課題2

1. $\int_0^5 2^x \, dx$

$k = 2^x$ とおくと

$\log k = \log 2^x$

$= x \log 2$

両辺をxで微分して

$\frac{1}{k} k' = \log 2$

$k' = k \log 2$

$\frac{k'}{\log 2} = k$

$\left(\frac{k}{\log 2}\right)' = k$

すなわち

$\left(\frac{2^x}{\log 2}\right)' = 2^x$

両辺を積分すると

$\frac{2^x}{\log 2} + C = \int 2^x \, dx$

したがって

$\int_0^5 2^x \, dx = \left[\frac{2^x}{\log 2}\right]_0^5$

$= \frac{2^5 - 1}{\log 2}$

$= \frac{31}{\log 2} \fallingdotseq 44.723\ldots$

2. $\int_0^1 \frac{1}{1+x^2} \, dx$

$x = \tan\theta$ とおくと

$dx = \frac{1}{\cos^2\theta} d\theta$

$x: 0 \to 1$

$t: 0 \to \frac{\pi}{4}$

$1 + x^2 = 1 + \tan^2\theta$

$= 1 + \frac{\sin^2\theta}{\cos^2\theta}$

$= \frac{\cos^2\theta + \sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$

$\therefore \frac{1}{1+x^2} = \cos^2\theta$

したがって

$\int_0^1 \frac{1}{1+x^2} \, dx$

$= \int_0^{\frac{\pi}{4}} \cos^2\theta \cdot \frac{1}{\cos^2\theta} \, d\theta$

$= \int_0^{\frac{\pi}{4}} d\theta$

$= \left[\theta\right]_0^{\frac{\pi}{4}} = \frac{\pi}{4} \fallingdotseq 0.78\ldots$