Master MIR – Underwater Acoustics

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Homework 2

Let us consider a water channel, 350 m in depth, and let us assume that the velocity c is constant and equal to $1500 m/s$. A signal is transmitted from a point-like source in the water and measured by a set of 9 receivers located along a vertical line at depths $25 * n$ meters, $1 \leqslant n \leqslant 9$. When hitting the (flat) boundaries, the acoustic waves are totally reflected. The file 'Received.mat' contains a 9 x 32000 array which provides the 9 received signals during 6.4 s with sampling path dt = 2.e-04 s. Line n corresponds to the receiver at depth $25 * n$ meters.

1. Plot the received signals and explain how you can get a rough estimation of the location of the transmitter from the various time delays.

   It is possible to get a rough idea of the transmitter's location by analyzing the time delay of the signal at each receiver. This can be done by identifying which receiver receives the signal first.

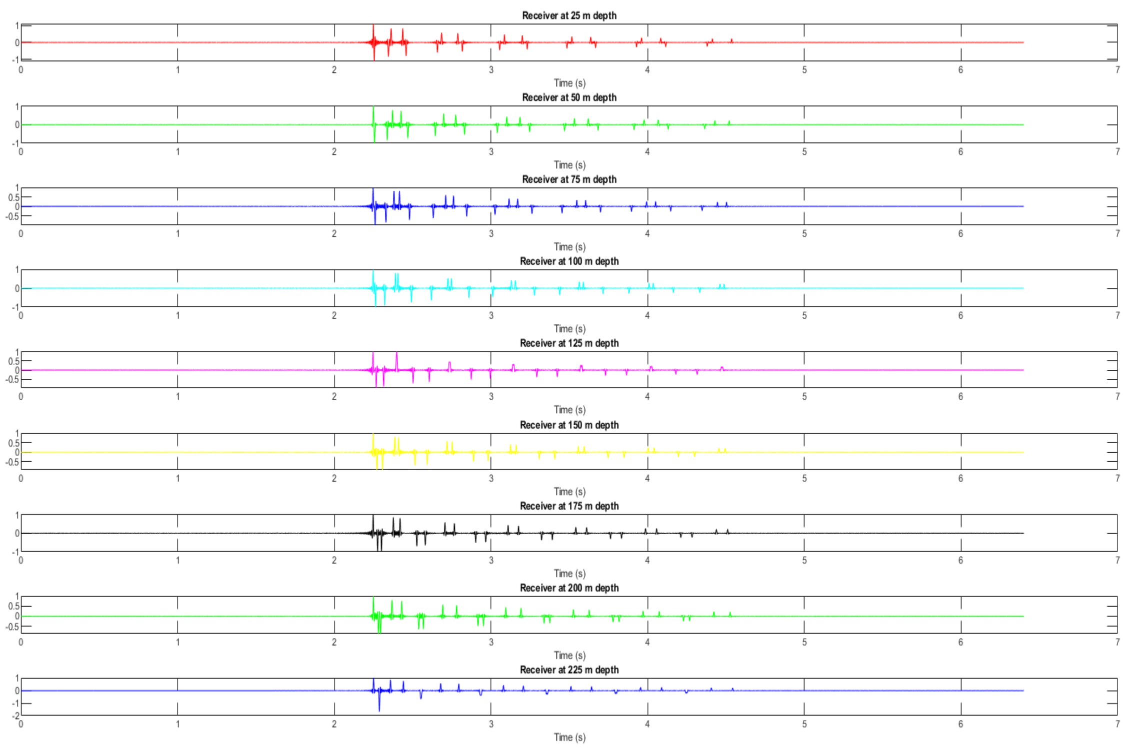

Figure 1: Graph with the received signals.

By analyzing the time delays between the signals, we can make an initial estimate of the location of the source. To determine the transmitter's depth, a limit was used to separate the signal from noise. In this way, any value greater than 0.5 will be set to zero. Thanks to this, we can identify the exact time of arrival of each signal to each transmitter. The receiver that has received the signal in the shortest time will be the closest to the depth at which the transmitter is located.

Time of arrival per receiver depth	
Depth (m)	time(s)
25	2.2472
50	2.2466
75	2.2450
100	2.2448
125	2.2444
150	2.2450
175	2.2454
200	2.2468
225	2.2484

From time of arrival in the receiver, we can determine that the transmitter is closer to the receiver at a depth of 125 m, followed by the receivers at 100 m and 150. The receiver at 100 m receives the signal slightly before the one at 150 m. Therefore, it can be inferred that the source is located in the range of $100\,m \leq zs \leq 125\,m$.

2. Improve the accuracy of this first estimate thanks to the simulation of the back propagation of the time-reversed received signals.

   To determine the horizontal location of the source (x-coordinate), x_a, the time delay between two receivers can be calculated and used. The receiver at $z_a = 125\,m$ depth is assumed to be receiving the signal via the direct path, and the receiver located at $z_b = 25\,m$ is used as the reflected path. The location of the source can be determined by treating the receivers and the source as the three points of a right-angled triangle.

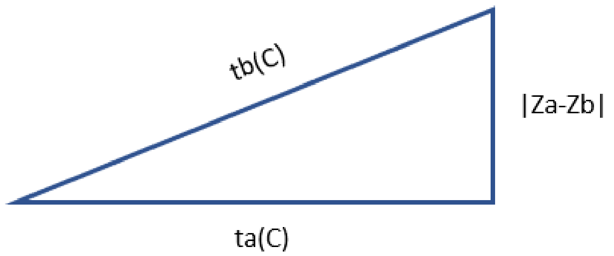

Figure 2: Pythagoras theorem applied to acoustics signals.

Given that, the difference of arrival time, Δt, between the two receivers is calculated as $\Delta t = 2.2472 - 2.2448 = 0.0028$.So we now have two equations with two unknowns (x and z coordinates of the source location) , which can be solved to get the exact location of the source.

$$t_b - t_a = \Delta t$$

$$t_b \cdot C = \sqrt{(t_{a \cdot C)^2 + (\Delta_z)^2}}$$

We can obtain the following by replacing an expression in the other:

$$(t_a + \Delta t) \cdot C = \sqrt{(t_a \cdot C)^2 + (\Delta_z)^2}$$

$$t_a = \frac{\sqrt{(t_a \cdot C)^2 + (\Delta_z)^2}}{C} - \Delta t$$

If we solve the equation, we find that the distance of the receiver is around, 1888 m.

To verify these results, we scan the location where the signals originated by transmitting the flipped signals and measuring the energy of the combined signals. This is because they will create constructive interference at the source location. The flipped signals $si(t)$ are represented by convolving them with the Green's function of the medium $si(t) * G(x_t, z_t, x_s, z_s, t)$, given by:

$$si(t) * G(x_t, z_t, x_s, z_s, t) = -\sum_j \in_j \frac{s(t - r_j/c)}{4 \cdot \pi \cdot r_j} \tag{1}$$

The reason for the summation is because we receive multiple impulses that come from the different reflections on the boundary layers of the medium. Thanks to the previous results, we can narrow down the search area to values close to a distance of 1188 and a depth of 125m. To do this, a search grid of the source is created between 1200 and 800, with a depth between 100 m and 150 m, searching each 2 m.

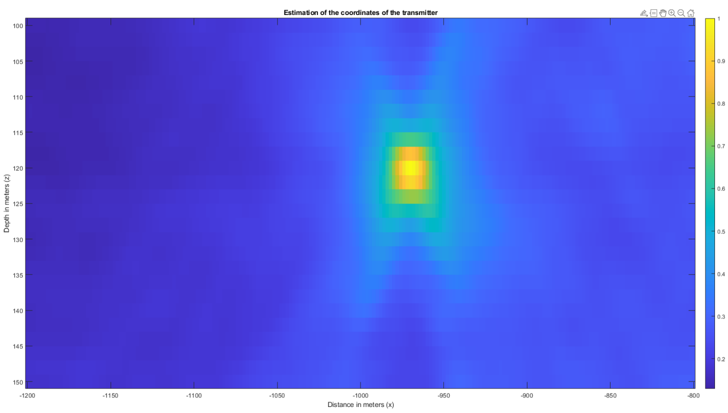

Figure 3: Right-angle triangle assumption.

As a result, we go that the maximum energy is at a depth of 120 m and 970 m of distance. are closer to the theoretical estimation.