(7)

COMPITO 9/01/2014

1 SVOLTO

$$f(x,y) = \sqrt{(x-y)^2(y^2-x^2-1)} \quad \frac{x+y^2-4}{\ln(4x^2+y^2-1)}$$

$$A = \{ (x,y) \in \mathbb{R}^2 \mid (x-y)^2(y^2-x^2-1) \geq 0, \ 4x^2+y^2-1>0, \ 4x^2+y^2-1 \neq 1 \}$$

DOMINIO e SEGNO

$f_1 = (x-y)^2 = (x-y)(x-y)$

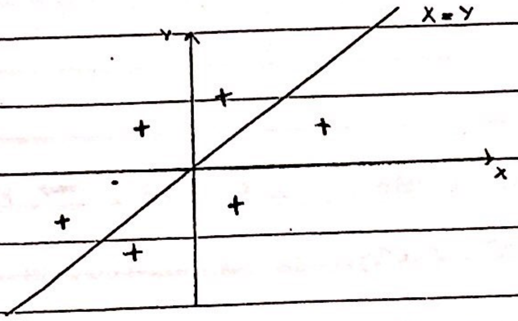
$f: x \rightarrow x \in \mathcal{C}(\mathbb{R})$ per Th COLLEGAMENTO $\in \mathcal{C}(\mathbb{R}^2)$

$f: y \rightarrow y \in \mathcal{C}(\mathbb{R})$ per Th COLLEGAMENTO $\in \mathcal{C}(\mathbb{R}^2)$

PER CRITERIO SOMMA $f: (x,y) \rightarrow x-y \in \mathcal{C}(\mathbb{R}^2)$

PER CRITERIO PRODOTTO $f: (x,y) \rightarrow (x-y)^2 \in \mathcal{C}(\mathbb{R}^2)$

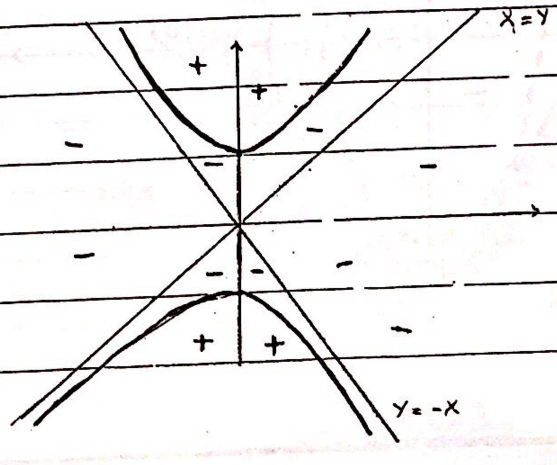
$\Longrightarrow$ MRC $(x-y)^2=0 \iff x=y$



$f_2 = (y^2-x^2-1)$

PER CRITERI ANALOGHI $f_2 \in \mathcal{C}(\mathbb{R}^2) \longrightarrow$ MRC

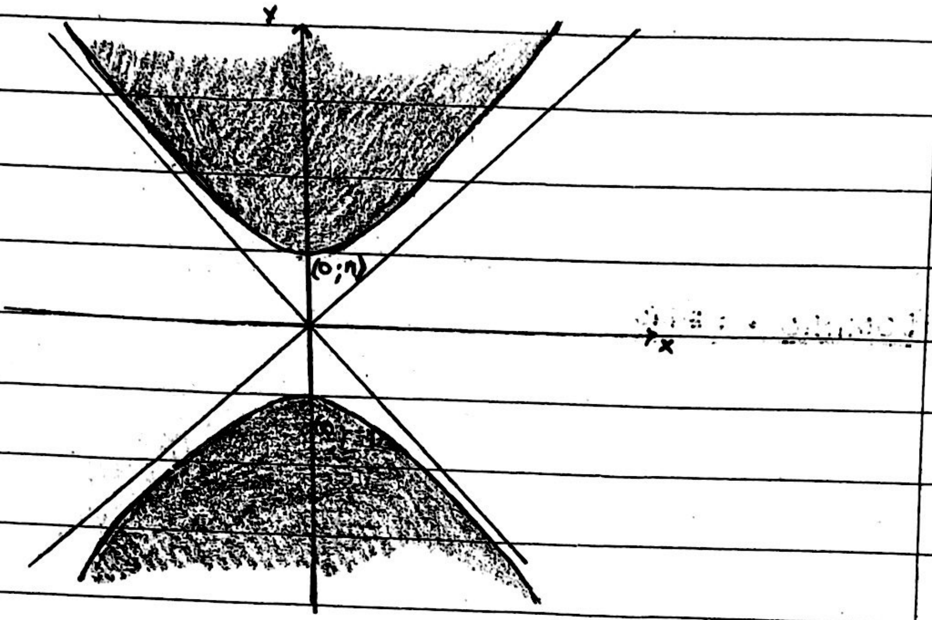
$y^2-x^2-1=0 \iff y^2-x^2=1$



$f(0;2) = f(0;-2) = 3>0$

$f(0;1/2) = f(0;-1/2) = -3/4<0$

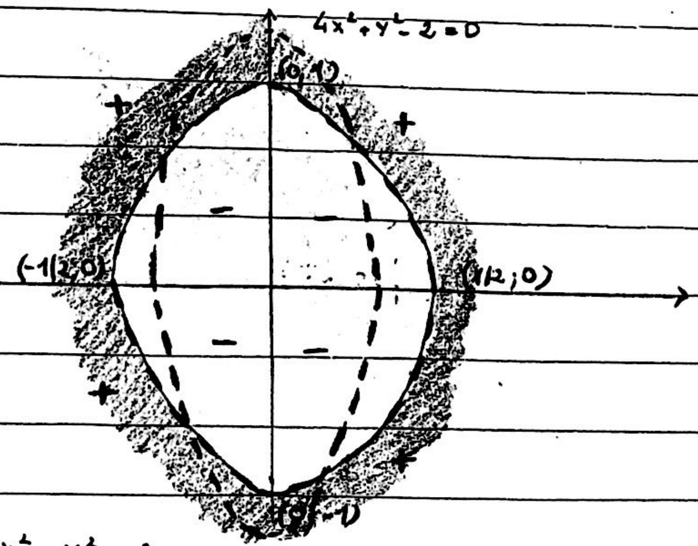
$\Longrightarrow f_1 \cdot f_2 \geq 0$



$f_3 = 4x^2+y^2-1$

PER MOTIVI ANALOGHI A PRIMA $f_3 \in \mathcal{C}(\mathbb{R}^2) \Longrightarrow$ MAC

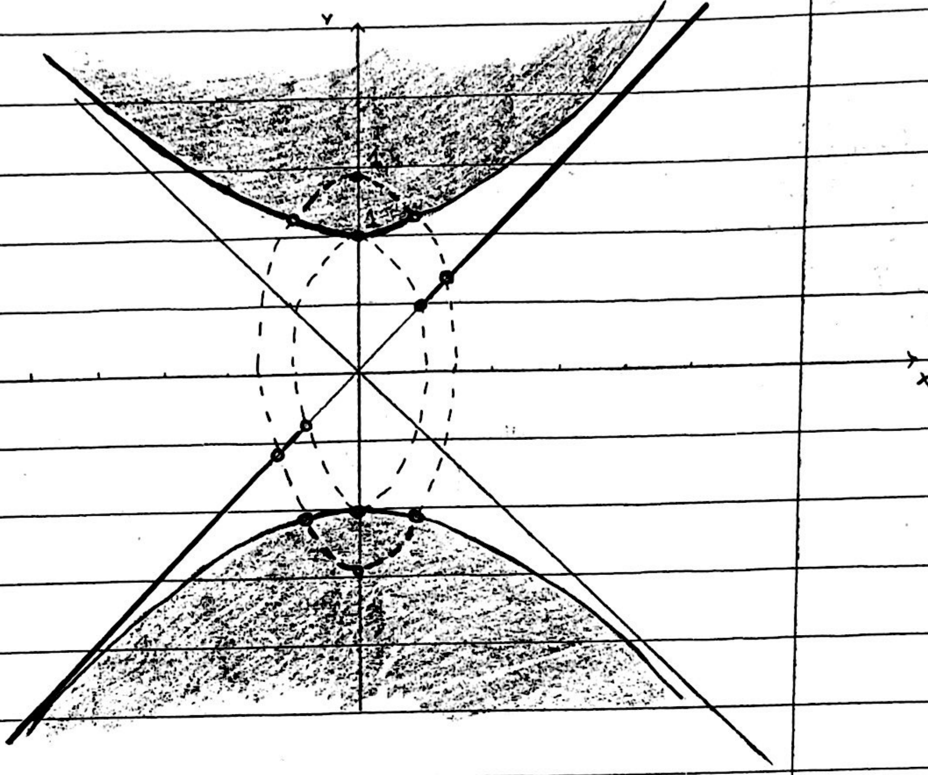
$4x^2+y^2-1=0 \iff 4x^2+y^2=1$



$f(0;0) = -1 < 0$
$f(0;5) = 24 > 0$

$f_4 = 1 \qquad 4x^2+y^2=2$

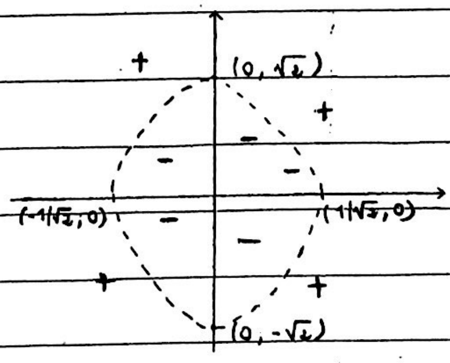
DOMINIO



$\sqrt{\cdot}$ NON INFLUENZA IL SEGNO di f POICHÈ è SEMPRE POSITIVA.

$\ln(4x^2+y^2-1) \in \mathcal{C}(\mathbb{R}^2)$ per criterio della COMPOSTA

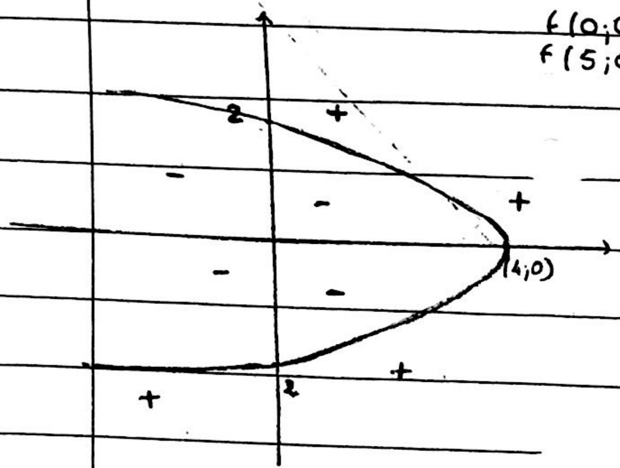
$\Longrightarrow$ MRC $\quad \ln(4x^2+y^2-1)=0 \iff 4x^2+y^2-1=1 \iff 4x^2+y^2=2$



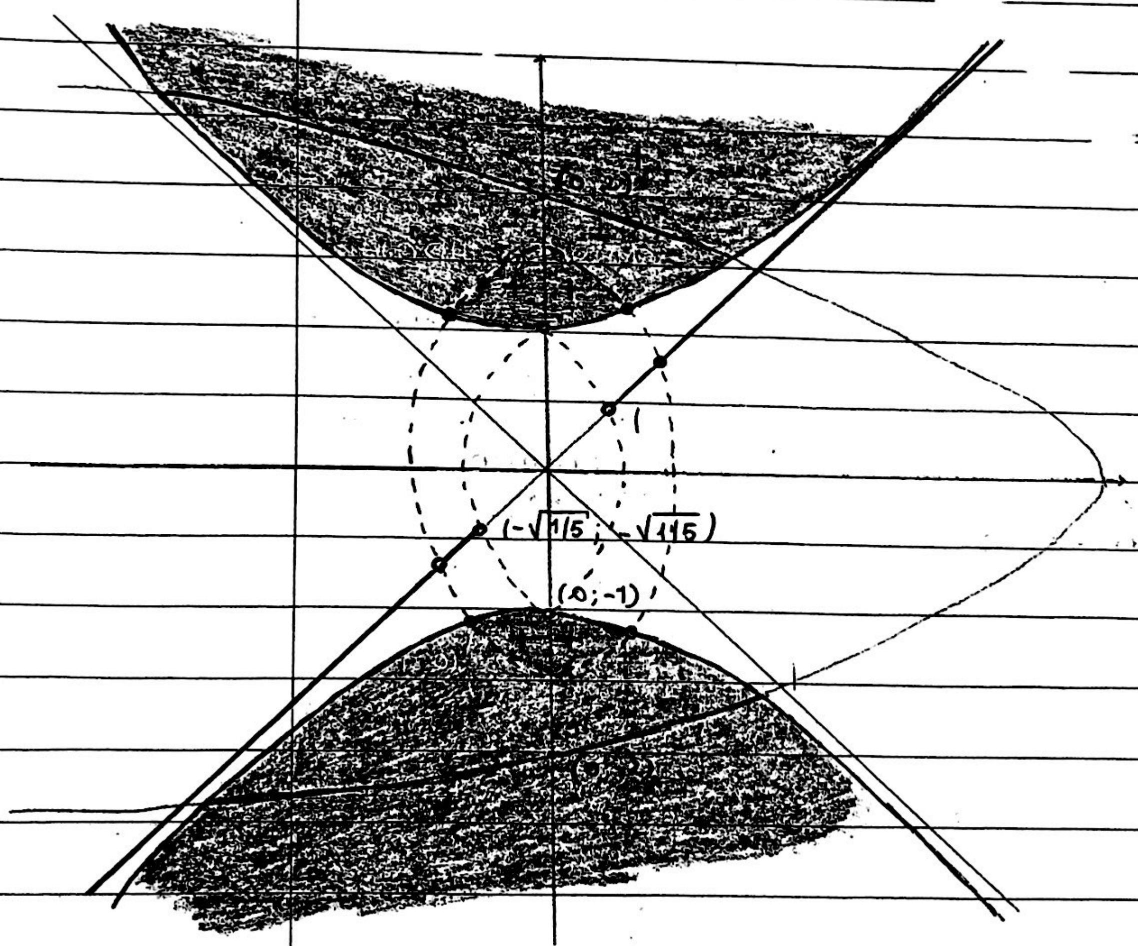
$f(0;0) = -2 < 0$
$f(0;2) = 2 > 0$

$f_4 = x + y^2 - 4 \in \mathcal{C}(\mathbb{R}^2)$ per criteri analoghi [illegible]

MBC $x + y^2 - 4 = 0 \iff x = 4 - y^2$



− e.e.o



LIMITI

$A\left(-\sqrt{1/5}\,;\ -\sqrt{1/5}\right)$

$A \in D(A)$

$$\lim_{(x,y)\to\left(-\sqrt{1/5};\,-\sqrt{1/5}\right)} \underbrace{\sqrt{(x-y)^2(y^2-x^2-1)}}_{\to 0 \text{ per continuità}} \ \underbrace{\frac{\overbrace{x+y^2-4}^{\to -\frac{19}{5}+\sqrt{5} \text{ per continuità}}}{\ln(\underbrace{4x^2+y^2-1}_{\to 0 \text{ per continuità}})}}_{\to -\infty \text{ per la composta}} = 0$$

B $(0;0)$ $\quad 0 \notin D(A)$ NON ha senso calcolare il limite

C $(0,-1)$

$C \in D(A)$

$$\lim_{(x,y)\to(0,-1)} \underbrace{\sqrt{(x-y)^2(y^2-x^2-1)}}_{\to 0 \text{ continuità}} \ \underbrace{\frac{\overbrace{x+y^2-4}^{\to -3 \text{ per continuità}}}{\ln(\underbrace{4x^2+y^2-1}_{\to 0})}}_{\to -\infty \text{ per la composta}} = 0$$

D $(0;-2)$

$D \in D(A)$

$\lim_{(x,y)\to(0;-2)} f(x,y) = 0$ POICHÈ f è continua in $(0;-2)$ il limite è proprio $f(0;-2)$

APPROSSIMANTE LINEARE:

$g(x,y) = (x+y^2-4)\cdot \ln(4x^2+y^2-1)$

$(-1;1) \in D(g)$

g deve essere differenziabile in $(-1;1)$. Per il Th del differenziale totale g'_x e g'_y devono esistere in un intorno del pto ed essere continue nel pto

$$g'_x = \ln(4x^2+y^2-1) + \frac{8x}{4x^2+y^2-1}(x+y^2-4)$$

$$g'_y = 2y\ln(4x^2+y^2-1) + \frac{2y}{4x^2+y^2-1}(x+y^2-4)$$

PER I SOLITI CRITERI $\in C(A)$ ENTRAMBE

$$\Longrightarrow f(x,y) = \underbrace{f(x_0,y_0) + g'_x(x_0,y_0)(x-x_0) + g'_y(x_0,y_0)(y-y_0)}_{P(x,y)} + T(x,y)$$

$$P(\underline{x}) = -4\ln 4 + (\ln(4)+8)(x+1) + (2\ln(4)-2)(y-1)$$

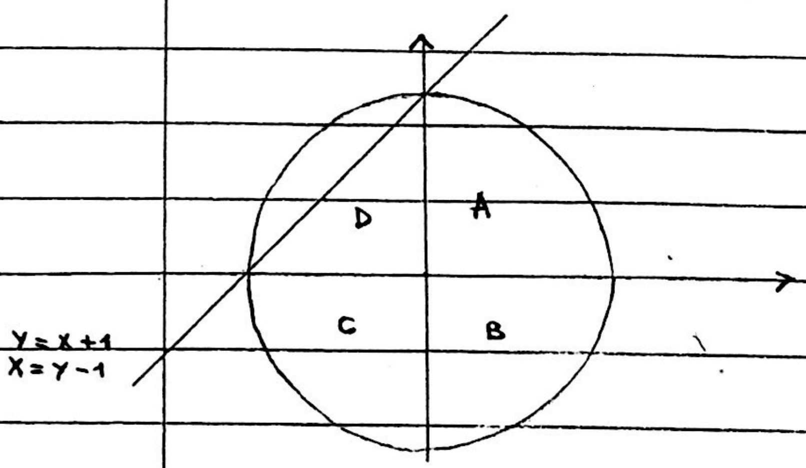
$$\boxed{z(x,y) = -4\ln 4 + (\ln(4)+8)(x+1) + (2\ln(4)-2)(y-1)}$$

3

2

$$\iint_E \left[x \operatorname{sen} y + x^2 y + \frac{1}{2}\right] dx\,dy$$

$$E = \{(x,y) \in \mathbb{R}^2 \mid y \leq x+1,\ x^2+y^2 \leq 1\}$$



$$= \iint_E x \,\text{sem}\, y \,dx\,dy + \iint_E x^2 y \,dx\,dy + \underbrace{\iint_E 1/2 \,dx\,dy}_{1/2 \cdot m(E)}$$

$\underline{\underline{A}}$: $\iint_E x \,\text{sem}\, y \,dx\,dy$

$f(x,-y) = x\,\text{sem}(-y) = -x\,\text{sem}\, y = -f(x,y)$ FUNZIONE DISPARI

$$\Rightarrow \iint_E x\,\text{sem}\, y\,dx\,dy = \iint_C x\,\text{sem}\, y\,dx\,dy + \iint_D x\,\text{sem}\, y\,dx\,dy$$

$$\iint_D x\,\text{sem}\, y\,dx\,dy = \int_{-1}^{0} x \left[\int_0^{x+1} \text{sem}\, y\,dy\right] dx =$$

$$\int_{-1}^{0} x\left[-\cos x\right]_0^{x+1} dx = \int_{-1}^{0} \left[-x\cos(x+1) + 1\right] dx =$$

$$\int_{-1}^{0} -x\cos(x+1)\,dx + \int_{-1}^{0} dx =$$

$$\left[-x\,\text{sem}(x+1) - \int \text{sem}(x+1)\,dx\right]_{-1}^{0} + 1 =$$

$$\left[-x\,\text{sem}(x+1) - \cos(x+1)\right]_{-1}^{0} + 1 = 2 - \cos(1)$$

$$\iint_C x\,\text{sem}\, y\,dx\,dy \qquad \begin{cases} x = \rho\cos\theta \\ y = \rho\,\text{sem}\,\theta \end{cases} \qquad \rho \in [0;1] \quad \theta \in [\pi; 3/2\pi]$$

$$= \iint \rho^2\cos\theta\,\text{sem}(\rho\,\text{sem}\,\theta)\,dx\,dy$$

$$\int_0^1 \rho\left[\int_\pi^{3/2\pi} \rho\cos\theta\,\text{sem}(\rho\,\text{sem}\,\theta)\,d\theta\right] d\rho = \int_0^1 \rho\left[-\cos(\rho\,\text{sem}\,\theta)\right]_\pi^{3/2\pi} =$$

$$= \int_0^1 -\rho\cos(\rho) + \rho \, d\rho =$$

$$\int_0^1 -\rho\cos(\rho)\, d\rho + \int_0^1 \rho \, d\rho =$$

$$\left| -\rho \operatorname{sem}(\rho) - \cos(\rho) \right|_0^1 + \left| \frac{\rho^2}{2} \right|_0^1 =$$

$$= -\operatorname{sem}(1) - \cos(1) + 1 + 1/2$$

$$\iint_E x \operatorname{sem} y \, dx\,dy = 2 - 2\cos(1) - \operatorname{sem}(1) + 1 + 1/2$$

$$= \boxed{7/2 - 2\cos(1) - \operatorname{sem}(1)}$$

B: $\iint_E x^2 y \, dx\,dy$

AB SIMMETRICO RISPETTO ASSE x; $f(x,-y) = -f(x,y)$ DISPARI

$$\Longrightarrow \iint_E x^2 y \, dx\,dy = \iint_C x^2 y \, dx\,dy + \iint_D x^2 y \, dx\,dy$$

SU C:

$$\begin{cases} x = \rho\cos\theta \\ y = \rho \operatorname{sem}\theta \end{cases} \qquad \begin{matrix} \rho \in [0;1] \\ \theta \in [\pi; 3/2\pi] \end{matrix}$$

$$\iint \rho^4 \cos^2\theta \operatorname{sem}\theta \, d\theta\, d\rho = \int_0^1 \rho^4 \left[\int_\pi^{3/2\pi} \cos^2\theta \operatorname{sem}\theta \, d\theta \right] d\rho =$$

$$\int_0^1 \rho^4 \left| \frac{-\cos^3\theta}{3} \right|_\pi^{3/2\pi} d\rho = \int_0^1 -\frac{\rho^4}{3} \, d\rho = \frac{1}{3} \left| \frac{\rho^5}{5} \right|_0^1 = \frac{1}{15}$$

SU D: $$\int_{-1}^0 x^2 \left[\int_0^{x+1} y \, dy \right] dx = \int_{-1}^0 x^2 \frac{(x+1)^2}{2} dx = \frac{1}{2} \left| \frac{x^5}{5} + \frac{x^4}{2} + \frac{x^3}{3} \right|_{-1}^0 = \frac{1}{60}$$

$$\iint_E x^2 y \, dx\, dy = \frac{1}{15} + \frac{1}{60} = \boxed{\frac{1}{12}}$$

C $\quad m(E) : \frac{3}{4}\pi + \frac{1}{2}$

$$\Rightarrow \iint_E \frac{1}{2} dx\, dy = \frac{1}{2}\left[\frac{3}{4}\pi + \frac{1}{2}\right] = \boxed{\frac{3}{8}\pi + \frac{1}{4}}$$

x)

$$\iint_E f(x,y)\, dx\, dy =$$ [redacted]

$\boxed{3}$

$$\sum_{m=1}^{+\infty} \left(\frac{1}{(m-1)!} - \frac{1}{m}\right)(2x - x^2)^m$$

$$t = 2x - x^2 \Longrightarrow \sum_{m=1}^{+\infty} \left[\frac{1}{(m-1)!} - \frac{1}{m}\right] t^m$$

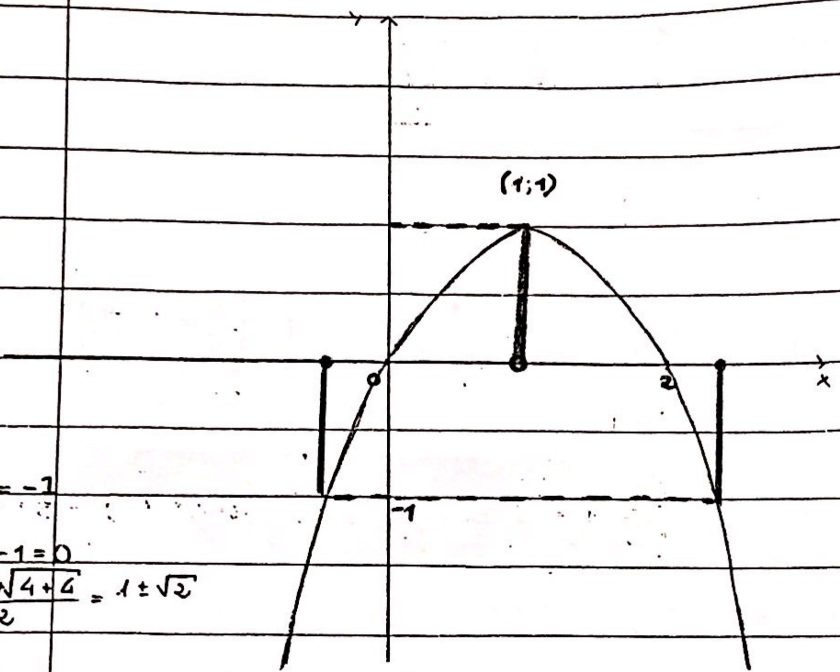
$$\lim_{m \to +\infty} \frac{m+1 - m!}{\cancel{m!}\,(m+1)} \cdot \frac{\cancel{(m-1)!\, m}}{m - (m-1)!} = \lim_{m \to +\infty} \frac{m}{m+1} = 1$$

$(-1;1) \subset \Gamma_t \subset [-1;1]$

per $t = -1$ TERMINE m-ESIMO INFINITESIMO
TERMINI DI SEGNO ALTERNATO
TERMINI DECRESCENTI] per LIEBNITZ la SERIE CONVERGE

per $t = 1$ $\quad \sum \frac{1}{(m+1)!} - \underbrace{\sum \frac{1}{m}}_{\text{DIVERGE}}$ la SERIE DIVERGE

$$\Longrightarrow \Gamma_t = [-1;1)$$



$2x - x^2 = -1$

$x^2 - 2x - 1 = 0$

$x_{1,2} = \frac{2 \pm \sqrt{4+4}}{2} = 1 \pm \sqrt{2}$

$y = 2x - x^2 = x(2-x)$

$\Gamma_x = [1-\sqrt{2}; 1) \cup (1; 1+\sqrt{2}]$

la SERIE CONVERGE PUNTUALMENTE IN OGNI pto di Γ_x
la SERIE CONVERGE UNIFORMEMENTE IN OGNI INTERVALLO CHIUSO $[a;b]$ COMPRESO IN Γ_x

SOMMA

$$\underbrace{\sum_{m=1}^{+\infty} \frac{1}{(m-1)!} t^m}_{A} \quad \underbrace{-\sum_{m=1}^{+\infty} \frac{t^m}{m}}_{B}$$

$\underline{\underline{A}} \quad t \sum_{m=1}^{+\infty} \frac{t^{m-1}}{(m-1)!} = t e^t$

$\underline{\underline{B}} \quad \ln(1-t)$

$$S_t = t e^t + \ln(1-t) = \boxed{(2x - x^2) e^{2x-x^2} + \ln(1-2x+x^2)}$$

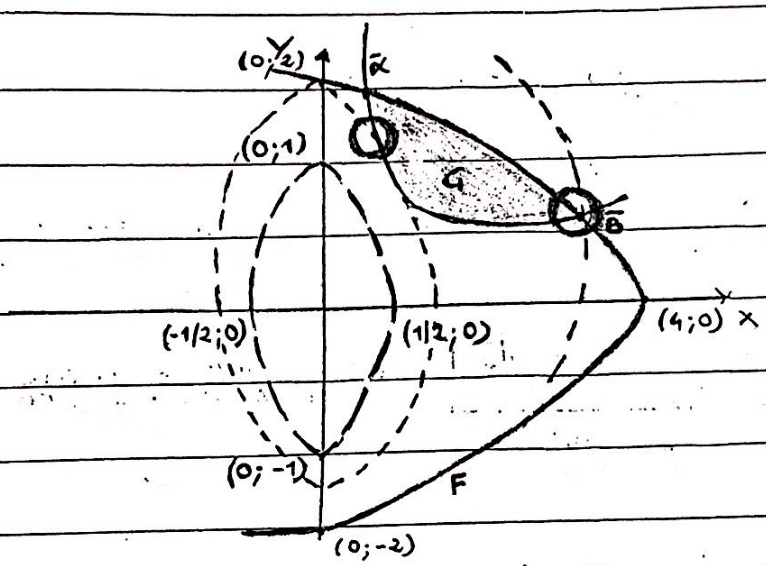
$\boxed{4}$ $h(x,y) = \ln(4x^2 + y^2 - 1)$

$F = \{(x,y) \in \mathbb{R}^2 \mid x + y^2 - 4 = 0\}$ $\quad x = 4 - y^2$

$G = \{(x,y) \in \mathbb{R}^2 \mid x + y^2 - 4 \leq 0, \quad x \cdot y \geq 1\}$

$A = \{(x,y) \in \mathbb{R}^2 \mid 4x^2 + y^2 - 1 > 0\}$

$\Longrightarrow$ MRC $\quad 4x^2 + y^2 - 1 = 0 \iff 4x^2 + y^2 = 1$



$$\max_{\substack{\min \\ F}} h(x,y) \equiv \max_{\substack{\min \\ F\cap A}} h(x,y) \equiv \max_{\substack{\min \\ F\cap A}} 4x^2 + y^2 - 1$$

$$\equiv \max_{\substack{\min \\ y^2 = 4-x}} 4x^2 + y^2 - 1 \equiv \max_{\min} 4x^2 + 4 - x - 1$$

$\longrightarrow 8x - 1 = 0 \iff x = 1/8$

$\left(1/8; \frac{\sqrt{62}}{4}\right)$

$\left(1/8; -\frac{\sqrt{62}}{4}\right)$

1/8

pti di MINIMO ASSOLUTO

POICHÈ h è ILLIMITATA SUPERIORMENTE NON CI SONO pti di MASSIMO ASSOLUTO

G chiuso e LIMITATO; f CONTINUA su G $\Longrightarrow$ per WEIERSTRASS C'È UN MASSIMO e UN MINIMO

$$\max_{\substack{\min \\ G}} h(x,y) \equiv \max_{\substack{\min \\ G}} 4x^2 + y^2 - 1$$

pti INTERNI: $g'_x = 8x = 0 \iff x = 0$

$g'_y = 2y = 0 \iff y = 0$

$(0;0) \notin A$
$(0;0) \notin G$ $\rightarrow$ NON CI SONO pti di OTTIMO INTERNI

pti di FRONTIERA:

A: $\max_{\substack{\min \\ xy=1}} 4x^2 + y^2 - 1 \equiv \max_{\min} 4x^2 + \frac{1}{x^2} - 1$

$$8x - \frac{2}{x^3} = \frac{8x^4 - 2}{x^3} = 0 \iff 8x^4 - 2 = 0 \iff x^4 = \frac{2}{8} = \frac{1}{4}$$

$$\iff \boxed{x = \frac{1}{\sqrt{2}} \quad y = \sqrt{2}}$$

$x = -\frac{1}{\sqrt{2}}$ N.A.

dalle CURVE di LIVELLO, pto di MINIMO ASSOLUTO

B: $\max_{\substack{\min \\ y^2 = 4-x}} 4x^2 + y^2 - 1$ $\left(1/8;\ \frac{\sqrt{62}}{4}\right)$ NON è NÈ MASSIMO NÈ MINIMO

$$\begin{cases} y^2 + x - 4 = 0 \\ xy = 1 \end{cases} \qquad y^2 + \frac{1}{y} - 4 = 0$$

$$\frac{y^3 + 1 - 4y}{y} = 0$$

$\rightarrow$ $\boxed{\bar{\beta} \text{ pto MASSIMO ASSOLUTO}}$

$\boxed{\left(\bar{\beta};\ \frac{1}{\bar{\beta}}\right) \text{ pto di MASSIMO ASSOLUTO}}$

dalle linee di livello