Multiple Travelling Officers Problem to Optimize the Patrolling Schedules

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***Abstract*—This study formulates the Multiple Travelling Officers Problem as a linear programming model for the collection of fines by patrolling officers in urban cities. The model ensures that parking bays are visited based on overstaying probability, distance between parking bays, and the maximum working hours of each officer. By balancing fine collection efficiency and deployment costs, the formulation provides a scalable solution for multi-officer patrol optimization offering practical insights for real time urban parking management systems.**

I. Introduction

Urbanisation has increased the demand for efficient management of public service, particularly in densely populated areas like Central Business Districts (CBDs). Parking management is a critical aspect of urban systems as inefficiencies and violations can not only disrupt traffic flow but also strain enforcement resources. With the rise of smart city technologies, IoT enabled parking sensors provide real time data on violations.

The study addresses multiple travelling officers problem through Mixed Integer Linear Programming formulation which seeks to maximize parking fine collection by minimising operational costs and taking into consideration the working hours of officers, travel distances and probability of vehicles overstaying at the parking bays. Using this approach the study provides a robust framework for urban parking management presenting actionable insights for optimizing multi-officer patrol schedules in smart city environments.

II. Problem Statement

We strive to optimize the routes and task assignment of multiple parking officers to maximize the parking fines collected while adhering to travel time, maximum working hours of officers and parking violation probabilities.

A. *Some notations mentioned in the formulation*

- i : refers to parking bays $\forall\ i \in N\ (Parking\ Bays)$
- k : refers to officers $\forall\ k \in M\ (Officers)$
- t : refers to the time at which officer k might be present at parking bay i $\forall t \in T(Available\ Time)$

B. *Parameters*

- Distance between the parking bays are denoted by $d_{ij}\ where\ i, j \in N$
- Fines collected at each parking bay is denoted by $f_i\ where\ i \in N$
- Base salary paid to each officer k is denoted by sal
- Probability of a car overstaying at a parking bay is denoted by $os_i\ where\ i\ \in N$
- Minimum threshold probability for prioritising visits to the parking bays with overstaying cars is denoted by mp

C. *Binary Variables*

- $x_{ikt} = \begin{cases} 1 & if\ officer\ k\ visits\ bay\ i\ at\ time\ t \\ 0 \end{cases}$
- $y_k = \begin{cases} 1 & if\ officer\ k\ is\ on\ patrol\ duty \\ 0 \end{cases}$

III. Mathematical Programming

A. *Single Objective*

We have employed a Mixed Integer Linear Programming model to maximize the collection of parking fines subjected to a few constraints

Objective Function

$$Maximize\ Z = \left\{ \sum_{i=1}^{N} \sum_{k=1}^{M} \sum_{t=0}^{T} f_i * x_{ikt} - \sum_{k=1}^{M} y_k * sal \right\}$$

Constraints

1) *Single Visit Constraint :*

This ensures that each parking bay i is visited at most once across all officers k and time periods t

$$\sum_{k=1}^{M} \sum_{t=0}^{T} x_{ikt} \le 1 \quad \forall\ i \in N$$

2) Time limit Constraint

This limits the total time travel for each officer k within their maximum working hours T. If an officer is not active ($y_k = 0$) they cannot accrue travel time

$$\sum_{i=1}^{N}\sum_{j=1}^{N} d_{ij} \,.\, \sum_{t=0}^{T} x_{ikt} \leq T * y(k) \qquad \forall\, k \in M$$

3) Officer Availability Constraint

This ensures that an officer k can only visit parking bay i at time t if they are actively employed ($y_k = 1$)

$$x_{ikt} \leq y_k \qquad \forall \;\; i \in N, k \in M, t \in T$$

4) Minimum Probability Threshold

This ensures that the parking bay i is visited only if the overstay probability for that bay os_i meets or exceeds the minimum threshold probability mp

$$\sum_{k=1}^{M}\sum_{t=0}^{T} x_{ikt} * os_i \;\; \geq \;\; mp\,.\, \sum_{k=1}^{M}\sum_{t=0}^{T} x_{ikt} \qquad \forall\; i \in N$$

B. Multi-Objective

We have employed a Mixed Integer Linear Programming model to minimize the travel time and maximize the fines collected by the patrolling officers

Objective Function

$$Z_o = \left\{ \alpha \sum_{i,j,k,t} d_{ij} * x_{ikt} - \beta \sum_{i,k,t} f_i * os_i * x_{ikt} - \sum_{k=1}^{M} y_k * sal \right\}$$

- α and β are the weights associated with the overall objective function, allowing a trade-off between minimization of travel distance and maximization of fine collection respectively
- The overall objective function is subjected to the same constraints as defined for the single objective problem

IV. Tables and Figures

Technique	No. of Variables		No. of Constraints
	Binary	Continuous	
Mathematical Programming	(10x3x10) +3=303	0	10+3+(10x3x10)+10+ 300+3=626

TABLE 1.

To summarize the variables and constraints defined using MILP formulation, we consolidate everything in the form of a table (Table 1).

- Binary variables include x_{ikt} and y_k which contribute to 303 constraints considering there are 10 parking bays, 3 officers and 10 instances of time spread over their working time
- Constraint *(1)* contributes 10 constraints considering there are 10 parking bays
- Constraint *(2)* contributes 3 constraints considering there are 3 officers who are employed to manage the parking violations
- Constraint *(3)* contributes 300 constraints considering it applies to 10 parking bays, 3 officers and 10 time periods
- Constraint *(4)* contributes 10 constraints considering overstay probability applies over all 10 parking bays

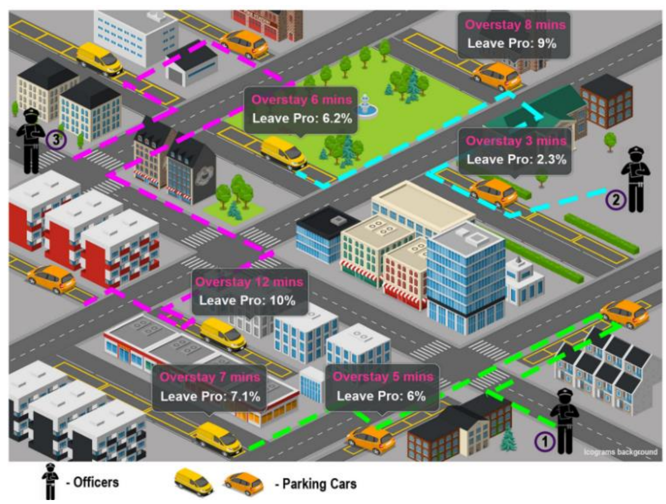

Fig. 1. 10 cars in parking violation with 3 officers in charge and the routes taken by them in accordance with the distance and the overstay time

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