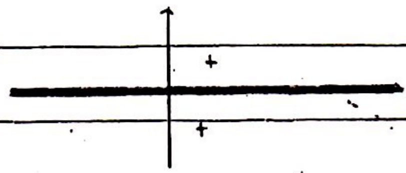
COMPITO del 6/02/2014

$\boxed{1}$ $$f(x,y) = e^{\sqrt{y^2(x^2+4y^2-4)}} \cdot \frac{x+y^2-4}{\ln(x^2-4y^2)}$$

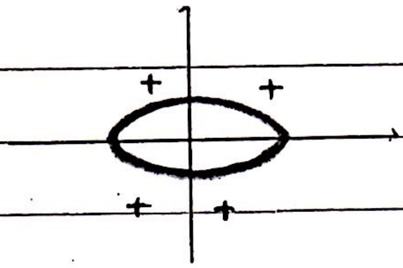
$$A = \{ (x,y) \in \mathbb{R}^2 \mid y^2(x^2+4y^2-4) \geq 0, \quad x^2-4y^2>0, \quad x^2-4y^2 \neq 1 \}$$

- $f_1(x,y) = y^2$
  $f_1 \in \mathcal{C}(\mathbb{R}^2)$ per criterio collegamento $\longrightarrow$ MRC



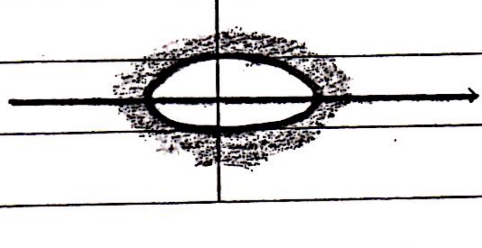
- $f_2(x,y) = x^2+4y^2-4$

$f(x) = x^2 \in \mathcal{C}(\mathbb{R})$ per criterio collegamento
$(x,y) \longrightarrow x^2 \in \mathcal{C}(\mathbb{R}^2)$
$f(y) = 4y^2 \in \mathcal{C}(\mathbb{R})$ per criterio collegamento
$(x,y) \longrightarrow 4y^2 \in \mathcal{C}(\mathbb{R}^2)$
$(x,y) \longrightarrow 4 \in \mathcal{C}(\mathbb{R}^2)$ per criterio collegamento
per criterio somma $(x,y) \longrightarrow x^2+4y^2-4 \in \mathcal{C}(\mathbb{R}^2)$
$\Longrightarrow$ MRC $\quad x^2+4y^2-4=0$

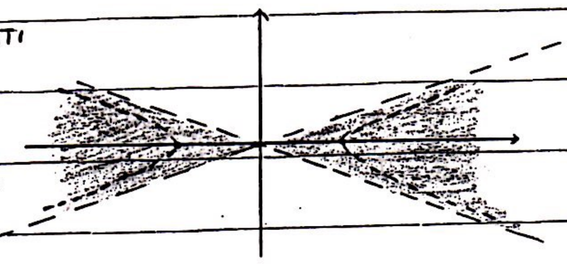


$f(0;0) = -4 < 0$
$f(0;2) = 12 > 0$

- $f_1 \cdot f_2 \geq 0$



- $f_3(x,y) = x^2-4y^2$
  $f_3 \in \mathcal{C}(\mathbb{R}^2)$ per soliti criteri $\longrightarrow$ MRC
  ~~$x^2-4y^2=0$~~ $\Longleftrightarrow$
  $y = \pm x/2$
  $f(0;2) = f(0;-2) < 0$
  $f(2;0) = f(-2;0) = 4 > 0$



- $f_4(x,y) = x^2-4y^2$
  $x^2-4y^2 = 1$

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ANALISI MATEMATICA

6/2/2014

(1) a) Data la funzione $f(x,y) = e^{\sqrt{y^2(x^2+4y^2-4)}} \quad \frac{x+y^2-4}{\ln(x^2-4y^2)}$

studiare l'insieme di definizione e il segno di f;

(b) rappresentare in grafico riportato le linee di livello zero e la frontiera dell'insieme di definizione di f.

(c) Si scriva, se esiste, il polinomio di Taylor del II° ordine della funzione $g(x,y) = e^{(xy-y^2)}$ relativamente al punto $(1,1)$. Determinare l'equazione del piano tangente al grafico di g in corrispondenza di tale punto.

(2) Si calcoli l'integrale $\iint_{F\cup G} \left(2 + x(y+e^{2y})\right) dx\,dy$

ove $F=\{(x,y)\in\mathbb{R}^2 \mid y-x\le 2,\ y\ge 0,\ x\le 0\}$, $G=\{(x,y)\in\mathbb{R}^2 \mid x^2+y^2-2y\le 0\}$

(3) Data la serie di funzioni

$$\sum_{n=1}^{+\infty}\left(\frac{1}{n(n+1)}-\frac{1}{2^n}\right)\left(\frac{x}{3x+1}\right)^n$$

(a) se ne studi la convergenza puntuale ed uniforme;

(b) se ne calcoli la somma.

(4) Data la funzione $h(x,y)=\sqrt{2y-x}$

(a) studiarne i massimi e minimi nella regione

$D=\{(x,y)\in\mathbb{R}^2 \mid y=\sin x,\ x\in[0,2\pi]\}$;

(b) studiarne i massimi e minimi nella regione

$E=\{(x,y)\in\mathbb{R}^2 \mid y\ge 0,\ y-\sin x\le 0\}$

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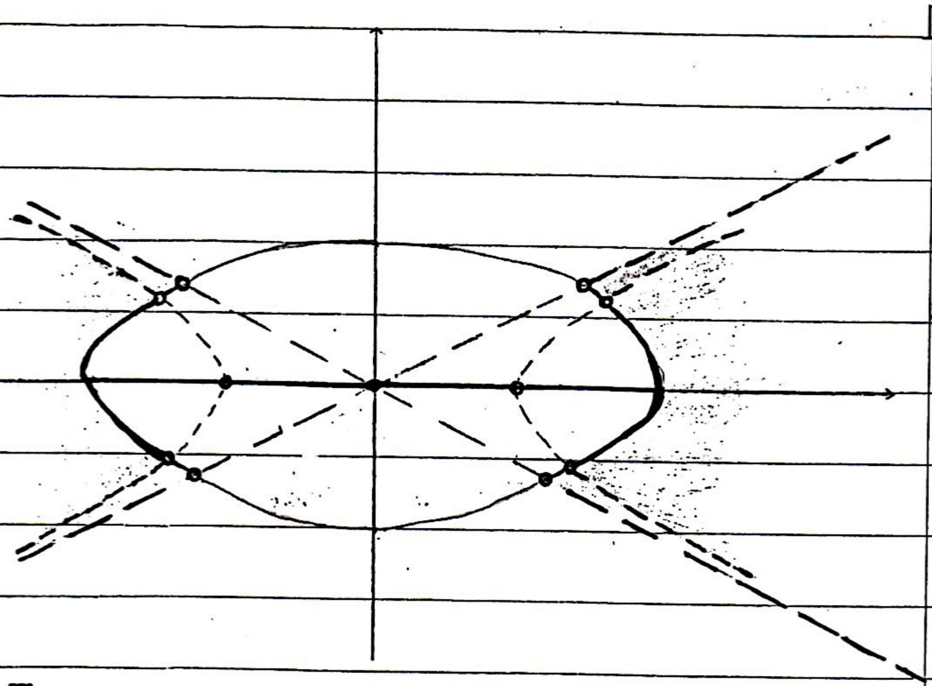
DOMINIO:

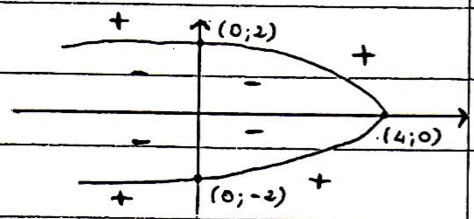
FRONTIERA



$\sqrt{\cdot}$ è DOVE ESISTE è SEMPRE POSITIVA

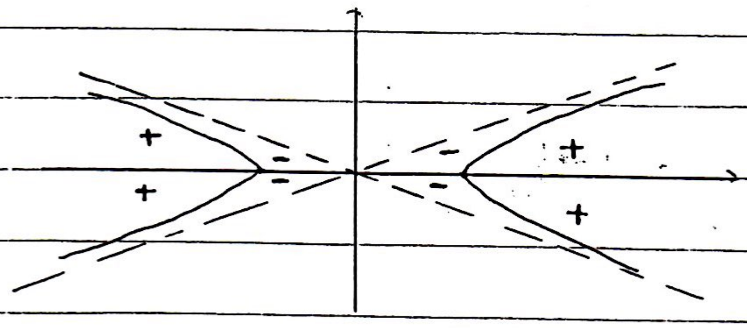
- $f_5(x,y) = x + y^2 - 4$
  $f_5 \in \mathcal{C}(\mathbb{R}^2) \Longrightarrow$ MRC
  $x + y^2 - 4 = 0 \iff x = 4 - y^2$

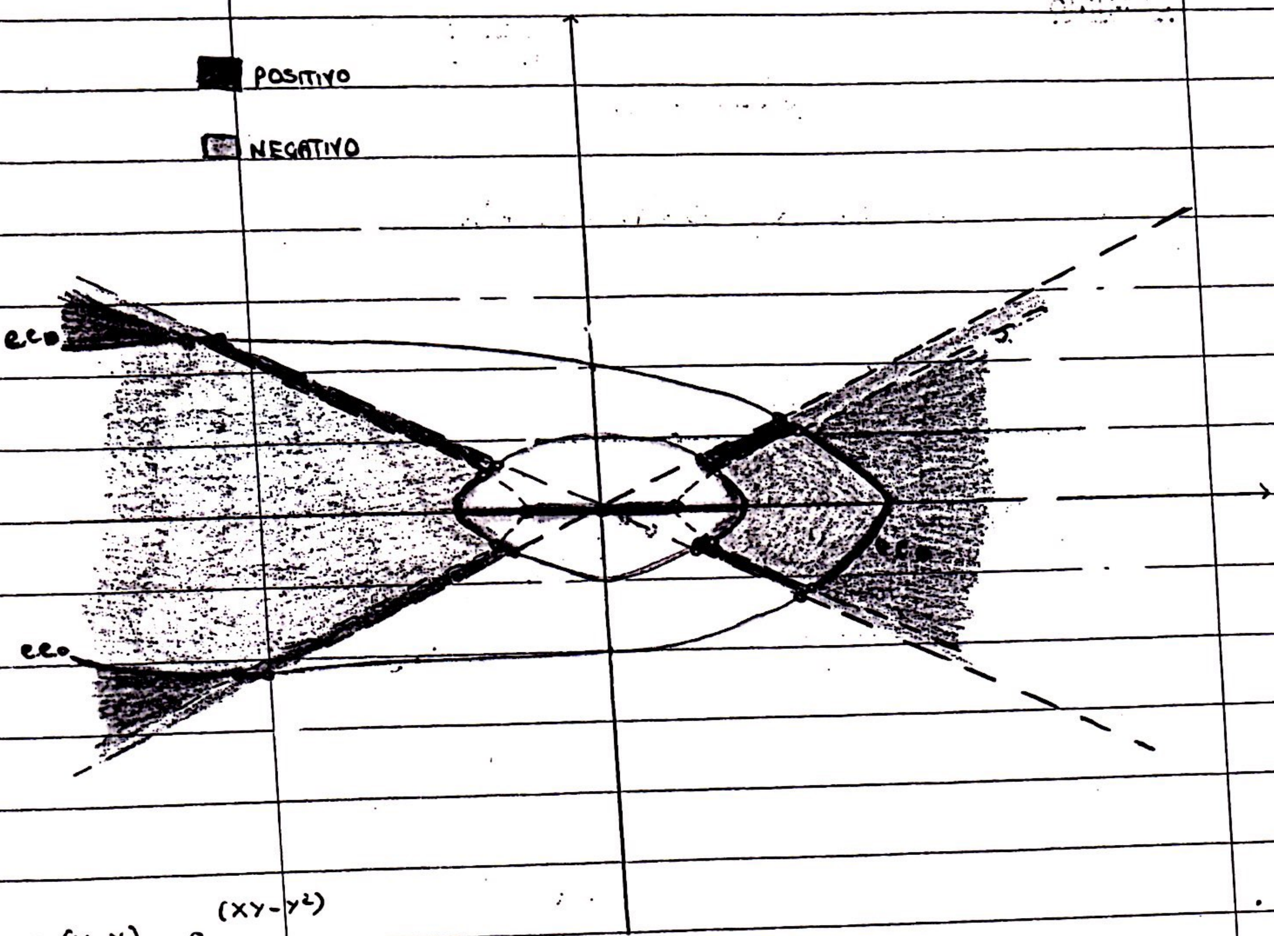
  $f(0;0) = -4 < 0$
  $f(5;0) = 1 > 0$



- $f_6(x,y) = \ln(x^2 - 4y^2) \in \mathcal{C}(A)$ per CRITERIO COLLEGAMENTO, SOMMA e COMPOSTA
  $\Longrightarrow$ MRC $\ln(x^2 - 4y^2) = 0 \iff x^2 - 4y^2 = 1$

  $f_6(0.5;0) < 0$
  $f_6(-0.5;0) < 0$
  $f_6(2;0) = f_6(-2;0) > 0$





$g(x,y) = e^{(xy-y^2)}$

AFFINCHÈ ESISTA IL POLINOMIO dell II° ORDINE g DEVE ESSERE DIFFERENZIABILE 2 VOLTE nel pto (1;1)
PER IL Th del DIFFERENZIALE TOTALE $g^{k}_{x,y}$ DEVE ESISTERE IN UN INTORNO del pto ed essere CONTINUA nel pto

$g'_x = e^{(xy-y^2)} y \cdot 1$

$g'_y = e^{(xy-y^2)} (x-2y) \cdot -1$

$g'_{xx} = y e^{(xy-y^2)} y$

$g'_{xy} = 1 e^{(xy-y^2)} y(x-2y) + e^{(xy-y^2)} \cdot 0$

$g'_{yx} = e^{(xy-y^2)} y(x-2y) + e^{(xy-y^2)} \cdot 0$

$g'_{yy} = e^{(xy-y^2)} (x-2y)^2 - 2 e^{(xy-y^2)} \cdot -1$

SIAMO nelle IPOTESI del DIFFERENZIALE TOTALE.

$$g(x,y) = g(1,1) + dg\big|_{(1;1)} + \frac{1}{2} d^2 g\big|_{(1;1)} + \varepsilon(x,y) \cdot \|x-1, y-1\|^2$$

$$p(x) = 1 + \langle \nabla g(1,1), (x-1, y-1) \rangle + \frac{1}{2}\left[g'_{xx}(1;1)(x-1)^2 + g'_{yy}(1;1)(y-1)^2 + 2g'_{xy}(1;1)(x-1)(y-1) \right]$$

$$\boxed{p(x) = 1 + (x-1) - (y-1) + \frac{1}{2}(x-1)^2 - \frac{1}{2}(y-1)^2}$$

EQ. PIANO TANGENTE $z = h(x,y) = \langle \nabla g(1,1), (x-1, y-1) \rangle$

$$\boxed{z = x - y}$$

$\boxed{3}$

$$\sum_{n=1}^{+\infty} \left[\frac{1}{n(n+1)} - \frac{1}{2^n}\right] \left(\frac{x}{3x+1}\right)^n$$

$$t = \frac{x}{3x+1} \longrightarrow \sum_{n=1}^{+\infty} \left[\frac{1}{n(n+1)} - \frac{1}{2^n}\right] t^n$$

$$\lim_{n \to +\infty} \frac{\frac{1}{(n+1)(n+2)} - \frac{1}{2^{n+1}}}{\frac{1}{(n+1)n} - \frac{1}{2^n}} \overset{\text{P.S. degli INFINITESIMI}}{=} \lim_{n \to +\infty} \frac{(n+1)n}{(n+1)(n+2)} = 1$$

$\rho = 1 \qquad (-1;1) \subset \Gamma_t \subset [-1;1]$

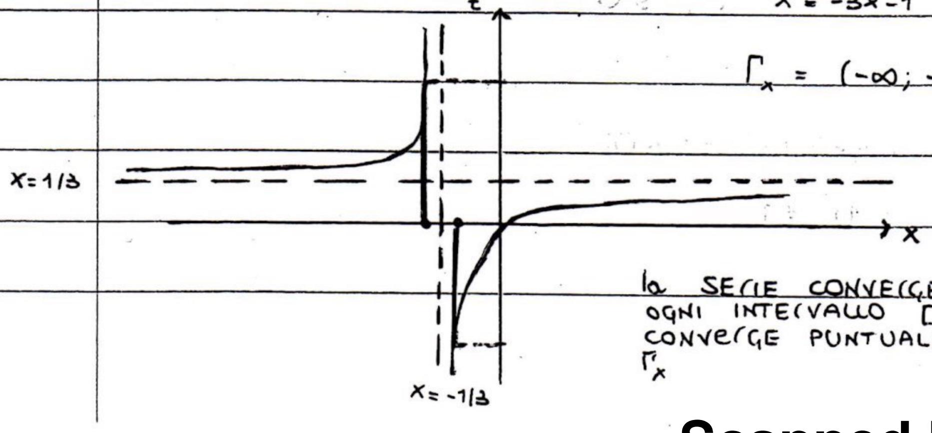
per $t = -1$ TERMINE n-ESIMO INFINITESIMO, TERMINI A SEGNO ALTERNATO, TERMINI DECRESCENTI — per LEIBNITZ la serie converge

per $t = 1$

$$\underbrace{\sum_{n\geq 1} \frac{1}{n(n+1)}}_{\text{SERIE di MENGOLI CONVERGE}} - \underbrace{\sum_{n \geq 1}\left(\frac{1}{2}\right)^n}_{\text{SERIE GEOMETRICA di RAGIONE 1/2 (ESCLUSO IL PRIMO TERMINE)} \longrightarrow \text{CONVERGE}} \quad \text{la SERIE CONVERGE}$$

$\longrightarrow \Gamma_t = [-1; 1]$

$x = 3x+1 \iff x = -1/2$
$x = -3x-1 \iff x = -1/4$



$\Gamma_x = (-\infty; -1/2] \cup [-1/4; +\infty)$

la SERIE CONVERGE UNIFORMEMENTE IN OGNI INTERVALLO $[a;b]$ CONTENUTO IN Γ_x; CONVERGE PUNTUALMENTE IN OGNI pto di Γ_x

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SOMMA:

$$\sum_{m\geq 1}\left[\frac{1}{m(m+1)}-\frac{1}{2^m}\right]t^m = \sum_{m\geq 1}\left[\frac{1}{m}-\frac{1}{m+1}-\frac{1}{2^m}\right]t^m =$$

$$\underbrace{\sum_{m\geq 1}\frac{t^m}{m}}_{A} - \underbrace{\sum_{m\geq 1}\frac{t^m}{m+1}}_{B} - \underbrace{\sum_{m\geq 1}\left(\frac{t}{2}\right)^m}_{C}$$

$\underline{\underline{A}}$: $\displaystyle\sum_{m\geq 1}\frac{t^m}{m} = -\ln(1-t)$

$\underline{\underline{B}}$: $\displaystyle\sum_{m\geq 1}\frac{t^m}{m+1} = \frac{1}{t}\sum_{m\geq 1}\frac{t^{m+1}}{m+1} = \frac{1}{t}\Big[-\ln(1-t)-t\Big] = -\frac{\ln(1-t)}{t}-1$

$\underline{\underline{C}}$: $\displaystyle\sum_{m\geq 1}\left(\frac{t}{2}\right)^m = \frac{1}{1-t/2}-1 = \frac{2}{2-t}-1 = \frac{t}{2-t}$

$$\boxed{S_t = -\ln(1-t)+\frac{\ln(1-t)}{t}+1+\frac{t}{t-2}}$$

per la SOMMA IN X SOSTITUIRE: $t = \dfrac{x}{3x+1}$

$\boxed{2}$ $\displaystyle\iint_{F\cup G}\left(2+x\left(y+e^{2y}\right)\right)dx\,dy$

$F = \{(x,y)\in\mathbb{R}^2 \mid y-x\leq 2,\ y\geq 0,\ x\leq 0\}$

$G = \{(x,y)\in\mathbb{R}^2 \mid x^2+y^2-2y\leq 0\}$

$$x^2+(y^2-2y+1)-1\leq 0$$

$$x^2+(y-1)^2\leq 1$$

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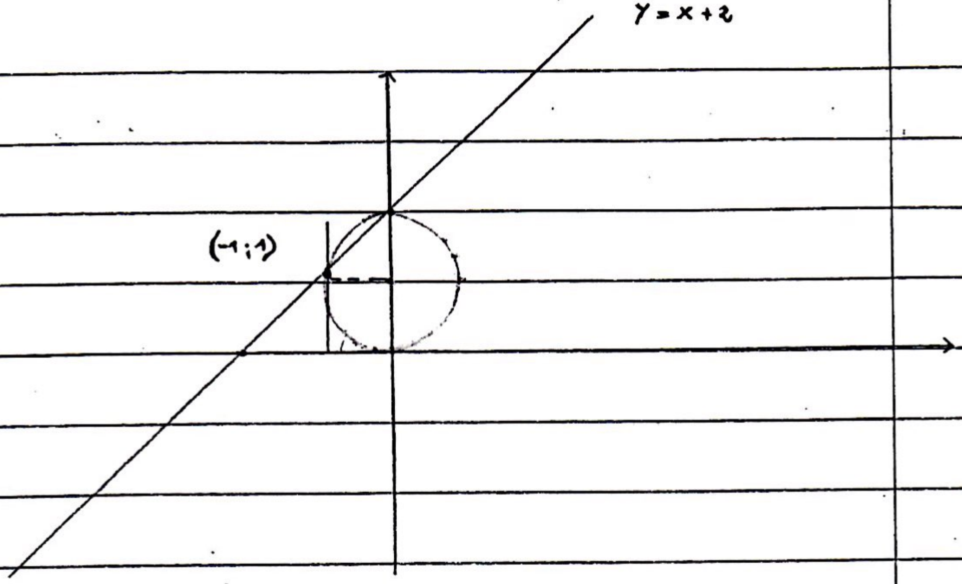

$$\iint_{F\cup G} h(x,y)\,dx\,dy = \underbrace{\iint_{F\cup G} 2\,dx\,dy}_{A} + \underbrace{\iint_{F\cup G} xy\,dx\,dy}_{B} + \underbrace{\iint_{F\cup G} x e^{2y}\,dx\,dy}_{C}$$

$\underline{\underline{C}}$: $\iint_{F\cup G} x e^{2y}\,dx\,dy$ G SIMMETRICO RISPETTO ASSE Y

$f(-x,y) = -x\,e^{2y} = -f(x,y)$ F è DISPARI

$\Longrightarrow \iint_G$ è NULLO.

$$\left[\begin{array}{l} x = \rho\cos\theta \\ y = 1 + \rho\,\mathrm{sen}\,\theta \end{array}\right.$$

$$= \iint_{\triangleleft} x e^{2y}\,dx\,dy + \iint_{\square} x e^{2y}\,dx\,dy - \iint_{\triangledown} x e^{2y}\,dx\,dy$$

$$= \int_0^1 e^{2y}\left[\int_{y-2}^{-1} x\,dy\right] dx + \int_{-1}^{0} x\left[\int_0^1 e^{2y}\,dy\right] dx -$$

$$\iint_{\triangledown} \rho^2\cos\theta\, e^{2(1+\rho\,\mathrm{sen}\,\theta)}\,d\theta\,d\rho =$$

$$= \int_0^1 e^{2y}\left[\frac{x^2}{2}\right]_{y-2}^{1} + \int_{-1}^{0} x\left[\frac{e^{2y}}{2}\right]_0^1 - \int_0^1 \rho\left[\frac{e^{2(1+\rho\,\mathrm{sen}\,\theta)}}{2}\right]_{\pi}^{\frac{3}{2}\pi} =$$

$$= \int_0^1 \frac{e^{2y}}{2} - \frac{y e^{2y}}{2} + 2ye^{2y} - 2e^{2y}\,dy + \left(\frac{e^2}{2} - \frac{1}{2}\right)\left|\frac{x^2}{2}\right|_{-1}^{0} - \int_0^1 \rho\left(\frac{e^{2(1+\rho)}}{2} - \frac{e^2}{2}\right) d\rho$$

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• $\int_0^1 \frac{-3e^{2y}}{2}dy - \int_0^1 \frac{y^2e^{2y}}{2}dy + \int_0^1 2ye^{2y}dy = \boxed{-\frac{1}{8}e^2+\frac{9}{8}}$

(1) $-\frac{3}{2}\left[\frac{e^{2y}}{2}\right]_0^1 = -\frac{3}{2}\left(\frac{e^2}{2}-\frac{1}{2}\right) = -\frac{3e^2}{4}+\frac{3}{4}$

(2) $\frac{1}{2}\left[y^2\frac{e^{2y}}{2} - \int 2y\frac{e^{2y}}{2}dy\right]_0^1 = \frac{1}{2}\left[y^2\frac{e^{2y}}{2} - \frac{1}{2}ye^{2y} + \int \frac{e^{2y}}{2}dy\right]_0^1 =$

$= \frac{1}{2}\left[y^2\frac{e^{2y}}{2} - \frac{1}{2}ye^{2y} + \frac{e^{2y}}{4}\right]_0^1 = \frac{1}{2}\left(\frac{e^2}{2}-\frac{e^2}{2}+\frac{e^2}{4}-\frac{1}{4}\right)$

$= \frac{e^2}{8}-\frac{1}{8}$

(3) $\left[e^{2y}\cdot y - \int e^{2y}dy\right]_0^1 = \left[ye^{2y}-\frac{e^{2y}}{2}\right]_0^1 = e^2-\frac{e^2}{2}+\frac{1}{2}$

$= \frac{e^2}{2}+\frac{1}{2}$

• $\left(\frac{e^2}{2}-1/2\right)\left|x^2/2\right|_{-1}^0 = \boxed{-\frac{e^2}{8}+1/4}$

• $\int_0^1 \rho\left(\frac{e^{2(1+\rho)}}{2}-\frac{e^2}{2}\right)d\rho = \frac{1}{2}\int_0^1 \rho e^{2(1+\rho)}d\rho - \frac{1}{2}e^2\int_0^1\rho\, d\rho$

(1) $\frac{1}{2}\left[\rho\frac{e^{2(1+\rho)}}{2} - \int\frac{e^{2(1+\rho)}}{2}d\rho\right]_0^1 = \frac{1}{2}\left[\rho\frac{e^{2(1+\rho)}}{2}-\frac{e^{2(1+\rho)}}{4}\right]_0^1 =$

$\frac{1}{2}\left(\frac{e^4}{2}-\frac{e^4}{4}+\frac{e^2}{4}\right) = \frac{e^4}{8}+\frac{e^2}{8}$

(2) $\frac{e^2}{2}\left|\frac{\rho^2}{2}\right|_0^1 = \frac{e^2}{4}$

$\Longrightarrow \boxed{\frac{e^4}{8}-\frac{1}{8}e^2}$

$\underline{\underline{C}}:\ -\frac{e^2}{8}+\frac{9}{8}-\cancel{\frac{e^2}{8}}+\frac{1}{4}-\frac{e^4}{8}+\cancel{\frac{e^2}{8}} = \boxed{\frac{11}{8}-\frac{e^2}{8}-\frac{e^4}{8}}$

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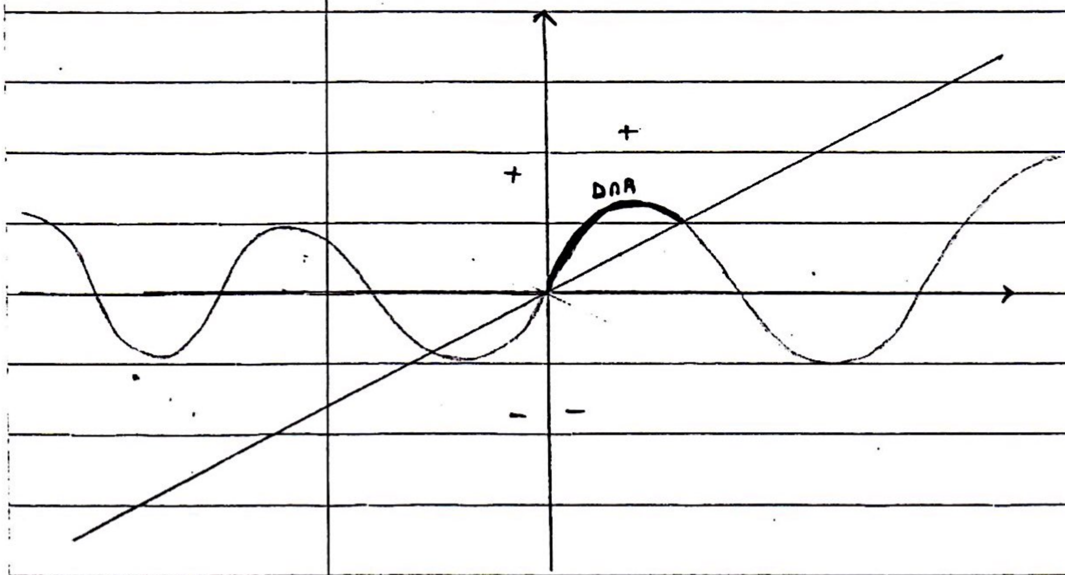
$\boxed{4}$ $h(x,y) = \sqrt{2y - x}$

$D = \{(x,y) \in \mathbb{R}^2 \mid y = \text{sen}\, x \quad x \in [0; 2\pi]\}$

$A = \{(x,y) \in \mathbb{R}^2 \mid 2y - x \geq 0\}$



$$\max_{D}\min h(x,y) \equiv \max_{D \cap A}\min\, 2y - x \equiv \max_{\substack{y = \text{sen}\, x \\ 0 \leq x \leq \bar{\alpha}}}\min\, 2y - x$$

$$\equiv \max_{0 \leq x \leq \bar{\alpha}}\min\, 2\,\text{sen}\, x - x \longrightarrow 2\cos x - 1 = 0 \iff \cos x = 1/2 \iff x = 60°$$

$\nearrow \ \frac{\pi}{3} \ \searrow$

$(\pi/3 ; \sqrt{3}/2)$ pto di MASSIMO ASSOLUTO sulla restrizione

$(0;0)$ pto di MINIMO ASSOLUTO sulla restrizione

$(\bar{\alpha} ; \text{sen}\,\bar{\alpha})$ pto di MINIMO ASSOLUTO sulla restrizione

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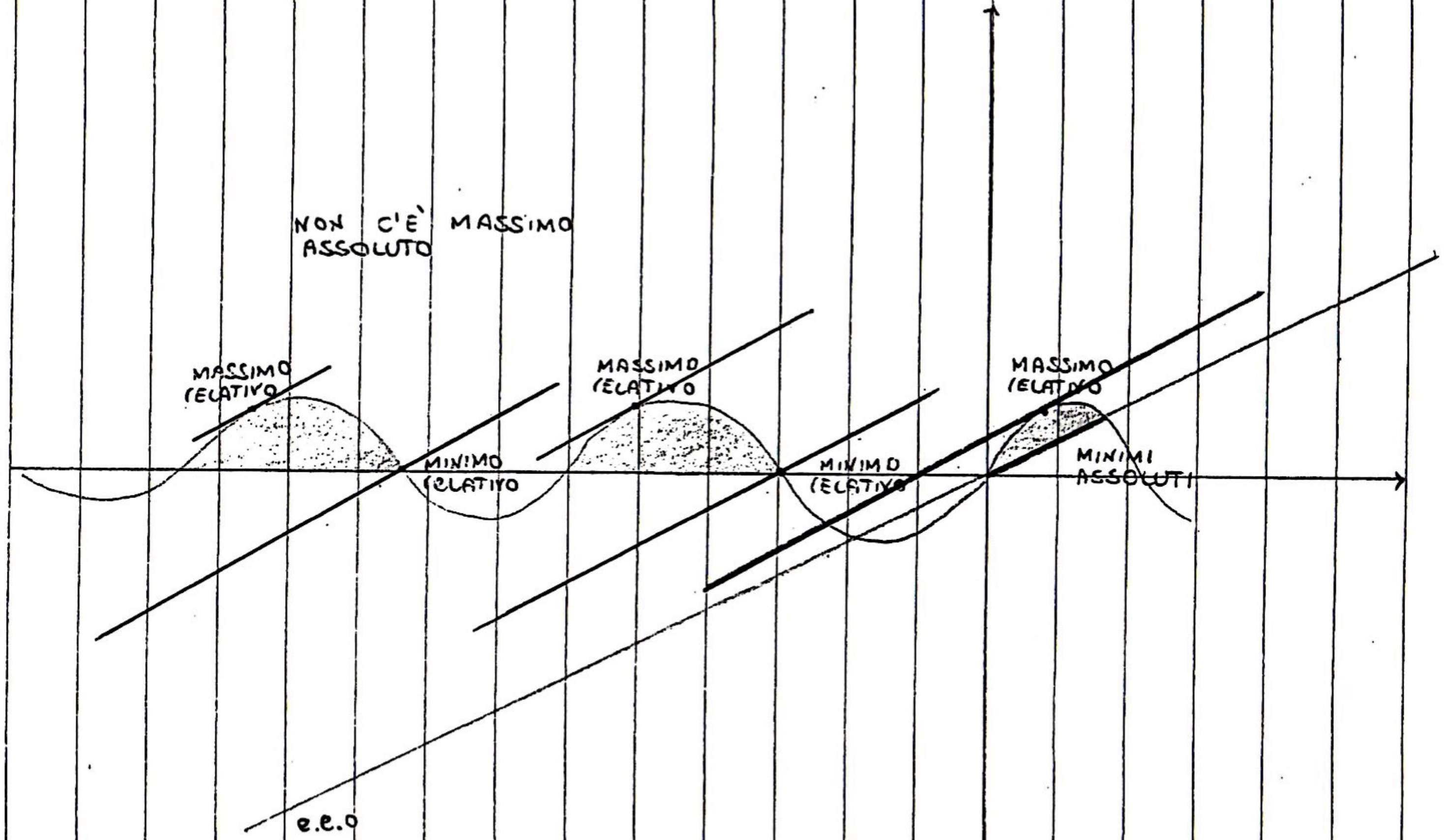

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$$\underline{\underline{B}}: \iint_{F \cup G} xy \, dx \, dy = \iint_{\triangle} xy \, dx \, dy + \iint_{\square} xy \, dx \, dy - \iint_{◜} xy \, dx \, dy$$

(1) $$\int_0^1 y \left[\int_{y-2}^{-1} x \, dx \right] dy = \int_0^1 y \left[\frac{x^2}{2} \right]_{y-2}^{-1} dy =$$

$$\int_0^1 y \left[\frac{1}{2} - \frac{y^2 - 4y + 4}{2} \right] = \frac{1}{2} \int_0^1 -y^3 + 4y^2 - 3y \, dy$$

$$= \frac{1}{2} \left[-\frac{y^4}{4} + \frac{4y^3}{3} - \frac{3y^2}{2} \right]_0^1 = \boxed{-5/24}$$

(2) $$\int_0^1 y \left[\int_{-1}^{0} x \, dx \right] dy = \int_0^1 y \left[\frac{x^2}{2} \right]_{-1}^{0} = -\frac{1}{2} \int_0^1 y \, dy = \boxed{-\frac{1}{4}}$$

(3) $$\begin{cases} x = \rho \cos \theta \\ y = 1 + \rho \operatorname{sen} \theta \end{cases} \qquad \begin{array}{l} \rho \in [0;1] \\ \theta \in [\pi; 3/2\pi] \end{array}$$

$$\iint \rho^2 \cos\theta \, (1 + \rho \operatorname{sen}\theta) \, d\theta \, d\rho = \iint \rho^2 \cos\theta \, d\theta \, d\rho + \iint \rho^3 \cos\theta \operatorname{sen}\theta$$

$$= \int_0^1 \rho^2 \left[\int_\pi^{3/2\pi} \cos\theta \, d\theta \right] d\rho + \int_0^1 \rho^3 \left[\int_\pi^{3/2\pi} \cos\theta \operatorname{sen}\theta \, d\theta \right] d\rho =$$

$$- \left[\frac{\rho^3}{3} \right]_0^1 + \int_0^1 \rho^3 \left[\frac{\operatorname{sen}^2\theta}{2} \right]_\pi^{3/2\pi} = -\frac{1}{3} + \frac{1}{4} = \boxed{-1/12}$$

$\underline{\underline{B}}$: $\boxed{-3/8}$

$$\underline{\underline{A}}: \quad 2 \, m(F \cup G) = 2 \left[\pi + \frac{1}{2} + 1 - \frac{\pi}{4} \right] = \boxed{\frac{3}{2}\pi + 3}$$

$$\iint_{F \cup G} h(x,y) \, dx \, dy = \boxed{4 - \frac{e^2}{8} - \frac{e^4}{8} + \frac{3\pi}{2}}$$

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