Robotics - Modeling of Kninematics Chains

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Modeling of a hexabot leg

1. According to the mission dedicated to this limb, what are desired controlled dof of the foot? The desired controlled degrees of freedom of the foot are 3.
   (a) One for the Hip
   (b) One for the Knee
   (c) One for the Ankle
2. The robot body may be considered as still(same orientation as the Ground frame), so place all the intermediate frames until the foot. Fill in the table as asked in the first exercise. Write all the homogeneous transform matrices from a frame.
   (a) In order to generate the transformation matrix that represents the direct kinematics of the hexabot leg, the frames of each joint are established as shown in the following image.

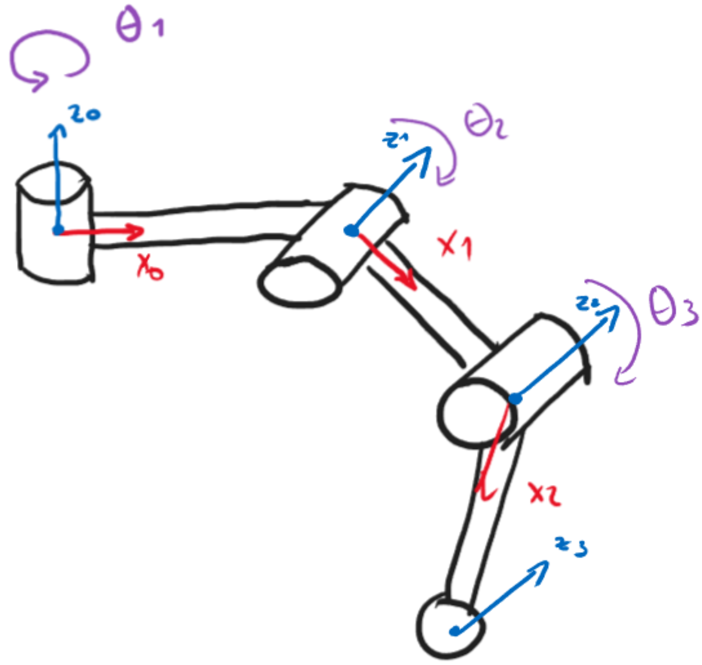

Figure 1: Hexapod leg frame assignment.

(b) Then the DH table is generated.

Link	a_i	α_i	d_i	θ_i
1	L_1	-90	0	θ_1^*
2	L_2	0	0	θ_2^*
3	L_3	0	0	θ_3^*

Table 1: DH parameters for a Hexapod leg

(c) Considering that the transformation matrix of A_i on each link is given by the product of the four transformations of the quantities $a_i, \alpha_i, d_i, \theta_i$, taking as reference the DH convention. We have that:

$$A_i = Rot_{z,\theta_i} Trans_{z,d_i} Trans_{x,a_i} Rot_{x,\alpha_i}$$

The transformation matrices for each link are as follows

$$T_1^0 = \begin{pmatrix} \cos(\theta_1) & -\sin(\theta_1) & 0 & 0 \\ \sin(\theta_1) & \cos(\theta_1) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & L1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$T_1^0 = \begin{pmatrix} \cos(\theta_1) & 0 & -\sin(\theta_1) & L_1\cos(\theta_1) \\ \sin(\theta_1) & 0 & \cos(\theta_1) & L_1\sin(\theta_1) \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$T_1^2 = \begin{pmatrix} \cos(\theta_2) & -\sin(\theta_2) & 0 & 0 \\ \sin(\theta_2) & \cos(\theta_2) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & L_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$T_1^2 = \begin{pmatrix} \cos(\theta_2) & -\sin(\theta_2) & 0 & L_2\cos(\theta_2) \\ \sin(\theta_2) & \cos(\theta_2) & 0 & L_2\sin(\theta_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$T_2^3 = \begin{pmatrix} \cos(\theta_3) & -\sin(\theta_3) & 0 & 0 \\ \sin(\theta_3) & \cos(\theta_3) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & L_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$T_2^3 = \begin{pmatrix} \cos(\theta_3) & -\sin(\theta_3) & 0 & L_3\cos(\theta_3) \\ \sin(\theta_3) & \cos(\theta_3) & 0 & L_3\sin(\theta_3) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Finally, we have that $T_3^0 = T_1^0 T_2^1 T_3^2 = T_2^0 T_3^2$

Where

$$T_2^0 = T_1^0 T_2^1 = \begin{pmatrix} \cos(\theta_1)\cos(\theta_2) & -\cos(\theta_1)\sin(\theta_2) & -\sin(\theta_1) & \cos(\theta_1)\,\sigma_1 \\ \cos(\theta_2)\sin(\theta_1) & -\sin(\theta_1)\sin(\theta_2) & \cos(\theta_1) & \sin(\theta_1)\,\sigma_1 \\ -\sin(\theta_2) & -\cos(\theta_2) & 0 & -L_2\sin(\theta_2) \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\text{where } \sigma_1 = L_1 + L_2\cos(\theta_2)$$

Therefore the homogeneous transformation matrix which defines the direct kinematics of hexabot leg is:

$$T_3^0 = \begin{pmatrix} \mathrm{c}(\theta_2+\theta_3)\,\mathrm{c}(\theta_1) & -\mathrm{s}(\theta_2+\theta_3)\,\mathrm{c}(\theta_1) & -\mathrm{s}(\theta_1) & \mathrm{c}(\theta_1)\,\sigma_1 \\ \mathrm{c}(\theta_2+\theta_3)\,\mathrm{s}(\theta_1) & -\mathrm{s}(\theta_2+\theta_3)\,\mathrm{s}(\theta_1) & \mathrm{c}(\theta_1) & \mathrm{s}(\theta_1)\,\sigma_1 \\ -\mathrm{s}(\theta_2+\theta_3) & -\mathrm{c}(\theta_2+\theta_3) & 0 & -L_3\,\mathrm{s}(\theta_2+\theta_3) - L_2\,\mathrm{s}(\theta_2) \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\text{where: } \sigma_1 = L_1 + L_3\,\mathrm{C}(\theta_2+\theta_3) + L_2\,\mathrm{C}(\theta_2)$$

3. By using direct method, calculate the Jacobian matrix which give the foot motion relative to the ground frame.

   The direct calculation of the Jacobian allows us to describe the contribution of each together with the motion of the end effector. In order to obtain a velocity map between the end effector expressed in linear and angular velocities and the linear and angular velocities of the joints. It is defined by:

$$J_n^0 = \begin{bmatrix} Jv \\ -- \\ Jw \end{bmatrix}$$

   For a revolute joint for the calculation of the direct Jacobian is given by:

$$Jv_i.\dot{q}_i = \omega_{i-1} \times^{i-1} \mathrm{t}_n = z_{i-1} \times (t_n - t_{i-1}).\dot{q}_i$$
$$J\omega_i.\dot{q}_i = \omega_{i-1} = z_{i-1}.\dot{q}_i$$

Thus we have that the Jacobian for the hexabot leg with 3 revolute joints is given by:

$$J = \begin{bmatrix} z_0 \times (t3 - t0) & z_1 \times (t3 - t1) & z_2 \times (t3 - t2) \\ z_0 & z_1 & z_2 \end{bmatrix}$$

where:

$$z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, t_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{aligned} z_1 &= \text{Rows 1,2,3. column 3 of the matrix } T_1^0; \\ z_2 &= \text{Rows 1,2,3. column 3 of the matrix } T_2^0; \\ z_3 &= \text{Rows 1,2,3. column 3 of the matrix } T_3^0; \end{aligned}$$

Therefore

$$z_1 = z_2 = z_3 = \begin{bmatrix} -sin(\theta_1) \\ cos(\theta_1) \\ 0 \end{bmatrix}$$

and

$$\begin{aligned} t_1 &= \text{Rows 1,2,3. column 4 of the matrix } T_1^0 \\ t_2 &= \text{Rows 1,2,3. column 4 of the matrix } T_2^0 \\ t_3 &= \text{Rows 1,2,3. column 4 of the matrix } T_3^0 \end{aligned}$$

Therefore

$$t_1 = \begin{pmatrix} L_1 \cos(\theta_1) \\ L_1 \sin(\theta_1) \\ 0 \end{pmatrix}$$

$$t_2 = \begin{pmatrix} L_1 \cos(\theta_1) + L_2 \cos(\theta_1)\,\cos(\theta_2) \\ L_1 \sin(\theta_1) + L_2 \cos(\theta_2)\,\sin(\theta_1) \\ -L_2 \sin(\theta_2) \end{pmatrix}$$

$$t_3 = \begin{pmatrix} \cos(\theta_1)\,(L_1 + L_3 \cos(\theta_2 + \theta_3) + L_2 \cos(\theta_2)) \\ \sin(\theta_1)\,(L_1 + L_3 \cos(\theta_2 + \theta_3) + L_2 \cos(\theta_2)) \\ -L_3 \sin(\theta_2 + \theta_3) - L_2 \sin(\theta_2) \end{pmatrix}$$

Hence, we have then that:

$$J = \begin{pmatrix} -\sin(\theta_1)\,\sigma_2 & -\cos(\theta_1)\,\sigma_1 & -L_3 \sin(\theta_2 + \theta_3)\,\cos(\theta_1) \\ \cos(\theta_1)\,\sigma_2 & -\sin(\theta_1)\,\sigma_1 & -L_3 \sin(\theta_2 + \theta_3)\,\sin(\theta_1) \\ 0 & -\sigma_3 - L_2 \cos(\theta_2) & -\sigma_3 \\ 0 & -\sin(\theta_1) & -\sin(\theta_1) \\ 0 & \cos(\theta_1) & \cos(\theta_1) \\ 1 & 0 & 0 \end{pmatrix}$$

where $\sigma_1 = L_3 \sin(\theta_2 + \theta_3) + L_2 \sin(\theta_2)$, $\sigma_2 = L_1 + \sigma_3 + L_2 \cos(\theta_2)$, $\sigma_3 = L_3 \cos(\theta_2 + \theta_3)$

4. Provided the main robot axis (longitudinal axis to move forwards) is lining with ground, can we control the leg to generate a strictly forward step(without any other motion). What would it imply about the robot locomotion?

Answer: By analyzing the determinant of the Jacobian it can be say that the range of the Jacobian matrix for hexapod leg will be between 2 and 3. This imply that about the robot locomotion:

(a) Under certain joint configurations the leg may only have 2 speeds that are directly controllable.

(b) Under certain joint configurations the leg can move instantaneously in all 3 directions independently

This means that it is possible to control the leg under certain joint configurations we can control the leg to generate a strictly forward step(without any other motion).

On the other hand, we can see that when calculating the determinant of the Jacobian.

$$det(J) = L_2\, L_3 \left(L_3\, \mathrm{s}\left(\theta_2\right)\, \mathrm{c}\left(\theta_3\right)^2 + L_3\, \mathrm{c}\left(\theta_2\right)\, \mathrm{s}\left(\theta_3\right)\, \mathrm{c}\left(\theta_3\right) + L_1\, \mathrm{s}\left(\theta_3\right) - L_3\, \mathrm{s}\left(\theta_2\right) + L_2\, \mathrm{c}\left(\theta_2\right)\, \mathrm{s}\left(\theta_3\right)\right)$$

We can see that the singularities of this for the hexapod leg do not depend on the variable θ_1

In the same way as the singular configurations of the robot only depends on θ_2 and θ_3. The following graph shows the family of points that represent singular configurations of the change of the angle of these 2 variables between -360 to 360 degrees.

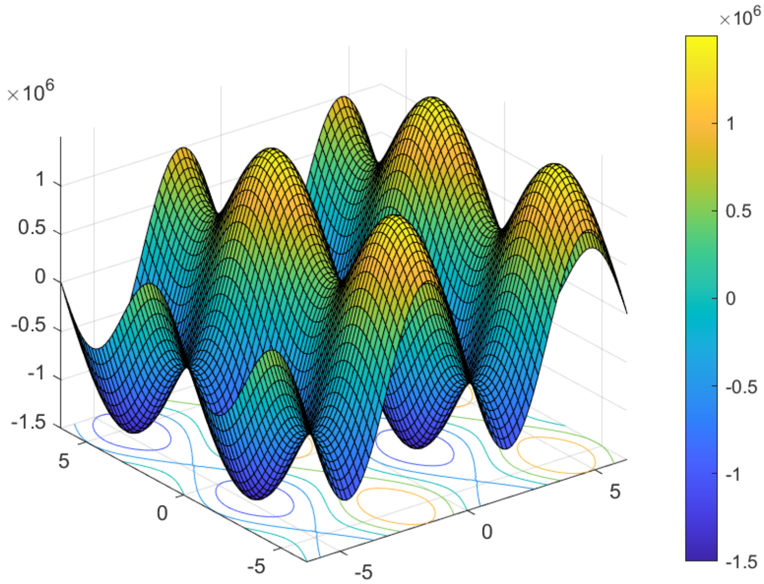

Figure 2: Representation of robot singularities given by the change of θ_2 and θ_3.

Modeling of a hexabot leg in Simscape

1. The model of the kinematic chain of the leg is supposed to include one base frame fixed on the robot body, one frame fixed on the foot (tip of the leg, but it is not necessary), and probably 3 other intermediate frames to express the 3 joints contributions.
   Note that for the simulated model, we will create many more frames to describe the geometry of each solid (at least one frame on the gravity centre of the concerned body, and another one on the tip of the body if linked to a joint).
   Do not be confused with these additional frames that do not appear in the modelling you created according to Khalil and DH principle. Hint: Keep an eye on your 3D model of the kinematic chain (question 2 of the exercise) to create the digital model with the same configuration related to the 3 dof q_1, q_2, q_3.

   - Digital Model.

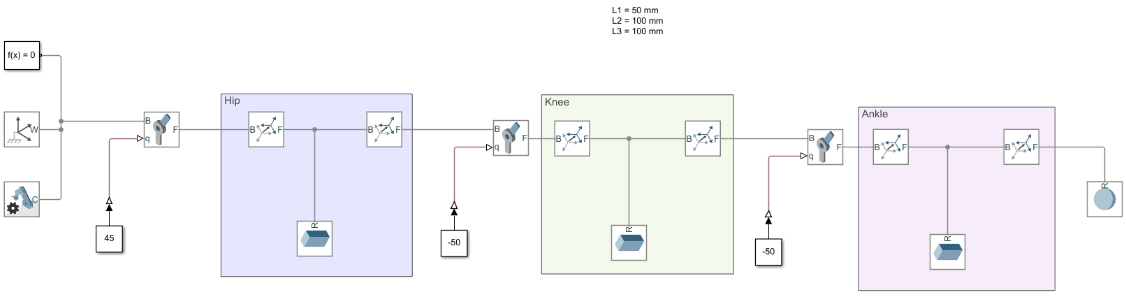

Figure 3: Digital model of a hexapod leg in Simscape Miltibody tool.

   - Redering mechanics Explorer Window.

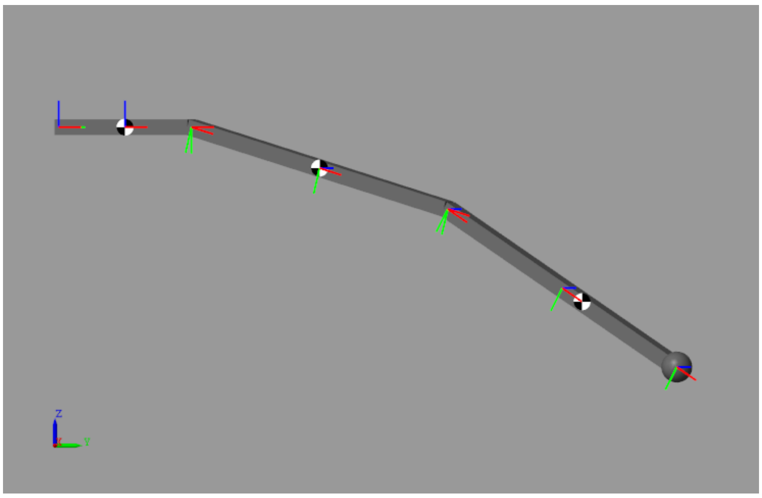

Figure 4: Rendered visualization of the hexapod leg.

2. Jacobian Each joint can provide its current position as output signal (parameter of the joint related to “Sensing”). These current values are necessary to compute the Jacobian matrix at each step time. According to the desired motion of the foot with the respect to the base frame, the inverse of the Jacobian matrix will give the joint velocities.
So we will create a Matlab function (Simulink library / User defined functions / Matlab Function) which takes the joint positions as the first 3 input signals (qi to build the Jacobian) and the desired motion (speed) of the foot relative to the base frame as the last 3 input signals (dXj). The Jacobian matrix and its inverse are computed within the function, then the product of the inverse Jacobian and the dXj give the joint speeds as the 3 output signals of this function. Connect these signals to the actuation input ports of each joint to make your robot move as desired (expected at least!)

   - Digital Model

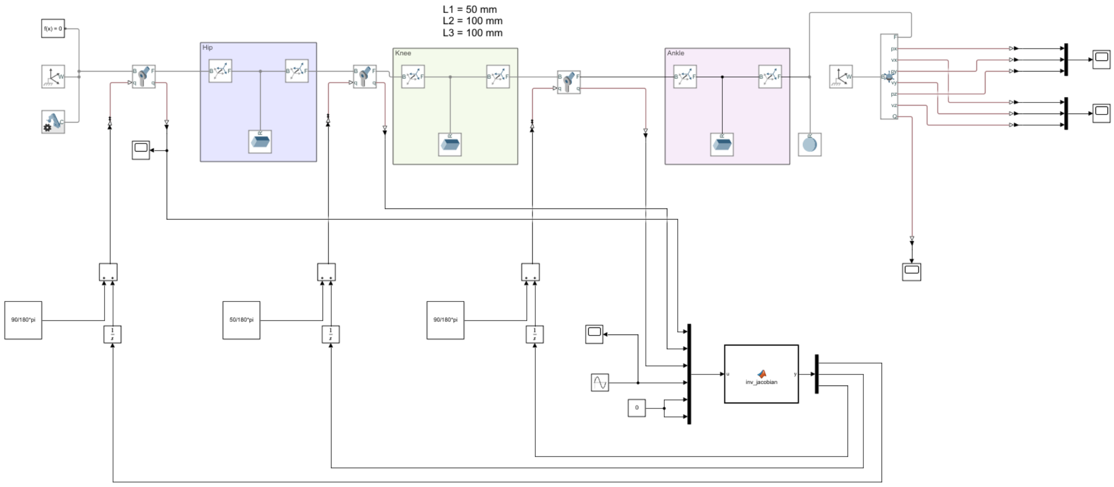

Figure 5: Matlab Function, for the inverse jacobian

   - Redering mechanics Explorer Window.

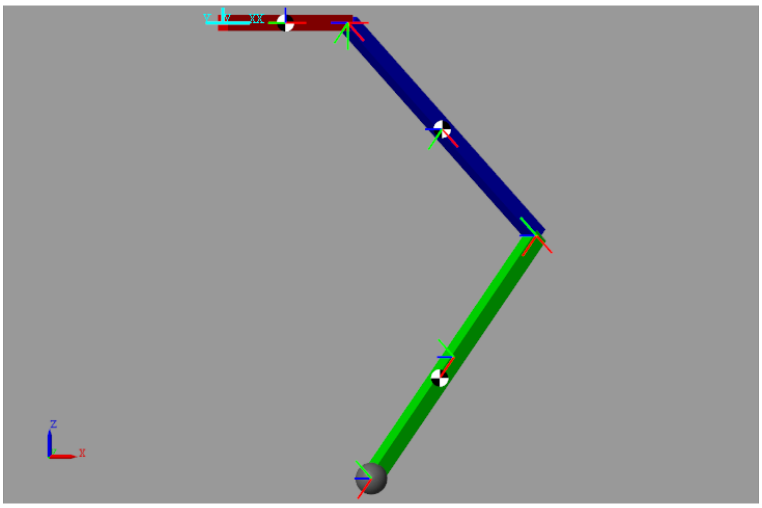

Figure 6: Matlab Function, for the inverse jacobian

- Matlab Function

```
function y = inv_jacobian(u)

L1 = 50
L2 = 100
L3 = 100


theta1 = u(1)
theta2 = u(2)
theta3 = u(3)

dx = u(4)
dy = u(5)
dz = u(6)

s1 = sin(theta1)
s2 = sin(theta2)
s3 = sin(theta3)

c1 = cos(theta1)
c2 = cos(theta2)
c3 = cos(theta3)

c23 = cos(theta2 + theta3)
s23 = sin(theta2 + theta3)


Jacobian = [-s1*(L1 + L3*c23 + L2*c2), -c1*(L3*s23 + L2*s2), -L3*s23*c1;
             c1*(L1 + L3*c23 + L2*c2), -s1*(L3*s23 + L2*s2), -L3*s23*s1;
             0, - L3*c23 - L2*c2, -L3*c23]


 y = inv(Jacobian)*[dx; dy; dz]
```

Figure 7: Matlab Function for the inverse jacobian

- Foot path

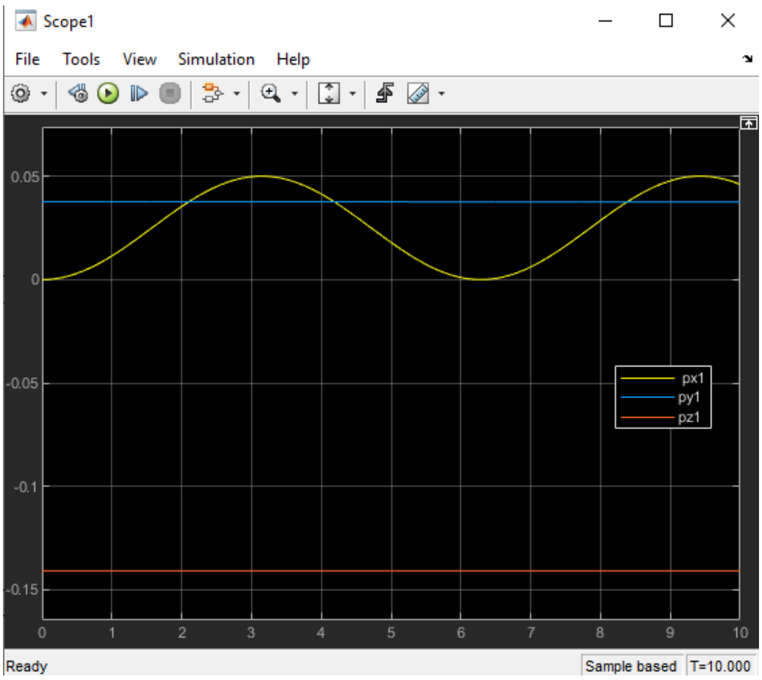

Figure 8: Foot path

3. Can we control the leg to generate a strictly forward step (without generating any other motion to the whole robot body)?

   By the Jacobian

$$J = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{w}_x \\ \dot{w}_y \\ \dot{w}_z \end{bmatrix} = \begin{bmatrix} -37.67 & 0 & 0 \\ 0 & -140.88 & -64.27 \\ 0 & 12.32 & 76.60 \\ 0 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

   Therefore, under this initial articular configuration.

   (a) The linear velocity in x depends only on joint 1.

   (b) The linear velocity in y and z depends only on joint 2 and 3.

   (c) The angular velocity in x can be controlled by the velocity of joint 2 and 3.

   (d) The leg cannot apply a rotation in the y direction.

   (e) The angular velocity in z can be controlled by the velocity of joint 1.

   Therefore we can say that if we intend to move the leg in one direction without any other movement it is possible, because the linear and angular velocities only depend on the variation of θ_1 under this particular configuration.