

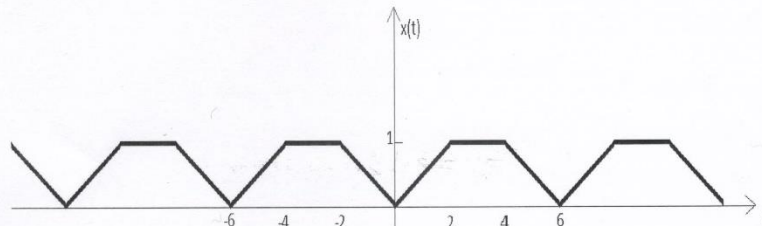
INSTITUTO POLITECNICO NACIONAL  
ESCUELA SUPERIOR DE COMPUTO  
Teoría de Comunicaciones y Señales

1er. Exámen departamental

NOMBRE: \_\_\_\_\_ TIPO: A

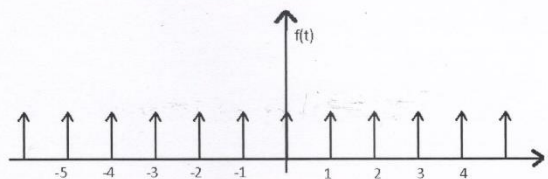
GRUPO: \_\_\_\_\_

Problema 1. Encuentre la Serie Trigonométrica de Fourier de  $x(t)$



Problema 2. A partir de la serie obtenida en el problema anterior, encuentra la Serie Exponencial de Fourier.

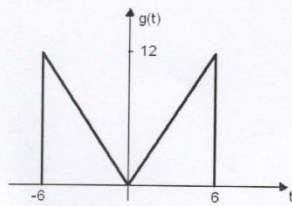
Problema 3. Encuentre la transformada de Fourier de la siguiente función periódica:



**Problema 4.** Usando propiedades, encuentre la transformada de Fourier de:

$$-6\delta[5t-10] \cdot \cos 15t + \frac{1}{3-jt} \cdot t^2 + e^{j4t}(t-1) \leftrightarrow ?$$

**Problema 5.** Usando Propiedades de la transformada de Fourier encuentre la transformada de  $g(t)$



Problema 1

$$x(t) = \begin{cases} \frac{1}{2}t & 0 < t \leq 2 \\ 1 & 2 < t \leq 4 \\ -\frac{1}{2}(t-6) & 4 < t \leq 6 \\ x(t+6) & \text{otro caso} \end{cases}$$

$$T = 6 \therefore \omega_0 = \frac{2\pi}{6} = \frac{\pi}{3}$$

como  $x(t)$  es par

$$\therefore b_n = 0$$

$$a_n = \frac{4}{T} \int_0^{\frac{T}{2}} x(t) \cos n\omega_0 t dt \quad ; \quad a_0 = \frac{2}{T} \int_0^{\frac{T}{2}} x(t) dt$$

$$a_n = \frac{4}{6} \int_0^3 x(t) \cos \frac{n\pi}{3} t dt$$

$$a_0 = \frac{2}{6} \int_0^3 x(t) dt$$

$$a_n = \frac{8}{3} \int_0^2 \frac{1}{2}t \cos \frac{n\pi}{3} t dt + \frac{2}{3} \int_2^3 \cos \frac{n\pi}{3} t dt$$

$$a_0 = \frac{1}{3} \int_0^2 \frac{1}{2}t dt + \frac{1}{3} \int_2^3 dt$$

$$u = t \quad dv = \cos \frac{n\pi}{3} t$$

$$du = dt \quad v = \frac{3}{n\pi} \sin \frac{n\pi}{3} t$$

$$a_n = \frac{1}{3} \left\{ \frac{3t}{n\pi} \sin \frac{n\pi}{3} t \Big|_0^2 - \frac{3}{n\pi} \int_0^2 \sin \frac{n\pi}{3} t dt \right\} + \left( \frac{2}{3} \right) \left( \frac{3}{n\pi} \right) \sin \frac{n\pi}{3} t \Big|_2^3$$

$$a_0 = \frac{1}{6} \left[ \frac{t^2}{2} \right]_0^2 + \frac{1}{3} t \Big|_2^3$$

$$a_0 = \frac{1}{12} [4 - 0] + \frac{1}{3} [3 - 2]$$

$$a_0 = \frac{1}{3} + \frac{1}{3}$$

$$a_0 = \underline{\underline{\frac{2}{3}}}$$

$$a_n = \frac{1}{3} \left\{ \frac{6}{n\pi} \sin \frac{2n\pi}{3} + \frac{9}{n^2\pi^2} \cos \frac{n\pi}{3} \right\} + \frac{2}{n\pi} \left( \sin \frac{3n\pi}{3} - \sin \frac{2n\pi}{3} \right)$$

$$a_n = \frac{2}{n\pi} \sin \frac{2n\pi}{3} + \frac{3}{n^2\pi^2} [\cos \frac{2n\pi}{3} - 1] - \frac{2}{n\pi} \sin \frac{2n\pi}{3} \quad \forall n \neq 0$$

$$a_n = \frac{3}{n^2\pi^2} [\cos \frac{2n\pi}{3} - 1]$$

Finalmente:

$$x(t) = \frac{2}{3} + \sum_{n=1}^{\infty} \frac{3}{n^2\pi^2} (\cos \frac{2n\pi}{3} - 1) \cdot \cos \frac{n\pi}{3} t$$

## Problema 2

$$\text{Si } x(t) = \frac{2}{3} + \sum_{n=1}^{\infty} \frac{3}{n^2 \pi^2} \left( \cos \frac{2}{3} n \pi t - 1 \right) \cdot \cos \frac{n \pi}{3} t$$

entonces su serie exponencial está dada por:

$$C_n = \frac{1}{2}(a_n - i b_n), \text{ con } b_n = 0$$

$$\text{así } C_n = \frac{1}{2} \cdot \frac{3}{n^2 \pi^2} \left( \cos \left( \frac{2}{3} n \pi \right) - 1 \right) \quad \forall n \neq 0$$

$$C_0 = a_0 = \frac{2}{3}$$

Finalmente:

$$x(t) = \frac{2}{3} + \sum_{n=-\infty}^{\infty} \frac{3}{2n^2 \pi^2} \left( \cos \frac{2}{3} n \pi - 1 \right) \cdot e^{i \frac{n \pi}{3} t}$$

### Problema 3

$$\mathcal{F}\{f(t)\} = 2\pi \sum_{n=-\infty}^{\infty} C_n \delta(\omega - n\omega_0)$$

$$\text{Si } T = 1 \quad \therefore \omega_0 = \frac{2\pi}{T} = 2\pi$$

$$2 \quad C_n = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} f(t) e^{-jn\omega_0 t} dt$$

$$C_n = \int_{-\frac{1}{2}}^{\frac{1}{2}} f(t) \cdot e^{-j2\pi n t} dt$$

$$C_n = \int_{0^-}^{0^+} \delta(t) e^{-j2\pi n t} dt$$

De la Propiedad de muestreo de  $\delta(t)$

$$C_n = e^0 = 1$$

Finalmente:

$$\mathcal{F}\{f(t)\} = 2\pi \sum_{n=-\infty}^{\infty} \delta(\omega - 2\pi n)$$

2.0 pts



# Problema 4

$$-6 \delta(5t-10) \cdot \cos 15t + \frac{1}{3-jt} t^2 + e^{j4t} (t-1) \leftrightarrow ?$$

(I)
(II)
(III)

SOLUCION:

(I) Si  $\delta(t) \leftrightarrow 1$   
 $\delta(t-10) \leftrightarrow e^{-j10\omega}$   
 $\delta(5t-10) \leftrightarrow \frac{1}{|5|} e^{-j\frac{10}{5}\omega}$

$$\delta(5t-10) \cdot \cos 15t \leftrightarrow \frac{2}{5} [e^{-j2(\omega+15)} + e^{-j2(\omega-15)}]$$

$$-6 \delta(5t-10) \cdot \cos 15t \leftrightarrow -\frac{12}{5} (e^{-j2(\omega+15)} + e^{-j2(\omega-15)})$$

(II) Si  $e^{-at} u(t) \leftrightarrow \frac{1}{a+j\omega}$   
 $\frac{1}{a+jt} \leftrightarrow 2\pi e^{-a(-\omega)} u(-\omega)$   
 $\frac{1}{3+jt} \leftrightarrow 2\pi e^{-3\omega} u(-\omega)$   
 $\frac{1}{3-jt} \leftrightarrow 2\pi e^{-3\omega} u(\omega)$

$$\frac{(-jt)^2}{3-jt} \leftrightarrow \frac{2\pi d^2}{d^2\omega} [e^{-3\omega} u(\omega)]$$

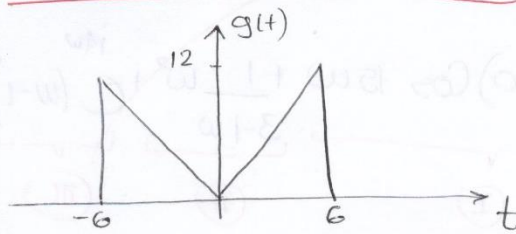
$$\frac{t^2}{3-jt} \leftrightarrow -\frac{2\pi d^2}{d^2\omega} [e^{-3\omega} u(\omega)]$$

(III) Si  $\delta(t) \leftrightarrow 1$   
 $1 \leftrightarrow 2\pi \delta(-\omega)$   
 $-jt \leftrightarrow 2\pi \frac{d}{d\omega} \delta(\omega)$   
 $t-1 \leftrightarrow 2\pi i \frac{d}{d\omega} \delta(\omega) \cdot e^{-j\omega}$   
 $e^{j4t} \cdot (t-1) \leftrightarrow 2\pi i \frac{d}{d\omega} \delta(\omega+4) e^{j4\omega}$

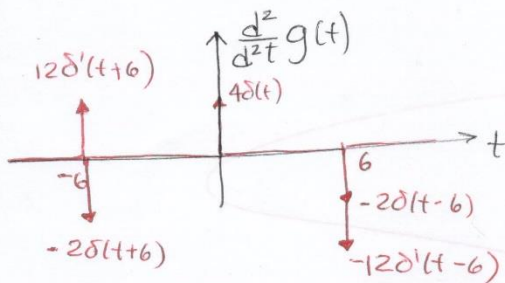
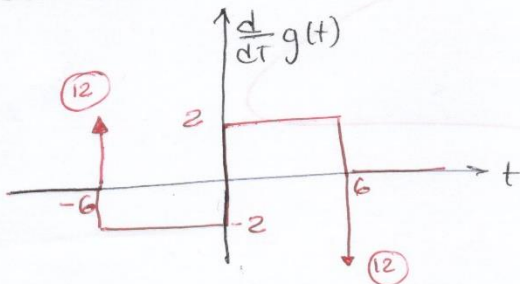
$$e^{j4t} \cdot (t-1) \leftrightarrow j2\pi e^{-j(\omega+4)} \frac{d}{d\omega} \delta(\omega+4)$$

# Problema 5

2.0 pts



SOLUCION:



$$\frac{d^2}{dt^2} g(t) = -2\delta(t+6) + 12\delta'(t+6) + 4\delta(t) - 2\delta(t-6) - 12\delta'(t-6)$$

$$\begin{aligned} \mathcal{F}\left\{\frac{d^2}{dt^2} g(t)\right\} &= -2\mathcal{F}\{\delta(t+6)\} \\ &+ 12\mathcal{F}\{\delta'(t+6)\} + 4\mathcal{F}\{\delta(t)\} \\ &- 2\mathcal{F}\{\delta(t-6)\} - 12\mathcal{F}\{\delta'(t-6)\} \end{aligned}$$

$$\mathcal{F}\left\{\frac{d^2}{dt^2} g(t)\right\} = -2e^{j\omega 6} + 12j\omega e^{j\omega 6} + 4 - 2e^{-j\omega 6} - 12j\omega e^{-j\omega 6}$$

$$= -2(e^{j\omega 6} + e^{-j\omega 6}) + 12j\omega(e^{j\omega 6} - e^{-j\omega 6}) + 4$$

$$= -4(\cos 6\omega) - 24\omega \sin 6\omega + 4$$

$$= 4(1 - \cos 6\omega) - 24\omega \sin 6\omega$$

$$= 8 \sin^2 3\omega - 24\omega \sin 6\omega$$

Así

$$\frac{d^2}{dt^2} g(t) \longleftrightarrow 8 \sin^2 3\omega - 24\omega \sin 6\omega$$

De la Prop. de diferenciación en t

$$\text{Si } g(t) \longleftrightarrow G(\omega)$$

$$\frac{d^2}{dt^2} g(t) \longleftrightarrow (j\omega)^2 G(\omega)$$

Finalmente

$$(j\omega)^2 G(\omega) = 8 \sin^2 3\omega - 24\omega \sin 6\omega$$

$$G(\omega) = \frac{-1}{\omega^2} [8 \sin^2 3\omega - 24\omega \sin 6\omega]$$

$$G(\omega) = \frac{24}{\omega} \sin 6\omega - \frac{8}{\omega^2} \sin^2 3\omega$$

$$G(\omega) = 144 \text{Sa}(\omega) - \frac{8 \sin 3\omega}{(3)\omega} \cdot \frac{\sin 3\omega}{(3)\omega}$$

$$G(\omega) = 144 \text{Sa}(\omega) - 72 \text{Sa}^2(3\omega)$$