Dynamic Programming

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Introduction

This is a replication of Adams, et al.(2015), mainly based on Chapter 7.

Overview

1. Replication
    1. Value Function Iteration
    2. Policy Function Iteration
    3. Finite Stochastic Dynamic Programming
2. Exercise

1. Replication

Setup

```
# initialize
rm(list = ls())

# package
if(!require(pacman)) install.packages("packman")
pacman::p_load(
  tidyverse,
  tictoc,
  RColorBrewer
)
```

1-1. Value Function Iteration

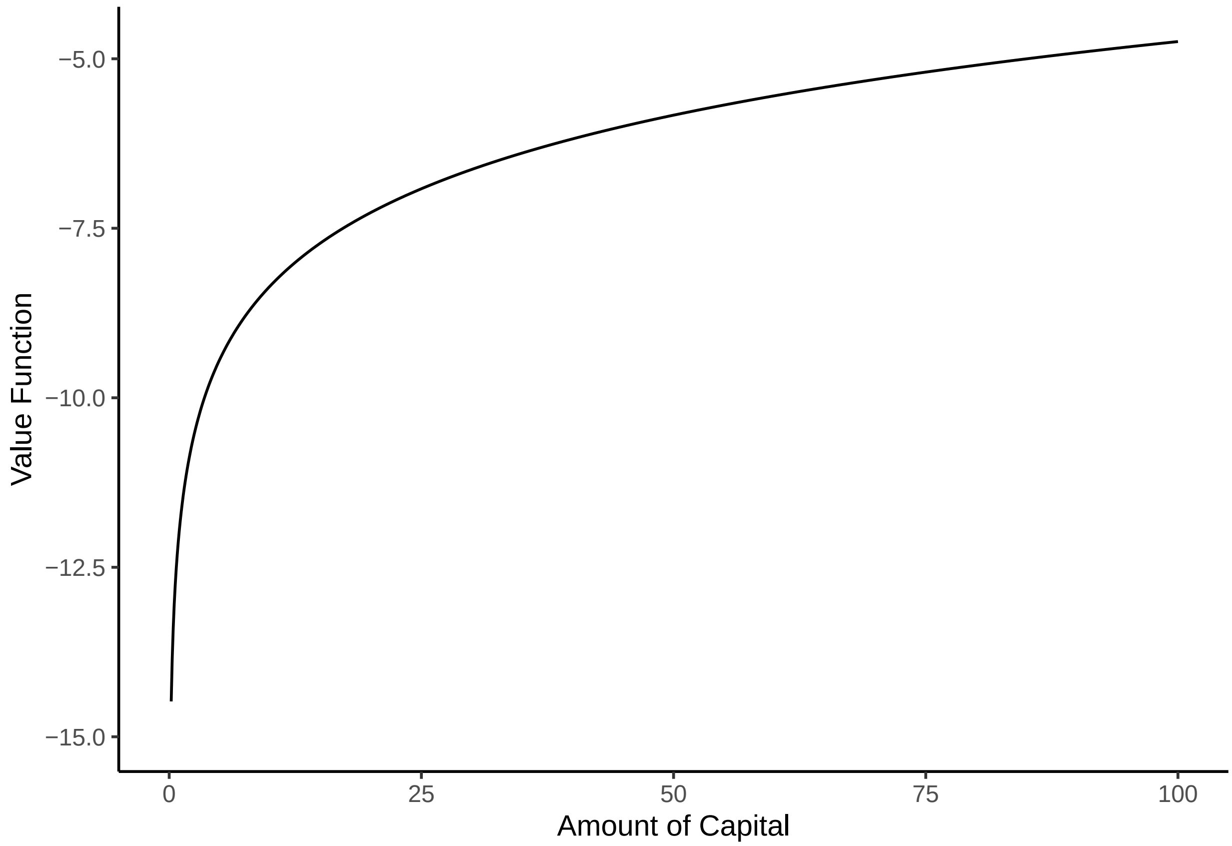
```
# one iteration of the value function
IterateVF <- function(V, maxK){
  # basic parameters
  Alpha <- 0.65
  Beta <- 0.9
```

```
  Theta <- 1.2

  grid <- length(V)
  K <- seq(from = 1e-6, to = maxK, length.out = grid)
  TV <- rep(0, length(V))
  optK <- rep(0, length(V))

  # loop through and create new value function for each possible capital value
  for(k in 1:grid){
    c <- rep(Theta*(K[k]^Alpha), grid) - K
    c[c<=0] <- rep(0, sum(c<=0))
    u <- log(c)
    candid <- u + Beta*V
    TV[k] <- max(candid)
    optK[k] <- which(candid == max(candid))
  }

  # time consuming method
  # candid <- rep(NA, 1000)
  # c <- rep(NA, grid)
  # u <- rep(NA, grid)
  #
  # for(k in 1:grid){
  #   for(k_tilde in 1:grid){
  #     c[k_tilde] <- Theta*(K[k]^Alpha)-K[k_tilde]
  #     c[k_tilde] <- ifelse(c[k_tilde] > 0, c[k_tilde], 0)
  #     u[k_tilde] <- log(c[k_tilde])
  #     candid[k_tilde] <- u[k_tilde] + Beta*V[k_tilde]
  #   }
  #   TV[k] <- max(candid)
  #   optK[k] <- K[which(candid == max(candid))]
  # }

  sol <- matrix(c(TV, optK), nrow = length(V), ncol = 2, byrow = FALSE)
  return(sol)
}
```

Analytical solution

```
# set parameters, plot analytical solution
Beta <- 0.9
Alpha <- 0.65
Theta <- 1.2
aB <- Alpha*Beta
K <- seq(from = 1e-6, to = 100, length.out = 1000)

E <- Alpha / (1 - aB)
f <- (1/(1-Beta))*(log(Theta*(1-aB))) + aB*log(aB*Theta)/((1-aB)*(1-Beta))
soln <- E*log(K) + f

ggplot() +
  geom_line(aes(x = K, y = soln)) +
  ylim(c(-15, NA)) +
```

```
  xlab("Amount of Capital") + ylab("Value Function") +
  theme_classic()
```



Iterated graph

```
tic()
n <- 15
# 10 iterations
TV <- matrix(rep(NA, 1000*n), ncol = n)
TV[, 1] <- rep(0, 1000)

for(iter in 1:n){
  cat("Iteration number:", iter, "\n")
  if(iter < n) TV[, iter+1] <- IterateVF(TV[, iter], 100)[, 1]
}
```

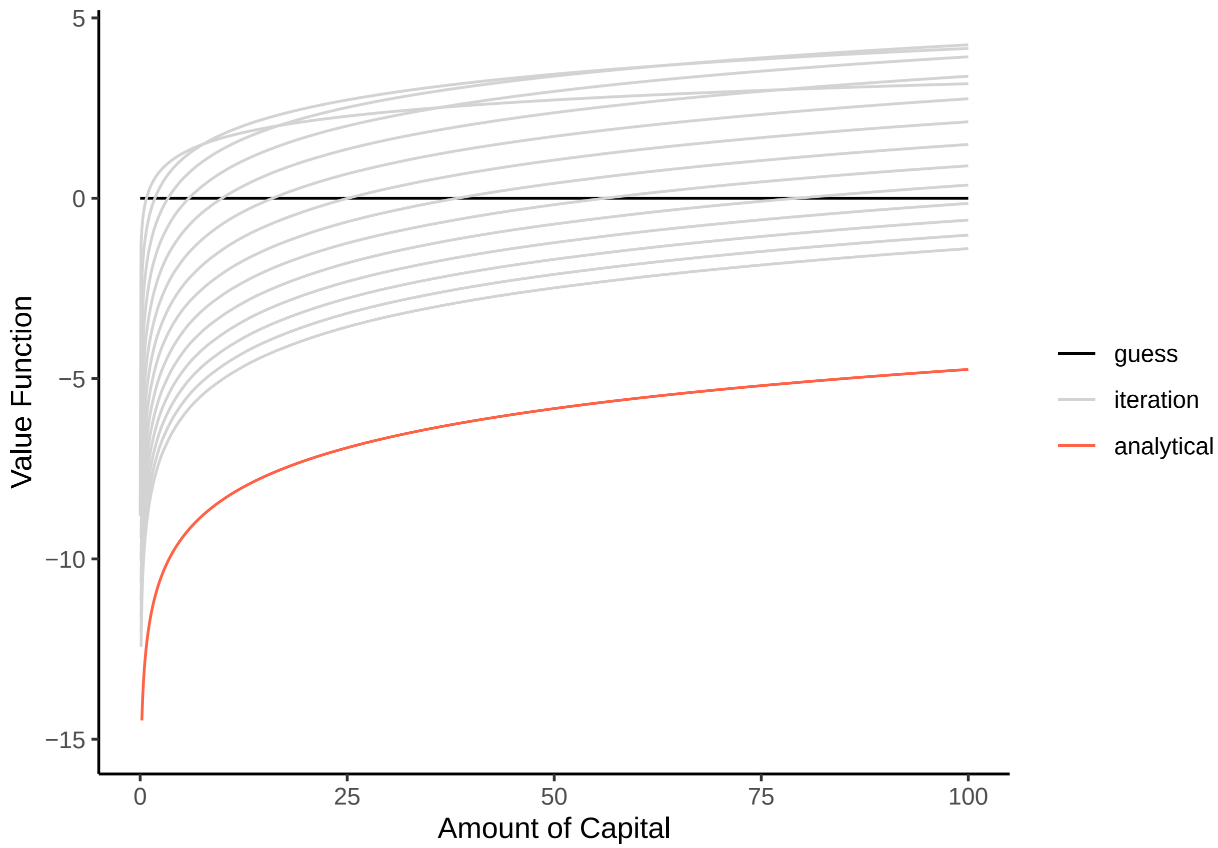
```
## Iteration number: 1
## Iteration number: 2
## Iteration number: 3
## Iteration number: 4
## Iteration number: 5
## Iteration number: 6
## Iteration number: 7
## Iteration number: 8
## Iteration number: 9
```

```
## Iteration number: 10
## Iteration number: 11
## Iteration number: 12
## Iteration number: 13
## Iteration number: 14
## Iteration number: 15
```

```
toc()
```

```
## 0.419 sec elapsed
```

```
# plot
tic()
p1 <- ggplot()
for(i in 1:n){
  df <- data.frame(k = K, tv = TV[, i]) # store TV in a data.frame to layer plots
  if(i == 1){
    p1 <- p1 + geom_line(data = df, aes(x = k, y = tv, color = 'guess'))
  } else{
    p1 <- p1 + geom_line(data = df, aes(x = k, y = tv, color = 'iteration'))
  }
}
p1 <- p1 +
  geom_line(data = df, aes(x = k, y = soln, color = "analytical")) +
  scale_color_manual(
    name = NULL,
    values = c("guess" = "black", "iteration" = "lightgray", "analytical" = "tomato"),
    labels = c("guess", "iteration", "analytical")
  ) +
  ylim(c(-15, NA)) +
  labs(x = "Amount of Capital", y = "Value Function") +
  theme_classic()
p1
```




```
toc()
```

```
## 0.192 sec elapsed
```

Convergence of Value Function

```
tic()
# setting
K <- seq(from = 1e-6, to = 100, length.out = 1000) # grid
V <- rep(0, 1000) # guess
conv <- 100 # criterion for convergence
crit <- 1e-2 # stopping threshold
Iter <- 0 # numbering iteration

# for plot
df <- data.frame(K = K, V = V)
p2 <- ggplot() +
  geom_line(data = df, aes(x = K, y = V, color = 'guess'))

# iteration
while(conv>crit && Iter<1000){
  Iter <- Iter + 1
  if(Iter %/% 10 == Iter/10) cat("Iteration number:", Iter, "\n")

  # mapping
```

```r
  sol <- IterateVF(V, 100)
  TV <- sol[, 1]

  # distance between TV and V
  conv <- max(abs(TV-V))

  # for plot
  df$TV <- TV # store TV in a data.frame to layer plots
  p2 <- p2 + geom_line(data = df, aes(x = K, y = TV, color = 'iteration'))

  # pass TV to next iteration
  V <- TV
}
```

```
## Iteration number: 10
## Iteration number: 20
## Iteration number: 30
## Iteration number: 40
## Iteration number: 50
## Iteration number: 60
```

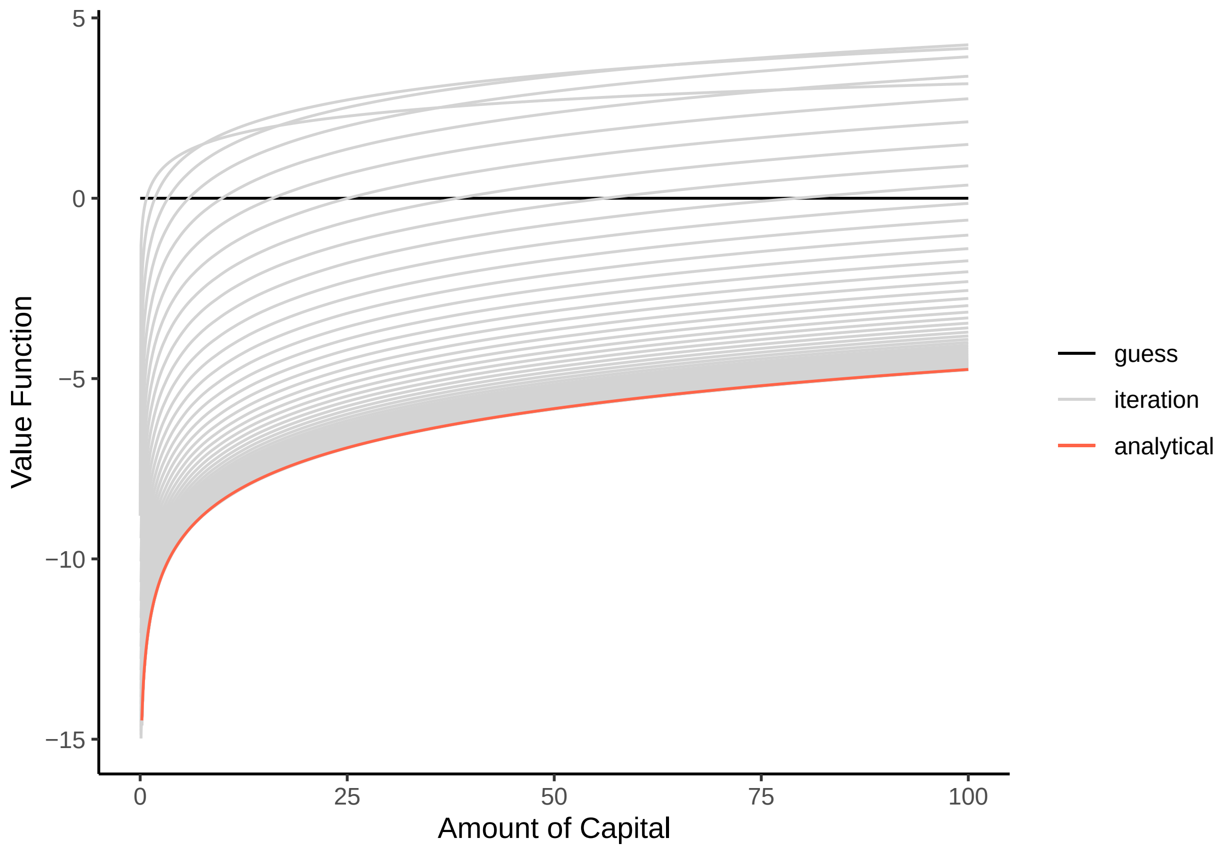
```r
toc()
```

```
## 2.032 sec elapsed
```

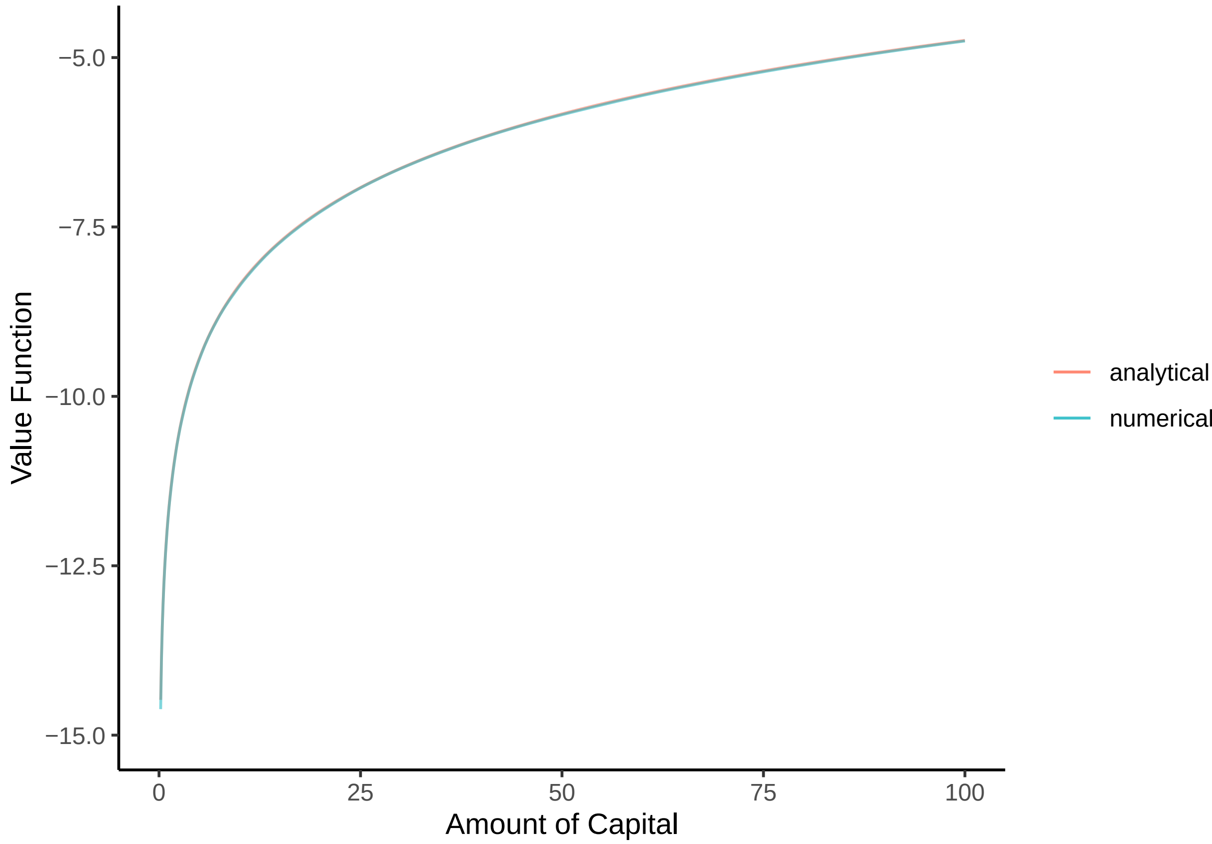
```r
cat("# of iterations:", Iter)
```

```
## # of iterations: 66
```

```r
# plot
p2 <- p2 +
  geom_line(aes(x = K, y = soln, color = 'analytical')) +
  scale_color_manual(
    name = NULL,
    values = c("guess" = "black", "iteration" = "lightgray", "analytical" = "tomato"),
    labels = c("guess", "iteration", "analytical")
  ) +
  ylim(c(-15, NA)) +
  labs(x = "Amount of Capital", y = "Value Function") +
  theme_classic()
p2
```

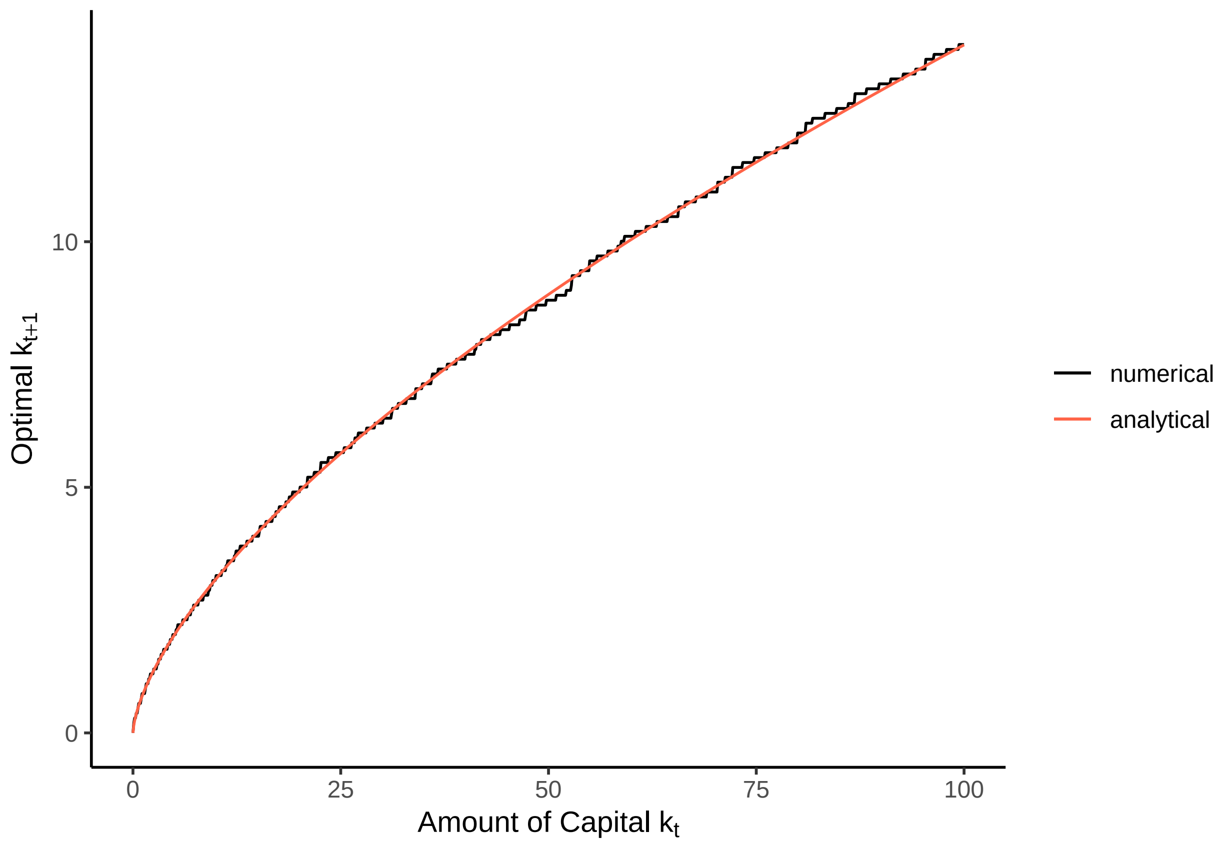



```
# another plot
p3 <-
  ggplot() +
  geom_line(aes(x = K, y = soln, color = 'soln'), alpha = 0.5) +
  geom_line(aes(x = K, y = TV, color = 'TV'), alpha = 0.5) +
  scale_color_manual(
    name = NULL,
    values = c("soln" = "tomato", "TV" = "#00AFBB"),
    labels = c("analytical", "numerical")
  ) +
  ylim(c(-15, NA)) +
  labs(x = "Amount of Capital", y = "Value Function") +
  theme_classic()
p3 # seems overlapped!
```



Calculated Best Path

```
ggplot() +
  geom_line(aes(x = K, y = K[sol[, 2]], color = "numerical")) +
  geom_line(aes(x = K, y = Theta*(K^Alpha)*(aB), color = "analytical")) +
  scale_color_manual(
    name = NULL,
    values = c("numerical" = "black", "analytical" = "tomato"),
    labels = c("numerical", "analytical")
  ) +
  labs(x = expression(paste("Amount of Capital ", k[t])),
       y = expression(paste("Optimal ", k[t+1]))) +
  theme_classic()
```



1-2. Policy Function Iteration

We are to solve the following functional equation:

$$V(k) = \max_{\tilde{k}} \{u(f(k) - \tilde{k}) + \beta V(\tilde{k})\}.$$

Note that

$$V_j = U_j + \beta Q_j V_j,$$

where V_j denotes a vector by which a value function with respect to discretized k_t is expressed, and Q_j denotes the transition matrix whose binary entry takes 1 if it is the optimal $\tilde{k}$ given k or not. U_j is a vactor whose each entry represents the maximized utility given today's capital k, i.e., $u(c_j(k))$. Solving this equation, we have

$$V_j = (I - \beta Q_j)^{-1} U_j.$$

This is the idea of the policy function iteration.

```
IteratePolicy <- function(V, maxK){
  # basic parameters
  Alpha <- 0.65
  Beta <- 0.9
  Theta <- 1.2

  grid <- length(V)
  K <- seq(from = 1e-6, to = maxK, length.out = grid)
  #TV <- rep(0, length(V))
```

```
  opt <- rep(0, length(V))

  # loop through and create new value function for each possible capital value
  for(k in 1:grid){
    c <- rep(Theta*(K[k]^Alpha), grid) - K
    c[c<=0] <- rep(0, sum(c<=0))
    u <- log(c)
    candid <- u + Beta*V
    #TV[k] <- max(candid)
    opt[k] <- which(candid == max(candid))
  }

  kopt <- K[opt]
  c <- Theta*K^Alpha - kopt
  u <- log(c)
  Q <- matrix(rep(0, grid*grid), ncol = grid)

  # create the transition matrix
  for(k in 1:grid){
    Q[k, opt[k]] <- 1
  }

  TV <- solve(diag(grid)-Beta*Q)%*%u
  sol <- matrix(c(TV, opt), ncol = 2)
  V <- TV
  return(sol)
}
```

```
tic()
# setting
K <- seq(from = 1e-6, to = 100, length.out = 1000) # grid
V <- rep(0, 1000) # guess
conv <- 100 # criterion for convergence
crit <- 1e-2 # stopping threshold
Iter <- 0 # numbering iteration


# for plot
df <- data.frame(K = K, V = V)
p4 <- ggplot()
# iteration
while(conv>crit && Iter<1000){
  Iter <- Iter + 1
  if(Iter %/% 10 == Iter/10) cat("Iteration number:", Iter, "\n")

  # mapping
  sol <- IteratePolicy(V, 100)
  TV <- sol[, 1]
  opt <- sol[, 2]

  # distance between TV and V
  conv<- max(abs(TV-V))
```

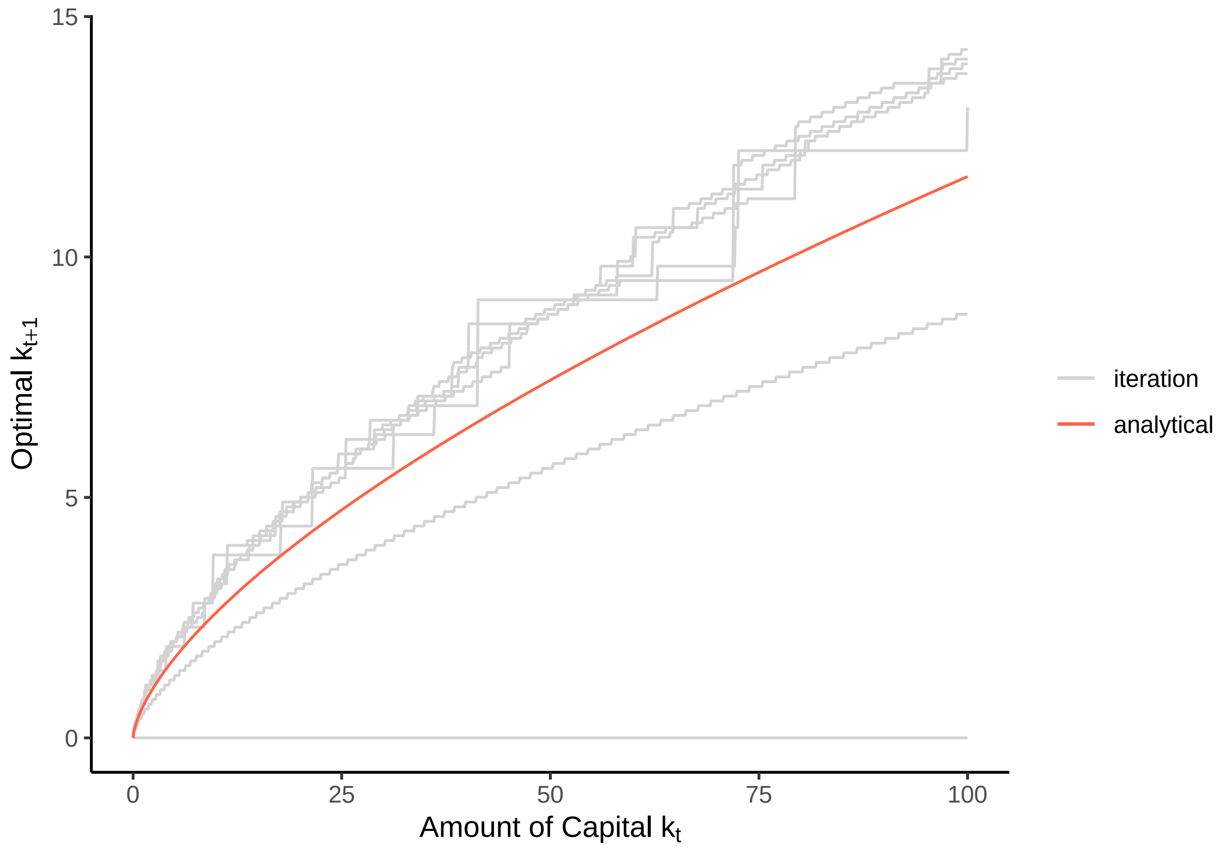
```r
  # for plot
  df$K_opt <- K[opt]
  p4 <- p4 + geom_line(data = df, aes(x = K, y = K_opt, color = 'iteration'))

  # pass TV to next iteration
  V <- TV
}
toc()
```

```
## 1.379 sec elapsed
```

```r
cat("# of outer iterations:", Iter)
```

```
## # of outer iterations: 7
```

```r
# plot
p4 <- p4 +
  geom_line(aes(x = K, y = aB*(K^Alpha), color = 'analytical')) +
  scale_color_manual(
    name = NULL,
    values = c("iteration" = "lightgray", "analytical" = "tomato"),
    labels = c("iteration", "analytical")
  ) +
  labs(x = expression(paste("Amount of Capital ", k[t])),
       y = expression(paste("Optimal ", k[t+1]))) +
  theme_classic()
p4
```



1-3. Finite Stochastic Dynamic Programming

This part is based on Chapter 6, "6.4 Stochastic Dynamic Programming".

```
tic()
# setup parameters
e <- c(-2, 2)
PI <- c(0.5, 0.5)
Beta <- 0.9
Theta <- 1.2
Alpha <- 0.98
Kl <- 100
grid <- 0.1
t <- 10
K <- seq(from = 0, to = Kl + max(e), by = grid)
V <- cbind(matrix(rep(NA, length(K)*t), ncol = t), matrix(rep(0, length(K)), ncol = 1))
aux <- array(rep(NA, length(K)*length(K)*t), dim = c(length(K), length(K), t))

# loop over preriods
for(t_iter in seq(from = t, to = 1, by = -1)){
  cat("Currently in period", t_iter, "\n")

  # loop over k_{t}
  for(inK in 1:length(K)){
  # for(inK in 1:length(seq(from = 0, to = Kl, by = grid))){ # seems mistaken
```

```r
    # loop over k_{t + 1}
    for(outK in 1:inK){
      c <- K[inK] - (K[outK]/Theta)^(1/Alpha) # note this is scaler
      nextKl <- Theta*(K[inK] - c)^Alpha + e[1]
      nextKh <- Theta*(K[inK] - c)^Alpha + e[2]
      nextKl[nextKl < 0] <- rep(0, sum(nextKl < 0))

      position_l <- ifelse((round(nextKl/grid) + 1) <= length(K),
                           (round(nextKl/grid) + 1),
                           length(K))
      position_h <- ifelse((round(nextKh/grid) + 1) <= length(K),
                           (round(nextKh/grid) + 1),
                           length(K))
      EnextV <-
        PI[1] * V[position_l, t_iter+1] + PI[2] * V[position_h, t_iter+1]
        #PI[1] * V[(round(nextKl/grid) + 1), t_iter+1] +
        #PI[2] * V[(round(nextKh/grid) + 1), t_iter+1] # seems mistaken

      #c <- ifelse(c <= 0, 1e-100, c) # if you want to avoid -Inf
      aux[inK, outK, t_iter] <- log(c) + Beta*EnextV
    }
  }
  V[, t_iter] <-
    apply(
      X = as.matrix(aux[, , t_iter]), MARGIN = 1, FUN = max, na.rm = TRUE
    )
}
```

```
## Currently in period 10
## Currently in period 9
## Currently in period 8
## Currently in period 7
## Currently in period 6
## Currently in period 5
## Currently in period 4
## Currently in period 3
## Currently in period 2
## Currently in period 1
```

```r
toc()
```

```
## 15.993 sec elapsed
```

Simulation

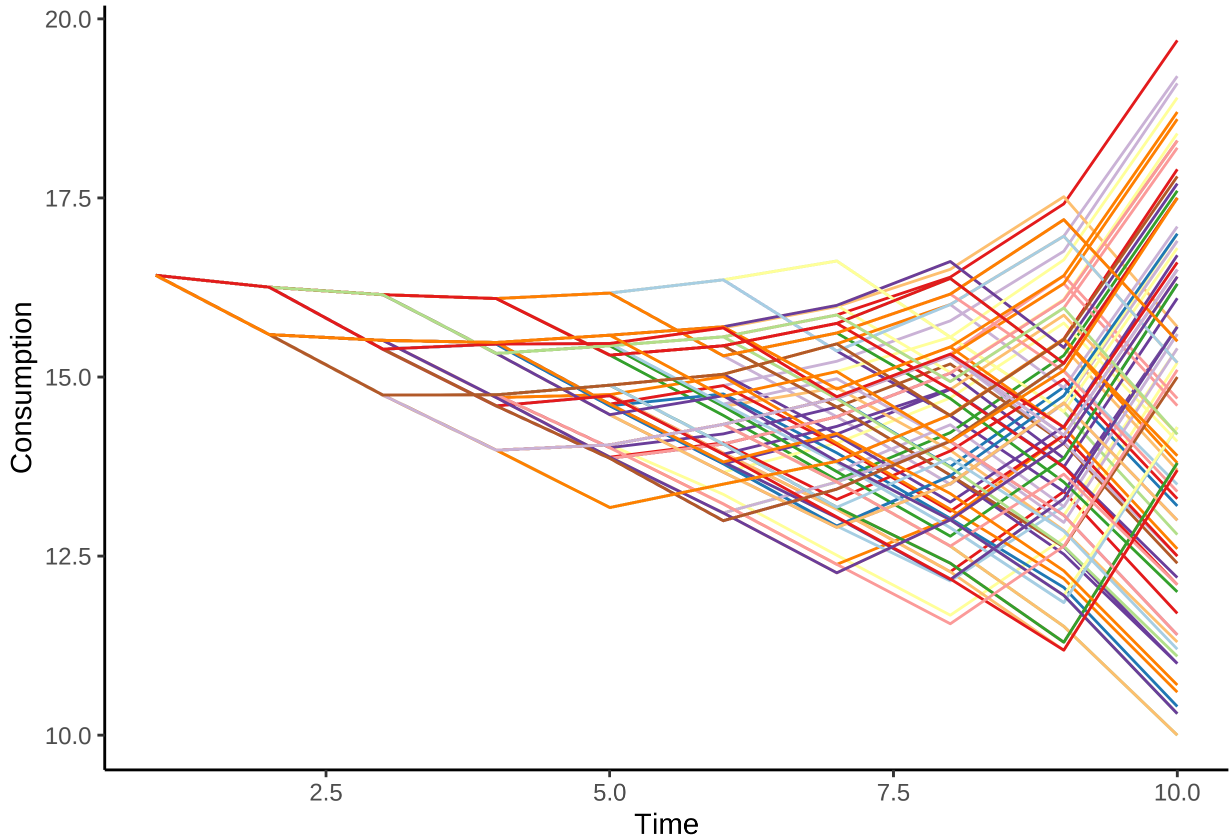
```r
tic()
# setup parameters and simulate shocks
set.seed(2022)
people <- 100
epsilon <-
  ifelse(purrr::rbernoulli(people*(t+1), 0.5), 2, -2) %>%
  matrix(ncol = t+1)
```

```r
vf <-
  rep(NA, people*t) %>%
  matrix(ncol = t)
kap <- # capital
  cbind(
    Kl*(rep(1, people)), # k_{0}
    matrix(rep(NA, people*t), ncol = t)
  )
con <- # consumption
  rep(NA, people*t) %>%
  matrix(ncol = t)

for(p in 1:people){
  for(t_iter in 1:t){
    position <- round(kap[p, t_iter]/grid + 1)
    vf[p, t_iter] <- V[position, t_iter]
    kap[p, t_iter + 1] <- K[which(aux[position, , t_iter] == vf[p, t_iter])]
    con[p, t_iter] <- kap[p, t_iter] - (kap[p, t_iter+1]/Theta)^(1/Alpha)
    kap[p, t_iter + 1] <- kap[p, t_iter + 1] + epsilon[p + t_iter + 1]
  }
}
toc()
```

```
## 0.026 sec elapsed
```

plot

```r
p_5 <- ggplot()
for(i in 1:people){
  df <- data.frame(time = 1:t, consumption = con[i, ])
  p_5 <- p_5 +
    geom_line(
      data = df, aes(x = time, y = consumption),
      color = sample(brewer.pal(12, "Paired"), 1)
    )
}
p_5 <- p_5 +
  labs(x = "Time", y = "Consumption") +
  theme_classic()
p_5
```



2. Exercise

(i)

We have

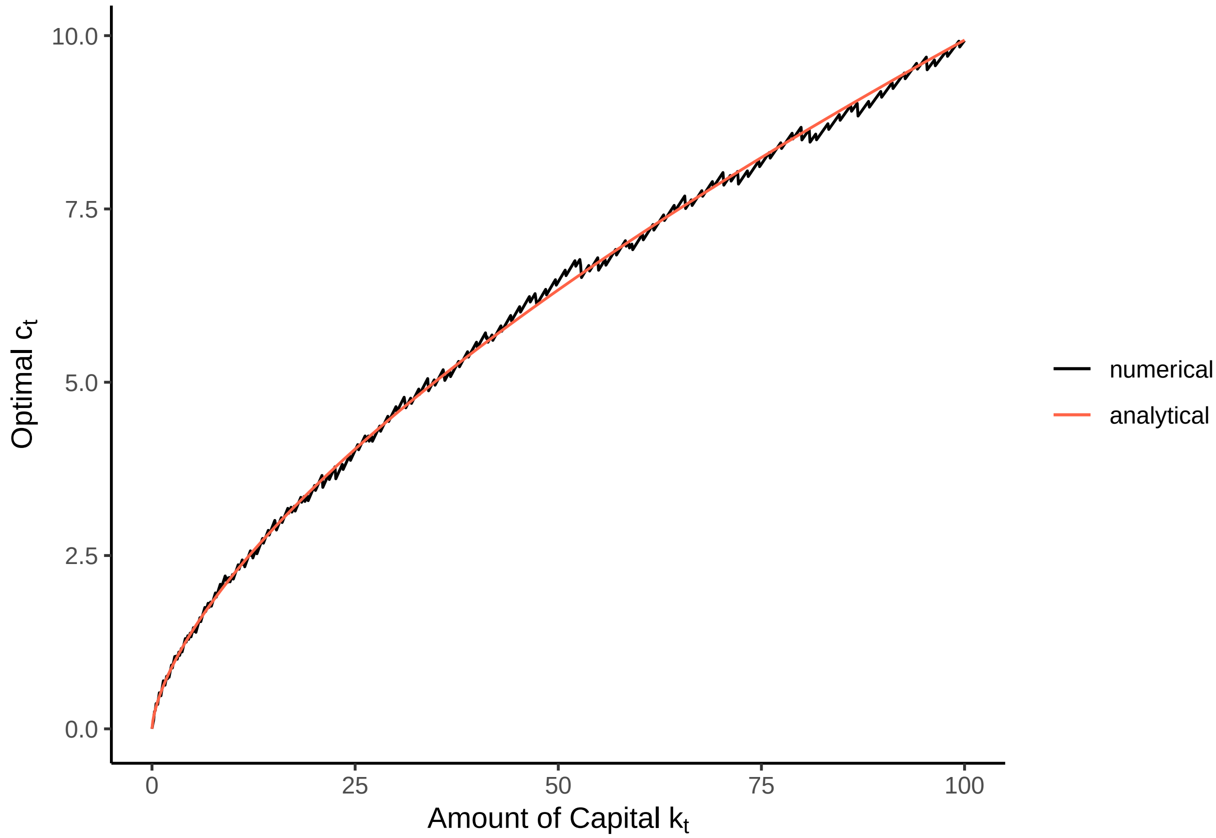
$$c_t = \theta k_t^\alpha - k_{t+1}.$$

```
Beta <- 0.9
Alpha <- 0.65
Theta <- 1.2
aB <- Alpha*Beta


K <- seq(from = 1e-6, to = 100, length.out = 1000) # grid
V <- rep(0, 1000) # guess
conv <- 100 # criterion for convergence
crit <- 1e-2 # stopping threshold
Iter <- 0 # numbering iteration

# for plot
df <- data.frame(K = K, V = V)

# iteration
while(conv>crit && Iter<1000){
```

```r
  Iter <- Iter + 1
  if(Iter %/% 10 == Iter/10) cat("Iteration number:", Iter, "\n")

  # mapping
  sol <- IterateVF(V, 100)
  TV <- sol[, 1]

  # distance between TV and V
  conv <- max(abs(TV-V))

  # for plot
  df$TV <- TV # store TV in a data.frame to layer plots

  # pass TV to next iteration
  V <- TV
}
```

```
## Iteration number: 10
## Iteration number: 20
## Iteration number: 30
## Iteration number: 40
## Iteration number: 50
## Iteration number: 60
```

```r
C_opt <- Theta*K^Alpha - K[sol[, 2]]
ggplot() +
  geom_line(aes(x = K, y = C_opt, color = "numerical")) +
  geom_line(aes(x = K, y = Theta*(K^Alpha)*(1-aB), color = "analytical")) +
  scale_color_manual(
    name = NULL,
    values = c("numerical" = "black", "analytical" = "tomato"),
    labels = c("numerical", "analytical")
  ) +
  labs(x = expression(paste("Amount of Capital ", k[t])),
       y = expression(paste("Optimal ", c[t]))) +
  theme_classic()
```



(ii)

Note that the transition equation is

$$k_{t+1} = f(k_t) - c_t + \varepsilon_{t+1},$$

where the production function is set as $f(k_t) = \theta k^{\alpha}$. The value function is written as

$$V(k_t) = \max_{c_t \in (0, k_t]} \{u(c_t) + \beta E_t[V(f(k_t) - c_t + \varepsilon_{t+1})]\}.$$

Here, discretization of consumption c_t is necessary to find the solution.

(ii)-(a)

```
# one iteration of the value function
IterateStochastic <- function(V, maxK){
  # basic parameters
  Alpha <- 0.65
  Beta <- 0.9
  Theta <- 1.2
  e <- c(-2, 2)
  PI <- c(0.5, 0.5)

  grid <- length(V)
  K <- seq(from = 1e-6, to = maxK, length.out = grid)
```

```r
  TV <- rep(0, length(V))
  optC <- rep(0, length(V))

  # loop through and create new value function for each possible capital value
  for(k in 1:grid){
    C <- seq(from = 1e-10, to = K[k], by = 0.1)
    #c <- rep(Theta*(K[k]^Alpha), grid) - K
    nextKl <- rep(Theta*K[k]^Alpha, length(C)) - C + e[1]
    nextKh <- rep(Theta*K[k]^Alpha, length(C)) - C + e[2]
    nextKl[nextKl < 0] <- rep(0, sum(nextKl < 0))


    EnextV <- rep(0, length(C))
    position_l <- rep(0, length(C))
    position_h <- rep(0, length(C))

    for(i in 1:length(C)){
      position_l[i] <- which(abs(K - nextKl[i]) == min(abs(K - nextKl[i])))[1]
      position_h[i] <- which(abs(K - nextKh[i]) == min(abs(K - nextKh[i])))[1]
      EnextV[i] <- PI[1] * V[position_l[i]] + PI[2] * V[position_h[i]]
    }

    u <- log(C)
    candid <- u + Beta*EnextV
    TV[k] <- max(candid)
    optC[k] <- which(candid == max(candid))
  }

  sol <- matrix(c(TV, optC), nrow = length(V), ncol = 2, byrow = FALSE)
  return(sol)
}
```

Convergence of Value Function

```r
tic()
# setting
K <- seq(from = 1e-6, to = 100, length.out = 1000) # grid
V <- rep(0, 1000) # guess
conv <- 100 # criterion for convergence
crit <- 1e-2 # stopping threshold
Iter <- 0 # numbering iteration

# for plot
df <- data.frame(K = K, V = V)
p7 <- ggplot() +
  geom_line(data = df, aes(x = K, y = V, color = 'guess'))

# iteration
while(conv>crit && Iter<1000){
  Iter <- Iter + 1
  if(Iter %/% 10 == Iter/10) cat("Iteration number:", Iter, "\n")

  # mapping
```

```
  sol <- IterateStochastic(V, 100)
  TV <- sol[, 1]

  # distance between TV and V
  conv <- max(abs(TV-V))

  # for plot
  df$TV <- TV # store TV in a data.frame to layer plots
  p7 <- p7 + geom_line(data = df, aes(x = K, y = TV, color = 'iteration'))

  # pass TV to next iteration
  V <- TV
}
```

```
## Iteration number: 10
## Iteration number: 20
## Iteration number: 30
## Iteration number: 40
## Iteration number: 50
## Iteration number: 60
```

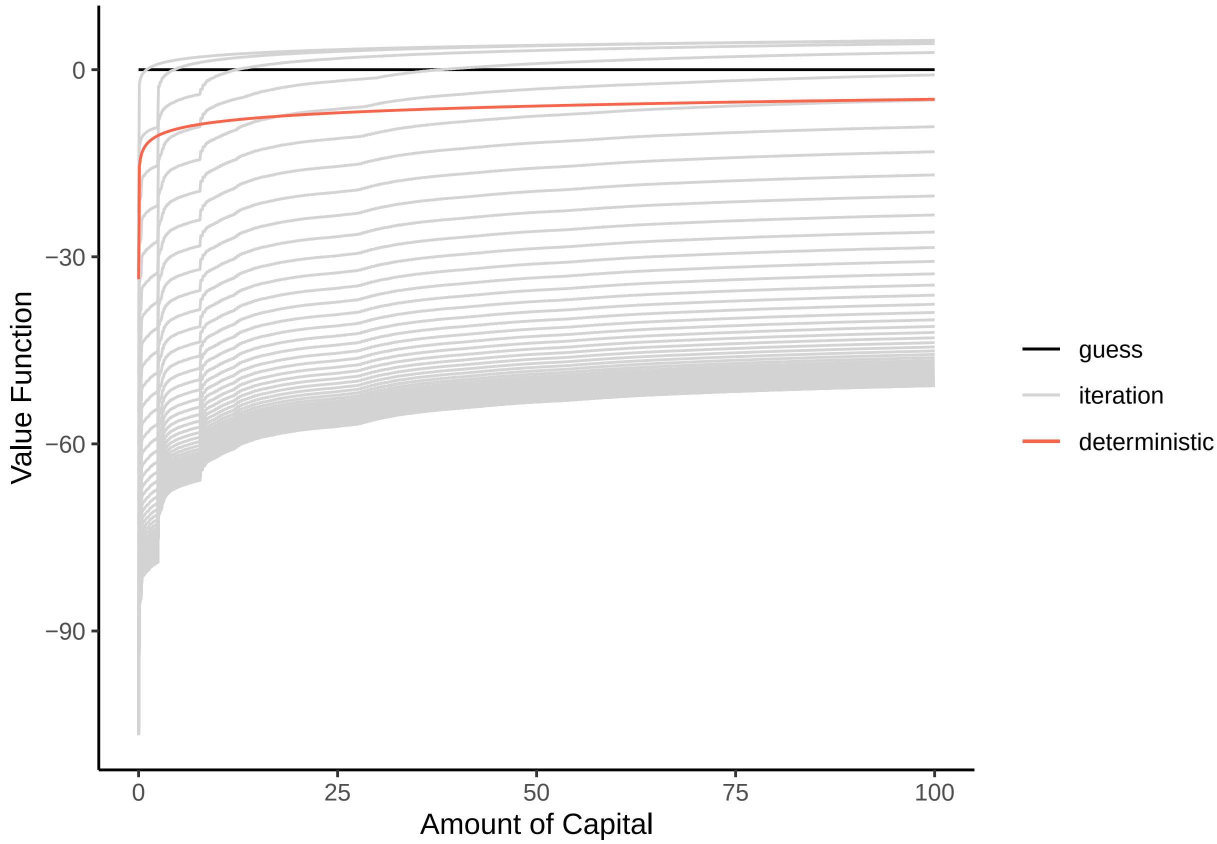
```
V_stoch <- V
optC_stoch <- sol[, 2]
toc()
```

```
## 776.204 sec elapsed
```

```
cat("# of iterations:", Iter)
```

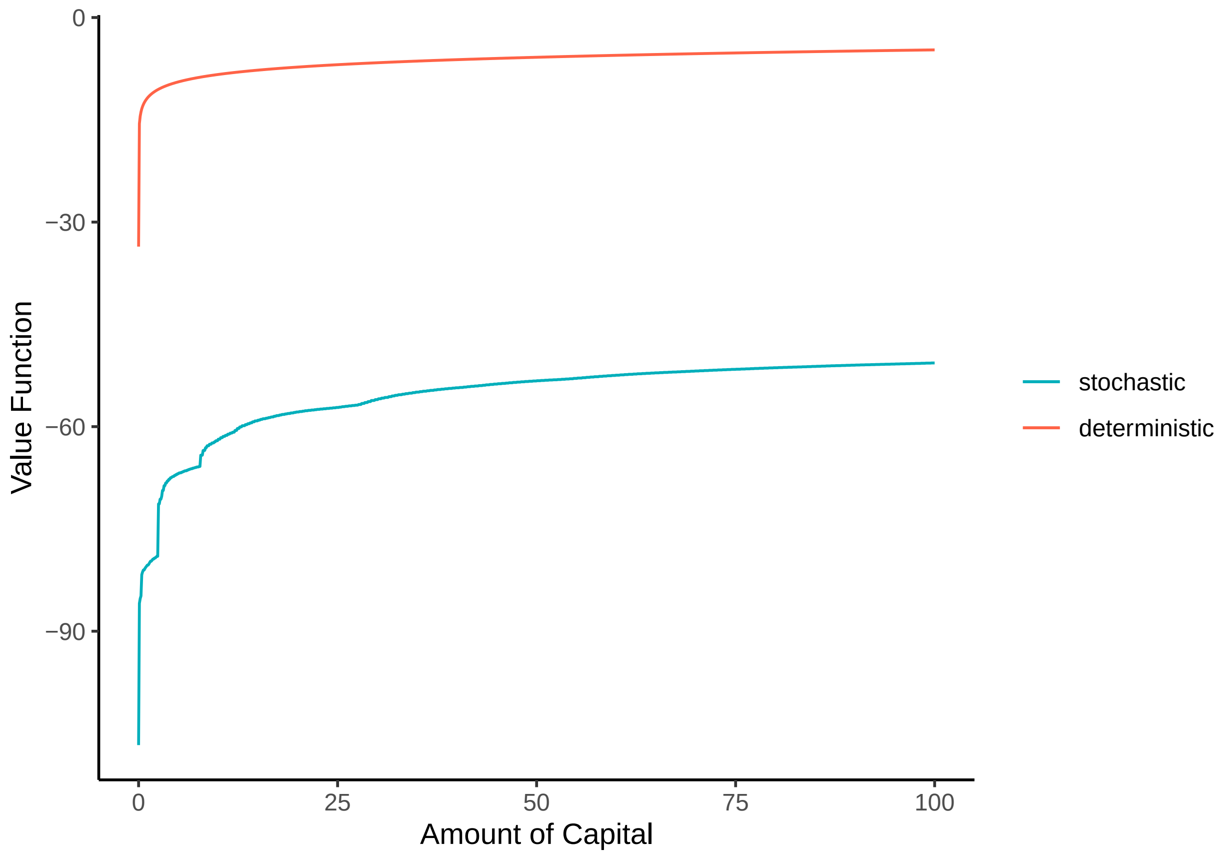
```
## # of iterations: 66
```

```
# plot
p7 <- p7 +
  geom_line(aes(x = K, y = soln, color = 'deterministic')) +
  scale_color_manual(
    name = NULL,
    values = c("guess" = "black", "iteration" = "lightgray", "deterministic" = "tomato"),
    labels = c("guess", "iteration", "deterministic")
  ) +
  labs(x = "Amount of Capital", y = "Value Function") +
  theme_classic()
p7
```



Comparison of stochastic and deterministic dynamic programming

```
ggplot() +
  geom_line(aes(x = K, y = V_stoch, color = "stochastic")) +
  geom_line(aes(x = K, y = soln, color = 'deterministic')) +
  scale_color_manual(
    name = NULL,
    values = c("stochastic" = "#00AFBB", "deterministic" = "tomato"),
    labels = c("stochastic", "deterministic")
  ) +
  labs(x = "Amount of Capital", y = "Value Function") +
  theme_classic()
```



Simulation

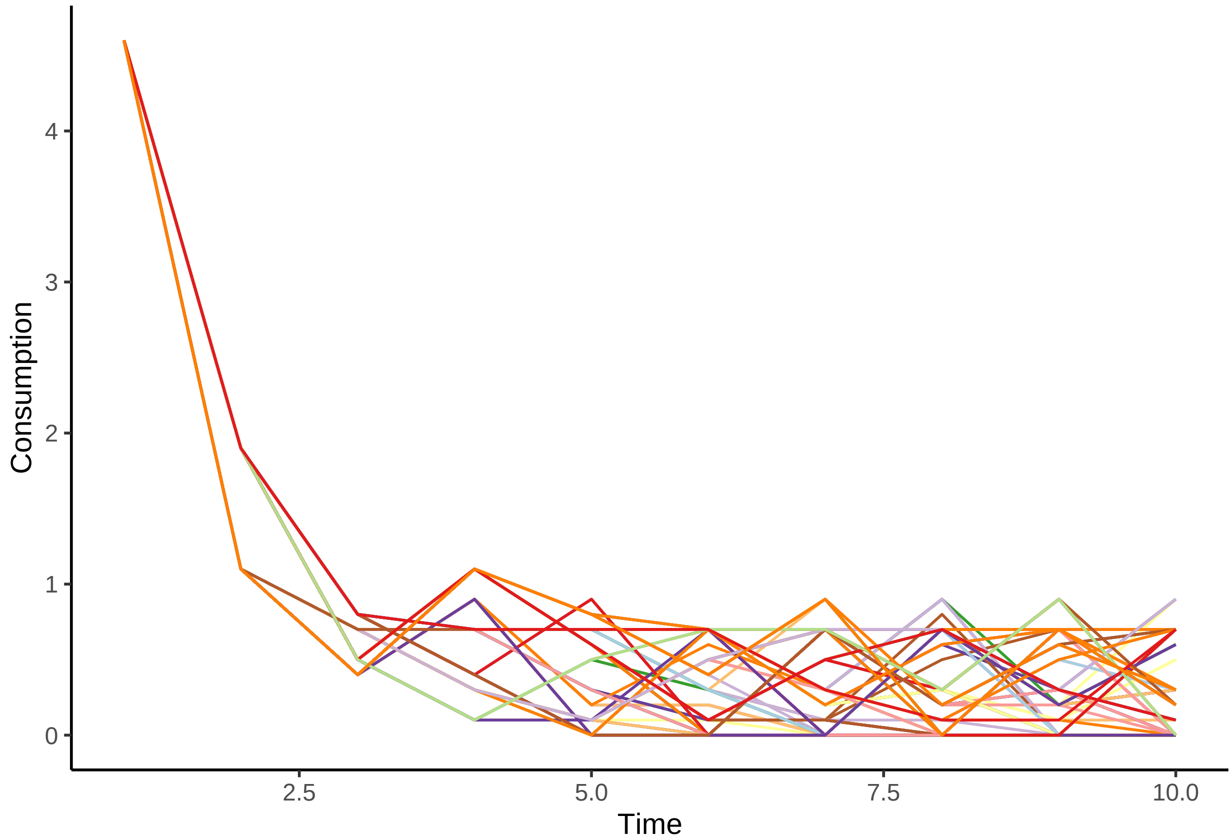
```
tic()
# setup parameters and simulate shocks
set.seed(2022)
t <- 10
people <- 100
K1 <- 100
V <- cbind(matrix(rep(NA, length(K)*t), ncol = t), matrix(rep(0, length(K)), ncol = 1))
epsilon <-
  ifelse(purrr::rbernoulli(people*(t+1), 0.5), 2, -2) %>%
  matrix(ncol = t+1)
vf <-
  rep(NA, people*t) %>%
  matrix(ncol = t)
kap <- # capital
  cbind(
    K1*(rep(1, people)), # k_{0}
    matrix(rep(NA, people*t), ncol = t)
  )
con <- # consumption
  rep(NA, people*t) %>%
  matrix(ncol = t)

for(p in 1:people){
  for(t_iter in 1:t){
```

```
    position <-
      which(abs(K - kap[p, t_iter]) == min(abs(K - kap[p, t_iter]), na.rm = TRUE))[1]
    vf[p, t_iter] <- V_stoch[position]

    con[p, t_iter] <-
      seq(from = 1e-10, to = K[position], by = 0.1)[optC_stoch[position]]
    k_tmp <- Theta*kap[p, t_iter]^(Alpha) - con[p, t_iter] + epsilon[p + t_iter + 1]
    kap[p, t_iter + 1] <- ifelse(k_tmp > 0, k_tmp, 0)
  }
}
toc()
```

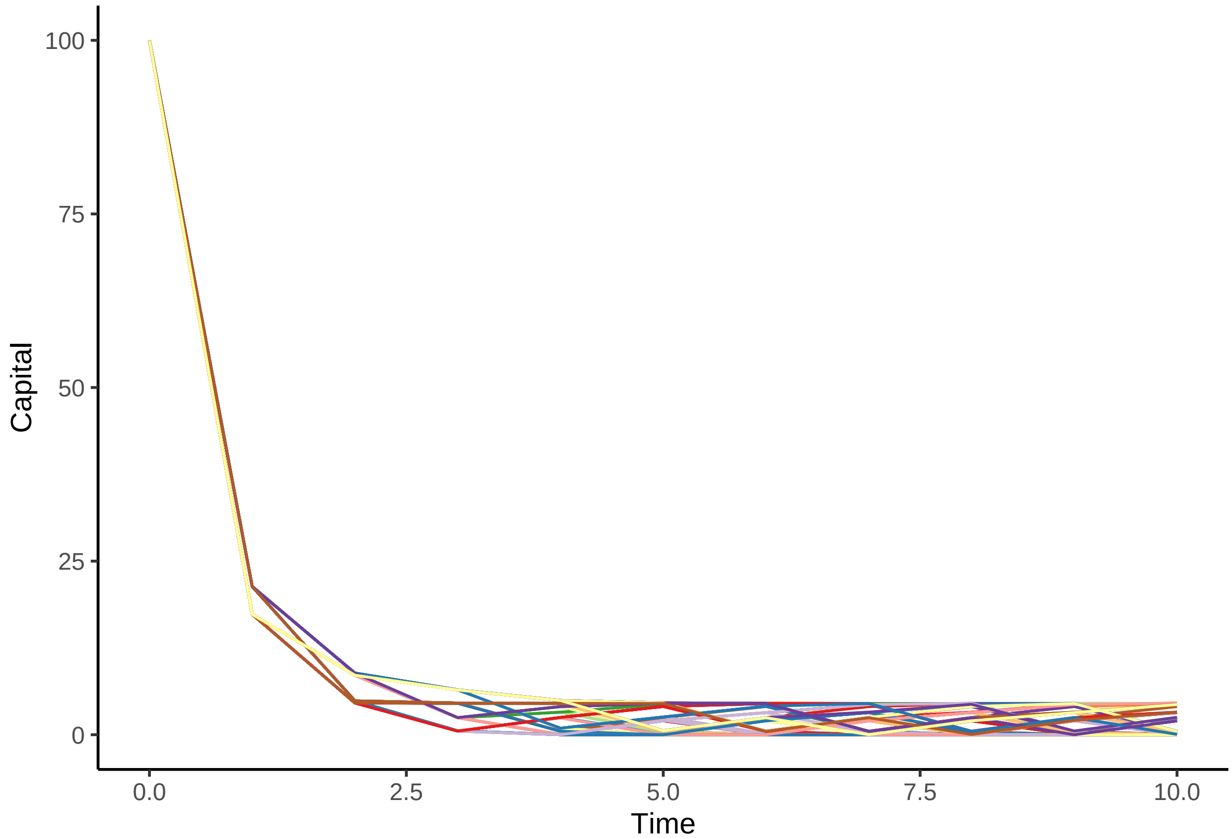
```
## 0.037 sec elapsed
```

plot consumption transition

```
p_8 <- ggplot()
for(i in 1:people){
  df <- data.frame(time = 1:t, consumption = con[i, ])
  p_8 <- p_8 +
    geom_line(
      data = df, aes(x = time, y = consumption),
      color = sample(brewer.pal(12, "Paired"), 1)
    )
}
p_8 <- p_8 +
  labs(x = "Time", y = "Consumption") +
  theme_classic()
p_8
```




```
p_9 <- ggplot()
for(i in 1:people){
  df <- data.frame(time = 0:t, capital = kap[i, ])
  p_9 <- p_9 +
    geom_line(
      data = df, aes(x = time, y = capital),
      color = sample(brewer.pal(12, "Paired"), 1)
    )
}
p_9 <- p_9 +
  labs(x = "Time", y = "Capital") +
  theme_classic()
p_9
```



(ii)-(b)

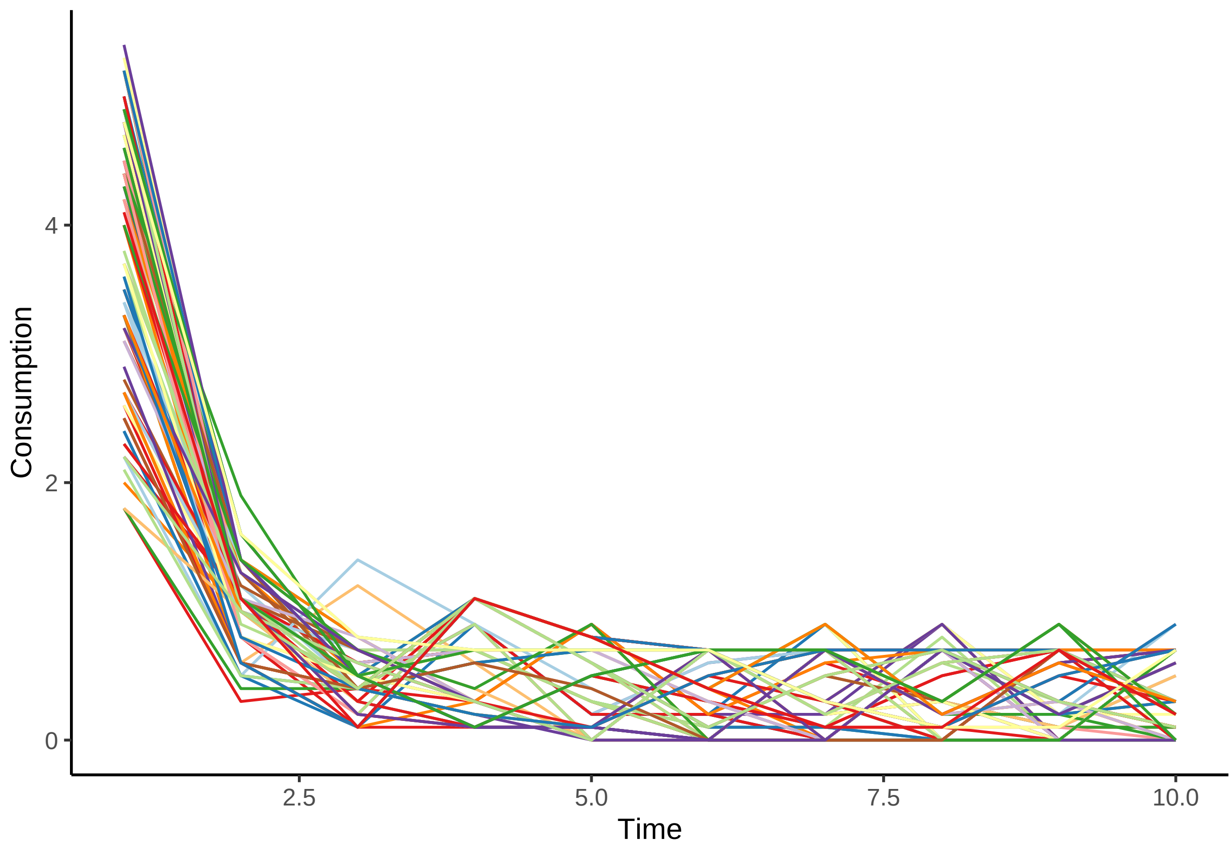
```
tic()
# setup parameters and simulate shocks
set.seed(2022)
t <- 10
people <- 100
Kl <- 100
V <- cbind(matrix(rep(NA, length(K)*t), ncol = t), matrix(rep(0, length(K)), ncol = 1))
epsilon <-
  ifelse(purrr::rbernoulli(people*(t+1), 0.5), 2, -2) %>%
  matrix(ncol = t+1)
vf <-
  rep(NA, people*t) %>%
  matrix(ncol = t)
kap <- # capital
  cbind(
    runif(people, min = 50, max = 100), # k_{0}
    matrix(rep(NA, people*t), ncol = t)
  )
con <- # consumption
  rep(NA, people*t) %>%
  matrix(ncol = t)
```

```
for(p in 1:people){
  for(t_iter in 1:t){
    position <-
      which(abs(K - kap[p, t_iter]) == min(abs(K - kap[p, t_iter]), na.rm = TRUE))[1]
    vf[p, t_iter] <- V_stoch[position]

    con[p, t_iter] <-
      seq(from = 1e-10, to = K[position], by = 0.1)[optC_stoch[position]]
    k_tmp <- Theta*kap[p, t_iter]^(Alpha) - con[p, t_iter] + epsilon[p + t_iter + 1]
    kap[p, t_iter + 1] <- ifelse(k_tmp > 0, k_tmp, 0)
  }
}
toc()
```

```
## 0.027 sec elapsed
```

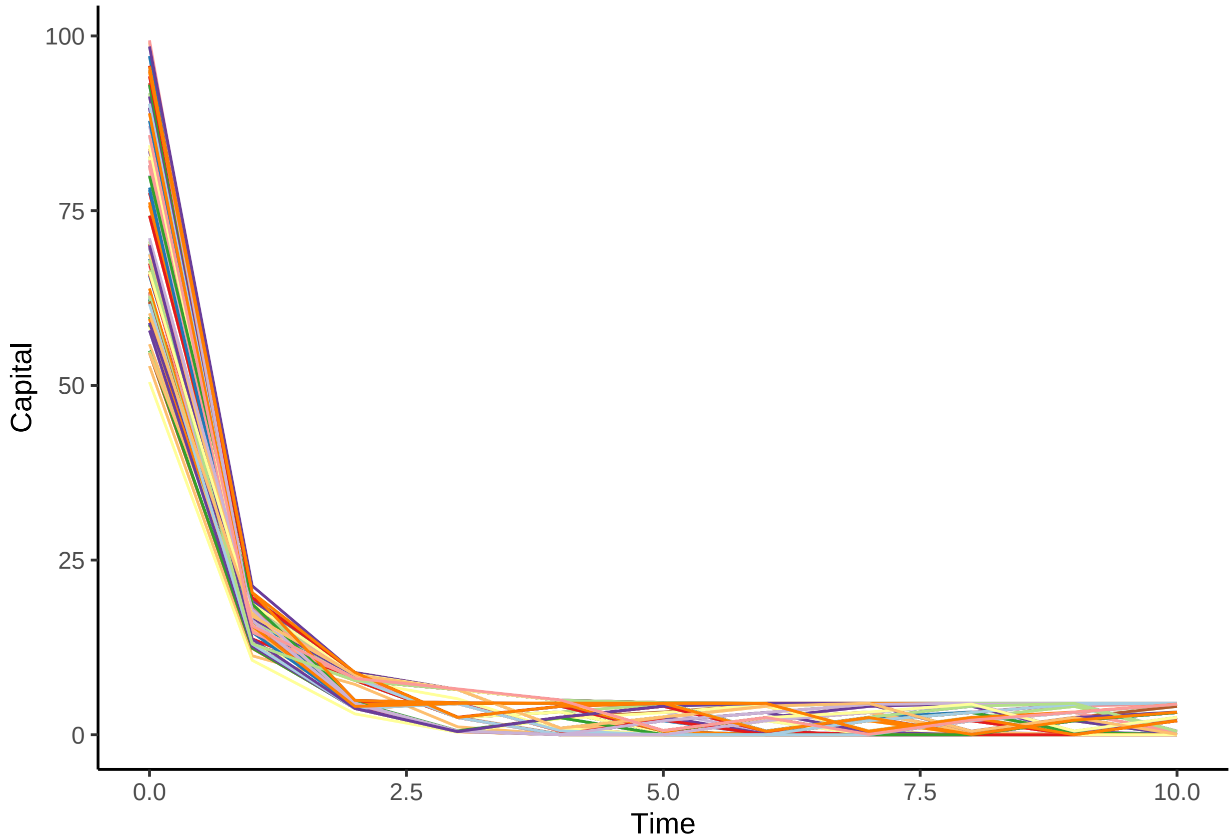
Transision of Consumption

```
p_10 <- ggplot()
for(i in 1:people){
  df <- data.frame(time = 1:t, consumption = con[i, ])
  p_10 <- p_10 +
    geom_line(
      data = df, aes(x = time, y = consumption),
      color = sample(brewer.pal(12, "Paired"), 1)
    )
}
p_10 <- p_10 +
  labs(x = "Time", y = "Consumption") +
  theme_classic()
p_10
```



Transision of Capital

```
p_11 <- ggplot()
for(i in 1:people){
  df <- data.frame(time = 0:t, capital = kap[i, ])
  p_11 <- p_11 +
    geom_line(
      data = df, aes(x = time, y = capital),
      color = sample(brewer.pal(12, "Paired"), 1)
    )
}
p_11 <- p_11 +
  labs(x = "Time", y = "Capital") +
  theme_classic()
p_11
```



(iii)

Reference

- Adams, A., D. Clarke, and S. Quinn. Microeconometrics and MATLAB. Oxford University Press, 2015.