Master MIR – Underwater Acoustics

Ismetcan SARAÇ
Yosef GUEVARA

January 22, 2023

Homework 2

Let us consider a water channel, 350 m in depth, and let us assume that the velocity c is constant and equal to $1500m/s$. A signal is transmitted from a point-like source in the water and measured by a set of 9 receivers located along a vertical line at depths $25 * n$ meters, $1 \leqslant n \leqslant 9$. When hitting the (flat) boundaries, the acoustic waves are totally reflected. The file 'Received.mat' contains a 9 x 32000 array which provides the 9 received signals during 6.4 s with sampling path dt = 2.e-04 s. Line n corresponds to the receiver at depth $25 * n$ meters.

1. Plot the received signals and explain how you can get a rough estimation of the location of the transmitter from the various time delays.

   It is possible to get a rough idea of the transmitter's location by analyzing the time delay of the signal at each receiver. This can be done by identifying which receiver receives the signal first.

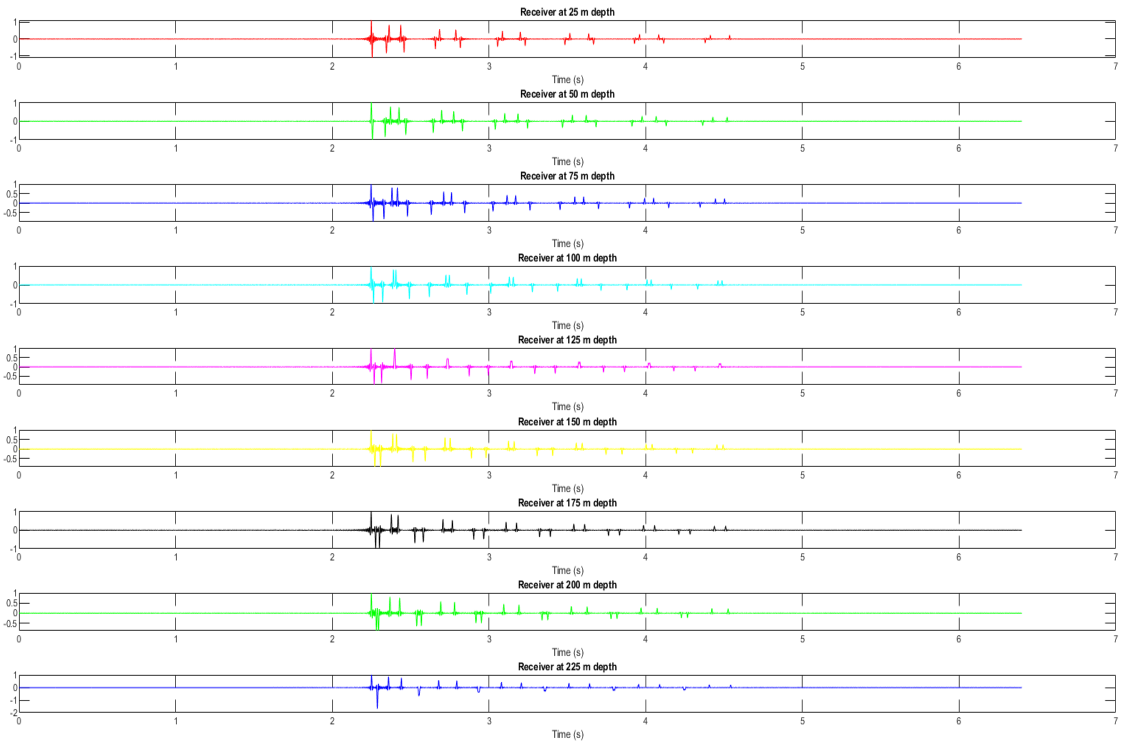

Figure 1: Graph with the received signals.

1 Recieved Signals

By analyzing the time delays between the signals, we can make an initial estimate of the location of the source. By finding the greater absolute value of the signal, we can isolate it and take the first peak of the signal and ignore any later peaks or noise.

To determine the depth of the source, we can examine the signal that was received first. If we zoom in on the signals and plot them together, we can observe the time delays between them and use that information to approximate the depth of the source.

FIGURE FIGURE FIGURE FIGURE FIGURE FIGURE FIGURE FIGURE FIGURE

From the figure, it can be observed that the receiver located at 125m is the first one to receive the signal, followed by the receivers at 100m and 150m. The receiver at 100m receives the signal slightly before the one at 150m. Therefore, it can be inferred that the source is located in the range of 100m ¡ zs ¡ 125m. The specific times at which the maximum value of the first peak is observed at each receiver are as follows:

FIGURE FIGURE FIGURE

2. Improve the accuracy of this first estimate thanks to the simulation of the back propagation of the time-reversed received signals.

   To determine the horizontal location of the source (x-coordinate), x_a, the time delay between two receivers can be calculated and used. One of the receivers is assumed to be receiving the signal via the direct path, and the other one is assumed to be receiving the signal via a reflected path. In this case, the receiver located at $z_a = 125$m is used as the direct path approximation and the receiver located at $z_b = 25$m is used as the reflected path. The location of the source can be determined by treating the receivers and the source as the three points of a right-angled triangle.

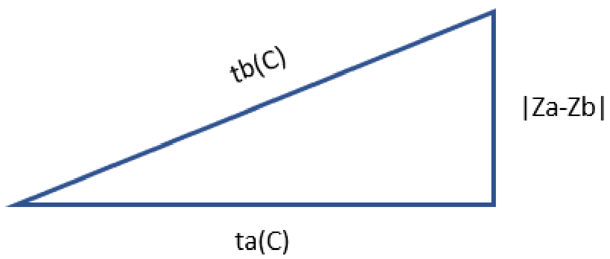

Figure 2: Pythagoras theorem applied to acoustics signals.

Given that, the difference of arrival time, Δt, between the two receivers is calculated as $\Delta t = 2.24292 - 2.2464 = 0.0028$.So we now have two equations with two unknowns (x and z coordinates of the source location) , which can be solved to get the exact location of the source.

$$\begin{aligned} t_b - t_a &= \Delta t \\ t_b \cdot C &= \sqrt{(t_{a\cdot C})^{2}+(\Delta_z)^{2}} \end{aligned} \tag{1}$$

We can obtain the following by replacing an expression in the other:

$$\begin{aligned}(t_a + \Delta t)\cdot &= \sqrt{(t_a \cdot C)^2 + (\Delta_z)^2} \\ t_a &= \frac{\sqrt{(t_a \cdot C)^2 + (\Delta_z)^2}}{C} - \Delta t\end{aligned} \tag{2}$$

If we solve the equation, we find that $t_a = 0.79225$s. Using this, we can calculate the distance as $t_a * C = 0.79225 * 1500 = 1188m$.

To verify these results, we can scan the location where the signals originated by transmitting the reverse of the received signals and measuring the power of the combined signals. This is because they will create constructive interference at the source location. The transmission of the inverse signals $si(t)$ is represented by convolving them with the Green's function of the medium $si(t) * G(x_t, z_t, x_s, z_s, t)$, which numerically corresponds to the following equation:

$$si(t) * G(x_t, z_t, x_s, z_s, t) = -\sum_j \in_j \frac{s(t - r_j/c)}{4 \cdot \pi \cdot r_j} \tag{3}$$

The reason for the summation is due to the fact that we receive multiple impulses that come from the different reflections on the boundary layers of the medium. By keeping the depths Z = 110, 115, 120, 125 constant while scanning, we can get a more precise idea of the coordinates while still keeping the computational resources used low.

We can observe that the maximum power was attained at a depth of 120m and that the x-coordinate was around 960m. These results are similar to the earlier estimation, particularly in terms of depth. If we now perform a scan across both x and z, we will get a more intuitive visualization of the source location. The picture below corresponds to a scan around the point z = 115, x = 970, with a pixel resolution of 5m and a grid size of 200m.

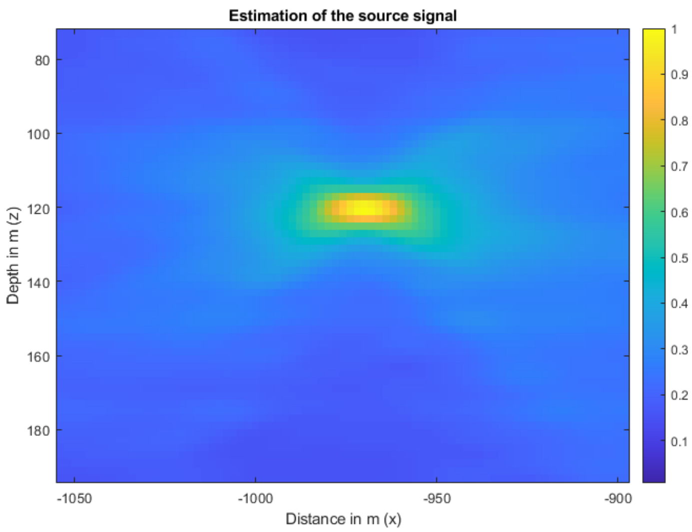

Figure 3: Right-angle triangle assumption.

In conclusion, we can see that the source location was accurately determined using a combination of experimental and analytical methods. The experimental methods were more precise, but they required significant computational resources if a large area needed to be scanned.