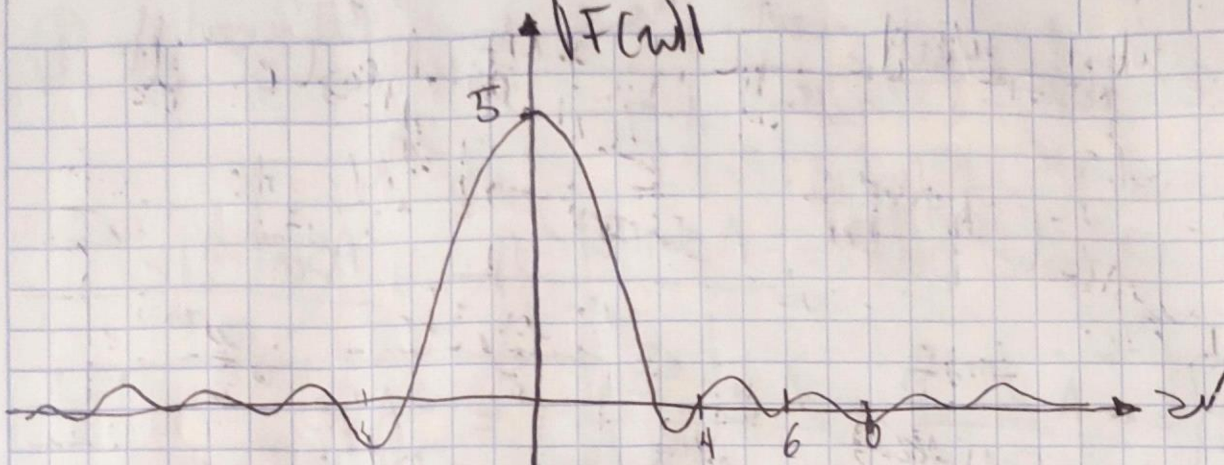
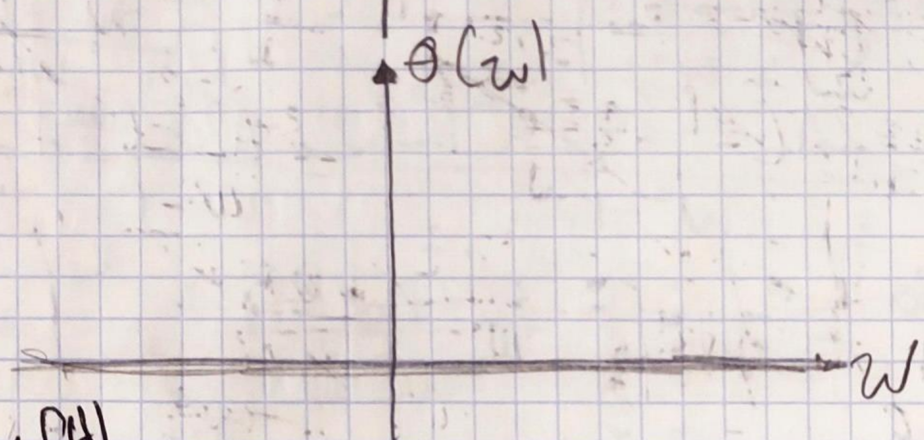
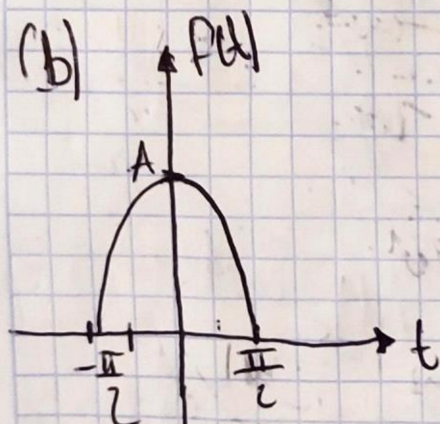




(b)



$$f(t) = \begin{cases} A\cos t & -\frac{\pi}{2} < t < \frac{\pi}{2} \\ 0 & \text{en otro caso} \end{cases}$$

$\int u\,dv = uv - \int v\,du$

$$\tilde{F}\{f(t)\} = A\int_{-\frac{\pi}{2}}^{\pi/2} \cos t\, e^{-iwt}\,dt$$

$u = \cos t$ $\qquad$ $\int dv = \int e^{-iwt}$

$du = \operatorname{sen} t$ $\qquad$ $v = -\frac{e^{-iwt}}{iw}$

$$\tilde{F}\{f(t)\} = A\int_{-\frac{\pi}{2}}^{\pi/2} \cos t\, e^{-iwt}\,dt = A\left\{ -\frac{\cos t\, e^{-iwt}}{iw}\Big|_{-\frac{\pi}{2}}^{\pi/2} - \int_{-\pi/2}^{\pi/2} \frac{e^{-iwt}}{iw}\operatorname{sen} t\,dt \right\}$$

$$= A\left\{ \cancel{-\frac{\cos(\frac{\pi}{2})e^{-iwt}}{iw} + \frac{\cos(\frac{\pi}{2})e^{-iwt}}{iw}}^{\,0} - \frac{1}{iw}\int_{-\frac{\pi}{2}}^{\pi/2} e^{-iwt}\operatorname{sen} t\,dt \right\} = -\frac{A}{iw}\left\{ -\frac{\operatorname{sen} t\, e^{-iwt}}{wi} + \int_{-\frac{\pi}{2}}^{\pi/2} \frac{\cos t\, e^{-iwt}}{iw}\,dt \right\}$$

$u = \operatorname{sen} t$

$du = \cos t$

$$= A\left\{ -\frac{1}{iw}\left[-\frac{\operatorname{sen} t\, e^{-iwt}}{iw}\Big|_{-\pi/2}^{\pi/2} + \frac{1}{iw}\int_{-\pi/2}^{\pi/2} \cos t\, e^{-iwt}\,dt \right] \right\}$$

$$= A\left\{ \frac{\operatorname{sen} t\, e^{-iwt}}{i^2 w^2}\Big|_{-\pi/2}^{\pi/2} + \frac{1}{w^2}\int_{-\pi/2}^{\pi/2} \cos t\, e^{-iwt}\,dt \right\} = -\frac{A \operatorname{sen} t\, e^{-iwt}}{w^2} + \frac{A}{w^2}\int_{-\frac{\pi}{2}}^{\pi/2} \cos t\, e^{-iwt}\,dt$$

$$A\int_{-\pi/2}^{\pi/2} \cos t\, e^{-i\omega t}\,dt = \frac{-A\sin t\, e^{-i\omega t}}{\omega^2}\Bigg|_{-\pi/2}^{\pi/2} + \frac{A}{\omega^2}\int_{-\pi/2}^{\pi/2} \cos t\, e^{-i\omega t}\,dt$$

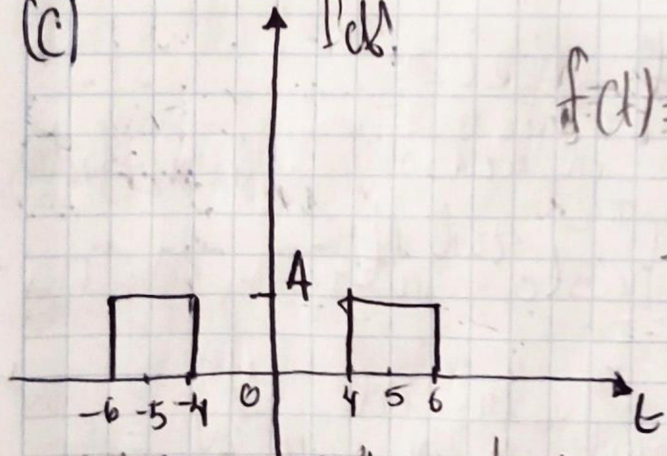
$$\left(1-\frac{1}{\omega^2}\right)A\int_{-\pi/2}^{\pi/2} \cos t\, e^{-i\omega t}\,dt = \frac{-A\sin(\frac{\pi}{2})e^{-i\omega\frac{\pi}{2}}}{\omega^2} + \frac{A\sin(-\frac{\pi}{2})e^{i\omega\frac{\pi}{2}}}{\omega^2}$$

$$\mathcal{F}\{f(t)\} = \frac{\frac{-Ae^{-i\omega\frac{\pi}{2}}}{\omega^2} - \frac{Ae^{i\omega\frac{\pi}{2}}}{\omega^2}}{\left(\frac{\omega^2-1}{\omega^2}\right)} = \frac{\frac{-A(e^{-i\omega\frac{\pi}{2}}+e^{i\omega\frac{\pi}{2}})}{\omega^2}}{\frac{\omega^2-1}{\omega^2}}$$

$$\mathcal{F}\{f(t)\} = \frac{-2A\left(\frac{e^{-i\omega\frac{\pi}{2}}+e^{i\omega\frac{\pi}{2}}}{2}\right)}{\omega^2-1} = \frac{-A2\cos(\frac{\omega\pi}{2})}{\omega^2-1}$$

$$\mathcal{F}\{f(t)\}\ \frac{-2A\cos(\frac{\omega\pi}{2})}{\omega^2-1}$$

(c)



$$f(t) = \begin{cases} A & -6<t<-4 \\ 0 & -4<t<4 \\ A & 4<t<6 \end{cases}$$

$$\mathcal{F}\{f(t)\} = \int_{-\infty}^{\infty} f(t)e^{-i\omega t}$$

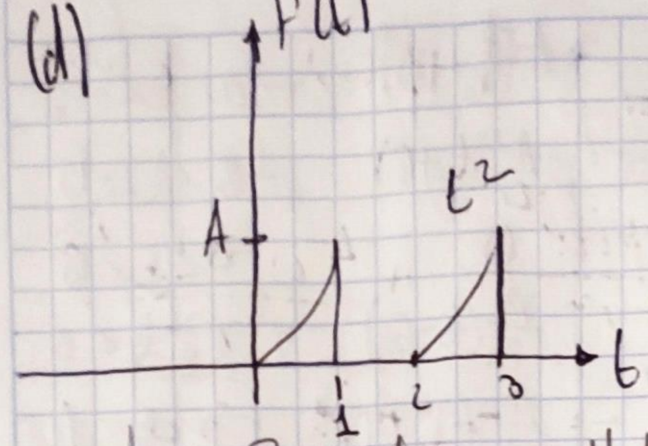
$$\mathcal{F}\{f(t)\} = \int_{-6}^{-4} Ae^{-i\omega t}\,dt + \cancel{\int_{-4}^{4} 0e^{-i\omega t}\,dt}^{\,0} + \int_{4}^{6} Ae^{-i\omega t}\,dt$$

$$= \frac{Ae^{-i\omega t}}{-i\omega}\Bigg|_{-6}^{-4} + \frac{Ae^{-i\omega t}}{-i\omega}\Bigg|_{4}^{6} = \frac{-A}{i\omega}\left(e^{+i\omega 4} - e^{i\omega 6} + e^{-i\omega 6} - e^{-i\omega 4}\right)$$

$$= \frac{-2A}{\omega}\left(\frac{e^{i\omega 4}-e^{-i\omega 4}}{2i} - \left(\frac{-e^{-i\omega 6}+e^{i\omega 6}}{2i}\right)\right) = -2A\left(\sin(4\omega) - \sin(6\omega)\right)$$

$$\mathcal{F}\{f(t)\} = 2A\left(\sin(6\omega) - \sin(4\omega)\right)$$

(d)



$$f(t)=\begin{cases} At^2 & 0<t<1 \\ 0 & 1<t<2 \\ A(t-2)^2 & 2<t<3 \end{cases}$$

$$\mathcal{F}\{f(t)\}=A\left\{\int_0^1 t^2 e^{-i\omega t}\,dt+0+\int_2^3 (t-2)^2 e^{-i\omega t}\,dt\right\}$$

$t^2 \quad +e^{-i\omega t}$

$2t \quad -e^{-i\omega t}/(-i\omega)$

$2 \quad +e^{-i\omega t}/(-i\omega)^2$

$e^{-i\omega t}/(-i\omega)^3$

$$A\left\{\frac{t^2 e^{-i\omega t}}{-i\omega}-\frac{2t e^{-i\omega t}}{(-i\omega)^2}+\frac{2e^{-i\omega t}}{(-i\omega)^3}\right|_0^1+\left\{\frac{(t-2)^2 e^{-i\omega t}}{-i\omega}-\frac{2(t-2)e^{-i\omega t}}{(-i\omega)^2}+\frac{2e^{-i\omega t}}{(-i\omega)^3}\right|_2^3$$

$$=A\left\{\frac{e^{-i\omega}}{-i\omega}-\frac{2e^{-i\omega}}{i^2\omega^2}+\frac{2e^{-i\omega}}{-i^2 i\omega}-\frac{2}{-i^2 i\omega}\right\}+\left\{\frac{e^{-i\omega 3}}{-i\omega}-\frac{2e^{-i\omega 3}}{i^2\omega^2}-\frac{2e^{-i\omega 3}}{i^3\omega^3}\right.$$

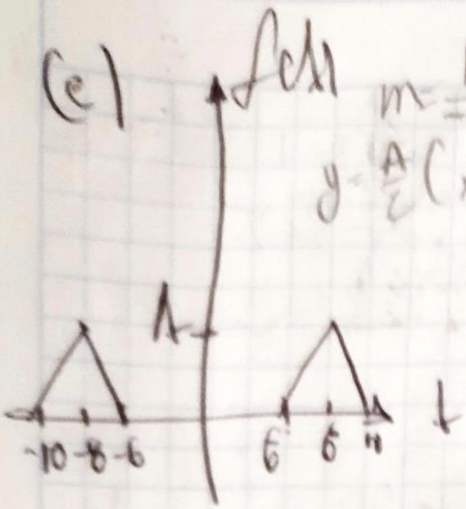
$$\left.-0-0+\frac{2e^{i\omega 2}}{i^3\omega^3}\right\}=A\left\{-\frac{e^{-i\omega}}{i\omega}+\frac{2e^{-i\omega}}{\omega^2}+\frac{2e^{-i\omega}}{i\omega^3}-\frac{e^{-i\omega 3}}{i\omega}+\frac{2e^{-i\omega 3}}{\omega^2}\right.$$

$$\left.+\frac{2e^{-i\omega 3}}{i\omega^3}-\frac{2e^{i\omega 2}}{i\omega^3}-\frac{2}{i\omega}\right\}$$

$$\mathcal{F}\{f(t)\}=A\left\{\frac{e^{-i\omega}}{\omega}i+\frac{e^{-i\omega 3}}{\omega}i+\frac{2e^{-i\omega}}{\omega^2}+\frac{2e^{-i\omega 3}}{\omega^2}-\frac{2e^{-i\omega}}{\omega^3}i\right.$$

$$\left.\frac{2e^{i\omega 2}i}{\omega^3}-\frac{2e^{-i\omega 3}}{\omega^3}+\frac{2}{\omega}i\right\}$$

(c)



$m = \frac{A-0}{-8+10} = \frac{A}{2}$

$y = \frac{A}{2}(x+10)$

$$f(t) = \begin{cases} \frac{A}{2}(t+10) & -10 < t < -8 \\ -\frac{A}{2}(t+6) & -8 < t < -6 \\ 0 & -6 < t < 6 \\ \frac{A}{2}(t-6) & 6 < t < 8 \\ \frac{A}{2}(-t+10) & 8 < t < 10 \end{cases}$$

$$\mathcal{F}\{f(t)\} = \int_{-10}^{-8} \frac{A}{2}(t+10)e^{-i\omega t}\,dt - \int_{-8}^{-6}\frac{A}{2}(t+6)e^{-i\omega t}\,dt + 0 + \int_{6}^{8}\frac{A}{2}(t-6)\,dt + \int_{8}^{10}\frac{A}{2}(-t+10)\,dt = \frac{A}{2}\Big\{ \frac{(t+10)e^{-i\omega t}}{-i\omega}\Big|_{-10}^{-8} + \frac{e^{-i\omega t}}{\omega^2}\Big|_{-10}^{-8}$$

$$+ \frac{(t+6)e^{-i\omega t}}{i\omega}\Big|_{-8}^{-6} - \frac{e^{-i\omega t}}{\omega^2}\Big|_{-8}^{-6} + \frac{(t-6)e^{-i\omega t}}{-i\omega}\Big|_{6}^{8} + \frac{e^{-i\omega t}}{\omega^2}\Big|_{6}^{8}$$

$$\frac{(-t+10)e^{-i\omega t}}{-i\omega}\Big|_{8}^{10} - \frac{e^{-i\omega t}}{\omega^2}\Big|_{8}^{10}\Big\}$$

$t+6 \quad + e^{-i\omega t}$

$1 \quad - e^{-i\omega t}/i\omega \quad e^{-i\omega t}/i\omega^2$

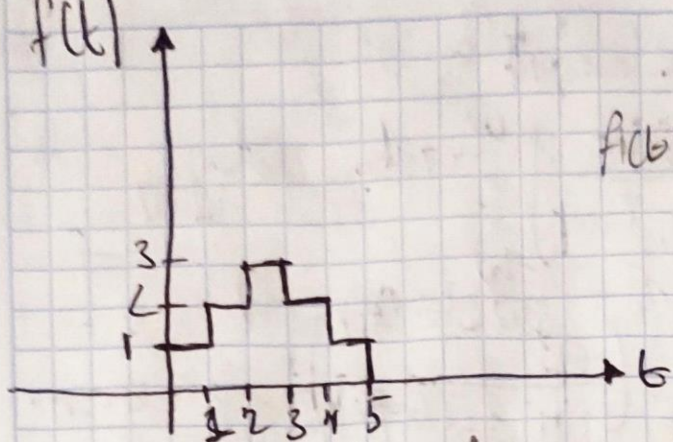
$$\mathcal{F}\{f(t)\} = \frac{A}{2}\Big\{\frac{2e^{i\omega 8}}{-i\omega} + \frac{e^{i\omega 8}}{\omega^2} + 0 - \frac{e^{i\omega 10}}{\omega^2} + 0 - \frac{e^{i\omega 6}}{\omega^2}$$

$$-\frac{2e^{i\omega 8}}{-8\omega} + \frac{e^{i\omega 6}}{\omega^2} \; \frac{2e^{-8i\omega}}{i\omega} + \frac{e^{-8i\omega}}{\omega^2} - \frac{e^{-6i\omega}}{\omega^2} \; 0 - \frac{e^{-i\omega 10}}{\omega^2}$$

$$+ \frac{e^{-8i\omega}}{\omega^2} - \frac{2i\,e^{-8i\omega}}{\omega}\Big\}$$

$$\mathcal{F}\{f(t)\} = A\Big\{\frac{4\,sen^2(\omega)\cos(8\omega)}{\omega^2}\Big\}$$

(b)



$$f(t) = \begin{cases} 1 & 0<t<1 \\ 2 & 1<t<2 \\ 3 & 2<t<3 \\ 2 & 3<t<4 \\ 1 & 4<t<5 \end{cases}$$

$$\mathcal{F}\{f(t)\} = \int_0^1 e^{-iwt}\,dt + 2\int_1^2 e^{-iwt}\,dt + 3\int_2^3 e^{-iwt}\,dt + 2\int_3^4 e^{-iwt}\,dt$$

$$+ \int_4^5 e^{-iwt} = \frac{e^{-iwt}\, i}{-iw \cdot -i}\Big|_0^1 + \frac{2e^{-iwt}\, i}{w}\Big|_1^2 + \frac{3e^{-iwt}\, i}{w}\Big|_2^3 + \frac{2e^{-iwt}\, i}{w}\Big|_3^4$$

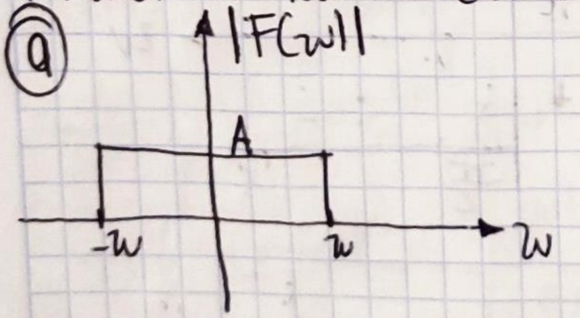
$$+ \frac{e^{-iwt}\, i}{w}\Big|_4^5 = \frac{i}{w}\Big(e^{-iw} + \cancel{2e^{-iw2}} - 2e^{-iw} + \cancel{3e^{-iw3}} - \cancel{3e^{-iw2}}$$

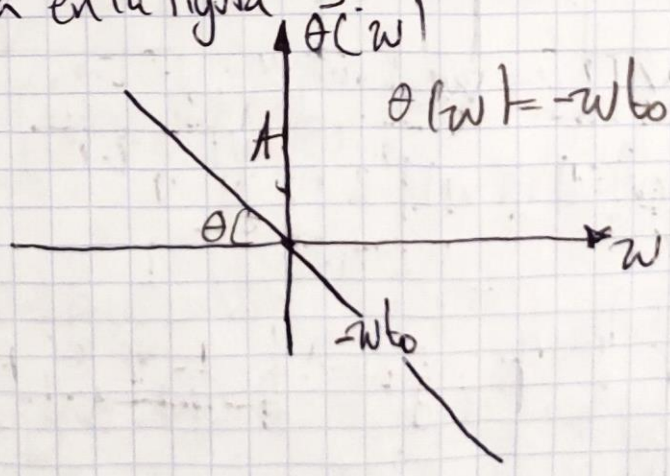
$$+ \cancel{2e^{-iw4}} - \cancel{2e^{-iw3}} + e^{-iw5} - \cancel{e^{-iw4}}\Big)$$

$$\mathcal{F}\{f(t)\} = \frac{i}{w}\left(-e^{-iw} - e^{-iw2} + e^{-iw3} + e^{-iw4} + e^{-iw5}\right)$$

Problema 6. Determinar cada una de las señales $f(t)$ cuya transformada de Fourier se ilustra en la figura 5.

(a)





$$F(w) = |F(w)|\, e^{i\theta(w)}$$

$$F(w) \begin{cases} Ae^{-iwt_0} & -W < w < W \\ 0 & \text{en otro caso} \end{cases}$$

$$F(w) = Ae^{-iwt_0} \qquad -W < t < W$$

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(w) e^{iwt}\, dw = \frac{A}{2\pi}\int_{-W}^{W} e^{-iwt_0} e^{iwt}\, dw$$

$$= \frac{A}{2\pi}\int_{-W}^{W} e^{iw(t-t_0)}\, dw = \frac{A}{2\pi}\left(\frac{e^{iw(t-t_0)}}{i(t-t_0)}\right)\Bigg|_{-W}^{W}$$

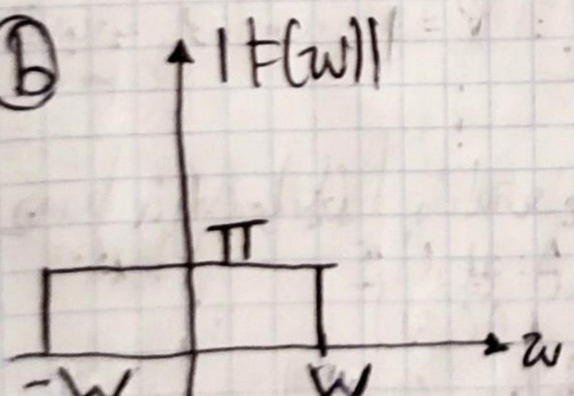
$$= \frac{A}{i2\pi(t-t_0)}\left(e^{iW(t-t_0)} - e^{-iW(t-t_0)}\right)$$

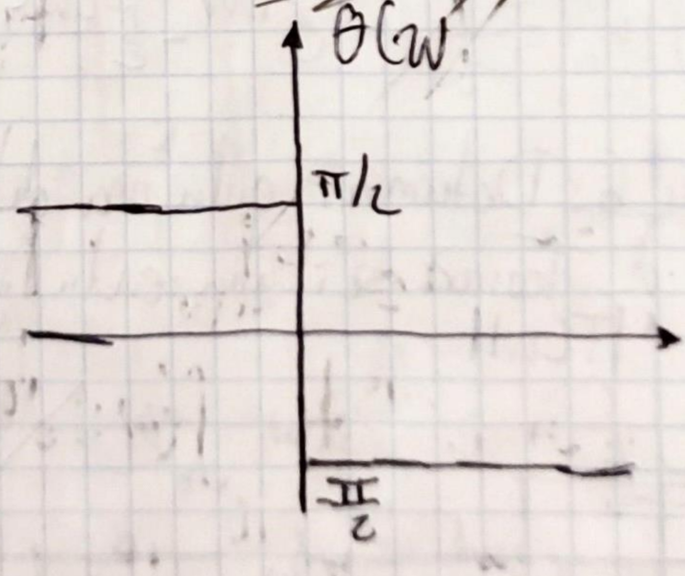
$$f(t) = \frac{A}{\pi(t-t_0)}\left[\frac{e^{iW(t-t_0)} - e^{-iW(t-t_0)}}{i2}\right]$$

$$f(t) = \frac{A}{\pi(t-t_0)}\frac{W}{W}\operatorname{Sen}(W(t-t_0)) = \frac{WA}{\pi}\operatorname{Sa}(W(t-t_0))$$

$$f(t) = \frac{WA}{\pi}\operatorname{Sa}(W(t-t_0))$$

(b)





$$F(w) = |F(w)|\, e^{i\theta(w)}$$

$$F(w) = \begin{cases} \pi e^{i\frac{\pi}{2}} & -W < w < 0 \\ \pi e^{-i\frac{\pi}{2}} & 0 < w < W \end{cases}$$

$$f(t) = \frac{\pi}{2\pi}\left\{\int_{-W}^{0} e^{i\frac{\pi}{2}} e^{iwt}\, dw + \int_{0}^{W} e^{-i\frac{\pi}{2}} e^{iwt}\, dw\right\} = \pi\left\{e^{i\frac{\pi}{2}}\int e^{iwt}\, dw\right.$$

$$e^{-i\frac{\pi}{2}} \int e^{iwt} dw \}$$

$$f(t) = \frac{\pi}{2\pi}\left\{ \frac{e^{i\frac{\pi}{2}} e^{iwt}}{it}\Big|_{-w}^{0} + \frac{e^{-i\frac{\pi}{2}} e^{iwt}}{it}\Big|_{0}^{w} \right\} = \frac{\pi}{it}\left\{ e^{i\frac{\pi}{2}}(1 - e^{-iwt}) \right.$$

$$\left. + e^{-i\frac{\pi}{2}}(e^{iwt} - 1) \right\} = \frac{\pi}{it}\left\{ e^{i\frac{\pi}{2}} - e^{i\frac{\pi}{2}} e^{-iwt} + e^{-i\frac{\pi}{2}} e^{iwt} - e^{-i\frac{\pi}{2}} \right\}$$

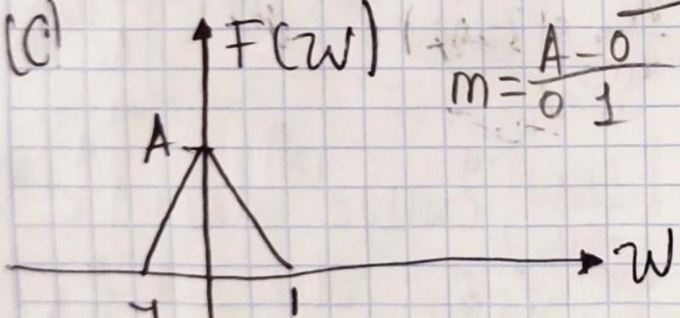
$$= \frac{\pi}{2\pi i t}\left\{ \cancel{\cos(\tfrac{\pi}{2})} + i\,sen(\tfrac{\pi}{2}) - \cancel{\cos(\tfrac{\pi}{2})} + i\,sen(\tfrac{\pi}{2}) + e^{-i\frac{\pi}{2}} e^{iwt} - e^{i\frac{\pi}{2}} e^{-iwt} \right\}$$

$$= \frac{\pi}{2\pi i t}\left\{ 2i - e^{iwt} i - e^{-iwt} i \right\} = \frac{\pi}{t}\left\{ 2 - e^{iwt} - e^{-iwt} \right\}$$

$$= \frac{2\pi}{2\pi t}\left\{ 2 - \left(\frac{e^{iwt} + e^{-iwt}}{2} \right) \right\} = \frac{2\cancel{\pi}}{\cancel{2\pi} t}\left\{ 2 - \cos(wt) \right\}$$

$$\therefore f(t) = \frac{2}{t} - \frac{2}{t}\cos(wt)$$

(c)



$$m = \frac{A - 0}{0 \;\; 1} = A$$

$$F(w) = \begin{cases} A(w+1) & -1 < w < 0 \\ -A(w-1) & 0 < w < 1 \end{cases}$$

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(w) e^{iwt} dw = \frac{1}{2\pi}\left\{ \int_{-1}^{0} A(w+1) e^{iwt} dw - \int_{0}^{1} A(w-1) e^{iwt} dw \right\}$$

$$= \frac{A}{2\pi}\left\{ \int_{-1}^{0} (w+1) e^{iwt} dw - \int_{0}^{1} (w+1) e^{iwt} dw \right\}$$

$$= \frac{A}{2\pi}\left\{ \left[\frac{(w+1)e^{iwt}}{it} - \frac{e^{iwt}}{(it)^2} \right]_{-1}^{0} - \left[\frac{(w-1)e^{iwt}}{it} - \frac{e^{iwt}}{(it)^2} \right]_{0}^{1} \right\}$$

$$= \frac{A}{2\pi}\left\{ \left[\frac{1}{it} + \frac{1}{t^2} - 0 - \frac{e^{-iw}}{t^2} \right] - \left[0 + \frac{e^{iw}}{t^2} + \frac{1}{it} - \frac{1}{t^2} \right] \right\}$$

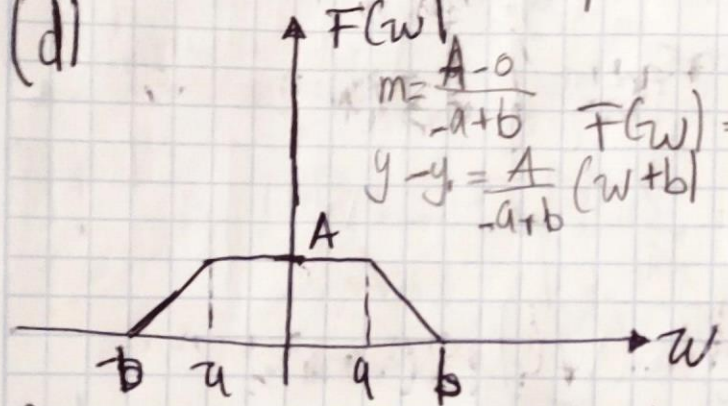
$$= \frac{A}{2\pi}\left\{ \frac{1}{\cancel{it}} + \frac{1}{t^2} - \frac{e^{iw}}{t^2} - \frac{e^{iw}}{t^2} - \frac{1}{it} + \frac{1}{t^2} \right\} = \frac{A}{2\pi}\left\{ \frac{2}{t^2} - \frac{e^{iw}}{t^2} - \frac{e^{-iw}}{t^2} \right\}$$

$$f(t) = \frac{A}{\pi}\left\{\frac{2}{2t^2} - \frac{1}{2t^2}\left(e^{it} + e^{-it}\right)\right\}$$

$$f(t) = \frac{A}{\pi}\left\{\frac{1}{t^2} - \frac{1}{t^2}\cos(t)\right\} = \frac{A}{\pi}\left\{\frac{1}{t^2} - \frac{1}{t^2}\cos(t)\right\}$$

$$f(t) = \frac{A}{\pi}\left\{\frac{1}{t^2} - \frac{\cos(t)}{t^2}\right\}$$

(d)



$$m = \frac{A-0}{-a+b}$$

$$y - y_1 = \frac{A}{-a+b}(\omega+b)$$

$$F(\omega) = \begin{cases} \frac{A}{-a+b}(\omega+b), & -b \le \omega < -a \\ A & -a \le \omega < a \\ \frac{-A}{-a+b}(\omega-b) & a \le \omega < b \end{cases}$$

$$f(t) = \frac{1}{2\pi}\int F(\omega)\, e^{i\omega t}\, d\omega = \frac{1}{2\pi}\left\{\frac{A}{-a+b}\int_{-b}^{-a}(\omega+b)\, e^{i\omega t}\, d\omega + \int_{-a}^{a} A e^{i\omega t}\, d\omega - \frac{A}{-a+b}\int_{a}^{b}(\omega-b)e^{i\omega t}\, d\omega\right\}$$

$$= \frac{A}{2\pi}\left\{\frac{1}{-a+b}\int_{-b}^{-a}(\omega+b)e^{i\omega t}\, d\omega + \int_{-a}^{a} e^{i\omega t}\, d\omega - \frac{1}{-a+b}\int_{a}^{b}(\omega-b)e^{i\omega t}\, d\omega\right\}$$

$$= \frac{A}{2\pi}\left\{\frac{1}{-a+b}\left[\frac{(\omega+b)e^{i\omega t}}{it} - \frac{e^{i\omega t}}{(it)^2}\right]_{-b}^{-a} + \frac{e^{i\omega t}}{it}\Big|_{-a}^{a} - \frac{1}{-a+b}\left[\frac{(\omega-b)e^{i\omega t}}{it} - \frac{e^{i\omega t}}{(it)^2}\right]_{a}^{b}\right\}$$

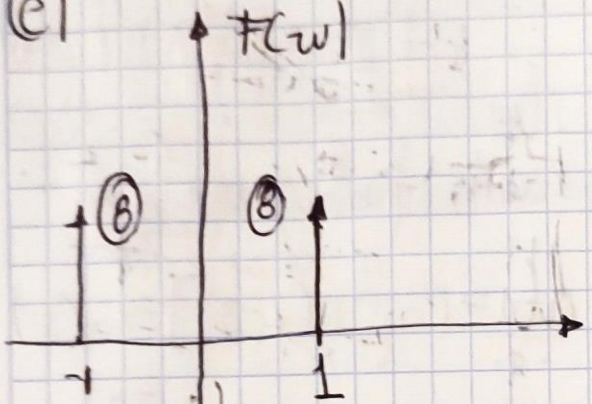
$$f(t) = \frac{A}{2\pi}\left\{\frac{1}{-a+b}\left[\frac{(-a+b)e^{-iat}}{it} + \frac{e^{-iat}}{t^2} - 0 - \frac{e^{-ibt}}{t^2}\right] + \frac{e^{iat}}{it} - \frac{e^{-iat}}{it} - \frac{1}{-a+b}\left[0 + \frac{e^{ibt}}{t^2} - \frac{(a-b)e^{iat}}{it} - \frac{e^{iat}}{t^2}\right]\right\}$$

$$f(t) = \frac{A}{2\pi}\left\{\frac{2\sin(at)}{t} + \frac{2\left(t(b-a)\sin(at) - \cos(at) + \cos(bt)\right)}{t^2(a-b)}\right\}$$

$$f(t) = \frac{A}{\cancel{2\pi}} \left\{ \frac{-\cancel{2}(\cos(at) - \cos(bt))}{t^2(a-b)} \right\}$$

$$f(t) = \frac{A}{\pi} \left\{ \frac{-\cos(at) + \cos(bt)}{t^2(a-b)} \right\}$$

(c)



$$F(w) = \begin{cases} 8\delta(w-1) & w = 1 \\ 8\delta(w+1) & w = -1 \end{cases}$$

$$f(t) = \frac{1}{2\pi} \left\{ \int_{-\infty}^{\infty} 8\delta(w-1)\, e^{iwt}\, dw + \int_{-\infty}^{\infty} 8\delta(w+1)\, e^{iwt}\, dw \right\}$$

$$f(t) = \frac{8}{2\pi} \left\{ e^{+iwt}\Big|_{w=1} + e^{iwt}\Big|_{w=-1} \right\} = 16\pi \left\{ \frac{e^{iw} + e^{-iw}}{2} \right\} \cdot 2$$

$$f(t) = \frac{8}{\cancel{2}\pi} \cos w \cdot \cancel{2}$$

$$f(t) = \frac{8}{\pi} \cos w$$

Problema 7. Por medio de la propiedad de muestreo de la función impulso, calcular las si integrales

a) $\displaystyle\int_{-\infty}^{\infty} \delta(t-5) \operatorname{Sen} 2t\, dt = \operatorname{Sen}(2*10) = \operatorname{Sen} 10$; $t = 5$

$$\therefore \int_{-\infty}^{\infty} \delta(t-5) \operatorname{Sen} 2t\, dt = \operatorname{Sen}(10)$$

b) $\int_{-\infty}^{+\infty} \delta(2-t)(t^5-3)\,dt = (2^5-3) = 29$

$\alpha = 2$

$\therefore \int_{-\infty}^{\infty} \delta(2-t)(t^5-3)\,dt = 29$

c) $\int_{-1}^{x} e^{-x^2} \delta(x)\,dx = 0$

Ya que necesitamos $x=0$ y no está en rango

d) $\int_{-\infty}^{\infty} \delta(t-2)\cos[\pi(t-3)]\,dt = \cos[\pi(2-3)] = \cos(\pi)$

$t=2$ $\qquad = -1$

$\therefore \int_{-\infty}^{\infty} \delta(t-2)\cos[\pi(t-3)] = -1$

e) $\int_{-\infty}^{\infty} \delta(t+2)\, e^{-2t}\,dt = e^{-2(-2)} = e^{4}$

$t=-2$

$\therefore \int_{-\infty}^{\infty} \delta(t+2)\, e^{-2t}\,dt = e^{4}$

f) $\int_{-\infty}^{\infty} e^{\cos t}\, \delta(t-\pi)\,dt = e^{\cos\pi} = e^{-1} = \frac{1}{e}$

$t=\pi$

$\therefore \int_{-\infty}^{\infty} e^{\cos t}\, \delta(t-\pi) = \frac{1}{e}$

g) $\int_{1}^{100} \log_{10}(t)\, \delta(t-10)\,dt = \log_{10}(10) = 1$

$t=10$

$\therefore \int_{1}^{100} \log_{10}(t)\, \delta(t-10) = 1$

Problema 8. Considerando que $f(t)$ y $F(w)$ forman un par de transformadas, USANDO LAS PROPIEDADES DE LA TRANSFORMADA, encontrar la transformada de Fourier de las siguientes expresiones.

a). $f(2-t) \longleftrightarrow ?$

Sol: Si $f(t) \longleftrightarrow F(w)$

∴ por la prop. de desplazamiento en tiempo

$$f(t)\Big|_{t=t+2} \longleftrightarrow ?$$

$$f(t+2) \longleftrightarrow F(w)e^{i2w}$$

Por la propiedad escalar con $a=-1$

$$f(t+2)\Big|_{t=-t} \longleftrightarrow ?$$

$$f(2-t) \longleftrightarrow F(w)e^{i2w}\Big|_{w=-w}$$

$$f(2-t) \longleftrightarrow F(-w)e^{-i2w}$$

b) $f[(t-3)-3]$

Sol:

Si $f(t) \longleftrightarrow F(w)$

prop. de desplazamiento en tiempo

$$f(t)\Big|_{t-3}$$

$$f[t-3] \longleftrightarrow F(w)e^{-i3w}$$

⇓

prop. de desplazamiento en tiempo

$$f(t)\Big|_{(t-3)}$$

$$f[(t-3)-3] \longleftrightarrow F(w)e^{-i3w}e^{-i3w}$$

c) $\left(\dfrac{df(t)}{dt}\right)(\operatorname{Sen} t) \longleftrightarrow ?$

$$f(t) \longleftrightarrow F(w)$$

$$f(t)\operatorname{Sen} t \longleftrightarrow \frac{i}{2}\left[F(w+1) - F(w-1)\right]$$

$$\left(\frac{d}{dt}f(t)\right)\operatorname{Sen} t \longrightarrow \frac{i}{2}\cdot iw\left[F(w+1) - F(w+1)\right]$$

$$\left(\frac{df(t)}{dt}\right)\operatorname{Sen} t \longleftrightarrow \frac{-w}{2}\left[F(w+1) - F(w-w_0)\right]$$

d) $\dfrac{d}{dt}\left[f(-2t)\right] \longleftrightarrow ?$

$$f(t) \longleftrightarrow F(w)$$

$$f(-2t) \longleftrightarrow \frac{1}{|-2|}F\left(\frac{w}{-2}\right)$$

$$f(-2t) \longleftrightarrow \frac{1}{2}F\left(-\frac{w}{2}\right)$$

$$\frac{d[f(-2t)]}{dt} \longleftrightarrow (iw)\frac{1}{2}F\left(-\frac{w}{2}\right)$$

e) $t\, f(3t) \longleftrightarrow$

$$f(t) \longleftrightarrow F(w)$$

$$f(3t) \longleftrightarrow \frac{1}{|3|}F\left(\frac{w}{3}\right)$$

$$f(3t) \longleftrightarrow \frac{1}{3}F\left(\frac{w}{3}\right)$$

$$(-it)f(3t) \longleftrightarrow \frac{d}{dw}\left(\frac{1}{3}F\left(\frac{w}{3}\right)\right)$$

$$-itf(3t) \longleftrightarrow \frac{1}{3}\frac{d}{dw}\left(F\left(\frac{w}{3}\right)\right)$$

$$tf(3t) \longleftrightarrow -\frac{1}{3i}\frac{d}{dw}\left(F\left(\frac{w}{3}\right)\right)$$

f). $(t-5)f(t) \leftrightarrow ?$

① $f(t) \longleftrightarrow F(w)$

$-5f(t) \longleftrightarrow -5F(w)$

② $f(t) \longleftrightarrow F(w)$

$(-it)\ f(t) \longleftrightarrow \frac{d}{dw} F(w)$

$t f(t) \longleftrightarrow \frac{1}{-i} \frac{d}{dw} F(w)$

①, + ②

$-5f(t) + t f(t) \longleftrightarrow -5F(w) - \frac{d}{i dw} F(w)$

$(t-5)f(t) \longleftrightarrow -\left[5F(w) + \frac{d}{i dw} F(w)\right]$ //

g) $(t-3)\, f(-3t) \longleftrightarrow$

$f(t) \longleftrightarrow F(w)$

$f(-3t) \longleftrightarrow \frac{1}{|-3|} F\left(\frac{w}{-3}\right)$

$f(-3t) \longleftrightarrow \frac{1}{3} F\left(-\frac{w}{3}\right)$

① $-3f(-3t) \longleftrightarrow -F\left(-\frac{w}{3}\right)$

\+

② $(-it)\ f(-3t) \longleftrightarrow \frac{d}{dw}\left(\frac{1}{3} F\left(-\frac{w}{3}\right)\right)$

$t f(-3t) \longleftrightarrow \frac{-1}{i} \cdot \frac{i}{i} \frac{d}{dw}\left(\frac{1}{3} F\left(-\frac{w}{3}\right)\right)$

$t f(-3t) \longleftrightarrow i \frac{d}{dw}\left(\frac{1}{3} F\left(-\frac{w}{3}\right)\right)$

$-3f(-3t) + t f(-3t) \longleftrightarrow -F\left(-\frac{w}{3}\right) + i\frac{d}{dw}\left(\frac{1}{3} F\left(-\frac{w}{3}\right)\right)$

$(t-3)f(-3t) \longleftrightarrow i \frac{d}{dw}\left(\frac{1}{3} F\left(-\frac{w}{3}\right)\right) - F\left(-\frac{w}{3}\right)$ //

h) $t\dfrac{df(t)}{dt} \longleftrightarrow$

$f(t) \longleftrightarrow F(w)$

$\dfrac{df(t)}{dt} \longleftrightarrow iw F(w)$

$(-it)\dfrac{df(t)}{dt} \longleftrightarrow i\dfrac{d}{dw} iwF(w)$

$t\dfrac{df(t)}{dt} \longleftrightarrow \dfrac{-i}{i}\dfrac{d}{dw}(iwF(w))$

$t\dfrac{df(t)}{dt} \longleftrightarrow -\dfrac{d}{dw}(iwF(w))$

i) $f(6-t) \longleftrightarrow ?$

$f(t) \longleftrightarrow F(w)$ $\Big|_{t=t+6}$

$f(6+t) \longleftrightarrow F(w)\, e^{i6w}$ $\Big|_{t=-t}$

$f(6-t) \longleftrightarrow \dfrac{1}{|-1|} F(w)\, e^{i6w}$ $\Big|_{w=\frac{w}{-1}}$

$f(6-t) \longleftrightarrow F(-w)\, e^{-i6w}$

j). $(2-t)f(8-t)$

$$f(t) \leftrightarrow F(w)$$

$$f(8+t) \leftrightarrow F(w)e^{i8w}$$

$$f(8-t) \leftrightarrow \frac{1}{|-1|}F(w)e^{i8w} \Big|_{w=\frac{w}{-1}}$$

$$f(8-t) \leftrightarrow F(-w)e^{-i8w}$$

① $2f(8-t) \leftrightarrow 2F(-w)e^{-i8w}$

② $f(8-t) \leftrightarrow F(-w)e^{-i8w}$

$$-it\, f(8-t) \leftrightarrow \frac{d}{dw}\left(F(-w)e^{-i8w}\right)$$

$$-t f(8-t) \leftrightarrow \frac{1}{i}\frac{d}{dw}\left(F(-w)e^{-i8w}\right)$$

$$2f(8-t) - tf(8-t) \leftrightarrow 2F(-w)e^{-i8w} + \frac{1}{i}\frac{d}{dw}\left(F(-w)e^{-i8w}\right)$$

$$(2-t)f(8-t) \leftrightarrow 2F(-w)e^{-i8w} + \frac{1}{i}\frac{d}{dw}\left(F(-w)e^{-i8w}\right)$$

Problema 9. Completa en tiempo o frecuencia el par de Transformada solicitado, usando las propiedades de la transformada de Fourier

a) $5\delta(t-1) \leftrightarrow ?$

$$\delta(t) \leftrightarrow 1$$

$$\delta(t-1) \leftrightarrow 1e^{-iw}$$

$$5\delta(t-1) \leftrightarrow 5e^{-iw}$$

b) $? \longleftrightarrow 8\,\delta(w+1) + 8\,\delta\,\cancel{}(w-1)$

$\delta(t) \longleftrightarrow 1$

$1 \longleftrightarrow 2\pi\,\delta(-w)$

$1 \longleftrightarrow 2\pi\,\delta(w)$ Prop. Modulación

$\mathrm{Cos}\,t \longleftrightarrow \frac{1}{2} \cdot 2\pi\left[\delta(w+1) - \delta(w-1)\right]$

$\mathrm{Cos}\,t \longleftrightarrow \pi\left[\delta(w+1) - \delta(w-1)\right]$

$\frac{8\,\mathrm{Cos}\,t}{\pi} \longleftrightarrow \frac{8\cancel{\pi}}{\cancel{\pi}}\left[\delta(w+1) - \delta(w-1)\right]$

$\frac{8\,\mathrm{Cos}\,t}{\pi} \longleftrightarrow 8\,\delta(w+1) - 8\,\delta(w-1)$ //

c) $t \longleftrightarrow ?$

$\delta(t) \longleftrightarrow 1$

$1 \longleftrightarrow 2\pi\,\delta(-w)$

$-it \longleftrightarrow 2\pi \frac{d}{dw}\,\delta(-w)$

$t \longleftrightarrow \frac{2\pi}{-i} \cdot \frac{i}{i} \frac{d}{dw}\,\delta(-w)$

$t \longleftrightarrow 2\pi i \frac{d}{dw}\,\delta(w)$ //

d) $t^2 \longleftrightarrow ?$

$\delta(t) \longleftrightarrow 1$

$1 \longleftrightarrow 2\pi\,\delta(-w)$

$(-it)^2 \longleftrightarrow 2\pi \frac{d}{dt}\,\delta(w)$

$i^2 t^2 \longleftrightarrow 2\pi \frac{d^2}{d^2 t}\,\delta(w)$

$t^2 \leftrightarrow -2\pi \frac{d^2}{d^2 t} \delta(w)$

e) $2C_2(t)\cos 1000t \leftrightarrow ?$

$AC_d(t) \leftrightarrow A d\, Sa \frac{wd}{2}$

$2C_2(t) \leftrightarrow 2 \cdot 2\, Sa \frac{w \cdot 2}{2}$

$2C_2(t) \leftrightarrow 4\, Sa\, w$

$2C_2(t)\cos 1000t \leftrightarrow \frac{1}{2} 4 [Sa(w+1000) + Sa(w-1000)]$

$2C_2(t)\cos 1000t \leftrightarrow 2[Sa(w+1000) + Sa(w-1000)]$

f) $? \leftrightarrow \cos 1000w$

$\delta(t) \leftrightarrow 1$

$\delta(t+1000) \leftrightarrow e^{i1000w}$

$+$

$\delta(t-1000) \leftrightarrow e^{-i1000w}$

$\frac{\delta(t+1000) + \delta(t-1000)}{2} \leftrightarrow \frac{e^{i1000w} + e^{-i1000w}}{2}$

$\frac{1}{2}[\delta(t+1000) + \delta(t-1000)] \leftrightarrow \cos(1000w)$

g) $? \leftrightarrow 5w$

$\delta(t) \leftrightarrow 1$

$5\delta(t) \leftrightarrow 5$

$\frac{d}{dt} 5\delta(t) \leftrightarrow 5 \cdot (iw)$

$\Downarrow$

$\frac{5}{i} \frac{d}{dt} \delta(t) \leftrightarrow 5w$

$-5i \frac{d}{dt} \delta(t) \leftrightarrow 5w$

h) $? \longleftrightarrow \delta(w)\, e^{-i5w}$

$\delta(t) \longleftrightarrow 1$

$1 \longleftrightarrow 2\pi\delta(-w)$

$1 \longleftrightarrow 2\pi\delta(w)$

$1 - 5 \longleftrightarrow 2\pi\delta(w)\, e^{-i5w}$

$-\frac{1}{i\pi} \longleftrightarrow \delta(w)\, e^{-i5w}$

$\frac{-2}{\pi} \longleftrightarrow \delta(w)\, e^{-i5w}$

Problema 10. A partir de los siguientes pares de transform

$\delta(t) \leftrightarrow 1,\ AG_d(t) \leftrightarrow AdSa\left(\frac{wd}{2}\right),\ u(t) \longleftrightarrow \pi\delta(w)$

$+\frac{1}{iw},\quad sgn(t) \longleftrightarrow \frac{2}{jw},$

Encuentre

a) $? \longleftrightarrow 3\, sgn(4w-2)$

$sgn(t) \longleftrightarrow \frac{2}{iw}$

$sgn(4t) \longleftrightarrow \frac{2}{\frac{iw}{4}} \cdot \frac{1}{|4|}$

$sgn(4t) \longleftrightarrow \frac{2\cdot\cancel{4}}{iw} \cdot \frac{1}{\cancel{4}}$

$sgn(4t) \longleftrightarrow \frac{2}{iw}$

$$\frac{2}{it} \longleftrightarrow 2\pi \, Sgn(-4w)$$

$$\frac{2}{i(-t)} \longleftrightarrow 2\pi \, Sgn(-(-4w))$$ Prop. reflejo

$$\frac{2i}{-t} \longleftrightarrow 2\pi Sgn(4w)$$

$$\frac{8i}{t8\pi} \longleftrightarrow Sgn(4w)$$

$$e^{i2t} \cdot \frac{i}{t\pi} \longleftrightarrow Sgn(4w-2)$$

$$i\frac{3e^{i2t}}{t\pi} \longleftrightarrow 3sgn(4w-2)$$

b) $G_2\left(\frac{2}{3}t\right) \leftrightarrow ?$

$$A\,G_d(t) \leftrightarrow A d Sa\left(\frac{wd}{2}\right)$$

$A = 1$ y $d = 2$

$$G_2(t)\Big|_{t=\frac{2}{3}t} \longleftrightarrow 2\,Sa\left(\frac{w2}{2}\right)$$

$$G_2\left(\frac{2}{3}t\right) \longleftrightarrow \frac{1}{\left|\frac{2}{3}\right|} 2Sa\left(\frac{2w}{2}\right)\Big|_{w=\frac{w}{\frac{2}{3}}}$$

$$G_2\left(\frac{2}{3}t\right) \leftrightarrow \frac{3}{\cancel{2}}\cancel{2}Sa\left(\frac{w}{\frac{2}{3}}\right)$$

$$G_2\left(\frac{2}{3}t\right) \leftrightarrow 3\,Sa\left(\frac{3w}{2}\right)$$

c) $2C_2(t)\cos 250t \longleftrightarrow ?$

$AC_d(t) \longleftrightarrow Ad\,Sa\left(\frac{wd}{2}\right)$

$2C_2(t) \longleftrightarrow 4\,Sa\left(w\frac{\cancel{2}}{\cancel{2}}\right)$

$2C_2(t) \longleftrightarrow 4\,Sa(w)$

$2C_2(t)\cos 250t \longleftrightarrow \frac{4}{2}[Sa(w+250)+Sa(w-250)]$

$2C_2(t)\cos 250t \longleftrightarrow 2[Sa(w+250)+Sa(w-250)]$

d) $u(10t-1)\,t \longleftrightarrow ?$

$u(t) \longleftrightarrow \pi\delta(w)+\frac{1}{iw}$

$u(t-1) \longleftrightarrow \left(\pi\delta(w)+\frac{1}{iw}\right)e^{-iw}$

$t=\cancel{}$ $\quad w=\frac{w}{10}$

$u(10t-1) \longleftrightarrow \frac{1}{|10|}\left(\pi\delta\left(\frac{w}{10}\right)+\frac{10}{iw}\right)e^{-i\frac{w}{10}}$

$u(10t-1) \longleftrightarrow \frac{1}{10}\left(\pi\delta\left(\frac{w}{10}\right)+\frac{10}{iw}\right)e^{-i\frac{w}{10}}$

$-it\,u(10t-1) \longleftrightarrow \frac{d}{dw}\left[\frac{1}{10}\left(\pi\delta\left(\frac{w}{10}\right)+\frac{10}{iw}\right)e^{-i\frac{w}{10}}\right]$

$t\,u(10t-1) \longleftrightarrow -\frac{d}{i\,dw}\left[\frac{1}{10}\left(\pi\delta\left(\frac{w}{10}\right)+\frac{10}{iw}\right)e^{-i\frac{w}{10}}\right]$

e) $e^{i7t}\,\delta(6t-1)\,t^3\,e^{i5t} \longleftrightarrow ?$

$\delta(t) \longleftrightarrow 1$

$\delta(t-1) \longleftrightarrow 1\cdot e^{-iw}$

$\delta(t-1) \longleftrightarrow e^{-iw}$

$t=2t \quad w=w-2$

$$e^{i12t}\delta(6t-1) \longleftrightarrow \frac{e^{-i\left(\frac{w-2}{6}\right)}}{16}$$

$$(-it)^3 e^{i7t}\delta(6t-1)e^{i5t} \longleftrightarrow \frac{d^3}{d^3w}\left\{\frac{e^{-i\left(\frac{w-12}{6}\right)}}{16}\right\}$$

$$t^3 e^{i7t}\delta(6t-1)e^{i5t} \longleftrightarrow \frac{1}{i^3}\frac{d^3}{d^3w}\left\{\frac{e^{-i\left(\frac{w-12}{6}\right)}}{16}\right\}$$

$$g(t)? \longleftrightarrow \left[\pi\delta\left(w+\frac{3}{4}\right) + \frac{1}{i\left(w+\frac{3}{4}\right)}\right](-w)e^{i1000w}$$

$$\delta(t) \longleftrightarrow 1$$

$$1 \longleftrightarrow 2\pi\delta(w)$$

$$e^{-i\frac{3}{4}t}\frac{1}{2} \longleftrightarrow \pi\delta\left(w+\frac{3}{4}\right)$$

$$\frac{d}{dt}\left\{\frac{e^{-i\frac{3}{4}(t+1000)}}{2}\right\} \longleftrightarrow \pi\delta\left(w+\frac{3}{4}\right)(iw)\frac{i}{i}e^{i1000w}$$

$$① \; i\frac{d}{dt}\left\{\frac{e^{-i\frac{3}{4}(t+1000)}}{2}\right\} \longleftrightarrow \pi\delta\left(w+\frac{3}{4}\right)(-w)e^{i1000w}$$

$$\mathrm{sgn}(t) \longleftrightarrow \left.\frac{2}{iw}\right|_{w=w+\frac{3}{4}}$$

$$\left.\frac{e^{-i\frac{3}{4}t}\,\mathrm{sgn}(t)}{2}\right|_{t=t+1000} \longleftrightarrow \frac{1}{i\left(w+\frac{3}{4}\right)}$$

$$② \; i\frac{d}{dt}\left\{\frac{e^{-i\frac{3}{4}(t+1000)}\,\mathrm{sgn}(t+1000)}{2}\right\} \longleftrightarrow \pi\frac{1}{i\left(w+\frac{3}{4}\right)}(-w)e^{i1000w}$$

① + ②

$$i\frac{d}{dt}\left\{\frac{e^{-i\frac{3}{4}(t+1000)}}{2}(1+\mathrm{sgn}(t+1000))\right\} \longleftrightarrow \left\{\pi\delta\left(w+\tfrac{3}{4}\right)+\frac{1}{i\left(w+\frac{3}{4}\right)}\right\} e^{i1000w}\, i w .$$

f) $? \longleftrightarrow \frac{4}{\pi} Sa(4w-2)$

$$A C_d(t) \longleftrightarrow A d\, Sa\left(\frac{wd}{2}\right)$$

$d = 8$

$A = \frac{1}{\pi}$

$$\frac{1}{\pi} C_8(t) \longleftrightarrow \frac{8}{\pi} Sa\left(\frac{w \cdot 8}{2}\right)$$

$$\frac{C_8(t)}{\pi} \longleftrightarrow \frac{8}{\pi} Sa(4w)\Big|_{w = w-\frac{1}{2}}$$

$$\frac{e^{i\frac{1}{2}t} C_8(t)}{2\pi} \longleftrightarrow \frac{8}{2\pi} Sa\left(4\left(w-\tfrac{1}{2}\right)\right)$$

$$\frac{e^{i\frac{t}{2}} C_8(t)}{2\pi} \longleftrightarrow \frac{4}{\pi} Sa(4w-2)$$

h) $C_{\frac{4}{3}}(t+6) \longleftrightarrow ?$

$AC_d(t) \leftrightarrow A d Sa\left(\frac{wd}{2}\right)$

$d = \frac{4}{3}$, $A = 1$

$C_{\frac{4}{3}}(t) \leftrightarrow \frac{4}{3} Sa\left(\frac{w \cdot \frac{4}{3}}{2}\right)$

$C_{\frac{4}{3}}(t) \leftrightarrow \frac{4}{3} Sa\left(\frac{2w}{3}\right)$

$C_{\frac{4}{3}}(t+6) \leftrightarrow \frac{4}{3} Sa\left(\frac{2w}{3}\right) e^{i6w}$

i) $(3\delta(t-1) - 3\delta(t+1)) \cdot \text{Cos} 18t \longleftrightarrow ?$

$\delta(t) \longleftrightarrow 1$

$\delta(t-1) \longleftrightarrow e^{-iw}$

$\delta(t+1) \longleftrightarrow e^{iw}$

$\delta(t-1) - \delta(t+1) \longleftrightarrow e^{-iw} - e^{iw}$

$3\delta(t-1) - 3\delta(t+1) \longleftrightarrow 3(e^{-iw} - e^{iw})$

$(3\delta(t-1) - 3\delta(t+1)) \text{Cos} 18t \leftrightarrow \frac{3}{2}\left[e^{-i(w+18)} - e^{i(w+18)} + e^{-i(w-18)} - e^{i(w-16)}\right]$

j) $? \longleftrightarrow 2\cos 500w$

$\delta(t) \longleftrightarrow 1$

$\frac{1}{2}[\delta(t+500) - \delta(t-500)] \leftrightarrow \cos 500w$

$\delta(t+500) - \delta(t-500) \leftrightarrow 2\cos 500w$

k) $t^2 + t + 1 \leftrightarrow ?$

$\delta(t) \leftrightarrow 1$

$1 \leftrightarrow 2\pi\delta(-w)$

$it^2 \leftrightarrow \frac{d^2}{d^2w} 2\pi\delta(w)$

$t^2 \leftrightarrow -\frac{d^2}{d^2w}\{2\pi\delta(w)\} \ldots ①$

Usando 1 como base

$t \leftrightarrow -\frac{d}{dw}\{2\pi\delta(w)\}$

$1 \leftrightarrow 2\pi\delta(w)$

$t^2 + t + 1 \leftrightarrow -\frac{d^2}{d^2w}\{2\pi\delta(w)\} - \frac{d}{dw}\{2\pi\delta(w)\} + 2\pi\delta(w)$

l) $i\frac{5}{t} \leftrightarrow ?$

$\text{sgn}(t) \leftrightarrow \frac{2}{iw}$

$\frac{2}{it} \leftrightarrow \text{sgn}(-w)$

$\frac{i}{-t} \leftrightarrow \frac{1}{2}\text{sgn}(-w)$

$\frac{i}{t} \leftrightarrow -\frac{1}{2}\text{sgn}(-w)$

$i\frac{5}{t} \leftrightarrow -\frac{5}{2}\text{sgn}(-w)$

m) $? \longleftrightarrow \frac{1}{w}$

$\frac{i}{2}\operatorname{sgn}(t) \longleftrightarrow \frac{2}{iw} \cdot \frac{i}{2}$

$\frac{i\operatorname{sgn}(t)}{2} \longleftrightarrow \frac{1}{w}$

n) $? \longleftrightarrow \frac{1}{w} e^{-i4w}$

Usando m

$\frac{i\operatorname{sgn}(t)}{2} \longleftrightarrow \frac{1}{w}$

$\frac{i\operatorname{sgn}(t-4)}{2} \longleftrightarrow \frac{1}{w} e^{-i4w}$

ñ) $5e^{-i\frac{7}{8}(t-3)} \longleftrightarrow ?$

$\delta(t) \longleftrightarrow 1$

$1 \longleftrightarrow 2\pi\delta(w)$

$5 \longleftrightarrow 10\pi\delta(w) \Big|_{w=w+1}$

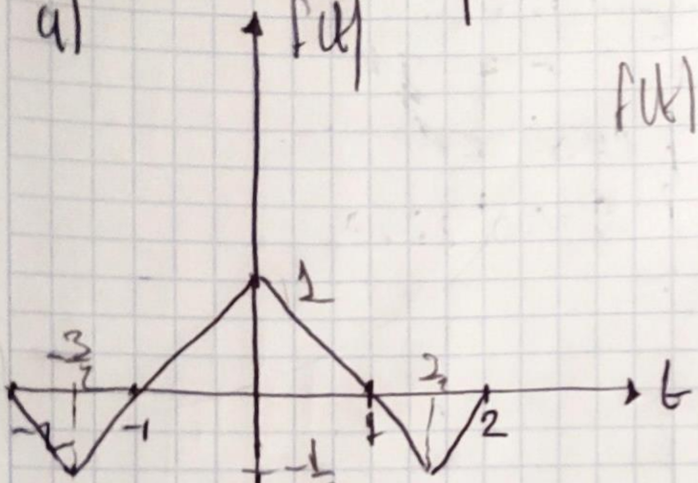
$5e^{-it} \Big|_{t=\frac{7}{8}t} \longleftrightarrow 10\pi\delta(w+1) \Big|_{w=\frac{7}{8}w} \cdot \frac{1}{\left|\frac{7}{8}\right|}$

$5e^{-i\frac{7}{8}t} \Big|_{t=t-3} \longleftrightarrow \frac{80}{7}\pi\delta\left(\frac{7}{8}w+1\right)$

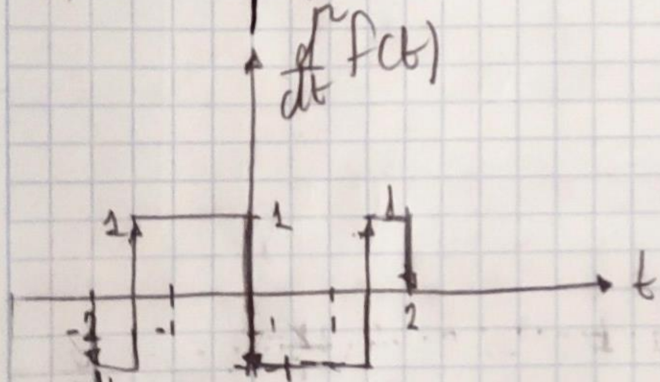
$5e^{-i\frac{7}{8}(t-3)} \longleftrightarrow \frac{80}{7}\pi\delta\left(\frac{7}{8}w+1\right)e^{-i3w}$

Problema 11. Aplicando las propiedades de la transformada Fourier determinar $F(w)$ para cada una de las señales

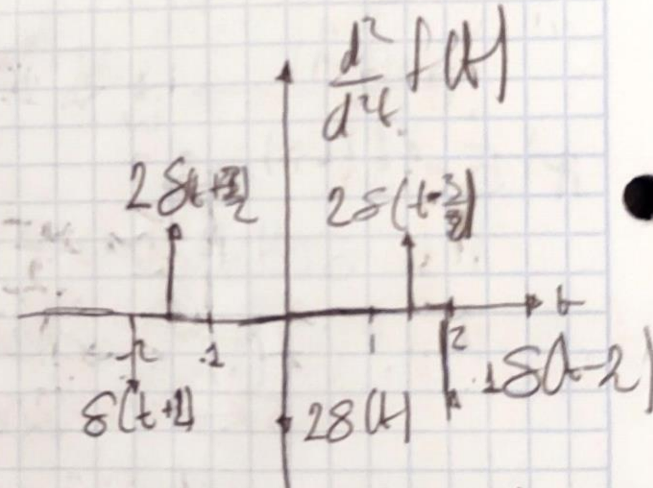
a)



$$f(t) = \begin{cases} -(t+2) & -2 < t < -\frac{3}{2} \\ t+1 & -\frac{3}{2} < t < 0 \\ -(t-1) & 0 < t < \frac{3}{2} \\ t-2 & \frac{3}{2} < t < 2 \end{cases}$$



$\Rightarrow$



$$\frac{d^2}{dt^2} f(t) = -\delta(t+2) + 2\delta(t+\tfrac{3}{2}) - 2\delta(t) + 2\delta(t-\tfrac{3}{2}) - \delta(t-2)$$

$$\mathcal{F}\left\{\frac{d^2}{dt^2} f(t)\right\} = \mathcal{F}\{\delta(t+2)\} + \mathcal{F}\{2\delta(t+\tfrac{3}{2})\} - 2\mathcal{F}\{\delta(t)\} + 2\mathcal{F}\{\delta(t-\tfrac{3}{2})\}$$

$$- \mathcal{F}\{\delta(t-2)\} = -e^{i2w} + 2e^{i\frac{3}{2}w} - 2 + 2e^{-i\frac{3}{2}w} - e^{i2w}$$

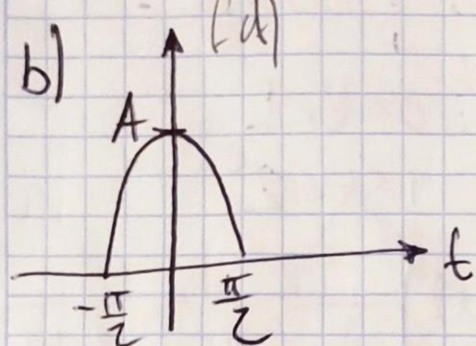
$$\frac{d^2}{dt^2} f(t) \longleftrightarrow -e^{i2w} + 2e^{i\frac{3}{2}w} - 2 + 2e^{-i\frac{3}{2}w} - e^{i2w}$$

$$\frac{d^2}{dt^2} f(t) \longleftrightarrow -2 - \frac{(e^{i2w} + e^{-i2w})2}{2} + \frac{2(e^{i\frac{3}{2}w} + e^{-i\frac{3}{2}w})2}{2}$$

$(iw)^2 f(t) \longleftrightarrow -2 - 2\cos(2w) + 4\cos(\frac{3}{2}w)$

$$f(t) \longleftrightarrow \frac{2 + 2\cos(2w) - 4\cos(\frac{3}{2}w)}{w^2}$$

b)



$$f(t) = \begin{cases} A\cos t & -1 < t < 1 \\ 0 & \text{en otro caso} \end{cases}$$

$A\cos t \cdot C_\pi t \longleftrightarrow ?$

$A\,C_d(t) \leftrightarrow A d\, Sa\,\frac{wd}{2}$

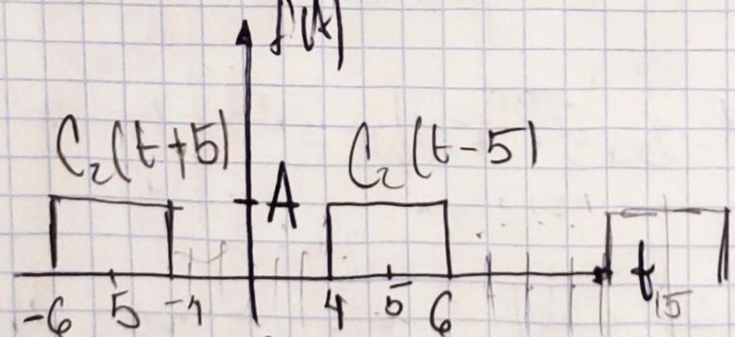
$A = A$ y $d = \pi$

$A\,C_\pi(t) \leftrightarrow A\pi\, Sa\,\frac{w\pi}{2}$

$A\cos t\, C_\pi(t) \longleftrightarrow \frac{A\pi}{2}\left[Sa\left(\frac{w\pi}{2}+1\right) + Sa\left(\frac{w\pi}{2}-1\right)\right]$

$$g(t) \longleftrightarrow \frac{A\pi}{2}\left[Sa\left(\frac{w\pi}{2}+1\right) + Sa\left(\frac{w\pi}{2}-1\right)\right]$$

c)

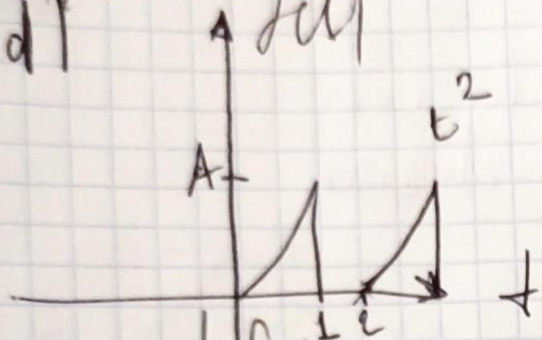


$$f(t) = \sum_{K=-\infty}^{\infty} C_2(t - 5K)$$

$K = 0$

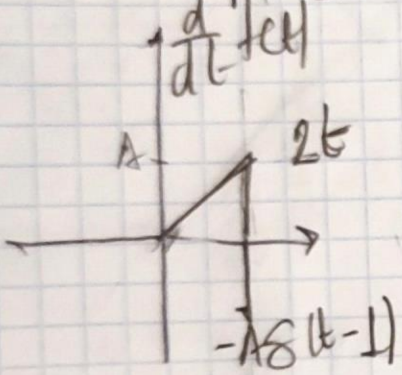
$A\,C_2(t) \longleftrightarrow A2\,Sa\,w \Rightarrow A\,C_2(t-5K) \longleftrightarrow A2\,Sa\,w\, e^{-i5Kw}$

$$A\sum_{\substack{k=-\infty\\ k\neq 0}}^{\infty} C_2(t-5k) \longleftrightarrow \sum_{\substack{k=-\infty\\ k\neq 0}}^{\infty} A\,2\,Sa\,w\,e^{-i4kw}$$

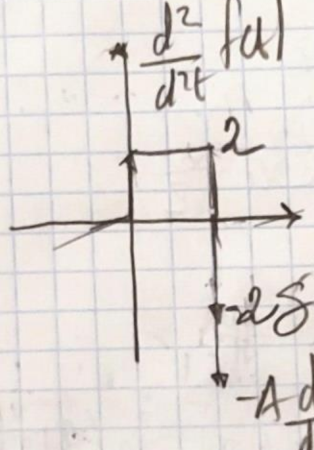


$$f(t)=\begin{cases} At^2 & 0<t<1 \\ 0 & 1<t<2 \end{cases}$$

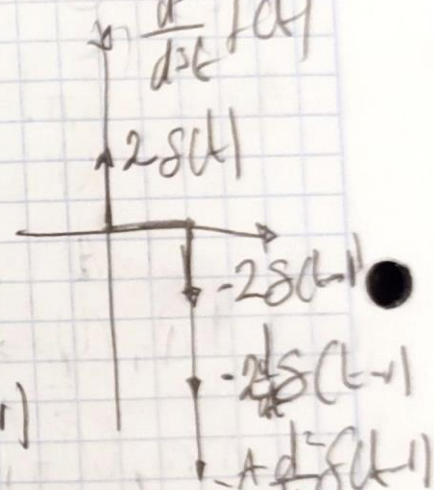
$$f(t)=f(t+2)$$



$\Rightarrow$



$\Rightarrow$



$$\frac{d^3}{d^3t}f(t)=A\{2\delta(t)-2\delta(t-1)-2\frac{d}{dt}\delta(t-1)-\frac{d^2}{dt}\delta(t-1)\}$$

$$\mathcal{F}\left\{\frac{d^3}{d^3}f(t)\right\}=A\{\mathcal{F}\{\delta(t)\}-2\mathcal{F}\{\delta(t-1)\}-2\mathcal{F}\{\frac{d}{dt}\delta(t-1)\}-A\mathcal{F}\{\frac{d^2}{dt}\delta(t-1)\}\}$$

$$\mathcal{F}\left\{\frac{d^3}{d^3t}f(t)\right\}=A\{2-2e^{-iw}-i2we^{-iw}+Aw^2e^{-iw}\}$$

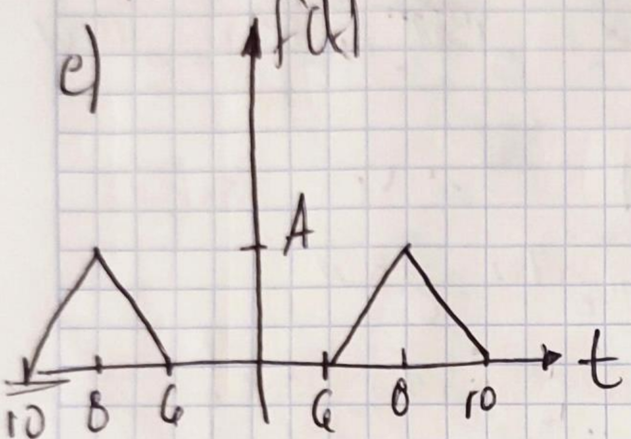
$$\frac{d^3}{d^3t}f(t)\longleftrightarrow(iw)^3f(t)$$

$$-iw^3F(w)=A\{2-2e^{-iw}-i2we^{-iw}+Aw^2e^{-iw}\}$$

$$F(w)=\frac{iA}{w^3}\{2-2e^{-iw}-i2we^{-iw}+w^2e^{-iw}\}$$

$$A\sum_{K=-\infty}^{\infty}(t+2K)^2 \longleftrightarrow \sum_{K=-\infty}^{\infty}\frac{iA}{w^3}\left\{2-2e^{-iw}-i2we^{-iw}+w^2e^{-iw}\right\}e^{-i2Kw}$$

e)



$$\Lambda\left(\frac{t}{d}\right) \longleftrightarrow \frac{Ad}{2}\,Sa^2\,\frac{wd}{4}$$

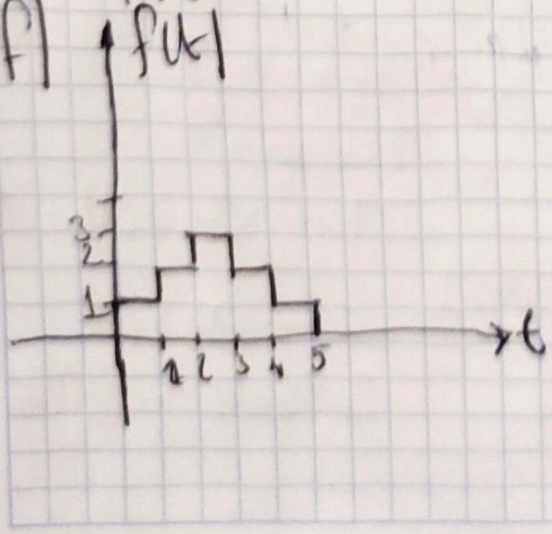
$d=4$

$$\Lambda\left(\frac{t}{4}\right) \longleftrightarrow \frac{A\,4}{2}\,Sa^2\,\frac{w\,\cancel{4}}{\cancel{4}}$$

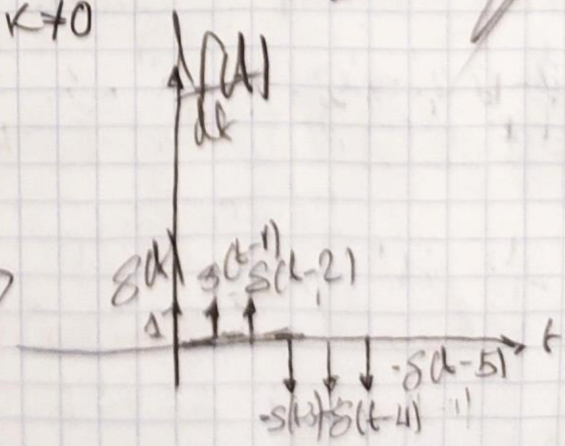
$$\Lambda\left(\frac{t}{4}\right) \longleftrightarrow 2A\,Sa^2(w)$$

$$\sum_{\substack{K=-\infty\\K\neq 0}}^{\infty}\Lambda\left(\frac{t}{4}-8K\right) \longleftrightarrow \sum_{\substack{K=-\infty\\K\neq 0}}^{\infty}2A\,Sa^2(w)\,e^{+i8Kw}$$

f)



$\Rightarrow$



$$\mathcal{F}\left\{\frac{d}{dt}f(t)\right\} = \mathcal{F}\{\delta(t)\} + \mathcal{F}\{\delta(t-1)\} + \mathcal{F}\{\delta(t-2)\} - \mathcal{F}\{\delta(t-3)\} - \mathcal{F}\{\delta(t-4)\} - \mathcal{F}\{\delta(t-5)\}$$

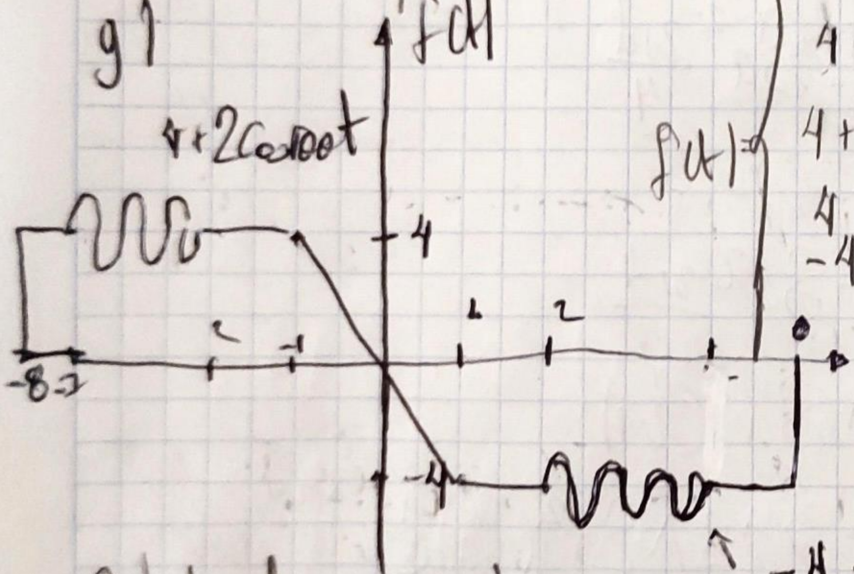
$$\mathcal{F}\left\{\frac{d}{dt}f(t)\right\} = 1 + e^{-i\omega} + e^{-i2\omega} - e^{-i3\omega} - e^{-i4\omega} - e^{-i5\omega}$$

$$\frac{d}{dt}f(t) \longleftrightarrow i\omega F(\omega)$$

$$i\omega F(\omega) = 1 + e^{-i\omega} + e^{-i2\omega} - e^{-i3\omega} - e^{-i4\omega} - e^{-i5\omega}$$

$$F(\omega) = \frac{1}{i\omega}\left\{1 + e^{-i\omega} + e^{-i2\omega} - e^{-3i\omega} - e^{-i4\omega} - e^{-i5\omega}\right\}$$

g)



$$f(t) = \begin{cases} 4 & -8 < t < -7 \\ 4 + 2\cos 100t & -7 < t < -2 \\ 4 & -2 < t < -1 \\ -4x & -1 < t < 1 \\ -4 & 1 < t < 2 \\ -4 - 2\cos 100t & \\ -4 & 7 < t < 8 \end{cases}$$

$$m = \frac{0 - 4}{0 + 1} = -4$$

$$y + 4 = -4(x+1)$$

$$y = -4x - 4 + 4$$

$$y = -4x$$

Calculando $F(\omega)$ por partes

$\pm 4 \pm \cos 100t$

$$\delta(t) \longleftrightarrow 1$$

$$4\delta(t) \longleftrightarrow 4$$

$$4 \longleftrightarrow 8\pi\,\delta(t)$$

$$2 \longleftrightarrow 4\pi\,\delta(t)$$

$$2\cos 100t \longleftrightarrow 2\pi[\delta(t+100) - \delta(t-100)]$$

$$F(\omega) = \begin{cases} 8\pi\delta(t) + 2\pi[\delta(t+100) - \delta(t-100)] & -7 < t < -2 \\ -8\pi\delta(t) - 2\pi[\delta(t+100) - \delta(t-100)] & 2 < t < 7 \end{cases}$$