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المدرسة التونسية للتقنيات

# Engineering Internship Report

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# Modeling & Predicting Funding Rates Using Stochastic Models & Quantifying Risk

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# *Abstract*

This internship project focuses on the intricate task of modeling and predicting funding rates for perpetual contracts, a critical aspect in risk quantification within the financial sector. The approach involves employing Monte Carlo simulations, specifically Merton's jump diffusion process for index price and the Ornstein-Uhlenbeck process for funding rates. These simulations are harnessed to generate time series data on liquidation prices, from which we derive two crucial metrics: the expected liquidation time and the probability of liquidation prior to an artificially defined maturity depicting the trader's intended exit date. The insights gleaned from this research are poised to significantly enhance risk assessment and management strategies within the domain of perpetual contracts trading.

*Keywords: Modeling, Predicting, Funding rates, Perpetual contracts, Risk quantification, Financial sector, Monte Carlo simulations, Merton's jump diffusion process, Ornstein-Uhlenbeck process, Index price, Expected liquidation time, Probability of liquidation, Risk management.*

# Outline

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# General Introduction

During my internship, conducted from July 3, 2023, to August 28, 2023, at Laevitas, a distinguished entity specializing in data analytics for cryptocurrency derivatives, I have experienced a comprehensive exposure to the complexities of perpetual contracts. These financial instruments, characterized by their absence of a predefined expiration date, present a unique set of challenges in risk assessment.

In this context, the present study aims to delve into the critical task of modeling and predicting funding rates for perpetual contracts. The research is situated at the crossroads of financial innovation, computational methodology, and risk management strategies within the cryptocurrency derivatives landscape. By exploring this area, the study endeavors to contribute to the broader discourse on risk assessment and management in contemporary financial markets.

The ensuing sections will provide an overview of the project's methodology. In chapter one, we will define Perpetual Contracts in detail. The second chapter we present the project setup and define the risk quantification metrics. In the third chapter, we take an in-depth look at the modeling methodology of the project. We will also discuss the findings and the backtesting of the results in chapter four and in the fifth chapter we delve into the importance and the use cases of the developed solution.

# Chapter 1

# Perpetual Contracts: A Comprehensive Overview

## Introduction

Perpetual contracts are dynamic instruments in cryptocurrency trading that replicate asset prices without the constraints of expiry dates, meaning that the position remains open until the trader closes it or it gets liquidated. Perpetuals are popular financial instruments and in the year 2022, the daily global trading volume of perpetual futures was around $101.9 billion which is about $2\times$ to $3\times$ the total spot trading volume. In this chapter, we will delve into a comprehensive overview of perpetual contracts.

## 1.1 The Significance of Perpetuals in the Crypto Market

Perpetual contracts emerged in the cryptocurrency market as a revolutionary financial instrument designed to address inherent limitations of traditional futures contracts. Unlike standard futures, perpetual contracts lack an expiration date, allowing traders to maintain positions indefinitely. This innovation was introduced to provide a seamless trading experience, eliminating the need for continuous contract rollovers. Additionally, perpetual contracts introduce a funding mechanism that ensures prices closely track the underlying asset's spot market. This design promotes market stability and minimizes the potential for large discrepancies between contract and spot prices.

## 1.2 Perpetual Contracts Breakdown

In this section we break down the key components to a Perpetuals contract and we establish a proper understanding:

### 1.2.1 The Initial & Maintenance Margin

The initial margin is the upfront amount the trader commits when opening a position. It acts as a collateral. This margin is a percentage of the value of the total value of the position and it differs from an exchange to another.

For instance, if a trader wants to open a position with a contract value of $10,000 and the initial margin requirement is 5%, the trader would need to deposit $500 to initiate the trade.

The maintenance margin is the minimum value required to keep the position open otherwise it gets liquidated, meaning that if the wallet balance of the trader goes below the maintenance margin, the exchange will issue a margin call or, in some cases, automatically liquidate the position to prevent further losses.

For example, if the maintenance margin for a $10,000 position is 3%, the trader needs to maintain a balance of at least $300 to avoid liquidation. If the balance falls below this threshold, the position is at risk of being closed.

### 1.2.2 Leverage

In the context of Perpetuals, leverage is a mechanism that enables traders to control a larger position size than what they could with their own capital alone. It is expressed as a ratio, such as 10x, which means that for every unit of capital a trader puts in, they can control 10 units of the underlying asset.

### 1.2.3 Funding Mechanism

Funding is the mechanism that keeps a tight correlation between the perpetuals' price and the underlying. Basically, it is a payment between long and short positions.

The goal is to make sure the Futures price remains almost identical to the underlying price, so that they are effective hedging tools for price fluctuations. This payment happens every 8 hours.

If the Perpetuals' price is higher than the spot price, the funding rate is then positive and long position holders need to pay the short positions. This makes people more likely to short their positions, which can bring the Futures price back in line with the underlying price, and vis versa.

## Conclusion

In this chapter, we have gained a comprehensive understanding of perpetual contracts and their significance in the cryptocurrency market. We explored key components like margin requirements, leverage, and the funding mechanism, which are crucial for effective trading strategies. This knowledge forms the basis for further discussions on model implementation and risk assessment in the following chapters.

# Chapter 2

# Project Setup & Risk Quantification Metrics

## Introduction

In this chapter, we establish the fundamental framework for our risk assessment methodology by making critical assumptions to streamline our computational process.

## 2.1 Assumptions

The disbursement of funding payments happens at intervals of every eight hours. However, upon inspecting the funding rate, we discovered a negligible degree of variability within diurnal periods. In most instances, it exhibits a consistent pattern over the course of the day. In light of this observation, we opted for a simplification, employing a daily granularity as opposed to the original eight-hour intervals.
Consequently, we posited that the funding payment equates to a daily disbursement of a fraction equivalent to three times the funding rate, initiated precisely at the beginning of each day. And that explains the 3 factor in the formulae mentioned in the following sections.
This refinement serves to render the model less computationally expensive, all the while maintaining its fidelity in capturing the underlying dynamics.

## 2.2 Key Terminologies & Vocabulary

Table 2.1, that follows, contains the necessary terminology and a description of the variables:

**Table 2.1:** *Terminologies & Variables Table.*

| ***Variable*** | ***Symbol*** | ***Description*** |
|---|---|---|
| Position Value | $P_t$ | This is equal to the notional amount of the contracts, i.e. marginal amount scaled up by leverage. |
| Funding Rate | $r_t$ | Funding is the primary mechanism to tether to spot price. The funding payment at time t ($F_t$) is given by: $F_t = r_t.P_t$ |
| Index Price | $S_t$ | The index price approximates the price of the underlying In our case, it's the $BTC$. A bitcoin is worth 1 $BTC = \$S_t$. |
| Liquidation Price | $S_t^L$ | The index price when the position gets liquidated. |
| Liquidation Time | $\tau$ | The stopping time $\tau$ is when our position gets liquidated. |
| Leverage | $L$ | Borrowings from the exchange that amplify gains and losses of the position. |
| Margin Size | $N$ | The total number of contracts bought. |
| Unrealized P&L | $PnL_t$ | The unrealized profit and loss of the position. |
| Wallet Balance | $M_t$ | Accounts for the funding cash flows of the position . |
| Initial Margin | $I$ | The amount the trader commits when opening a position. |
| Maintenance Margin | $U_t$ | The minimum value required to keep the position open. |

## 2.3 Exchange Selection Process

In this Project, Bybit, OKEX, Binance, and Deribit were chosen due to their significant influence in this domain, collectively accounting for approximately 80% of the Open Interest (OI) in perpetual contracts. By focusing on these four exchanges and their main contracts, we can draw upon a vast pool of data and experiences, ensuring that our findings and analyses are representative of a substantial portion of the market.

Moreover, these exchanges boast robust infrastructures, liquidity, and trading volumes, making them pivotal players in the perpetual contracts landscape. This strategic choice allows us to capture a holistic view of the market's dynamics, providing valuable insights and in-depth analyses for traders, researchers, and enthusiasts alike.

| Exchange | Maximum Leverage | MMR (%) | TF (%) | MA | Symbol |
|---|---|---|---|---|---|
| BINANCE | 125 | 0.5 | 0.050 | 0 | BTCUSD_PERP |
| OKEX | 125 | 0.4 | 0.100 | - | BTC-USDT |
| BYBIT | 100 | 0.5 | 0.055 | - | BTCUSD |
| DERIBIT | 50 | 1 - 0.005$\frac{N}{S0}$ | 0.050 | - | BTC-PERPETUAL |

**Table 2.2:** *Featured Contracts*

## 2.4 Risk Metrics & Significance

The goal of this project is to derive the expected liquidation time and the probability of liquidation before a given date T since they stand out as critical tools for effective risk assessment and mitigation:

- **The Expected Liquidation Time**, $E(\tau)$, provides traders with a valuable estimate of the timeframe during which their leveraged position might be at risk of liquidation. This insight allows traders to align their strategies, make informed decisions about leverage levels, and optimize their risk exposure.

- **The Probability of Liquidation Before the Artificial Maturity** $T$, $P(\tau < T)$, quantifies the likelihood of early liquidation, enabling traders to adjust their strategies and risk management techniques accordingly.

These metrics are not just numerical figures, they are strategic guides that empower traders to navigate the complexities of the cryptocurrency market. By incorporating these metrics into our risk management pipeline, we are equipped to make agile and calculated decisions that strike a balance between potential gains and acceptable risk levels.

## Conclusion

This chapter forms the bedrock of our risk assessment endeavor. By refining our assumptions, establishing a common vocabulary, and introducing the critical metrics we've created a sturdy foundation for the subsequent analyses. With these elements in place, we are well-prepared to embark on a thorough risk assessment journey.

# Chapter 3

# Methodology and Modeling

## Introduction

The entire project revolves around one random variable and that is the liquidation time $\tau$, and in the following sections we will delve into the modeling of $\tau$ along with deriving the risk metrics based on it.

## 3.1 Data Collection & Preprocessing

For this study, comprehensive data sets were collected from various exchanges, encompassing the funding rates spanning from July 19, 2021, to July 19, 2023. Simultaneously, data pertaining to the index price and global funding rates were gathered from March 26, 2021, to March 20, 2023. All the data was meticulously organized in CSV files to facilitate efficient handling and processing.

To ensure the quality and accuracy of the data, a preprocessing step was implemented. Specifically, the funding rates underwent a thorough cleaning process to mitigate the influence of outliers. This was achieved through the application of the Interquartile Range (IQR) method, which effectively identified and treated any extreme values that may have skewed the dataset. This process significantly enhanced the reliability and robustness of the subsequent analyses conducted in this study. The data was then transformed into daily granularity, providing a comprehensive temporal perspective for the subsequent modeling and simulation endeavors.

## 3.2 Mathematical Modeling of the Liquidation Process

Suppose a trader opens a position (long or short) at time $t = 0$ with N contracts for a given leverage L at a specific exchange. To keep this position open the wallet balance has to be greater than the maintenance margin, meaning:

$$PnL_t + M_t \geq U_t$$

Where:

$$PnL_t = \begin{cases} NL(\frac{1}{S_0} - \frac{1}{S_t}) \text{ for Long Position Holders} \\ NL(\frac{1}{S_t} - \frac{1}{S_0}) \text{ for Short Position Holders} \end{cases}$$

$$M_t = \begin{cases} \frac{N}{S_0} - 3\sum_{i=1}^{t} \frac{NL}{S_i} r_i \text{ for Long Position Holders} \\ \frac{N}{S_0} + 3\sum_{i=1}^{t} \frac{NL}{S_i} r_i \text{ for Short Position Holders} \end{cases}$$

$U_t$ is the Maintenance Margin at time t and it is unique to each exchange.Meaning, the liquidation occurs when the previous condition is no longer met, e.g.:

$$PnL_t + M_t < U_t$$

So we can easily notice that:

$$\tau = inf\{t \geq 0 | PnL_t + M_t < U_t\}$$

And that the liquidation threshold is given by:

$$PnL_t + M_t = U_t$$

And by injecting the respective formulas for $PnL_t$ and $M_t$ and rearranging, this equation allows us to derive the liquidation price for both positions:

$$S_t^L = \begin{cases} \frac{L^2 S_0}{L^2(1-3\sum_{i=1}^{t} \frac{S_0}{S_i} r_i) + L(1-\frac{S_0}{N} U_t)} \text{ for Long Position Holders} \\ \frac{L^2 S_0}{L^2(1-3\sum_{i=1}^{t} \frac{S_0}{S_i} r_i) - L(1-\frac{S_0}{N} U_t)} \text{ for Short Position Holders} \end{cases}$$

And we can rewrite $\tau$ as:

$$\tau = \begin{cases} inf\{t \geq 0 | S_t < S_t^{L,\ long}\} \text{ for Long Position Holders} \\ inf\{t \geq 0 | S_t > S_t^{L,\ short}\} \text{ for Short Position Holders} \end{cases}$$

## 3.3 Methodology of the Project: Monte Carlo Simulation

The method employed for computing our metrics relies on the application of Monte Carlo Simulation to both the Funding Rate and the Index Price. This choice is underpinned by several compelling reasons:

Firstly, financial markets are inherently influenced by stochastic elements such as price fluctuations, market sentiment, and unforeseeable events, all of which elude precise mathematical formulations. The Monte Carlo method adeptly accommodates this inherent unpredictability by simulating numerous random scenarios to project potential outcomes.

Secondly, this approach adeptly addresses the probabilistic nature of liquidation metrics. It accommodates the dynamic nature of market conditions, as well as the intricate interplay of various variables, providing a robust framework for accurate evaluation and risk assessment.

### 3.3.1 Stochastic Process Selection

#### 3.3.1.1 The Funding Rate Modelling Process: Ornstein-Uhlenbeck Process

During the analysis of the Global Funding Rate data (which is constructed in a linear fashion from the funding rates of different exchanges), a distinct mean-reverting pattern became evident.

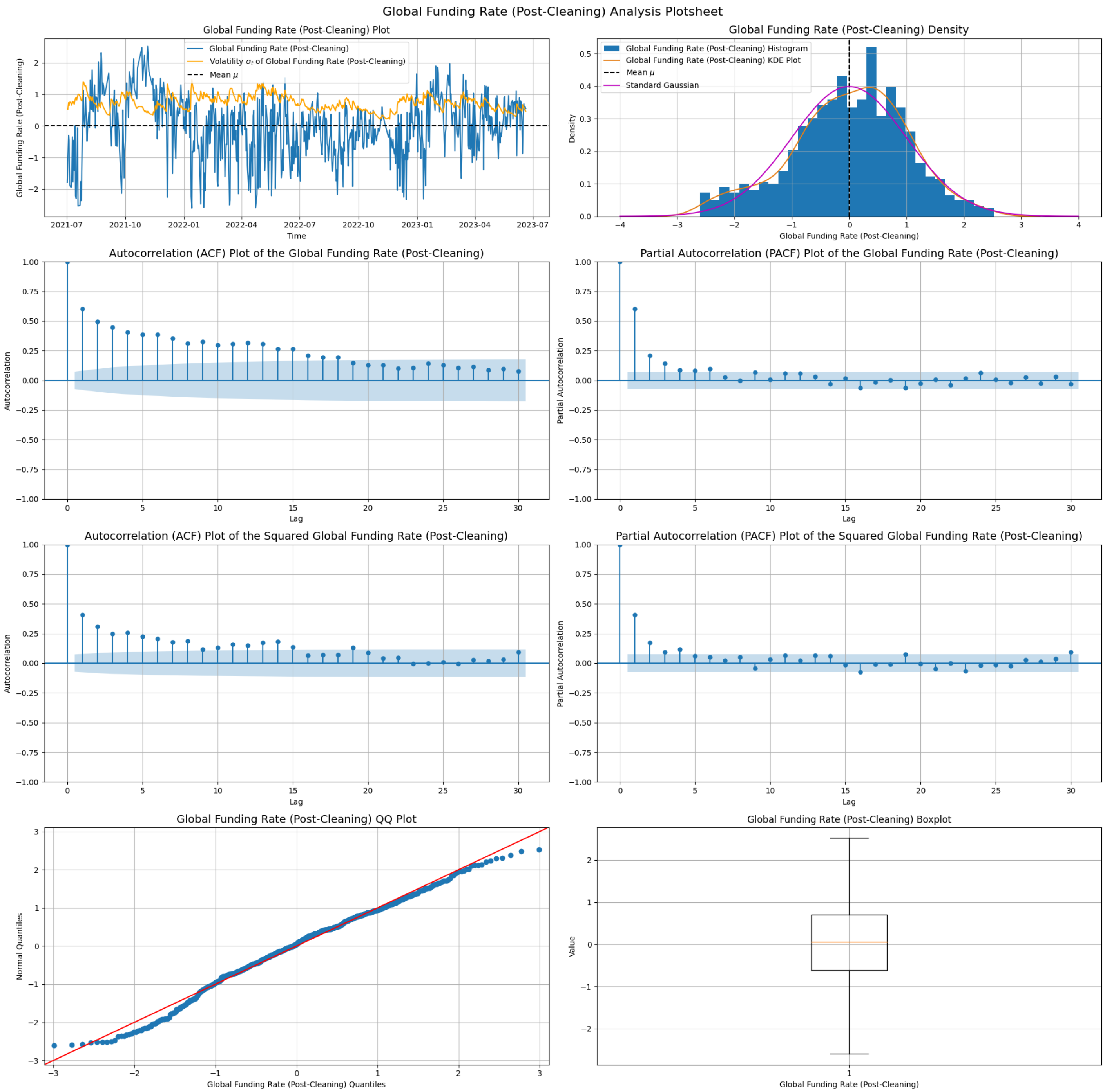


**Figure 3.1:** *Post-Cleaning Global Funding Rate Time Series Analysis Plots (Scaled for Visual Purposes)*

This observation is further substantiated by the positive estimated mean reversion parameter ($\theta = 0.398$), which quantifies the rate at which deviations from the mean revert back over time.

Moreover, the global funding rate is stationary. In fact, the results of the Advanced Dickey-Fuller Test, applied to the Global Funding Rate, yielded a test statistic of -5.536. This value indicates a pronounced deviation from the null hypothesis, which posits non-stationarity. The accompanying p-value of 0.0000 provides compelling evidence against the null hypothesis, further corroborating the stationary nature of the series. Furthermore, the critical value for a 5% significance level, set at -2.866, substantiates the rejection of the null hypothesis. This collective evidence substantiates the assertion that the Global Funding Rate, post-cleaning, exhibits stationary characteristics, implying sustained constancy in its statistical properties over time.

This observation justified the utilization of the Ornstein-Uhlenbeck process within the Monte Carlo framework, as it accurately captures such reverting behaviors.

The Ornstein-Uhlenbeck process is defined using the following Stochastic Differential Equation:

$$dr_t = \theta(\mu - r_t)dt + \sigma dW_t$$

Where:

- $r_t$ represents the process at time t, in our case it is the Funding Rate.
- $dr_t$ represents the infinitesimal change in the process at time t.
- $\theta$ is the mean-reversion rate, indicating how quickly the process reverts to the mean.
- $\mu$ is the long-term mean or equilibrium value.
- $\sigma$ is the volatility of the process.
- $dW_t$ represents a differential of a standard Wiener process (or Brownian motion) at time t, which represents the stochastic noise (or random shocks).

The solution to the previous SDE is given by this formula:

$$r_t = r_0 e^{-\theta t} + \mu(1 - e^{-\theta t}) + \sigma \int_0^t e^{-\theta(t-s)} dW_s$$

And the statistic properties of the process are:

$$E[r_t] = r_0 e^{-\theta t} + \mu(1 - e^{-\theta t})$$

$$V[r_t] = \frac{\sigma^2}{2\theta}(1 - e^{-2\theta t})$$

And one can easily derive the longterm mean and variance of the process:

$$E[r_\infty] = \lim_{t\to\infty} r_0 e^{-\theta t} + \mu(1 - e^{-\theta t}) = \mu$$

$$V[r_\infty] = \lim_{t\to\infty} \frac{\sigma^2}{2\theta}(1 - e^{-2\theta t}) = \frac{\sigma^2}{2\theta}$$

And that is what we will be using estimate the parameters $\mu$, $\sigma$ and $\theta$. Our approach relies on the Euler-Maruyama Discretization for the OU SDE since $dr_t \approx r_{t+dt} - r_t$ :

$$r_{t+\Delta t} \approx r_t + \theta(\mu - r_t)\Delta t + \sigma \Delta t Z_t$$

Where:

$$Z_t \sim N(0, 1)$$

So by denoting the first difference as $\Delta r_t = r_{t+\Delta t} - r_t$ we get:

$$\Delta r_t \approx \theta(\mu - r_t)\Delta t + \sigma \Delta t Z_t$$

And by rearranging the equation we get the following regression problem:

$$\Delta r_t \approx \theta\mu\Delta t - \theta\Delta t.r_t + \sigma\Delta t.Z_t = \alpha + \beta r_t + \epsilon_t$$

And by determining the $\beta$ coefficient we get an estimate of $\theta = \frac{-\beta}{\Delta t}$. And for the rest of the parameters $\mu$ and $\sigma$, we know that the transformed funding rate is a stationary standard gaussian. That being said, the longterm mean and variance of the simulated process $r_t$ has to match those statistical properties, hence the equations:

$$\begin{cases} E[r_\infty] = \mu \\ V[r_\infty] = \frac{\sigma^2}{2\theta} \\ \theta = \frac{-\beta}{\Delta t} \end{cases} \iff \sigma = \sqrt{2\theta.V[r_\infty]} = \sqrt{2\theta}.\sigma_{r_\infty}$$

#### 3.3.1.2 The Index Price Modelling Process: Merton's Jump-Diffusion Process

In our comprehensive analysis of the Index Price data, we observed two prominent characteristics that distinctly influence its behavior. The first is its propensity for continuous diffusion, indicating a gradual and continuous evolution of prices over time, influenced by various market forces. This continuous diffusion component encapsulates the regular fluctuations and movements in the index price, driven by factors such as market sentiment, trading activities, and economic indicators.

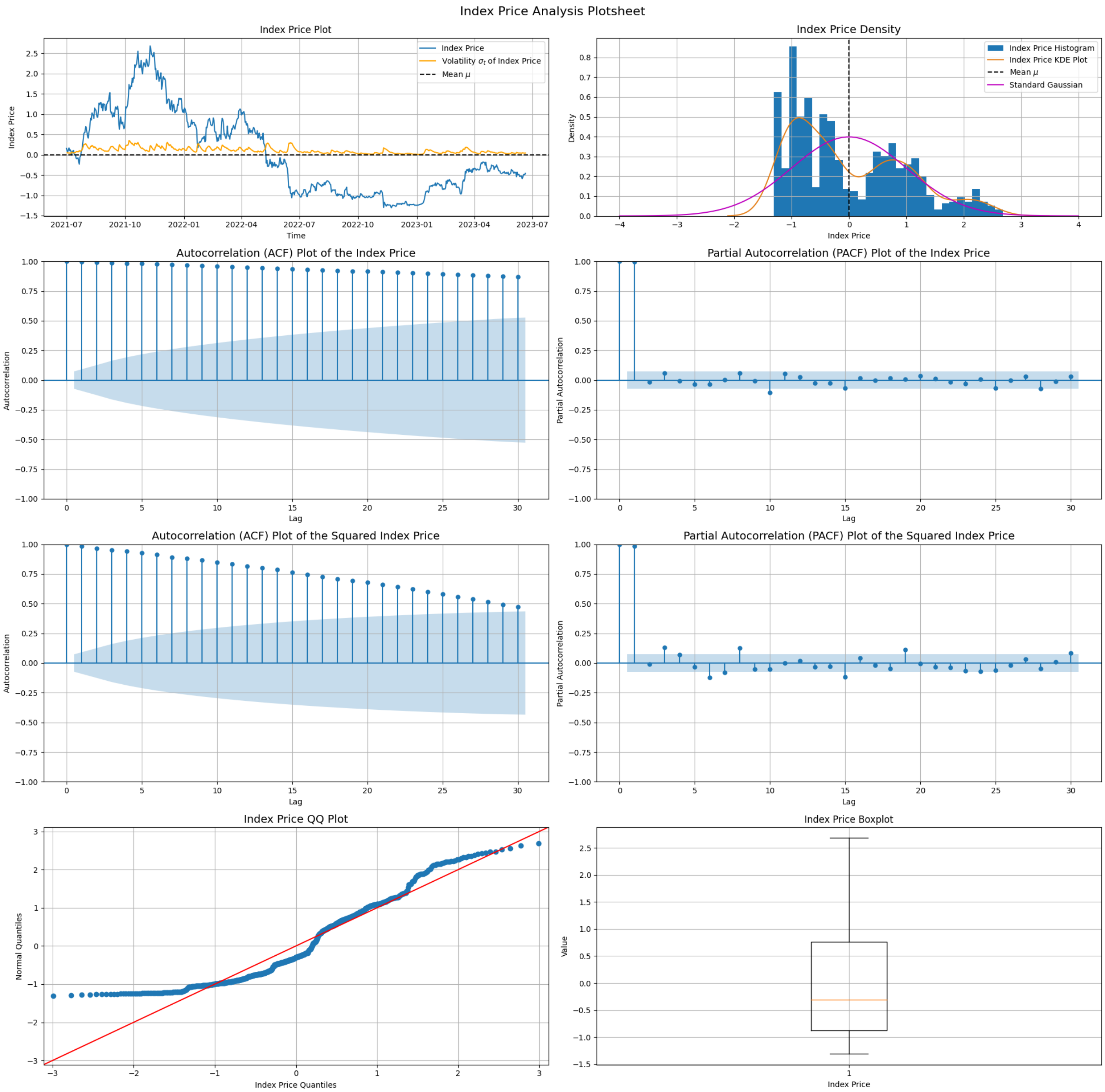


**Figure 3.2:** *Index Price Time Series Analysis Plots (Scaled for Visual Purposes)*

In addition to this continuous diffusion, we also identified instances of occasional jumpiness in the Index Price. These abrupt, discrete movements represent significant market events or sudden shifts in investor sentiment, leading to rapid and sizeable changes in the price level. These jumps can be triggered by a variety of factors, including unexpected news releases, geopolitical events, or large institutional trades.

Given the interplay of these two distinct characteristics - continuous diffusion and occasional jumpiness - it became imperative to employ a modeling approach that could effectively capture this intricate behavior. Consequently, we opted for the Merton jump diffusion process as the most suitable framework for our Monte Carlo simulation. This process incorporates both continuous diffusion and jump components, allowing us to simulate a range of scenarios that mirror the dynamic and multifaceted nature of the index price movements. By integrating these elements into our modeling framework, we aim to provide a more accurate and comprehensive representation of the underlying dynamics driving the Index Price in the context of our analysis.

The Merton Jump-Diffusion process is defined using the following Stochastic Differential Equation:

$$dS_t = \mu S_t dt + \sigma S_t dW_t + dJ_t$$

Where:

- $S_t$ represents the process at time $t$, in our case it is the Index Price.
- $dS_t$ is the infinitesimal change in the process at time $t$.
- $\mu$ is the drift coefficient.
- $\sigma$ is the volatility coefficient.
- $dW_t$ represents a differential of a standard Wiener process at time $t$.
- $dJ_t$ is the increment of a jump process at time $t$, representing the sudden jumps in the Index Price.

The jump process $J_t$ is modeled as a compound Poisson process with intensity $\lambda$, where the number of jumps follows a Poisson distribution, and the jump sizes are independently and identically distributed. The jump sizes can be assumed to follow a log-normal distribution with mean $\mu_{\text{Jump}}$ and standard deviation $\sigma_{\text{Jump}}$, which accounts for the fact that jumps are often proportional to the asset's current price.

The $\mu$, $\sigma$, $\mu_{\text{Jump}}$, $\sigma_{\text{Jump}}$ and $\lambda$ parameters' estimation method for Merton's jump diffusion model is as follows. We start by computing logarithmic returns within index price sequences $S_t$:

$$\text{Logarithmic Return: } R_t = \log\left(\frac{S_t}{S_{t-1}}\right) \text{ for } t \in [1, n]$$

And then we determine the continuous component's parameters which is governed by a Geometric Brownian Motion with a drift parameter $\mu_{\text{gbm}}$ and a volatility parameter $\sigma_{\text{gbm}}$, using $R_t$:

$$\mu_{\text{gbm}} = \frac{1}{n}\sum_{t=1}^{n} R_t, \quad \sigma_{\text{gbm}} = \sqrt{\frac{1}{n}\sum_{t=1}^{n}(R_t - \mu_{\text{gbm}})^2}$$

By subtracting the continuous drift component from the logarithmic returns, we isolate the effects of jump events and we denote it as $C_t$:

$$\text{Jump Size: } C_t = R_t - \mu_{\text{gbm}}$$

And then using the jumps, we estimate the mean $\mu_{\text{jump}}$ and volatility $\sigma_{\text{jump}}$ that intricately govern these ascending perturbations:

$$\mu_{\text{jump}} = \frac{1}{n}\sum_{t=1}^{n} C_t, \quad \sigma_{\text{jump}} = \sqrt{\frac{1}{n}\sum_{t=1}^{n}(C_t - \mu_{\text{jump}})^2}$$

And finally we provide an estimate of the jump intensity $\lambda$ using the method of moments. We employ enumeration of jumps exceeding a specified threshold (determined via visual inspection of the jumps in the historical data), rendering the jump intensity as the ratio of such jumps to the entirety of the observation time:

$$\lambda = \frac{\text{Number of Jumps}}{\text{Observation Time}} = \frac{N_{\text{Jumps}}}{n \cdot \Delta t}$$

### 3.3.2 Monte Carlo Simulations of Processes

To simulate the Ornstein-Uhlenbeck and the Merton Jump-Diffusion processes numerically we used the Euler-Maruyama method. For the Funding Rate, the following discrete-time update rule is applied:

$$r_{t+\Delta t} = r_t + \theta(\mu - r_t)\Delta t + \sigma \Delta W_t$$

Where:

- $r_{t+\Delta t}$ is the estimated value of the OU process at the next time step, $t+\Delta t$.
- $r_t$ is the current value of the OU process.
- $\theta(\mu - X_t)\Delta t$ represents the mean-reversion component.
- $\sigma \Delta W_t$ represents the stochastic component.

Similarly, we used the discrete version of the Merton process's SDE to simulate the Index Price:

$$S_{t+\Delta t} = S_t + \mu S_t \Delta t + \sigma S_t \Delta W_t + \Delta J_t$$

Where:

- $S_{t+\Delta t}$ is the estimated value of the process at the next time step, $t + \Delta t$.
- $S_t$ is the current value of the process.
- $\mu S_t \Delta t$ represents the deterministic drift component.
- $\sigma S_t \Delta W_t$ represents the stochastic component due to Brownian motion.
- $\Delta J_t$ represents the impact of jumps on the process.

The two processes are bound with the same white noise process $W_t$, and each time we generate a new Brownian Motion path we get a new scenario for the Funding Rate and the Index Price.

## 3.4 Metrics Computation Process

Following the simulation of the Funding Rate and Index Price, the focus shifts to calculating the time series of liquidation prices for each scenario. By monitoring the Index Price's behavior relative to these liquidation prices, liquidation times are determined when the index breaches the specific threshold for that specific position. The aggregation of these times, coupled with averaging across each path (which leverages the Central Limit Theorem), allows us to derive the expected liquidation time for both long and short positions separately.

$$E(\tau) = \frac{1}{Card(\Gamma)} \sum_{i \in \Gamma} \tau_{P_i}$$

Where:

- $P_i$ is the $i^{th}$ path, $i \in [1, m]$ ($m$ is the total number of paths).
- $\tau_{P_i}$ is the liquidation time of the path $P_i$.
- $\Gamma = \{P_i | \tau_{P_i} \leq T\}$ is the set of liquidated paths.

The computation of the probability of liquidation before a specific time T follows a straightforward process. It involves dividing the number of liquidated paths before T by the total number of simulations performed. This ratio effectively quantifies the likelihood of encountering a liquidation event within the designated timeframe.

$$P(\tau \leq T) = \frac{Card(\Gamma)}{m}$$

## 3.5 Model Implementation & Pipeline Design

The pipeline was structured into distinct stages to ensure a comprehensive understanding of the underlying dynamics. Initially, we obtained CSV files containing the historical records of funding rates and index prices through a datafetching process. Subsequently, we implemented a robust data cleaning step, focusing primarily on the funding rates. This entailed the application of an Interquartile Range method to identify and rectify outliers, ensuring that the dataset was devoid of anomalous values that could potentially distort subsequent analyses.

Following the data cleaning phase, our attention shifted towards parameter estimation for the Ornstein-Uhlenbeck process applied to the funding rate data. Concurrently, we estimated the parameters of the Merton model for the index price.

The subsequent stage entailed the simulation of Monte Carlo paths for both the funding rate and index price, utilizing the derived parameters. This enabled us to generate a diverse range of scenarios, each reflecting plausible trajectories that these variables might assume over time. These simulated paths served as the foundation for subsequent analyses.

In the final steps of the pipeline, we derived the liquidation prices for specific positions based on the simulated Monte Carlo paths. Additionally, we computed our risk assessment metrics; The expected liquidation time $E(\tau)$ and the probability of liquidation before a specified artificial maturity date $P(\tau \leq T)$.

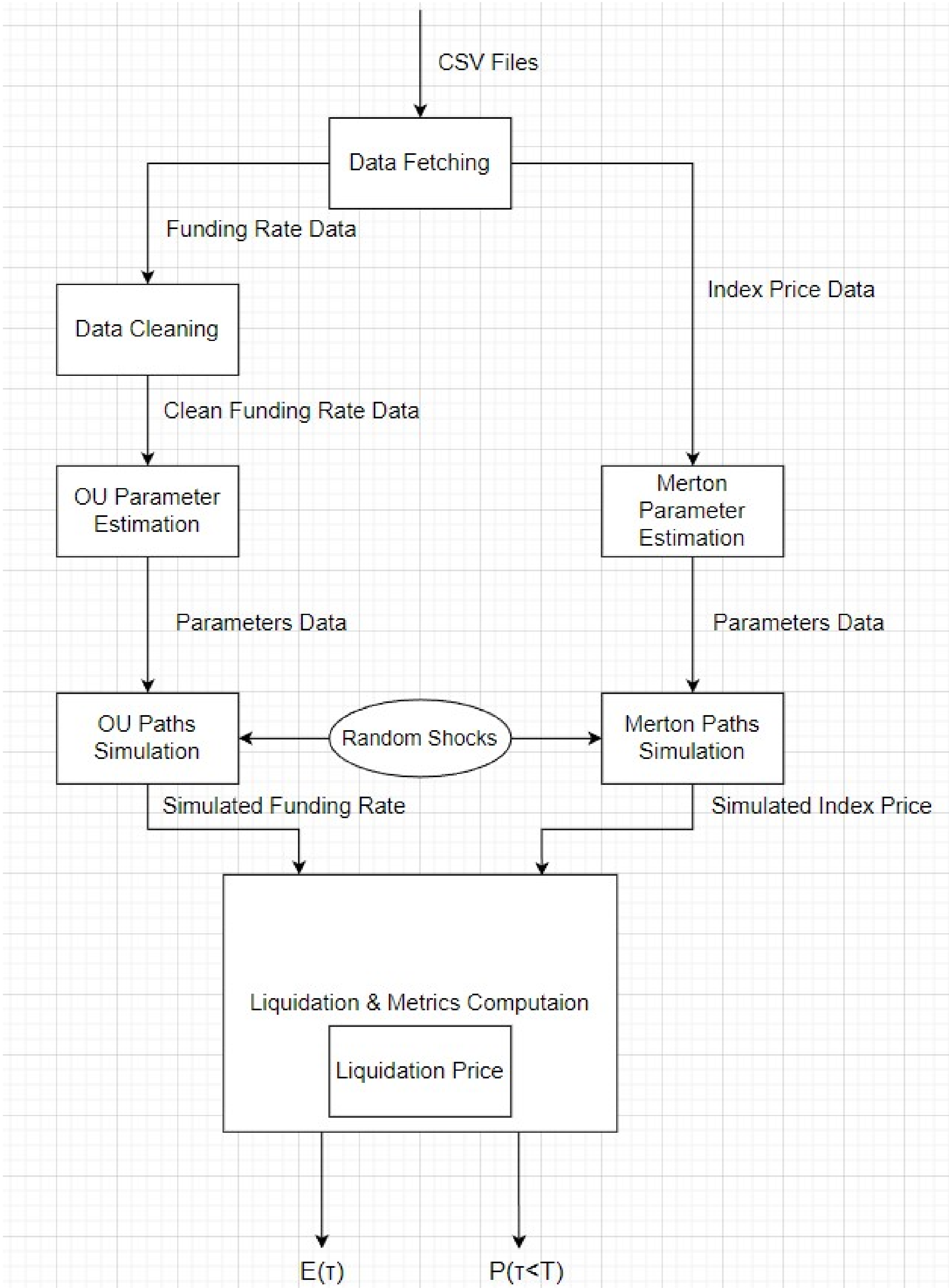


**Figure 3.3:** *Pipeline Design*

## Conclusion

In summary, this chapter has established a robust methodology and modeling framework for assessing risk in perpetual contracts trading. By employing Monte Carlo Simulation and mathematical modeling, we've generated diverse scenarios that capture the dynamic nature of the market. The computation of key risk metrics further equips traders with invaluable insights.

As we move forward, the next chapter will delve into the empirical results and their interpretations, validating the effectiveness of our approach.

# Chapter 4

# Findings, Interpretations & Comparison

## Introduction

This pivotal chapter sheds light on the findings, interpretations, and rigorous backtesting to underpin its efficacy. By examining the empirical evidence and drawing insightful conclusions, we pave the way for a deeper understanding of its practical applications and strategic implications.

## 4.1 Key Findings & Visualization

The analysis of the liquidation pipeline's outcomes reveals significant insights into the impact of leverage on risk and liquidation dynamics. As leverage increases, the inherent risk amplifies for both long and short positions.

This heightened risk is manifested by a decrease in the expected liquidation time as leverage grows. Moreover, the probability of liquidation before the predefined maturity time exhibits an upward trend with escalating leverage.

Notably, the relationship between leverage and expected liquidation time is characterized by a convex pattern, where the rate of decrease in expected liquidation time accelerates with higher leverage. Conversely, the probability of liquidation before maturity displays a concave behavior, indicating that the incremental increase in leverage leads to a diminishing rise in the likelihood of early liquidation.

These findings underscore the intricate interplay between leverage, risk, and liquidation metrics, providing traders with valuable insights for optimizing their positions and strategies in the perpetual contracts market.

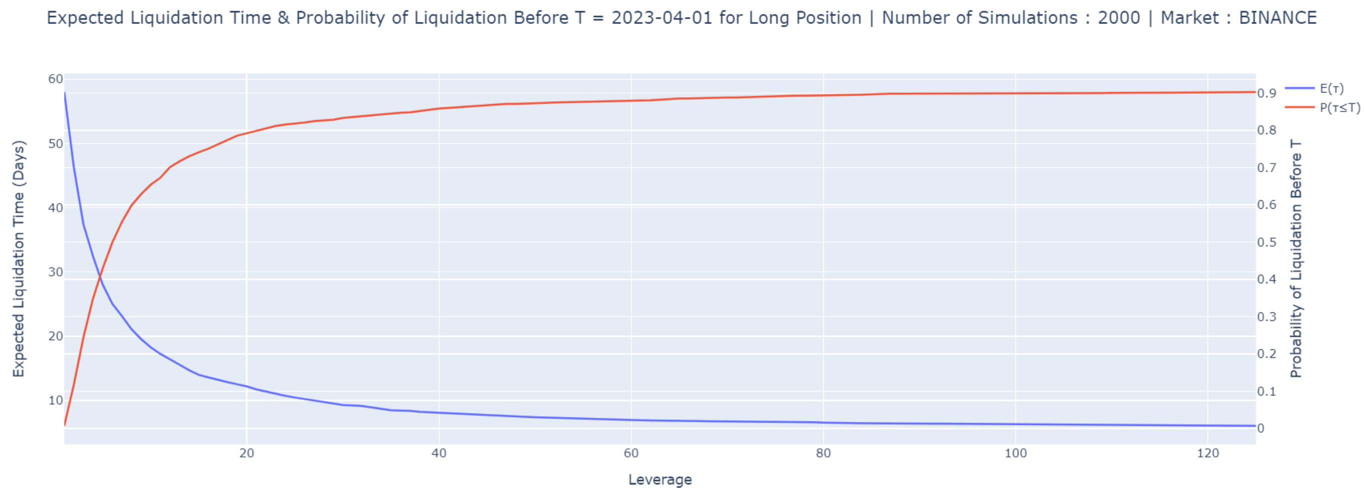


**Figure 4.1:** *$E(\tau)$ & $P(\tau \leq T)$ for Binance, Long Position*

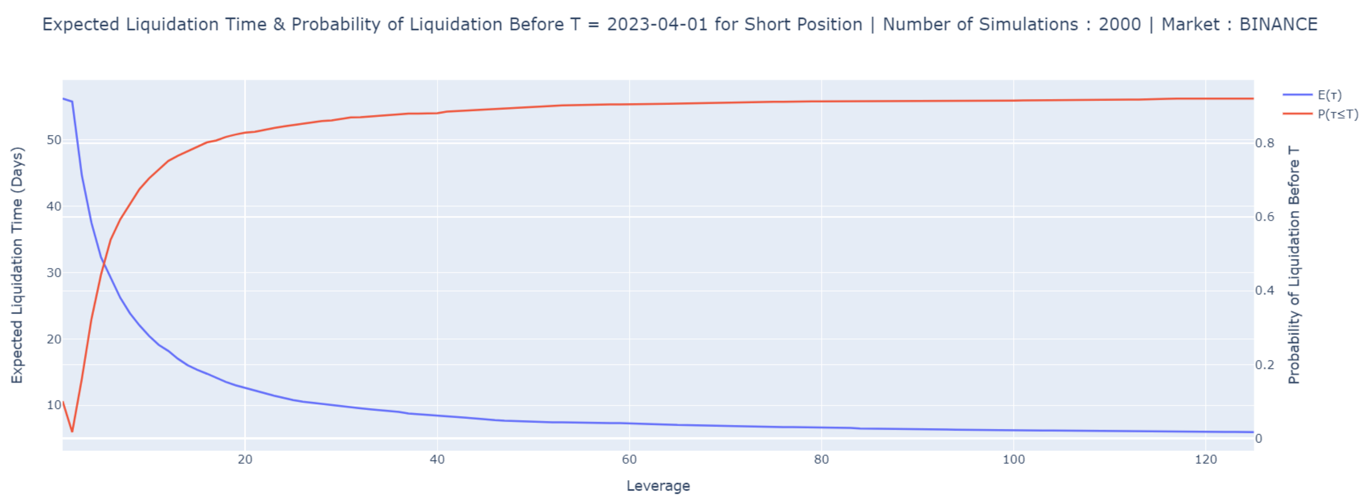


**Figure 4.2:** *$E(\tau)$ & $P(\tau \leq T)$ for Binance, Short Position*

## 4.2 Backtesting Results

The backtesting phase of the liquidation pipeline validates its effectiveness and sheds light on its key strengths and potential improvements.

Notably, the comparison between the plots of expected and actual liquidation times reaffirms the accuracy of our methodology. Both plots exhibit similar trends and maintain the same scale, pointing out good insights in this approach.

The backtesting results also underscore the relative performance of the long and short positions. While the short side demonstrates a notably successful outcome, the long side exhibits decent but not so accurate performance.

Recognizing this discrepancy, the primary avenue for future enhancement lies in refining the plots related to the long position.

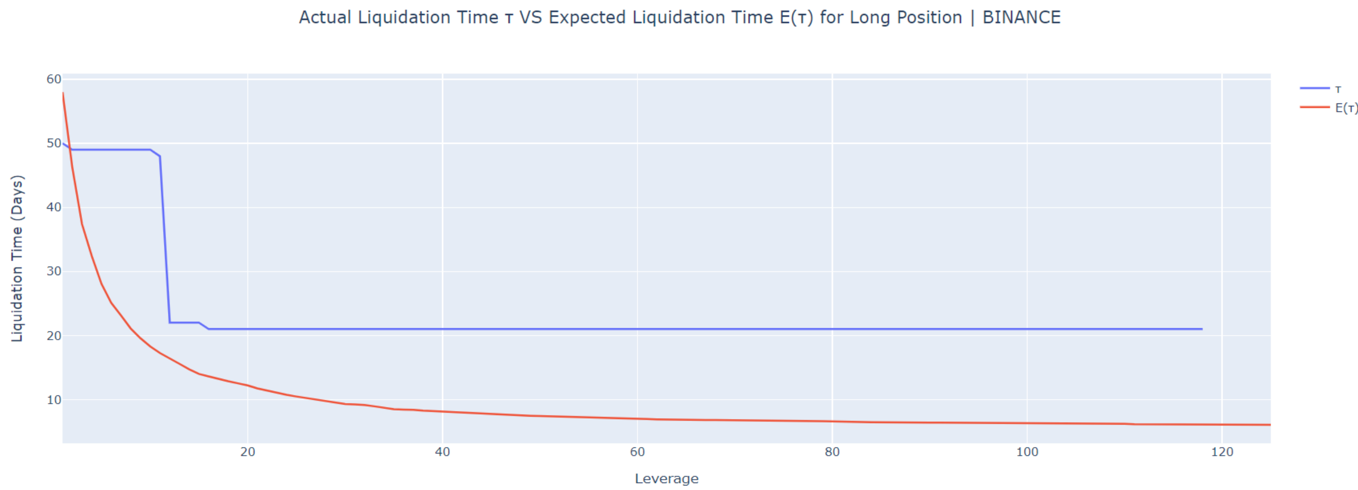


**Figure 4.3:** *Expected Liquidation Time $E(\tau)$ & Actual Liquidation Time $\tau$ for Binance, Long Position*

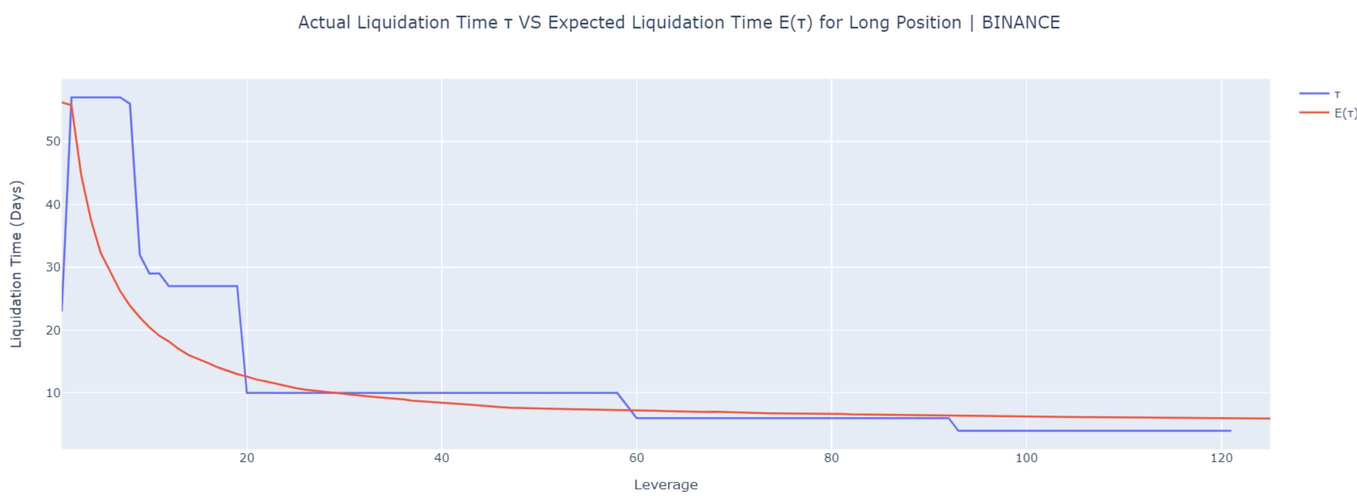


**Figure 4.4:** *Expected Liquidation Time $E(\tau)$ & Actual Liquidation Time $\tau$ for Binance, Short Position*

# Conclusion

The findings and backtesting results validate the liquidation pipeline's effectiveness in perpetual contracts trading. The relationship between leverage, risk, and liquidation metrics offers critical insights for decision-making. While the methodology proves accurate, there is room for refinement, especially for long positions. This highlights opportunities for future development.

In the next chapter, we'll explore practical use cases, showcasing the pipeline's real-world applications and its significance in the perpetual contracts market.

# Chapter 5

# Use Cases & Importance of the Solution

## Introduction

This chapter presents the use cases of the powerful pipeline tailored for perpetual contracts trading. We will take a look at how it acts as an advanced tools for risk assessment, exchange evaluation, leverage optimization, and strategy refinement. These quantitative insights provide a competitive edge in the dynamic trading landscape.

## 5.1 Advanced Risk Assessment and Decision-Making Capabilities

The comprehensive pipeline provides traders with a sophisticated toolkit for risk assessment in the realm of perpetual contracts. Through intricate computations, it furnishes crucial metrics such as expected liquidation time and the likelihood of liquidation before the predetermined maturity period. Armed with this granular data, traders can make judicious decisions, gaining profound insights into the potential exposure and vulnerability of their positions. This empowers them to dynamically adjust their trading strategies in response to changing market conditions.

## 5.2 Strategic Exchange Selection and Fine-Tuning

Operating across diverse exchanges with varying funding mechanisms and terms demands a nuanced approach. The pipeline equips traders with a robust quantitative framework to systematically evaluate and compare these exchanges. By simulating a spectrum of scenarios across different platforms, traders gain a comprehensive understanding of how each impacts the risk profiles of their positions. This meticulous evaluation process culminates in the selection of the most optimal exchange tailored to their specific trading objectives, enabling them to navigate the market with precision.

## 5.3 Precision Leverage Optimization and Strategic Formulation

Recognizing the pivotal role of leverage in shaping risk and potential returns, the pipeline offers traders a dynamic platform to experiment with various leverage levels. This enables them to observe the corresponding shifts in expected liquidation times and probabilities. Such a feature is instrumental in fine-tuning leverage strategies, ensuring a well-balanced approach that harmonizes with individual risk tolerance and profit targets. Traders can navigate the complexities of leverage with confidence, maximizing their potential for success.

## 5.4 Agile Strategy Validation and Adaptive Refinement

In the ever-evolving landscape of financial markets, traders must continuously refine their strategies to stay ahead. The pipeline serves as an indispensable validation tool, providing a controlled simulation environment for rigorous testing and fine-tuning of strategies. Through meticulous assessment of performance across a spectrum of scenarios, traders can refine their approaches, enhancing their decision-making prowess. This iterative process ensures that traders are equipped to not only weather changing market dynamics but also seize opportunities for sustained success.

## Conclusion

The use cases presented in this chapter underscore the profound impact of the developed solution on perpetual contracts trading. Its versatility in risk assessment, strategic refinement, and decision-making amplifies the trader's capabilities in navigating the market.

# Conclusion

In conclusion, this work stands as a robust tool for risk assessment, strategy evaluation, and decision-making in the realm of perpetual contracts trading. By estimating the expected liquidation time and probability of liquidation before artificial maturity, the pipeline empowers traders to navigate the complexities of cryptocurrency markets more effectively using Perpetuals.

Looking ahead, several promising avenues for enhancement come to light. Foremost, refining the accuracy of the long position side simulation remains a priority, as it enhances the pipeline's ability to provide comprehensive risk evaluations and it can be done by including more downward behavior in the Index Price's Monte Carlo Simulation. Additionally, customizing the simulation of Funding Rates for different exchanges would add a layer of realism by accounting for each exchange's distinct characteristics.

Furthermore, streamlining data acquisition through automated processes, perhaps utilizing APIs, would expedite the pipeline's efficiency. This enhancement would ensure real-time access to contract data and market indicators, enabling traders to stay current with the rapidly evolving crypto landscape. These enhancements collectively bolster the pipeline's capabilities, making it an indispensable asset for traders seeking to navigate the dynamic world of perpetual contracts trading.

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