UNDERWATER ACOUSTICS

Homework 2

Lab Report

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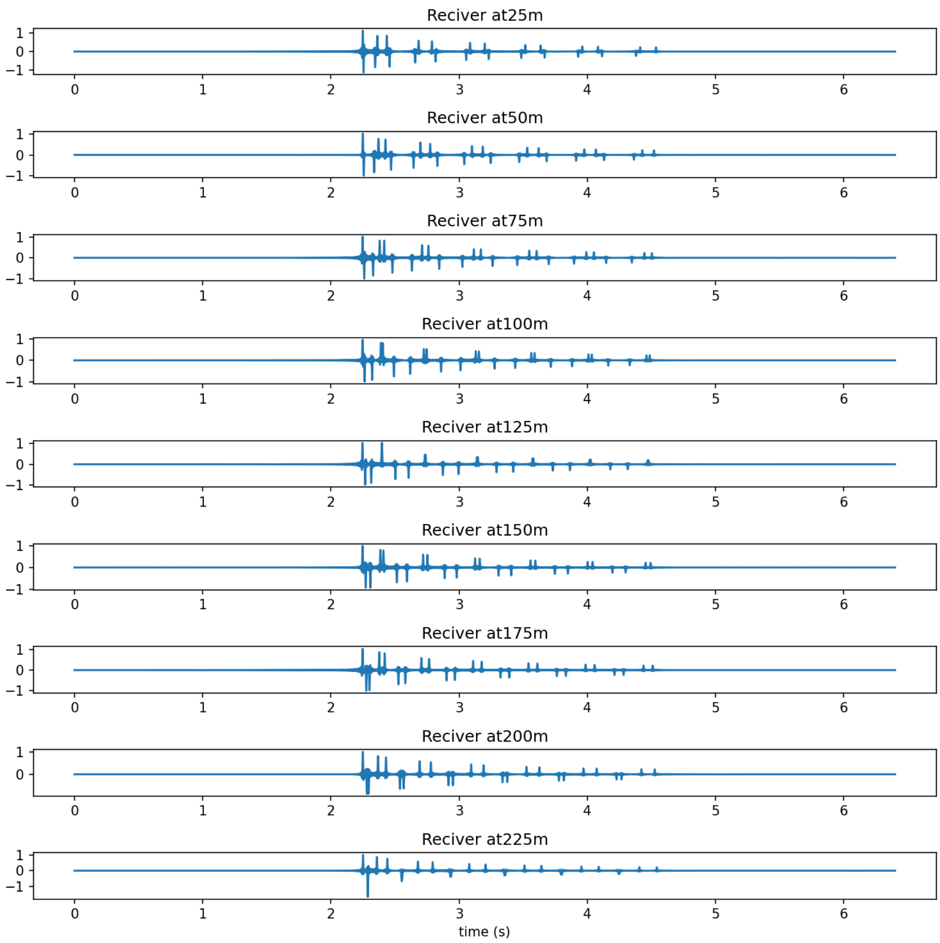
Introduction

Let us consider a water channel, 350 m in depth, and let us assume that the velocity c is constant and equal to 1500 m/s. A signal is transmitted from a point-like source in the water and measured by a set of 9 receivers located along a vertical line at depths 25*n meters, $1 \leq n \leq 9$. When hitting the (flat) boundaries, the acoustic waves are totally reflected.

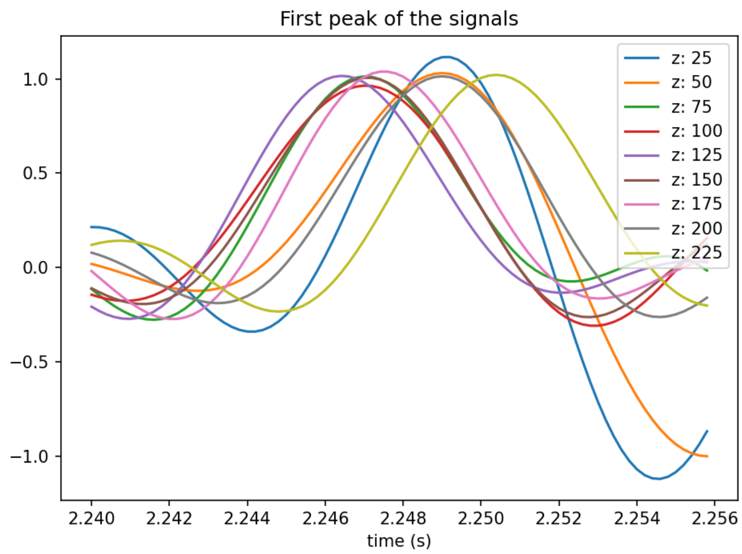
Received signals

We receive 9 signals during 6.4s with sampling path $dt = 2e^{-4}s$, and bandwidth, $B = 200HZ$, if we plot the them we obtain the following figure:



Source localization

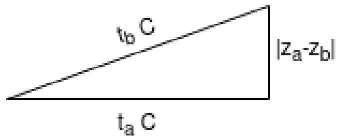
By looking at the various time delays we can get a first approximation of the source location. To get an approximation of the depth of the source we can look at the receiver that got first the impulse, if we zoom enough in the signals and we plot them together we can appreciate their time delays:



In the figure above we can see how the receiver at 125m is the first one to receive the signal, and then the receivers at 100m and 150m, being the receiver at 100m a bit ahead, therefore, the source should be located on $100 << z_s < 125$. The exact times of the maximum value in the first peak are the followings:

Depth	Time
25	2.2492
50	2.249
75	2.247
100	2.247
125	2.2464
150	2.2472
175	2.2476
200	2.249
225	2.2504

To locate the x coordinate, x_s, we can compute it from the time delay between two receivers, supposing that one of them is the direct path. For the direct path approximation we can use the receiver located at $z_a = 125$ and for the second one the receiver located at $z_b = 25$. Then we know that a rectangular triangle is shaped between the location of the source and each receiver.



And we also know that the difference of arrival time, Δt, between both receiver is $\Delta t = 2.2492 - 2.2464 = 0.0028$. So now we have two equations and two incognitas.

$$t_b - t_a = \Delta t;$$

$$t_b \cdot C = \sqrt{(t_a \cdot C)^2 + (\Delta z)^2};$$

By substituting one expression in the other we get:

$$(t_a + \Delta t) \cdot C = \sqrt{(t_a \cdot C)^2 + (\Delta z)^2};$$

$$t_a = \frac{\sqrt{(t_a \cdot C)^2 + (\Delta z)^2}}{C} - \Delta t.$$

And solving the equation, we get that $t_a = 0.79225s$, then the distance would be $t_a * C = 0.79225 * 1500 = 1188m$.

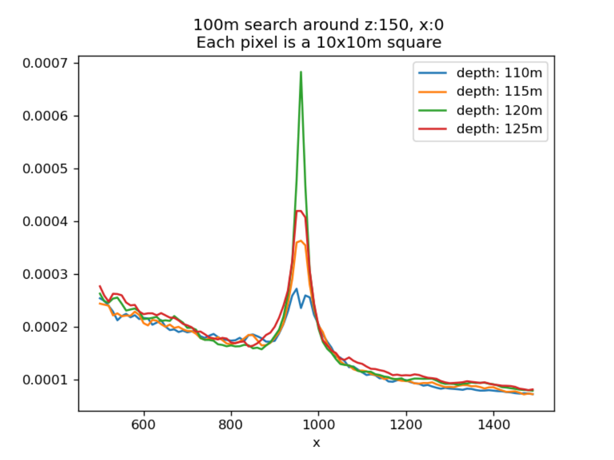
To check this results, we can scan the source location transmitting the inverse of the received signals and computing the power of their superposition, since they will create constructive interference at the source location. The transmission of the inverse signals $si(t)$, is represented as the convolution with the Green's function of the media, $si(t) * G(x_t, z_t, x_s, z_s, t)$, that numerically correspond to the following expression:

$$si(t) * G(x_t, z_t, x_s, z_s, t) = -\sum_j \epsilon_j \frac{s(t - r_j/c)}{4 \cdot \pi \cdot r_j}$$

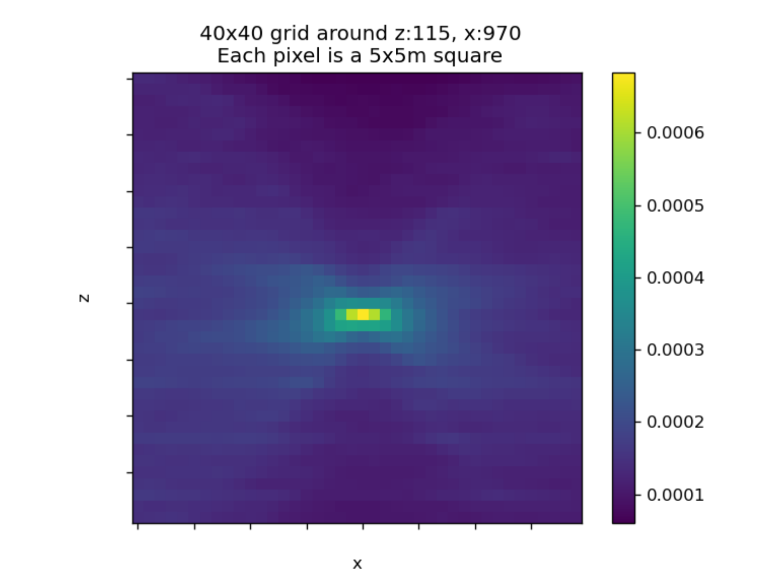
The summation comes from the fact that we receive multiple impulses coming from the different bounces on the boundary layers of the media.

If we scan maintaining constant depths $Z = 110, 115, 120, 125$, we will get a more accurate idea of the coordinates while keeping the used computational resources low.

The following figure represent a scan over x with different depths.



We can see how the maximum power was obtaining for a depth of 120m and we can also check that the x coordinate was around 960m, these results are close to the estimation that we got before, especially on depth. If we now do a scan over x and z we will get a more intuitive visualization of the location of the source. The following picture correspond to a scan around the point $z = 115, x = 970$, with a pixel resolution of 5m and a grid size of 200m.



To conclude we can see that the source location was properly estimated using both experimental and analytical methods, being the experimental methods more accurate but needing large computing power if we would like to scan a big area.