A thick dark blue vertical bar runs down the left side of the page. A blue arrow-shaped banner points to the right from this bar, containing the date. In the bottom-left corner, several thin, curved lines in dark blue and light grey sweep upwards and to the right.

7-5-2021

Examen 1

Martínez Coronel Brayan Yosafat

① $T=6$ $\omega_0 = \pi/3$

$$x(t) = \begin{cases} 1 & -3 \leq t \leq -2 \\ -\frac{t}{2} & -2 \leq t \leq 0 \\ \frac{t}{2} & 0 \leq t \leq 2 \\ 1 & 2 \leq t \leq 3 \end{cases}$$

$x(t)$ es par $b_n = 0$

$$a_0 = \frac{1}{T} \int_{t_0}^{t_0+T} f(t) dt$$

$$a_n = \frac{2}{T} \int_{t_0}^{t_0+T} f(t) \cos(n\omega_0 t) dt$$

$$a_0 = \frac{2}{6} \int_{-2}^2 dt + \frac{2}{6} \int_0^2 \frac{t}{2} dt = \frac{1}{3} t \Big|_{-2}^2 + \frac{1}{12} t^2 \Big|_0^2 = \frac{4}{3} + \frac{1}{3} = \frac{5}{3}$$

$$a_n = \frac{2}{3} \int_0^2 \frac{t}{2} \cos(n\pi t/3) dt + \frac{2}{3} \int_2^3 \cos(n\pi t/3) dt$$

$$= \frac{1}{3} \left[\frac{3}{n\pi} t \sin\left(\frac{n\pi t}{3}\right) + \frac{9}{n^2\pi^2} \cos\left(\frac{n\pi t}{3}\right) \right]_0^2 + \frac{2}{n\pi} \sin\left(\frac{n\pi t}{3}\right) \Big|_2^3$$

$$= \frac{1}{n\pi} \left[2 \sin\left(\frac{2n\pi}{3}\right) + \frac{3}{n\pi} \cos\left(\frac{n\pi 2}{3}\right) - 0 - \frac{3}{n\pi} \right] + 0 - \frac{2}{n\pi} \left(\frac{2n\pi}{3} \right)$$

$$= \frac{3}{n^2\pi^2} \left(\cos\left(\frac{n\pi 2}{3}\right) - 1 \right)$$

$$x(t) = \frac{2}{3} + \sum_{n=1}^{\infty} \frac{3}{n^2\pi^2} \left(\cos\left(\frac{2n\pi}{3}\right) - 1 \right) \cos\left(\frac{n\pi t}{3}\right)$$

Marlín
Coronel
Drogon
Lasalet

② La función ya está en forma de suma de cosenos y senos, es ella misma

$$f(t) = \cos t$$

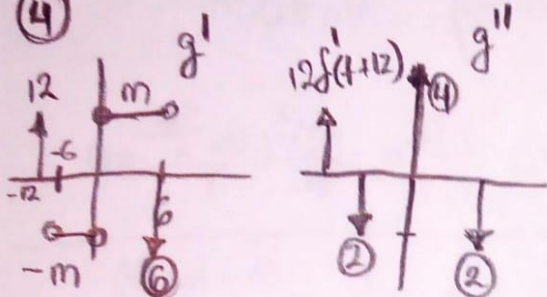
$$\textcircled{3} \quad AC_d(t) \leftrightarrow Ad \text{Sa}\left(\frac{\omega d}{2}\right)$$

$$C_y(t) \leftrightarrow 4 \text{Sa}(2\omega)$$

$$C_y(t) \cos 20t \leftrightarrow 2[\text{Sa}(2\omega + 40) + \text{Sa}(2\omega - 40)]$$

$$10 C_y(t) \cos 20t \leftrightarrow 20[\text{Sa}(2\omega + 40) + \text{Sa}(2\omega - 40)]$$

$\textcircled{4}$



$$m = 2 = \left| \frac{0-12}{0+6} \right|$$

$$g''(t) = 12\delta'(t+12) - 2\delta(t+6) + 4\delta(t) - 2\delta(t-6) - 6\delta'(t-6)$$

$$(i\omega)^2 G(\omega) = 12(i\omega)e^{12i\omega} - 2e^{6i\omega} + 4 - 2e^{-6i\omega} - 6i\omega e^{-6i\omega}$$

$$G(\omega) = \frac{-1}{\omega^2} [12i\omega(e^{12i\omega} - e^{-6i\omega}) - 2e^{6i\omega} - 2e^{-6i\omega} + 4]$$

$\textcircled{5}$

$$e^{-at} u(t) \leftrightarrow \frac{1}{a+i\omega}$$

$$\delta(t) \leftrightarrow 1$$

$$e^{-3t} u(t) \leftrightarrow \frac{1}{3+i\omega}$$

$$\delta'(t) \leftrightarrow i\omega$$

$$\frac{d^2}{dt^2} [e^{-3t} u(t)] \leftrightarrow \frac{(i\omega)^2}{3+i\omega}$$

$$-i\delta'(t)e^{it} \leftrightarrow (\omega-1)$$

$$-\frac{d^2}{dt^2} [e^{+3t} u(t)] \leftrightarrow \frac{\omega^2}{3-i\omega}$$

$$-i\delta'(t+4)e^{i(t+4)} \leftrightarrow (\omega-1)e^{i4\omega}$$

$$-\frac{d^2}{dt^2} [e^{3t} u(t)] - i\delta'(t+4)e^{i(t+4)} \leftrightarrow \frac{\omega^2}{3-i\omega} + e^{i4\omega}(\omega-1)$$

⑥

$$F(\omega) = \frac{1}{(1+i\omega)^2} = \frac{1e^0}{(1+\omega^2)e^{2\arctan(\omega)}} = \frac{1}{1+\omega^2} e^{-2\arctan(\omega)}$$

$$|F(\omega)| = \frac{1}{1+\omega^2} \quad \theta = -2\arctan(\omega)$$

