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Section 1

Question 1

A

Given

identical twins	$P(\text{identical twins}) = \frac{1}{300}$
fraternal twins	$P(\text{fraternal twins}) = \frac{1}{125}$
Boy = gril	$P(boy) = 0.5$

$P(\text{identical twins}|\text{twins boys}) = ?$

Equation

$P(A \cap B) = P(A) \cdot P(B)$
$\mathrm{P}(A\|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B\|A) \cdot P(A)}{P(B\|A) \cdot P(A) + P(B\|\bar{A}) \cdot P(\bar{A})}$
$P(A \cap B) = P(A\|B) * P(B) = P(B\|A) * P(A)$

Calculation

$$P(A) = P(\text{identical twins } \cap \text{Boy}) = P(\text{identical twins}) \cdot P(\text{Boy}) = \frac{1}{300} * \frac{1}{2}$$

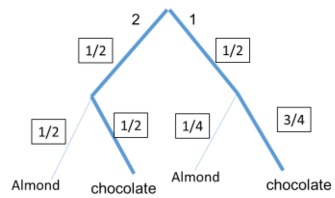
$$P(B) = P(\text{fraternal twins } \cap \text{Boy}) = P(\text{fraternal twins}) \cdot P(\text{Boys}) = \frac{1}{125} * \frac{1}{4}$$

$$\mathrm{P}(A|\mathrm{B}) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{300} * \frac{1}{2}}{\frac{1}{300} * \frac{1}{2} + \frac{1}{125} * \frac{1}{4}}$$

$$\boxed{\mathrm{P}(A|\mathrm{B}) = \frac{5}{11}}$$

B

Given



$$P(Bowl\ 1|chocolate) = ?$$

Equation

$P(A \cap B) = P(A) \cdot P(B)$
$\text{P}(A\vert B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{P(B\vert A) \cdot P(A)}{P(B\vert A) \cdot P(A) + P(B\vert \bar{A}) \cdot P(\bar{A})}$
$P(A \cap B) = P(A\vert B) * P(B) = P(B\vert A) * P(A)$

Calculation

$$P(A \cap B) = P(Bowl\ 1 \cap chocolate) = \frac{1}{2} * \frac{3}{4}$$

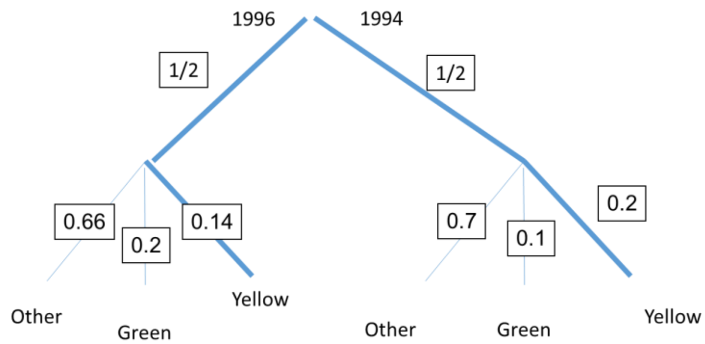
$$P(B) = P(chocolate) = \frac{1}{2} * \frac{1}{2} + \frac{1}{2} * \frac{3}{4}$$

$$\text{P}(A\ |\text{B}) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{2} * \frac{3}{4}}{\frac{1}{2} * \frac{1}{2} + \frac{1}{2} * \frac{3}{4}}$$

$$\boxed{\text{P}(A\ |\text{B}) = \frac{3}{5}}$$

Question 2

Given



$$P(1994|\text{Yellow}) = ?$$

Equation

$P(A \cap B) = P(A) \cdot P(B)$
$\mathrm{P}(A\|B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{P(B\|A) \cdot P(A)}{P(B\|A) \cdot P(A) + P(B\|\bar{A}) \cdot P(\bar{A})}$
$P(A \cap B) = P(A\|B) * P(B) = P(B\|A) * P(A)$

Calculation

$$P(1994 \cap \text{Yellow}) = P(\text{Yellow}|1994) * \mathrm{P}(1994) = 0.2 * 0.5$$

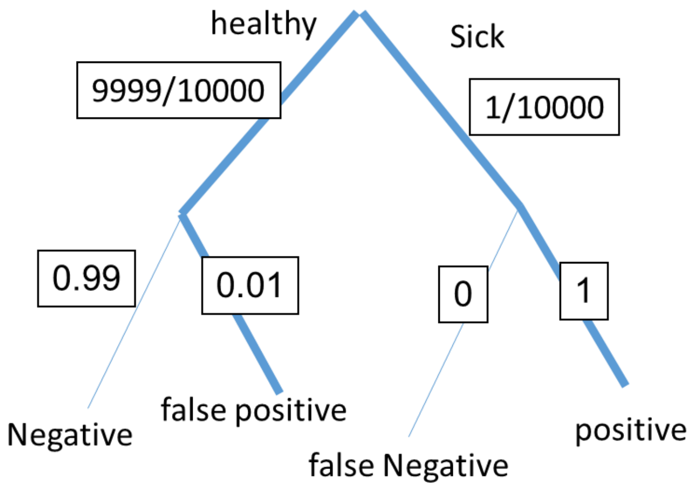
$$P(1994|\text{Yellow}) = \frac{P(A \cap B)}{P(B)} = \frac{0.2 * 0.5}{0.2 * 0.5 * 0.5 * 0.14}$$

$$\boxed{P(1994|\text{Yellow}) = \frac{10}{17}}$$

Question 3

A

Given



$$P(\text{true positive}|\text{positive}) = ?$$

Equation

$P(A \cap B) = P(A) \cdot P(B)$
$\mathrm{P}(A\|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B\|A) \cdot P(A)}{P(B\|A) \cdot P(A) + P(B\|\bar{A}) \cdot P(\bar{A})}$
$P(A \cap B) = P(A\|B) * P(B) = P(B\|A) * P(A)$

Calculation

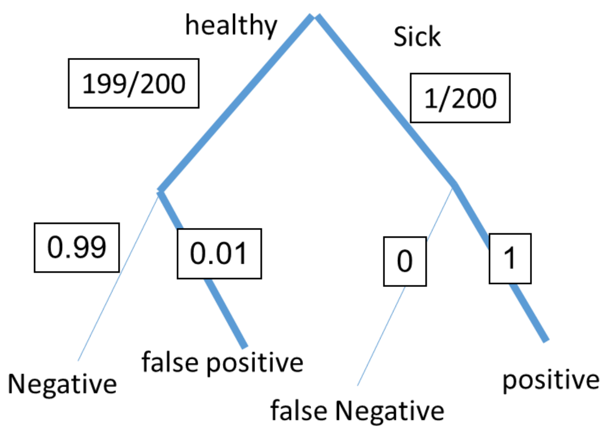
$$P(\text{true positive} \cap \text{positive}) = P(\text{positive}|\text{true positive}) * \mathrm{P}(\text{sick}) = \frac{1}{10000} * 1$$

$$P(\text{true positive}|\text{positive}) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{10000} * 1}{\frac{1}{10000} * 1 + \frac{9999}{10000} * 0.01}$$

$$\boxed{P(\text{true positive}|\text{positive}) = \frac{100}{10099}}$$

B

Given



$$P(\text{true positive}|\text{positive}) = ?$$

Equation

$P(A \cap B) = P(A) \cdot P(B)$
$\mathrm{P}(A\|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B\|A) \cdot P(A)}{P(B\|A) \cdot P(A) + P(B\|\bar{A}) \cdot P(\bar{A})}$
$P(A \cap B) = P(A\|B) * P(B) = P(B\|A) * P(A)$

Calculation

$$P(\text{true positive} \cap \text{positive}) = P(\text{positive}|\text{true positive}) * \mathrm{P}(\text{sick}) = \frac{1}{10000} * 1$$

$$P(\text{true positive}|\text{positive}) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{200} * 1}{\frac{1}{200} * 1 + \frac{199}{200} * 0.01}$$

$$\boxed{P(\text{true positive}|\text{positive}) = \frac{100}{299}}$$

Question 4

Given

2 six-sided dice

divisible by 3-> 6$

not divisible by 3->-3$

Equation

$E(x) = \sum x_i P(x_i)$

x is the outcome of the event

P(x) is the probability of the event occurring

Calculation

<table>
<tr><td>6$</td><td>-3$</td></tr>
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<tr><td></td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr>
<tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td></tr>
<tr><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td></tr>
<tr><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td></tr>
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</table>
</td><td></td></tr>
<tr><td>$P = \frac{12}{36} = \frac{1}{3}$</td><td>$P = \frac{36-12}{36} = \frac{2}{3}$</td></tr>
</table>

$E(x) = \sum x_i P(x_i) = \frac{1}{3} * 6 + \frac{2}{3} * (-3)$

$\boxed{E(x) = 0}$

Question 5

Given

Alex :

more than 12, he will win 5$

sum is less than 12, he will lose 6$.

exactly 12, he will break even **-> 0$ (assumption)**

Equation

$E(x) = \sum x_i P(x_i)$

x is the outcome of the event

P(x) is the probability of the event occurring

Calculation

	1	**2**	**3**	**4**	**5**
6	7	8	9	10	11
7	8	9	10	11	12
8	9	10	11	12	13
9	10	11	12	13	14
10	11	12	13	14	15

$$P(sum < 12) = \frac{15}{25}$$
$$P(sum > 12) = \frac{6}{25}$$
$$P(sum = 12) = \frac{4}{25}$$

$$E(x) = \sum x_i P(x_i) = 5 * \frac{6}{25} + (-6) * \frac{15}{25} + \frac{4}{25} * 0$$

$$\boxed{E(x) = -\frac{12}{5}}$$

Question 6

Given

$n = 8$

$200\ people$

$P = 40\%\ man$

Equation

$\mu = \sum x_i P(x_i)$

$$\sigma = \sqrt{\frac{P - P^2}{n}}$$

Calculation

$\mu = nP = 0.4 * 8$

$\boxed{\mu = 3.2}$

$$\sigma = \sqrt{\frac{P - P^2}{n}} = \sqrt{\frac{0.4 - (0.4)^2}{8}} = \frac{\sqrt{3}}{10} \sim 0.2$$

$\boxed{\sigma \sim 0.2}$

Question 7

Given

$\mu = 26$

$\sigma = 2$

$P(26 < X < 30) = ?$

Equation

$$Z = \frac{X - \mu}{\sigma}$$

Calculation

$$P(26 < X < 30) = P(\frac{26 - 26}{2} < \frac{X - \mu}{\sigma} < \frac{30 - 26}{2})$$

$$P(26 < X < 30) = P(0 < Z < 2)$$

Using : https://www.statisticshowto.com/tables/z-table/

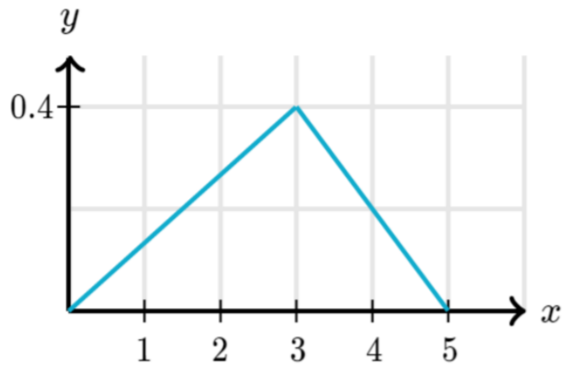
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817

$$P(26 < X < 30) = 0.4772 \sim 0.48$$

$$\boxed{P(26 < X < 30) \sim 0.48 = 48\%}$$

Question 8

Given



$P(X > 3) = ?$

With help of

https://en.wikipedia.org/wiki/Probability_density_function

Explanation

Assuming that the y-axis is Probability density function then the way to calculate $P(X > 3)$ is the area below the triangle

Calculation

$$P(X > 3) = 0.4 * \frac{5-3}{2}$$

$$\boxed{P(X > 3) = 0.4}$$

Question 9

Given

$k = 3$

$n = 4$

$P = 0.6$

$q = 1 - P = 0.4$

$P(x = k) = ?$

Equation

$$P(X = k) = C_n^k P^k q^{(n-k)}$$

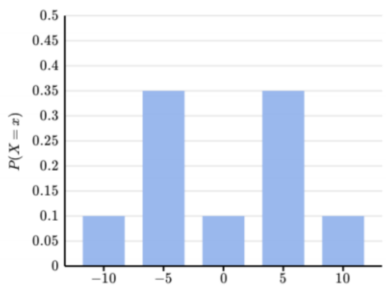
Calculation

$$P(X = k) = C_4^3 (0.6)^3 (0.4)^{4-3}$$

$$\boxed{P(X = k) = \frac{216}{625} \sim 0.34}$$

Question 10

Given



Equation

$E(x) = \sum x_i P(x_i)$

x is the outcome of the event

P(x) is the probability of the event occurring

Calculation

$E(x) = \sum x_i P(x_i) = 0.1 * -10 + 0.35 * (-5) + 0.1 * 0 + 0.35 * 5 + 0.1 * 10$

$\boxed{E(x) = 0}$