$y - A = m(x-0)$

$y = -\frac{A}{\pi}x + A$ $\quad \omega_0 = 1$

$$f(t) = \begin{cases} A(\frac{t}{\pi}+1) & -\pi<t<0 \\ A(-\frac{t}{\pi}+1) & 0<t<\pi \end{cases}$$

Par, $b_n = 0$

$$a_0 = \frac{2}{2\pi}\int_0^{\pi} A(-\frac{t}{\pi}+1)\,dt = \frac{\cancel{2}}{\cancel{2}\pi}A\left(\frac{-t^2}{2\pi}\Big|_0^{\pi} + t\Big|_0^{\pi}\right)$$

$$a_0 = \frac{A}{\pi}\left(-\frac{\pi^2}{2\pi} - 0 + \pi - 0\right) = \frac{A}{\pi}\left(-\frac{\pi}{2}+\pi\right) = \frac{A}{\pi}\left(\frac{\pi}{2}\right) = \frac{A}{2}$$

$$a_0 = \frac{A}{2}$$

$$a_n = \frac{4}{2\pi}\int_0^{\pi} A(-\frac{t}{\pi}+1)\cos(nt)\,dt = \frac{4A}{2\pi}\int_0^{\pi} -\frac{t}{\pi}\cos nt + \cos nt\,dt$$

$$\cancel{\begin{matrix} t & \cos nt \\ -1 & \frac{\operatorname{sen} nt}{n} \\ 0 & -\frac{\cos nt}{n^2}\end{matrix}}$$

$$a_n = \frac{2A}{\pi}\left(\frac{\operatorname{sen} nt}{n}\Big|_0^{\pi} + \frac{1}{\pi}\left(\frac{t \operatorname{sen} nt}{n}\Big|_0^{\pi} - \frac{\cos nt}{n^2}\Big|_0^{\pi}\right)\right)$$

$$a_n = \frac{2A}{\pi}\left(0 + \frac{1}{\pi}\left(\frac{\pi \cdot \cancel{0}}{n} + \frac{0 \cdot \cancel{\operatorname{sen} 0}}{n} - \frac{\cos n\pi}{n^2} + \frac{1}{n^2}\right)\right)$$

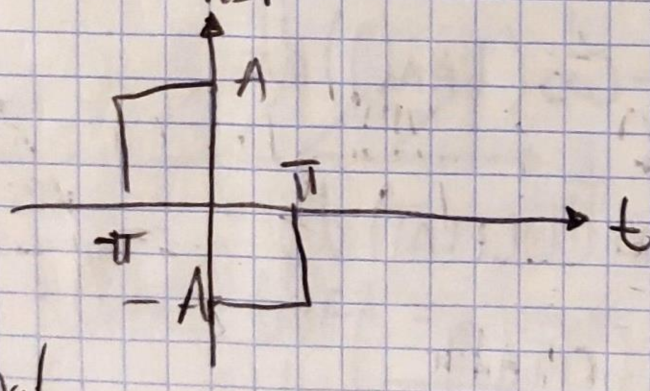
$$a_n = \frac{2A}{\pi}\left(\frac{-\cos n\pi + 1}{\pi n^2}\right) = -\frac{2A}{\pi}\left(\frac{(-1)^n - 1}{\pi n^2}\right)$$

$$a_n = -\frac{2A\left((-1)^n - 1\right)}{\pi^2 n^2}$$

$$\therefore f(t) = \frac{A}{2} + \sum_{n=1}^{\infty} -\frac{2A\left((-1)^n - 1\right)}{\pi^2 n^2} \cos(nt)$$

~~$-\pi < t < \pi$~~

③



$T = 2\pi$

$w_0 = 1$

$$g(t) = \begin{cases} A & -\pi < t < 0 \\ -A & \pi < t < 0 \end{cases}$$

Impar

$$b_n = \frac{4}{2\pi}\int_0^{\pi} -A \operatorname{sen} nt\,dt = \frac{-4}{2\pi} A \int_0^{\pi} \operatorname{sen} nt\,dt = \frac{-2A}{\pi}\int_0^{\pi} \operatorname{sen} nt\,dt$$

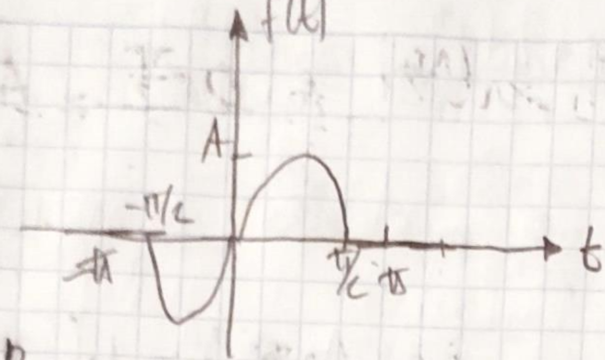
$$= \frac{-2A}{\pi}\left(-\frac{\cos(nt)}{n}\right) = \frac{2A}{\pi}\left(\frac{\cos(nt)}{n}\Big|_0^{\pi}\right) = \frac{2A}{\pi}\left(\frac{(-1)^n}{n} - \frac{1}{n}\right)$$

$$b_n = \frac{2A}{\pi n}\left((-1)^n - 1\right)$$

$$\therefore f(t) = \sum_{n=1}^{\infty} \frac{2A\left((-1)^n - 1\right)}{\pi n} \operatorname{sen}(n\pi)$$

~~$-\pi < t < \pi$~~

(4)



$$f(t) = \begin{cases} 0 & -\pi < t < -\frac{\pi}{2} \\ A\,\text{Sen}(t) & -\frac{\pi}{2} < t < \frac{\pi}{2} \\ 0 & \frac{\pi}{2} < t < \pi \end{cases}$$

$T = 2\pi$

$$w_0 = \frac{2\pi}{2\pi} = 1$$

Calculando b_n

$$b_n = \frac{4}{4\pi}\int_0^{\pi} f(t)\,\text{Sen}\left(\frac{nt}{2}\right)dt = \frac{2}{\pi}\int_0^{\pi/2} A\,\text{sen}\,t\,\text{Sen}(nt)\,dt$$

$$= \frac{2A}{\pi}\int_0^{\pi/2} \frac{1}{2}\left(\text{Cos}(t-nt) - \text{Cos}(t+nt)\right)dt$$

$$= \frac{\cancel{2}A}{\pi}\int_0^{\pi/2} \text{Cos}(t(1-n)) - \text{Cos}(t(1+n))\,dt$$

$$= \frac{A}{\pi}\left(\frac{\text{Sen}(t(1-n))}{1-n} - \frac{\text{Sen}(t(1+n))}{1+n}\right)\Bigg|_0^{\pi/2}$$

$$= \frac{A}{\pi}\left(\frac{\text{Sen}\left(\frac{\pi}{2} - \frac{n\pi}{2}\right)}{1-n} - \frac{\text{Sen}\left(\frac{\pi}{2} + \frac{n\pi}{2}\right)}{1+n}\right)$$

$$= \frac{A}{\pi}\left(\frac{\text{Sen}\left(\frac{\pi}{2}\right)\text{Cos}\left(\frac{n\pi}{2}\right) + \cancel{\text{Cos}\left(\frac{\pi}{2}\right)\text{Sen}\left(\frac{n\pi}{2}\right)}^{0}}{1-n} - \frac{\text{sen}\left(\frac{\pi}{2}\right)\text{Cos}\left(\frac{n\pi}{2}\right) + 0}{1+n}\right)$$

$$= \frac{A}{\pi}\left(\frac{\cancel{\text{Cos}\left(\frac{n\pi}{2}\right)} + n\,\text{Cos}\left(\frac{n\pi}{2}\right) - \cancel{\text{Cos}\left(\frac{n\pi}{2}\right)} + n\,\text{Cos}\left(\frac{n\pi}{2}\right)}{\cancel{1-n}\ \ 1-n^2}\right)$$

$$b_n = \frac{A}{\pi}\left(\frac{n\,\text{Cos}\left(\frac{n\pi}{2}\right)}{1-n^2}\right)$$ Cal

Calculando b_1

$$b_1 = \frac{A}{\pi} \lim_{n\to 1} \frac{\cos(\frac{n\pi}{2}) + \frac{\pi}{2} n \operatorname{sen}(\frac{n\pi}{2})}{1-2n} = \frac{A}{\pi} \frac{0 - \frac{\pi}{2}}{2} = \frac{A}{\pi}\frac{\frac{\pi}{2}}{2}$$

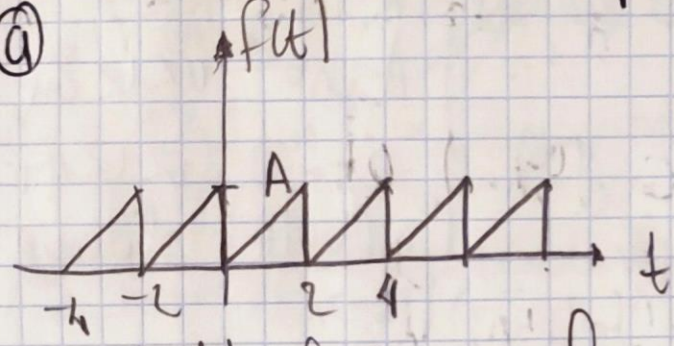
$$b_1 = \frac{A}{\pi}\cdot\frac{\pi}{4} = \frac{A}{4}$$

$$\therefore f(t) = \frac{A}{4}\operatorname{sen} t + \sum_{n=2}^{\infty} \frac{An}{\pi}\left(\frac{\cos(\frac{n\pi}{2})}{1-n^2}\right)\operatorname{sen}(nt)$$

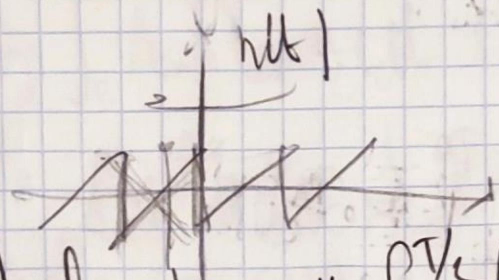
$-\pi < t < \pi$

Problema 3. Determinar la serie trigonométrica Fourier de cada una de las señales periódicas

(a)



Sea $h(t) = f(t) - 1 \Rightarrow$ función impar



$h(t) = t - 1$

Calculando $b_n = \frac{4}{T}\int_0^{T/2} f(t)\operatorname{sen} n\omega_0 t\,dt$

$T = 2$

$\omega_0 = \pi$

$$b_n = \frac{4}{2}\int_0^{T/2}(t-1)\operatorname{sen} n\pi t\,dt = 2\left[\int_0^1 t\operatorname{sen}(n\pi t)\,dt - \int_0^1 \operatorname{sen}(n\pi t)\,dt\right]$$

$t \searrow \operatorname{sen} n\pi t$
$1 \searrow -\cos n\pi t / n\pi$
$0 \searrow -\operatorname{sen}(n\pi t)/(n\pi)^2$

$$b_n = 2\left[-\frac{t\cos n\pi t}{n\pi} + \frac{\operatorname{sen}(n\pi t)}{n^2\pi^2} + \frac{\cos(n\pi t)}{n\pi}\right]_0^1$$

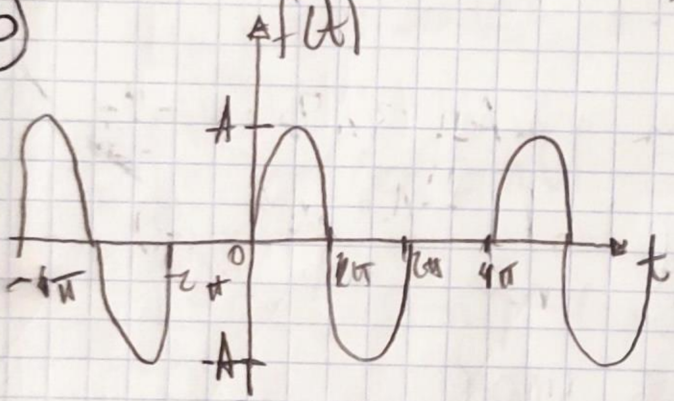
$$b_n = 2\left[\cancel{\frac{-\cos(n\pi)}{n\pi}} + \cancel{\frac{\sin(n\pi)}{n^2\pi^2}} + \cancel{\frac{\cos(n\pi)}{n\pi}} + \cancel{\frac{0\cos(0)}{n\pi}} \cancel{\frac{\sin(0)}{n^2\pi^2}} - \frac{\cos(0)}{n\pi} \right] = 2\left[-\frac{1}{n\pi} \right)$$

$$b_n = -\frac{2}{n\pi}$$

Como $h(t) = f(t) - 1 \Rightarrow f(t) = h(t) + 1$

$$f(t) = 1 - \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{\operatorname{sen}(n\pi t)}{n}$$

(b)



$$g(t) = \begin{cases} A\sin t & 0 < t < 2\pi \\ 0 & 2\pi < t < 4\pi \\ g(t+4\pi) & \text{otro caso} \end{cases}$$

$$T = 4\pi \rightarrow w_0 = \frac{2\pi}{4\pi} = \frac{1}{2}$$

$$a_n = \frac{2}{4\pi} \int_0^{4\pi} g(t) \cos\left(\frac{n}{2}t\right) dt = \frac{1}{2\pi}\left[\int_0^{2\pi} A\sin t \cos\left(\frac{n}{2}t\right) dt + 0 \right]$$

$$a_n = \frac{A}{2\pi} \int_0^{2\pi} \sin t \cos\left(\frac{n}{2}t\right) dt = \frac{A}{2\pi} \int_0^{2\pi} \sin\left(t\left(1-\frac{n}{2}\right)\right) + \sin\left(t\left(1+\frac{n}{2}\right)\right) dt$$

$$a_n = \frac{A}{4\pi}\left[\frac{-\cos\left(t\left(1-\frac{n}{2}\right)\right)}{1-\frac{n}{2}} - \frac{\cos\left(t\left(1+\frac{n}{2}\right)\right)}{1+\frac{n}{2}} \right]_0^{2\pi}$$

$$a_n = -\frac{A}{4\pi}\left[\frac{1}{\frac{2-n}{2}} \left(\cos\left(2\pi\left(1-\frac{n}{2}\right)\right) - \cos(0) \right) + \frac{1}{\frac{2+n}{2}} \left(\cos\left(2\pi\left(1+\frac{n}{2}\right)\right) - 1 \right) \right.$$

$$a_n = \frac{-2A}{4\pi}\left[\frac{1}{2-n} \left(\cos(2\pi)\cos(\pi n) + \cancel{\sin 2\pi} \sin n\pi - 1 \right) + \frac{1}{2+n} \left(\cos 2\pi \right.\right.$$

$\cos \pi n - \sin 2\pi \cancel{\operatorname{sen} n\pi} - 1]$

$$a_n = \frac{-A}{2\pi}\left[\frac{(2+n)(\cos n\pi - 1) + (2-n)(\cos n\pi - 1)}{4-n^2}\right]$$

$$a_n = \frac{-A}{2\pi}\left[\frac{2\cos n\pi - 2 + \cancel{n\cos n\pi} - \cancel{n} + 2\cos n\pi - 2 - \cancel{n\cos n\pi} + \cancel{n}}{4-n^2}\right]$$

$$a_n = \frac{-A}{2\pi}\left\{\frac{4\cos n\pi - 4}{4-n^2}\right\} = \frac{-A}{\pi}\left\{\frac{2(\cos n\pi - 1)}{4-n^2}\right\}$$

$$a_n = \frac{2A}{\pi(n^2-4)}\left((-1)^n - 1\right) \quad n \neq 2$$

$$a_2 = \frac{2A}{\pi}\lim_{n\to 2} \to \frac{2An\operatorname{sen} n\pi}{2\pi n} = -\frac{An}{\pi}\sin 2\pi = 0 = a_2$$

$$a_0 = \lim_{n\to 0}\frac{2A}{\pi(n^2-4)}(\cos(n\pi)-1) = \lim_{n\to 0}\frac{\cancel{A\pi\operatorname{sen} n\pi}}{2n} = -0$$

$$a_0 = 0$$

$$b_n = \frac{1}{2\pi}\int_0^{4\pi} A\sin t \sin \tfrac{n}{2}t\,dt \quad t_0$$

Usando $\sin A \sin B = \frac{1}{2}(\cos(A-B) - \cos(A+B))$

$$b_n = \frac{1}{4\pi}\int_0^{2\pi}\cos\left(\left(1-\tfrac{n}{2}\right)t\right) - \cos\left(\left(1+\tfrac{n}{2}\right)t\right)dt$$

$$b_n = \frac{A}{4\pi}\left(\frac{\sin(2\pi - \pi n)}{\frac{2-n}{2}} - \frac{\sin(2\pi + n\pi) + \sin 0 - \sin 0}{\frac{2+n}{2}}\right)$$

$$b_n = \frac{A}{2\pi}\left(\frac{\sin 2\pi\cos n\pi - \sin n\pi \cos 2\pi}{2+n} - \frac{\sin 2\pi\cos n\pi + \operatorname{sen} n\pi\cos}{2-n}\right)$$

$$b_n = \emptyset \quad \forall n \neq 2$$

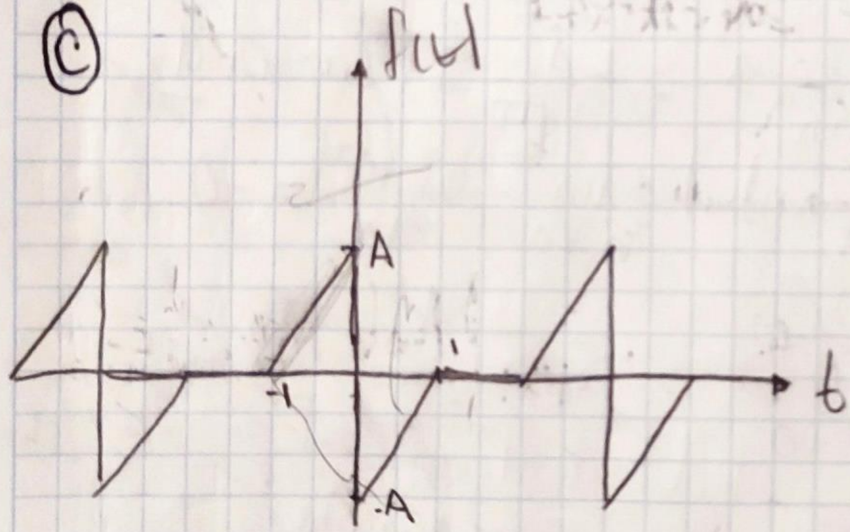
$$b_2=\frac{1}{2\pi}\int_0^{2\pi} A\sin t\sin t\,dt=\frac{A}{2\pi}\int_0^{2\pi}\sin^2 t\,dt=\frac{A}{2\pi}\int_0^{2\pi}\frac{1}{2}[1-\cos 2t]\,dt$$

$$=\frac{A}{4\pi}\int_0^{2\pi}1-\cos 2t\,dt=\frac{A}{4\pi}\left[t-\frac{\sin 2t}{2}\right]_0^{2\pi}=A$$

$$b_n=\frac{A}{2}$$

$$\therefore g(t)=\frac{A}{2}\sin t+\frac{4A}{3\pi}\cos\left(\frac{t}{2}\right)+\sum_{n=3}^{\infty}\frac{2A[(-1)^n-1]}{\pi(n^2-4)}\cos\left(\frac{n}{2}t\right)$$

ⓒ



$$g(t)=\begin{cases}A(x+1) & -1<t<0\\ A(x-1) & 0<t<1\\ 0 & 1<t<2\end{cases}$$

Impar $\Rightarrow a_n=a_0=0$

$T=3$

$\omega_0=\frac{2\pi}{3}$

$$b_n=\frac{4}{3}\int_0^{3/2}f(t)\sin n\frac{2\pi}{3}t\,dt=\frac{4}{3}\int_0^{1}(A(t-1)\sin n\frac{2\pi t}{3})\,dt+0$$

$$b_n=\frac{4A}{3}\int_0^{1}t\sin\frac{n2\pi}{3}t-\sin\frac{n2\pi}{3}t\,dt=\frac{4A}{3}\left[\frac{-3t\cos\left(\frac{2n\pi}{3}t\right)}{2n\pi}+\frac{9\sin\left(\frac{2n\pi t}{3}\right)}{4n^2\pi^2}+\frac{3\cos\left(\frac{n2\pi}{3}t\right)}{n2\pi}\right]\Bigg|_0^1$$

$$b_n = \frac{4}{3} A\left[\frac{-3\cos\left(n\frac{2\pi}{3}\right)}{2\pi n} + 0 + \frac{9\sin\left(\frac{n2\pi}{3}\right)}{4\pi^2 n^2} - 0 + \frac{3\cos\left(\frac{n2\pi}{3}\right)}{2\pi n} - \frac{3}{2\pi}\right]$$

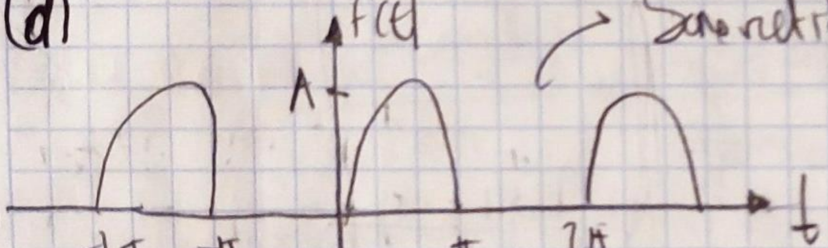
$$b_n = \frac{4}{3} A\left[\frac{9\sin\left(\frac{2n\pi}{3}\right)}{4\pi^2} - \frac{3}{2\pi}\right] = \frac{3A\sin\left(\frac{2n\pi}{3}\right)}{n^2\pi^2} - \frac{2A}{n\pi}$$

$$b_n = \frac{3A\sin\left(\frac{2n\pi}{3}\right)}{\pi^2 n^2} - \frac{2A}{n\pi}$$

Finalmente

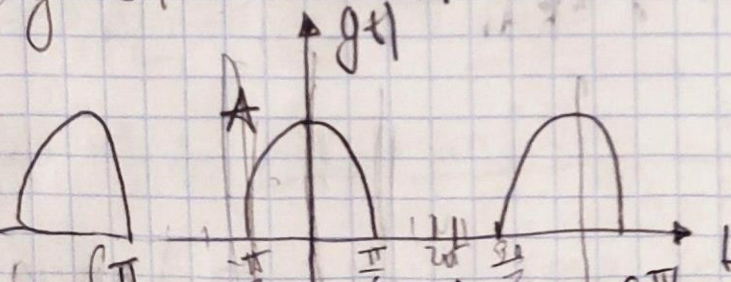
$$g(t) = \sum_{n=1}^{\infty}\left(\frac{3A\sin\left(\frac{2n\pi}{3}\right)}{\pi^2 n^2} - \frac{2A}{n\pi}\right)\sin\left(\frac{2n\pi}{3}t\right)$$

(d) Seno rectificado



Sea $g(t) = f\left(t - \frac{\pi}{2}\right)$

$$g(t) \begin{cases} A\cos t & -\frac{\pi}{2} < t < \frac{\pi}{2} \\ 0 & \frac{\pi}{2} < t < \frac{3\pi}{2} \end{cases}$$



$T = 2\pi$

$\omega_0 = \frac{2\pi}{2\pi} = 1$

$$a_n = \frac{4}{2\pi}\int_0^{\pi} g(t)\cos nt\, dt = \frac{2}{\pi}\left[\int_0^{\pi/2}\cos t\cos nt\, dt + 0\right]$$

$$a_n = \frac{2}{\pi}\int_0^{\pi/2}\frac{1}{2}\left(\cos(t(1-n)) + \cos(t(1+n))\right)dt$$

$$a_n = \frac{1}{\pi}\left[\int_0^{\pi/2}\cos(t(1-n))\,dt + \int_0^{\pi/2}\cos(t(1+n))\,dt\right] = \frac{1}{\pi}\left[\frac{\operatorname{sen}(t(1-n))}{1-n} + \frac{\operatorname{sen}(t(1+n))}{1+n}\right]$$

$$a_n = \frac{1}{\pi}\left[\frac{\operatorname{sen} t\cos tn - \cos t\operatorname{sen} tn}{1-n} + \frac{\operatorname{sen} t\cos tn + \cos t\operatorname{sen} tn}{1+n}\right]_0^{\pi/2}$$

$$a_n = \frac{1}{\pi}\left[\frac{\sin(\frac{\pi}{2})\cos(\frac{n\pi}{2})+0}{1-n} - 0 + \frac{\sin(\frac{\pi}{2})\cos(\frac{n\pi}{2})+0}{1+n} - 0\right]$$

$$a_n = \frac{1}{\pi}\left(\frac{\cos(\frac{n\pi}{2})}{1-n} + \frac{\cos(\frac{n\pi}{2})}{1+n}\right) = \frac{1}{\pi}\left(\frac{\cos(\frac{n\pi}{2}) + \cancel{n\cos(\frac{n\pi}{2})} + \cos(\frac{n\pi}{2}) \cancel{- n\cos(\frac{n\pi}{2})}}{1-n^2}\right)$$

$$a_n = \frac{1}{\pi}\left(\frac{2\cos(\frac{n\pi}{2})}{1-n^2}\right)$$

$$a_n = \frac{1}{\pi}\left(\frac{2\cos(\frac{n\pi}{2})}{1-n^2}\right) \quad n \neq 1$$

Calcular a_0

$$a_0 = \frac{1}{\pi}\left(\frac{2}{1-0}\right) = \frac{2}{\pi}$$

$$a_0 = \frac{2}{\pi}$$

Calculando a_1

L'Hopital

$$a_1 = \frac{1}{\pi}\lim_{n\to 1} \frac{2\cos(\frac{n\pi}{2})}{1-n^2} = \frac{1}{\pi}\lim_{n\to 1}\frac{\cancel{-}2(\frac{\pi}{2})\sin(\frac{n\pi}{2})}{\cancel{-}2n}$$

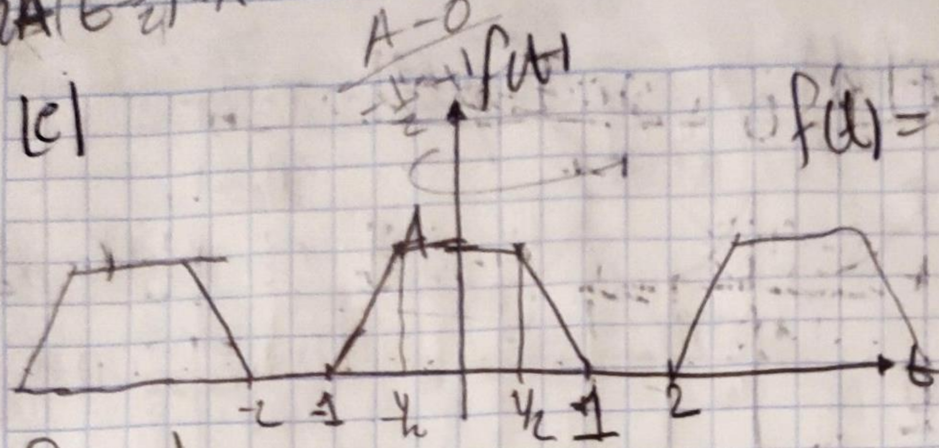
$$a_1 = \frac{1}{\pi}\left(\frac{\pi}{2}\right) = \frac{1}{2}$$

$$g(t) = \frac{2}{\pi} + \frac{1}{2}\cos\left(t - \frac{\pi}{2}\right) + \sum_{n=2}^{\infty}\frac{1}{\pi}\left(\frac{2\cos(\frac{n\pi}{2})}{1-n^2}\right)\cos\left(n\left(t-\frac{\pi}{2}\right)\right)$$

$t - A = m(x - \frac{1}{2})$ $\quad m = \frac{0-A}{1-\frac{1}{2}} = -\frac{A}{\frac{1}{2}} = -2A$ $\qquad 2A(t+1) \quad -1<t<-\frac{1}{2}$

$t = -2A(t-\frac{1}{2}) + A$

(c)



$$f(t) = \begin{cases} A & -\frac{1}{2}<t<\frac{1}{2} \\ -2A(t-\frac{1}{2})+A & \frac{1}{2}<t<1 \\ 0 & 1<t<2 \end{cases}$$

$T = 3$

$\omega_0 = \frac{2\pi}{3}$

Par $\quad b_n = 0$

$$a_n = \frac{4}{T}\int_0^{T/2} f(t)\cos n\omega_0 t\,dt = \frac{4}{3}\int_0^{3/2} f(t)\cos\left(\frac{2n\pi}{3}\right)t\,dt$$

$$a_n = \frac{4}{3}\int_0^1 -2A(t-2)\cos\left(\frac{2n\pi t}{3}\right)dt = -\frac{8}{3}A\int_0^1 t\cos\left(\frac{2n\pi t}{3}\right) - 2\cos\left(\frac{2n\pi t}{3}\right)$$

$$a_n = -\frac{8}{3}A\left[-\frac{6\sin\left(\frac{2n\pi t}{3}\right)}{2n\pi} + \frac{3t\sin\left(\frac{2n\pi t}{3}\right)}{2n\pi} + \frac{9\cos\left(\frac{2n\pi t}{3}\right)}{4\pi^2n^2}\right]_0^1$$

$$a_n = -\frac{8}{3}A\left[-\frac{6\sin\left(\frac{2n\pi}{3}\right)}{2n\pi} + \frac{3\sin\left(\frac{2n\pi}{3}\right)}{2n\pi} + \frac{9\cos\left(\frac{2n\pi}{3}\right)}{4\pi^2n^2} + 0 - 0 - \frac{9}{4\pi^2n^2}\right]$$

$$a_n = -\frac{8}{3}A\left[\frac{-3\sin\left(\frac{2n\pi}{3}\right)}{2n\pi} + \frac{9\cos\left(\frac{2n\pi}{3}\right)}{4\pi^2n^2} - \frac{9}{4\pi^2n^2}\right]$$

$$a_n = -\frac{8}{\cancel{3}}A\left(\frac{\cancel{3}(2n\pi\sin\left(\frac{2n\pi}{3}\right) - 3\cos\left(\frac{2\pi n}{3}\right) + 3)}{4\pi^2n^2}\right)$$

$$a_n = -2A\left(\frac{2n\pi\sin\left(\frac{2\pi n}{3}\right) - 3\cos\left(\frac{2\pi n}{3}\right) + 3}{\pi^2n^2}\right)$$

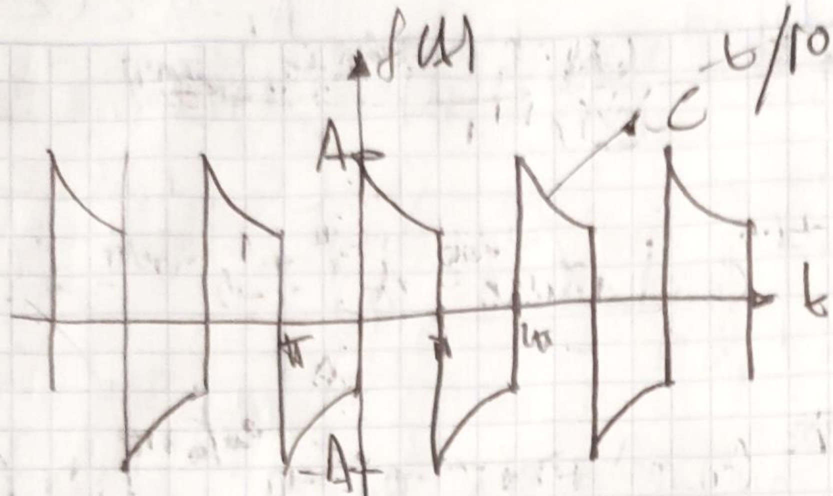
$$a_0 = \frac{2}{3}\int_0^1 -2A(t-2)\,dt = -\frac{4}{3}A\left[\frac{t^2}{2} - 2t\right]_0^1$$

$$a_0 = -\frac{4}{3}A\left(\frac{1}{2} - 2\right) = -\frac{2A}{3} + \frac{8A}{3} = \frac{6A}{3} = 2A$$

$$a_0 = 2A$$

$$\therefore f(t) = 2A + \sum_{n=1}^{\infty} -2A\left(\frac{2n\pi\sin\left(\frac{2\pi n}{3}\right) - 3\cos\left(\frac{2\pi n}{3}\right) + 3}{\pi^2n^2}\right)$$

(f)



$T = 2\pi$

$w_0 = 1$

$$f(t) = \begin{cases} e^{-t/10} & 0<t<\pi \\ -e^{-t/10+\pi} & \pi<t<2\pi \\ f(t+\pi) & \text{otro caso} \end{cases}$$

$$a_0 = \int_0^{\pi} e^{-t/10}\,dt - \int_{\pi}^{2\pi} e^{-t/10+\pi}\,dt = -10e^{-t/10}\Big|_0^{\pi} + 10e^{-\frac{t}{10}+\pi}\Big|_{\pi}^{2\pi}$$

$$= -10e^{-\pi/10} + 10 + 10e^{-\frac{2\pi}{10}+\pi} - 10e^{-\frac{\pi}{10}+\pi} = 10 - 10e^{-\pi/10} + 10e^{9\pi/10}$$
$$+10^{4/5\pi} \approx -42.0714$$

$$\int u\,dv = uv - \int v\,du$$

$$a_0 = -42.0714$$

$$a_n = \frac{1}{\pi}\int_0^{\pi} e^{-t/10}\cos nt\,dt - \int_{\pi}^{2\pi} e^{-t/10+\pi}\cos nt\,dt =$$

$u = \cos nt$ $\quad \int dv = \int e^{-t/10}$ $\quad u = \operatorname{sen} nt$

$du = -n\operatorname{sen} nt$ $\quad v = -10e^{-t/10}$ $\quad du = n\cos nt$

$$\int e^{-t/10}\cos nt\,dt = -10e^{-t/10}\cos nt - \int 10e^{-t/10} n\operatorname{sen} nt\,dt = -10e^{-t/10}\cos nt$$
$$-10n\left(-10e^{-t/10}\operatorname{sen} nt + \int 10e^{-t/10} n\cos nt\,dt\right)$$

$$\int e^{-t/10}\cos nt\,dt = -10e^{-t/10}\cos nt + 100n e^{-t/10}\operatorname{sen} nt - 100n^2\int e^{-t/10} n\cos nt\,dt$$

$$(100n^2+1)\int e^{-t/10}\cos nt\,dt = -10e^{-t/10}\cos nt + 100n e^{-t/10}\operatorname{sen} nt$$

$$\int e^{-t/10}\cos nt\,dt = \frac{-10e^{-t/10}\cos nt + 100n e^{-t/10}\operatorname{sen} nt}{100n^2+1}$$

$$\int e^{-t/10+\pi} = \frac{-10e^{-t/10+\pi}\cos nt + 100n e^{-t/10+\pi}\operatorname{sen} nt}{100n^2+1}$$

$$a_n = \frac{1}{\pi(100n^2+1)}\left\{\left[-10e^{-t/10}\cos nt + 100n e^{-t/10}\operatorname{sen} nt\right]_0^{\pi} - \left[-10e^{-t/10+\pi}\cos nt + 100n e^{-t/10+\pi}\operatorname{sen} nt\right]_{\pi}^{2\pi}\right\}$$

$$a_n = \frac{1}{\pi(100n^2+1)}\left\{-10e^{-\pi/10}(\cos n\pi) + 0 + 10 - 0 - \left[-10e^{-2\pi/10+\pi}(1) + 0\right.\right.$$
$$\left.\left. + 10e^{-\pi/10+\pi}\cos n\pi + 0\right]\right\}$$

$$a_n = \frac{1}{100n^2+1}\left\{-10e^{-\pi/10}(-1)^n + 10 + 10^{8\pi/10} + 10^{9/10\pi}(-1)^n\right\}$$

$$a_n = \frac{-10e^{-\frac{\pi}{10}}(-1)^n + 10 + 10^{\frac{8\pi}{10}} + 10^{\frac{9\pi}{10}}(-1)^n}{\pi(100n^2+1)}$$

$$b_n = \frac{1}{\pi}\left[\int_0^{\pi} e^{-t/10}\operatorname{sen} nt\,dt - \int_{\pi}^{2\pi} e^{-t/10+\pi}\operatorname{sen} t\,dt\right]$$

$$\int e^{-t/10}\operatorname{sen} nt\,dt = -10\operatorname{sen} nt\, e^{-t/10} + \int 10e^{-t/10} n\cos nt\,dt = -10\operatorname{sen} nt\, e^{-t/10}$$
$$+10n\left(-10e^{-t/10}\cos nt - \int 10e^{-t/10} n\operatorname{sen} nt\,dt\right.$$

$u = \operatorname{sen} nt$ $\quad \int dv = \int e^{-t/10}$

$du = n\cos nt$ $\quad v = -10e^{-t/10}$

$\int u\,dv = uv - \int v\,du$

$u = \cos nt$

$du = -n\operatorname{sen} nt$

$$= -10\operatorname{sen} nt\, e^{-t/10} + 10n\left(-10\cos nt\, e^{-t/10} - 10n\int e^{-t/10}\operatorname{sen} nt\,dt\right)$$
$$= -10\operatorname{sen} nt\, e^{-t/10} - 100n\cos nt\, e^{-t/10} - 100n^2\int e^{-t/10}\operatorname{sen} nt\,dt$$
$$(100n^2+1)\int e^{-t/10}\operatorname{sen} nt\,dt = -10e^{-t/10}\operatorname{sen} nt - 100n e^{-t/10}\cos nt$$

$$\int e^{-t/10}\operatorname{Sen} nt\,dt = \frac{-10e^{-t/10}\operatorname{Sen} nt - 100n e^{-t/10}\cos nt}{100n^2+1}$$

$$\int e^{-t/10+\pi}\operatorname{Sen} nt\,dt = \frac{-10e^{-t/10+\pi}\operatorname{sen} nt - 100n e^{-t/10+\pi}\cos nt}{100n^2+1}$$

$$b_n = \frac{-1}{\pi(100n^2+1)}\left\{\left[10e^{-t/10}\operatorname{sen} nt - 100n e^{-t/10}\cos nt\right]_0^{\pi} + \left[10e^{-t/10+\pi}\operatorname{Sen} nt\right.\right.$$

$$\left.\left. + 100n e^{-t/10+\pi}\cos nt\right]_{\pi}^{2\pi}\right\} = -\frac{1}{\pi(100n^2+1)}\left\{0 - 100n e^{-\pi/10}(-1)^n - 0 + 100n\right.$$

$$\left. + 0 + 100n e^{-2\pi/10+\pi} - 0 - 100n e^{-\pi/10+\pi}(-1)^n\right\}$$

$$b_n = \frac{-1}{\pi(100n^2+1)}\left\{100n + 100n e^{\frac{4}{5}\pi} - 100n e^{\frac{9}{10}\pi}(-1)^n - 100n e^{-\pi/10}(-1)^n\right.$$

$$b_n = -\frac{100n + 100n e^{\frac{4}{5}\pi} - 100n e^{\frac{9}{10}\pi}(-1)^n - 100n e^{-\frac{\pi}{10}}(-1)^n}{\pi(100n^2+1)}$$

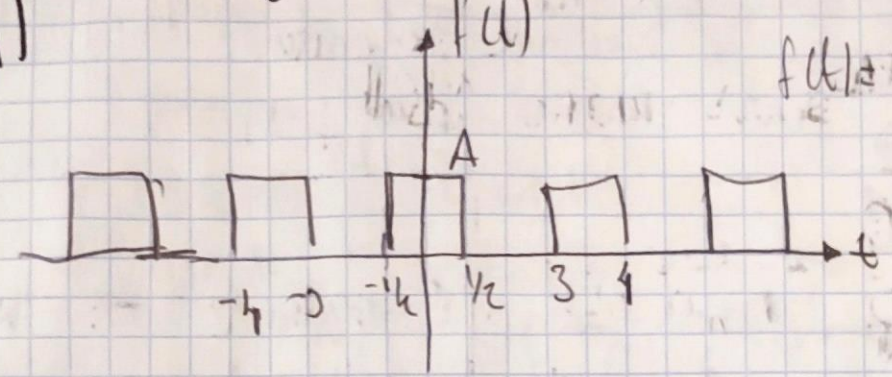
$$\therefore f(t) = -42.8714 + \sum_{n=1}^{\infty}\frac{1}{\pi(100n^2+1)}\left\{\left[-10e^{-\frac{\pi}{10}}(-1)^n + 10 + 10e^{\frac{8}{10}\pi} + 10e^{\frac{9}{10}\pi}(-1)^n\right]\right.$$

$$\left.\operatorname{Cos}(nt) - \left[100n + 100n e^{\frac{4}{5}\pi} - 100n e^{\frac{9}{10}\pi}(-1)^n - 100n e^{-\frac{\pi}{10}}(-1)^n\right]\operatorname{Sen}(nt)\right\}$$

Norma

Problema 4. Calcular la serie exponencial de Fourier de cada una de las señales periódicas que se ilustran en la figura 3 y graficar los espectros de magnitud y fase

(a)



$$f(t) = \begin{cases} A & -1/2 < t < 1/2 \\ 0 & 1/2 < t < 3 \end{cases}$$

$$T = \frac{7}{2}$$

$$W_0 = \frac{4\pi}{7}$$

$$f(t) = \sum_{n=-\infty}^{\infty} C_n e^{inW_0 t}$$

$$C_n = \frac{1}{T}\int_{t_0}^{t_0+T} f(t) e^{-inW_0 t}\, dt$$

$$C_n = \frac{1}{\frac{7}{2}} \int_{-1/2}^{3} f(t) e^{-in\frac{4\pi}{7}t} dt = \frac{2}{7}\left\{\int_{-1/2}^{1/2} A e^{-in\frac{4\pi}{7}t} dt + 0\right\}$$

$$C_n = \frac{2}{7} A \int_{-1/2}^{1/2} e^{-in\frac{4\pi}{7}t} dt = \frac{2}{7} \frac{A \cdot e^{-in\frac{4\pi}{7}t}}{\frac{-in4\pi}{7}}\Bigg|_{-1/2}^{1/2} = \frac{2}{7} \frac{7 \cdot e^{-in\frac{4\pi}{7}t}}{-in4\pi}\left(\frac{-1}{-1}\right)$$

$$C_n = \frac{-A e^{-in\frac{4\pi}{7}t}}{in2\pi}\Bigg|_{-1/2}^{1/2} = \frac{-A}{in2\pi}\left(e^{-in\frac{2\pi}{7}} - e^{in\frac{2\pi}{7}}\right)$$

$$C_n = \frac{A}{n\pi}\left(\frac{e^{in\frac{2\pi}{7}} - e^{-in\frac{2\pi}{7}}}{2i}\right) = \frac{A}{n\pi}\operatorname{Sen}\left(\frac{n2\pi}{7}\right)$$

$$C_n = \frac{A}{n\pi}\operatorname{Sen}\left(\frac{2n\pi}{7}\right) \quad \forall n \neq 0$$

$$C_0 = \frac{A}{\pi} \lim_{n\to 0} \frac{\operatorname{Sen}\left(\frac{2\pi n}{7}\right)}{n\pi} = \frac{A}{\pi} \cdot \frac{\frac{2\pi}{7}\operatorname{Cos}\left(\frac{2\pi n}{7}\right)}{1} = \frac{A}{\pi} \cdot \frac{2\pi}{7} = \frac{2}{7}A$$

L'Hospital

$$\therefore f(t) = \frac{2}{7}A + \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{A}{\pi n}\operatorname{Sen}\left(\frac{2n\pi}{7}\right) e^{in\frac{4\pi}{7}t}$$

Norma

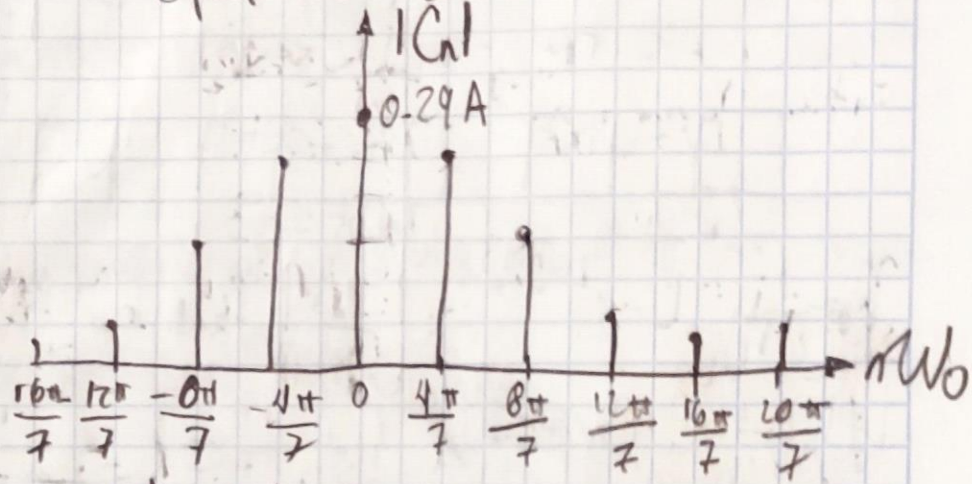
$$|C_n| = \sqrt{\left(\frac{A}{n\pi}\operatorname{sen}\left(\frac{2n\pi}{7}\right)\right)^2 + 0} = \frac{A}{n\pi}\operatorname{sen}\left(\frac{2n\pi}{7}\right)$$

$$\theta_n = \tan^{-1}\left(\frac{0}{\frac{A}{n\pi}\operatorname{sen}\left(\frac{2n\pi}{7}\right)}\right) = 0$$

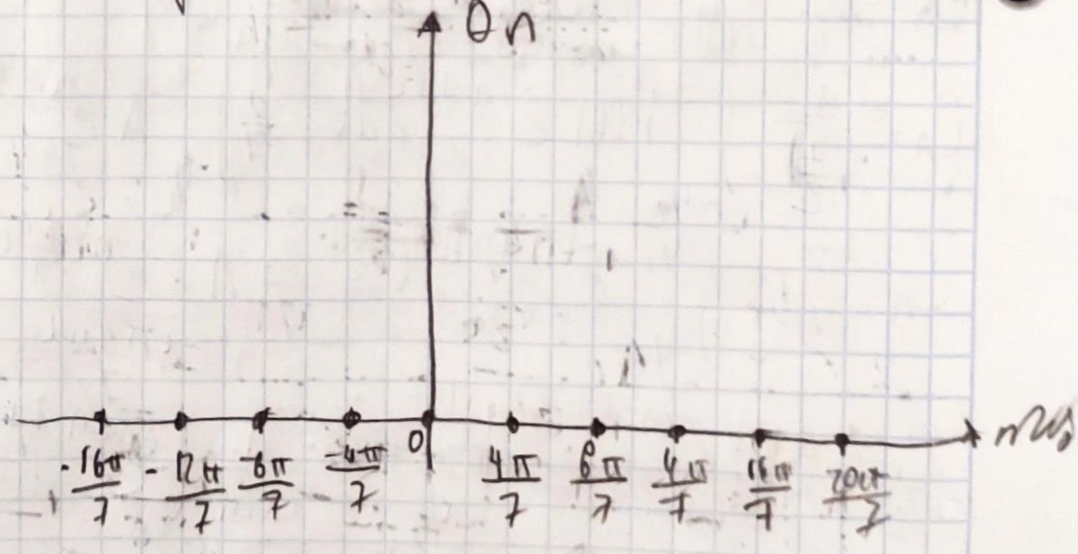
$$|C_n| = |C_{-n}|$$

n	$n\omega_0$	$\lvert C_n \rvert$	θ_n
⋮			
0	0	0.29A	0
1	$4\pi/7$	0.25A	0
2	$8\pi/7$	0.15A	0
3	$12\pi/7$	0.05A	0
4	$16\pi/7$	0.03A	0
5	$20\pi/7$	0.06A	0

Espectro de Frecuencia



Espectro de Fase

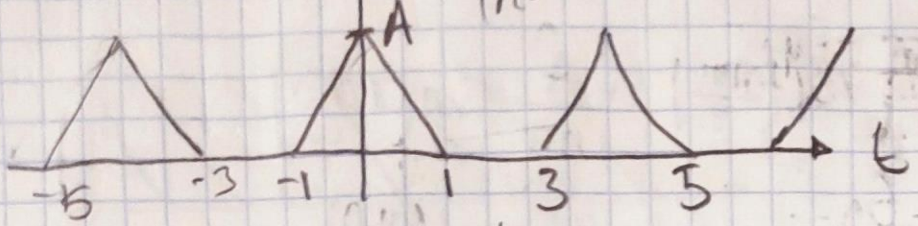


$y_t - y_1 = m(x - x_1)$ $\qquad m = \frac{A - A}{1 - 0}$

(b)

$m = \frac{A - 0}{0 + 1}$ $\qquad T = 4$

$m = A$ $\qquad W_0 = \frac{2\pi}{4} = \frac{\pi}{2}$



$$f(t) = \begin{cases} At + A & -1 < t < 0 \\ -At + A & 0 < t < 1 \\ 0 & 1 < t < 3 \end{cases}$$

$$C_n = \frac{1}{T}\int_{t_0}^{t_0+T} f(t) \cdot e^{-in w_0 t} = \frac{1}{4}\left\{\int^{3} f(t) \cdot e^{-in\frac{\pi}{2}t}\, dt\right\} = \frac{1}{4}\left\{\int_{-1}^{0} A(t+1) e^{-in\frac{\pi}{2}t}\, dt\right.$$

$$\left. + \int_0^1 A(-t+1) e^{-in\frac{\pi}{2}t}\, dt + 0\right\} = \frac{A}{4}\left\{\left[\frac{2te^{-in\frac{\pi}{2}t}}{-in\pi} - \frac{4e^{-in\frac{\pi}{2}t}}{(-in\pi)^2} + \frac{e^{-in\frac{\pi}{2}t}}{-in\pi}\right]_{-1}^{0} + \right.$$

$$\left.\left[\frac{-2te^{-in\frac{\pi}{2}t}}{-in\pi} + \frac{4e^{-in\frac{\pi}{2}t}}{(-in\pi)^2} + \frac{e^{-in\frac{\pi}{2}t}}{-in\pi}\right]_0^1\right\}$$

$$C_n = \frac{A}{4}\left\{0 + \frac{4}{n^2\pi^2} - \frac{1}{in\pi} - \left[\frac{2e^{in\frac{\pi}{2}}}{in\pi} + \frac{4e^{in\frac{\pi}{2}}}{n^2\pi^2} - \frac{e^{in\frac{\pi}{2}}}{in\pi}\right]\right.$$

$$\left.\frac{-2e^{-in\frac{\pi}{2}}}{in\pi} - \frac{4e^{-in\frac{\pi}{2}t}}{n^2\pi^2} - \frac{e^{-in\frac{\pi}{2}}}{in\pi} - \left[0 - \frac{4}{n^2\pi^2} - \frac{1}{in\pi}\right]\right\}$$

$$C_n = \frac{A}{4}\left\{\frac{4}{n^2\pi^2} - \cancel{\frac{1}{in\pi}} - \cancel{\frac{2e^{in\frac{\pi}{2}}}{in\pi}} - \frac{4e^{in\frac{\pi}{2}}}{n^2\pi^2} + \cancel{\frac{e^{in\frac{\pi}{2}}}{in\pi}} + \cancel{\frac{2e^{-in\frac{\pi}{2}}}{in\pi}} - \frac{4e^{-in\frac{\pi}{2}t}}{n^2\pi^2}\right.$$

$$\left. - \cancel{\frac{e^{-in\frac{\pi}{2}}}{in\pi}} + \frac{4}{n^2\pi^2} + \cancel{\frac{1}{in\pi}}\right\}$$

$$C_n = \frac{A}{4}\left\{\frac{8}{n^2\pi^2} + \frac{4e^{in\frac{\pi}{2}}}{n^2\pi^2} - \frac{4e^{-in\frac{\pi}{2}}}{n^2\pi^2} - \frac{e^{in\frac{\pi}{2}}}{in\pi} + \frac{e^{-in\frac{\pi}{2}}}{in\pi}\right\}$$

$$C_n = \frac{2A}{n^2\pi^2} - \frac{A2e^{in\frac{\pi}{2}}}{2n^2\pi^2} - \frac{A2e^{-in\frac{\pi}{2}}}{2n^2\pi^2} - \frac{Ae^{in\frac{\pi}{2}}}{4in\pi} + \frac{Ae^{-in\frac{\pi}{2}}}{4in\pi}$$

$$C_n = \frac{2A}{n^2\pi^2} + \frac{2A}{n^2\pi^2}\left(\frac{e^{in\frac{\pi}{2}} + e^{-in\frac{\pi}{2}}}{2}\right) + \frac{A}{n\pi}\left(\frac{e^{in\frac{\pi}{2}} - e^{-in\frac{\pi}{2}}}{2i}\right)$$

$$C_n = \frac{2A}{n^2\pi^2} - \frac{2A}{n^2\pi^2}\cos\left(\frac{n\pi}{2}\right) + \frac{A}{n\pi}\sin\left(\frac{n\pi}{2}\right)$$

$$a_0 = C_0$$

$$C_n = \frac{4A\sin^2\left(\frac{\pi n}{4}\right)}{\pi^2 n^2}$$

$$a_0 = \lim_{n\to 0} \frac{4A\sin^2\left(\frac{\pi n}{4}\right)}{\pi^2 n^2} = \frac{A}{4}$$

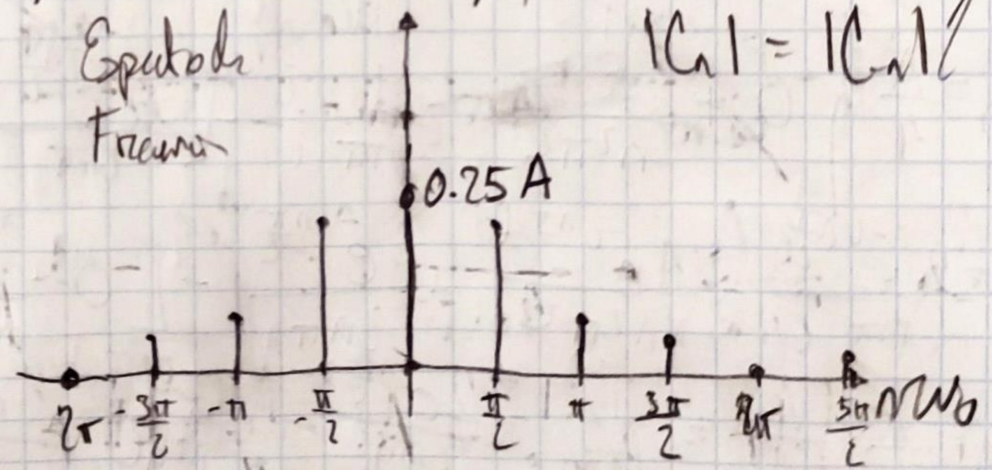
$$a_0 = \frac{A}{4}$$

$$f(t) = \frac{1}{4}A + \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{4A\sin^2\left(\frac{\pi n}{4}\right)}{\pi^2 n^2}$$

n	$n\omega_0$	$\lvert C_n \rvert$	θ_n
⋮			
0	0	0.25A	0
1	$\pi/2$	0.20A	0
2	π	0.10A	0
3	$3\pi/2$	0.02A	0
4	2π	0 A	0
5	$5\pi/2$	0.600A	0

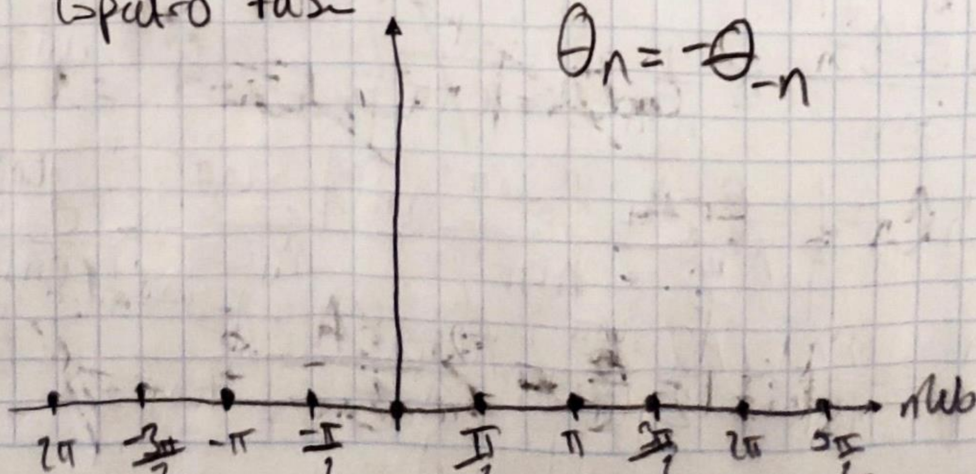
Espectro Frecuencia

$$|C_n| = |C_{-n}|$$



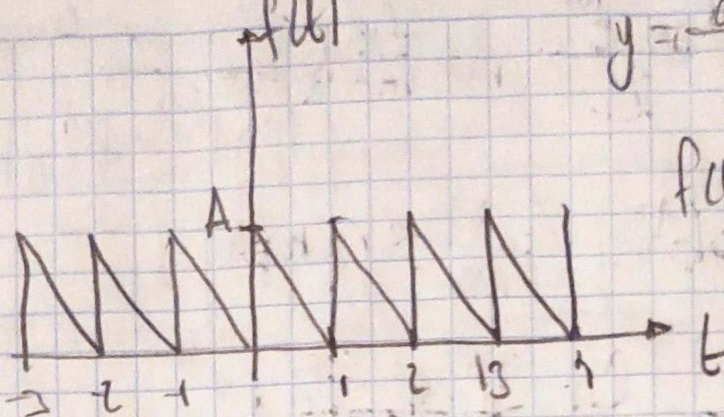
Espectro Fase

$$\theta_n = -\theta_{-n}$$



$y - y_2 = m(x - x_2)$

(c)



$y = \frac{0-A}{1-0} = -A$

$f(t) = \begin{cases} -At + A & 0<t<1 \\ f(t+1) & \text{otherwise} \end{cases}$

$T=1 \quad \omega_0 = 2\pi \quad f(t) = \sum_{n=-\infty}^{\infty} C_n e^{-in2\pi t}$

$C_n = \frac{1}{T}\int_{t_0}^{t_0+T} f(t)\, e^{-in\omega_0 t}\, dt$

$C_n = \int_0^1 (-At + A) e^{-in2\pi t}\, dt = A\left[\int_0^1 (-t+1) e^{-in2\pi t}\, dt\right]$

$C_n = A\left[\frac{-te^{-in2\pi t}}{-i2n\pi} + \frac{e^{-in2\pi t}}{(-i2n\pi)^2} + \frac{e^{-in2\pi t}}{-in2\pi}\right]_0^1$

$C_n = A\left[\cancel{\frac{e^{-i2n\pi}}{i2n\pi}} + \frac{e^{-i2n\pi}}{i^2 4n^2\pi^2} - \cancel{\frac{e^{-in2\pi}}{in2\pi}} - 0 - \frac{1}{i^2 4n^2\pi^2} + \frac{1}{i2n\pi}\right]$

$C_n = \frac{-iA}{2n\pi} - \frac{1}{4n^2\pi^2}\left(e^{-i2n\pi} - 1\right)$

$C_0 = \int_0^1 (-At + A)\, dt = A\left(\frac{-t^2}{2} + t\right)\Big|_0^1 = A\left(-\frac{1}{2} + 1\right) = \frac{1}{2}A$

$e^{-i2n\pi} = \cancel{\cos(2n\pi)}^{1} - i\cancel{\sin(2n\pi)}^{0} = 1$

$C_n = \frac{-iA}{2n\pi}$

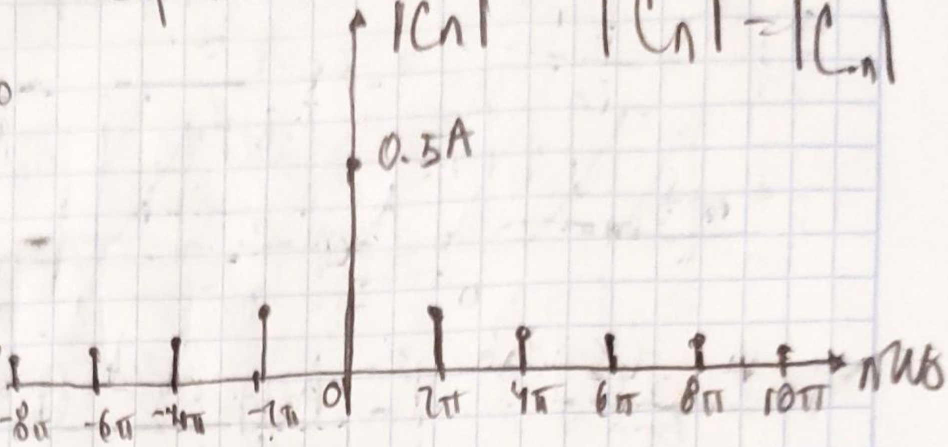
$f(t) = \frac{A}{2} - \sum \frac{iA}{2n\pi} e^{-in2\pi t}$ //

$|C_n| = \sqrt{0 + \frac{}{(2n\pi)}}$

n	nW_0	$\lvert C_n \rvert$	nW_0
⋮			
0	0	0.5A	0
1	2π	0.16A	$-90°$
2	4π	0.08A	$-90°$
3	6π	0.05A	$-90°$
4	8π	0.04A	$-90°$
5	10π	0.03A	$-90°$

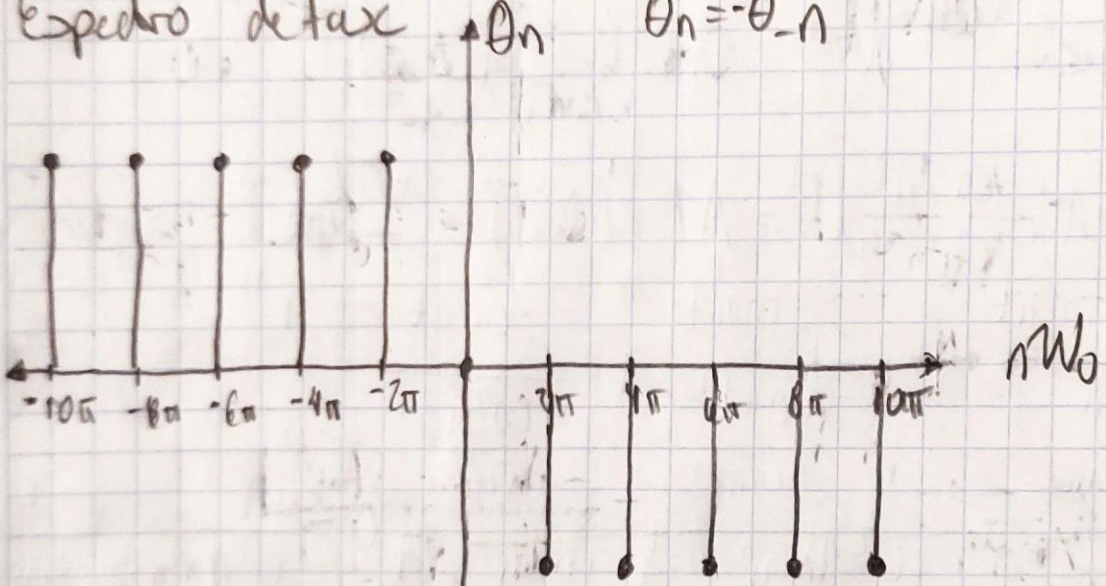
Espectro de frecuencia

$|C_n| = |C_{-n}|$

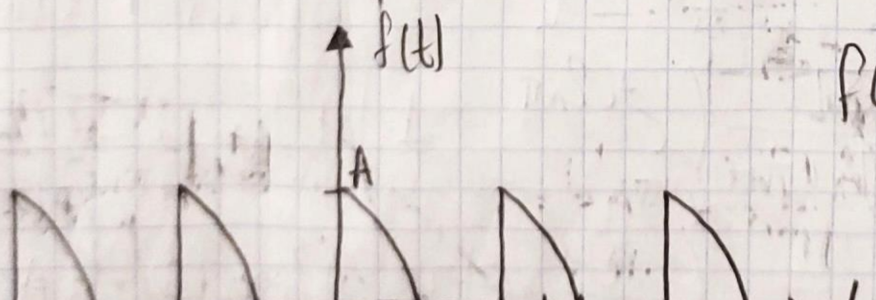


Espectro de fase

$\theta_n = -\theta_{-n}$



(d)



$$f(t) = \begin{cases} A\cos t & 0 < t < \pi/2 \\ 0 & \frac{\pi}{2} < t < \pi \end{cases}$$

$T = \pi$

$W_0 = \frac{2\pi}{\pi} = 2$

$$f(t) = \sum_{n=-\infty}^{\infty} C_n e^{inW_0 t} \qquad C_n = \frac{1}{T}\int_{t_0}^{t_0+T} f(t)\, e^{-inW_0 t}\, dt$$

$u = \cos t$

$du = -\operatorname{sen} t$

$\int dv = \int e^{-i2nt}$

$v = -\frac{e^{-i2nt}}{i2n}$

$$C_n = \frac{A}{\pi}\int_0^{\pi} f(t) e^{-i2nt}\, dt = \frac{A}{\pi}\left[\int_0^{\pi/2} \cos t\, e^{-i2nt}\, dt + 0\right]$$

$$C_n = \frac{A}{\pi}\int_0^{\pi/2} \cos t\, e^{-i2nt}\, dt = \frac{A}{\pi}\left\{ -\frac{\cos t\, e^{-i2nt}}{i2n} - \int_0^{\pi/2} \frac{e^{-i2nt}}{i2n} \operatorname{sen} t\, dt \right\}$$

$u = \sin t$

$du = \cos t$

$$= \frac{A}{\pi}\left\{ \frac{-\cos t\, e^{-i2nt}}{i2n} - \frac{1}{i2n}\left[\frac{-\sin t\, e^{-i2nt}}{i2n} + \int_0^{\pi/2} \frac{e^{-i2nt}}{i2n}\cos t\, dt \right]\right\}$$

$$= \frac{A}{\pi}\left\{ \frac{-\cos t\, e^{-i2nt}}{i2n} + \frac{\sin t\, e^{-i2nt}}{(i2n)^2} + \frac{1}{(i2n)^2}\int_0^{\pi/2} \cos t\, dt \right\}$$

$$= \left.\frac{-A\cos t\, e^{-i2nt}}{i\pi 2n}\right|_0^{\pi/2} + \left.\frac{A\sin t\, e^{-i2nt}}{\pi(i2n)^2}\right|_0^{\pi/2} + \frac{A}{\pi(i2n)^2}\int_0^{\pi/2} \cos t\, e^{-i2nt}\, dt$$

$$= -\left.\frac{A\cos t\, e^{-i2nt}}{i2\pi n}\right|_0^{\pi/2} - \left.\frac{A\sin t\, e^{-i2nt}}{\pi 4n^2}\right|_0^{\pi/2} + \frac{A}{\pi 4n^2}\int_0^{\pi/2} \cos t\, e^{-i2nt}\, dt$$

$$\left(1 - \frac{4}{4n^4}\right)\frac{A}{\pi}\int_0^{\pi/2} \cos t\, e^{-i2nt}\, dt = 0 + \frac{A}{i2\pi n} - \frac{A e^{-in\pi}}{\pi 4n^2} + 0$$

$$C_n = \frac{\left(\frac{-iA}{2\pi n} - \frac{A e^{-in\pi}}{4\pi n^2}\right)}{\left(1 - \frac{4}{n^2 4}\right)}$$

$$e^{-in\pi} = \cos(n\pi) - i\cancel{\sin(n\pi)}$$

$$C_n = \frac{\frac{A(-2ni - (-1)^n)}{4\pi n^2}}{\left(\frac{4n^2 - 1}{4n^2}\right)} = \frac{-A(2ni + (-1)^n)}{\pi(4n^2 - 1)}$$

$$C_n = \frac{A(2ni + (-1)^n)}{\pi(1 - 4n^2)}$$

$$|C_n| = \sqrt{\frac{A^2(4n^2 + (-1)^{2n})}{(\pi(1 - 4n^2))^2}}$$

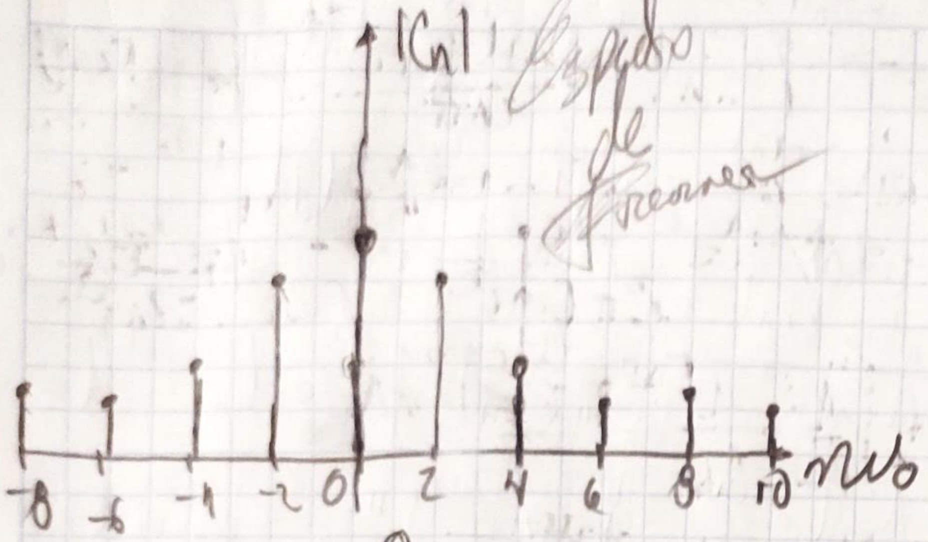
$$|C_n| = \sqrt{\frac{A^2(4n^2 + 1)}{(\pi(1 - 4n^2))^2}}$$

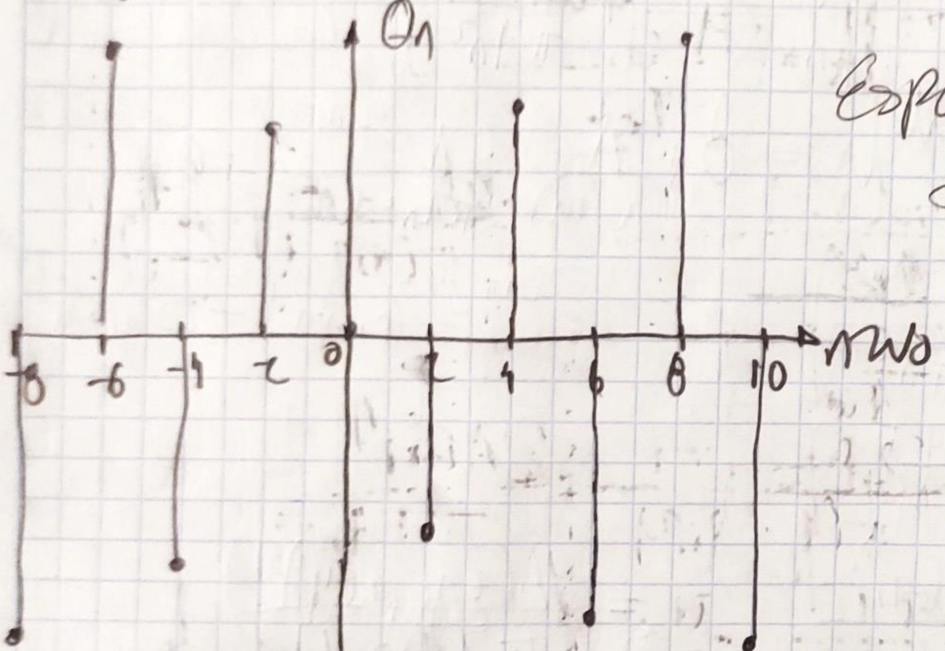
$$f(t) = \sum_{n=-\infty}^{\infty} \frac{A(2ni + (-1)^n)}{\pi(1 - 4n^2)} e^{in2t}$$

$$\tan^{-1}\left(\frac{\frac{A(2n)}{\pi(4n^2-1)}}{\frac{A(-1)^n}{\pi(4n^2-1)}}\right)$$

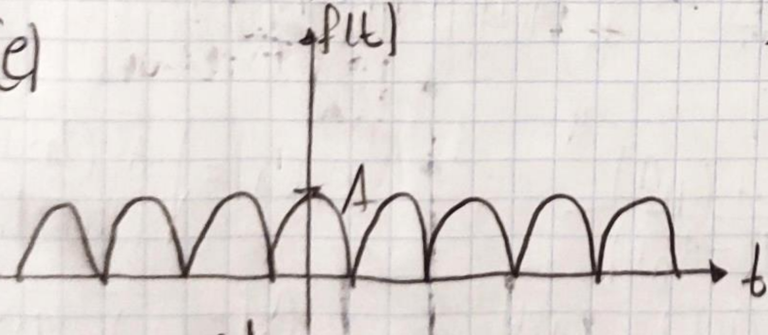
$$\tan\left(\frac{2n}{(-1)^n}\right)$$

n	$n\omega_0$	$\lvert C_n\rvert$	θ_n
0	0	A/π	0
1	2	$A\sqrt{5}/3\pi$	$-63.43°$
2	4	$A\sqrt{17}/15\pi$	$75.96°$
3	6	$A\sqrt{37}/35\pi$	$-80.54°$
4	8	$A\sqrt{65}/63\pi$	$82.87°$
5	10	$A\sqrt{101}/99\pi$	$-84.29°$





(e)



$T = 2$

$W = \frac{2\pi}{2} = \pi$

$f(t) = A\left|\cos\left(\frac{t\pi}{2}\right)\right|$

$u = \cos\left(\frac{t\pi}{2}\right) \qquad \int dv = \int e^{-in\pi t}$

$du = -\frac{\pi}{2}\sin\left(\frac{t\pi}{2}\right) \qquad v = -\frac{e^{-in\pi t}}{in\pi}$

$$C_n = \frac{A}{2}\int_{-1}^{1} \cos\left(\frac{t\pi}{2}\right) e^{-in\pi t}\, dt$$

$$C_n = \frac{A}{2}\left[-\frac{\cos\left(\frac{t\pi}{2}\right) e^{-in\pi t}}{in\pi} - \int_{-1}^{1} \sin\left(\frac{t\pi}{2}\right) \cdot \frac{\pi}{2} \cdot \frac{e^{-in\pi t}}{in\pi}\, dt\right.$$

$$C_n = \frac{A}{2}\left[-\frac{\cos\left(\frac{t\pi}{2}\right) e^{-in\pi t}}{in\pi}\Bigg|_{-1}^{1} - \frac{1}{2in}\int_{-1}^{1} \sin\left(\frac{t\pi}{2}\right) e^{-in\pi t}\, dt\right]$$

$u = \sin\left(\frac{t\pi}{2}\right)$

$du = \frac{\pi}{2}\cos\left(\frac{t\pi}{2}\right)$

$$C_n = \frac{A}{2}\left[\frac{-\cos\left(\frac{t\pi}{2}\right)e^{-in\pi t}}{in\pi}\Bigg|_{-1}^{1} + \frac{1}{i2n}\left(\frac{-\sin\left(\frac{t\pi}{2}\right)e^{-in\pi t}}{in\pi}\Bigg|_{-1}^{1} + \int_{-1}^{1}\frac{\frac{\pi}{2}\cos\left(\frac{t\pi}{2}\right)e^{-in\pi t}}{in\pi}\,dt\right)\right.$$

$$C_n = \frac{A}{2}\left[\cancel{\frac{-\cos\left(\frac{\pi}{2}\right)e^{in\pi}}{i2n\pi}} + \frac{2\sin\left(\frac{t\pi}{2}\right)e^{-in\pi t}}{2\pi(in)^2} - \int_{-1}^{1}\frac{\cos\left(\frac{t\pi}{2}\right)e^{-in\pi t}}{(i2n)^2}\,dt\right.$$

$$C_n = -\frac{Ae^{-in\pi}}{2n^2\pi} + \int_{-1}^{1}\frac{A\cos\left(\frac{t\pi}{2}\right)e^{-in\pi t}}{2\cdot 4n^2}\,dt$$

$$\left(1-\frac{1}{4n^2}\right)\frac{A}{2}\int_{-1}^{1}\cos\left(\frac{t\pi}{2}\right)e^{-in\pi t}\,dt = \frac{-Ae^{-in\pi}}{2n^2}$$

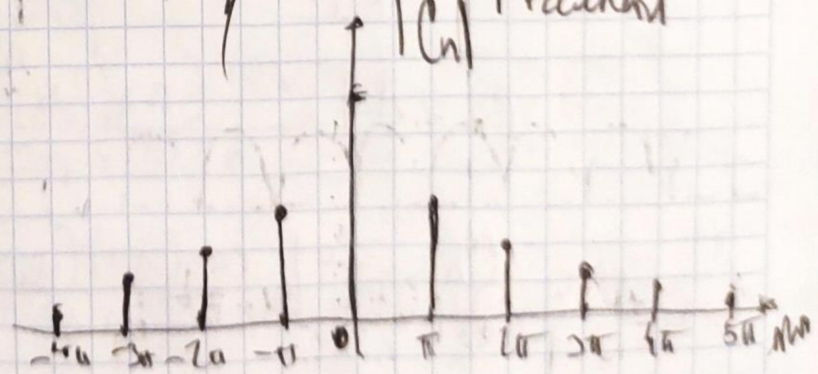
$$C_n = \frac{\frac{A\cos(n\pi)}{\pi\cancel{2n^2}}}{\left(\frac{4n^2-1}{4\cancel{n^2}}\right)} = \frac{-2A\cos(n\pi)}{\pi(4n^2-1)}\frac{(-1)}{(-1)}$$

$$C_n = \frac{2A\cos(n\pi)}{\pi(1-4n^2)} = \frac{2A(-1)^n}{\pi(1-4n^2)}$$

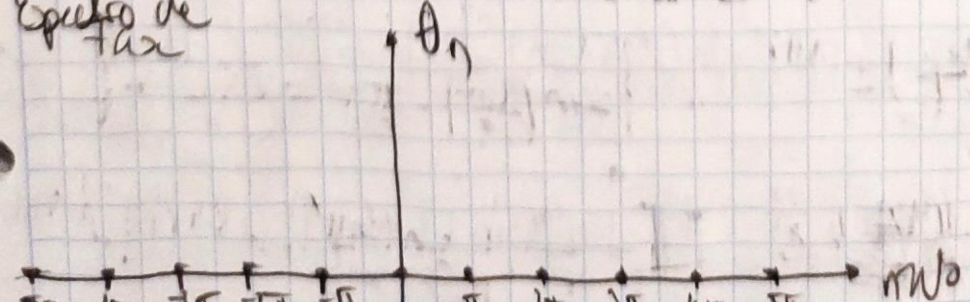
$$f(t) = \sum_{n=-\infty}^{\infty}\frac{2A(-1)^n}{\pi(1-4n^2)}e^{in\pi t}$$

Espectro de Frecuencia

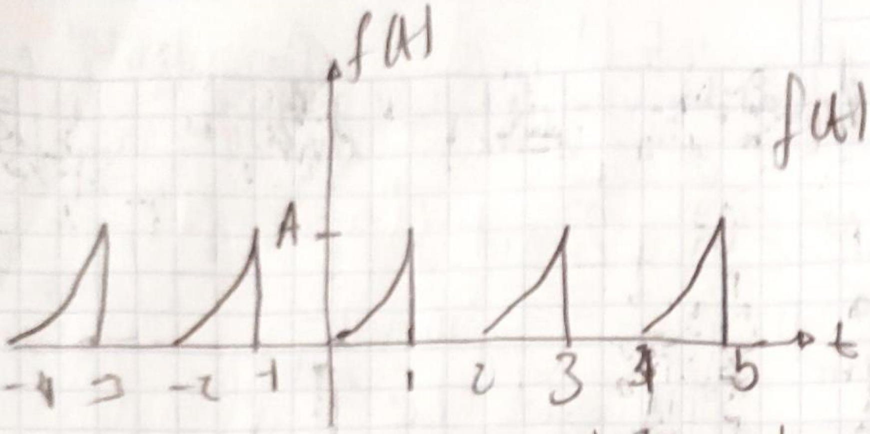
n	$n\omega_0$	$\lvert C_n\rvert$	θ_n
0	0	$2A/\pi$	0
1	π	$2A/3\pi$	0
2	2π	$2A/15\pi$	0
3	3π	$2A/35\pi$	0
4	4π	$2A/63\pi$	0
5	5π	$2A/99\pi$	0



Espectro de fase



(f)



$$f(t) = \begin{cases} At^2 & 0<t<1 \\ 0 & 1<t<2 \end{cases}$$

$T=2$

$\omega_0 = \pi$

$$C_n = \frac{A}{2}\int_0^1 t^2 e^{-in\pi t}\,dt = \frac{A}{2}\left\{ \frac{-t^2 e^{-in\pi t}}{in\pi} - \frac{2t e^{-in\pi t}}{(-in\pi)^2} + \frac{2e^{-in\pi t}}{(-in\pi)^3} \right\}_0^1$$

t^2	$e^{-in\pi t}$
$2t$	$-\frac{e^{-in\pi t}}{in\pi}$
2	$\frac{e^{-in\pi t}}{(in\pi)^2}$
0	$-\frac{e^{-in\pi t}}{(in\pi)^3}$

$$C_n = \frac{A}{2}\left\{ \frac{-e^{-in\pi}}{in\pi} + \frac{2e^{-in\pi}}{n^2\pi^2} + \frac{2e^{-in\pi t}}{n^3\pi^3 i} - 0 - 0 - \frac{2}{in^3\pi^3} \right\}$$

$$C_n = \frac{A}{2}\left[\frac{e^{-i\pi n}(\pi n(2+i\pi n)) + 2ie^{i\pi n} - 2i}{\pi^3 n^3} \right]$$

$$C_n = \frac{A}{2}\left[\frac{e^{-i\pi n}(\pi n(2+i\pi n)+2i) + 2i}{\pi^3 n^3} \right]$$

$$C_n = \frac{A}{2}\left[\frac{\cos(n\pi)(\pi n(2+i\pi n)+2i) + 2i}{\pi^3 n^3} \right]$$

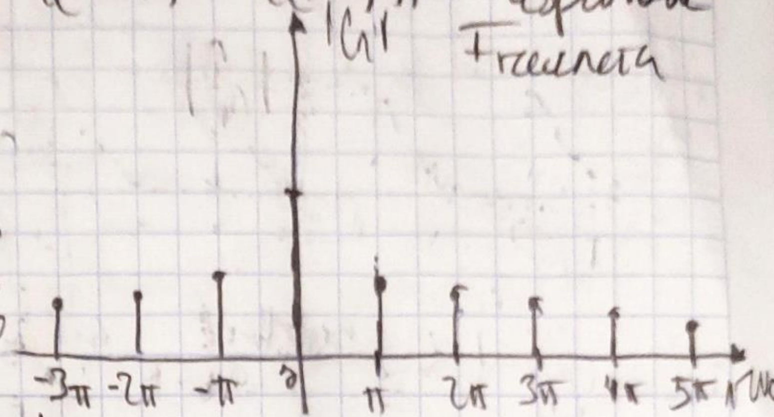
$$C_n = \frac{A}{2}\left[\frac{(-1)^n(\pi n(2+i\pi n)-2i) + 2i}{\pi^3 n^3} \right]$$

$$\therefore f(t) = \frac{1}{6}A + \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{A}{2}\left[\frac{(-1)^n(\pi n(2+i\pi n)-2i) + 2i}{\pi^3 n^3} \right] e^{in\pi t}$$

$$|C_n| = \sqrt{\left(\frac{2(-1)^n}{2(\pi^2 n^2)}\right)^2 + \left(\frac{-2(-1)^n}{2(\pi^3 n^3)} + \frac{2}{2(\pi^3 n^3)} + \frac{(-1)^n}{2(\pi n)}\right)^2}$$

n	$n\omega_0$	$\lvert C_n\rvert$	θ_n
0	0	$\frac{1}{6}A$	0
1	π	0.07 A	21.52°
2	2π	0.06 A	36.17°
3	3π	0.05 A	38.74°
4	4π	0.04 A	40.49°
5	5π	0.03 A	41.31°

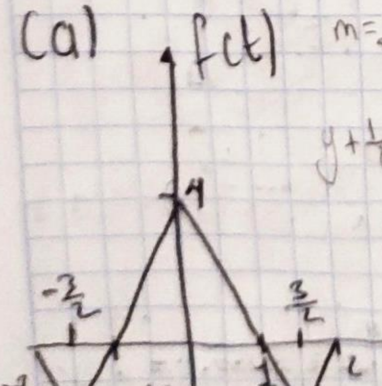
Espectro de Frecuencia



Espectro de Fase

Problema 5.- Obtener la transformada de Fourier de cada una de las señales de la figura 4.

(a)



$m = \frac{4+\frac{1}{2}}{0+\frac{3}{2}} = \frac{\frac{9}{2}}{\frac{3}{2}} = 3$

$y + \frac{1}{2} = m(x - x_1)$

$y = 3x + \frac{9}{2} - \frac{1}{2} = 3x + 4$

$$f(t) = \begin{cases} -(t+2) & -2 < t < -\frac{3}{2} \\ 3t+4 & -\frac{3}{2} < t < 0 \\ -(3t-4) & 0 < t < \frac{3}{2} \\ (t-2) & \frac{3}{2} < t < 2 \end{cases}$$

$$\mathcal{F}\{f(t)\} = F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-i\omega t}\,dt$$

$$\mathcal{F}\{f(t)\} = -\int_{-2}^{-3/2}(t+2)e^{-i\omega t}\,dt + \int_{-3/2}^{0}(3t+4)e^{-i\omega t}\,dt - \int_{0}^{3/2}(3t-4)e^{-i\omega t}\,dt + \int_{3/2}^{2}(t-2)e^{-i\omega t}\,dt$$

$(t+2)e^{-i\omega t}$

$1 \searrow \frac{-e^{-i\omega t}}{i\omega}$

$0 \searrow \frac{e^{-i\omega t}}{(i\omega)^2}$

$$F\{f(t)\} = -\int_{-2}^{-\frac{3}{2}} (t+2)e^{-i\omega t}\,dt + \int_{-\frac{3}{2}}^{0} (3t+4)e^{-i\omega t}\,dt - \int_{0}^{3/2} (3t-4)e^{-i\omega t}\,dt$$

$$+ \int_{\frac{3}{2}}^{2} (t-2)e^{-i\omega t}\,dt = -\left\{ \frac{-(t+2)e^{-i\omega t}}{i\omega} - \frac{t e^{-i\omega t}}{i^2\omega^2} \right\}\Bigg|_{-2}^{-\frac{3}{2}} + \left\{ \frac{-(3t+4)e^{-i\omega t}}{i\omega} - \frac{3e^{-i\omega t}}{i^2\omega^2} \right\}\Bigg|_{-\frac{3}{2}}^{0}$$

$$- \left\{ \frac{-(3t-4)e^{-i\omega t}}{i\omega} - \frac{3e^{-i\omega t}}{i^2\omega^2} \right\}\Bigg|_{0}^{3/2} + \left\{ \frac{-(t-2)e^{-i\omega t}}{i\omega} - \frac{e^{-i\omega t}}{i^2\omega^2} \right\}\Bigg|_{3/2}^{2}$$

$$F\{f(t)\} = -\left\{ \frac{-(-\frac{3}{2}+2)e^{+i\omega\frac{3}{2}}}{i\omega} + \frac{e^{i\omega\frac{3}{2}}}{\omega^2} + 0 - \frac{e^{i\omega 2}}{\omega^2} \right\} + \left\{ -\frac{4}{i\omega} + \frac{3}{\omega^2} \right.$$

$$\left. - \frac{0.5e^{i\omega\frac{3}{2}}}{i\omega} - \frac{3e^{i\omega\frac{3}{2}}}{\omega^2} \right\} - \left\{ \frac{-0.5e^{-i\omega\frac{3}{2}}}{i\omega} + \frac{3e^{-i\omega\frac{3}{2}}}{\omega^2} - \frac{4}{i\omega} \right.$$

$$\left. - \frac{3}{\omega^2} \right\} + \left\{ \frac{e^{-i\omega 2}}{\omega^2} - \frac{e^{-i\omega\frac{3}{2}}}{\omega^2} + \frac{0.5e^{-i\omega\frac{3}{2}}}{i\omega} \right\}$$

$$F\{f(t)\} = \frac{6}{\omega^2} + \frac{e^{+i\omega 2}}{\omega^2} + \frac{e^{-i\omega 2}}{\omega^2} - \frac{4e^{+i\omega\frac{3}{2}}}{\omega^2} - \frac{4e^{-i\omega\frac{3}{2}}}{\omega^2}$$

$$F\{f(t)\} = \frac{6}{\omega^2} + \frac{2}{\omega^2}\left\{ \frac{e^{i\omega 2} + e^{-i\omega 2}}{2} \right\} - \frac{8}{\omega^2}\left\{ \frac{e^{i\omega\frac{3}{2}} + e^{-i\omega\frac{3}{2}}}{2} \right\}$$

$$F\{f(t)\} = \frac{6}{\omega^2} + \frac{2}{\omega^2}\cos(2\omega) - \frac{8}{\omega^2}\cos(\tfrac{3}{2}\omega)$$

$$F\{f(t)\} = f(\omega) = \frac{2(3 + \cos(2\omega) - 4\cos(\frac{3}{2}\omega))}{\omega^2}$$

$$|F(\omega)| = \frac{2(3 + \cos(2\omega) - 4\cos(\frac{3}{2}\omega))}{\omega^2}$$

$$\theta(\omega) = \arctan\left(\frac{0}{f(\omega)}\right) \Rightarrow \arctan(0) = 0$$

$$\theta(\omega) = 0$$