It IS only rocket science!

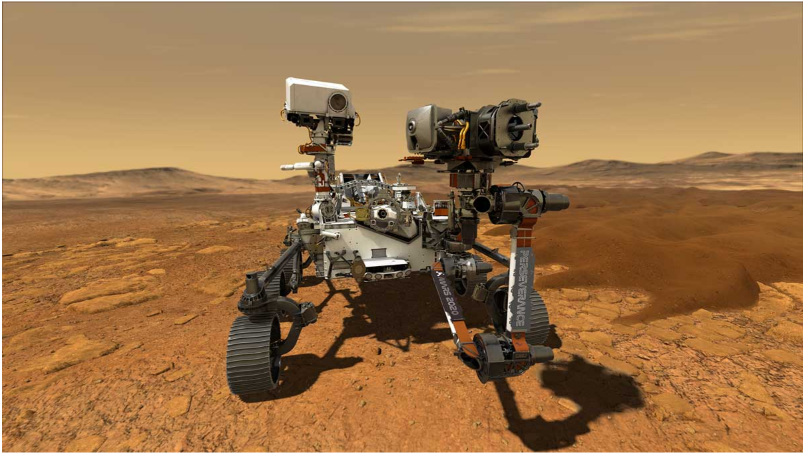
Maths Club - March 2021

Jorge Romero

University of Liverpool, Jyväskylän Yliopisto

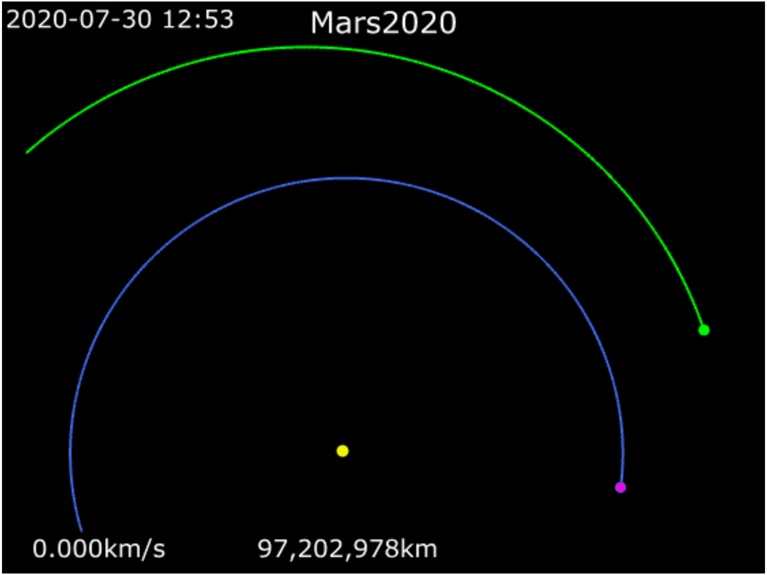
Rocket or arugula (*Eruca vesicaria*) is an edible annual plant in the family Brassicaceae used as a leaf vegetable for its fresh, tart, bitter, and peppery flavour. *Eruca sativa*, which is widely popular as a salad vegetable, is a species of *Eruca* native to the Mediterranean region, from Morocco and Portugal in the west to Syria, Lebanon, Egypt and Turkey in the east.

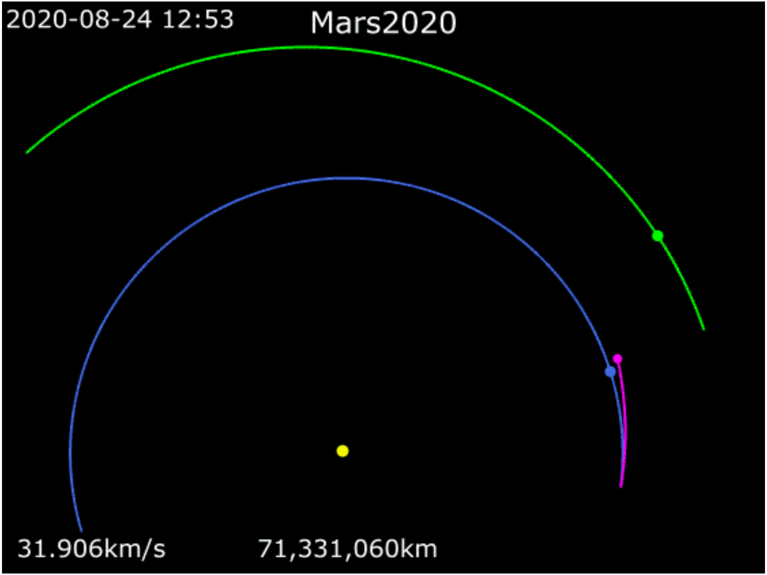
Picture: NASA

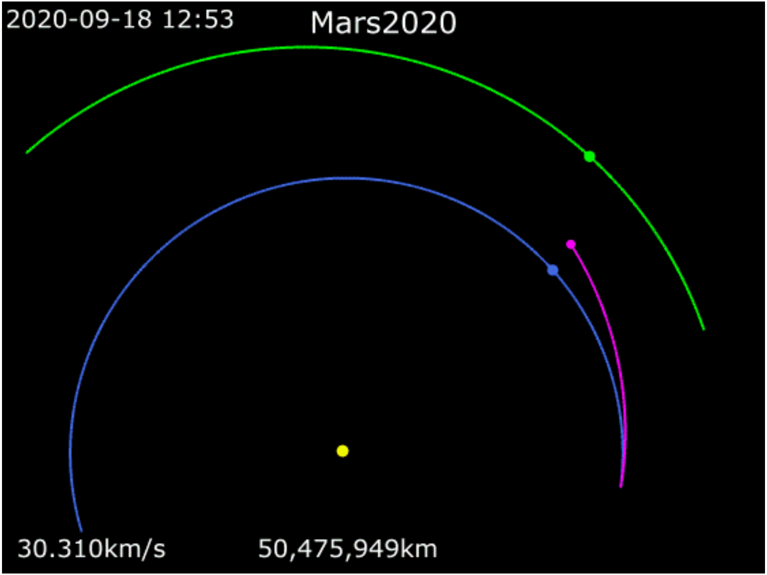


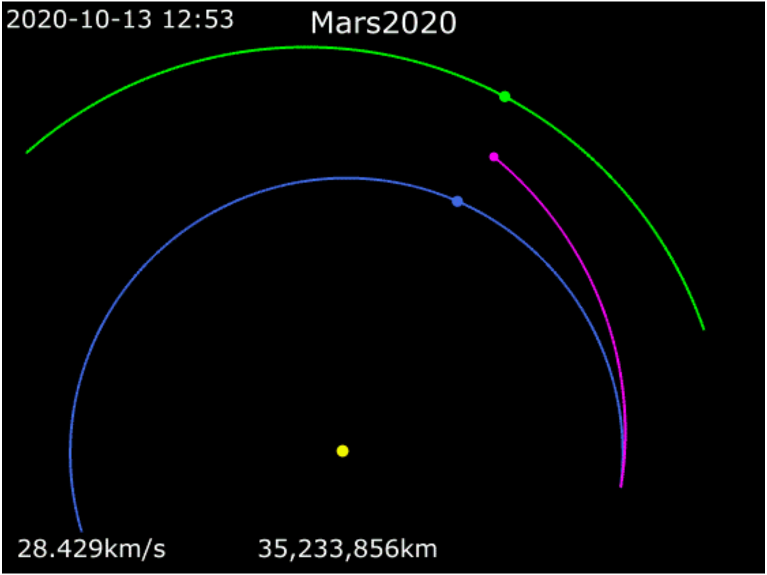
Perseverance is a rover within the NASA Mars 2020 Mission. It was launched on 30 July 2020, and landed successfully in Mars on 18 February 2021.

Trip to Mars

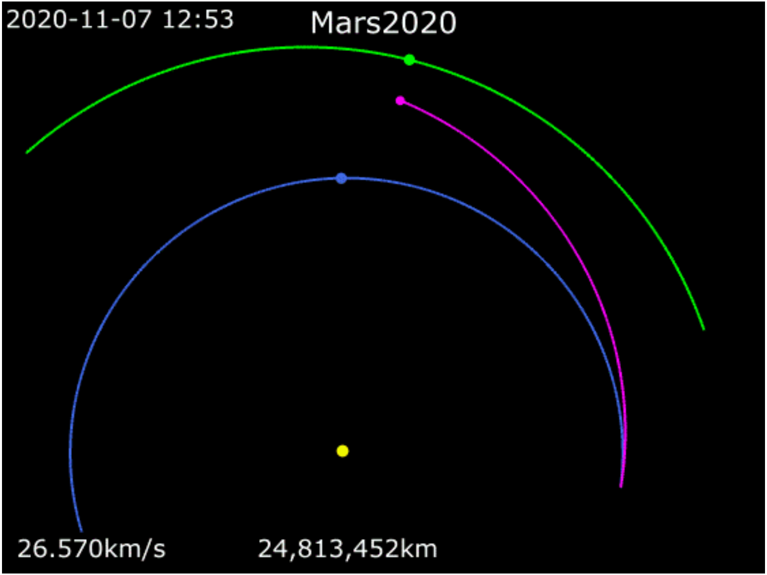


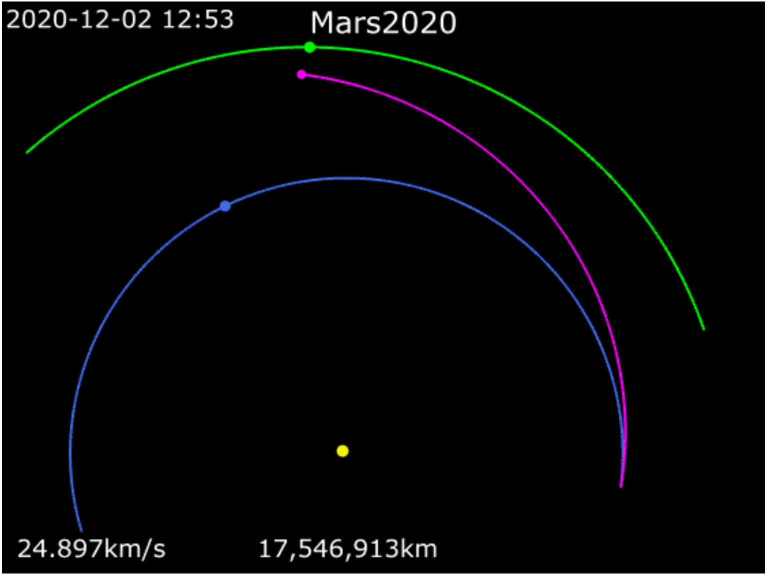

2020-08-24 12:53
Mars2020
31.906km/s
71,331,060km

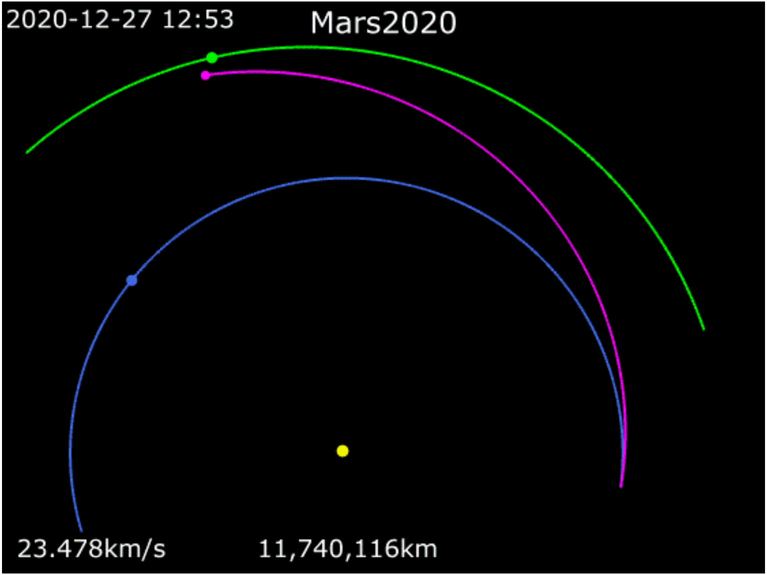

2020-09-18 12:53
Mars2020
30.310km/s
50,475,949km

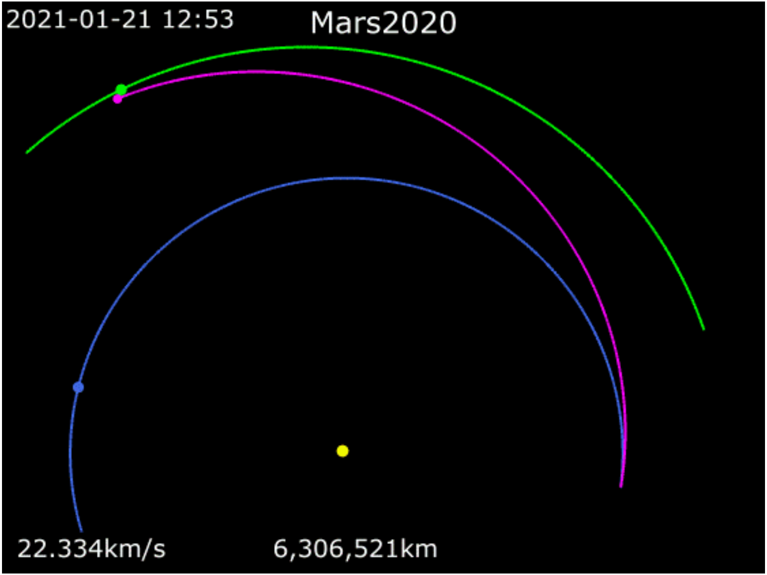


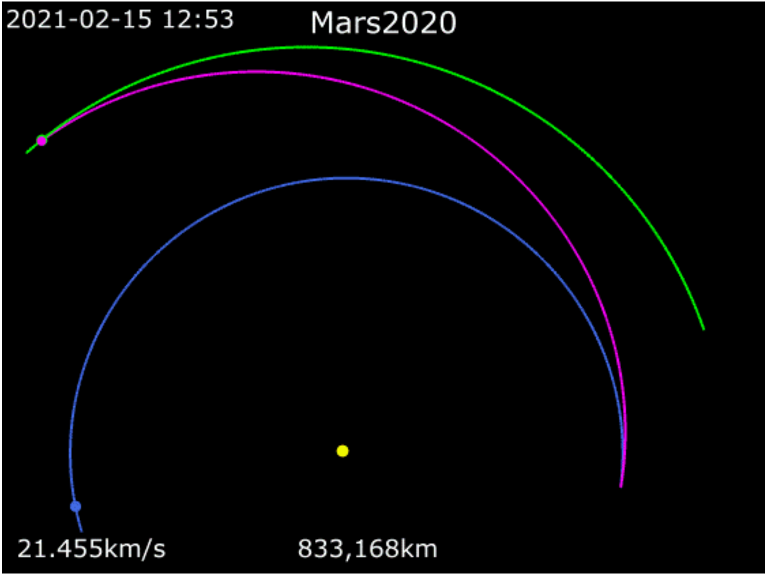
2020-10-13 12:53
Mars2020
28.429km/s
35,233,856km

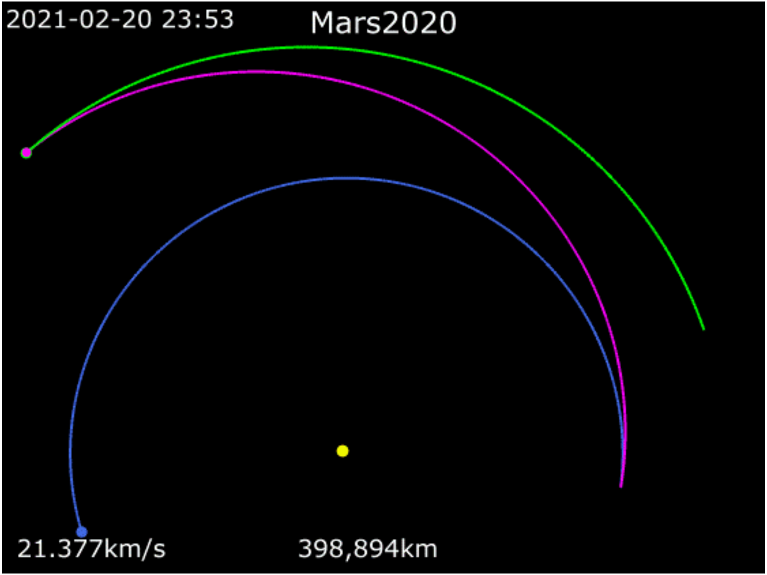

2020-11-07 12:53
Mars2020
26.570km/s
24,813,452km


2020-12-02 12:53
Mars2020
24.897km/s
17,546,913km


2020-12-27 12:53
Mars2020
23.478km/s
11,740,116km


2021-01-21 12:53
Mars2020
22.334km/s
6,306,521km

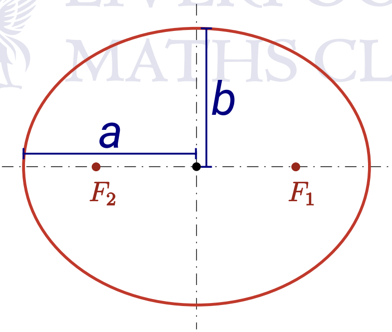

2021-02-15 12:53
Mars2020
21.455km/s
833,168km


2021-02-20 23:53
Mars2020
21.377km/s
398,894km

- The orbit of a planet is an ellipse with the Sun at one of the two foci.

- The orbit of a planet is an ellipse with the Sun at one of the two foci.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

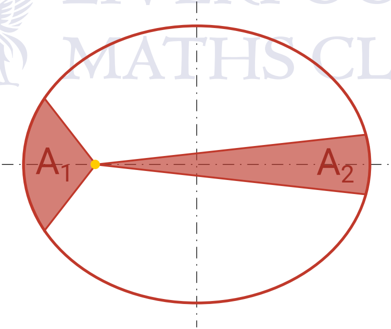


Kepler's Laws

- The orbit of a planet is an ellipse with the Sun at one of the two foci.
- A line segment joining a planet and the Sun sweeps out equal areas during equal intervals of time.

- The orbit of a planet is an ellipse with the Sun at one of the two foci.
- A line segment joining a planet and the Sun sweeps out equal areas during equal intervals of time.

$$A_1 = A_2 \Rightarrow t_1 = t_2$$

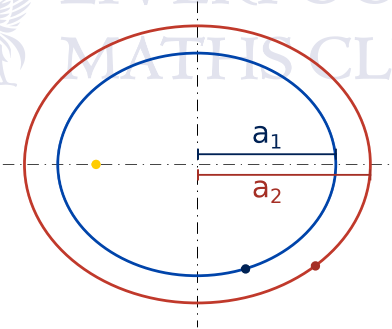


Kepler's Laws

- The orbit of a planet is an ellipse with the Sun at one of the two foci.
- A line segment joining a planet and the Sun sweeps out equal areas during equal intervals of time.
- The square of a planet's orbital period is proportional to the cube of the length of the semi-major axis of its orbit.

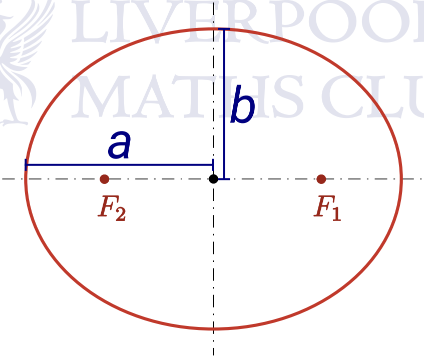
- The orbit of a planet is an ellipse with the Sun at one of the two foci.
- A line segment joining a planet and the Sun sweeps out equal areas during equal intervals of time.
- The square of a planet's orbital period is proportional to the cube of the length of the semi-major axis of its orbit.

$$\frac{T_1^2}{a_1^3} = \frac{T_2^2}{a_2^3}$$



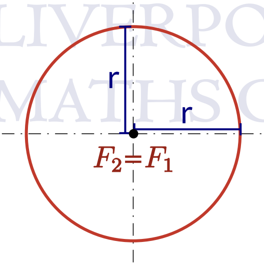
$$a = r; b = r$$

$$\frac{x^2}{r^2} + \frac{y^2}{r^2} = 1$$



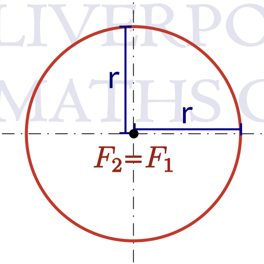
$$a = r; b = r \Rightarrow F_1 = F_2$$

$$\frac{x^2}{r^2} + \frac{y^2}{r^2} = 1 \Rightarrow \boxed{x^2 + y^2 = r^2}$$



$$a = r; b = r \Rightarrow F_1 = F_2$$

$$\frac{x^2}{r^2} + \frac{y^2}{r^2} = 1 \Rightarrow \boxed{x^2 + y^2 = r^2}$$



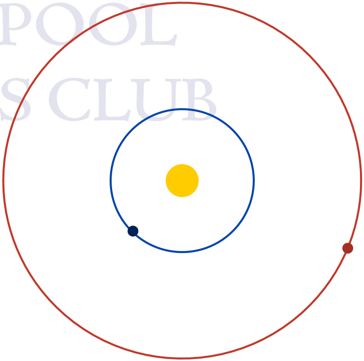
$$e = \sqrt{1 - \frac{b^2}{a^2}}$$

$$e = \sqrt{1 - \frac{b^2}{a^2}}$$

Planet	Excentricity
Earth	0.0167
Mars	0.0934

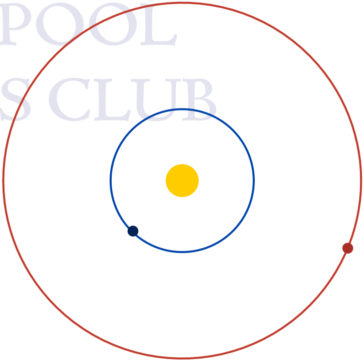
$$e = \sqrt{1 - \frac{b^2}{a^2}}$$

Planet	Excentricity
Earth	0.0167
Mars	0.0934

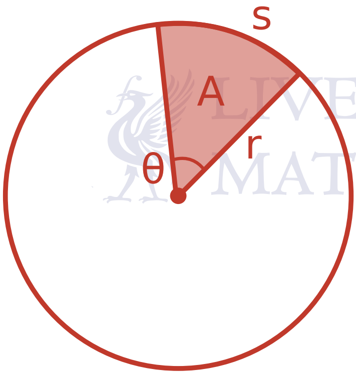


$$e = \sqrt{1 - \frac{b^2}{a^2}}$$

Planet	Excentricity	Radius [AU]
Earth	0.0167	1.00
Mars	0.0934	1.52



- A line segment joining a planet and the Sun sweeps out equal areas during equal intervals of time.



$$A = \frac{1}{2} \mathrm{b} \times \mathrm{h}$$

$$s = r\theta$$

$$A = \frac{1}{2} r \times (r\theta)$$

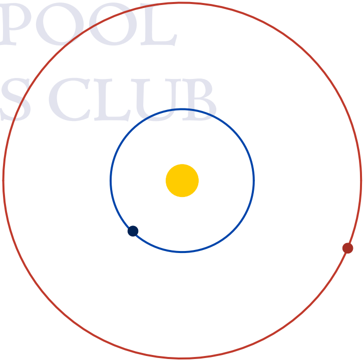
$$A = \frac{1}{2} \theta r^2$$

$$\frac{\Delta A}{\Delta t} = \frac{1}{2} \frac{\Delta \theta}{\Delta t} r^2$$

$$\frac{\Delta A}{\Delta t} = \frac{1}{2} r^2 \omega = \frac{L}{2m}$$

$$\frac{\Delta A}{\Delta t} = \text{constant}$$

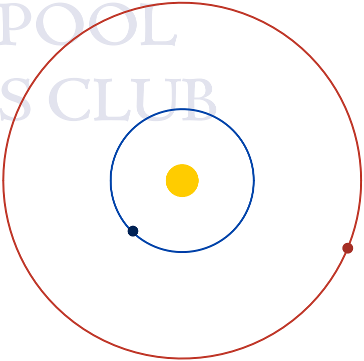
Planet	Excentricity	Radius [AU]
Earth	0.0167	1.00
Mars	0.0934	1.52



Martian Year

$$\frac{T^2_{\text{Earth}}}{r^3_{\text{Earth}}} = \frac{T^2_{\text{Mars}}}{r^3_{\text{Mars}}}$$

Planet	Excentricity	Radius [AU]
Earth	0.0167	1.00
Mars	0.0934	1.52



$$\frac{T^2_{\text{Earth}}}{r^3_{\text{Earth}}} = \frac{T^2_{\text{Mars}}}{r^3_{\text{Mars}}}$$

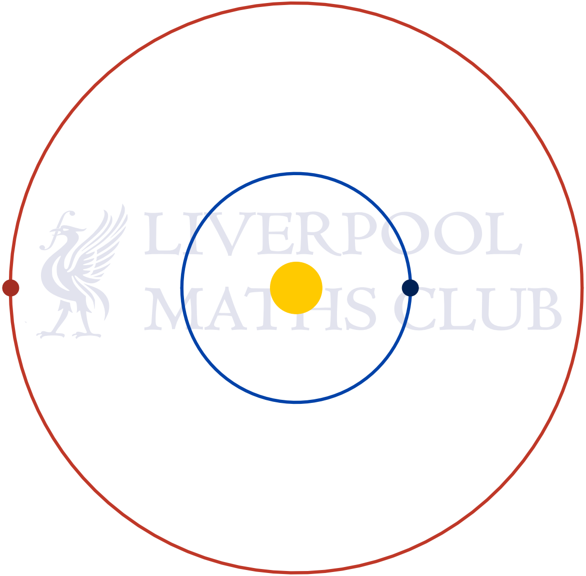
Planet	Excentricity	Radius [AU]
Earth	0.0167	1.00
Mars	0.0934	1.52

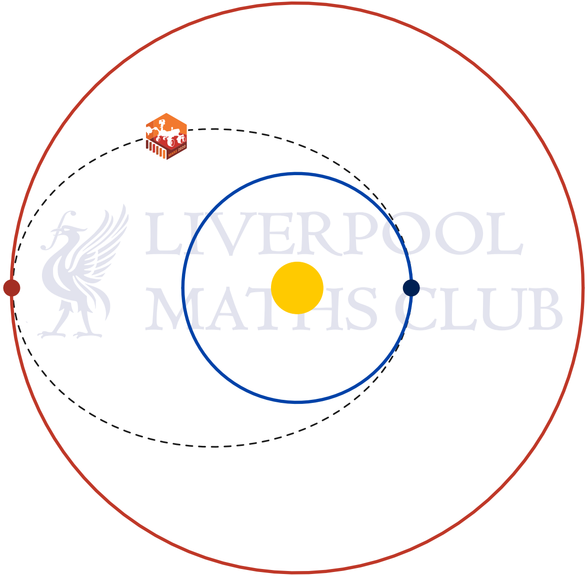
$$T^2_{\text{Mars}} = \frac{T^2_{\text{Earth}}}{r_{\text{Earth}^3}} r^3_{\text{Mars}}$$

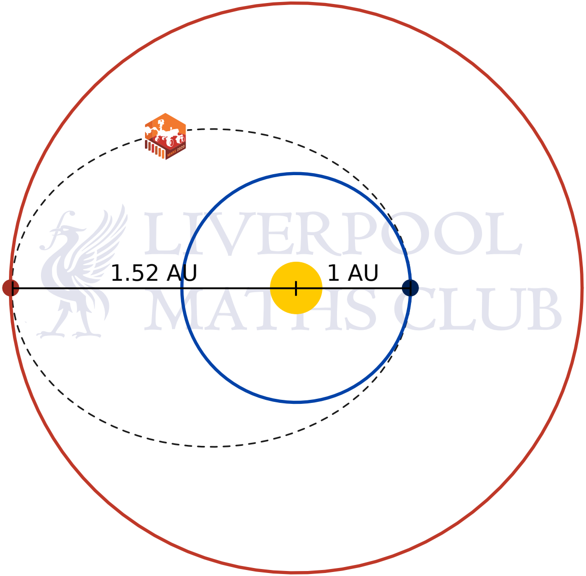
$$T_{\text{Mars}} = \sqrt{\frac{T^2_{\text{Earth}}}{r_{\text{Earth}^3}} r^3_{\text{Mars}}}$$

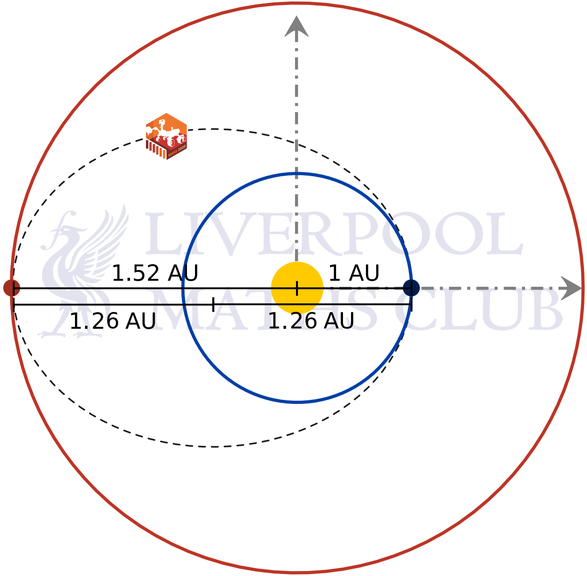
$$T_{\text{Mars}} = \sqrt{\frac{365^2}{1^3}(1.52)^3}$$

$$T_{\text{Mars}} = 684 \text{ days}$$

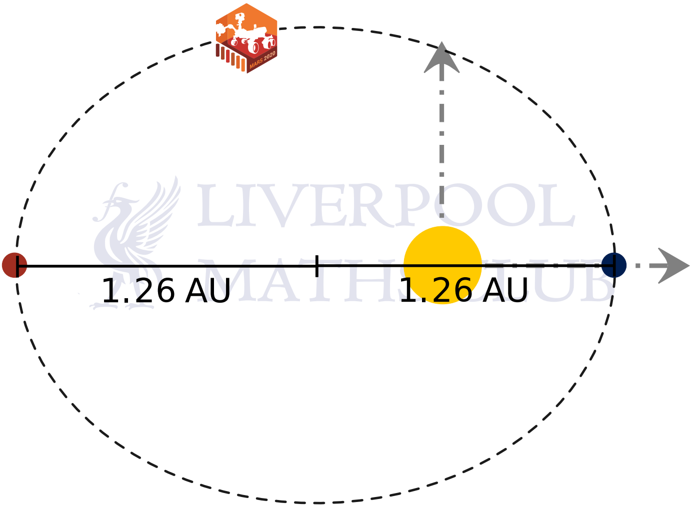




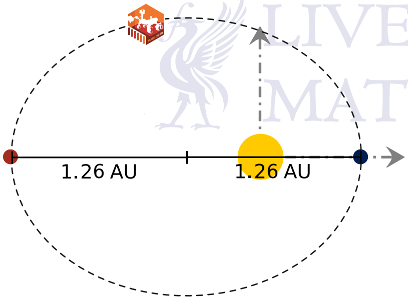

1.52 AU
1 AU



1.52 AU
1 AU
1.26 AU
1.26 AU


MARS 2020
1.26 AU
1.26 AU

$$\frac{T_{\text{Earth}}^2}{r_{\text{Earth}}^3} = \frac{T^2}{a^3}$$



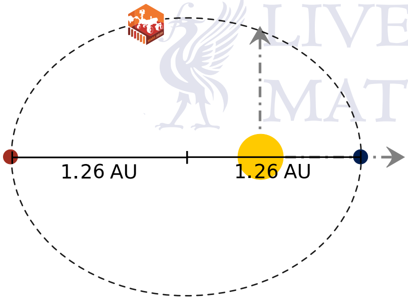
$$T^2 = \frac{T_{\text{Earth}}^2}{r_{\text{Earth}}^3} \times a^3$$

$$T = \sqrt{\frac{T_{\text{Earth}}^2}{r_{\text{Earth}}^3} \times a^3}$$

$$T = \sqrt{\frac{365^2}{1^3} \times (1.26)^3}$$

$$T = 516 \text{ days}$$

$$\frac{T^2_{\text{Earth}}}{r^3_{\text{Earth}}} = \frac{T^2}{a^3}$$



$$T^2 = \frac{T^2_{\text{Earth}}}{r^3_{\text{Earth}}} \times a^3$$

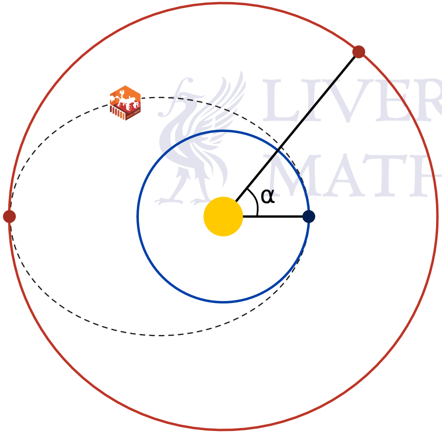
$$T = \sqrt{\frac{T^2_{\text{Earth}}}{r^3_{\text{Earth}}} \times a^3}$$

$$T = \sqrt{\frac{365^2}{1^3} \times (1.26)^3}$$

$$T = 516 \text{ days}$$

$$\boxed{T/2 = 258 \text{ days}}$$

$$\omega_{\text{Earth}} = \frac{360}{365} = 0.986°/\text{day} \qquad \omega_{\text{Mars}} = \frac{360}{684} = 0.526°/\text{day}$$



After N = 258 days, Mars has to be at 180°with respect to the Earth's initial position.

Assuming that Mars starts off at an angle α from the Earth:

$$N \times \omega_{\text{Mars}} = 180 - \alpha$$
$$\alpha = 180 - (N \times \omega_{\text{Mars}})$$
$$\alpha = 180 - (258 \times 0.526)$$
$$\boxed{\alpha = 44.3°}$$