

1.2 Continuous Distributions

	Notation	$F_X(x)$	$f_X(x)$	$\mathbb{E}[X]$	$\mathbb{V}[X]$	$M_X(s)$
Uniform	$\text{Unif}(a, b)$	$\begin{cases} 0 & x < a \\ \frac{x-a}{b-a} & a < x < b \\ 1 & x > b \end{cases}$	$\frac{I(a < x < b)}{b-a}$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$	$\frac{e^{sb} - e^{sa}}{s(b-a)}$
Normal	$\mathcal{N}(\mu, \sigma^2)$	$\Phi(x) = \int_{-\infty}^x \phi(t) dt$	$\phi(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\}$	μ	σ^2	$\exp\left\{\mu s + \frac{\sigma^2 s^2}{2}\right\}$
Log-Normal	$\ln \mathcal{N}(\mu, \sigma^2)$	$\frac{1}{2} + \frac{1}{2} \operatorname{erf}\left[\frac{\ln x - \mu}{\sqrt{2\sigma^2}}\right]$	$\frac{1}{x\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{(\ln x - \mu)^2}{2\sigma^2}\right\}$	$e^{\mu + \sigma^2/2}$	$(e^{\sigma^2} - 1)e^{2\mu + \sigma^2}$	
Multivariate Normal	$\text{MVN}(\mu, \Sigma)$		$(2\pi)^{-k/2} \Sigma ^{-1/2} e^{-\frac{1}{2}(x-\mu)^T \Sigma^{-1}(x-\mu)}$	μ	Σ	$\exp\left\{\mu^T s + \frac{1}{2} s^T \Sigma s\right\}$
Student's t	$\text{Student}(\nu)$	$I_x\left(\frac{\nu}{2}, \frac{\nu}{2}\right)$	$\frac{\Gamma(\frac{\nu+1}{2})}{\sqrt{\nu\pi}\Gamma(\frac{\nu}{2})} \left(1 + \frac{x^2}{\nu}\right)^{-(\nu+1)/2}$	0	0	
Chi-square	χ_k^2	$\frac{1}{\Gamma(k/2)} \gamma\left(\frac{k}{2}, \frac{x}{2}\right)$	$\frac{1}{2^{k/2}\Gamma(k/2)} x^{k/2} e^{-x/2}$	k	$2k$	$(1-2s)^{-k/2} s < 1/2$
F	$F(d_1, d_2)$	$I_{\frac{d_1 x}{d_1 x + d_2}}\left(\frac{d_1}{2}, \frac{d_1}{2}\right)$	$\frac{\sqrt{\frac{(d_1 x)^{d_1} d_2^{d_2}}{(d_1 x + d_2)^{d_1 + d_2}}}}{x B\left(\frac{d_1}{2}, \frac{d_1}{2}\right)}$	$\frac{d_2}{d_2 - 2}$	$\frac{2d_2^2(d_1 + d_2 - 2)}{d_1(d_2 - 2)^2(d_2 - 4)}$	
Exponential	$\text{Exp}(\beta)$	$1 - e^{-x/\beta}$	$\frac{1}{\beta} e^{-x/\beta}$	β	β^2	$\frac{1}{1 - \beta s} (s < 1/\beta)$
Gamma	$\text{Gamma}(\alpha, \beta)$	$\frac{\gamma(\alpha, x/\beta)}{\Gamma(\alpha)}$	$\frac{1}{\Gamma(\alpha)\beta^\alpha} x^{\alpha-1} e^{-x/\beta}$	$\alpha\beta$	$\alpha\beta^2$	$\left(\frac{1}{1 - \beta s}\right)^\alpha (s < 1/\beta)$
Inverse Gamma	$\text{InvGamma}(\alpha, \beta)$	$\frac{\Gamma(\alpha, \frac{\beta}{x})}{\Gamma(\alpha)}$	$\frac{\beta^\alpha}{\Gamma(\alpha)} x^{-\alpha-1} e^{-\beta/x}$	$\frac{\beta}{\alpha - 1} \alpha > 1$	$\frac{\beta^2}{(\alpha - 1)^2(\alpha - 2)^2} \alpha > 2$	$\frac{2(-\beta s)^{\alpha/2}}{\Gamma(\alpha)} K_\alpha(\sqrt{-4\beta s})$
Dirichlet	$\text{Dir}(\alpha)$		$\frac{\Gamma(\sum_{i=1}^k \alpha_i)}{\prod_{i=1}^k \Gamma(\alpha_i)} \prod_{i=1}^k x_i^{\alpha_i - 1}$	$\frac{\alpha_i}{\sum_{i=1}^k \alpha_i}$	$\frac{\mathbb{E}[X_i](1 - \mathbb{E}[X_i])}{\sum_{i=1}^k \alpha_i + 1}$	
Beta	$\text{Beta}(\alpha, \beta)$	$I_x(\alpha, \beta)$	$\frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1} (1-x)^{\beta-1}$	$\frac{\alpha}{\alpha + \beta}$	$\frac{\alpha\beta}{(\alpha + \beta)^2(\alpha + \beta + 1)}$	$1 + \sum_{k=1}^{\infty} \left(\prod_{r=0}^{k-1} \frac{\alpha + r}{\alpha + \beta + r}\right) \frac{s^k}{k!}$
Weibull	$\text{Weibull}(\lambda, k)$	$1 - e^{-(x/\lambda)^k}$	$\frac{k}{\lambda} \left(\frac{x}{\lambda}\right)^{k-1} e^{-(x/\lambda)^k}$	$\lambda \Gamma\left(1 + \frac{1}{k}\right)$	$\lambda^2 \Gamma\left(1 + \frac{2}{k}\right) - \mu^2$	$\sum_{n=0}^{\infty} \frac{s^n \lambda^n}{n!} \Gamma\left(1 + \frac{n}{k}\right)$
Pareto	$\text{Pareto}(x_m, \alpha)$	$1 - \left(\frac{x_m}{x}\right)^\alpha \quad x \geq x_m$	$\alpha \frac{x_m^\alpha}{x^{\alpha+1}} \quad x \geq x_m$	$\frac{\alpha x_m}{\alpha - 1} \alpha > 1$	$\frac{x_m^\alpha}{(\alpha - 1)^2(\alpha - 2)} \alpha > 2$	$\alpha(-x_m s)^\alpha \Gamma(-\alpha, -x_m s) \quad s < 0$