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1; Expresa las señales e^{-t}, t^2 y $2t$ como series Trigonométrica de Fourier en el intervalo $(0,1)$

a) e^{-t} $\qquad T=1 \qquad \omega_0 = 2\pi$

$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + b_n \operatorname{sen} n\omega_0 t$$

$$a_n = 2\int_0^1 e^{-t}\cos 2\pi n t\,dt \qquad \begin{matrix} u=\cos 2\pi n t & dv = e^{-t} \\ du = -2\pi n \operatorname{sen}(2\pi n t)\,dt & v=-e^{-t}\end{matrix}$$

$$a_n = 2\left[-e^{-t}\cos 2\pi n t\Big|_0^1 - 2\pi n\int_0^1 e^{-t}\operatorname{sen}(2\pi n t)\,dt\right]$$

$$\begin{matrix} u=\operatorname{sen} 2\pi n t & dv = e^{-t}dt \quad v=-e^{-t} \\ du = 2\pi n\cos 2\pi n t\,dt & \end{matrix}$$

$$a_n = 2\left[-e^{-t}\cos 2\pi n t\Big|_0^1 - 2\pi n\left(-e^{-t}\operatorname{sen} 2\pi n t\Big|_0^1 + 2\pi n\int_0^1 e^{-t}\cos 2\pi n t\,dt\right)\right]$$

$$a_n = -2\left[e^{-t}\cos 2\pi n t\Big|_0^1 + 2\pi n e^{-t}\operatorname{sen} 2\pi n t\Big|_0^1 - 4\pi^2 n^2\int_0^1 e^{-t}\cos 2\pi n t\,dt\right]$$

$$\int_0^1 e^{-t}\cos 2\pi n t\,dt = 2\left[(1+4\pi^2 n^2)^{-1}\left(e^{-t}\cos 2\pi n t\Big|_0^1 + 2\pi n e^{-t}\operatorname{sen} 2\pi n t\Big|_0^1\right)\right]$$

$$= -(e^{-1}\cos 2\pi n - 1) + 2\pi n\left(e^{-1}\operatorname{sen} 2\pi n\right)$$

$$\int_0^1 e^{-t}\cos 2\pi n t\,dt = \frac{-2e^{-1}\cos 2\pi n + 2 + \cancel{4\pi n e^{-1}\operatorname{sen} 2\pi n}^{\,0}}{1+4\pi^2 n^2}$$

$$= \frac{-2e^{-1}+2}{2+8\pi^2 n^2} = \frac{2(1-e^{-1})}{1+4\pi^2 n^2}$$

$$a_0 = 2\int_0^1 e^{-t}\,dt = 2\left(-e^{-t}\right)\Big|_0^1 = -(e^{-1}-1)$$

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$$b_n = 2\int_0^1 e^{-t}\,\text{sen}\,2\pi n t\,dt \qquad \begin{matrix} u = \text{sen}\,2\pi n t \\ du = 2\pi n \cos 2\pi n t\,dt \end{matrix}$$

$$dv = e^{-t}\,dt \qquad v = -e^{-t}$$

$$b_n = 2\left[-e^{-t}\,\text{sen}\,2\pi n t\Big|_0^1 + 2\pi n \int_0^1 e^{-t}\cos 2\pi n t\,dt \right]$$

$$u = \cos 2\pi n t \qquad dv = e^{-t}\,dt$$
$$du = -2\pi n\,\text{sen}\,2\pi n t \qquad v = -e^{-t}$$

$$b_n = -2e^{-t}\,\text{sen}\,2\pi n t\Big|_0^1 + 4\pi n\left[-e^{-t}\cos 2\pi n t - 2\pi n \int_0^1 e^{-t}\,\text{sen}\,2\pi n t\,dt \right]$$

$$= -2e^{-t}\,\text{sen}\,2\pi n t\Big|_0^1 - 4\pi n e^{-t}\cos 2\pi n t\Big|_0^1 - 8\pi^2 n^2 \int_0^1 e^{-t}\,\text{sen}\,2\pi n t\,dt$$

$$(1 + 4\pi n^2)\int_0^1 e^{-t}\,\text{sen}\,2\pi n t = \cancel{-2(e^{-1}\,\text{sen}\,2\pi n)}^{0} - 4\pi n(e^{-1}\cancel{\cos 2\pi n}^{1} - 1)$$

$$\int_0^1 e^{-t}\,\text{sen}\,2\pi n t\,dt = \frac{-4\pi n(e^{-1} - 1)}{1 + 4\pi^2 n^2}$$

$$f(t) = -2(e^{-1} + 1) + \sum_{n=1}^{\infty}\left(\frac{-2e^{-1} + 2}{1 + 4\pi^2 n^2}\right)\cos 2\pi n t - \frac{4\pi n(e^{-1} - 1)}{1 + 4(\pi n)^2}$$

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2) t^2 $T=1$ $\omega_0 = 2\pi$

$$a_n = 2\int_0^1 t^2 \cos 2\pi n t \, dt$$

$$\int t^2 \cos 2\pi n t \, dt \qquad u = t^2 \quad du = 2t\,dt \qquad dv = \cos 2\pi n t \, dt \quad v = \frac{\operatorname{sen} 2\pi n t}{2\pi n}$$

$$\int t^2 \cos 2\pi n t \, dt = \frac{t^2 \operatorname{sen} 2\pi n t}{2\pi n} - \frac{1}{\pi n}\int t \operatorname{sen} 2\pi n t \, dt$$

$$u = t \quad du = dt \qquad dv = \operatorname{sen} 2\pi n t \, dt \quad v = -\frac{\cos 2\pi n t}{2\pi n}$$

$$\int t^2 \cos 2\pi n t \, dt = \frac{t^2 \operatorname{sen} 2\pi n t}{2\pi n} - \frac{1}{\pi n}\left(-\frac{t\cos 2\pi n t}{2\pi n} + \frac{1}{2\pi n}\int \cos 2\pi n t \, dt\right)$$

$$= \frac{t^2 \operatorname{sen} 2\pi n t}{2\pi n} - \frac{1}{\pi n}\left[-\frac{t\cos 2\pi n t}{2\pi n} + \frac{1}{2\pi n}\left(\frac{\operatorname{sen} 2\pi n t}{2\pi n}\right)\right]$$

$$= \frac{t^2 \operatorname{sen} 2\pi n t}{2\pi n} + \frac{t\cos 2\pi n t}{2(\pi n)^2} - \frac{\operatorname{sen} 2\pi n t}{4(\pi n)^3}$$

$$= \frac{2(\pi n)^2 t^2 \operatorname{sen} 2\pi n t + 2\pi n t \cos 2\pi n t - \operatorname{sen} 2\pi n t}{4(\pi n)^3} + C$$

$$2\int_0^1 t^2 \cos 2\pi n t \, dt = \frac{1}{4(\pi n)^3}\left[2\pi n\right] = \frac{1}{(\pi n)^2}$$

$$a_0 = \int_0^1 t^2 \, dt = 2\left(\frac{t^3}{3}\right)\Big|_0^1 = \frac{2}{3}(1) = \frac{1}{3}$$

$$b_n = 2\int_0^1 t^2 \operatorname{sen} 2\pi n t \, dt$$

$$\int$$

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Dados los siguientes procesos, sus tiempos de llegada y servicios. Utiliza RR con un cuanto de 3. Calcula el tiempo de finalización, retorno, espera y el giro ponderado de cada proceso, así como sus promedios. Muestra el desarrollo del algoritmo, mostrando el proceso e cada momento

Proceso	1	2	3	4	5
tll	0	2	3	5	9
ts	4	2	5	3	6

$\longrightarrow$ A B C D E

A BCD E

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	24
A	A	A	B	B	C	C	C	A	D	D	D	C	C	E	E	E	E	E	E		

~~tf~~	A	B	C	D	E	prom
tf	9	5	14	12	20	-
tr	9	3	11	7	11	8.2
te	5	1	6	4	5	-
gp	2.25	1.5	2.2	2.33	1.83	2.022

$$\int t^2 \operatorname{sen} 2\pi nt\, dt \qquad u = t^2 \quad du = 2t\,dt \qquad dv = \operatorname{sen} 2\pi nt\,dt \quad v = -\frac{\cos 2\pi nt}{2\pi n}$$

$$= -\frac{t^2\cos 2\pi nt}{2\pi n} + \frac{1}{\pi n}\int t\cos 2\pi nt\,dt \qquad u = t \quad du = dt \qquad dv = \cos 2\pi nt \quad v = \frac{\operatorname{sen} 2\pi nt}{2\pi n}$$

$$= -\frac{t^2\cos 2\pi nt}{2\pi n} + \frac{1}{\pi n}\left[\frac{t\operatorname{sen} 2\pi nt}{2\pi n} - \frac{1}{2\pi n}\int \operatorname{sen} 2\pi nt\,dt\right]$$

$$= -\frac{t^2\cos 2\pi nt}{2\pi n} + \frac{t\operatorname{sen} 2\pi nt}{2(\pi n)^2} + \frac{\cos 2\pi nt}{4(\pi n)^3}$$

$$= \frac{-2(\pi n)^2 t^2\cos 2\pi nt + 2\pi nt\operatorname{sen} 2\pi nt + \cos 2\pi nt}{4(\pi n)^3}$$

$$2\int_0^1 t^2 \operatorname{sen} 2\pi nt\,dt = \frac{2}{4(\pi n)^{\cancel{3}}}\left[-2\cancel{(\pi n)^2} + 1 - 1\right] = -\frac{1}{\pi n}$$

$$f(t) = \frac{1}{\ } + \sum^{\infty}\left(\frac{1}{\ }\cos 2\pi nt - \frac{1}{\ }\operatorname{sen} 2\pi nt\right)$$

c) $2t$ $\quad T=1 \quad \omega_0 = 2\pi$

$a_n = 2\int_0^1 2t \cos 2\pi n t \; dt$

$\int t\cos 2\pi n t \; dt \qquad u = t \quad du = dt \qquad dv = \cos 2\pi n t \; dt \quad v = \frac{\operatorname{sen} 2\pi n t}{2\pi n}$

$$\int = \frac{t \operatorname{sen} 2\pi n t}{2\pi n} - \frac{1}{2\pi n}\int \operatorname{sen} 2\pi n t \; dt$$

$$= \frac{t\operatorname{sen}2\pi n t}{2\pi n} + \frac{\cos 2\pi n t}{(2\pi n)^2} = \frac{2\pi n t \operatorname{sen} 2\pi n t + \cos 2\pi n t}{(2\pi n)^2}$$

$$\int_0^1 t\cos 2\pi n t \; dt = \frac{1}{(2\pi n)^2}\left[1 - 1\right] = \underline{0}$$

$$a_0 = \int_0^1 2t \; dt = \frac{2t^2}{2} = \underline{1}$$

$b_n = 4\int_0^1 t \operatorname{sen} 2\pi n t \; dt$

$\int t\operatorname{sen} 2\pi n t \qquad u = t \quad du = dt \qquad dv = \operatorname{sen} 2\pi n t \quad v = -\frac{\cos 2\pi n t}{2\pi n}$

$$= -\frac{t\cos 2\pi n t}{2\pi n} + \frac{1}{2\pi n}\int \cos 2\pi n t \; dt$$

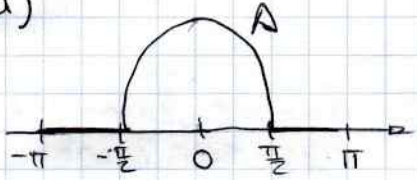
$$= -\frac{t\cos 2\pi n t}{2\pi n} + \frac{\operatorname{sen} 2\pi n t}{(2\pi n)^2} = \frac{-2\pi n t\cos 2\pi n t + \operatorname{sen} 2\pi n t}{(2\pi n)^2}$$

$$4\int_0^1 t\operatorname{sen} 2\pi n t = \frac{4}{(2\pi n)^2}\left[-2\pi n\right] = -\frac{2}{\pi n}$$

$$f(t) = 1 - 2\sum_{n=1}^{\infty} \frac{\operatorname{sen} 2\pi n t}{\pi n}$$

Problema 2: Encuentra las SFT de las sig. funciones de $-\pi$ a π

a)



$$f(t) = \begin{cases} 0, & -\pi < t < -\frac{\pi}{2} \\ A\cos t, & -\frac{\pi}{2} < t < \frac{\pi}{2} \\ 0, & \pi/2 < t < \pi \end{cases}$$

$T = 2\pi \qquad \omega_0 = 1$

La función es par. Entonces:

$$a_0 = \frac{2}{T}\int_0^{T/2} f(t)\,dt \qquad a_n = \frac{4}{T}\int_0^{T/2} f(t)\cos n\omega_0 t\,dt$$

$$b_n = 0$$

$$a_n = \frac{4}{2\pi}\int_0^{\pi} f(t)\cos n\omega_0 t\,dt = \frac{4}{2\pi}\int_0^{\frac{\pi}{2}} A\cos t\cos nt\,dt$$

$$= \frac{2A}{\pi}\int_0^{\frac{\pi}{2}} \cos t\cos nt\,dt$$

$$\int \cos t\cos nt\,dt = \int \frac{1}{2}\left[\cos(nt-t) + \cos(nt+t)\right]dt$$

$$= \frac{1}{2}\int \cos\left[t(n-1)\right]dt + \frac{1}{2}\int \cos\left[t(n+1)\right]dt$$

$$= \frac{1}{2}\left(\frac{\operatorname{sen}[t(n-1)]}{n-1}\right) + \frac{1}{2}\left(\frac{\operatorname{sen}[t(n+1)]}{n+1}\right)$$

$$= \frac{1}{2}\left[\frac{(n+1)\operatorname{sen}[t(n-1)] + (n-1)\operatorname{sen}[t(n+1)]}{n^2-1}\right]$$

$$= \frac{1}{2}\left[\frac{(n+1)(\operatorname{sen}(nt)\cos(t) - \operatorname{sen}(t)\cos(nt)) + (n-1)(\operatorname{sen}(nt)\cos t + \operatorname{sen} t\cos nt)}{n^2-1}\right]$$

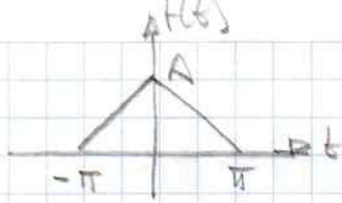
$$= \frac{1}{2}\left[\frac{n\operatorname{sen} nt\cos t - \cancel{n\operatorname{sen} t\cos nt} + \cancel{\operatorname{sen} nt\cos t} - \operatorname{sen} t\cos nt + n\operatorname{sen} nt\cos t + \cancel{n\operatorname{sen} t\cos nt} - \cancel{\operatorname{sen} nt\cos t} - \operatorname{sen} t\cos nt}{n^2-1}\right]$$

$$= \frac{1}{\cancel{2}}\left[\frac{\cancel{2}n\operatorname{sen} nt\cos t - \cancel{2}\operatorname{sen} t\cos nt}{n^2-1}\right] = \frac{n\operatorname{sen} nt\cos t - \operatorname{sen} t\cos nt}{n^2-1}$$

$$\frac{2A}{\pi}\int_0^{\frac{\pi}{2}} \cos t\cos nt\,dt = \frac{1}{n^2-1}\left[-\cos\frac{n\pi}{2}\right] = \frac{-2A\cos\frac{n\pi}{2}}{\pi(n^2-1)}$$

$$m = \frac{A}{-\pi} = -\frac{A}{\pi}$$

2)



$$f(t)=\begin{cases} \frac{A}{\pi}t + A, & -\pi<t<0 \\ -\frac{A}{\pi}t + A, & 0<t<\pi \end{cases} \qquad T = 2\pi \quad \omega_0 = 1$$

La serie es par.

$$a_0 = \frac{4}{T}\int_0^\pi f(t)\,dt \qquad a_n = \frac{2}{T}\int_0^\pi f(t)\cos n\omega_0 t\,dt$$

$$b_n = 0$$

$$a_n = \frac{2}{2\pi}\int_0^\pi \left(-\frac{A}{\pi}t + A\right)\cos nt\,dt = \frac{1}{\pi}\left(\int_0^\pi -\frac{A}{\pi}t\cos nt\,dt + \int_0^\pi A\cos nt\,dt\right)$$

$$\int_0^\pi -\frac{A}{\pi}t\cos nt\,dt = -\frac{A}{\pi}\int_0^\pi t\cos nt\,dt \qquad \begin{matrix} u = t & dv = \cos nt\,dt \\ du = dt & v = \frac{\text{sen}\,nt}{n} \end{matrix}$$

$$= -\frac{A}{\pi}\left(\frac{t\,\text{sen}\,nt}{n}\Big|_0^\pi - \int_0^\pi \frac{\text{sen}\,nt}{n}\,dt\right) = \frac{A}{\pi}\left(-\frac{t\,\text{sen}\,nt}{n}\Big|_0^\pi + \frac{1}{n}\int_0^\pi \text{sen}\,nt\,dt\right)$$

$$\frac{A}{\pi}\left(-\frac{t\,\text{sen}\,nt}{n}\Big|_0^\pi + \frac{1}{n}\left(-\frac{\cos nt}{n}\right)\Big|_0^\pi\right) = \left(-\frac{t\,\text{sen}\,nt}{n}\Big|_0^\pi - \frac{\cos nt}{n^2}\Big|_0^\pi\right)\frac{A}{\pi}$$

$$= -\frac{1}{n}\left(\cancel{\pi\,\text{sen}\,\pi n} - 0\right) - \frac{1}{n^2}\left(\overset{(-1)^n}{\cancel{\cos \pi n}} - 1\right) = -\frac{(-1)^n - 1}{n^2}\left(\frac{A}{\pi}\right)$$

$$\int_0^\pi A\cos nt\,dt = A\int_0^\pi \cos nt\,dt = A\left(\frac{\text{sen}\,nt}{n}\right)\Big|_0^\pi$$

$$= \frac{A}{n}\left(\cancel{\text{sen}\,\pi n - \text{sen}\,0}\right) = 0$$

$$a_n = \frac{1}{\pi}\left(\frac{(-1)^n - 1}{n^2}\right) = \frac{(-1)^n - 1}{\pi n^2}\left(\frac{A}{\pi}\right)$$

$$a_0 = \frac{4}{2\pi}\int_0^\pi \left(-\frac{A}{\pi}t + A\right)dt = \frac{2}{\pi}\left(\int_0^\pi -\frac{A}{\pi}t\,dt + \int_0^\pi A\,dt\right) = \frac{2}{\pi}\left(-\frac{A}{\pi}\int_0^\pi t\,dt + A\int_0^\pi dt\right)$$

$$= \frac{2}{\pi}\left(-\frac{A}{\pi}\frac{t^2}{2}\Big|_0^\pi + At\Big|_0^\pi\right) = \frac{2}{\cancel{\pi}}\left[-\frac{A}{2\cancel{\pi}}(\pi^{\cancel{2}}) + A(\cancel{\pi})\right] = 2\left(-\frac{A}{2} + A\right)$$

$$= -A + 2A = A$$

3) $f(t) = \begin{cases} A, & -\pi < t < 0 \\ -A, & 0 < t < \pi \end{cases}$ La función es impar

$T = 2\pi \qquad \omega_0 = 1$

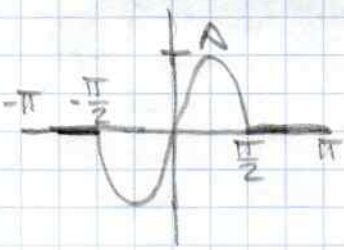
$$b_n = \frac{4}{T}\int_0^{\pi} g(t)\,\text{sen}\,nt\,dt = \frac{4}{2\pi}\int_0^{\pi} -A\,\text{sen}\,nt\,dt = -\frac{2A}{\pi}\int_0^{\pi}\text{sen}\,nt\,dt$$

$$= -\frac{2A}{\pi}\left(-\frac{\cos nt}{n}\right)\Bigg|_0^{\pi} = \frac{2A}{\pi n}\left(\overset{(-1)^n}{\cancel{\cos \pi n}} - \overset{1}{\cancel{\cos 0}}\right) = \frac{2A[(-1)^n - 1]}{\pi n}$$

$a_n = 0$

$$f(t) = \sum_{n=1}^{\infty}\frac{2A[(-1)^n - 1]}{\pi n}\,\text{sen}\,nt = \frac{2A}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n - 1}{n}\,\text{sen}\,nt$$

4)



$f(t) = \begin{cases} 0 & ; -\pi < t < -\frac{\pi}{2} \\ A\,\text{sen}\,t & ; -\frac{\pi}{2} < t < \frac{\pi}{2} \\ 0 & ; \frac{\pi}{2} < t < \pi \end{cases}$

$T = 2\pi$

La función es impar $\qquad \omega_0 = 1$

$$b_n = \frac{4}{T}\int_0^{\pi} f(t)\,\text{sen}\,nt\,dt = \frac{4}{T}\int_0^{\frac{\pi}{2}} A\,\text{sen}\,t\,\text{sen}\,nt\,dt + \cancel{\frac{4}{T}\int_{\pi/2}^{\pi} 0\,dt}$$

$$= \frac{4}{T}\int_0^{\frac{\pi}{2}} A\,\text{sen}\,t\,\text{sen}\,nt\,dt = \frac{4A}{2\pi}\int_0^{\frac{\pi}{2}}\text{sen}\,t\,\text{sen}\,nt\,dt$$

$$= \frac{2A}{\pi}\int_0^{\pi/2}\frac{1}{2}\left(\cos(t-nt) - \cos(t+nt)\right)dt$$

$$= \frac{A}{\pi}\int_0^{\pi/2}\cos(t-nt)\,dt - \frac{A}{\pi}\int_0^{\pi/2}\cos(t+nt)\,dt$$

$$\int_0^{\pi/2}\cos(t-nt)\,dt = \frac{\text{sen}(t-nt)}{1-n}\Bigg|_0^{\frac{\pi}{2}} = \frac{1}{1-n}\left(\text{sen}\left(\frac{\pi}{2} - \frac{n\pi}{2}\right) - 0\right)$$

$$= \frac{\text{sen}\left(\frac{\pi}{2} - \frac{n\pi}{2}\right)}{1-n} = \frac{\text{sen}\left(\frac{\pi}{2}(1-n)\right)}{1-n}$$

$$\int_0^{\pi/2} \cos(t+nt)\,dt = \frac{\operatorname{sen}(t(n+1))}{n+1}\Bigg|_0^{\pi/2} = \frac{\operatorname{sen}\left(\frac{\pi}{2}(n+1)\right)}{n+1}$$

$$\frac{\operatorname{sen}\left(\frac{\pi}{2}(1-n)\right)}{1-n} = \frac{\operatorname{sen}\left(-(n-1)\frac{\pi}{2}\right)}{-(n-1)} = \frac{\cancel{-}\operatorname{sen}\left(\frac{\pi}{2}(n-1)\right)}{\cancel{-}(n-1)} = \frac{\operatorname{sen}\left(\frac{\pi}{2}(n-1)\right)}{n-1}$$

$$b_n = \frac{A}{\pi}\left[\frac{\operatorname{sen}\left(\frac{\pi}{2}(n-1)\right)}{n-1} - \frac{\operatorname{sen}\left(\frac{\pi}{2}(n+1)\right)}{n+1}\right]$$

$$= \frac{A}{\pi}\left[\frac{(n+1)\operatorname{sen}\left(\frac{\pi}{2}(n-1)\right) - (n-1)\operatorname{sen}\left(\frac{\pi}{2}(n+1)\right)}{n^2-1}\right]$$

$$b_0 = \lim_{n\to 0} b_n = 0$$

$$f(t) = \sum_{n=2}^{\infty} \frac{A}{\pi}\left[\frac{(n+1)\operatorname{sen}\left(\frac{\pi}{2}(n-1)\right) - (n-1)\operatorname{sen}\left(\frac{\pi}{2}(n+1)\right)}{n^2-1}\right]\operatorname{sen} nt$$

$$\frac{1}{2}+\frac{3}{8}=\frac{4+3}{8}=\frac{7}{8}$$

Restando (2) y (3)

$$\frac{3}{2\pi n}\left[\operatorname{sen}\frac{2\pi n}{3}-\operatorname{sen}\frac{\pi n}{3}\right]-\frac{1}{(2\pi n)^2}\left[6\pi n\operatorname{sen}\frac{2\pi n}{3}+9\cos\frac{2\pi n}{3}-3\pi n\operatorname{sen}\frac{\pi n}{3}-9\cos\frac{\pi n}{3}\right]$$

$$=\frac{\cancel{6\pi n\operatorname{sen}\frac{2\pi n}{3}}-6\pi n\operatorname{sen}\frac{\pi n}{3}-\cancel{6\pi n\operatorname{sen}\frac{2\pi n}{3}}-9\cos\frac{2\pi n}{3}+3\pi n\operatorname{sen}\frac{\pi n}{3}+9\cos\frac{\pi n}{3}}{(2\pi n)^2}$$

$$=\frac{-3\pi n\operatorname{sen}\frac{\pi n}{3}-9\cos\frac{2\pi n}{3}+9\cos\frac{\pi n}{3}}{4\pi^2n^2}$$

$$\int_{1/2}^{1}2A(1-t)\cos\frac{2\pi n}{3}t\,dt=\frac{A}{2}\left(\frac{-3\pi n\operatorname{sen}\frac{\pi n}{3}-9\cos\frac{2\pi n}{3}+9\cos\frac{\pi n}{3}}{\pi^2n^2}\right)$$

$$\int_{0}^{3/2}f(t)\cos\frac{2\pi n}{3}t\,dt=\frac{3A}{2\pi n}\operatorname{sen}\frac{n\pi}{3}+\frac{A}{2\pi^2n^2}\left(-3\pi n\operatorname{sen}\frac{\pi n}{3}-9\cos\frac{2\pi n}{3}+9\cos\frac{\pi n}{3}\right)$$

$$=\frac{\cancel{3A\pi n\operatorname{sen}\frac{n\pi}{3}}-\cancel{3A\pi n\operatorname{sen}\frac{\pi n}{3}}-9A\cos\frac{2\pi n}{3}+9A\cos\frac{\pi n}{3}}{2\pi^2n^2}$$

$$=\frac{9A\left(\cos\frac{\pi n}{3}-\cos\frac{2\pi n}{3}\right)}{2\pi^2n^2}$$

$$a_n=\frac{4}{3}\left(\frac{9A\left(\cos\frac{\pi n}{3}-\cos\frac{2\pi n}{3}\right)}{2\pi^2n^2}\right)=2\left(\frac{3A\left(\cos\frac{\pi n}{3}-\cos\frac{2\pi n}{3}\right)}{\pi^2n^2}\right)$$

$$=\frac{6A\left(\cos\frac{\pi n}{3}-\cos\frac{2\pi n}{3}\right)}{\pi^2n^2}$$

$$a_0=\frac{1}{2}\left(\frac{6A}{\pi^2}\right)\lim_{n\to 0}\frac{\cos\frac{\pi n}{3}-\cos\frac{2\pi n}{3}}{n^2}\xrightarrow{L'H}\frac{3A}{\pi^2}\lim_{n\to 0}\frac{-\frac{\pi}{3}\operatorname{sen}\frac{\pi n}{3}+\frac{2\pi}{3}\operatorname{sen}\frac{2\pi n}{3}}{2n}$$

$$\xrightarrow{L'H}\frac{3A}{\pi^2}\lim_{n\to 0}\frac{-\frac{\pi^2}{9}\cos\frac{\pi n}{3}+\frac{4\pi^2}{9}\cos\frac{2\pi n}{3}}{2}=\frac{3A}{\pi^2}\left(\frac{-\frac{\pi^2}{9}+\frac{4\pi^2}{9}}{2}\right)$$

$$=\frac{\cancel{3}A}{2\cancel{\pi^2}}\left(\frac{\cancel{3\pi^2}}{\cancel{9}}\right)=\frac{A}{2}$$

$$f(t)=\frac{A}{\ }+\frac{6A}{\ }\sum^{\infty}\left(\cos\frac{\pi n}{3}-\cos\frac{2\pi n}{3}\right.$$

3d) $f(t) = \begin{cases} A, & -\frac{1}{2} < t < \frac{1}{2} \\ -2At + 2A, & \frac{1}{2} < t < 1 \\ 0, & \frac{3}{2} < t < 1 \end{cases}$ $\quad 2At + 2A, \; -1 < t < -\frac{1}{2}$ $\quad 0, \; 1 < t < \frac{3}{2}$

Función par $\quad T = 3 \quad \omega_0 = \frac{2\pi}{3} \quad b_n = 0$

$$a_n = \frac{4}{T}\int_0^{3/2} f(t)\cos n\omega_0 t \, dt$$

$$\int_0^{3/2} f(t)\cos n\omega_0 t \, dt = \int_0^{1/2} A\cos n\omega_0 t \, dt + \int_{1/2}^{1} (-2At + 2A)\cos n\omega_0 t \, dt + \cancel{\int_1^{3/2} 0 \, dt} \quad \text{— (A)}$$

$$\int_0^{1/2} A\cos n\omega_0 t = A\int_0^{1/2} \cos n\omega_0 t = A\left(\frac{\operatorname{sen} n\omega_0 t}{n\omega_0}\right)\Bigg|_0^{1/2} =$$

$$= \frac{A}{n\omega_0}\left(\operatorname{sen}\frac{n\omega_0}{2} - \cancel{\operatorname{sen} 0}\right) = \frac{3A}{2\pi n}\left(\operatorname{sen}\frac{n}{2}\left(\frac{2\pi}{3}\right)\right)$$

$$= \underline{\frac{3A}{2\pi n}\left(\operatorname{sen}\frac{n\pi}{3}\right)} \quad \text{— (1)}$$

$$\int_{1/2}^{1} (-2At + 2A)\cos n\omega_0 t \, dt = \int 2A(1-t)\cos\frac{2\pi n}{3}t \, dt$$

$$= 2A\left[\int \cos\frac{2\pi n}{3}t \, dt - \int t\cos\frac{2\pi n}{3}t \, dt\right] \quad \text{— (B)}$$

$$\int_{1/2}^{1} \cos\frac{2\pi n}{3}t \, dt = \frac{3\operatorname{sen}\frac{2\pi n}{3}t}{2\pi n}\Bigg|_{1/2}^{1} = \underline{\frac{3}{2\pi n}\left(\operatorname{sen}\frac{2\pi n}{3} - \operatorname{sen}\frac{\pi n}{3}\right)} \quad \text{— (2)}$$

$$\int t\cos\frac{2\pi n}{3}t \, dt \qquad u = t \quad du = dt \qquad dv = \cos\frac{2\pi n}{3} dt \quad v = \frac{3\operatorname{sen}\frac{2\pi n}{3}t}{2\pi n}$$

$$= \frac{3t\operatorname{sen}\frac{2\pi n}{3}}{2\pi n} - \frac{3}{2\pi n}\int \operatorname{sen}\frac{2\pi n}{3} dt = \frac{3t\operatorname{sen}\frac{2\pi n}{3}t}{2\pi n} - \frac{3}{2\pi n}\left(-\frac{3\cos\frac{2\pi n}{3}t}{2\pi n}\right)$$

$$= \frac{3t\operatorname{sen}\frac{2\pi n}{3}t}{2\pi n} + \frac{9\cos\frac{2\pi n}{3}t}{(2\pi n)^2} = \frac{6\pi n t\operatorname{sen}\frac{2\pi n}{3}t + 9\cos\frac{2\pi n}{3}t}{(2\pi n)^2}\Bigg|_{1/2}^{1}$$

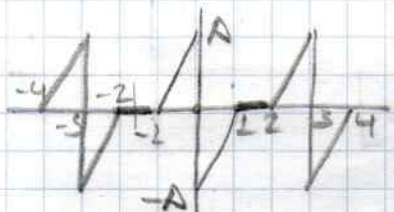
$$= \underline{\frac{1}{4\pi^2 n^2}\left[6\pi n\operatorname{sen}\frac{2\pi n}{3} + 9\cos\frac{2\pi n}{3} - 3\pi n\operatorname{sen}\frac{\pi n}{3} - 9\cos\frac{\pi n}{3}\right]} \quad \text{— (3)}$$

$$g(t) = -\frac{A}{2}\operatorname{sen} t + \frac{4A}{\pi}\sum_{\substack{n=1\\ n\neq 2}}^{\infty} \frac{\operatorname{sen}\frac{\pi n}{2}}{n^2-4}\left(\operatorname{sen}\frac{n}{2}t\right)$$

Como $f(t) = g(t-\pi)$

$$f(t) = \frac{-A}{2}\operatorname{sen}(t-\pi) + \frac{4A}{\pi}\sum_{\substack{n=1\\ n\neq 2}}^{\infty} \frac{\operatorname{sen}\frac{\pi n}{2}}{n^2-4}\left(\operatorname{sen}\frac{n}{2}(t-\pi)\right)$$

$$b_1 = \frac{4A}{\pi}\left(\frac{\operatorname{sen}\frac{\pi}{2}}{1-4}\right) = \frac{4A}{-3\pi} = -\frac{4A}{3\pi}\operatorname{sen}\frac{t-\pi}{2}$$

c) 

$$f(t) = \begin{cases} 0 & , \; -1.5 < t < -1 \\ At + A & , \; -1 < t < 0 \\ At - A & , \; 0 < t < 1 \\ 0 & , \; 1 < t < 1.5 \end{cases}$$

$T = 3 \quad \omega_0 = \frac{2\pi}{3}$ La función es impar.

$$a_n = 0 \qquad b_n = \int_0^{3/2} f(t)\operatorname{sen} n\omega_0 t \, dt \left(\frac{4}{T}\right)$$

$$b_n = \left(\int_0^1 (At - A)\operatorname{sen} n\omega_0 t \, dt + \cancel{\int_1^{3/2} 0 \, dt}\right)\left(\frac{4}{3}\right)$$

$$= \frac{4}{3}\int_0^1 A(t-1)\operatorname{sen} n\omega_0 t \, dt = \frac{4A}{3}\int_0^1 (t-1)\operatorname{sen} n\omega_0 t \, dt$$

$$\int_0^1 (t-1)\operatorname{sen} n\omega_0 t \, dt = \int_0^1 t\operatorname{sen} n\omega_0 t \, dt - \int_0^1 \operatorname{sen} n\omega_0 t \, dt$$

$$\int t \operatorname{sen} n\omega_0 t \, dt \qquad \begin{matrix} u = t \\ du = dt \end{matrix} \qquad \begin{matrix} dv = \operatorname{sen} n\omega_0 t \, dt \\ v = -\frac{\cos n\omega_0 t}{n\omega_0} \end{matrix}$$

$$= -\frac{t\cos n\omega_0 t}{n\omega_0} + \frac{1}{n\omega_0}\int \cos n\omega_0 t \, dt$$

$$= -\frac{t\cos n\omega_0 t}{n\omega_0} + \frac{1}{n\omega_0}\left(\frac{\operatorname{sen} n\omega_0 t}{n\omega_0}\right) = \frac{1}{n\omega_0}\left(\frac{\operatorname{sen} n\omega_0 t}{n\omega_0} - t\cos n\omega_0 t\right)$$

$$b_n = -\frac{A}{\pi}\int_0^{\pi} \text{sen}\, t\ \text{sen}\,\frac{n}{2}t\,dt$$

$$\int \text{sen}\, t\ \text{sen}\,\frac{n}{2}t\,dt = \int \frac{1}{2}\left[\cos\left(t-\frac{n}{2}t\right) - \cos\left(t+\frac{n}{2}t\right)\right]dt$$

$$= \frac{1}{2}\left[\int \cos\left(t-\frac{n}{2}t\right)dt - \int \cos\left(t+\frac{n}{2}t\right)dt\right]$$

$$= \frac{1}{2}\left[\int \cos\left(\frac{2t-nt}{2}\right)dt - \int \cos\left(\frac{2t+nt}{2}\right)dt\right]$$

$$= \frac{1}{2}\left[\int \cos\left(\frac{t(2-n)}{2}\right)dt - \int \cos\left(\frac{t(2+n)}{2}\right)dt\right]$$

$$= \frac{1}{2}\left[\int \cos\left(\frac{-t(n-2)}{2}\right)dt - \int \cos\left(\frac{t(n+2)}{2}\right)dt\right]$$

$$= \frac{1}{2}\left[\int \cos\left(\frac{t(n-2)}{2}\right)dt - \int \cos\left(\frac{t(n+2)}{2}\right)dt\right]$$

$$= \frac{1}{\cancel{2}}\left[\frac{\cancel{2}\,\text{sen}\left(\frac{t(n-2)}{2}\right)}{n-2} - \frac{\cancel{2}\,\text{sen}\left(\frac{t(n+2)}{2}\right)}{n+2}\right]$$

$$= \frac{(n+2)\,\text{sen}\left(\frac{t(n-2)}{2}\right) - (n-2)\,\text{sen}\left(\frac{t(n+2)}{2}\right)}{n^2-4}\Bigg|_0^{\pi}$$

$$= \frac{1}{n^2-4}\left[(n+2)\,\text{sen}\left(\frac{\pi n}{2}-\pi\right) - (n-2)\,\text{sen}\left(\frac{\pi n}{2}+\pi\right) - \cancel{(n+2)\,\text{sen}\,0 - (n-2)\,\text{sen}\,0}\right]$$

$$= \frac{1}{n^2-4}\left[(n+2)\left(\text{sen}\,\frac{\pi n}{2}\overset{-1}{\cancel{\cos\pi}} - \cancel{\cos\frac{\pi n}{2}\,\text{sen}\,\pi}\right) - (n-2)\left(\text{sen}\,\frac{\pi n}{2}\overset{-1}{\cancel{\cos\pi}} + \cancel{\cos\frac{\pi n}{2}\,\text{sen}\,\pi}\right)\right]$$

$$= \frac{1}{n^2-4}\left[(n+2)\left(-\text{sen}\,\frac{\pi n}{2}\right) - (n-2)\left(-\text{sen}\,\frac{\pi n}{2}\right)\right] = \frac{1}{n^2-4}\left[\left(-\text{sen}\,\frac{\pi n}{2}\right)(\cancel{n}+2-\cancel{n}+2)\right]$$

$$= -\frac{4\,\text{sen}\,\frac{\pi n}{2}}{n^2-4}\left(-\frac{A}{\pi}\right) = \frac{4A\,\text{sen}\,\frac{\pi n}{2}}{\pi(n^2-4)}$$

$$b_2 = \lim_{n\to 2} b_n \xrightarrow{L'H} \frac{4A\cos\frac{\pi n}{2}}{2n\pi}\left(\frac{\pi}{2}\right) = \frac{A\cos\frac{\pi n}{2}}{n}\Bigg|_2 = \frac{\cos\pi}{2} = \frac{-A}{2}\,\text{sen}\,t$$

$$g(t) = \frac{-A}{2}\,\text{sen}\,t + \frac{4A}{\pi}\sum_{n=1}^{\infty}\frac{\text{sen}\,\frac{\pi n}{2}}{n^2-4}$$

$$\int_0^2 \text{sen}\,\frac{\pi n t}{2}\,dt = -\frac{\cos\frac{\pi n}{2}t}{\frac{\pi n}{2}} = -\frac{2\cos\frac{\pi n}{2}t}{\pi n}\Bigg|_0^2 = -\frac{2}{\pi n}\left(\cos \pi n - \cos 0\right)$$

$$= -\frac{2}{\pi n}\left(\cos \pi n - 1\right)$$

$$b_n = \frac{A}{2}\left[\int_0^2 t\,\text{sen}\,\frac{\pi n}{2}t\,dt - \int_0^2 \text{sen}\,\frac{\pi n t}{2}\,dt\right]$$

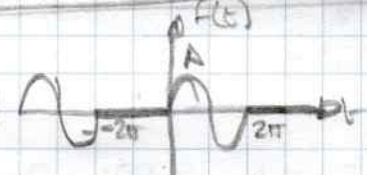
$$= \frac{A}{2}\left[-\frac{4\cos \pi n}{\pi n} + \frac{2(\cos \pi n - 1)}{\pi n}\right] = \frac{A}{2}\left[\left(\frac{2}{\pi n}\right)\left(\cos \pi n - 1 - 2\cos \pi n\right)\right]$$

$$= \frac{A}{\cancel{2}}\left[\frac{\cancel{2}}{\pi n}\left(-1 - \cos \pi n\right)\right] = -\frac{A}{\pi n}\left(\cos \pi n + 1\right)$$

$$g(t) = \sum_{n=1}^{\infty} -\frac{A}{\pi n}\left(\cos \pi n + 1\right)\text{sen}\,\frac{n\pi}{2}t = -\frac{A}{\pi}\sum_{n=1}^{\infty}\frac{\left(\cos \pi n + 1\right)\text{sen}\,\frac{\pi n}{2}t}{n}$$

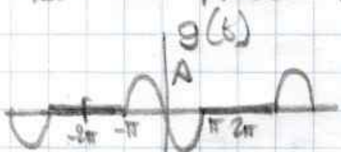
Como $f(t) = g(t) + \frac{A}{2}$

$$f(t) = \frac{A}{2} - \frac{A}{n}\sum_{n=1}^{\infty}\frac{\left(\cos \pi n + 1\right)\text{sen}\,\frac{\pi n}{2}t}{n}$$

b) 

$$f(t) = \begin{cases} A\,\text{sen}\,t, & 0 < t < 2\pi \\ 0, & 2\pi < t < 4\pi \\ f(t+T) & \text{otro} \end{cases}$$

Si desplazamos la función a la izq, podemos calcular la SFT más rápido



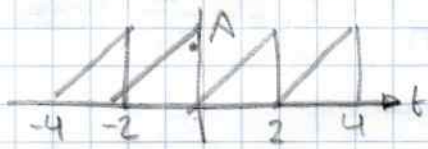
$f(t) = g(t - \pi)$ Si obtenemos la SFT de g(t), una función impar, obtenemos la SFT de f(t).

$$g(t) = \begin{cases} -A\,\text{sen}\,t, & -\pi < t < \pi \\ 0 & \pi < t < 3\pi \\ g(t+T) & \end{cases} \qquad T = 4\pi \qquad \omega_0 = \frac{2\pi}{4\pi} = \frac{1}{2}$$

$$b_n = \frac{4}{T}\int_0^{2\pi} g(t)\,\text{sen}\,\frac{n}{2}t\,dt = \frac{1}{\pi}\left[\int_0^{\pi} -A\,\text{sen}\,t\,\text{sen}\,\frac{n}{2}t\,dt + \cancel{\int_{\pi}^{2\pi} 0\,dt}\right]$$

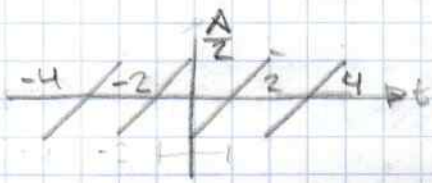
3; Determina las SFT de las siguientes señales

a)



$$f(t) = \begin{cases} \frac{A}{2}t, & 0<t<2 \\ f(t+T), & \end{cases}$$

Si $g(t) = f(t) - \frac{A}{2}$ podemos obtener la S.F.T con mayor facilidad



$g(t)$ ahora es una función impar

$$T = 4 \qquad \omega_0 = \frac{\pi}{2}$$

$$b_n = \frac{4}{T}\int_0^2 \left(\frac{A}{2}t - \frac{A}{2}\right) sen\, \frac{\pi n}{2} t\, dt = \int_0^2 \frac{A}{2}(t-1)\, sen\, \frac{\pi n t}{2}\, dt$$

$$= \frac{A}{2}\int_0^2 (t-1)\, sen\, \frac{\pi n t}{2}\, dt = \frac{A}{2}\left[\int_0^2 t\, sen\, \frac{\pi n}{2}t\, dt - \int_0^2 sen\, \frac{\pi n t}{2}\, dt\right]$$

$$\int t\, sen\, \frac{\pi n}{2} t\, dt \qquad \begin{matrix} u = t \\ du = dt \end{matrix} \qquad \begin{matrix} dv = sen\, \frac{\pi n}{2} t\, dt \\ v = -\frac{\cos \frac{\pi n}{2} t}{\frac{\pi n}{2}} = -\frac{2\cos \frac{\pi n}{2} t}{\pi n} \end{matrix}$$

$$= -\frac{2t\cos\frac{\pi n}{2}t}{\pi n} + \frac{2}{\pi n}\int \cos\frac{\pi n}{2}t\, dt$$

$$= -\frac{2}{\pi n}\left[t\cos\frac{\pi n}{2}t + \frac{2\, sen\, \frac{\pi n}{2}t}{\pi n}\right] = \frac{2}{\pi n}\left[\frac{-\pi n t\cos\frac{\pi n}{2}t + 2\, sen\, \frac{\pi n}{2}t}{\pi n}\right]$$

$$= \frac{2}{(\pi n)^2}\left[2\, sen\, \frac{\pi n}{2}t - \pi n t\cos\frac{\pi n}{2}t\right]\Big|_0^2$$

$$= \frac{2}{(\pi n)^2}\left[\cancel{2\, sen\, \pi n} - 2\pi n\cos\pi n - \cancel{2\, sen\, 0} + 0\right] = \frac{2}{(\pi n)^2}\left[-2\pi n\cos\pi n\right]$$

$$= -\frac{4\cos\pi n}{\pi n}$$

$$\int_0^1 t \operatorname{sen} n\omega_0 t \, dt = \frac{1}{n\omega_0}\left(\frac{\operatorname{sen} n\omega_0 t - n\omega_0 t \cos n\omega_0 t}{n\omega_0}\right)\Big|_0^1$$

$$= \frac{1}{(n\omega_0)^2}\Big[\operatorname{sen} n\omega_0 t - n\omega_0 t \cos n\omega_0 t\Big]_0^1$$

$$= \frac{1}{(n\omega_0)^2}\left(\operatorname{sen} n\omega_0 - n\omega_0 \cos n\omega_0 - \cancel{\operatorname{sen} 0 + 0}\right)$$

$$= \frac{1}{\left(\frac{2\pi n}{3}\right)^2}\left(\operatorname{sen}\frac{2\pi n}{3} - \frac{2\pi n}{3}\cos\frac{2\pi n}{3}\right) = \frac{9}{4\pi^2 n^2}\;(*)$$

$$\int_0^1 \operatorname{sen} n\omega_0 t \, dt = -\frac{\cos n\omega_0 t}{n\omega_0}\Big|_0^1 = -\frac{1}{n\omega_0}\left(\cos n\omega_0 - \cos 0\right)$$

$$= -\frac{1}{n\omega_0}\left(\cos n\omega_0 - 1\right) = -\frac{\cos n\omega_0 - 1}{n\omega_0}$$

$$\int_0^1 (t-1)\operatorname{sen} n\omega_0 t \, dt = \frac{9}{(2\pi n)^2}\left(\operatorname{sen}\frac{2\pi n}{3} - \frac{2\pi n}{3}\cos\frac{2\pi n}{3}\right) + \frac{3\left(\cos\frac{2\pi n}{3} - 1\right)}{2\pi n}$$

$$= \frac{9\left(\operatorname{sen}\frac{2\pi n}{3} - \frac{2\pi n}{3}\cos\frac{2\pi n}{3}\right) + 6\pi n\left(\cos\frac{2\pi n}{3} - 1\right)}{(2\pi n)^2}$$

$$= \frac{1}{(2\pi n)^2}\left(9\operatorname{sen}\frac{2\pi n}{3} - \cancel{6\pi n\cos\frac{2\pi n}{3}} + \cancel{6\pi n\cos\frac{2\pi n}{3}} - 6\pi n\right)$$

$$= \frac{1}{(2\pi n)^2}\left(9\operatorname{sen}\frac{2\pi n}{3} - 6\pi n\right) = \frac{3}{(2\pi n)^2}\left(3\operatorname{sen}\frac{2\pi n}{3} - 2\pi n\right)$$

$$b_n = \frac{4A}{\cancel{3}}\left[\frac{\cancel{3}}{4\pi^2 n^2}\left(3\operatorname{sen}\frac{2\pi n}{3} - 2\pi n\right)\right] = \frac{A}{\pi^2}\left(\frac{3\operatorname{sen}\frac{2\pi}{3}n - 2\pi n}{n^2}\right)$$

$$f(t) = \frac{A}{\pi^2}\sum_{n=1}^{\infty}\frac{3\operatorname{sen}\frac{2\pi n}{3} - 2\pi n}{n^2}\operatorname{sen}\frac{2\pi n}{3}t$$

4; Obtén la SEF y las gráficas de magnitud y fase

a) $f(t) = \begin{cases} A & 0 < t < 1/2 \\ 0 & 1/2 < t < 3 \\ A & 3 < t < 7/2 \\ & \text{otro caso} \end{cases}$ $\quad T = 7/2$

$\omega_0 = \frac{2\pi}{7/2} = \frac{4\pi}{7}$

$f(t) = \sum_{n=-\infty}^{\infty} C_n e^{2\pi i n t}$ $\quad C_n = \frac{1}{T}\int_{t_0}^{t_0+T} f(t) e^{-inw_0 t} dt$

$C_n = \frac{2}{7}\int_0^{7/2} f(t) e^{-inw_0 t} dt = \frac{2}{7}\left[\int_0^{1/2} A e^{-inw_0 t} dt + \int_3^{7/2} A e^{-inw_0 t} dt\right]$

$= \frac{2A}{7}\left[\int_0^{1/2} e^{-inw_0 t} dt + \int_3^{7/2} e^{-inw_0 t} dt\right]$

$\int e^{-inw_0 t} dt = -\frac{e^{-inw_0 t}}{inw_0} = \frac{ie^{-inw_0 t}}{nw_0} = \frac{7ie^{-\frac{4\pi i n t}{7}}}{4\pi n}$

$\int_0^{1/2} = \frac{7i}{4\pi n}\left(e^{-\frac{4\pi i n}{7}\left(\frac{1}{2}\right)} - e^0\right) = \frac{7i}{4\pi n}\left(e^{-\frac{2\pi i n}{7}} - 1\right)$

$= \frac{7i}{4\pi n}\left(\cos\left(-\frac{2\pi n}{7}\right) + i\,\text{sen}\left(-\frac{2\pi n}{7}\right) - 1\right)$

$= \frac{7i}{4\pi n}\left[\cos\frac{2\pi n}{7} - i\,\text{sen}\frac{2\pi n}{7} - 1\right]$ — 1

$\int_3^{7/2} = \frac{7i}{4\pi n}\left(e^{-\frac{4\pi i n}{7}\left(\frac{7}{2}\right)} - e^{-\frac{4\pi i n}{7}(3)}\right)$

$= \frac{7i}{4\pi n}\left[\overset{1}{\cancel{\cos 2\pi n}} - \cancel{i\,\text{sen}\, 2\pi n} - \cos\frac{12\pi n}{7} + i\,\text{sen}\frac{12\pi n}{7}\right]$

$= \frac{7i}{4\pi n}\left[1 - \cos\frac{12\pi n}{7} + i\,\text{sen}\frac{12\pi n}{7}\right]$ — 2

Sumando 1 y 2

$$\frac{7i}{4\pi n}\left[\cos\frac{2\pi n}{7}-i\,\text{sen}\frac{2\pi n}{7}-\cancel{1}+\cancel{1}-\cos\frac{12\pi n}{7}+i\,\text{sen}\frac{12\pi n}{7}\right]$$

$$\frac{7i}{4\pi n}\left[\cos\frac{2\pi n}{7}-\cos\frac{12\pi n}{7}+i\left(\text{sen}\frac{12\pi n}{7}-\text{sen}\frac{2\pi n}{7}\right)\right]$$

$$\frac{7i}{4\pi n}\left[\cancel{-2\,\text{sen}\,\pi n\,\text{sen}\frac{5\pi n}{7}}+i\left(2\,\text{sen}\frac{5\pi n}{7}\cos\pi n\right)\right]$$

$$\frac{7i}{4\pi n}\left[2i\,\text{sen}\frac{5\pi n}{7}\cos\pi n\right]=-\frac{7\,\text{sen}\frac{5\pi n}{7}\cos\pi n}{2\pi n}$$

$$C_n=\frac{\cancel{2}A}{\cancel{7}}\left(-\frac{\cancel{7}\,\text{sen}\frac{5\pi n}{7}\cos\pi n}{\cancel{2}\pi n}\right)=-\frac{A\,\text{sen}\frac{5\pi n}{7}\cos\pi n}{\pi n}$$

$$C_0=\frac{2A}{7}\left[\int_0^{1/2}dt+\int_3^{7/2}dt\right]=\frac{2A}{7}\left[t\Big|_0^{1/2}+t\Big|_3^{7/2}\right]$$

$$=\frac{2A}{7}\left[\frac{1}{2}+\frac{7}{2}-3\right]=\frac{2A}{7}\left[4-3\right]=\frac{2A}{7}$$

$$f(t)=\sum_{\substack{n=-\infty\\ n\neq 0}}^{n=\infty}-\frac{A\,\text{sen}\frac{5\pi n}{7}\cos\pi n}{\pi n}$$

$$|C_n|=\left|-\frac{A\,\text{sen}\frac{5\pi n}{7}\cos\pi n}{\pi n}\right|=\frac{A\,\text{sen}\frac{5\pi n}{7}\cos\pi n}{\pi n}$$

$$\Theta=\emptyset$$

$$4.2\quad f(t) = \begin{cases} 0 & -2 < t < -1 \\ A(t+1) & -1 < t < 0 \\ A(t-1) & 0 < t < 1 \\ 0 & 1 < t < 2 \end{cases} \qquad T = 4 \qquad \omega_0 = \frac{\pi}{2}$$

$$C_n = \frac{1}{4}\left[\int_{-1}^{0} A(t+1)e^{-in\omega_0 t}dt + \int_0^1 -A(t-1)e^{-in\omega_0 t}dt\right]$$

$$= \frac{A}{4}\left[\int_{-1}^{0}(t+1)e^{-in\omega_0 t}dt - \int_0^1 (t-1)e^{-in\omega_0 t}dt\right]$$

$$= \frac{A}{4}\left[\int_{-1}^{0} te^{-in\omega_0 t}dt + \int_{-1}^{0} e^{-in\omega_0 t}dt - \int_0^1 te^{-in\omega_0 t}dt + \int_0^1 e^{-in\omega_0 t}dt\right]$$

$$= \frac{A}{4}\left[\int_{-1}^{0} te^{-in\omega_0 t} - \int_0^1 te^{-in\omega_0 t}dt + \int_{-1}^{1} e^{-in\omega_0 t}dt\right]$$

$$\int te^{-in\omega_0 t}dt \quad \begin{matrix} u = t \\ du = dt \end{matrix} \quad v = -\frac{e^{-in\omega_0 t}}{in\omega_0} \quad dv = e^{-in\omega_0 t}dt$$

$$= -\frac{te^{-in\omega_0 t}}{in\omega_0} + \frac{1}{in\omega_0}\int e^{-in\omega_0 t}dt$$

$$\int e^{-in\omega_0 t}dt = -\frac{e^{-in\omega_0 t}}{in\omega_0} = \frac{ie^{-in\omega_0 t}}{n\omega_0} \quad *$$

$$\int te^{-in\omega_0 t}dt = \frac{ite^{-in\omega_0 t}}{n\omega_0} - \frac{i}{n\omega_0}\left(\frac{ie^{-in\omega_0 t}}{n\omega_0}\right)$$

$$= \frac{ie^{-in\omega_0 t}}{n\omega_0}\left(t - \frac{i}{n\omega_0}\right) = \frac{ie^{-in(\frac{\pi}{2})t}}{n(\frac{\pi}{2})}\left(t - \frac{i}{n(\frac{\pi}{2})}\right)$$

$$= \frac{2ie^{-in(\frac{\pi}{2})t}}{\pi n}\left(t - \frac{2i}{\pi n}\right)$$

$$\int_{-1}^{0} te^{-in\omega_0 t}dt = \frac{2i\cancel{e^{0}}^{1}}{\pi n}\left(0 - \frac{2i}{\pi n}\right) - \frac{2ie^{\frac{\pi i n}{2}}}{\pi n}\left(-1 - \frac{2i}{\pi n}\right)$$

$$= \frac{2i}{\pi n}\left(-\frac{2i}{\pi n}\right) + \frac{2ie^{\pi i n/2}}{\pi n}\left(\frac{\pi n + 2i}{\pi n}\right) = \frac{2i}{\pi n}\left[-\frac{2i}{\pi n} + e^{\frac{\pi i n}{2}}\left(1 + \frac{2i}{\pi n}\right)\right]$$

$$\int_0^1 te^{-in\omega_0 t}dt = \frac{2ie^{-in(\frac{\pi}{2})}}{\pi n}\left(1 - \frac{2i}{\pi n}\right) - \frac{2i\cancel{e^{0}}^{1}}{\pi n}\left(-\frac{2i}{\pi n}\right) =$$

$\operatorname{sen}\theta = \dfrac{e^{x}-e^{-x}}{2i}$

$$\int_{-1}^{1} e^{-in\omega_0 t}\,dt = \frac{ie^{-in\omega_0 t}}{n\omega_0}\Big|_{-1}^{1} = \frac{i}{n\omega_0}\left(e^{-in\omega_0}-e^{in\omega_0}\right)$$

$$= \frac{i}{n\omega_0}\left(\cos(-n\omega_0)+i\operatorname{sen}(-n\omega_0)-\cos(n\omega_0)-i\operatorname{sen}(n\omega_0)\right)$$

$$= \frac{i}{n\omega_0}\left(\cancel{\cos n\omega_0}-i\operatorname{sen}n\omega_0\cancel{-\cos n\omega_0}-i\operatorname{sen}n\omega_0\right)$$

$$= \frac{i}{n\omega_0}\left(-2i\operatorname{sen}n\omega_0\right) = \underline{\frac{2}{n\omega_0}\operatorname{sen}\frac{\pi n}{2}} = \frac{4}{\pi n}\operatorname{sen}\frac{\pi n}{2}$$

$$C_n = \frac{A}{4}\left[\frac{2i}{\pi n}\left(-\frac{2i}{\pi n}+e^{\pi i n/2}\left(1+\frac{2i}{\pi n}\right)\right)-\frac{2i}{\pi n}\left(\frac{2i}{\pi n}+e^{-\pi i n/2}\left(1-\frac{2i}{\pi n}\right)\right)+\frac{4}{\pi n}\operatorname{sen}\frac{\pi n}{2}\right]$$

$$= \frac{A}{4}\left[\frac{2i}{\pi n}\left(-\frac{2i}{\pi n}+e^{\pi i n/2}\left(1+\frac{2i}{\pi n}\right)-\frac{2i}{\pi n}-e^{-\pi i n/2}\left(1-\frac{2i}{\pi n}\right)\right)+\frac{4}{\pi n}\operatorname{sen}\frac{\pi n}{2}\right]$$

$$= \frac{A}{4}\left[\frac{2i}{\pi n}\left(-\frac{4i}{\pi n}+\left(\cos\frac{\pi n}{2}+i\operatorname{sen}\frac{\pi n}{2}\right)\left(1+\frac{2i}{\pi n}\right)-\left(\cos-\frac{\pi n}{2}+i\operatorname{sen}-\frac{\pi n}{2}\right)\left(1-\frac{2i}{\pi n}\right)\right)+\frac{4}{\pi n}\operatorname{sen}\frac{\pi n}{2}\right]$$

$$= \frac{A}{4}\left[\frac{2i}{\pi n}\left(-\frac{4i}{\pi n}+\left(\cos\frac{\pi n}{2}+i\operatorname{sen}\frac{\pi n}{2}\right)\left(1+\frac{2i}{\pi n}\right)-\left(\cos\frac{\pi n}{2}-i\operatorname{sen}\frac{\pi n}{2}\right)\left(1-\frac{2i}{\pi n}\right)\right)+\frac{4}{\pi n}\operatorname{sen}\frac{\pi n}{2}\right]$$

$$= \frac{A}{4}\left[\frac{2i}{\pi n}\left(-\frac{4i}{\pi n}\cancel{+\cos\frac{\pi n}{2}}+\frac{2i}{\pi n}\cos\frac{\pi n}{2}+i\operatorname{sen}\frac{\pi n}{2}\cancel{-\frac{2}{\pi n}\operatorname{sen}\frac{\pi n}{2}}\right.\right.$$
$$\left.\left.\cancel{-\cos\frac{\pi n}{2}}+\frac{2i}{\pi n}\cos\frac{\pi n}{2}+i\operatorname{sen}\frac{\pi n}{2}\cancel{+\frac{2}{\pi n}\operatorname{sen}\frac{\pi n}{2}}\right)+\frac{4}{\pi n}\operatorname{sen}\frac{\pi n}{2}\right)$$

$$= \frac{A}{4}\left[\frac{2i}{\pi n}\left(-\frac{4i}{\pi n}+\frac{4i}{\pi n}\cos+2i\operatorname{sen}\frac{\pi n}{2}\right)+\frac{4}{\pi n}\operatorname{sen}\frac{\pi n}{2}\right]$$

$$= \frac{A}{4}\left(\frac{2}{\pi n}\right)\left[i\left(-\frac{4i}{\pi n}+\frac{4i}{\pi n}\cos\frac{\pi n}{2}+2i\operatorname{sen}\frac{\pi n}{2}\right)+2\operatorname{sen}\frac{\pi n}{2}\right]$$

$$= \frac{A}{2\pi n}\left[\frac{4}{\pi n}-\frac{4}{\pi n}\cos\frac{\pi n}{2}\cancel{-2\operatorname{sen}\frac{\pi n}{2}+2\operatorname{sen}\frac{\pi n}{2}}\right] = \frac{A}{2\pi n}\left(\frac{4}{\pi n}\right)\left[1-\cos\frac{\pi n}{2}\right]$$

$$= \underline{\frac{2A}{(\pi n)^2}\left(1-\cos\frac{\pi n}{2}\right)}$$

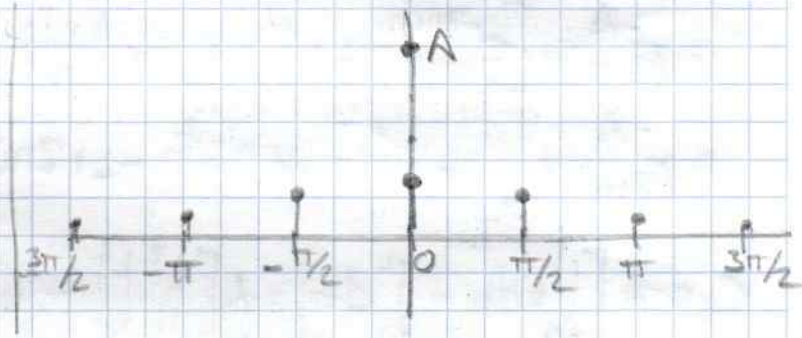
$$C_0 = \frac{A}{4}\left[\int_{-1}^{0} t\,dt-\int_{0}^{1} t\,dt+\int_{-1}^{1} dt\right] = \frac{A}{4}\left[\frac{t^2}{2}\Big|_{-1}^{0}-\frac{t^2}{2}\Big|_{0}^{1}+t\Big|_{-1}^{1}\right]$$

$$= \frac{A}{4}\left[\frac{1}{2}(0-1)-\frac{1}{2}(1)+(1+1)\right] = \frac{A}{4}\left[-\frac{1}{2}-\frac{1}{2}+2\right] = \frac{A}{4}\left[-1+2\right] = \underline{\frac{A}{4}}$$

$$f(t) = \frac{A}{4} + \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{2A}{(\pi n)^2}\left(1-\cos\frac{\pi n}{2}\right) e^{\frac{\pi i n}{2}t} = \frac{A}{4} + \frac{2A}{\pi^2}\sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{1-\cos\frac{\pi n}{2}}{n^2} e^{\frac{\pi i n}{2}t}$$

$$|C_n| = \frac{2A}{\pi^2}\left(\frac{1-\cos\frac{\pi n}{2}}{n^2}\right) \qquad \Theta = \emptyset \qquad n\omega = \frac{n\pi}{2}$$

n	$n\omega_0$	$\lvert C_n \rvert$	Θ
-3	$-3\pi/2$	$2A/9\pi^2$	0
-2	$-\pi$	A/π^2	0
-1	$-\pi/2$	$2A/\pi^2$	0
0	0	$A/4$	0
1	$\pi/2$	$2A/\pi^2$	0
2	π	A/π^2	0
3	$3\pi/2$	$2A/9\pi^2$	0



4.3 $f(t) = \begin{cases} -A(t-1), & 0<t<1 \\ f(t+T), & \text{otro caso} \end{cases} \qquad T=1 \qquad \omega_0 = 2\pi$

$$C_n = \int_0^1 -A(t-1)e^{-in\omega_0 t}\,dt = -A\left[\int_0^1 te^{-in\omega_0 t} - \int_0^1 e^{-in\omega_0 t}\,dt\right]$$

$$\int te^{-in\omega_0 t}\,dt \qquad \begin{matrix} u=t \\ du=dt \end{matrix} \qquad \begin{matrix} dv=e^{-in\omega_0 t}\,dt \\ v= \end{matrix} \qquad v=\frac{ie^{-in\omega_0 t}}{n\omega_0}$$

$$= \frac{ite^{-in\omega_0 t}}{n\omega_0} - \frac{i}{n\omega_0}\int e^{-in\omega_0 t}\,dt = \frac{ite^{-in\omega_0 t}}{n\omega_0} - \frac{i}{n\omega_0}\left(\frac{ie^{-in\omega_0 t}}{n\omega_0}\right)$$

$$= \frac{ite^{-in\omega_0 t}}{n\omega_0} + \frac{e^{-in\omega_0 t}}{(n\omega_0)^2} = \frac{in\omega_0 t e^{-in\omega_0 t} + e^{-in\omega_0 t}}{(n\omega_0)^2}$$

$$= \frac{e^{-in\omega_0 t}(in\omega_0 t+1)}{(n\omega_0)^2} = \frac{e^{-2\pi i n t}(2\pi i n t+1)}{(2\pi n)^2}$$

$$\int_0^1 te^{-in\omega_0 t} = \frac{1}{(2\pi n)^2}\left[e^{-2\pi i n}(2\pi i n+1) - e^0(0+1)\right] = \frac{1}{(2\pi n)^2}\left[e^{-2\pi i n}(2\pi i n+1)-1\right]$$

$$\int_0^1 e^{-in\omega_0 t} = \frac{ie^{-in\omega_0 t}}{n\omega_0}\Big|_0^1 = \frac{i}{n\omega_0}\left[e^{-in\omega_0} - e^0\right] = \frac{i}{n\omega_0}\left[e^{-in\omega_0}-1\right]$$

$$= \frac{i}{\pi}\left[e^{-2\pi i n}-1\right]$$

$$C_n = -A\left[\frac{1}{(2\pi n)^2}\left(e^{-2\pi i n}(2\pi i n+1)-1\right) - \frac{i}{2\pi n}\left(e^{-2\pi i n}-1\right)\right]$$

$$= -A\left[\frac{e^{-2\pi i n}(2\pi i n+1) - 1 - 2\pi i n\left(e^{-2\pi i n}-1\right)}{(2\pi n)^2}\right]$$

$$= -A\left[\frac{\cancel{2\pi i n e^{-2\pi i n}} + e^{-2\pi i n} - 1 - \cancel{2\pi i n e^{-2\pi i n}} + 2\pi i n}{(2\pi n)^2}\right]$$

$$= \frac{-A}{(2\pi n)^2}\left[e^{-2\pi i n} - 1 + 2\pi i n\right] = \frac{A}{(2\pi n)^2}\left[\cos(-2\pi n) + i\,\mathrm{sen}(-2\pi n) - 1 + 2\pi i n\right]$$

$$= \frac{-A}{(2\pi n)^2}\left[\cancel{\cos 2\pi n}^{\,1} - \cancel{i\,\mathrm{sen}\,2\pi n} - 1 + 2\pi i n\right] = \frac{A}{(2\pi n)^2}\left[2\pi i n\right] = \frac{Ai}{2\pi n}$$

$$C_0 = -A\left[\int_0^1 t\,dt - \int_0^1 dt\right] = A\left[\frac{t^2}{2}\Big|_0^1 - t\Big|_0^1\right] = A\left[\frac{1}{2}(1) - (1)\right] = A\left[\frac{1}{2} - 1\right]$$

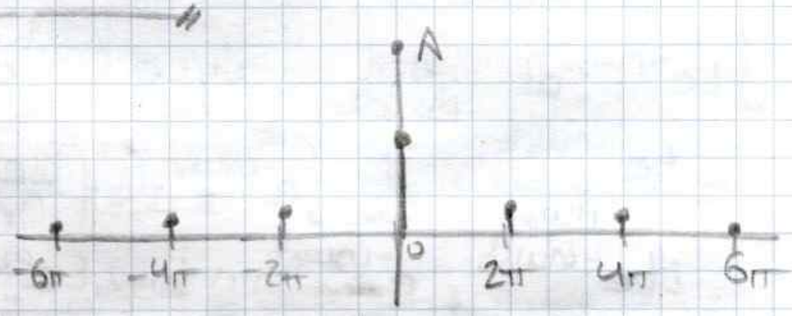
$$= \frac{A}{2}$$

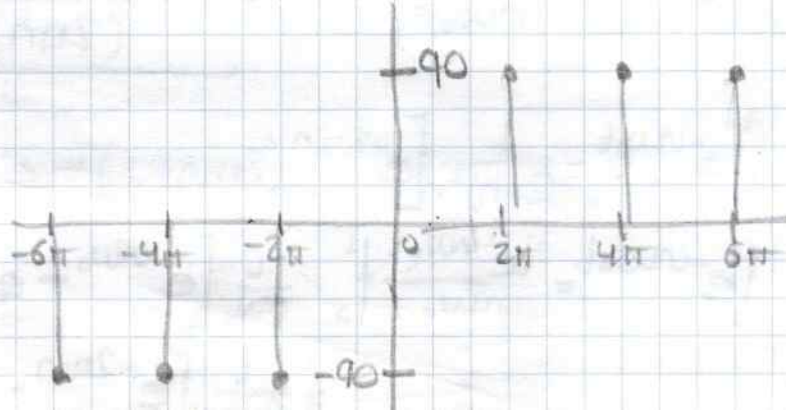
$$|C_n| = \frac{A}{2\pi n}$$

$$f(t) = \frac{A}{2} - \frac{Ai}{2\pi}\sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{e^{2\pi i n t}}{n}$$

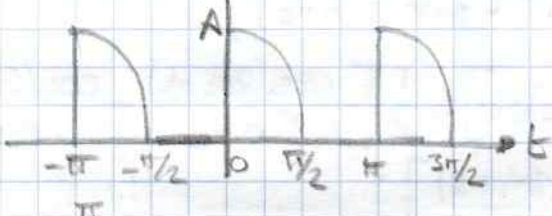
$$\Theta = \begin{cases} -90^\circ, & n<0 \\ +90, & n>0 \end{cases}$$

n	$n\omega_0$	$\lvert C_n \rvert$	Θ
-3	-6π	0.053A	-90
-2	-4π	0.08A	-90
-1	-2π	0.16A	-90
0	0	0.5A	
1	2π	0.16A	90
2	4π	0.08A	90
3	6π	0.053A	90





4d)



$$f(t) = \begin{cases} A|\cos t| & , 0<t<\pi/2 \\ 0 & , \pi/2<t<\pi \\ f(t+T) & , \text{otro caso} \end{cases}$$

$$T = \pi \qquad \omega_0 = \frac{2\pi}{\pi} = 2$$

$$C_n = \frac{1}{T}\int_0^T f(t)e^{-in\omega_0 t}dt = \frac{1}{\pi}\int_0^\pi f(t)e^{-in\omega_0 t}dt$$

$$= \frac{1}{\pi}\left(\int_0^{\pi/2} A|\cos t| e^{-2int}dt + \cancel{\int_{\pi/2}^{0} 0\, dt}\right) = \frac{A}{\pi}\int_0^{\pi/2} \cos t\, e^{-2int}dt$$

$$= \frac{A}{\pi}\int_0^{\pi/2} \cos t\left(\cos(-2nt) + i\,\text{sen}(-2nt)\right)dt =$$

$$= \frac{A}{\pi}\int_0^{\pi/2}(\cos t \cos 2nt - i\cos t\,\text{sen}\,2nt)dt$$

$$= \frac{A}{\pi}\left[\int_0^{\pi/2}\cos t\cos 2nt\,dt - i\int_0^{\pi/2}\cos t\,\text{sen}\,2nt\,dt\right] \quad \text{—} \; 1$$

$$\int \cos t \cos 2nt\,dt = \int \frac{1}{2}\left(\cos(t-2nt) + \cos(t+2nt)\right)dt$$

$$= \frac{1}{2}\left[\int \cos(t-2nt)dt + \int\cos(t+2nt)dt\right] = \frac{1}{2}\left[\frac{\text{sen}(t-2nt)}{1-2n} + \frac{\text{sen}(t+2nt)}{1+2n}\right]$$

$$= \left[\frac{(1+2n)\,\text{sen}(t-2nt) + (1-2n)\,\text{sen}(t+2nt)}{(1-2n)(1+2n)}\right]\frac{1}{2}$$

$$= \frac{1}{2(1-4n^2)}\Big[(1+2n)(\text{sen}\,t\cos 2nt - \cos t\,\text{sen}\,2nt) + (1-2n)(\text{sen}\,t\cos 2nt + \cos t\,\text{sen}\,2nt)\Big]\Big|_0^{\pi/2}$$

$$= \frac{1}{2(1-4n^2)}\Big[(1+2n)(\text{sen}\,\tfrac{\pi}{2}\cos n\pi - \cancel{\cos\tfrac{\pi}{2}\,\text{sen}\,n\pi}) + (1-2n)(\text{sen}\,\tfrac{\pi}{2}\cos n\pi + \cos\tfrac{\pi}{2}\,\text{sen}\,\pi n) - \cancel{(1+2n)(0) - (1-2n)(0)}\Big]$$

$$= \frac{1}{2(1-4n^2)}\Big[(1+2n)(\cos n\pi) + (1-2n)(\cos n\pi)\Big] = \frac{\cos n\pi}{2(1-4n^2)}\Big[1+\cancel{2n}+1-\cancel{2n}\Big]$$

$$= \frac{2\cos n\pi}{2(1-4n^2)} = \frac{\cos n\pi}{1-4n^2} \quad \text{I}$$

$$\int \cos t \,\text{sen}\, 2nt \, dt = \int \frac{1}{2}\left(\text{sen}(2nt-t) + \text{sen}(2nt+t)\right) dt$$

$$= \frac{1}{2}\left[\int \text{sen}(2nt-t)\,dt + \int \text{sen}(2nt+t)\,dt\right] = \frac{1}{2}\left[-\frac{\cos(2nt-t)}{2n-1} - \frac{\cos(2nt+t)}{2n+1}\right]$$

$$= -\frac{1}{2}\left[\frac{(2n+1)\cos(2nt-t) + (2n-1)\cos(2nt+t)}{\underset{(1-4n^2)(-1)}{4n^2-1}}\right]$$

$$= \frac{1}{2(1-4n^2)}\left[(2n+1)\cos(2nt-t) + (2n-1)\cos(2nt+t)\right]$$

$$= \frac{1}{2(1-4n^2)}\left[(2n+1)(\cos 2nt\cos t + \text{sen}\,2nt\,\text{sen}\,t) + (2n-1)(\cos 2nt \cos t - \text{sen}\,2nt\,\text{sen}\,t)\right]\Big|_0^{\pi/2}$$

$$= \frac{1}{2(1-4n^2)}\Big[\cancel{(2n+1)(\cos n\pi \cos \tfrac{\pi}{2} + \text{sen}\,n\pi\,\text{sen}\,\tfrac{\pi}{2}) + (2n-1)(\cos n\pi \cos\tfrac{\pi}{2} - \text{sen}\,n\pi\,\text{sen}\,\tfrac{\pi}{2})}$$
$$- (2n+1)(\cos 0 \cos 0 + \text{sen}\,0\,\text{sen}\,0) - (2n-1)(\cos 0 \cos 0 - 0)\Big]$$

$$= \frac{1}{2(1-4n^2)}\left[-(2n+1)(1) - (2n-1)(1)\right] = \ast\left[-2n - \cancel{1} - 2n + \cancel{1}\right] = \frac{1}{2(1-4n^2)}(-4n)$$

$$= \frac{-2n}{1-4n^2} = -\frac{2n}{1-4n^2} \quad \text{— (II)}$$

Sustituyendo (I) y (II) en (1),

$$C_n = \frac{A}{\pi}\left[\frac{\cos n\pi}{1-4n^2} + \frac{2in}{1-4n^2}\right] = \frac{A\cos n\pi}{\pi(1-4n^2)} + \frac{2Ain}{\pi(1-4n^2)}$$

$$C_0 = \frac{A}{\pi}\int_0^{\pi/2} \cos t \, dt = \frac{A}{\pi}(\text{sen}\, t)\Big|_0^{\pi/2} = \frac{A}{\pi}\left(\text{sen}\,\frac{\pi}{2} - \cancel{\text{sen}\,0}\right) = \frac{A}{\pi}$$

$$f(t) = \frac{A}{\pi(1-4n^2)} \sum_{n=-\infty}^{\infty} (\cos n\pi + 2in)\, e^{2int}$$

$$|C_n| = \sqrt{\left(\frac{A\cos n\pi}{\pi(1-4n^2)}\right)^2 + \left(\frac{2An}{\pi(1-4n^2)}\right)^2} = \sqrt{\frac{(A\cos n\pi)^2 + (2An)^2}{(\pi(1-4n^2))^2}}$$

$$= \frac{\sqrt{(A\cos n\pi)^2 + (2An)^2}}{|\pi(1-4n^2)|}$$

[illegible]

5; Obtén las transformadas de Fourier de las sig. funciones

a) $f(t)=\begin{cases} -m(x-1)-3 & , -2<t<-1.5 \\ m(x+1) & , -1.5<t<0 \\ -m(x-1) & , 0<t<1.5 \\ m(x+1)-3 & , 1.5<t<2 \end{cases}$

$$\mathcal{F}\{f(t)\}=\int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt$$

$$=\int_{-2}^{-3/2}(-m(t-1)-3)e^{-i\omega t}dt+\int_{-3/2}^{0}m(t+1)e^{-i\omega t}dt$$

$$+\int_{0}^{3/2}-m(t-1)e^{-i\omega t}dt+\int_{3/2}^{2}(m(t+1)-3)e^{-i\omega t}dt$$

(1) $$\int_{-2}^{-3/2}-(m(t-1)+3)e^{-i\omega t}dt=-\int(mt-m+3)e^{-i\omega t}dt$$

$$=-\int mte^{-i\omega t}dt+\int me^{-i\omega t}dt-\int 3e^{-i\omega t}dt$$

$$=-m\int_{-2}^{-3/2}te^{-i\omega t}dt+m\int_{-2}^{-3/2}e^{-i\omega t}dt-3\int_{-2}^{-3/2}e^{-i\omega t}dt$$

$\int te^{-i\omega t}dt$ $\quad u=t \quad du=dt \quad dv=e^{-i\omega t}dt \quad v=-\frac{e^{-i\omega t}}{i\omega}=\frac{ie^{-i\omega t}}{\omega}$ — (A)

$$=\frac{ite^{-i\omega t}}{\omega}-\frac{i}{\omega}\int e^{-i\omega t}dt=\frac{ite^{-i\omega t}}{\omega}-\frac{i}{\omega}\left(\frac{ie^{-i\omega t}}{\omega}\right)$$

$$=\frac{ite^{-i\omega t}}{\omega}+\frac{e^{-i\omega t}}{\omega^2}=\frac{i\omega te^{-i\omega t}+e^{-i\omega t}}{\omega^2}=\frac{i\omega t+1}{\omega^2}e^{-i\omega t}$$ — (B)

$$\int_{-2}^{-3/2}te^{-i\omega t}dt=\frac{1}{\omega^2}\left[(i\omega(-\tfrac{3}{2})+1)e^{-i\omega(-\frac{3}{2})}-(i\omega(-2)+1)e^{-i\omega(-2)}\right]$$

$$=\frac{1}{\omega^2}\left[(-\tfrac{3}{2}i\omega+1)e^{\frac{3i\omega}{2}}-(-2i\omega+1)e^{2i\omega}\right]$$

$$\int_{-2}^{-3/2} e^{-i\omega t}\,dt = \frac{i}{\omega} e^{-i\omega t}\Big|_{-2}^{-3/2} = \frac{i}{\omega}\left(e^{\frac{3i\omega}{2}} - e^{2i\omega}\right)$$

$$\int_{-2}^{-3/2} [-m(t-1)-3 = -\frac{m}{\omega^2}\left[\left(-\frac{3}{2}i\omega+1\right)e^{\frac{3i\omega}{2}} - (-2i\omega+1)e^{2i\omega}\right]$$

$$+ \frac{im}{\omega}\left(e^{3i\omega/2} - e^{2i\omega}\right) - \frac{3i}{\omega}\left(e^{3i\omega/2} - e^{2i\omega}\right)$$

$$= -\frac{m}{\omega^2}\left[-\frac{3}{2}i\omega e^{\frac{3i\omega}{2}} + e^{\frac{3i\omega}{2}} + 2i\omega e^{2i\omega} - e^{2i\omega}\right] + *$$

$$= \frac{3m}{2\omega^2} i\omega e^{\frac{3i\omega}{2}} - \frac{m}{\omega^2}e^{\frac{3i\omega}{2}} - \frac{2m}{\omega^2}i\omega e^{2i\omega} + \frac{m}{\omega^2}e^{2i\omega}$$

$$+ \frac{im}{\omega}e^{\frac{3i\omega}{2}} - \frac{im}{\omega}e^{2i\omega} - \frac{3i}{\omega}e^{3i\omega/2} + \frac{3i}{\omega}e^{2i\omega}$$

$$= e^{\frac{3i\omega}{2}}\left[\frac{3im}{2\omega} - \frac{m}{\omega^2} + \frac{im}{\omega} - \frac{3i}{\omega}\right] + e^{2i\omega}\left[\frac{2im}{\omega} + \frac{m}{\omega^2} - \frac{im}{\omega} + \frac{3i}{\omega}\right]$$

$$= e^{3i\omega/2}\left[\frac{-2m + 3im\omega + 2im\omega - 6i\omega}{2\omega^2}\right]$$

$$+ e^{2i\omega}\left[\frac{m + 2im\omega - im\omega + 3i\omega}{\omega^2}\right]$$

$$= e^{\frac{3i\omega}{2}}\left[\frac{5im\omega - 6i\omega - 2m}{2\omega^2}\right] + e^{2i\omega}\left[\frac{im\omega + m + 3i\omega}{\omega^2}\right]$$

$$= e^{3i\omega}\left[\frac{-i\omega - 2}{2\omega^2}\right] + e^{2i\omega}\left[\frac{4i\omega + 1}{\omega^2}\right]$$

(2) $$\int m(t+1)e^{-i\omega t}\,dt = \int_{-3/2}^{0} t e^{-i\omega t}\,dt + \int_{-3/2}^{0} e^{-i\omega t}\,dt$$

$$\int_{-3/2}^{0} t e^{-i\omega t}\,dt = \frac{i\omega t + 1}{\omega^2}e^{-i\omega t}\Big|_{-3/2}^{0} = \frac{1}{\omega^2}\left[1 - \left(i\omega\left(-\frac{3}{2}\right)+1\right)e^{-i\omega\left(-\frac{3}{2}\right)}\right]$$

$$= \frac{1}{\omega^2}\left[1 - \left(-\frac{3}{2}i\omega + 1\right)e^{3i\omega/2}\right] \quad - 1$$

$-mt+m = -t+1$

$$\int_{3/2}^{0} e^{-i\omega t}dt = \frac{i}{\omega}\left(1-e^{\frac{3i\omega t}{2}}\right)$$

$$\int(t+1)e^{-i\omega t}dt = \frac{1}{\omega^2}\left[1+\frac{3}{2}i\omega e^{\frac{3i\omega}{2}}-e^{\frac{3i\omega}{2}}\right]+\frac{i}{\omega}\left(1-e^{3i\omega t/2}\right)$$

$$=\frac{1}{\omega^2}+\frac{3}{2\omega}ie^{\frac{3i\omega}{2}}-\frac{1}{\omega^2}e^{\frac{3i\omega}{2}}+\frac{i}{\omega}-e^{3i\omega t/2}\left(\frac{i}{\omega}\right)$$

$$=e^{\frac{3i\omega}{2}}\left(\frac{3i}{2\omega}-\frac{1}{\omega^2}-\frac{i}{\omega}\right)+\frac{i}{\omega}+\frac{1}{\omega^2}=$$

$$=e^{\frac{3i\omega}{2}}\left(\frac{3i\omega-2-2i\omega}{2\omega^2}\right)+\frac{i\omega+1}{\omega^2}=e^{\frac{3i\omega}{2}}\left(\frac{i\omega-2}{2\omega^2}\right)+\frac{i\omega+1}{\omega^2} - (2)$$

$$\int_0^{3/2}-m(t-1)e^{-i\omega t}dt = -\int te^{-i\omega t}dt+\int e^{-i\omega t}dt$$

$$\int_0^{3/2} te^{-i\omega t}dt = \frac{1}{\omega^2}\left[\left(\frac{3}{2}i\omega+1\right)e^{-\frac{3i\omega}{2}}-1\right]$$

$$\int_0^{3/2} e^{-i\omega t}dt = \frac{i}{\omega}\left(e^{-\frac{3i\omega}{2}}-1\right)$$

$$-\int_0^{3/2}(t-1)e^{-i\omega t} = -\frac{1}{\omega^2}\left[\frac{3}{2}i\omega e^{-\frac{3i\omega}{2}}+e^{-\frac{3i\omega}{2}}-1\right]+\frac{i}{\omega}\left(e^{-\frac{3i\omega}{2}}-1\right)$$

$$=-\frac{3i}{2\omega}e^{-\frac{3i\omega}{2}}-\frac{1}{\omega^2}e^{-\frac{3i\omega}{2}}+\frac{1}{\omega^2}+\frac{i}{\omega}e^{-3i\omega/2}-\frac{i}{\omega}$$

$$=e^{-3i\omega/2}\left(-\frac{3i}{2\omega}-\frac{1}{\omega^2}+\frac{i}{\omega}\right)+\frac{1}{\omega^2}-\frac{i}{\omega}$$

$$=e^{-\frac{3i\omega}{2}}\left(\frac{-3i\omega-2+2\omega i}{2\omega^2}\right)+\frac{1-i\omega}{\omega^2}=-e^{-\frac{3i\omega}{2}}\left(\frac{i\omega+2}{2\omega^2}\right)+\frac{1-i\omega}{\omega^2} - (3)$$

$$\int_{3/2}^{2}((t+1)-3)e^{-i\omega t}dt = \int te^{-i\omega t}dt-2\int e^{-i\omega t}dt$$

$$\int_{3/2}^{2} te^{-i\omega t}dt = \frac{i}{\omega^2}\left[(2i\omega+1)e^{-2i\omega}-\left(\frac{3}{2}i\omega+1\right)e^{-\frac{3i\omega}{2}}\right]$$

$$\int_{3/2}^{2} e^{-i\omega t}dt = \frac{i}{\omega}\left(e^{-2i\omega}-e^{-\frac{3}{2}i\omega t}\right)$$

$$\int_{3/2}^{2}(t-2)e^{-i\omega t}\,dt = \frac{1}{\omega^2}\left[(2i\omega+1)e^{-2i\omega} - \left(\frac{3}{2}i\omega+1\right)e^{-\frac{3i\omega}{2}}\right] - \frac{2i}{\omega}\left(e^{-2i\omega} - e^{-\frac{3}{2}i\omega}\right)$$

$$= \frac{1}{\omega^2}\left[2i\omega e^{-2i\omega} + e^{-2i\omega} - \frac{3}{2}i\omega e^{-\frac{3i\omega}{2}} - e^{-\frac{3i\omega}{2}}\right] -$$

$$= \frac{2i}{\omega}e^{-2i\omega} + \frac{1}{\omega^2}e^{-2i\omega} - \underline{\frac{3i}{2\omega}e^{-\frac{3i\omega}{2}}} - \underline{\frac{1}{\omega^2}e^{-\frac{3i\omega}{2}}} - \frac{2i}{\omega}e^{-2i\omega} + \underline{\frac{2i}{\omega}e^{-\frac{3}{2}i\omega}}$$

$$= e^{-\frac{3i\omega}{2}}\left(-\frac{3i}{2\omega} - \frac{1}{\omega^2} + \frac{2i}{\omega}\right) + e^{-2i\omega}\left(\cancel{\frac{2i}{\omega}} + \frac{1}{\omega^2} - \cancel{\frac{2i}{\omega}}\right) \quad (4)$$

$$= e^{-\frac{3i\omega}{2}}\left(\frac{-3i\omega - 2 + 4i\omega}{2\omega^2}\right) + e^{-2i\omega}\left(\frac{1}{\omega^2}\right) = e^{-\frac{3i\omega}{2}}\left(\frac{i\omega-2}{\omega^2}\right) + e^{-2i\omega}\left(\frac{1}{\omega^2}\right)$$

Sumando (1), (2), (3) y (4)

$$= \cancel{-e^{\frac{3i\omega}{2}}\left[\frac{i\omega+2}{2\omega^2}\right]} + e^{2i\omega}\left[\frac{4i\omega+1}{\omega^2}\right] + \cancel{e^{3i\omega/2}\left(\frac{i\omega-2}{2\omega^2}\right)} + \frac{i\omega+1}{\omega^2}$$

$$\cancel{-e^{-\frac{3i\omega}{2}}\left(\frac{i\omega+2}{2\omega^2}\right)} + \frac{1-i\omega}{\omega^2} + \cancel{e^{-\frac{3i\omega}{2}}\left(\frac{i\omega-2}{2\omega^2}\right)} + e^{-2i\omega}\left(\frac{1}{\omega^2}\right)$$

$$= e^{-\frac{3i\omega}{2}}\left(-\frac{i\omega+2}{2\omega^2} + \frac{i\omega-2}{2\omega^2}\right) + e^{2i\omega}\left(\frac{4i\omega+1}{\omega^2}\right) - e^{-2i\omega}\left(\frac{1}{\omega^2}\right) + \frac{1-i\omega}{\omega^2} + \frac{i\omega}{\omega}$$

$$= e^{-3i\omega}\left(\frac{-2}{\omega^2}\right) + e^{2i\omega}\left(\frac{4i\omega+1}{\omega^2}\right) - e^{-2i\omega}\left(\frac{1}{\omega^2}\right) + \frac{2}{\omega^2}$$

5b) $f(t)=\begin{cases} A\cos t, & -\frac{\pi}{2}<t<\frac{\pi}{2} \\ 0 & , \text{o.c} \end{cases}$

$$\mathcal{F}\{f(t)\}=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} A\cos t\, e^{-i\omega t}dt = A\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos t\,(\cos\omega t - i\,\text{sen}\,\omega t)dt$$

$$= A\left[\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos t\cos\omega t\, dt - i\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos t\,\text{sen}\,\omega t\, dt\right]$$

$$\int \cos t\cos\omega t\, dt = \int \frac{1}{2}(\cos(t-\omega t)+\cos(t+\omega t))dt$$

$$=\frac{1}{2}\left[\int \cos(t-\omega t)+\int\cos(t+\omega t)dt\right] = \frac{1}{2}\left[\frac{\text{sen}(t-\omega t)}{1-\omega}+\frac{\text{sen}(t+\omega t)}{1+\omega}\right]$$

$$=\frac{1}{2}\left[\frac{(1+\omega)\text{sen}(t-\omega t)+(1-\omega)\text{sen}(t+\omega t)}{(1-\omega^2)}\right]$$

$$=\frac{1}{2(1-\omega^2)}\left[(1+\omega)(\text{sen}\,t\cos\omega t-\cos t\,\text{sen}\,\omega t)+(1-\omega)(\text{sen}\,t\cos\omega t+\cos t\,\text{sen}\,\omega t)\right]$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos t\cos\omega t\, dt =$$

$$\frac{1}{2(1-\omega^2)}\Big[(1+\omega)(\overset{1}{\cancel{\text{sen}\tfrac{\pi}{2}}}\cos\omega\tfrac{\pi}{2}-\cancel{\cos\tfrac{\pi}{2}\text{sen}\,\omega\tfrac{\pi}{2}})+(1-\omega)(\overset{1}{\cancel{\text{sen}\tfrac{\pi}{2}}}\cos\omega\tfrac{\pi}{2}+\cancel{\cos\tfrac{\pi}{2}\text{sen}\,\omega\tfrac{\pi}{2}})$$
$$-(1+\omega)(-\overset{1}{\cancel{\text{sen}\tfrac{\pi}{2}}}\cos\omega\tfrac{\pi}{2}+\cancel{\cos\tfrac{\pi}{2}\text{sen}\,\omega\tfrac{\pi}{2}})-(1-\omega)(-\overset{1}{\cancel{\text{sen}\tfrac{\pi}{2}}}\cos\omega\tfrac{\pi}{2}-\cancel{\cos\tfrac{\pi}{2}\text{sen}\,\omega\tfrac{\pi}{2}})\Big]$$

$$=\frac{1}{2(1-\omega^2)}\left[(1+\omega)(\cos\omega\tfrac{\pi}{2})+(1-\omega)(\cos\omega\tfrac{\pi}{2})+(1+\omega)(\cos\omega\tfrac{\pi}{2})+(1-\omega)(\cos\omega\tfrac{\pi}{2})\right]$$

$$=\frac{1}{2(1-\omega^2)}\left[\cos\frac{\pi\omega}{2}(1+\cancel{\omega}+1-\cancel{\omega}+1+\cancel{\omega}+1-\cancel{\omega})\right]=\frac{1}{2(1-\omega^2)}\left[4\cos\frac{\pi\omega}{2}\right]$$

$$=\frac{2\cos\frac{\pi\omega}{2}}{1-\omega^2}$$

$$\int \cos t\,\text{sen}\,\omega t\, dt=\int\frac{1}{2}(\text{sen}(\omega t-t)+\text{sen}(\omega t+t))dt$$

$$=\frac{1}{2}\left[\int\text{sen}(\omega t-t)\,dt+\int\text{sen}(\omega t+t)\,dt\right]=\frac{1}{2}\left(-\frac{\cos(\omega t-t)}{\omega-1}-\frac{\cos(\omega t+t)}{\omega+1}\right)$$

$$=-\frac{1}{2}\left(\frac{(\omega+1)\cos(\omega t-t)+(\omega-1)\cos(\omega t+t)}{-(1-\omega^2)}\right)$$

$$=\frac{1}{2}\left[\frac{(\omega+1)\cos(\omega t-t)+(\omega-1)\cos(\omega t+t)}{1-\omega^2}\right]\Bigg|^{\frac{\pi}{2}}$$

$$(\omega+1)\left(\operatorname{sen}\frac{\omega\pi}{2}\right)-(\omega-1)\left(\operatorname{sen}\frac{\omega\pi}{2}\right)-(\omega+1)\left(-\operatorname{sen}\frac{\omega\pi}{2}\right)+(\omega-1)($$

$$=\frac{1}{2(1-\omega^2)}\left[(\omega+1)(\cos\omega t\cos t+\operatorname{sen}\omega t\operatorname{sen}t)+(\omega-1)(\cos\omega t\cos t-\operatorname{sen}\omega t\operatorname{sen}t)\right]\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$=\frac{1}{2(1-\omega^2)}\Big[\omega\cos\omega t\cos t+\cancel{\omega\operatorname{sen}\omega t\operatorname{sen}t}+\cancel{\cos\omega t\cos t}+\operatorname{sen}\omega t\operatorname{sen}t$$
$$+\omega\cos\omega t\cos t-\cancel{\omega\operatorname{sen}\omega t\operatorname{sen}t}-\cancel{\cos\omega t\cos t}+\operatorname{sen}\omega t\operatorname{sen}t\Big]\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

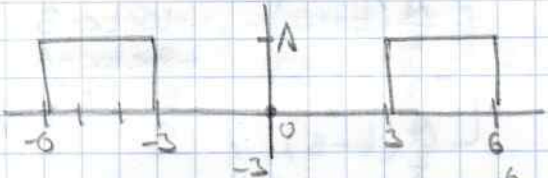
$$=\frac{1}{\cancel{2}(1-\omega^2)}\left[\cancel{2}\omega\cos\omega t\cos t+\cancel{2}\operatorname{sen}\omega t\operatorname{sen}t\right]\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$=\frac{1}{1-\omega^2}\left[\cancel{\omega\cos\frac{\pi\omega}{2}\cos\frac{\pi}{2}}+\cancel{\operatorname{sen}\frac{\pi\omega}{2}\operatorname{sen}\frac{\pi}{2}}-\cancel{\omega\cos\frac{\pi\omega}{2}\cos\frac{\pi}{2}}-\cancel{\operatorname{sen}\frac{\pi\omega}{2}\operatorname{sen}\frac{\pi}{2}}\right]$$

$=0$

$$\mathfrak{F}\{f(t)\}=A\left[\frac{2\cos\frac{\pi\omega}{2}}{1-\omega^2}\right]$$

5c



$$f(t) = \begin{cases} A & ; -6 < t < -3 \\ A & ; 3 < t < 6 \\ 0 & ; \text{o.c.} \end{cases}$$

$$\mathcal{F}\{f(t)\} = \int_{-6}^{-3} A e^{-i\omega t} dt + \int_{3}^{6} A e^{-i\omega t} dt = A\left[\int_{-6}^{-3} e^{-i\omega t} dt + \int_{3}^{6} e^{-i\omega t} dt\right]$$

$$\int e^{-i\omega t} dt = \frac{e^{-i\omega t}}{-i\omega} = \frac{i e^{-i\omega t}}{\omega}$$

$$\left.\frac{i e^{-i\omega t}}{\omega}\right|_{-6}^{-3} = \frac{i}{\omega}\left(e^{3i\omega} - e^{6i\omega}\right) \qquad \left.\frac{i e^{-i\omega t}}{\omega}\right|_{3}^{6} = \frac{i}{\omega}\left(e^{-6i\omega} - e^{-3i\omega}\right)$$

$$= A\left[\frac{i}{\omega}\left(e^{3i\omega} - e^{6i\omega}\right) + \frac{i}{\omega}\left(e^{-6i\omega} - e^{-3i\omega}\right)\right]$$

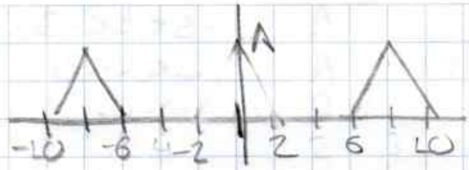
$$= \frac{Ai}{\omega}\left[\cos 3\omega + i\,\text{sen}\,3\omega - \cos 6\omega - i\,\text{sen}\,6\omega + \cos(-6\omega) + i\,\text{sen}(-6\omega) - \cos(-3\omega) - i\,\text{sen}(-3\omega)\right]$$

$$= \frac{Ai}{\omega}\left[\cancel{\cos 3\omega} + i\,\text{sen}\,3\omega - \cancel{\cos 6\omega} - i\,\text{sen}\,6\omega + \cancel{\cos 6\omega} - i\,\text{sen}\,6\omega - \cancel{\cos 3\omega} + i\,\text{sen}\,3\omega\right]$$

$$= \frac{Ai}{\omega}\left[2i\,\text{sen}\,3\omega - 2i\,\text{sen}\,6\omega\right] = -\frac{A}{\omega}\left[2\,\text{sen}\,3\omega - 2\,\text{sen}\,6\omega\right] = \frac{2A}{\omega}\left[\text{sen}\,3\omega - \text{sen}\,6\omega\right]$$

$$\frac{A}{-8+10}=\frac{A}{2} \quad \frac{-A}{-6+8}=-\frac{A}{2} \quad \frac{A}{8-6}=\frac{A}{2} \quad \frac{-A}{10-8}=-\frac{A}{2}$$

5e;



$$f(t)=\begin{cases} \frac{A}{2}(t+10) & , -10<t<-8 \\ -\frac{A}{2}(t+6) & , -8<t<-6 \\ \frac{A}{2}(t-6) & , 6<t<8 \\ -\frac{A}{2}(t+10) & , 8<t<10 \end{cases} \quad , 0, \text{o.c}$$

$$\mathcal{F}=\int_{-10}^{-8}\frac{A}{2}(t+10)e^{-i\omega t}dt+\int_{-8}^{-6}-\frac{A}{2}(t+6)e^{-i\omega t}dt+\int_{6}^{8}\frac{A}{2}(t-6)e^{-i\omega t}dt+\int_{8}^{10}\frac{A}{2}(t+10)e^{-i\omega t}$$

$$=\frac{A}{2}\left[\int_{-10}^{-8}(t+10)e^{-i\omega t}dt-\int_{-8}^{-6}(t+6)e^{-i\omega t}dt+\int_{6}^{8}(t-6)e^{-i\omega t}-\int_{8}^{10}(t+10)e^{-i\omega t}dt\right]$$

$$=\frac{A}{2}\left[\int_{-10}^{-8}te^{-i\omega t}dt+10\int_{-10}^{-8}e^{-i\omega t}dt-\int_{-8}^{-6}te^{-i\omega t}dt-6\int_{-8}^{-6}e^{-i\omega t}dt+\int_{6}^{8}te^{-i\omega t}-6\int_{6}^{8}e^{-i\omega t}dt-\int_{8}^{10}te^{-i\omega t}+10\int_{8}^{10}e^{-i\omega t}dt\right.$$

$$\int te^{-i\omega t}dt \qquad u=-i\omega t \qquad t=-\frac{u}{i\omega}=\frac{iu}{\omega} \qquad dt=\frac{i}{\omega}du$$

$$\rightarrow \int\left(\frac{iu}{\omega}\right)e^{u}\left(\frac{i}{\omega}du\right)=\frac{i^2}{\omega^2}\int ue^{u}du=-\frac{1}{\omega^2}\int ue^{u}du$$

$$=-\frac{1}{\omega^2}(u-1)e^{u}=-\frac{1}{\omega^2}(-i\omega t-1)e^{-i\omega t}$$

$$=\frac{i\omega t+1}{\omega^2}e^{-i\omega t}$$

$$\int e^{-i\omega t}dt=-\frac{e^{-i\omega t}}{i\omega}=\frac{ie^{-i\omega t}}{\omega}$$

$$e^{i\theta}=\cos\theta+i\,\mathrm{sen}\,\theta$$
$$e^{-i\theta}=\cos\theta-i\,\mathrm{sen}\,\theta$$
$$e^{i\theta}+e^{-i\theta}=2\cos\theta$$
$$e^{i\theta}-e^{-i\theta}=2i\,\mathrm{sen}\,\theta$$

Ahora, a evaluar

$$\int_{-10}^{-8}te^{-i\omega t}dt=\frac{1}{\omega^2}\left[(-8i\omega+1)e^{8i\omega}-(-10i\omega+1)e^{10i\omega}\right]$$

$$\int_{8}^{10}=\frac{1}{\omega^2}\left[(10i\omega+1)e^{-10i\omega}-(8i\omega+1)e^{-8i\omega}\right]$$

$$=\frac{1}{\omega^2}\left[-8i\omega e^{8i\omega}+e^{8i\omega}+10i\omega e^{10i\omega}-e^{10i\omega}\right]$$

$$-\frac{1}{\omega^2}\left[-8i\omega e^{-8i\omega}-e^{-8i\omega}+10i\omega e^{-10i\omega}+e^{-10i\omega}\right]$$

$$\int_{-10}^{-8}te^{-i\omega t}dt-\int_{8}^{10}te^{-i\omega t}dt=\frac{1}{\omega^2}\left[-8i\omega(2i\,\mathrm{sen}\,8\omega)+2\cos 8\omega+10i\omega(2i\,\mathrm{sen}\,10\omega)-2\cos 10\omega\right]$$

$$=\frac{2}{}\left[8\omega\,\mathrm{sen}\,8\omega+\cos 8\omega-10\omega\,\mathrm{sen}\,10\omega-\cos 10\omega\right] \rightarrow (1)$$

$$10\int_{-10}^{-8} e^{-i\omega t}\,dt = \frac{10i}{\omega}\left(e^{8i\omega} - e^{10i\omega}\right) \qquad 10\int_{8}^{10} dt = \frac{10i}{\omega}\left(e^{-10i\omega} - e^{-8i\omega}\right)$$

$$10\int_{-10}^{-8} + 10\int_{8}^{10} = \frac{10i}{\omega}\left(2i\,\text{sen}\,8\omega - 2i\,\text{sen}\,10\omega\right) = -\frac{20}{\omega}\left(\text{sen}\,8\omega - \text{sen}\,10\omega\right) \quad \text{— (2)}$$

$$\int_{6}^{8} te^{-i\omega t}\,dt = \frac{1}{\omega^2}\left[(8i\omega+1)e^{-8i\omega} - (6i\omega+1)e^{-6i\omega}\right]$$

$$= \frac{1}{\omega^2}\left[8i\omega e^{-8i\omega} + e^{-8i\omega} - 6i\omega e^{-6i\omega} - e^{-6i\omega}\right]$$

$$\int_{-8}^{-6} = \frac{1}{\omega^2}\left[(-6i\omega+1)e^{6i\omega} - (-8i\omega+1)e^{8i\omega}\right]$$

$$= \frac{1}{\omega^2}\left[8i\omega e^{8i\omega} - e^{8i\omega} - 6i\omega e^{6i\omega} + e^{6i\omega}\right]$$

$$\int_{6}^{8} - \int_{-8}^{-6} = \frac{1}{\omega^2}\left[-8i\omega(2i\,\text{sen}\,8\omega) + 2\cos 8\omega + 6i\omega(2i\,\text{sen}\,6\omega) - 2\cos 6\omega\right]$$

$$= \frac{2}{\omega^2}\left[8\omega\,\text{sen}\,8\omega + \cos 8\omega - 6\omega\,\text{sen}\,6\omega - \cos 6\omega\right] \quad \text{— (3)}$$

$$-6\int_{-8}^{-6} e^{-i\omega t}\,dt = -\frac{6i}{\omega}\left(e^{6i\omega} - e^{8i\omega}\right) \qquad -6\int_{6}^{8} e^{-i\omega t}\,dt = -\frac{6i}{\omega}\left(e^{-8i\omega} - e^{-6i\omega}\right)$$

$$-6\int_{-8}^{-6} + \left(-6\int_{6}^{8}\right) = -\frac{6i}{\omega}\left(2i\,\text{sen}\,6\omega - 2i\,\text{sen}\,8\omega\right) = \frac{12}{\omega}\left(\text{sen}\,6\omega - \text{sen}\,8\omega\right) \quad \text{— (4)}$$

Sumando (1) y (3) y multiplicando por $\frac{A}{2}$

$$\text{(I)} — = \frac{A}{\omega^2}\left[16\omega\,\text{sen}\,8\omega + 2\cos 8\omega - 10\omega\,\text{sen}\,10\omega - 6\omega\,\text{sen}\,6\omega - \cos 10\omega - \cos 6\omega\right]$$

Sumando (2) y (4) y multiplicando por $\frac{A}{2}$

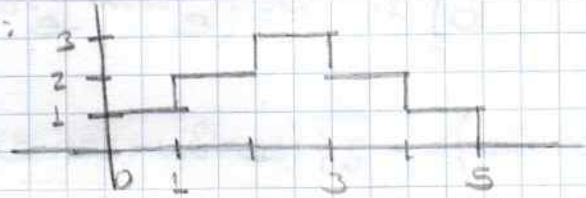
$$= -\frac{10A}{\omega}\left(\text{sen}\,8\omega - \text{sen}\,10\omega\right) + \frac{6A}{\omega}\left(\text{sen}\,6\omega - \text{sen}\,8\omega\right)$$

$$= \frac{A}{\omega}\left(-10\,\text{sen}\,8\omega + 10\,\text{sen}\,10\omega + 6\,\text{sen}\,6\omega - 6\,\text{sen}\,8\omega\right)$$

$$\text{(II)} \quad = \frac{A}{\omega}\left(-16\,\text{sen}\,8\omega + 10\,\text{sen}\,10\omega + 6\,\text{sen}\,6\omega\right)$$

La suma de (I) y (II) es el resultado final

5f.



$$f(t) = \begin{cases} 1, & 0<t<1 \\ 2, & 1<t<2 \\ 3, & 2<t<3 \\ 2, & 3<t<4 \\ 1, & 4<t<5 \\ 0, & \text{o.c.} \end{cases}$$

$$\mathcal{F}\{f(t)\} = \int_0^1 1e^{-i\omega t}dt + \int_1^2 2e^{-i\omega t} + \int_2^3 3e^{-i\omega t}dt + \int_3^4 2e^{-i\omega t}dt + \int_4^5 1e^{-i\omega t}dt$$

$$= \int_0^1 e^{-i\omega t}dt + 2\int_1^2 e^{-i\omega t}dt + 3\int_2^3 e^{-i\omega t}dt + 2\int_3^4 e^{-i\omega t}dt + \int_4^5 e^{-i\omega t}dt$$

$$\int e^{-i\omega t}dt = \frac{ie^{-i\omega t}}{\omega}$$

$$\left.\frac{ie^{-i\omega t}}{\omega}\right|_0^1 = \frac{i}{\omega}\left(e^{-i\omega}-1\right) \qquad \left.\frac{ie^{-i\omega t}}{\omega}\right|_1^2 = \frac{i}{\omega}\left(e^{-2i\omega}-e^{-i\omega}\right)$$

$$\left.\frac{ie^{-i\omega t}}{\omega}\right|_2^3 = \frac{i}{\omega}\left(e^{-3i\omega}-e^{-2i\omega}\right) \qquad \left.\frac{ie^{-i\omega t}}{\omega}\right|_3^4 = \frac{i}{\omega}\left(e^{-4i\omega}-e^{-3i\omega}\right)$$

$$\left.\frac{ie^{-i\omega t}}{\omega}\right|_4^5 = \frac{i}{\omega}\left(e^{-5i\omega t}-e^{-4i\omega}\right)$$

$$\mathcal{F}\{\} = \frac{i}{\omega}(e^{-i\omega}-1) + \frac{2i}{\omega}(e^{-2i\omega}-e^{-i\omega}) + \frac{3i}{\omega}(e^{-3i\omega}-e^{-2i\omega}) + \frac{2i}{\omega}(e^{-4i\omega}-e^{-3i\omega}) + \frac{i}{\omega}(e^{-5i\omega}-e^{-4i\omega})$$

$$= \frac{i}{\omega}\left[\cancel{e^{-i\omega}} - 1 + \cancel{2e^{-2i\omega}} - \cancel{2e^{-i\omega}} + \cancel{3e^{-3i\omega}} - 3e^{-2i\omega} + 2e^{-4i\omega} - \cancel{2e^{-3i\omega}} + e^{-5i\omega} - \cancel{e^{-4i\omega}}\right]$$

$$= \frac{i}{\omega}\left[-e^{-i\omega} - e^{-2i\omega} + e^{-3i\omega} + e^{-4i\omega} + e^{-5i\omega}\right]$$ //

6a) $|F(\omega)| = \begin{cases} A, & -W < \omega < W \\ 0, & o.c. \end{cases}$ $\quad \theta = -t_0 \omega$

$F(\omega) = Ae^{-t_0 i \omega}, \quad -W < \omega < W$

$f(t) = \int_{-W}^{W} Ae^{-t_0 i \omega} e^{i\omega t} dt = A \int_{-W}^{W} e^{i\omega t - t_0 i \omega} dt$

$= A \int_{-W}^{W} e^{(t-t_0)i\omega} dt = A \left(\frac{e^{(t-t_0)i\omega}}{(t-t_0)i} \right) \Big|_{-W}^{W}$

$= -\frac{Ai}{t-t_0} \left(e^{(t-t_0)iW} - e^{(t-t_0)iW} \right)$

$= -\frac{Ai}{t-t_0} \left[2i \operatorname{sen}((t-t_0)W) \right] = \frac{2AW}{(t-t_0)W} \operatorname{sen}((t-t_0)W)$

$= \underline{\left[2AW \operatorname{Sa}((t-t_0)W) \right]} \left(\frac{1}{2\pi} \right) = \frac{AW}{\pi} \operatorname{Sa}((t-t_0)W)$

6b) $|F(\omega)| = \begin{cases} \pi, & -W < \omega < W \\ 0, & o.c. \end{cases}$ $\quad \theta = \begin{cases} \pi/2, & \omega < 0 \\ -\pi/2, & \omega \geq 0 \end{cases}$

$F(\omega) = \begin{cases} \pi e^{i\frac{\pi}{2}}, & -W < \omega < 0 \\ \pi e^{-\frac{\pi}{2}i}, & 0 < \omega < W \end{cases}$

$f(t) = \int_{-W}^{0} \pi e^{i\frac{\pi}{2}} e^{i\omega t} + \int_{0}^{W} \pi e^{-\frac{\pi}{2}i} e^{i\omega t} dt$

$= \pi e^{\frac{\pi}{2}i} \int_{-W}^{0} e^{i\omega t} d\omega + \pi e^{-\frac{\pi}{2}i} \int_{0}^{W} e^{i\omega t} d\omega$

$= \pi e^{\pi/2 i} \left(\frac{e^{i\omega t}}{it} \right)_{-W}^{0} + \pi e^{-\pi/2 i} \left(\frac{e^{i\omega t}}{it} \right)_{0}^{W}$

$= \frac{\pi e^{\pi/2 i}}{it} \left(1 - e^{-itW} \right) + \frac{\pi e^{-\pi/2 i}}{it} \left(e^{iWt} - 1 \right)$

$$= \frac{\pi i}{t}\left[e^{\frac{\pi}{2}i} - e^{\frac{\pi}{2}i}e^{-wit} + e^{-\frac{\pi}{2}i}e^{wit} - e^{-\frac{\pi}{2}i}\right]$$

$$= \frac{\pi i}{t}\left[e^{\frac{\pi}{2}i} - e^{-\frac{\pi}{2}i} - \left(e^{i(\frac{\pi}{2}-wt)} - e^{-i(\frac{\pi}{2}-wt)}\right)\right]$$

$$= \frac{\pi i}{t}\left[2i - 2i\,\mathrm{sen}\left(\tfrac{\pi}{2}-wt\right)\right] = -\frac{2\pi}{t}\left[1 - \mathrm{sen}\tfrac{\pi}{2}\cos wt + \cancel{\cos\tfrac{\pi}{2}\,\mathrm{sen}\,wt}\right]$$

$$= \left(-\frac{2\pi}{t}\left[1-\cos wt\right]\right)\left(\frac{1}{2\pi}\right) = -\frac{1-\cos wt}{t}$$

6c) $|F(\omega)| = \begin{cases} A\omega & , -1<\omega<0 \\ -A\omega & , 0<\omega<1 \end{cases}$, $\emptyset$ en otro caso

$$f(t) = A\left[\int_{-1}^{0} \omega e^{i\omega t}\,d\omega - \int_{0}^{1} \omega e^{i\omega t}\,d\omega\right]$$

$\int \omega e^{i\omega t}\,d\omega$ $\quad u = i\omega t$ $\quad \omega = \frac{u}{it}$ $\quad d\omega = -\frac{i}{t}du$

$$= \int \frac{u}{it}e^{u}\left(-\frac{i}{t}\right)du = -\frac{1}{t^2}\int ue^{u}du = -\frac{1}{t^2}(u-1)e^{u}$$

$$= -\frac{1}{t^2}(i\omega t - 1)e^{i\omega t}$$

$$\int_{-1}^{0} = -\frac{1}{t^2}\left[(-1) - (-it-1)e^{-it}\right] = -\frac{1}{t^2}\left[-1 + (it+1)e^{-it}\right]$$

$$\int_{0}^{1} = -\frac{1}{t^2}\left[(it-1)e^{it} - (-1)\right] = -\frac{1}{t^2}\left[(it-1)e^{it} + 1\right]$$

Restando,

$$-\frac{1}{t^2}\left[\cancel{-1} + (it+1)e^{-it} + (it-1)e^{it} + \cancel{1}\right]$$

$$= -\frac{1}{t^2}\left[ite^{-it} + e^{-it} + ite^{it} - e^{it}\right] = \frac{-1}{t^2}\left[it(2\cos t) - 2i\,\mathrm{sen}\,t\right]$$

$$= -\frac{2i}{t^2}\left[t\cos t - \mathrm{sen}\,t\right]\left(\frac{A}{2\pi}\right) = -\frac{Ai}{\pi t^2}\left[t\cos t - \mathrm{sen}\,t\right]$$

6d. $$|F(\omega)| = \begin{cases} \frac{A}{b-a}\omega + b \,, & -b<\omega<-a \\ A \,, & -a<\omega<a \\ -\frac{A}{b-a}\omega - b \,, & a<\omega<b \end{cases}$$

$$f(t) = \frac{1}{2\pi}\left[\int_{-b}^{-a}\left(\frac{A}{b-a}\omega + Ab\right)e^{i\omega t}d\omega + \int_{-a}^{a}Ae^{i\omega t}d\omega - \int_{a}^{b}\left(\frac{A}{b-a}\omega - Ab\right)d\omega\right]$$

$$= \frac{1}{2\pi}\left[\frac{A}{b-a}\int_{-b}^{-a}\omega e^{i\omega t}d\omega + Ab\int_{-b}^{-a}e^{i\omega t}d\omega + A\int_{-a}^{a}e^{i\omega t}d\omega - \frac{A}{b-a}\int_{a}^{b}\omega + Ab\int_{a}^{b}e^{i\omega t}d\omega\right]$$

$$= \frac{A}{2\pi}\left[\frac{1}{b-a}\int_{-b}^{-a}\omega e^{i\omega t}d\omega + b\int_{-b}^{-a}e^{i\omega t}d\omega + \int_{-a}^{a}e^{i\omega t}d\omega - \frac{1}{b-a}\int_{a}^{b}\omega e^{i\omega t} + b\int_{a}^{b}e^{i\omega t}d\omega\right]$$

$$\int_{-b}^{-a}\omega e^{i\omega t}d\omega = -\frac{1}{t^2}(i\omega t - 1)e^{i\omega t}\Big|_{-b}^{-a} = -\frac{1}{t^2}\left[(-i\omega a - 1)e^{-ait} - (-i\omega b - 1)e^{-bit}\right]$$

$$= \frac{1}{t^2}\left[(ita+1)e^{-ait} - (itb+1)e^{-bit}\right] \quad -(1)$$

$$\int_{a}^{b}\omega e^{i\omega t}d\omega = -\frac{1}{t^2}(i\omega t - 1)e^{i\omega t}\Big|_{a}^{b} = -\frac{1}{t^2}\left[(ita-1)e^{ait} - (itb-1)e^{bit}\right] \quad (2)$$

$$\int e^{i\omega t}d\omega = \frac{e^{i\omega t}}{it} = -\frac{ie^{i\omega t}}{t}$$

$$\int_{-b}^{-a} = -\frac{i}{t}\left[e^{-ait} - e^{-bit}\right] \quad -(3) \qquad \int_{-a}^{a} = -\frac{i}{t}\left[e^{ait} - e^{bit}\right] \quad -(4)$$

$$\int_{a}^{b} = -\frac{i}{t}\left[e^{bit} - e^{ait}\right] \quad -(5)$$

Sumando $b((3)+(5))$

$$b\int_{-b}^{-a}e^{i\omega t}d\omega + b\int_{a}^{b}e^{i\omega t}d\omega = -\frac{ib}{t}\left[e^{-ait} - e^{ait} - e^{-bit} + e^{bit}\right]$$

$$= -\frac{ib}{t}\left[-2i\,\mathrm{sen}\,at + 2i\,\mathrm{sen}\,bt\right] = -\frac{2b}{t}\left(\frac{ab}{ab}\right)(\mathrm{sen}\,at + \mathrm{sen}\,bt)$$

$$= -2ab^2\left(\frac{\mathrm{sen}\,at}{abt} + \frac{\mathrm{sen}\,bt}{abt}\right) = -2ab^2\left(\frac{\mathrm{Sa}\,at}{b} + \frac{\mathrm{Sa}\,bt}{a}\right)$$

$$= -2\cancel{a}b^{\cancel{2}}\left(\frac{a\mathrm{Sa}\,at + b\mathrm{Sa}\,bt}{\cancel{ab}}\right) = -2b(a\mathrm{Sa}\,at + b\mathrm{Sa}\,bt) \quad -(II)$$

Restando $\frac{1}{b-a}(① - ②)$

$$\frac{1}{t^2(b-a)}\left[(ait+1)e^{-ait}-(bit+1)e^{-bit}+(ait-1)e^{ait}-(bit-1)e^{bit}\right]$$

$$\frac{1}{(b-a)t^2}\left[\cancel{aite^{-ait}}+e^{-ait}-\cancel{bite^{-bit}}-e^{-bit}+\cancel{aite^{ait}}-e^{ait}-\cancel{bite^{bit}}+e^{bit}\right]$$

$$=\frac{1}{(b-a)t^2}\left[2i\,\mathrm{sen}\,at+2i\,\mathrm{sen}\,bt+2ait\cos at-2bit\cos bt\right]$$

$$=\frac{2i}{(b-a)t^2}\left[\mathrm{sen}\,at+\mathrm{sen}\,bt+at\cos at-bt\cos bt\right] \quad \text{—— (I)}$$

Sumando (I), (II) y (4) obtenemos el resultado final

$$\int_{-\infty}^{\infty} f(t)\,\delta(t-t_0)\,dt = f(t_0)$$

7; Mediante la propiedad de muestreo de $\delta(t)$, calcular las sig. integrales

a) $\int_{-\infty}^{\infty} \delta(t-5)\,\text{sen}\,2t\,dt$ $\quad f(t)=\text{sen}\,2t \quad t_0=5$

$= \text{sen}\,2t\big|_5 = \underline{\text{sen}\,10}$

b) $\int_{-\infty}^{\infty} \delta(2-t)(t^5-3)\,dt$ $\quad f(t)=t^5-3 \quad t_0=2$

$\delta(2-t)=\delta(-(t-2))=\delta(t-2)$

$f(t_0)=(2)^5-3=\underline{13}$

c) $\int_{1}^{x} e^{-x^2}\delta(x)\,dx$ $\quad$ $\delta(x)$ está centrado en $\emptyset$. La integral está definida en un intervalo mayor a 0, por lo que el resultado de la integral es cero.

$=0$

d) $\int_{-\infty}^{\infty} \delta(t-2)\cos[\pi(t-3)]\,dt$ $\quad f(t)=\cos(\pi t-3\pi) \quad t_0=2$

$f(t_0)=\cos(2\pi-3\pi)=\cos(-\pi)=\cos\pi=\underline{-1}$

e) $\int_{-\infty}^{\infty} \delta(t+2)e^{-2t}\,dt$ $\quad f(t)=e^{-2t} \quad t_0=-2$

$\underline{f(t_0)=e^4 \approx 54.6}$

f) $\int_{-\infty}^{\infty} e^{\cos t}\,\delta(t-\pi)\,dt$ $\quad f(t)=e^{\cos t} \quad t_0=\pi$

$f(t_0)=e^{\cos\pi}=\underline{e^{-1}}$

g) $\int_{1}^{100} \log_{10}(t)\,\delta(t-10)\,dt$

$$\int_{-\infty}^{\infty} f(t)\,\delta(t-t_0)\,dt = \cancel{\int_{-\infty}^{t_0^-} 0\,dt} + \int_{t_0^-}^{t_0^+} \delta(t-t_0)\,f(t_0)\,dt + \cancel{\int_{t_0^+}^{\infty} 0\,dt}$$

$$= \int_{10^-}^{10^+} \delta(t-10)\log_{10}(t)\,dt = \log_{10}(t)\big|_{10} = \underline{\log_{10}10 = 1}$$

8. Usa las propiedades de la transf. de Fourier para encontrar $\mathcal{F}\{f(t)\}$

8a) $f(2-t) \longleftrightarrow ?$

$$f(t) \longleftrightarrow F(\omega)$$
$$f(t+2) \longleftrightarrow F(\omega)e^{2i\omega}$$
$$f(-t+2) \longleftrightarrow e^{-2i\omega}F(-\omega)$$

8b) $f[(t-3)-3] \longleftrightarrow ?$

$$f(t) \longleftrightarrow F(\omega)$$
$$f(t-3) \longleftrightarrow F(\omega)e^{-3i\omega}$$
$$f(t-3-3) \longleftrightarrow F(\omega)e^{-3i\omega}e^{-3i\omega}$$
$$= F(\omega)e^{-6i\omega}$$

$f(t-6)$

$$f(t) \longleftrightarrow F(\omega)$$
$$f(t-6) \longleftrightarrow F(\omega)e^{-6i\omega}$$

8c) $\frac{df(t)}{dt}\operatorname{sen} t \longleftrightarrow ?$

$$f(t) \longleftrightarrow F(\omega)$$
$$f'(t) \longleftrightarrow i\omega F(\omega)$$
$$f'(t)\operatorname{sen} t \longleftrightarrow \frac{1}{2}\left[i(\omega+1)F(\omega+1) - i(\omega-1)F(\omega-1)\right]$$
$$= -\frac{1}{2}\left[(\omega+1)F(\omega+1) - (\omega-1)F(\omega-1)\right]$$

8d) $\frac{d}{dt}\left[f(-2t)\right]$ $\qquad f(t) \longleftrightarrow F(t)$

$$f(-2t) = \frac{1}{|-2|}F\left(\frac{\omega}{-2}\right) = \frac{1}{2}F\left(-\frac{\omega}{2}\right)$$

$$f'(-2t) = \frac{i\omega}{2}F\left(-\frac{\omega}{2}\right)$$

8e) $tf(3t) \longleftrightarrow ?$

$$f(t) = F(t)$$
$$f(3t) \longleftrightarrow \frac{1}{3}F\left(\frac{\omega}{3}\right)$$
$$-itf(3t) \longleftrightarrow \frac{1}{3}\frac{d}{d\omega}F\left(\frac{\omega}{3}\right)$$
$$tf(3t) \longleftrightarrow \frac{i}{3}\frac{d}{d\omega}F\left(\frac{\omega}{3}\right)$$

8f) $(t-5)f(t) \longleftrightarrow$

$tf(t) - 5f(t) \longleftrightarrow ?$

$f(t) \longleftrightarrow F(\omega)$

$-itf(t) \longleftrightarrow \frac{d}{d\omega}F(\omega)$

$tf(t) \longleftrightarrow i\frac{d}{d\omega}F(\omega)$ //

$-5f(t) = -5F(\omega)$

$(t-5)f(t) \longleftrightarrow i\frac{d}{d\omega}F(\omega) - 5F(\omega)$ //

8g) $(t-3)f(-3t) = tf(-3t) - 3f(-3t)$

$f(t) = F(\omega)$

$f(-3t) = \frac{1}{3}F(-\frac{\omega}{3})$

$-itf(-3t) = \frac{1}{3}\frac{d}{d\omega}F(-\frac{\omega}{3})$

$tf(-3t) = \frac{i}{3}\frac{d}{d\omega}F(-\frac{\omega}{3})$ //

$3f(-3t) \longleftrightarrow F(-\frac{\omega}{3})$ //

$(t-3)f(-3t) \longleftrightarrow \frac{i}{3}\frac{d}{d\omega}F(-\frac{\omega}{3}) - F(-\frac{\omega}{3})$ //

8h) $t\frac{df(t)}{dt} \longleftrightarrow ?$

$f(t) \longleftrightarrow F(\omega)$

$f'(t) \longleftrightarrow i\omega F(\omega)$

$-itf'(t) \longleftrightarrow i\frac{d}{d\omega}\omega F(\omega)$

$tf'(t) \longleftrightarrow -\frac{d}{d\omega}\omega F(\omega)$ //

8i) $f(6-t) \longleftrightarrow e^{-6i\omega}F(-\omega)$

8j) $(2-t)f(8-t) = 2f(8-t) - tf(8-t) \longleftrightarrow ?$

$f(8-t) \longleftrightarrow e^{-8i\omega}F(-\omega)$

$itf(8-t) \longleftrightarrow \frac{d}{d\omega}e^{-8i\omega}F(-\omega) \longrightarrow tf(8-t) = -i\frac{d}{d\omega}e^{-8i\omega}F(-\omega)$

$(2-t)f(8-t) = 2e^{-8i\omega}F(-\omega) + i\frac{d}{d\omega}e^{-8i\omega}F(-\omega)$ //

9; Completa los pares de transformada

9a) $5\delta(t-1)$

$$\delta(t) \longleftrightarrow 1$$
$$\delta(t-1) \longleftrightarrow e^{-i\omega}$$
$$5\delta(t-1) \longleftrightarrow 5e^{-i\omega}$$

9b) $? \longleftrightarrow 8\delta(\omega+1)+8\delta(\omega-1)$

$$\cos t \longleftrightarrow \pi\left[\delta(\omega+1)+\delta(\omega-1)\right]$$
$$\frac{8}{\pi}\cos t \longleftrightarrow 8\left[\delta(\omega+1)+\delta(\omega-1)\right]$$
$$\frac{8}{\pi}\cos t \longleftrightarrow 8\delta(\omega+1)+8\delta(\omega-1)$$

9c) $t \longleftrightarrow ?$

$$\delta(t) \longleftrightarrow 1$$
$$1 \longleftrightarrow 2\pi\,\delta(\omega)$$
$$-it \longleftrightarrow 2\pi\,\delta'(\omega)$$
$$t \longleftrightarrow 2\pi i\,\delta'(\omega)$$

$(-it)(-it) = i^2t^2 = t^2$

9d) $t^2 \longleftrightarrow ?$

$$\delta(t) \longleftrightarrow 1$$
$$1 \longleftrightarrow 2\pi\,\delta(\omega)$$
$$-t^2 \longleftrightarrow 2\pi\,\delta'(\omega)$$
$$t^2 \longleftrightarrow -2\pi\,\delta'(\omega)$$

9e) $2C_2(t)\cos 1000t \longleftrightarrow ?$

$$2C_2(t) \longleftrightarrow 4Sa(\omega)$$
$$2C_2(t)\cos 1000t \longleftrightarrow \frac{1}{2}\left[4Sa(\omega-1000)+4Sa(\omega+1000)\right]$$
$$2C_2(t)\cos 1000t \longleftrightarrow 2Sa(\omega-1000)+2Sa(\omega+1000)$$

9f) $? \longleftrightarrow \cos 1000\omega$

$$\cos 1000t \longleftrightarrow \pi\left[\delta(\omega+1000)+\delta(\omega-1000)\right]$$
$$\pi\left[\delta(t+1000)+\delta(t-1000)\right] \longleftrightarrow 2\pi\cos 1000\omega$$
$$\frac{1}{2}\left[\delta(t+1000)+\delta(t-1000)\right] \longleftrightarrow \cos 1000\omega$$

9g) $? \longleftrightarrow 5\omega$

$\delta(t) \longleftrightarrow 1$
$5\delta(t) \longleftrightarrow 5$
$5\delta'(t) \longleftrightarrow 5i\omega$
$-5i\delta'(t) \longleftrightarrow 5\omega$

9h) $? \longleftrightarrow \delta(\omega)e^{-j5\omega}$

$\delta(t) \longleftrightarrow 1$
$1 \longleftrightarrow 2\pi\delta(\omega)$
$1 \longleftrightarrow 2\pi\delta(\omega)e^{-5i\omega}$
$\frac{1}{2\pi} \longleftrightarrow \delta(\omega)e^{-5i\omega}$

10; Encuentra las sig. transformadas.

10a) $? \longrightarrow 3\,\mathrm{sgn}(4\omega-2)$

$\mathrm{sgn}(t) \longleftrightarrow \frac{2}{i\omega}$
$\mathrm{sgn}(t+2) \longleftrightarrow \frac{2}{i\omega}e^{+2i\omega}$
$\mathrm{sgn}(4t+2) \longleftrightarrow \frac{1}{|4|}\frac{2}{i\left(\frac{\omega}{4}\right)}e^{+2i\left(\frac{\omega}{4}\right)} = \frac{2}{i\omega}e^{+\frac{i\omega}{2}}$

$3\,\mathrm{sgn}(4t+2) \longleftrightarrow \frac{6}{i\omega}e^{+\frac{i\omega}{2}}$
$\frac{6}{it}e^{\frac{it}{2}} \longleftrightarrow 6\pi\,\mathrm{sgn}(-4t+2) = 6\pi\,\mathrm{sgn}(-(4\omega-2))$
$-\frac{6}{it}e^{\frac{it}{2}} \longleftrightarrow 6\pi\,\mathrm{sgn}(4\omega-2)$

10b) $G_2\left(\frac{2}{3}t\right) \longleftrightarrow$

$G_2(t) \longleftrightarrow 2\,Sa\,\omega$
$G_2\left(\frac{2}{3}t\right) \longleftrightarrow \frac{1}{\left|\frac{2}{3}\right|}2\,Sa\,\frac{\omega}{\frac{2}{3}} = \frac{3}{\cancel{2}}\cancel{2}\,Sa\,\frac{3\omega}{2} = 3\,Sa\,\frac{3\omega}{2}$

c) $2G_2(t)\cos 250t \longleftrightarrow ?$

$2G_2(t) \longleftrightarrow 4\,Sa\,\omega$

$2G_2(t)\cos 250t \longleftrightarrow \frac{1}{2}\left[4Sa(\omega+250)+4Sa(\omega-250)\right]$

$2G_2(t)\cos 250t \longleftrightarrow \underline{2\left[Sa(\omega+250)+Sa(\omega-250)\right]}$ //

10d) $tu(10t-1) \longleftrightarrow ?$

$u(t) \longleftrightarrow \pi\delta(\omega)+\frac{1}{i\omega}$

$u(t-1) \longleftrightarrow \left(\pi\delta(\omega)+\frac{1}{i\omega}\right)e^{-i\omega}$

$u(10t-1) \longleftrightarrow \frac{1}{10}\left(\pi\delta\left(\frac{\omega}{10}\right)+\frac{10}{i\omega}\right)e^{-\frac{i\omega}{10}}$

$-itu(10t-1) \longleftrightarrow \frac{1}{10}\frac{d}{d\omega}\left[\pi\delta\left(\frac{\omega}{10}\right)+\frac{10}{i\omega}\right]e^{-\frac{i\omega}{10}}$

$tu(10t-1) \longleftrightarrow \underline{\frac{i}{10}\frac{d}{d\omega}\left[\pi\delta\left(\frac{\omega}{10}\right)+\frac{10}{i\omega}\right]e^{-\frac{i\omega}{10}}}$ //

10e) $t^3\delta(6t-1)e^{12it} \longleftrightarrow ?$

$(-i)(-i)$
i^2
$-1(i)$

$\delta(t) \longleftrightarrow 1$

$\delta(t-1) \longleftrightarrow e^{-i\omega}$

$\delta(6t-1) \longrightarrow \frac{1}{6}e^{-i\left(\frac{\omega}{6}\right)}$

$\delta(6t-1)e^{12it} \longleftrightarrow \frac{1}{6}e^{-\frac{i}{6}(\omega-12)}$

$-it^3\delta(6t-1)e^{12it} \longleftrightarrow \frac{1}{6}\frac{d}{d\omega^3}e^{-\frac{i(\omega-12)}{6}}$

$t^3\delta(6t-1)e^{12it} \longleftrightarrow \underline{\frac{i}{6}\frac{d}{d\omega^3}e^{-\frac{i(\omega-12)}{6}}}$ //

f) $? \longleftrightarrow \frac{4}{\pi} Sa(4\omega - 2)$

$$\left(\frac{1}{2\pi}\right) C_8(t) \longleftrightarrow \left(\frac{1}{2\pi}\right)(8) Sa\left(\frac{8\omega}{2}\right) = \frac{4}{\pi} Sa(4\omega)$$

$$\left(e^{\frac{1}{2}it}\right) \frac{1}{2\pi} C_8(t) \longleftrightarrow \frac{4}{\pi} Sa\left(4\left(\omega - \frac{1}{2}\right)\right) = \frac{4}{\pi} Sa(4\omega - 2)$$

10g) $u(t) \longleftrightarrow \pi \delta(\omega) + \frac{1}{j(\omega)}$

$$u(t) e^{-\frac{3}{4}it} \longleftrightarrow \pi \delta\left(\omega + \frac{3}{4}\right) + \frac{1}{j\left(\omega + \frac{3}{4}\right)}$$

$$u(t+1000) e^{-\frac{3}{4}i(t+1000)} \longleftrightarrow \left(\pi \delta\left(\omega + \frac{3}{4}\right) + \frac{1}{j\left(\omega + \frac{3}{4}\right)}\right) e^{1000ti}$$

$$\frac{d}{dt} u(t+1000) e^{-\frac{3}{4}i(t+1000)} \longleftrightarrow i\omega \left(\pi \delta\left(\omega + \frac{3}{4}\right) + \frac{1}{i\left(\omega + \frac{3}{4}\right)}\right) e^{1000ti}$$

$$i \frac{d}{dt} u(t+1000) e^{-\frac{3}{4}i(t+1000)} \longleftrightarrow -\omega \left[\pi \delta\left(\omega + \frac{3}{4}\right) + \frac{1}{i\left(\omega + \frac{3}{4}\right)}\right] e^{1000it}$$

10h) $C_{\frac{4}{3}}(t) \longleftrightarrow \frac{4}{3} Sa\left(\frac{2\omega}{3}\right)$

$$C_{\frac{4}{3}}(t+6) \longleftrightarrow \frac{4}{3} Sa\left(\frac{2\omega}{3}\right) e^{6i\omega}$$

10i) $[3\delta(t-1) - 3\delta(t+1)] \cos 18t \longleftrightarrow ?$

$$\text{sen}\, t \longleftrightarrow -i\pi [\delta(\omega - 1) - \delta(\omega + 1)]$$

$$\frac{i\,\text{sen}\, t}{\pi} \longleftrightarrow \delta(\omega - 1) - \delta(\omega + 1)$$

$$\frac{3i\,\text{sen}\, t}{\pi} \longleftrightarrow 3\delta(\omega - 1) - 3\delta(\omega + 1)$$

$$3\delta(t-1) - 3\delta(t+1) \longleftrightarrow 2\pi\left(\frac{3i}{\pi}\right) \text{sen} -\omega = -6i\, \text{sen}\, \omega$$

$$[3\delta(t-1) - 3\delta(t+1)] \cos 18t \longleftrightarrow \frac{1}{2}\left[-6i\,\text{sen}(\omega + 18) - 6i\,\text{sen}(\omega - 18)\right]$$

10j) $? \longleftrightarrow 2\cos 500\omega$

$$\cos 500t \longleftrightarrow \pi[\delta(\omega+500)+\delta(\omega-500)]$$
$$\pi[\delta(t+500)+\delta(t-500)] \longleftrightarrow 2\pi\cos(-500\omega)$$
$$\delta(t+500)+\delta(t-500)] \longleftrightarrow 2\cos 500\omega$$

10k) $t+t^2+1 \longleftrightarrow ?$

$$1 \longleftrightarrow 2\pi\delta(\omega) \quad -① \qquad (-it)^2 \longleftrightarrow 2\pi\delta''(\omega)$$
$$-it \longleftrightarrow 2\pi\delta'(\omega) \qquad -t^2 \longleftrightarrow 2\pi\delta''(\omega)$$
$$t \longleftrightarrow 2\pi i\delta'(\omega) \quad -② \qquad t^2 \longleftrightarrow -2\pi\delta''(\omega) \quad -③$$
$$t+t^2+1 \longleftrightarrow 2\pi[\delta(\omega)+i\delta'(\omega)-\delta''(\omega)]$$

10l) $j\frac{5}{t} \longleftrightarrow ?$

$$sg(t) \longleftrightarrow \frac{2}{i\omega} = -\frac{2i}{\omega}$$
$$-\frac{2i}{t} \longleftrightarrow 2\pi\,sgn(-\omega)$$
$$-\frac{2i}{t}\Big|_{t=-\frac{2}{5}t} = \frac{\cancel{-}2i}{\cancel{-}\frac{2}{5}t} = \frac{5i}{t} \longleftrightarrow \frac{1}{|-\frac{2}{5}|}2\pi\,sgn\left(-\frac{\omega}{-\frac{2}{5}}\right)$$
$$\frac{5i}{t} \longleftrightarrow \frac{5}{\cancel{2}}\cancel{2}\pi\,sgn\left(\frac{5\omega}{2}\right) \longrightarrow \frac{5i}{t} \longleftrightarrow 5\pi\,sgn\left(\frac{5\omega}{2}\right)$$

10m) $? \longleftrightarrow \frac{1}{\omega}$

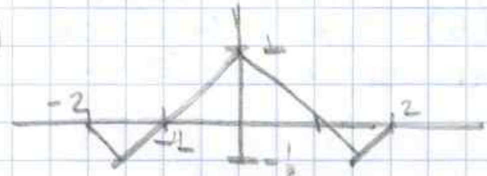
$$Sgn(t) \longleftrightarrow \frac{2}{j\omega} \longrightarrow \frac{j}{2}sgn(t) \longleftrightarrow \frac{1}{\omega}$$

10n) $? \longleftrightarrow \frac{1}{\omega}$

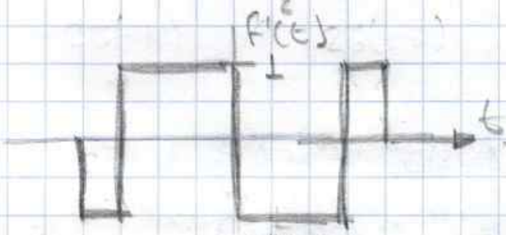
$$\frac{j}{2}sgn(t) \longleftrightarrow \frac{1}{\omega} \longrightarrow \frac{j}{2}sgn(t-4) \longleftrightarrow \frac{1}{\omega}e^{-4i\omega}$$

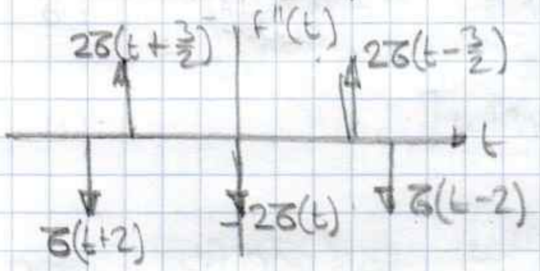
11; Determina $F(\omega)$ de las sig. funciones

11a.)



$$f(t)=\begin{cases} t+1, & -1.5<t<0 \\ -t+1, & 0<t<1.5 \\ t-2, & 1.5<t<2 \\ -t+2, & -2<t<-1.5 \end{cases}$$





$f''(t)=$

$$f''(t)=-\delta(t+2)+2\delta(t+\tfrac{3}{2})-2\delta(t)+2\delta(t-\tfrac{3}{2})-\delta(t-2)$$

$$\mathcal{F}\{f''(t)\}=-e^{2i\omega}+2e^{\frac{3}{2}i\omega}-2+2e^{-\frac{3}{2}i\omega}-e^{-2i\omega}$$

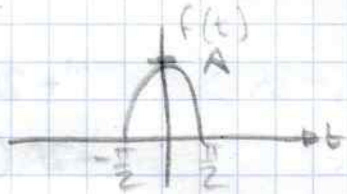
$$=-(e^{2i\omega}+e^{-2i\omega})+2(e^{\frac{3}{2}i\omega}+e^{-\frac{3}{2}i\omega})-2$$

$$=-2\cos 2\omega+4\cos\tfrac{3}{2}\omega-2$$

$$\mathcal{F}\left\{\frac{d^2}{dt^2}f(t)\right\}=i^2\omega^2F(\omega)$$

$$F(\omega)=\frac{-2\cos 2\omega+4\cos\frac{3}{2}\omega-2}{i^2\omega^2}=\underline{\frac{2\cos 2\omega-4\cos\frac{3}{2}\omega+2}{\omega^2}}$$

11b)



$$f(t)=\begin{cases} A\cos t, & -\frac{\pi}{2}<t<\frac{\pi}{2} \\ 0, & \text{o.c.} \end{cases}$$

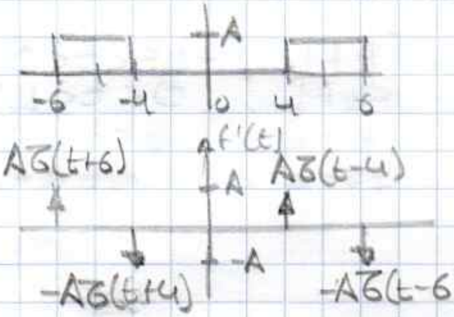
$$f(t)=A\cos t\cdot C_\pi(t)$$

$$C_\pi(t)=\pi Sa\left(\frac{\pi\omega}{2}\right)$$

$$AC_\pi(t)\cos t=\frac{A}{2}\left[\pi Sa\left(\frac{\pi(\omega+1)}{2}\right)+\pi Sa\left(\frac{\pi(\omega-1)}{2}\right)\right]$$

$$\frac{A\pi}{2}\left[Sa\left(\frac{\pi(\omega+1)}{2}\right)+Sa\left(\frac{\pi(\omega-1)}{2}\right)\right]$$

11c)



$$f(t)=\begin{cases} A, & -6<t<-4 \\ A, & 4<t<6 \\ 0 & \text{o.c.} \end{cases}$$

$$f'(t)=A\delta(t+6)-A\delta(t+4)+A\delta(t-4)-A\delta(t-6)$$

$$\mathscr{F}\{f'(t)\}=Ae^{6i\omega}-Ae^{4i\omega}+Ae^{-4i\omega}-Ae^{-6i\omega}$$

$$=A\left[e^{6i\omega}-e^{-6i\omega}-(e^{4i\omega}-e^{-4i\omega})\right]$$

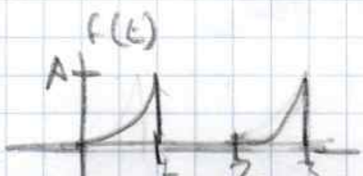
$$=\underline{A\left[2i\operatorname{sen}6\omega-2i\operatorname{sen}4\omega\right]}$$

$$\mathscr{F}\{f'(t)\}=i\omega F(\omega)$$

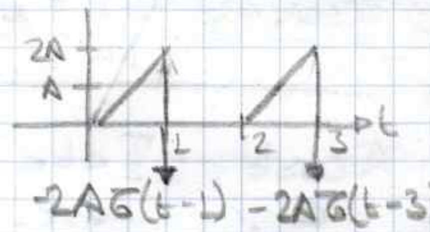
$$F(\omega)=\frac{2Ai(\operatorname{sen}6\omega-\operatorname{sen}4\omega)}{i\omega}=2A\left(\frac{\operatorname{sen}6\omega}{\omega}-\frac{\operatorname{sen}4\omega}{\omega}\right)$$

$$=2A\left(\frac{6\operatorname{sen}6\omega}{6\omega}-\frac{4\operatorname{sen}4\omega}{4\omega}\right)=\underline{2A(6Sa\,6\omega-4Sa\,4\omega)}\,/\!/$$

11d)

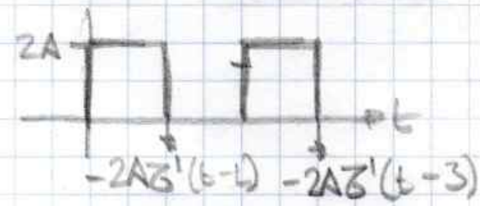


$$f(t)=\begin{cases} At^2, & 0<t<1 \\ At^2, & 2<t<3 \\ 0, & \text{o.c.} \end{cases}$$

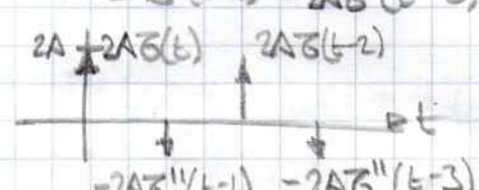


$$f'''(t)=2A\delta(t)-2A\delta''(t-1)+2A\delta(t-2)-2A\delta''(t-3)$$

$$\mathscr{F}\{f'''(t)\}=2A-2A(i\omega)^2e^{-i\omega}+2Ae^{-2i\omega}-2A(i\omega)^2e^{-3i\omega}$$



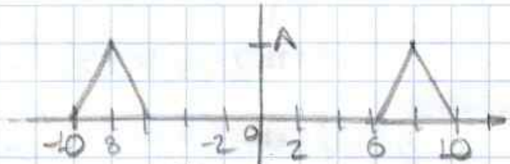
$$=2A\left[1+\omega^2e^{-i\omega}+e^{-2i\omega}+\omega^2e^{-3i\omega}\right]$$



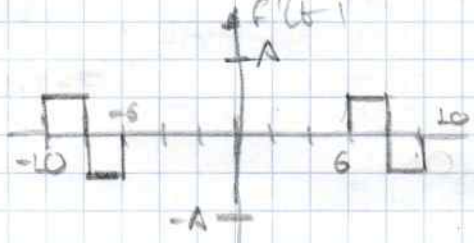
$$f'''(t)=(i\omega)^3F(\omega)=-i\omega^3F(\omega)$$

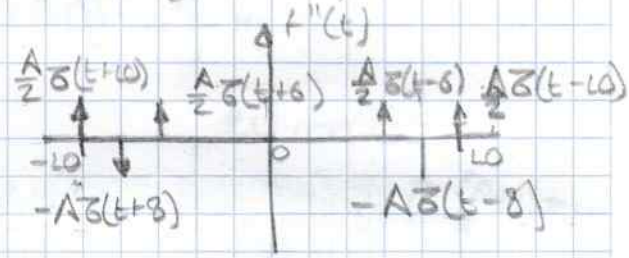
$$F(\omega)=\frac{2Ai\left[1+e^{-2i\omega}+\omega^2(e^{-i\omega}+e^{-3i\omega})\right]}{\omega^3}$$

11e)



$$f(t) = \begin{cases} \frac{A}{2}t + 10 & , \\ \frac{A}{2}t - 6 & , \\ -\frac{A}{2}t + 6 & , \\ -\frac{A}{2}t - 10 & , \end{cases}$$





$$f''(t) = \frac{A}{2}\delta(t+10) - A\delta(t+8) + \frac{A}{2}\delta(t+6) + \frac{A}{2}\delta(t-6) - A\delta(t-8) + \frac{A}{2}\delta(t-10)$$

$$\mathcal{F}\{f''(t)\} = \frac{A}{2}e^{10i\omega} - Ae^{8i\omega} + \frac{A}{2}e^{6i\omega} + \frac{A}{2}e^{-6i\omega} - Ae^{-8i\omega} + \frac{A}{2}e^{-10i\omega}$$

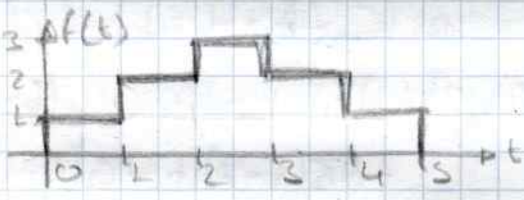
$$= \frac{A}{2}\left(e^{10i\omega} + e^{-10i\omega}\right) - A\left(e^{8i\omega} + e^{-8i\omega}\right) + \frac{A}{2}\left(e^{6i\omega} + e^{-6i\omega}\right)$$

$$= A\cos 10\omega - 2A\cos 8\omega + A\cos 6\omega$$

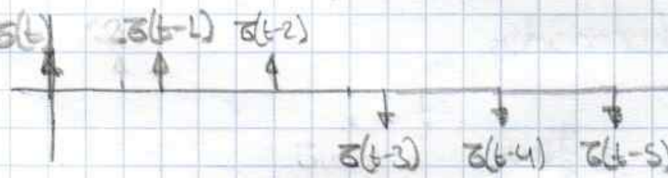
$$\mathcal{F}\{f''(t)\} = i^2\omega^2 F(\omega) = -\omega^2 F(\omega)$$

$$F(\omega) = -\frac{A\cos 10\omega - 2A\cos 8\omega + A\cos 6\omega}{\omega^2}$$

11f)



f'(t)



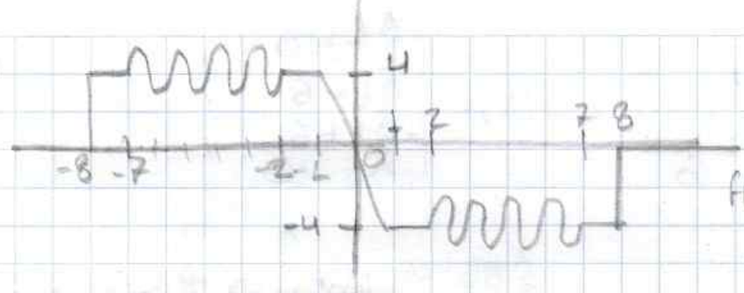
$$f'(t) = \delta(t) + \delta(t-1) + \delta(t-2) - \delta(t-3) - \delta(t-4) - \delta(t-5)$$

$$\mathcal{F}\{f'(t)\} = 1 + e^{-i\omega} + e^{-2i\omega} - e^{-3i\omega} - e^{-4i\omega} - e^{-5i\omega}$$

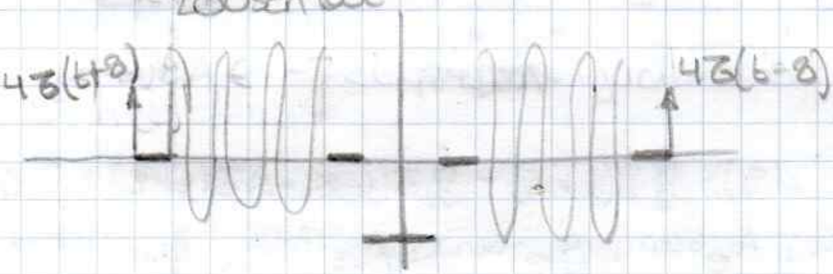
$$\mathcal{F}\{f'(t)\} = i\omega F(\omega)$$

$$F(\omega) = \frac{1 + e^{-i\omega} + e^{-2i\omega} - e^{-3i\omega} - e^{-4i\omega} - e^{-5i\omega}}{\ldots}$$

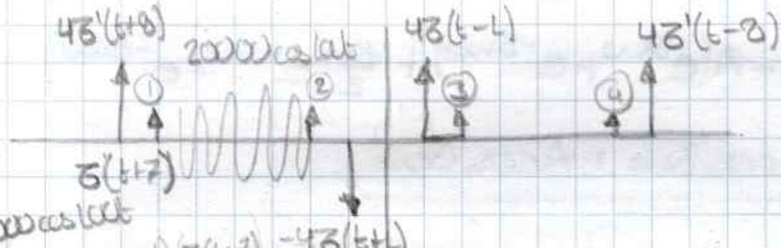
4g)



$$f(t)=\begin{cases} 4, & -8<t<-7 \\ 4+2\cos 100t, & -7<t<-2 \\ 4, & -2<t<-1 \\ -4t, & -1<t<1 \\ -4, & 1<t<2 \\ -4-2\cos 100t, & 2<t<7 \\ -4, & 7<t<8 \\ 0, & \text{o.c.} \end{cases}$$



$f'(t)$



$\lim_{t\to 7^+} 20000\cos 100t =$
$= 18{,}793.85 = A$

$\lim_{t\to -2^-} 20000\cos 100t =$
$= -18{,}793.85 = -A$

① $\lim_{t\to -7^+} 20000\cos 100t$ $\delta(t+7)$

② $\lim_{t\to -2^-} 20000\cos 100t$ $\delta(t+7)$ $-4\delta(t+1)$

③ $\lim_{t\to 2^+} 20000\cos 100t = -18{,}793.85 = -A$ ④ $\lim_{t\to 7^-} = A$

$$f''(t) = 4\delta'(t+8) + A\delta(t+7) + 20000\cos 100t\,(C_5(t+4.5)) - A\delta(t+2) - 4\delta(t+1) + 4\delta(t-1) - A\delta(t-2) + 20000\cos 100t\,[C_5(t-4.5)] + A\delta(t-7) + 4\delta'(t-8)$$

Obteniendo estas transformadas

$C_5(t) \leftrightarrow 5Sa\left(\frac{5\omega}{2}\right)$

$C_5(t \pm \frac{9}{2}) \leftrightarrow 5Sa\left(\frac{5\omega}{2}\right)e^{\pm\frac{9}{2}i\omega}$

$\cos 100t \cdot C_5(t \pm \frac{9}{2}) = \frac{1}{2}\left[5Sa\left(\frac{5(\omega+100)}{2}\right)e^{\pm\frac{9}{2}i(\omega+100)} + 5Sa\left(\frac{5(\omega-100)}{2}\right)e^{\pm\frac{9}{2}i(\omega-100)}\right]$

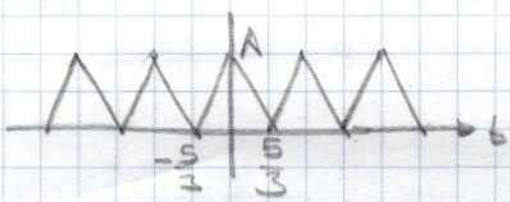
$20000\cos 100t \cdot C_5(t \pm \frac{9}{2}) = 10000\left[5Sa\left(\frac{5(\omega+100)}{2}\right)e^{\pm\frac{9}{2}i(\omega+100)} + 5Sa\left(\frac{5(\omega-100)}{2}\right)e^{\pm\frac{9}{2}i(\omega-100)}\right]$

$\frac{A}{2(4n^2-1)}\left[(2n+1)(-\cos\pi n)+(2n-1)(\cos\pi n)\right] = \frac{A\cos\pi n}{2(4n^2-1)}\left[-\cancel{2n}-1+\cancel{2n}-1\right]$

$= -\frac{4A\cos\pi n}{\pi(4n^2-1)}$

$C_n = -\frac{2A\cos\pi n}{\pi(4n^2-1)}$

$\mathcal{F}\{f(t)\} = 2\pi\sum_{n=-\infty}^{\infty} -\frac{2A\cos\pi n}{\pi(4n^2-1)} = -4A\sum_{n=-\infty}^{\infty}\frac{\cos\pi n}{4n^2-1}\delta(\omega-2n)$

iii) 

$f(t) = \begin{cases} \frac{3A}{5}t+\frac{5A}{3}, & -\frac{5}{3}<t<0 \\ \frac{3A}{5}t+\frac{5A}{3}, & 0<t<\frac{5}{3} \\ f(t+T), & \text{o.c.} \end{cases}$

$\mathcal{F}\{f(t)\} = 2\pi\sum_{n=-\infty}^{\infty} C_n\,\delta(\omega-n\omega_0)$ $\qquad T=\frac{10}{3}$

$\omega_0 = \frac{2\pi}{\frac{10}{3}} = \frac{6\pi}{10} = \frac{3\pi}{5}$

$C_n = \frac{1}{2}a_n$, porque es par

$a_n = \frac{4}{T}\int_0^{5/3}\left(-\frac{3A}{5}t+\frac{5A}{3}\right)\cos n\omega_0 t$

$= \frac{4}{\frac{10}{3}}\left[\int_0^{5/3}-\frac{3A}{5}t\cos n\omega_0 t + \int_0^{5/3}\frac{5}{3}A\cos n\omega_0 t\,dt\right]$

$\frac{6A}{5}\left[-\frac{3}{5}\int_0^{5/3}t\cos n\omega_0 t\,dt + \frac{5}{3}\int_0^{5/3}\cos n\omega_0 t\,dt\right]$

$\int t\cos n\omega_0 t\,dt \qquad \begin{matrix} u=t \\ du=dt \end{matrix} \qquad \begin{matrix} dv=\cos n\omega_0 t\,dt \\ v=\frac{\operatorname{sen} n\omega_0 t}{n\omega_0} \end{matrix}$

$= \frac{t\operatorname{sen} n\omega_0 t}{n\omega_0} - \frac{1}{n\omega_0}\int \operatorname{sen} n\omega_0 t\,dt = \frac{t\operatorname{sen} n\omega_0 t}{n\omega_0} - \frac{1}{n\omega_0}\left(-\frac{\cos n\omega_0 t}{n\omega_0}\right)$

$= \frac{t\operatorname{sen} n\omega_0 t}{n\omega_0} + \frac{\cos n\omega_0 t}{n\omega_0^2} = \frac{n\omega_0 t\operatorname{sen} n\omega_0 t + \cos n\omega_0 t}{(n\omega_0)^2}\Big|_0^{5/3}$

$= \frac{1}{(n\omega_0)^2}\left[n\left(\frac{3\pi}{5}\right)\left(\frac{5}{3}\right)\operatorname{sen}\left(\left(\frac{3\pi}{5}\right)\left(\frac{5}{3}\right)n\right) + \cos\left(n\left(\frac{3\pi}{5}\right)\left(\frac{5}{3}\right)\right) - \cos 0\right]$

$$F\{f''(t)\} = 4i\omega e^{8i\omega} + Ae^{7i\omega} + \overbrace{50000\left[Sa\left(\frac{5(\omega+100)}{2}\right)e^{\frac{9}{2}i(\omega+100)} + Sa\left(\frac{5(\omega-100)}{2}\right)e^{\frac{9}{2}i(\omega-100)}\right]}^{I}$$

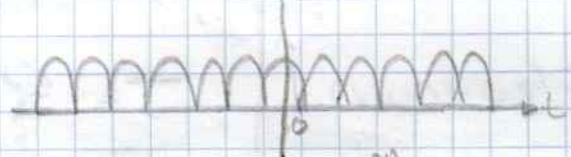
$$- Ae^{2i\omega} - 4e^{i\omega} + 4e^{-i\omega} - Ae^{-2i\omega} + \underbrace{+}_{II} + Ae^{-7i\omega} + 4i\omega e^{-8i\omega}$$

$$F\{f''(t)\} = 4i\omega(2\cos 8\omega) + A(2\cos 7\omega) - A(2\cos 2\omega) - 4(2i\,\mathrm{sen}\,\omega) + I + II$$

$$F\{f''(t)\} = -\omega^2 F(\omega)$$

$$F\{f(t)\} = \frac{8i\omega\cos 8\omega + 2A\cos 7\omega - 2A\cos 2\omega - 8i\,\mathrm{sen}\,\omega + I + II}{-\omega^2}$$

11h)



$$f(t) = \begin{cases} A\cos t, & -\frac{\pi}{2} < t < \frac{\pi}{2} \\ f(t+T), & \text{o.c.} \end{cases}$$

$T = \pi \quad \omega_0 = 2$

$$\mathcal{F}\{f(t)\} = 2\pi \sum_{n=-\infty}^{\infty} C_n\, \delta(\omega - n\omega_0)$$

$C_n = \frac{1}{2}(a_n)$, porque $f(t)$ es par

$$a_n = \int_0^{\frac{\pi}{2}} A\cos t \cos 2nt\, dt = A\int_0^{\frac{\pi}{2}} \cos t \cos 2nt\, dt \cdot \frac{4}{T}$$

$$= A\left[\int_0^{\frac{\pi}{2}} \frac{1}{2}\left(\cos(2nt-t) + \cos(2nt+t)\right)dt\right]$$

$$= \frac{A}{2}\left[\int_0^{\frac{\pi}{2}} \cos(2nt-t)\,dt + \int_0^{\pi/2} \cos(2nt+t)\,dt\right]$$

$$= \frac{A}{2}\left[\frac{\mathrm{sen}(2nt-t)}{2n-1}\Big|_0^{\pi/2} + \frac{\mathrm{sen}(2nt+t)}{2n+1}\Big|_0^{\pi/2}\right]$$

$$= \frac{A}{2}\left[\frac{(2n+1)(\mathrm{sen}(2nt-t)) + (2n-1)\mathrm{sen}(2nt+t)}{(2n-1)(2n+1)}\right]\Big|_0^{\pi/2}$$

$$= \frac{A}{2(4n^2-1)}\left[(\mathrm{sen}\,2nt\cos t - \cos 2nt\,\mathrm{sen}\,t)(2n+1) + (2n-1)(\mathrm{sen}\,2nt\cos t + \cos 2nt\,\mathrm{sen}\,t)\right]$$

$$\int t\cos n\omega_0 t\,dt = \frac{-1}{(n\omega_0)^2}\left[\cancel{n\pi \operatorname{sen}\pi n} + \cos \pi n - 1\right] = \frac{\cos \pi n - 1}{(n\omega_0)^2}$$

$$= \frac{\cos \pi n - 1}{n^2\left(\frac{3\pi}{5}\right)^2} = \frac{4}{n^2\left(\frac{9\pi^2}{25}\right)} = \frac{25}{9\pi^2}\left(\frac{\cos \pi n - 1}{n^2}\right)$$

$$\int_0^{5/3} \cos n\omega_0 t\,dt = \frac{\operatorname{sen} n\omega_0 t}{n\omega_0}\Big|_0^{5/3} = \frac{1}{n\omega_0}\left(\operatorname{sen}\left(\frac{3\pi}{5}\right)\left(\frac{5}{3}\right)n - \cancel{\operatorname{sen} 0}\right)$$

$$= \cancel{\frac{1}{n\omega_0}(\operatorname{sen}\pi n} = 0$$

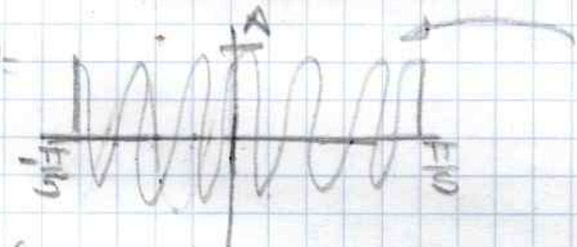
$$a_n = \frac{6A}{5}\left[-\frac{3}{5}\left(\frac{25}{9\pi^2}\left(\frac{\cos \pi n - 1}{n^2}\right)\right)\right] = -\frac{2A}{\pi^2}\left(\frac{\cos \pi n - 1}{n^2}\right)$$

$$F(t) = 2\pi \sum_{n=-\infty}^{\infty} -\frac{2A}{\pi^2}\left(\frac{\cos \pi n - 1}{n^2}\right)\delta(\omega - n\omega_0)$$

$$F(t) = -\frac{4A}{\pi}\sum_{n=-\infty}^{\infty}\left(\frac{\cos \pi n - 1}{n^2}\right)\delta(\omega - n\omega_0)$$

12; Con el teorema de modulación, encuentra las sig. transformadas

12a)



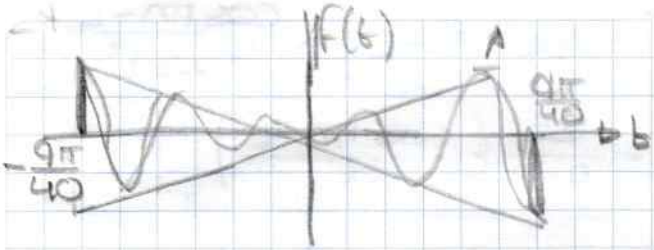
$A\cos 20t$

La función mostrada es el resultado del producto de $A\cos 20t$ con una función compuerta

$$A\cos 20t\, C_{\frac{2\pi}{5}}(t) \longleftrightarrow ?$$

$$A C_{\frac{2\pi}{5}} \longleftrightarrow \frac{2A\pi}{5} Sa\left(\frac{\pi\omega}{5}\right)$$

$$A C_{\frac{2\pi}{5}}(t)\cos 20t \longleftrightarrow \frac{A\pi}{5}\left[Sa\left(\frac{\pi(\omega+20)}{5}\right) + Sa\left(\frac{\pi(\omega-20)}{5}\right)\right]$$



$$f(t) = \begin{cases} -\frac{40A}{9\pi} t\cos 20t, & -\frac{9\pi}{40} < t < \frac{9\pi}{40} \\ 0, & \text{o.c.} \end{cases}$$

Podemos acotar la función con una compuerta

$$f(t)\,C_{\frac{9\pi}{20}}(t) = -\frac{40A}{9\pi} t\cos 20t\, C_{\frac{9\pi}{20}}(t)$$

$$-\frac{40A}{9\pi} C_{\frac{9\pi}{20}} \longleftrightarrow -\frac{40}{9\pi}\left(\frac{9\pi}{20}\right) Sa\left(\frac{9\pi\omega}{40}\right) = 2Sa\left(\frac{9\pi\omega}{40}\right)$$

$$-\frac{40A}{9\pi} C_{\frac{9\pi}{20}}(t)\cos 20t \longleftrightarrow Sa\left(\frac{9\pi(\omega+20)}{40}\right) + Sa\left(\frac{9\pi(\omega-20)}{40}\right)$$

$$-\frac{40A}{9\pi} t\, C_{\frac{9\pi}{20}}(t)\cos 20t \longleftrightarrow (i\omega)\left[Sa\left(\frac{9\pi(\omega+20)}{40}\right) + Sa\left(\frac{9\pi(\omega-20)}{40}\right)\right]$$