MARINE & MARITIME INTELLIGENT ROBOTICS

UNIVERSITÉ DE TOULON

Master MIR – Underwater Acoustics

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Homework 1

Here, the fluctuations of the velocity c with respect to horizontal coordinates are neglected and we focus on the trajectory of the rays in a vertical plane, taking into account the dependence of c with depth z. In the following, the reference velocity c_0 is the velocity at the water surface, $z = 0$.

Artic Ocean

The simplest model for the velocity profile in this ocean is an affine function of depth :

$$c(z) = C_0 + \gamma_0 \, |z|$$

where $c_o = 1450ms^{-1}$ and $\gamma_0 = 1.63 \times 10^{-2}s^{-1}$ with a seafloor at depth $h = 3.5$ km.

1. Let us consider a sonar placed at the depth $z_S = 600m$, emitting a beam of angular width 4° Around the horizontal direction. With the help of the software, plot the trajectory of the rays constituting the beam over a period of 30 s.

   Given the initial conditions, we obtain the following graph showing the paths followed by the beams between -2° and 2° with steps of 0.1°.

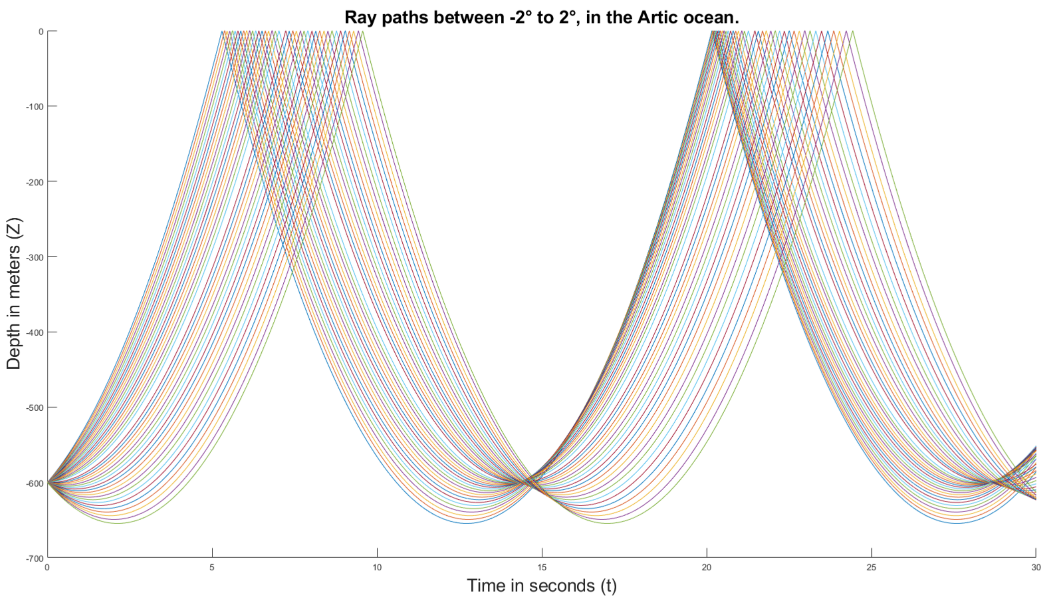

Figure 1: Ray paths at 600m depth, angular width 4°, constant gradient.

The following velocity profile is obtained for all beams, where the gradient remains constant.

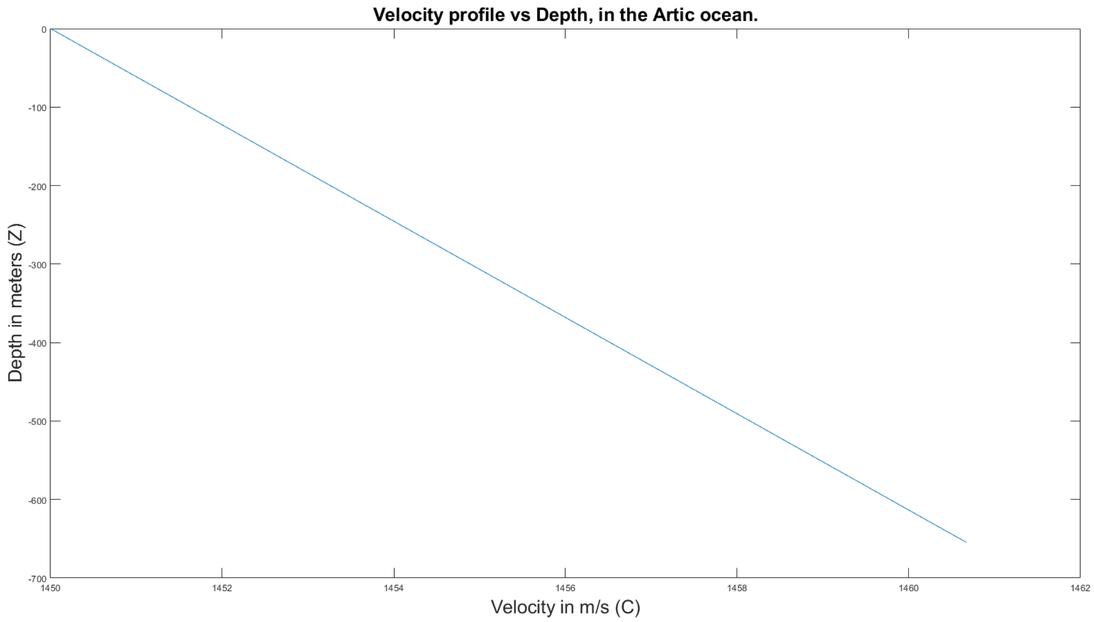

Figure 2: Velocity profile for the Artic Ocean.

What is the maximum depth reached by this beam?
The max maximum depth reached by the beam is meters $654.5890\,m$ at -2°, for the given $dt = 0.005$.

2. What is the maximum depth reached by a sound ray as a function of z_S and of the incidence angle θ_0. Check the formula with the software for the example of question 1/.

   Knowing that:

$$R = \frac{C(Z_s)}{cos(\theta_s)} \frac{1}{\left|\frac{dc}{dz}\right|} \tag{1}$$

$$Z_c = Z_s - \frac{C(Z_s)}{\frac{dc}{dz}} \tag{2}$$

We can say that:

$$Z_{max} = Z_c + R$$

then

$$Z_{max} = Z_s - \frac{C(Z_s)}{\frac{dc}{dz}} + \frac{C(Z_s)}{cos(\theta_s)} \frac{1}{\left|\frac{dc}{dz}\right|}$$

Hence:

$$Z_{max} = Z_s - \frac{C_0 + \gamma_0 \left|Z_s\right|}{\gamma_0} + \frac{C_0 + \gamma_0 \left|Z_s\right|}{cos(\theta_s)} \frac{1}{\gamma_0}$$

For the previous exercise, we know that the maximum depth is at 2°, thus:

$$Z_{max} = 600 - \frac{1450 + 0.0163\,|600|}{0.0163} + \frac{1450 + 0.0163\,|600|}{cos(\frac{-2\pi}{180})}\frac{1}{0.0163}$$

Then

$$Z_{max} = 654.890\,m$$

To find the maximum depth reached for the entire set of angles, the following cycle is generated by the software for an angle width of 4° between -2° and 2° with a step of 0.1°.

```
% Call velocity function for a constant gradient
c = velocity(zs,Co,gamma);
z = zeros(1,length(angle));

% Calculates the maximum depth for an angle width

for i = 1:length(angle)
    z(i) = zs - (c/gamma) + (c/(cos(angle(i)*pi/180)*abs(gamma)));
end

% show the maximum depth storage on the z array
z_max = max(z)
```

```
z_max = 654.5890
```

```
% Finds the angles in degrees which reach the maximum depth
% for the initial conditions with a constant gradient
angle_max = find(z == z_max);
angle(angle_max)
```

```
ans = 1x2
     -2     2
```

Figure 3: Maximum depth loop for a given set of angles.

Thanks to the previous loop, we can verify that the maximum depth is reached not only for -2°, but also for 2°. This means that the more vertical the angle is, the deeper the beam will reach.

3. Let us consider an echo sounder measuring the depth h of the ocean by measuring the travel time of short pulses. Determine the expression for the round-trip time t of a sound wave emitted under vertical incidence from the surface.

   We know that:

$$\frac{dz}{dt} = C_o + \gamma_o\ |z| \tag{3}$$

Therefore, to obtain the time to reach the seafloor:

$$\int dt = \int_0^h \frac{1}{c_o + \gamma_o \ |z|} dz$$

Hence:

$$t = \left. \frac{z \ln\left(|\gamma_0 \, |z| + C_o|\right)}{\gamma_0 \, |z|} \right|_0^h$$

Meaning that:

$$\int dt = \int_0^{3500} \frac{1}{1450 + 0.0163z} dz$$

$$t = \left. \frac{10000z \ln\left(163 \, |z| + 14500000\right)}{163 \, |z|} \right|_0^{3500}$$

As a result, we have that the time to reach the seafloor is:

$$t = 2.367 \ s$$

Because is a round trip, we need to multiply the time by 2:

$$Time \, round \, trip = 2 \times 2.367 \ = \ 4.735$$

If we modify the function for the first point with $Z_s = 0 \, m$ and an angle of 90°, we can check the path of the light beam for a round-trip path and get the same results.

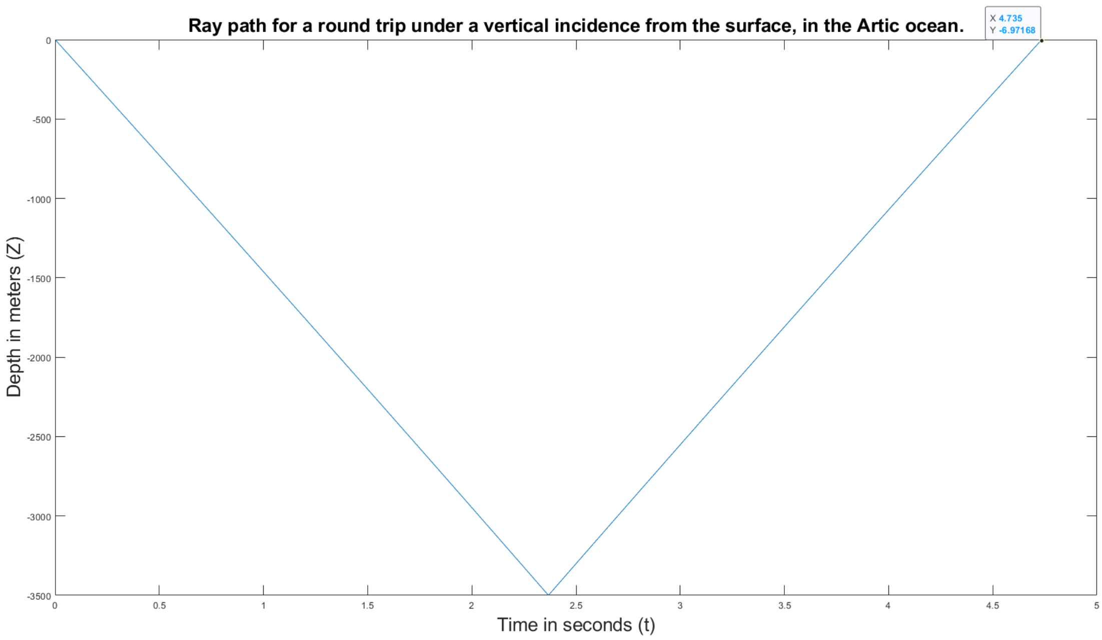

Figure 4: Ray path for a round trip, vertical incidence from the surface, in the Artic ocean.

What is the error on the depth, if it is estimated from the measurement of t assuming that the velocity is uniform, equal to c_0.

$$Estimated\ depth = C_o \times t$$

In our case:

$$Estimated\ depth = 1450 \times 2.367 = 3432\, m$$

Therefore, the error in the depth measurement is given by:

$$Error\ depth = h - Estimated\ depth$$

Thus, error in the measurement for our conditions is:

$$Error\ depth = 3500 - 3432 = 68\, m$$

Meaning an approximate error of:

$$100 \times (1 - \frac{3432}{3500}) = 2\%$$

Mediterranean Sea

Because of the higher water temperature at the surface, we observe a decrease in the sound velocity over 700 meters, with $-0.026\, s^{-1}$ gradient, before returning to a pressure-driven behavior, as described above for the Arctic Ocean ($\gamma_0 = +1.63 \times 10^{-2} s^{-1}$).

4. As in question 1/, using the software, plot the trajectory of the rays constituting the beam over a period of 15 s. The transmitter remains the same and is still located at 600m depth.

   Given the initial conditions, we obtain the following graph showing the paths followed by the beams between -2° and 2° with steps of 0.1°, given a variant gradient.

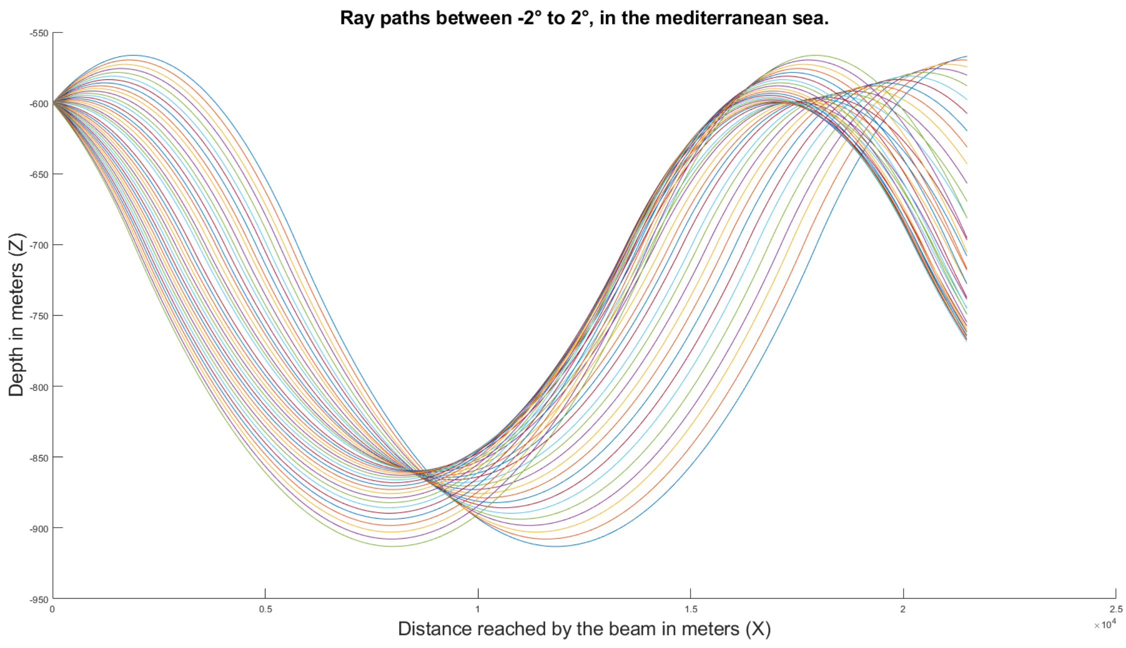

Figure 5: Ray paths at 600m depth, angular width 4°, variant gradient.

The following velocity profile is obtained for all beams, where the changes at 700m.

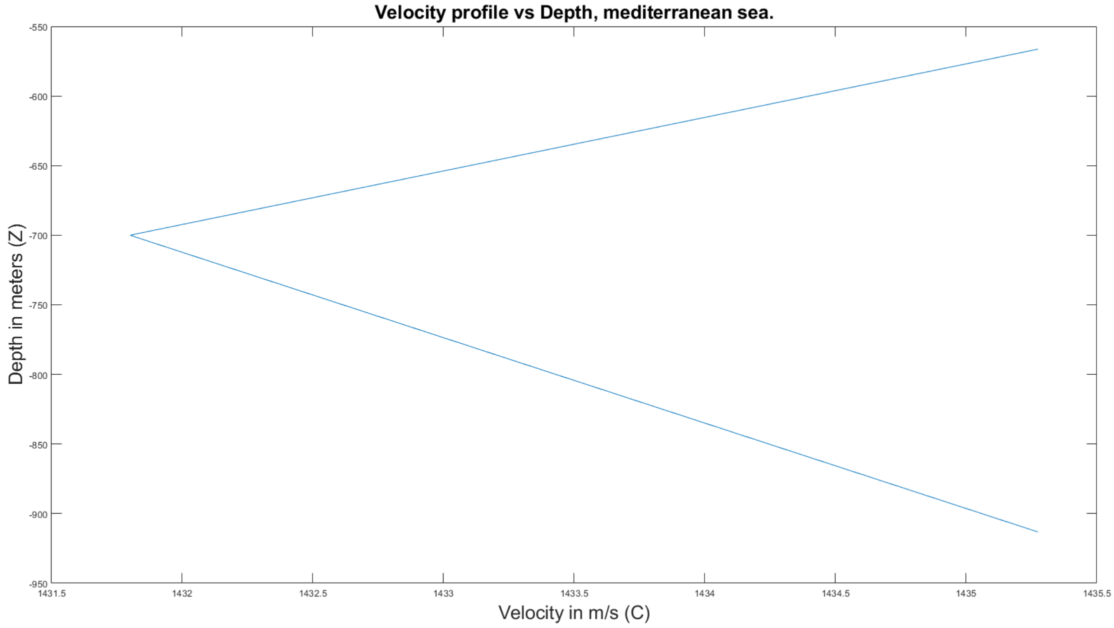

Figure 6: Velocity profile for the Mediterranean Sea.

5. It is planned to establish a communication with another antenna located at 10 km. At which depth should this antenna be located to receive a signal?

   For the same transmitter at 600 m depth with an angular width of 4° between -2° to 2° to reach the antenna, the antenna must be located between a depth of 844.391 m and 893.465 m to reach, 10000 m.

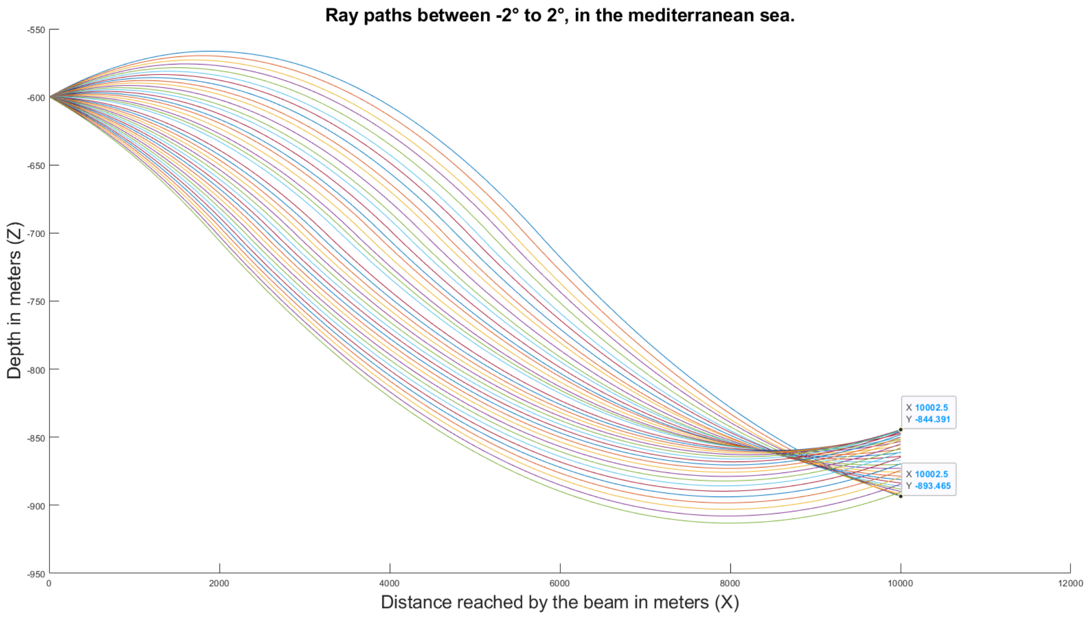

Figure 7: Velocity profile for the Arctic Ocean until reach, 10000 m.

The previous graph represents the beams sent by the transmitter to the antenna up to 10 km.