A thick dark blue vertical bar runs down the left side of the page. A blue arrow-shaped banner points to the right from this bar, containing the date. In the bottom left corner, several thin, curved lines in dark blue and light grey sweep upwards and to the right.

7-5-2021

Examen 1

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① $T=6$ $\omega_0 = \pi/3$

$$x(t) = \begin{cases} 1 & -3 \leq t \leq -2 \\ -\frac{t}{2} & -2 \leq t \leq 0 \\ \frac{t}{2} & 0 \leq t \leq 2 \\ 1 & 2 \leq t \leq 3 \end{cases}$$

$x(t)$ es par $b_n = 0$

$$a_0 = \frac{1}{T} \int_{t_0}^{t_0+T} f(t) dt$$

$$a_n = \frac{2}{T} \int_{t_0}^{t_0+T} f(t) \cos(n\omega_0 t) dt$$

$$a_0 = \frac{2}{6} \int_{-2}^2 dt + \frac{2}{6} \int_0^2 \frac{t}{2} dt = \frac{1}{3} t \Big|_{-2}^2 + \frac{1}{12} t^2 \Big|_0^2 = \frac{4}{3} + \frac{1}{3} = \frac{5}{3}$$

$$a_n = \frac{2}{3} \int_0^2 \frac{t}{2} \cos(n\pi t/3) dt + \frac{2}{3} \int_2^3 \cos(n\pi t/3) dt$$

$$= \frac{1}{3} \left[\frac{3}{n\pi} t \sin\left(\frac{n\pi t}{3}\right) + \frac{9}{n^2\pi^2} \cos\left(\frac{n\pi t}{3}\right) \right]_0^2 + \frac{2}{n\pi} \sin\left(\frac{n\pi t}{3}\right) \Big|_2^3$$

$$= \frac{1}{n\pi} \left[2 \sin\left(\frac{2n\pi}{3}\right) + \frac{3}{n\pi} \cos\left(\frac{n\pi 2}{3}\right) - 0 - \frac{3}{n\pi} \right] + 0 - \frac{2}{n\pi} \sin\left(\frac{2n\pi}{3}\right)$$

$$= \frac{3}{n^2\pi^2} \left(\cos\left(\frac{n\pi 2}{3}\right) - 1 \right)$$

$$x(t) = \frac{2}{3} + \sum_{n=1}^{\infty} \frac{3}{n^2\pi^2} \left(\cos\left(\frac{2n\pi}{3}\right) - 1 \right) \cos\left(\frac{n\pi t}{3}\right)$$

Marlín
Coronel
Drogon
Lasalet

② La función ya está en forma de suma de cosenos y senos, es ella misma

$$f(t) = \cos t$$

Lo que se ve con flash dice:

$$x(t) = \frac{2}{3} + \sum_{n=1}^{\infty} \frac{3}{n^2\pi^2} \left(\cos\left(\frac{2n\pi}{3}\right) - 1 \right) \cos\left(\frac{n\pi t}{3}\right)$$

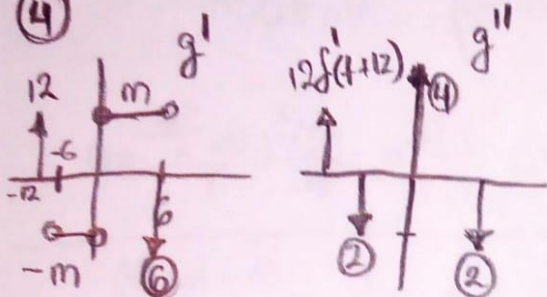
$$\textcircled{3} \quad AC_d(t) \leftrightarrow Ad \text{Sa}\left(\frac{\omega d}{2}\right)$$

$$C_y(t) \leftrightarrow 4 \text{Sa}(2\omega)$$

$$C_y(t) \cos 20t \leftrightarrow 2[\text{Sa}(2\omega + 40) + \text{Sa}(2\omega - 40)]$$

$$10 C_y(t) \cos 20t \leftrightarrow 20[\text{Sa}(2\omega + 40) + \text{Sa}(2\omega - 40)]$$

$\textcircled{4}$



$$m=2 = \left\lfloor \frac{0-12}{0+6} \right\rfloor$$

$$g''(t) = 12\delta'(t+12) - 2\delta(t+6) + 4\delta(t) - 2\delta(t-6) - 6\delta'(t-6)$$

$$(i\omega)^2 G(\omega) = 12(i\omega)e^{12i\omega} - 2e^{6i\omega} + 4 - 2e^{-6i\omega} - 6i\omega e^{-6i\omega}$$

$$G(\omega) = \frac{-1}{\omega^2} [12i\omega(e^{12i\omega} - e^{-6i\omega}) - 2e^{6i\omega} - 2e^{-6i\omega} + 4]$$

$\textcircled{5}$

$$e^{-at} u(t) \leftrightarrow \frac{1}{a+i\omega}$$

$$\delta(t) \leftrightarrow 1$$

$$e^{-3t} u(t) \leftrightarrow \frac{1}{3+i\omega}$$

$$\delta'(t) \leftrightarrow i\omega$$

$$\frac{d^2}{dt^2} [e^{-3t} u(t)] = \frac{(i\omega)^2}{3+i\omega}$$

$$-i\delta'(t)e^{it} \leftrightarrow (\omega-1)$$

$$-\frac{d^2}{dt^2} [e^{+3t} u(t)] \leftrightarrow \frac{\omega^2}{3-i\omega}$$

$$-i\delta'(t+4)e^{i(t+4)} \leftrightarrow (\omega-1)e^{i4\omega}$$

$$-\frac{d^2}{dt^2} [e^{3t} u(t)] - i\delta'(t+4)e^{i(t+4)} \leftrightarrow \frac{\omega^2}{3-i\omega} + e^{i4\omega}(\omega-1)$$

⑥

$$F(\omega) = \frac{1}{(1+i\omega)^2} = \frac{1e^0}{(1+\omega^2)e^{2\arctan(\omega)}} = \frac{1}{1+\omega^2} e^{-2\arctan(\omega)}$$

$$|F(\omega)| = \frac{1}{1+\omega^2} \quad \theta = -2\arctan(\omega)$$

