## SERIES DE FOURIER

| TRIGONOMETRICA | EXPONENCIAL O COMPLEJA |
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| $f(t)=a_0+\sum_{n=1}^{\infty}\left[a_n\cos(n\omega_0 t)+b_n\operatorname{sen}(n\omega_0 t)\right]$ <br> $a_n=\frac{2}{T}\int_{t_0}^{t_0+T}f(t)\cos(n\omega_0 t)\,dt$ <br> $b_n=\frac{2}{T}\int_{t_0}^{t_0+T}f(t)\operatorname{sen}(n\omega_0 t)\,dt$ <br> $a_0=\frac{1}{T}\int_{t_0}^{t_0+T}f(t)\,dt$ ; $\omega_0=\frac{2\pi}{T}$ | $f(t)=\sum_{n=-\infty}^{\infty}C_n\,e^{jn\omega_0 t}$ <br> $C_n=\frac{1}{T}\int_{t_0}^{t_0+T}f(t)\,e^{-jn\omega_0 t}\,dt$ <br> Módulo: $\lVert C_n\rVert=\sqrt{\mathrm{Re}^2\{C_n\}+\mathrm{Im}^2\{C_n\}}$ <br> Fase: $\theta=\arctan\frac{\mathrm{Im}\{C_n\}}{\mathrm{Re}\{C_n\}}$ |

## TRANSFORMADA DE FOURIER

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| DEFINICION | DIRECTA: $F(\omega)=\int_{-\infty}^{\infty}f(t)\,e^{-j\omega t}dt$ | |
| | INVERSA: $f(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}F(\omega)\,e^{j\omega t}\,d\omega$ | |
| PROPIEDADES | Linealidad: | Si $f_1(t)\leftrightarrow F_1(\omega)$ Y $f_2(t)\leftrightarrow F_2(\omega)$ <br> $\Rightarrow f_1(t)+f_2(t)\leftrightarrow F_1(\omega)+F_2(\omega)$ |
| | Desplazamiento en tiempo: | Si $f(t)\leftrightarrow F(\omega)$ <br> $\Rightarrow f(t\pm a)\leftrightarrow F(\omega)e^{\pm ja\omega}$ siendo $a\in\mathbb{R}$. |
| | Diferenciación en tiempo: | Si $f(t)\leftrightarrow F(\omega)$ <br> $\Rightarrow \frac{d^n f(t)}{dt^n}\leftrightarrow (j\omega)^n F(\omega)$ |
| | Escalamiento: | Si $f(t)\leftrightarrow F(\omega)$ <br> $\Rightarrow f(at)\leftrightarrow \frac{1}{\lvert a\rvert}F\left(\frac{\omega}{a}\right)$ siendo $a\in\mathbb{R}$. |
| | Simetría: | Si $f(t)\leftrightarrow F(\omega)$ <br> $\Rightarrow F(t)\leftrightarrow 2\pi f(-\omega)$ |
| | Desplazamiento en frecuencia: | Si $f(t)\leftrightarrow F(\omega)$ <br> $\Rightarrow f(t)e^{\mp jat}\leftrightarrow F(\omega\pm a)$ siendo $a\in\mathbb{R}$. |
| | Diferenciación en frecuencia: | Si $f(t)\leftrightarrow F(\omega)$ <br> $\Rightarrow (-jt)^n f(t)\leftrightarrow \frac{d^n F(\omega)}{d\omega^n}$ |
| | Modulación: | Si $f(t)\leftrightarrow F(\omega)$ <br> $\Rightarrow f(t)\cos(\omega_0 t)\leftrightarrow \frac{1}{2}\left[F(\omega+\omega_0)+F(\omega-\omega_0)\right]$ <br> Y $f(t)\operatorname{sen}(\omega_0 t)\leftrightarrow \frac{j}{2}\left[F(\omega+\omega_0)-F(\omega-\omega_0)\right]$ |

**IDENTIDADES TRIGONOMETRICAS:**

$$\operatorname{sen}(2A)=2\operatorname{sen}(A)\cos(A)$$
$$\cos(2A)=\cos^2(A)-\operatorname{sen}^2(A)$$
$$\operatorname{sen}(A\pm B)=\operatorname{sen}(A)\cos(B)\pm\cos(A)\operatorname{sen}(B)$$
$$\cos(A\pm B)=\cos(A)\cos(B)\mp\operatorname{sen}(A)\operatorname{sen}(B)$$
$$\operatorname{sen}^2(A)=\tfrac{1}{2}\left[1-\cos(2A)\right]$$
$$\cos^2(A)=\tfrac{1}{2}\left[1+\cos(2A)\right]$$
$$e^{\pm jA}=\cos(A)\pm j\operatorname{sen}(A)$$

$$\operatorname{sen}(A)\operatorname{sen}(B)=\tfrac{1}{2}\left[\cos(A-B)-\cos(A+B)\right]$$
$$\cos(A)\cos(B)=\tfrac{1}{2}\left[\cos(A-B)+\cos(A+B)\right]$$
$$\operatorname{sen}(A)\cos(B)=\tfrac{1}{2}\left[\operatorname{sen}(A-B)+\operatorname{sen}(A+B)\right]$$

$$\cos(A)=\frac{e^{jA}+e^{-jA}}{2}$$
$$\operatorname{sen}(A)=\frac{e^{jA}-e^{-jA}}{2j}$$

$$\cos(2n\pi)=1 \quad \forall n$$
$$\operatorname{sen}(n\pi)=0 \quad \forall n$$
$$\cos(n\pi)=(-1)^n \quad \forall n$$

**TABLA DE INTEGRALES:**

$$\int a\,dt=at$$
$$\int t^n\,dt=\frac{1}{n+1}t^{n+1}$$
$$\int e^{at}\,dt=\tfrac{1}{a}e^{at}$$
$$\int \operatorname{sen}(at)\,dt=-\tfrac{1}{a}\cos(at)$$
$$\int \cos(at)\,dt=\tfrac{1}{a}\operatorname{sen}(at)$$
$$\int \frac{dt}{t}=\ln|t|$$
$$\int \ln(t)\,dt=t\ln|t|-t$$

$$\int \operatorname{sen}(at)\operatorname{sen}(bt)\,dt=\frac{\operatorname{sen}\left[(a-b)t\right]}{2(a-b)}-\frac{\operatorname{sen}\left[(a+b)t\right]}{2(a+b)}$$
$$\int \cos(at)\cos(bt)\,dt=\frac{\operatorname{sen}\left[(a-b)t\right]}{2(a-b)}+\frac{\operatorname{sen}\left[(a+b)t\right]}{2(a+b)}$$
$$\int \operatorname{sen}(at)\cos(bt)\,dt=-\frac{\cos\left[(a-b)t\right]}{2(a-b)}-\frac{\cos\left[(a+b)t\right]}{2(a+b)}$$

$$\int e^{at}\operatorname{sen}(bt)\,dt=\frac{e^{at}}{a^2+b^2}\left[a\operatorname{sen}(bt)-b\cos(bt)\right]$$
$$\int e^{at}\cos(bt)\,dt=\frac{e^{at}}{a^2+b^2}\left[a\cos(bt)+b\operatorname{sen}(bt)\right]$$

$$\int (t\pm a)\cos(bt)\,dt=\frac{1}{b}(t\pm a)\operatorname{sen}(bt)+\frac{1}{b^2}\cos(bt)$$
$$\int (t\pm a)\operatorname{sen}(bt)\,dt=\frac{1}{b}(t\pm a)\cos(bt)-\frac{1}{b^2}\operatorname{sen}(bt)$$
$$\int (t\pm a)e^{bt}\,dt=\frac{1}{b}(t\pm a)e^{bt}-\frac{1}{b^2}e^{bt}$$