INSTITUTO POLITECNICO NACIONAL

ESCUELA SUPERIOR DE COMPUTO

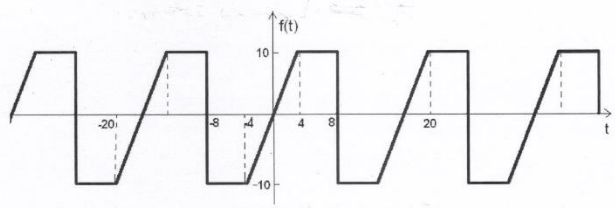
Teoría de Comunicaciones y Señales

1er. Exámen departamental

NOMBRE:____ TIPO: B

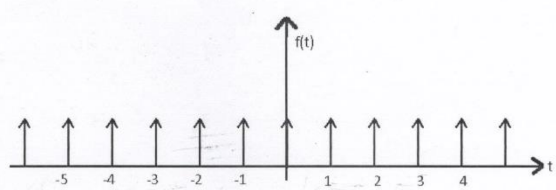
GRUPO:______________________

Problema 1. Encuentre la Serie Trigonométrica de Fourier de x(t)



Problema 2. A partir de la serie obtenida en el problema anterior, encuentra la Serie Exponencial de Fourier.

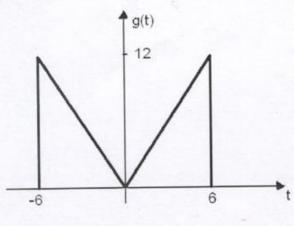
Problema 3. Encuentre la transformada de Fourier de la siguiente función periódica:



Problema 4. Usando propiedades, encuentre la transformada inversa de Fourier de:

$$? \leftrightarrow 6\delta(3\omega - 10)cos(15\omega) + \frac{1}{3 - j\omega}\omega^2 + e^{j4\omega}(\omega - 1)$$

Problema 5. Usando Propiedades de la transformada de Fourier encuentre la transformada de f(t)



$1^{\underline{er}}$ Exámen Departamental

TIPO B

Problema 1. (2.0 pts)

$$f(t) = \begin{cases} -10 & -8<t<-4 \\ \frac{5}{2}t & -4<t<4 \\ 10 & 4<t<8 \\ f(t+16) & \text{otro caso} \end{cases}$$

$T = 16$

$\omega_0 = \frac{2\pi}{16} = \frac{\pi}{8}$

$f(t)$ es impar ∴

$a_0 = a_n = 0$

&

$$b_n = \frac{4}{T}\int_0^{\frac{T}{2}} f(t)\, \text{Sen}\, n\omega_0 t\, dt$$

$$b_n = \frac{4}{16}\int_0^{8} f(t)\, \text{Sen}\, \frac{n\pi}{8}t\, dt$$

$$b_n = \frac{1}{4}\int_0^{4} \frac{5}{2}t\, \text{Sen}\frac{n\pi}{8}t\, dt + \frac{1}{4}\int_4^{8} 10\, \text{Sen}\, \frac{n\pi}{8}t\, dt$$

$u = t \quad dv = \text{sen}\, \frac{n\pi}{8}t\, dt$

$du = dt \quad v = -\frac{8}{n\pi}\cos\frac{n\pi}{8}t$

$$b_n = \frac{5}{8}\left\{ \frac{-8}{n\pi}t \cdot \cos\frac{n\pi}{8}t \Big|_0^4 + \frac{8}{n\pi}\int_0^4 \cos\frac{n\pi}{8}t\, dt \right\}$$

$$+\left(\frac{5}{2}\right)\left(-\frac{8}{n\pi}\right)\text{Cos}\, \frac{n\pi}{8}t\, \Big|_4^8$$

$$b_n = \frac{5}{8}\left\{ \frac{-32}{n\pi}\cos\frac{n\pi}{2} + \phi + \frac{64}{n^2\pi^2}\text{sen}\, \frac{n\pi}{8}t \Big|_0^4 - \frac{20}{n\pi}\cos\frac{n\pi}{8}t \Big|_4^8 \right.$$

$$b_n = -\frac{20}{n\pi}\cos\frac{n\pi}{2} + \frac{40}{n^2\pi^2}\left[\text{sen}\, \frac{n\pi}{2} - \phi\right] - \frac{20}{n\pi}\left[\cos n\pi - \cos\frac{n\pi}{2}\right]$$

$$= \cancel{\frac{-20}{n\pi}\cos\frac{n\pi}{2}} + \frac{40}{n^2\pi^2}\text{sen}\, \frac{n\pi}{2} - \frac{20}{n\pi}\cos n\pi + \cancel{\frac{20}{n\pi}\cos\frac{n\pi}{2}}$$

$$b_n = \frac{40}{n^2\pi^2}\text{sen}\, \frac{n\pi}{2} - \frac{20}{n\pi}(-1)^n$$

Finalmente:

$$f(t) = \sum_{n=1}^{\infty}\left[\frac{40}{n^2\pi^2}\text{Sen}\, \frac{n\pi}{2} - \frac{20}{n\pi}(-1)^n\right]\text{Sen}\frac{n\pi}{8}t$$

TIPO B

Problema 2 (1.5 pts)

Si $f(t) = \sum_{n=1}^{\infty} \left[\frac{40}{n^2\pi^2} \operatorname{sen} \frac{n\pi}{2} - \frac{20}{n\pi}(-1)^n \right] \cdot \operatorname{Sen} \frac{n\pi}{8} t$

Entonces, su serie exponencial, está dada por:

$$f(t) = \sum_{n=\infty}^{\infty} C_n \, e^{j\frac{n\pi}{8}t} \quad \text{con} \quad C_n = \frac{1}{2}(a_n - i b_n)$$

Pero como $a_n = 0$

$$\therefore \; C_n = -\frac{j}{2} b_n = -j \left[\frac{20}{n^2\pi^2} \operatorname{sen} \frac{n\pi}{2} - \frac{10}{n\pi}(-1)^n \right]$$

$$C_n = j \left[\frac{10}{n\pi}(-1)^n - \frac{20}{n^2\pi^2} \operatorname{sen} \frac{n\pi}{2} \right]$$

Finalmente:

$$f(t) = \sum_{n=\infty}^{\infty} j \left[\frac{10}{n\pi}(-1)^n - \frac{20}{n^2\pi^2} \operatorname{Sen} \frac{n\pi}{2} \right] \cdot e^{j\frac{n\pi}{8}t}$$

$$f(t) = \sum_{n=-\infty}^{\infty} j\frac{10}{n\pi} \left[(-1)^n - \frac{2}{n\pi} \operatorname{sen} \frac{n\pi}{2} \right] e^{j\frac{n\pi}{8}t}$$

1.5

Problema 3

$$\mathcal{F}\{f(t)\} = 2\pi \sum_{n=-\infty}^{\infty} C_n \, \delta(\omega - n\omega_0)$$

Si $T = 1 \therefore \omega_0 = \frac{2\pi}{T} = 2\pi$

& $$C_n = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)\, e^{-jn\omega_0 t}\, dt$$

$$C_n = \int_{-\frac{1}{2}}^{\frac{1}{2}} f(t) \cdot e^{-j2\pi n t}\, dt$$

$$C_n = \int_{0^-}^{0^+} \delta(t)\, e^{-j2\pi n t}\, dt$$

De la Propiedad de muestreo de $\delta(t)$

$$C_n = e^{0} = 1$$

Finalmente:

$$\mathcal{F}\{f(t)\} = 2\pi \sum_{n=-\infty}^{\infty} \delta(\omega - 2\pi n)$$

2.0 pts

Problema 4 (2,5 pts)

$$? \longleftrightarrow \underbrace{6\delta(3\omega-10)\cdot \cos 15\omega}_{\mathrm{I}} + \underbrace{\frac{1}{3-i\omega}\,\omega^2}_{\mathrm{II}} + \underbrace{e^{i4\omega}(\omega-1)}_{\mathrm{III}}$$

Solución:

I Si $\delta(t) \longleftrightarrow 1$

$$\frac{1}{2\pi} \longleftrightarrow \delta(-\omega)$$

$$\frac{1}{2\pi}e^{i10t} \longleftrightarrow \delta(\omega-10)$$

$$\frac{1}{2\pi}e^{i\frac{10}{3}t} \longleftrightarrow \frac{1}{\left|\frac{1}{3}\right|}\delta\left(\frac{\omega}{\frac{1}{3}}-10\right)$$

$$\frac{1}{2\pi}e^{i\frac{10}{3}t} \longleftrightarrow 3\delta(3\omega-10)$$

$$\frac{1}{\pi}e^{i\frac{10}{3}t} \longleftrightarrow 6\delta(3\omega-10)$$

$$\frac{1}{2\pi}\left[e^{i\frac{10}{3}(t+15)}+e^{i\frac{10}{3}(t-15)}\right] \longleftrightarrow 6\delta(3\omega-10)\cdot\cos 15\omega$$

II Si

$$e^{-at}u(t) \longleftrightarrow \frac{1}{a+i\omega}$$

$$e^{-3t}u(t) \longleftrightarrow \frac{1}{3+i\omega}$$

$$\frac{d^2}{dt^2}\left[e^{-3t}u(t)\right] \longleftrightarrow \frac{(j\omega)^2}{3+i\omega}$$

$$-\frac{d^2}{d^2t}\left[e^{+3t}u(-t)\right] \longleftrightarrow \frac{\omega^2}{3-i\omega}$$

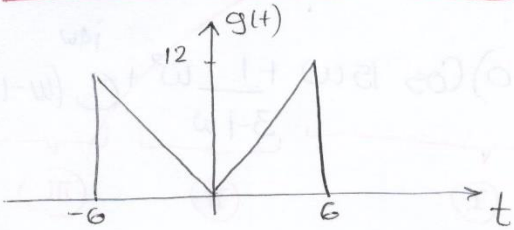
III Si $\delta(t) \longleftrightarrow 1$

$$\frac{d}{dt}\delta(t) \longleftrightarrow i\omega$$

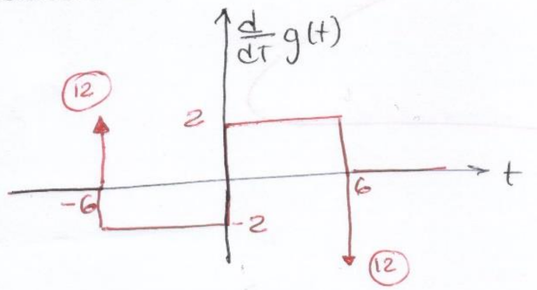
$$-j\frac{d}{dt}\delta(t)\cdot e^{it} \longleftrightarrow (\omega-1)$$

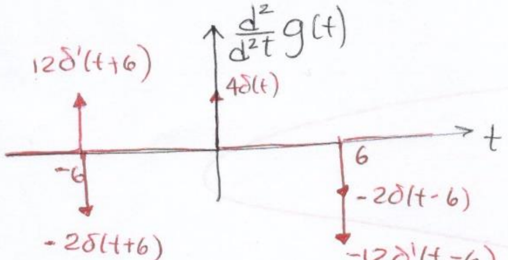
$$-i\frac{d}{dt}\delta(t+4)\cdot e^{i(t+4)} \longleftrightarrow (\omega-1)e^{i4\omega}$$

Problema 5 (2.0 pts)



SOLUCION:





$$\frac{d^2}{d^2t}g(t) = -2\delta(t+6) + 12\delta'(t+6) + 4\delta(t) - 2\delta(t-6) - 12\delta'(t-6)$$

$$\mathcal{F}\left\{\frac{d^2}{d^2t}g(t)\right\} = -2\mathcal{F}\{\delta(t+6)\} + 12\mathcal{F}\{\delta'(t+6)\} + 4\mathcal{F}\{\delta(t)\} - 2\mathcal{F}\{\delta(t-6)\} - 12\mathcal{F}\{\delta'(t-6)\}$$

$$\mathcal{F}\left\{\frac{d^2}{d^2t}g(t)\right\} = -2e^{i6\omega} + 12i\omega e^{i6\omega} + 4 - 2e^{-i6\omega} - 12j\omega e^{-i6\omega}$$

$$= -2(e^{i6\omega} + e^{-i6\omega}) + 12j\omega(e^{i6\omega} - e^{-i6\omega}) + 4$$

$$= -4\cos 6\omega - 24\omega \operatorname{Sen} 6\omega + 4$$

$$= 4(1 - \cos 6\omega) - 24\omega \operatorname{Sen} 6\omega$$

$$= 8 \cdot \operatorname{sen}^2 3\omega - 24\omega \operatorname{Sen} 6\omega$$

Así

$$\frac{d^2}{d^2t}g(t) \longleftrightarrow 8\operatorname{Sen}^2 3\omega - 24\omega \operatorname{Sen} 6\omega$$

De la Prop. de diferenciación en t

Si $g(t) \longleftrightarrow G(\omega)$

$$\frac{d^2}{d^2t}g(t) \longleftrightarrow (j\omega)^2 G(\omega)$$

Finalmente

$$(j\omega)^2 G(\omega) = 8\operatorname{sen}^2 3\omega - 24\omega \operatorname{Sen} 6\omega$$

$$G(\omega) = \frac{-1}{\omega^2}\left[8\operatorname{sen}^2 3\omega - 24\omega \operatorname{Sen} 6\omega\right]$$

$$G(\omega) = \frac{24}{\omega}\operatorname{Sen} 6\omega - \frac{8}{\omega^2}\operatorname{sen}^2 3\omega$$

$$G(\omega) = 144\, Sa\, 6\omega - 8(9)\frac{\operatorname{sen} 3\omega}{(3)\omega} \cdot \frac{\operatorname{sen} 3\omega}{(3)\omega}$$

$$G(\omega) = 144\, Sa\, 6\omega - 72\, Sa^2\, 3\omega$$