TAREA 6' EVALUE: TA-10-6'

$$I=\int_0^{2\pi}\frac{d\theta}{(5-3\,\text{sen}\,\theta)^2}$$

SOL: UTILIZANDO (2) y (3):

$$I=\int_0^{2\pi}\frac{1}{(5-3\,\text{sen}\,\theta)^2}\,d\theta=\int_\gamma\frac{1}{\left[5-3\left(\frac{z^2-1}{2iz}\right)\right]^2}\frac{dz}{iz}\quad \text{con } \gamma:|z|=1.$$

$$=\int_\gamma\frac{dz}{iz\left[5-\frac{3z^2-3}{2iz}\right]^2}=\int_\gamma\frac{dz}{iz\left[\frac{10iz-3z^2+3}{2iz}\right]^2}=\int_\gamma\frac{dz}{\frac{iz(10iz-3z^2+3)^2}{-4z^2}}$$

$$=\int_\gamma\frac{-4z\,dz}{i(-3z^2+10iz+3)^2}=\int_\gamma\frac{-4z\,dz}{i\left[-3\left(z^2-\frac{10}{3}iz-1\right)\right]^2}=\int_\gamma\frac{-4z\,dz}{9i\left(z^2-\frac{10}{3}iz-1\right)^2}$$

SINGULARIDADES:

$z^2-\frac{10}{3}iz-1=0$ con $a=1$, $b=-\frac{10}{3}i$ y $c=-1$.

$$z=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{\frac{10}{3}i\pm\sqrt{-\frac{100}{9}-4(1)(-1)}}{2}=\frac{\frac{10}{3}i\pm\sqrt{-\frac{100}{9}+4}}{2}=\frac{\frac{10}{3}i\pm\sqrt{-\frac{64}{9}}}{2}$$

$$z=\frac{\frac{10}{3}i\pm\frac{8}{3}i}{2}=\frac{10i\pm 8i}{6}$$

$\Rightarrow z_0=\frac{18i}{6}=3i\notin\gamma$ y $z_1=\frac{2i}{6}=\frac{i}{3}\in\gamma$.

SE UTILIZA $z_1=\frac{i}{3}\in\gamma$ con $K=2$.

POR EL T. DEL RESIDUO $R=\frac{1}{(K-1)!}\lim_{z\to z_1}\frac{d^{K-1}}{dz^{K-1}}\left[(z-z_1)^K f(z)\right]$

$$R=R_{z_1}=\frac{1}{1}\lim_{z\to i/3}\frac{d}{dz}\left[(z-i/3)^2\frac{z}{(z-i/3)^2(z-3i)^2}\right]=\lim_{z\to i/3}\frac{d}{dz}\left[\frac{z}{(z-3i)^2}\right]$$

$$R_{i/3} = \lim_{z \to i/3} -\left[\frac{z+3i}{(z-3i)^3}\right] = -\left[\frac{\frac{i}{3}+3i}{(\frac{i}{3}-3i)^3}\right]$$

TA-11 -6'

$$R_{i/3} = -\frac{45}{256}$$

FINALMENTE: POR EL T. DEL RESIDUO.

$$I = -\frac{4}{9i}\int_\gamma \frac{z\,dz}{(z^2-\frac{10}{3}iz-1)} = -\frac{4}{9i}2\pi i R_{i/3} = -\frac{4}{9i}2\pi i\left(-\frac{45}{256}\right)$$

$$I = \frac{5}{32}\pi \quad \therefore I = \frac{5}{32}\pi = 0.156\pi = 0.49 \simeq 0.5$$

b) $I = \int_0^{2\pi} \frac{\cos\theta}{3+2\cos\theta}\,d\theta.$

SOL: POR (1) Y (3):

$$I = \int_0^{2\pi}\frac{\cos\theta}{3+2\cos\theta}d\theta = \int_\gamma \frac{\frac{1}{2}\left(\frac{z^2+1}{z}\right)}{3+\frac{2}{2}\left(\frac{z^2+1}{z}\right)}\frac{dz}{iz} \quad \text{si } \gamma: |z|=1.$$

$$= \frac{1}{2}\int_\gamma \frac{\frac{z^2+1}{z}}{z^2+3z+1}\frac{dz}{i} = \int_\gamma \frac{z^2+1}{2iz(z^2+3z+1)}dz$$

$$= \frac{1}{2i}\int_\gamma \frac{z^2+1}{z(z^2+3z+1)}dz$$

SINGULARIDADES.

$$z(z^2+3z+1) = 0 \Rightarrow z_0 = 0 \in \gamma$$

$$z^2+3z+1 = 0 \Rightarrow z_1 = \frac{-3+\sqrt{5}}{2} = -0.38 \in \gamma \simeq -0.4 \in \gamma$$

$$z_2 = \frac{-3-\sqrt{5}}{2} = -2.6 \notin \gamma. \Rightarrow R_{z_2} = R_{-2.6} = 0$$

COMO TIENE UNA SINGULARIDAD $z_0 = 0, R=1$.

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$P(z) = z^2+1$

$Q(z) = z(z^2+3z+1) = z^3+3z^2+z$

$Q'(z) = 3z^2+6z+1$

LOS RESIDUOS SON:

$$R_{z_0} = R_0 = \left.\frac{z^2+1}{3z^2+6z+1}\right|_0 = \frac{0^2+1}{3(0)^2+6(0)+1} = \frac{1}{1} = 1$$

$$R_{z_1} = R_{-0.4} = \frac{(-0.4)^2+1}{3(-0.4)^2+6(-0.4)+1} = \frac{0.16+1}{0.48-2.4+1} = \frac{1.16}{-0.92} = -1.3.$$

POR EL T DEL RESIDUO.

$$I = \frac{1}{2i}\int_\gamma \frac{z^2+1}{z(z^2+3z+1)}dz = \frac{1}{2i}2\pi i\left[R_0 + R_{-0.4}\right]$$

$$= \pi\left[1-1.3\right] = -0.3\pi = -0.94.$$

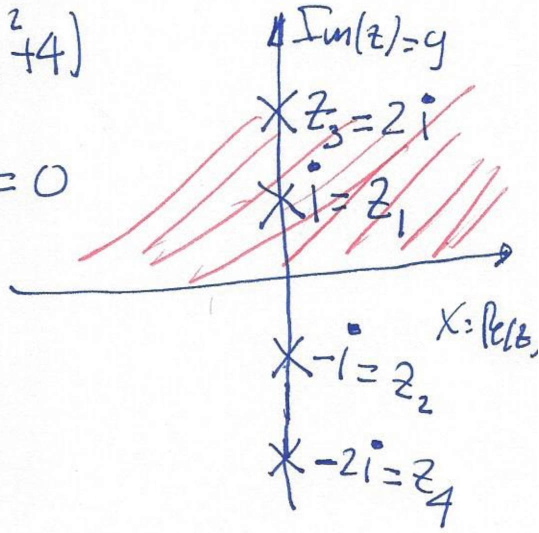
$$\therefore I = -0.3\pi = -0.94$$

① ⓐ $I=\int_{-\infty}^{\infty}\frac{x^2}{(x^2+1)(x^2+4)}\,dx$

TA-13-6'

Sol:

$$f(x)=\frac{x^2}{(x^2+1)(x^2+4)} \Rightarrow f(z)=\frac{z^2}{(z^2+1)(z^2+4)}$$



$$(z^2+1)(z^2+4)=(z+i)(z-i)(z+2i)(z-2i)=0$$

$$(z+i)(z-i)=0 \Rightarrow z_1=i,\ z_2=-i$$

$$(z+2i)(z-2i)=0 \Rightarrow z_3=2i,\ z_4=-2i$$

SINGULARIDADES.

$z_1=i$ y $z_3=2i$ ESTAN EN EL SEMIPLANO SUPERIOR.

$z_2=-i$ y $z_4=-2i$ NO ESTAN EN EL SEMIPLANO SUPERIOR. $R_{z_2}=R_{z_4}=0$.

UTILIZAMOS: $\boxed{R=\lim_{z\to z_0}(z-z_0)f(z)}$ CON K=1

$z_1=i$ CON K=1

$$R_{z_1}=R_i=\lim_{z\to i}(z-i)\frac{z^2}{(z-i)(z+i)(z^2+4)}=\lim_{z\to i}\frac{z^2}{(z+i)(z^2+4)}=\frac{i^2}{(i+i)(i^2+4)}$$

$$=\frac{-1}{2i(-1+4)}=\frac{-1}{2i(3)}=-\frac{1}{6i}\ \therefore R_i=-\frac{1}{6i}$$

$$R_{z_3}=R_{2i}=\lim_{z\to 2i}(z-2i)\frac{z^2}{(z^2+1)(z+2i)(z-2i)}=\lim_{z\to 2i}\frac{z^2}{(z^2+1)(z+2i)}$$

$$=\frac{(2i)^2}{((2i)^2+1)(2i+2i)}=\frac{4i^2}{(4i^2+1)(4i)}=\frac{-4}{(-4+1)4i}=\frac{-4}{(-3)4i}=\frac{1}{3i};\ R_{z_3}=\frac{1}{3i}$$

TA-14-61

$$I = \int_{-\infty}^{\infty} \frac{x^2}{(x^2+1)(x^2+4)}\,dx = 2\pi i\left[R_{z_1} + R_{z_3}\right]$$

$$= 2\pi i\left[-\frac{1}{6i} + \frac{1}{3i}\right] = 2\pi\left[-\frac{1}{6} + \frac{1}{3}\right] = 2\pi\left[\frac{-1+2}{6}\right] = 2\pi\left[\frac{1}{6}\right] = \frac{\pi}{3}.$$

AHORA:

$$f(x) = \frac{x^2}{(x^2+1)(x^2+4)} \quad \text{ES PAR.} \quad f(-x) = f(x).$$

$$f(-x) = \frac{(-x)^2}{((-x)^2+1)((-x)^2+4)} = \frac{x^2}{(x^2+1)(x^2+4)} = f(x)$$

. APLICANDO:

, SI $f(x)$ ES PAR $\Rightarrow \int_{-\infty}^{\infty} f(x)\,dx = 2\int_0^{\infty} f(x)\,dx$

. SI $f(x)$ ES IMPAR $\Rightarrow \int_{-\infty}^{\infty} f(x)\,dx = 0.$

$$\int_{-\infty}^{\infty} \frac{x^2}{(x^2+1)(x^2+4)}\,dx = 2\int_0^{\infty} \frac{x^2}{(x^2+1)(x^2+4)}\,dx$$

$$2\int_0^{\infty} \frac{x^2}{(x^2+1)(x^2+4)}\,dx = \frac{\pi}{3} \quad \therefore \int_0^{\infty} \frac{x^2}{(x^2+1)(x^2+4)}\,dx = \frac{\pi}{6}$$

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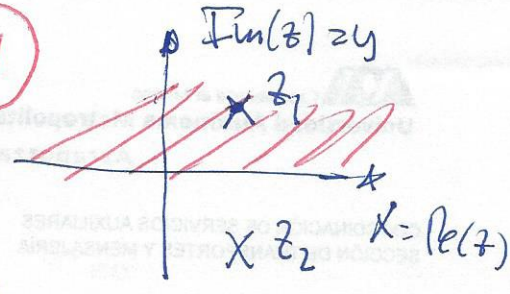
(B) $I=\int_{-\infty}^{\infty}\frac{dx}{x^2-2x+2}$



Sol.

$$f(x)=\frac{1}{x^2-2x+2} \Rightarrow f(z)=\frac{1}{z^2-2x+2}=\frac{1}{(z-1-i)(z-1+i)}$$

$z^2-2x+2=0 \Rightarrow z_1=1+i \in$ EN EL PLANO SUPERIOR

$z^2-2x+2=0\ (z-z_1)(z-z_2)\quad z_2=1-i \notin$ " " "

$$R_{z_1=1+i}=\lim_{z\to z_1}(z-z_1)\frac{1}{(z-z_1)(z-z_2)}$$

$$=\lim_{z\to z_1}\frac{1}{(z-z_2)}=\frac{1}{(1+i-1+i)}=\frac{1}{(2i)}$$

$$=\frac{1}{2i}$$

$$I=\int_{-\infty}^{\infty}\frac{dx}{x^2-2x+2}=2\pi i\left[\frac{1}{2i}\right]=\pi$$

$$\therefore I=\pi.$$