7-5-2021

Examen 1

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① $T=6 \qquad \omega_0 = \pi/3$

$$x(t) = \begin{cases} 1 & -3 \leq t \leq -2 \\ -\frac{t}{2} & -2 \leq t \leq 0 \\ \frac{t}{2} & 0 \leq t \leq 2 \\ 1 & 2 \leq t \leq 3 \end{cases}$$

$x(t)$ es par $b_n = 0$

$$a_0 = \frac{1}{T}\int_{t_0}^{t_0+T} f(t)\,dt$$

$$a_n = \frac{2}{T}\int_{t_0}^{t_0+T} f(t)\cos(n\omega_0 t)\,dt$$

$$a_0 = \frac{2}{6}\int_2^3 dt + \frac{2}{6}\int_0^2 \frac{t}{2}\,dt = \frac{1}{3}t\Big|_2^3 + \frac{1}{12}t^2\Big|_0^2 = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$$

$$a_n = \frac{2}{3}\int_0^2 \frac{t}{2}\cos(n\pi t/3)\,dt + \frac{2}{3}\int_2^3 \cos(n\pi t/3)\,dt$$

$$= \frac{1}{3}\left[\frac{3}{n\pi} t\,\mathrm{sen}\left(\frac{n\pi t}{3}\right) + \frac{9}{n^2\pi^2}\cos\left(\frac{n\pi t}{3}\right)\right]_0^2 + \frac{2}{n\pi}\,\mathrm{sen}\left(\frac{n\pi t}{3}\right)\Big|_2^3$$

$$= \frac{1}{n\pi}\left[\cancel{2\,\mathrm{sen}\,\frac{2n\pi}{3}} + \frac{3}{n\pi}\cos\left(\frac{n\pi 2}{3}\right) - 0 - \frac{3}{n\pi}\right] + 0 - \cancel{\frac{2}{n\pi}\,\mathrm{sen}\left(\frac{2n\pi}{3}\right)}$$

$$= \frac{3}{n^2\pi^2}\left(\cos\left(\frac{n\pi 2}{3}\right) - 1\right)$$

$$x(t) = \frac{2}{3} + \sum_{n=1}^{\infty} \frac{3}{n^2\pi^2}\left(\cos\left(\frac{2n\pi}{3}\right) - 1\right)\cos\left(\frac{n\pi t}{3}\right)$$

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② La Función ya está en forma de suma de cosenos y senos, es ella misma

$$f(t) = \cos t$$

(3) $AC_d(t) \leftrightarrow Ad\,Sa\left(\frac{\omega d}{2}\right)$

$C_4(t) \leftrightarrow 4\,Sa(2\omega)$

$C_4(t)\cos 20t \leftrightarrow 2\left[Sa(2\omega+40)+Sa(2\omega-40)\right]$

$10C_4(t)\cos 20t \leftrightarrow 20\left[Sa(2\omega+40)+Sa(2\omega-40)\right]$

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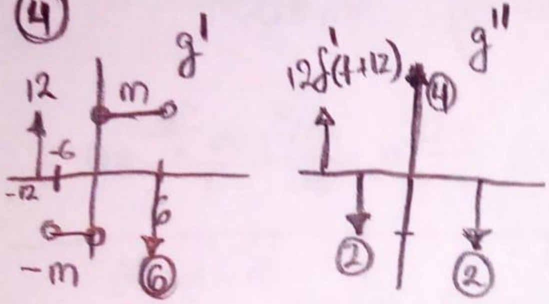
(4)



$m = 2 = \left|\frac{0-12}{0+6}\right|$

$g''(t) = 12\delta'(t+12) - 2\delta(t+6)$
$+ 4\delta(t) - 2\delta(t-6)$
$- 6\delta'(t-6)$

$(i\omega)^2 G(\omega) = 12(i\omega)e^{12i\omega} - 2e^{6i\omega} + 4 - 2e^{-6\omega i} - 6i\omega e^{-6i\omega}$

$G(\omega) = \frac{-1}{\omega^2}\left[12i\omega\left(e^{12i\omega} - e^{-6i\omega}\right) - 2e^{6i\omega} - 2e^{-6i\omega} + 4\right]$

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(5) $e^{-at}u(t) \leftrightarrow \frac{1}{a+i\omega}$

$\delta(t) \leftrightarrow 1$

$e^{-3t}u(t) \leftrightarrow \frac{1}{3+i\omega}$

$\delta'(t) \leftrightarrow i\omega$

$\frac{d^2}{dt^2}\left[e^{-3t}u(t)\right] = \frac{(i\omega)^2}{3+i\omega}$

$-i\delta'(t)e^{it} \leftrightarrow (\omega-1)$

$-\frac{d^2}{dt^2}\left[e^{+3t}u(t)\right] \leftrightarrow \frac{\omega^2}{3-i\omega}$

$-i\delta'(t+4)e^{i(t+4)} \leftrightarrow (\omega-1)e^{i4\omega}$

$-\frac{d^2}{dt^2}\left[e^{3t}u(t)\right] - i\delta'(t+4)e^{i(t+4)} \leftrightarrow \frac{\omega^2}{3-i\omega} + e^{i4\omega}(\omega-1)$

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$$F(\omega) = \frac{1}{(1+i\omega)^2} = \frac{1e^{0}}{(1+\omega^2)e^{2\arctan(\omega)}} = \frac{1}{1+\omega^2}e^{-2\arctan(\omega)}$$

$$|F(\omega)| = \frac{1}{1+\omega^2} \qquad \theta = -2\arctan(\omega)$$