PROVA SCRITTA DI ANALISI

(APPELLO DEL 12/6/2013)

1) Data la funzione

$$f(x,y) = \frac{\ln(4y^2-x^2)}{x-y}\sqrt{(x+y)^2(16-4y^2-x^2)}$$

(a) studiare l'insieme di definizione di f e il suo segno;

(b) rappresentare in grafico separato la linea di livello zero e la frontiera dell'insieme di definizione di f;

(c) studiare i limiti di f nei punti $(-4,0)$ $(-4,4)$, $(1,1)$, $(2\sqrt{2},\sqrt{2})$, $(0,0)$

(d) Si calcoli la derivata direzionale, se esiste, della funzione $g(x,y) = \dfrac{\ln(x^2-3y^4)}{x-y}$ nel punto $(2,1)$ lungo la direzione ortogonale alla retta di equazione $x+2y+7=0$; si scriva poi l'equazione del piano tangente al grafico di g in corrispondenza di tale punto.

2) Data la funzione $h(x,y) = x^2y + x\cos 2y + \frac{1}{2}$ si calcolino gli integrali

$$\iint_E h(x,y)\,dx\,dy, \quad \iint_F h(x,y)\,dx\,dy, \quad \iint_{E\cup F} h(x,y)\,dx\,dy$$

ove $E = \{(x,y)\in\mathbb{R}^2 \mid x^2+y^2 \le 1,\ y \ge 0\}$, F = triangolo di vertici $(0,0)$, $(1,1)$, $(1,-1)$

3) (a) Studiare la convergenza puntuale ed uniforme della serie di funzioni

$$\sum_{n\ge 1} \frac{1}{n-\ln(n^2)}\left(-x^2+2x\right)^n$$

(b) Calcolare la somma della serie di potenze

$$\sum_{n\ge 3}\left(\frac{1}{n-1} + (n+1)(n+2)\right)x^n$$

4) Data la funzione $f(x,y) = \ln(xy+1)$

(a) determinare i massimi e minimi nell'insieme

$$G = \{(x,y)\in\mathbb{R}^2 \mid x+y^2-y=0\}$$

(b) determinare i massimi e minimi nell'insieme

$$F = \{(x,y)\in\mathbb{R}^2 \mid x+y^2-y\le 0,\ x-y+1\ge 0\}$$

COMPITO 10/06/2013

$\boxed{1}$ SVOLTO

$$f(x,y) = \frac{\ln(4y^2-x^2)}{x-y} \sqrt{(x+y)^2(16-4y^2-x^2)}$$

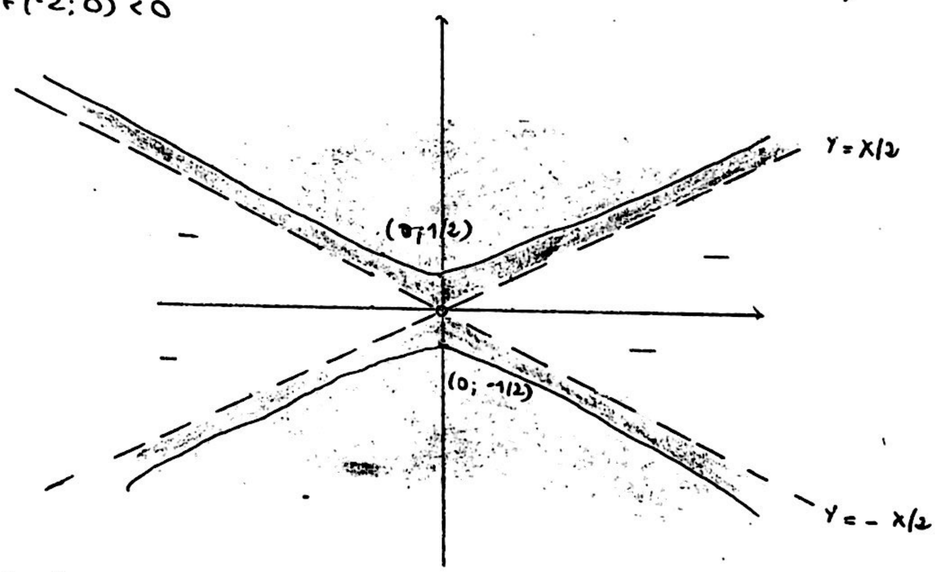
$$A = \{ (x,y) \in \mathbb{R}^2 \mid 4y^2-x^2>0,\ x \neq y,\ (x+y)^2(16-4y^2-x^2) \geq 0 \}$$

$f_1(x,y) = 4y^2-x^2$

$f: y \to 4y^2 \in \mathcal{C}(\mathbb{R})$ per criterio collegamento $\in \mathcal{C}(\mathbb{R}^2)$
$f: x \to -x^2 \in \mathcal{C}(\mathbb{R})$ per criterio collegamento $\in \mathcal{C}(\mathbb{R}^2)$
per criterio somma $f_1 \in \mathcal{C}(\mathbb{R}^2) \longrightarrow$ MRC

$4y^2-x^2=0 \iff (2y+x)(2y-x)=0 \iff y = \pm x/2$

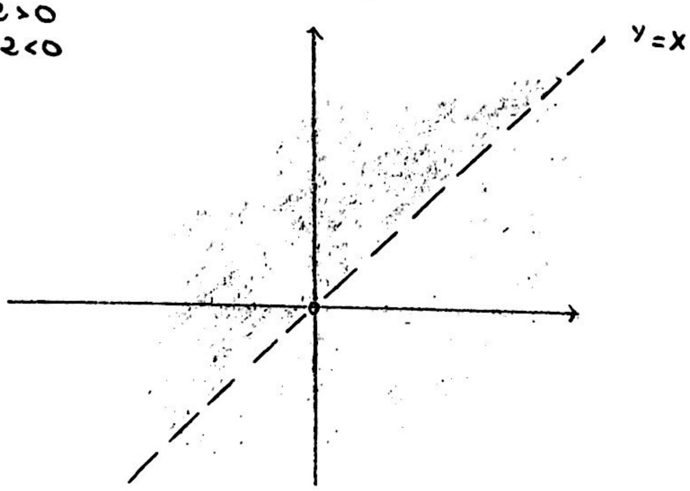
$f(0;2) = f(0;-2) > 0$
$f(2;0) = f(-2;0) < 0$



SEGNO: $4y^2-x^2=1$

$f(0;1) = \ln(4) > 0$
$f(0;-1) = \ln(4) > 0$
$f(0;1/4) = f(0;-1/4) = \ln(1/4) < 0$

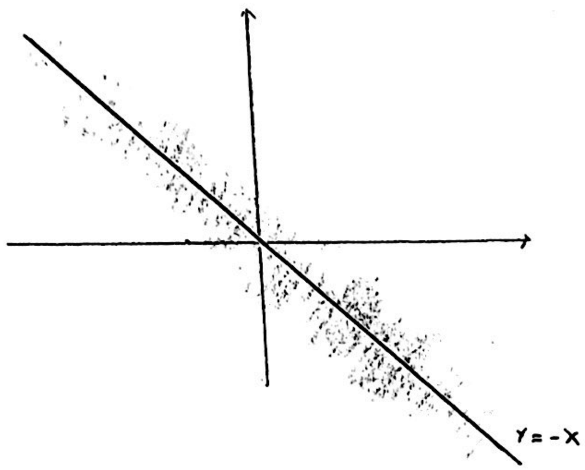
$f_2(x,y) = x-y$
$f_2 \in \mathcal{C}(\mathbb{R}^2)$ per collegamento/somma
MRC:
$f(2;0) = 2 > 0$
$f(0;2) = -2 < 0$



$f_3(x,y) = (x+y)^2$

$f_3 \in C(\mathbb{R}^2)$ per COLLEGAMENTO|SOMMA|PRODOTTO

MRC: $(x+y)^2 = 0 \iff y = -x$



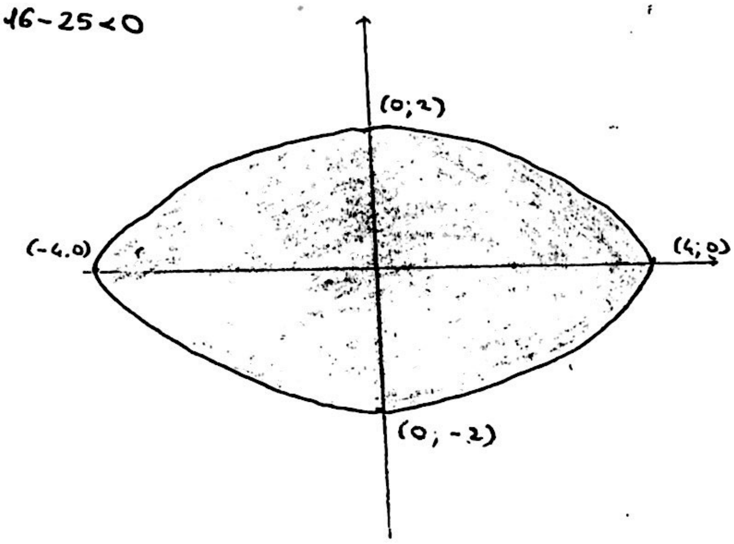
$f_4(x,y) = (16 - 4y^2 - x^2)$

$f_4 \in C(\mathbb{R}^2)$ per COLLEGAMENTO /SOMMA

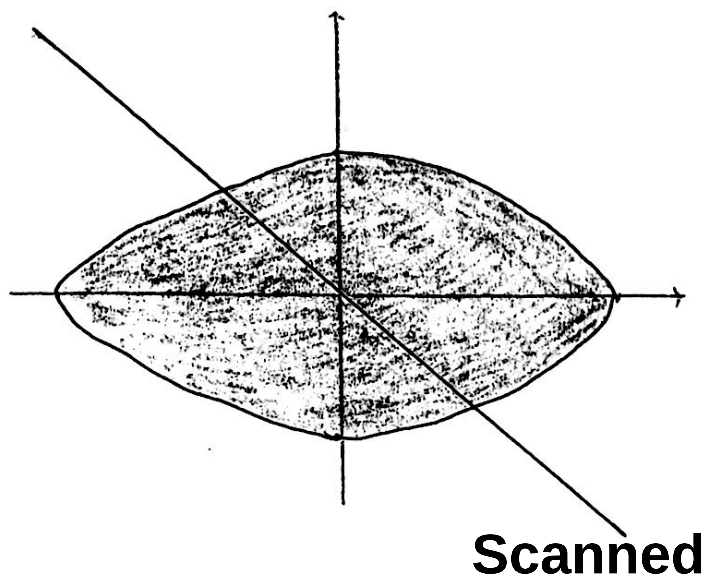
MRC: $16 - 4y^2 - x^2 = 0 \iff x^2 + 4y^2 = 16$

$f(0;0) = 16 > 0$

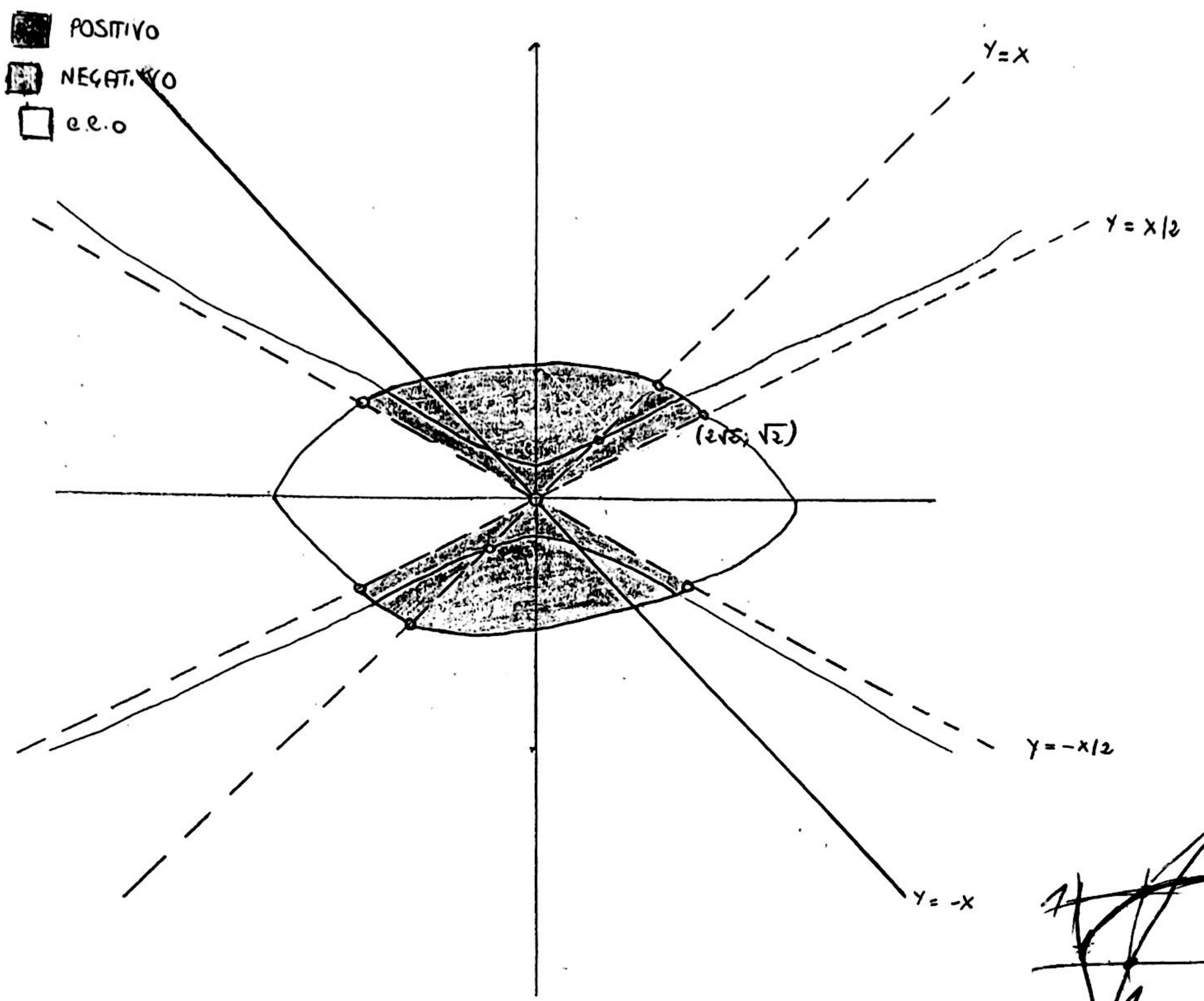
$f(5;0) = 16 - 25 < 0$

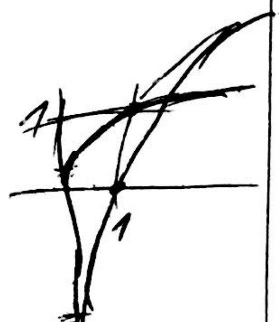


$f_3 \cdot f_4 \geq 0$



ove definita, $\sqrt{\cdot}$ è sempre ≥ 0





$(-4;0) \notin D(A)$ NON ha SENSO CALCOLARE IL LIMITE

$(-4;4) \in D(A)$ e $(-4;4) \in A$; f è CONTINUA nel SUO DOMINIO

$$\lim_{(x,y)\to(-4;4)} f(x,y) = f(-4;4) = 0$$

$(1;1) \in D(A)$

$$\lim_{(x,y)\to(1;1)} \underbrace{\left\{ \frac{\overbrace{\ln(\overbrace{4y^2 - x^2}^{\to 3})}^{\to \ln(3) \text{ PER CONTINUITÀ e LIMITE COMPOSTA}}}{\underbrace{x-y}_{\to 0 \text{ per CONTINUITÀ}}} \right.}_{\text{LIMITATA}} \underbrace{\sqrt{\underbrace{(x+y)^2(16-4y^2-x^2)}_{\to 4\cdot 13}}}_{\to \sqrt{4\cdot 13} \text{ per CONTINUITÀ e LIMITE COMPOSTA}} \Big\} \text{ LIMITATA} = \pm\infty$$

$(2\sqrt{2}; \sqrt{2}) \in D(A)$

$$\lim_{(x,y)\to(2\sqrt{2};\sqrt{2})} \frac{\overbrace{\ln(\overbrace{4y^2-x^2}^{\to 0 \text{ per CONTINUITÀ}})}^{\to -\infty \text{ LIMITE COMPOSTA}}}{\underbrace{x-y}_{\to \sqrt{2} \text{ per CONTINUITÀ}}} \Big\} \text{ LIMITATA} \; \underbrace{\sqrt{\underbrace{(x+y)^2(16-4y^2-x^2)}_{\to 0 \text{ per CONTINUITÀ}}}}_{\to 0 \text{ per LIMITE COMPOSTA}}$$

$$\lim_{(x,y)\to(2\sqrt{2};\sqrt{2})} f(x,y)\Big|_{16-4y^2=x^2} = 0$$

$\nexists$ IL LIMITE

$$\lim_{(x,y)\to(2\sqrt{2};\sqrt{2})} f(x,y)\Big|_{2y+x=0} = -\infty$$

IN MODO ANALOGO NON ESISTE IL LIMITE per $(x,y) \to (0;0)$

$$g(x,y) = \frac{\ln(x^2-3y^2)}{x-y} \qquad (2;1) \quad x+2y+7=0$$

SE $g(x,y)$ è DIFFERENZIABILE IN $(2;1)$

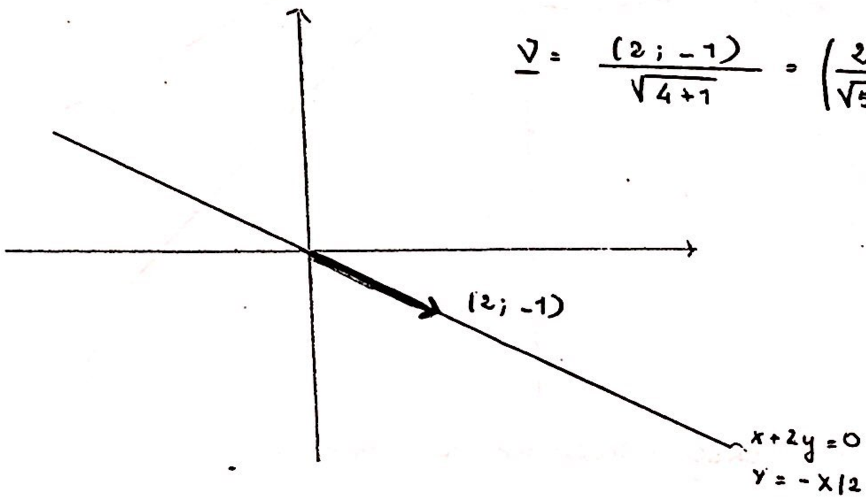
$$\frac{\partial g}{\partial \underline{v}}(2;1) = \langle \nabla g(2;1), \underline{v} \rangle$$

$$g'_x = \frac{\frac{2x(x-y)}{x^2-3y^2} - \ln(x^2-3y^2)}{(x-y)^2} = 4 = g'_x(2,1)$$

$$g'_y = \frac{\frac{-6y(x-y)}{x^2-3y^2} + \ln(x^2-3y^2)}{(x-y)^2} = -6 = g'_y(2,1)$$

ESISTONO IN UN INTORNO di $(2;1)$ CONTINUE nel pto

$\Longrightarrow$ Th DIFFERENZIALE TOTALE g è DIFFERENZIABILE



$$\underline{v} = \frac{(2;-1)}{\sqrt{4+1}} = \left(\frac{2}{\sqrt5} ; -\frac{1}{\sqrt5}\right)$$

$$\frac{\partial g}{\partial \underline{v}}(2;1) = \langle (4;6), (2/\sqrt5, -1/\sqrt5) \rangle = \frac{8}{\sqrt5} + \frac{6}{\sqrt5} = \boxed{\frac{14}{\sqrt5}}$$

POICHÈ g è DIFFERENZIABILE

$$z = f(2,1) + g'_x(2,1)(x-2) + g'_y(2,1)(y-1) =$$

$$4(x-2) - 6(y-1)$$

$$\boxed{z = 4x - 6y - 2}$$

$\boxed{3}$

$$\sum_{m\geq 1} \frac{1}{m-\ln(m^2)} (-x^2+2x)^m$$

$$t = -x^2+2x = -x(x-2)$$

$$\lim_{m\to+\infty} \frac{m-\ln(m^2)}{m+1-\ln((m+1)^2)} = \lim_{m\to+\infty} \frac{m}{m+1} = 1$$

$$(-1;1) \subset \Gamma_t \subset [-1;1]$$

per $t=1$ $\quad \underbrace{\sum_{m\geq 1} \frac{1}{m-\ln(m^2)}}_{\text{è una maggiorante della serie armonica, diverge}} \geq \underbrace{\sum_{m\geq 1} \frac{1}{m}}_{\text{diverge}}$

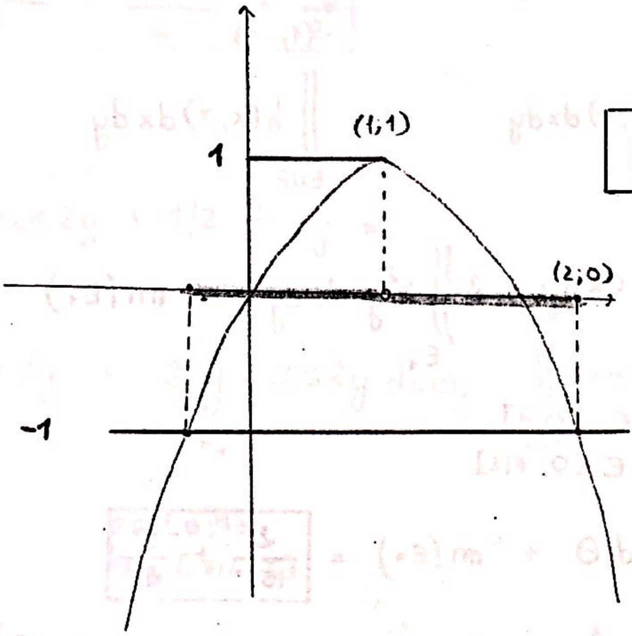
per $t=-1$ $\quad$ termini a segno alternato, termini decrescenti, termine m-esimo infinitesimo $\Big\}$ per Leibnitz la serie converge

$$\Longrightarrow \Gamma_t = [-1;1)$$

$-x^2+2x=-1$

$x^2-2x-1=0$

$x_{1,2} = \dfrac{2\pm\sqrt{4+4}}{2} = 1\pm\sqrt{2}$



$$\boxed{\Gamma_x = (1-\sqrt{2};1)\cup(1;1+\sqrt{2}]}$$

la serie converge puntualmente in ogni pto di Γ_x e converge uniformemente in ogni intervallo chiuso e limitato contenuto in Γ_x.

$$\sum_{m\geq 3}\left(\frac{1}{m-1}+(m+1)(m+2)\right)x^m = x\sum_{m\geq 3}\frac{x^{m-1}}{m-1} + \sum_{m\geq 3}\frac{d''}{dx}x^{m+2} =$$

$$x\cdot\left(-\ln(1-x)+1\right) + \frac{d''}{dx}\left(\frac{1}{1-x}\cancel{-1}\cancel{-x}-x^2-x^3-x^4\right) =$$

$$-x\ln(1-x)+x+\frac{2}{(1-x)^3}-2-6x-12x^2 =$$

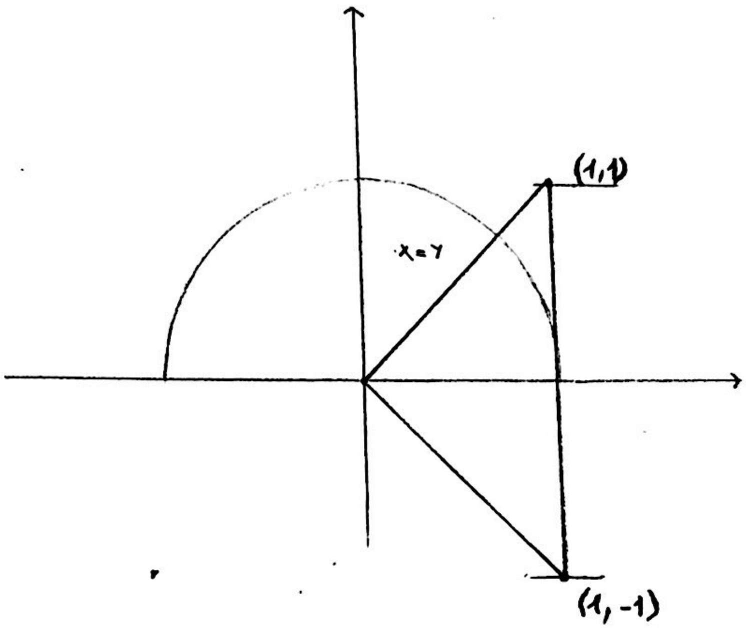
$$\boxed{\frac{2}{(1-x)^3}-x\ln(1-x)-12x^2-5x-2}$$

$\boxed{2}$

$$h(x,y) = x^2y + x\cos 2y + 1/2$$

$$E = \{ (x,y) \in \mathbb{R}^2 \mid x^2+y^2 \leq 1 \ , \ y \geq 0 \}$$

$F = \Delta \quad (0;0) \quad (1;1) \quad (1;-1)$



$$\iint_E h(x,y)\,dx\,dy \qquad \iint_F h(x,y)\,dx\,dy \qquad \iint_{E\cup F} h(x,y)\,dx\,dy$$

$$\iint_E x^2y + x\cos 2y + 1/2 \; dx\,dy = 2\iint_{E_1} x^2y \; dx\,dy + m(E_1)$$

$x = \rho\cos\theta \qquad \rho \in [0;1]$

$y = \rho \operatorname{sen}\theta \qquad \theta \in [0;\pi/2]$

$$= 2\iint_{E_1} \rho^4\cos^2\theta \operatorname{sen}\theta \; d\rho\, d\theta + m(E_1) = \boxed{\frac{2}{15} + \frac{\pi}{4}}$$

(i) $2\int_0^1 \rho^4 \left[\int_0^{\pi/2} \cos^2\theta \operatorname{sen}\theta \; d\theta \right] d\rho = 2\int_0^1 \rho^4 \left[-\frac{\cos^3\theta}{3} \right]_0^{\pi/2} =$

$$\frac{2}{3}\left[\frac{\rho^5}{5}\right]_0^1 = \frac{2}{15}$$

(18)

$$\iint_F x^2y + x\cos 2y + 1/2 \; dx\,dy = 2\iint_{F_1} x\cos 2y \; dx\,dy + m(F_1)$$

$$2\int_0^1 x\left[\int_0^x \cos 2y \; dy\right]dx + m(F_1) =$$

$$2\int_0^1 x\left[\frac{\operatorname{sen} 2y}{2}\right]_0^x dx + 1/2 =$$

$$\int_0^1 x \operatorname{sen} 2x \; dx + 1/2 = \left[-x\frac{\cos 2x}{2} + \int \frac{\cos 2x}{2}\,dx\right]_0^1 + 1/2 =$$

$$\left[-\frac{x\cos 2x}{2} + \frac{\operatorname{sen} 2x}{4}\right]_0^1 + 1/2 =$$

$$\boxed{-\frac{\cos 2}{2} + \frac{\operatorname{sen} 2}{4} + \frac{1}{2}}$$

$$\iint_{E\cup F} x^2y + x\cos 2y + 1/2 \; dx\,dy =$$

$$\iint x^2y \; dx\,dy + 2\iint_{F_1} x\cos 2y \; dx\,dy - \iint_{\triangle} x\cos 2y \; dx\,dy + \frac{m(E\cup F)}{2}$$

(1) $x = \rho\cos\theta$ $\quad \rho \in [0;1]$

$y = \rho\operatorname{sen}\theta$ $\quad \theta \in [\pi/4;\pi]$

$$\iint \rho^4\cos^2\theta \operatorname{sen}\theta \; d\rho\,d\theta = \int_0^1 \rho^4\left[-\frac{\cos^3\theta}{3}\right]_{\pi/4}^{\pi} d\rho =$$

$$\int_0^1 \rho^4\left(\frac{1}{3} + \frac{\sqrt{2}}{12}\right)d\rho = \boxed{\frac{1}{15} + \frac{\sqrt{2}}{60}}$$

(2) $$\iint \rho^2\cos\theta\cos(2\rho\operatorname{sen}\theta)\,d\rho\,d\theta = \int_0^1 \rho\left[\int_0^{\pi/4} \rho\cos\theta\cos(2\rho\operatorname{sen}\theta)\,d\theta\right]d\rho$$

$$= \int_0^1 \rho\left[\frac{\operatorname{sen}(2\rho\operatorname{sen}\theta)}{2}\right]_0^{\pi/4} d\rho = \frac{1}{2}\int_0^1 \rho\operatorname{sen}(\sqrt{2}\rho)\,d\rho =$$

$$1/2\left[-\rho\frac{\cos(\sqrt{2}\rho)}{\sqrt{2}} + \int\frac{\cos(\sqrt{2}\rho)}{\sqrt{2}}\,d\rho\right] = 1/2\left[-\rho\frac{\cos(\sqrt{2}\rho)}{\sqrt{2}} + \frac{\operatorname{sen}(\sqrt{2}\rho)}{2}\right]_0^1 =$$

$$\boxed{-\frac{\cos\sqrt{2}}{2\sqrt{2}} + \frac{\operatorname{sen}\sqrt{2}}{4}}$$

(3) $\dfrac{m(E\cup F)}{2} = \left(\dfrac{\pi}{4} + \dfrac{\pi}{8} + 1\right)\cdot\dfrac{1}{2} = \boxed{\dfrac{1}{2} + \dfrac{3}{16}\pi}$

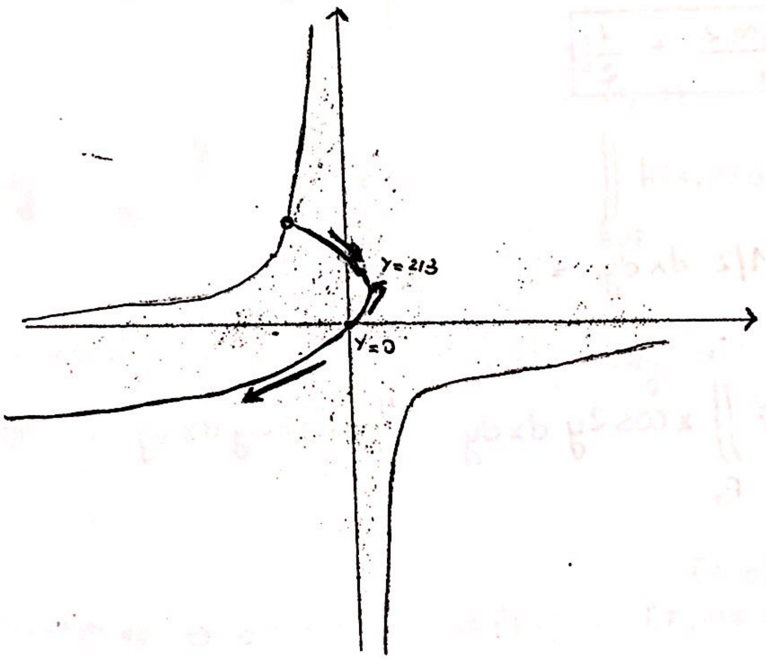
$$\iint_{E\cup F} h(x,y)\,dx\,dy = \frac{1}{15} + \frac{\sqrt{2}}{60} - \frac{\cos 2}{2} + \frac{\operatorname{sen} 2}{4} + \frac{\cos\sqrt{2}}{2\sqrt{2}} - \frac{\operatorname{sen}\sqrt{2}}{4} + \frac{1}{2} + \frac{3}{16}\pi$$

$\boxed{4}$

$f(x,y) = \ln(xy+1)$

$G = \{(x,y)\in\mathbb{R}^2 \mid x + y^2 - y = 0\}$ $\qquad x = -y^2 + y = -y(y-1)$

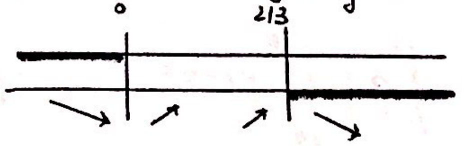
$A = \{(x,y)\in\mathbb{R}^2 \mid xy + 1 > 0\}$



$$\max_{G}\min f(x,y) \equiv \max_{x+y^2-y=0}\min\ xy+1 \equiv \max\min\ (-y^2+y)y+1 \equiv$$

$$\max\min\ -y^3+y^2+1 \longrightarrow -3y^2+2y=0 \iff y=0,\ y=2/3$$

$$-y(3y-2)=0$$



(0;0) pto di MINIMO relativo

(2/9; 2/3) pto di MASSIMO relativo

la Fz è illimitata sia superiormente che inferiormente: non ci sono pti di MAX/MIN ASSOLUTO sulla restrizione

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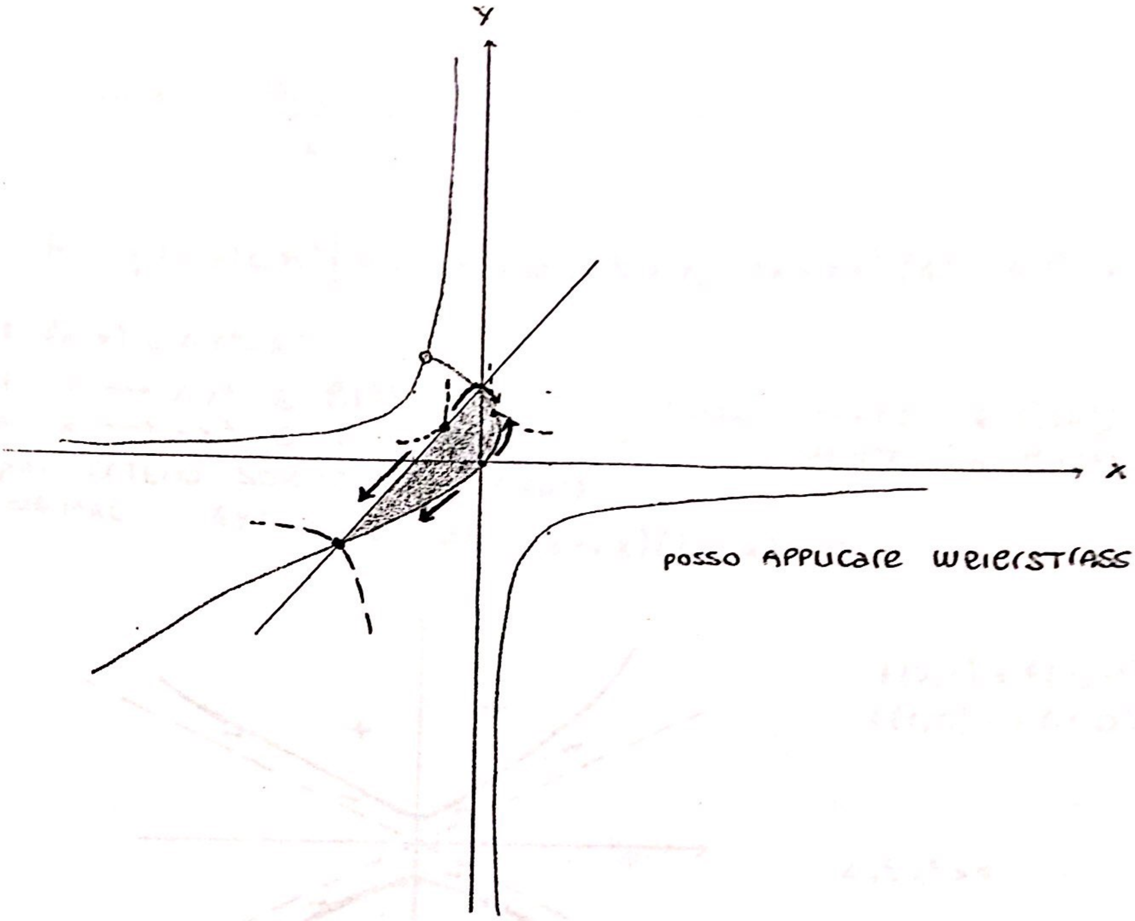


posso applicare Weierstrass

$$\max_{\substack{min \\ G}} f(x,y) \equiv \max_{\substack{min \\ A\cap G}} xy+1$$

pti interni: $g'_x = y = 0$ $\quad$ $g'_y = x = 0$ $\quad$ $\boxed{\begin{matrix} x=0 \\ y=0 \end{matrix}}$ N.A.

pti frontiera: $\boxed{(2/3;\ 2/3)}$ pto di MASSIMO RELATIVO

$$\max_{\substack{min \\ y=x+1}} xy+1 \equiv \max_{min} x^2+x+1 \longrightarrow 2x+1=0$$ $\boxed{\begin{matrix} x=-1/2 \\ y=1/2 \end{matrix}}$ pto di MINIMO ASSOLUTO

$-1/2$

$$\begin{cases} x = -y^2+y \\ x = y-1 \end{cases}$$

$+\ y^2 = 1$

$\boxed{\begin{matrix} y=-1 \\ x=-2 \end{matrix}}$ pto di MASSIMO ASSOLUTO

10/06/2013

$\boxed{1}$

$$f(x,y) = \frac{\ln(4y^2 - x^2)}{x - y} \sqrt{(x+y)^2(16 - 4y^2 - x^2)}$$

$$A = \{ (x,y) \in \mathbb{R}^2 \mid 4y^2 - x^2 > 0,\ x \neq y,\ (x+y)^2(16 - 4y^2 - x^2) \geq 0 \}$$

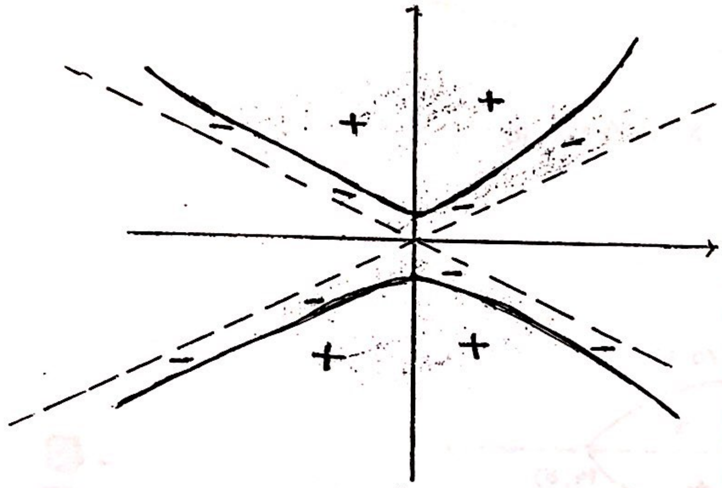
- $f_1(x,y) = 4y^2 - x^2$

$f: y \longrightarrow 4y^2 \in \mathcal{C}(\mathbb{R})$ per criterio collegamento $\in \mathcal{C}(\mathbb{R}^2)$

$f: x \longrightarrow -x^2 \in \mathcal{C}(\mathbb{R})$ per criterio collegamento $\in \mathcal{C}(\mathbb{R}^2)$

per criterio somma $f_1 \in \mathcal{C}(\mathbb{R}^2)$

$\Rightarrow$ MRC $4y^2 - x^2 = 0 \iff (2y + x)(2y - x) = 0$



$f(0;1) = f(0;-1) = 4 > 0$

$f(1;0) = f(-1;0) = -1 < 0$

$4y^2 - x^2 > 1$

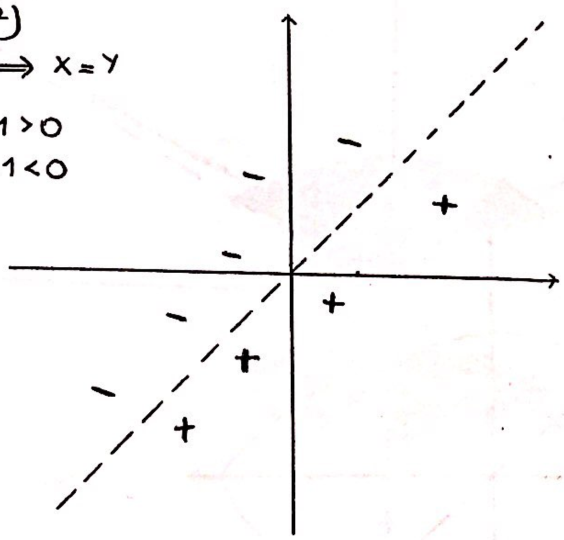
- $f_2(x,y) = x - y$

$f_2 \in \mathcal{C}(\mathbb{R}^2)$

$f_2 = 0 \iff x = y$

$f(1;0) = 1 > 0$

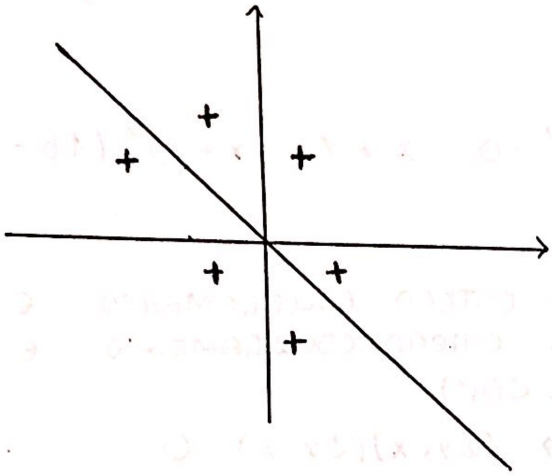
$f(0;1) = -1 < 0$



- $f_3(x,y) = (x+y)^2$

$f_3 \in \mathcal{C}(\mathbb{R}^2)$ per COLLEGAMENTO e PRODOTTO

MRC: $(x+y)^2=0 \iff -x=y$



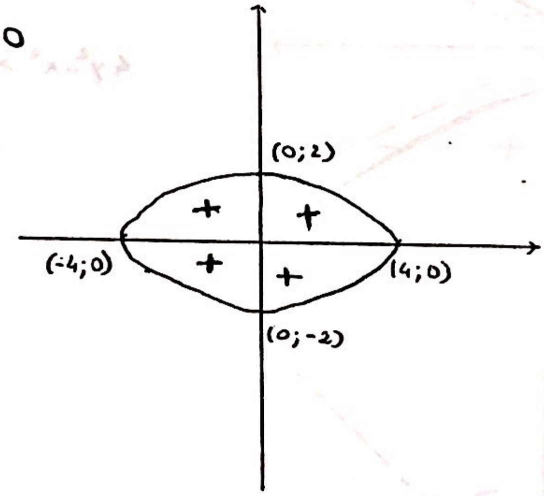
$f_4(x,y) = 16-4y^2-x^2$

$f_4 \in \mathcal{C}(\mathbb{R}^2)$

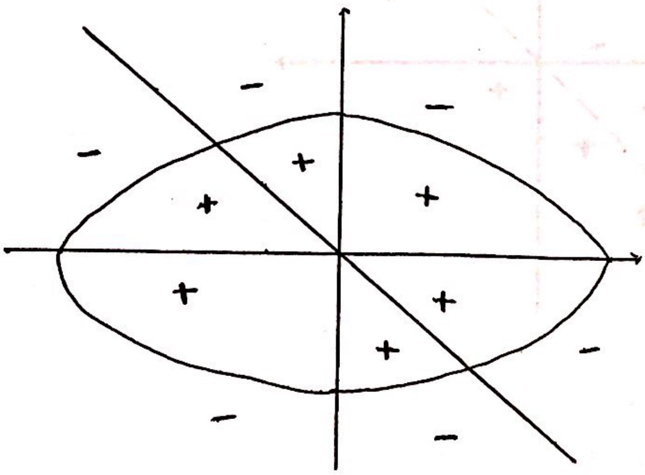
MRC: $16-4y^2-x^2=0 \iff x^2+4y^2=16$

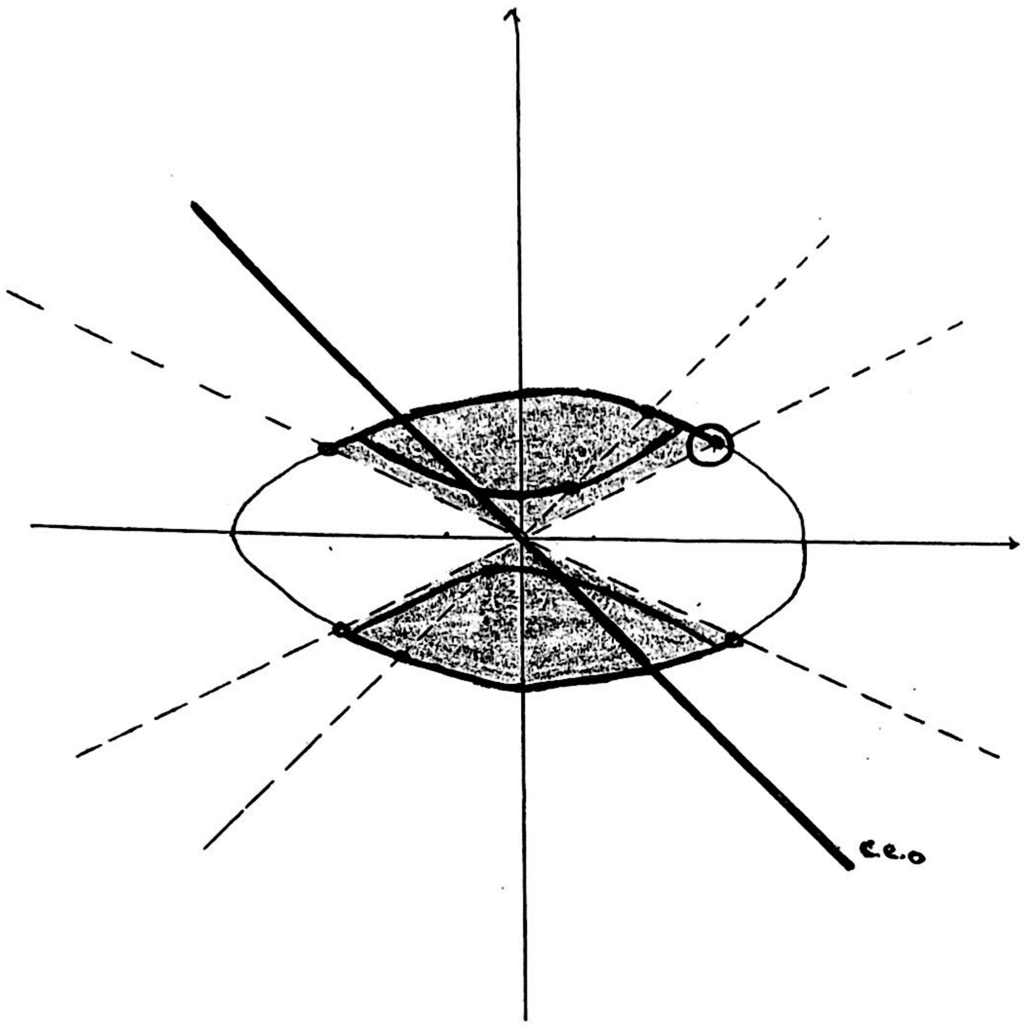
$f(0;0) = 16>0$

$f(0;3) = 16-36<0$



$f_3 \cdot f_4 \geq 0$

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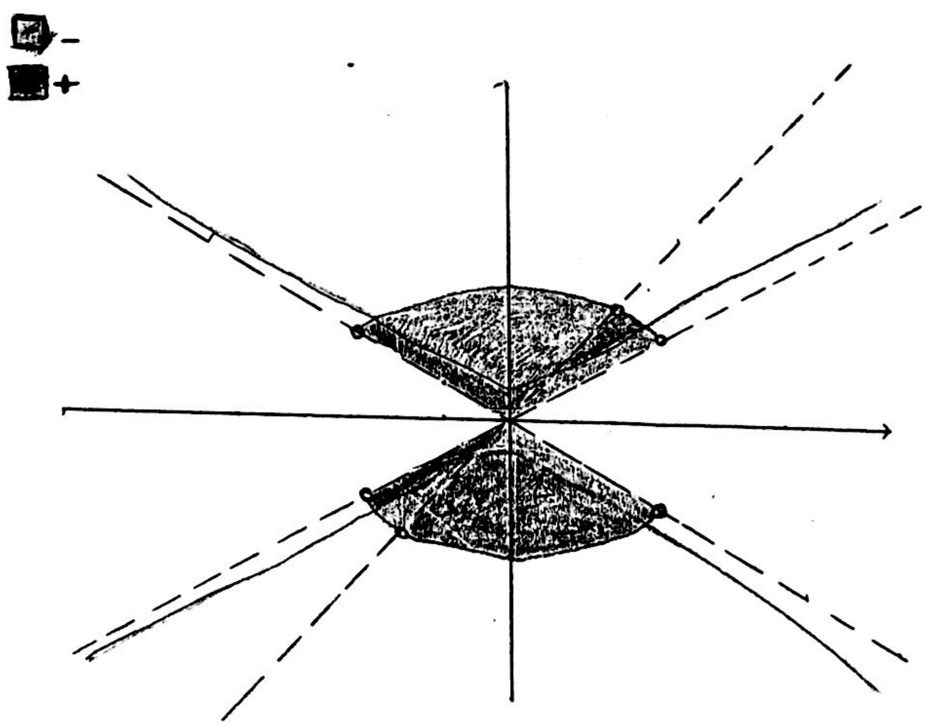
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LIMITI:

$(-4;0) \notin D(A)$

$(-4;4) \in D(A)$ $\quad f(x,y)$ è CONTINUA: IL LIMITE è $f(-4;4)=0$

$(1;1) \in D(A)$

$$\lim_{(x,y)\to(1;1)} \frac{\overbrace{\ln(4y^2-x^2)}^{\substack{\text{CONTINUITÀ e} \\ \text{LIMITE COMPOSTA} \\ \to \ln(3)}}}{\underbrace{x-y}_{\substack{\to 0 \\ \text{per CONTINUITÀ}}}} \underbrace{\sqrt{(x+y)^2(16-4y^2-x^2)}}_{\substack{\to \sqrt{22} \quad \text{CONTINUITÀ e LIMITE COMPOSTA} \\ \text{LIMITATA}}} = +\infty$$

LIMITATA

$(2\sqrt{2};\sqrt{2})$

$$\lim_{(x,y)\to(2\sqrt{2},\sqrt{2})} \frac{\overbrace{\ln(4y^2-x^2)}^{\substack{\text{CONTINUITÀ e} \\ \text{LIMITE COMPOSTA} \\ \to -\infty}}}{\underbrace{x-y}_{\substack{\to \sqrt{2} \text{ per} \\ \text{CONTINUITÀ} \\ \text{LIMITATA}}}} \cdot \underbrace{\sqrt{(x+y)^2(16-4y^2-x^2)}}_{\substack{\to 0 \text{ per LIMITE} \\ \text{COMPOSTA e} \\ \text{CONTINUITÀ}}}$$

$\nexists$ LIMITE per SOLITI MOTIVI

$(0;0) \in D(A)$ per MOTIVI ANALOGHI al pto PRIMA NON ESISTE IL LIMITE

$$g(x,y) = \frac{\ln(x^2-3y^2)}{x-y}$$

$(2;1)$

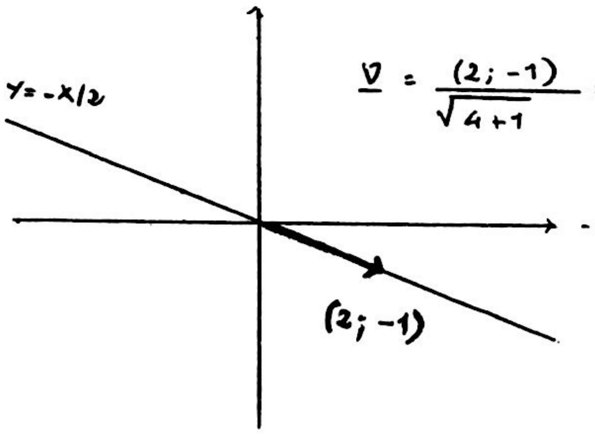
$x+2y+7=0$

$$g'_x = \frac{\frac{2x(x-y)}{x^2-3y^2} - \ln(x^2-3y^2)}{(x-y)^2} = 4$$

$$g'_y = \frac{\frac{-6x(x-y)}{x^2-3y^2} - \ln(x^2-3y^2)}{(x-y)^2} = -6$$

ESISTONO IN UN INTORNO di $(2;1)$ e SONO CONTINUE nel pto $\longrightarrow$ per IL Th del DIFFERENZIALE TOTALE g è DIFFERENZIABILE

$$\longrightarrow \exists \frac{\partial g}{\partial \underline{v}}(2;1) = \langle \nabla g(2;1), \underline{v} \rangle = \langle (4,-6), (2/\sqrt{5}, -1/\sqrt{5}) \rangle = \frac{8}{\sqrt{5}} + \frac{6}{\sqrt{5}} = \boxed{\frac{14}{\sqrt{5}}}$$



$$\underline{v} = \frac{(2;-1)}{\sqrt{4+1}} = \left(\frac{2}{\sqrt{5}}; -\frac{1}{\sqrt{5}}\right)$$

$$z = \langle \nabla g(2,1), (x-2, y-1) \rangle$$

$$\boxed{z = 4x - 6y - 2}$$

$$z = 4(x-2) - 6(y-1)$$
$$z = 4x - 8 - 6y + 6$$

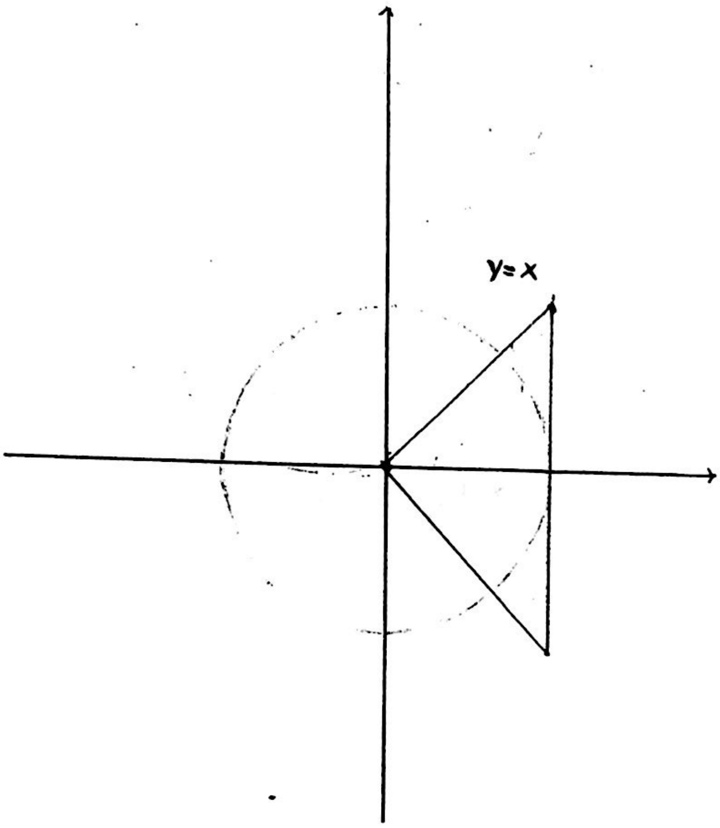
$h(x,y) = x^2y + x\cos 2y + 1/2$ (12)

$E = \{(x,y) \in \mathbb{R}^2 \mid x^2+y^2 \leq 1,\ y \geq 0\}$

$= (0;0)$

$(1;1)$

$(1;-1)$



$$\iint_E x^2y + x\cos 2y + 1/2 \, dxdy = \boxed{\frac{2}{15} + \frac{\pi}{4}}$$

$$= 2\iint_{E_1} x^2y \, dxdy + m(E_1) = 2\iint_{E_1} \rho^4\cos^2\theta \,\text{sen}\,\theta \, d\rho \, d\theta + m(E_1)$$

(a) $2\int_0^1 \rho^4\left[\int_0^{\pi/2}\cos^2\theta\,\text{sen}\,\theta\, d\theta\right] d\rho = 2\int_0^1 \rho^4\left[-\frac{\cos^3\theta}{3}\right]_0^{\pi/2} d\rho =$

(b) $\frac{2}{3}\int_0^1 \rho^4 \, d\rho = \frac{2}{15}$

$m(E_1) = \frac{\pi}{4}$

$$\iint_F x^2y + x\cos 2y + 1/2 \, dx\,dy = \boxed{\frac{-\operatorname{sen}2}{4} \; \frac{-\cos 2 + 1}{2}}$$

$$2\iint_{F_1} x\cos 2y \, dx\,dy + m(F_1)$$

$$2\int_0^1 \cos 2y \left[\int_y^1 x\,dx \right] dy =$$

$$2\int_0^1 \cos 2y \left[\frac{x^2}{2} \right]_y^1 dy = \int_0^1 \cos 2y \, dy - \int_0^1 y^2 \cos 2y \, dy =$$

$$\left[\frac{\operatorname{sen}2y}{2} \right]_0^1 - \left[y^2 \frac{\operatorname{sen}2y}{2} - \int y \operatorname{sen}2y \, dy \right]_0^1 =$$

$$\frac{\operatorname{sen}2}{2} - \left[y^2 \frac{\operatorname{sen}2y}{2} + y\frac{\cos 2y}{2} - \int \frac{\cos 2y}{2} dy \right]_0^1 =$$

$$\frac{\operatorname{sen}2}{2} - \left[y^2 \frac{\operatorname{sen}2y}{2} + y\frac{\cos 2y}{2} - \frac{\operatorname{sen}2y}{4} \right]_0^1 =$$

$$\cancel{\frac{\operatorname{sen}2}{2}} - \cancel{\frac{\operatorname{sen}2}{2}} - \frac{\cos 2}{2} + \frac{\operatorname{sen}2}{4} = -\frac{\operatorname{sen}2}{4} - \frac{\cos 2}{2}$$

$$m(F_1) = \frac{1}{2}$$

$$\iint_{E\cup F} x^2y + x\cos 2y + 1/2 \, dx\,dy$$

$$\iint x^2 y \, dx\,dy + \iint_F x\cos 2y \, dx\,dy - \iint x\cos 2y \, dx\,dy + \frac{m(E\cup F)}{2}$$

(1) $$\iint \rho^4 \cos^2\theta \operatorname{sen}\theta \, d\rho \, d\theta = \int_0^1 \rho^4 \left[-\frac{\cos^3\theta}{3} \right]_{\pi/4}^{\pi} d\rho = \left[\frac{\rho^5}{5} \right]_0^1 \left(\frac{1}{3} + \frac{\sqrt{2}}{12} \right)$$

$\rho \in [0;1]$
$\theta \in [\pi/4;\pi]$

$$= \frac{1}{15} + \frac{\sqrt{2}}{60}$$

(ii) $$\iint x\cos 2y \, dx\,dy = \iint \rho^2 \cos\theta \cos(2\rho \,\text{sen}\,\theta)\, d\rho\, d\theta$$

$\rho \in [0;1]$
$\theta \in [0;\pi/4]$

$$\int_0^1 \rho \left[\frac{\text{sen}(2\rho\,\text{sen}\,\theta)}{2} \right]_0^{\pi/4} d\rho = \int_0^1 \rho \, \frac{\text{sen}(\sqrt{2}\,\rho)}{2}\, d\rho =$$

$$\frac{1}{2}\int_0^1 \rho \;\text{sen}(\sqrt{2}\,\rho)\, d\rho = \frac{1}{2}\left[-\rho\,\frac{\cos(\sqrt{2}\,\rho)}{\sqrt{2}} + \int \frac{\cos(\sqrt{2}\,\rho)}{\sqrt{2}}\, d\rho \right]_0^1$$

$$= \frac{1}{2}\left[-\rho\,\frac{\cos(\sqrt{2}\,\rho)}{\sqrt{2}} + \frac{\text{sen}(\sqrt{2}\,\rho)}{2} \right]_0^1 =$$

$$\frac{1}{2}\left(-\frac{\cos\sqrt{2}}{\sqrt{2}} + \frac{\text{sen}\sqrt{2}}{2} \right) = \boxed{-\frac{\cos\sqrt{2}}{2\sqrt{2}} + \frac{\text{sen}\sqrt{2}}{4}}$$

(iii) $$m(E\cup F) = \frac{1 + 3/8\,\pi}{2}$$

$$\iint_{E\cup F} =$$ [redacted]

$$\sum_{n\geq 1} \frac{1}{n - \ln(n^2)} \left(-x^2+2x\right)^n$$

$t = -x^2 + 2x$

$$\lim_{n\to+\infty} \frac{n - \ln(n^2)}{n+1 - \ln(n+1)^2} = \lim_{n\to+\infty} \frac{n}{n+1} = 1$$

$\rho = 1 \qquad (-1;1) \subset \Gamma_t \subset [-1;1]$

per $t = -1$
SEGNI ALTERNATI
TERMINE n-ESIMO INFINITESIMO
DECRESCENTI
} per LEIBNITZ CONVERGE

per $t = 1$ $\quad \dfrac{1}{n - \ln(n^2)} \geq \underbrace{\dfrac{1}{n}}_{\text{DIVERGE}}$ per criterio del CONFRONTO DIVERGE

$\Gamma_t = [-1; 1)$

$-x(x-2)=0$

$-x^2+2x=+1$

$x^2-2x+1=0$

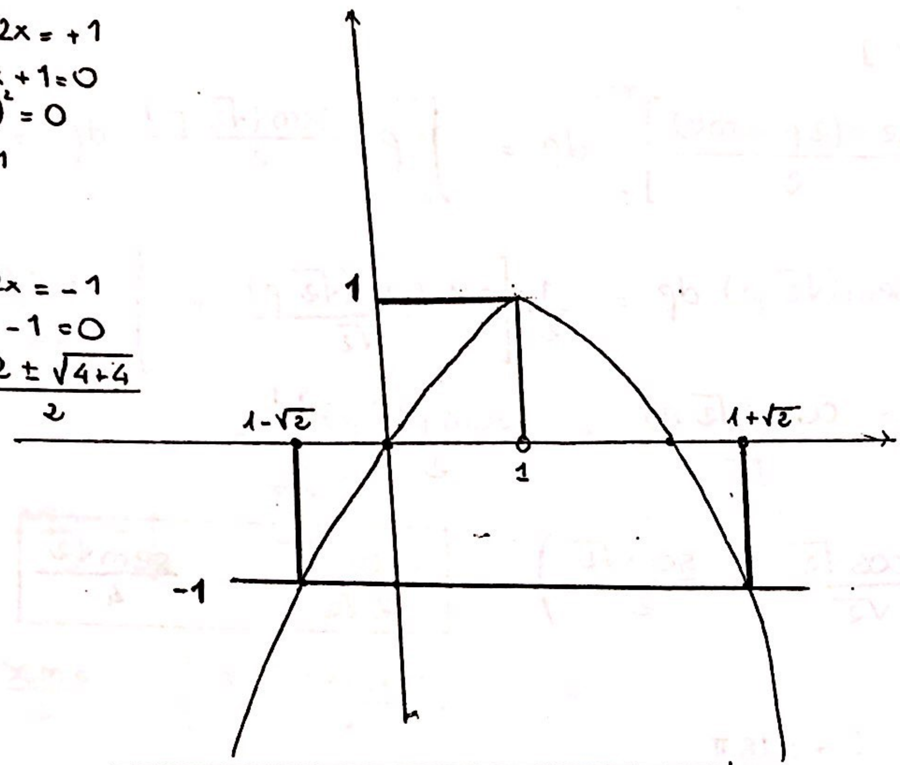
$(x-1)^2=0$

$x=1$

$-x^2+2x=-1$

$x^2-2x-1=0$

$x_{1,2} = \dfrac{2 \pm \sqrt{4+4}}{2}$



$\boxed{\Gamma'_x = [1-\sqrt{2}; 1) \cup (1; 1+\sqrt{2}]}$

$$\sum_{m\geq 3}\left(\frac{1}{m-1} + (m+1)(m+2)\right)x^m$$

$$\sum_{m\geq 3}\frac{x^m}{m-1} + \sum_{m\geq 3}(m+1)(m+2)\,x^m =$$

$$x\sum_{m\geq 3}\frac{x^{m-1}}{m-1} + \frac{d''}{dx}\sum_{m\geq 3}x^{m+2} =$$

$$x\left(-\ln(1-x) \cdots +1\right) + \frac{d''}{dx}\left(\frac{1}{1-x} - \cancel{1} - \cancel{x} - x^2 - x^3 - x^4\right)$$

$$\boxed{= -x\ln(1-x) + x + \frac{2}{(1-x)^3} - 2 - 6x - 12x^2}$$

$\ln(xy+1)$

$G = \{(x,y)\in \mathbb{R}^2 \mid x + y^2 - y = 0\}$ $x = -y^2 + y = -y(y-1)$

$A = \{(x,y)\in \mathbb{R}^2 \mid xy + 1 > 0\}$

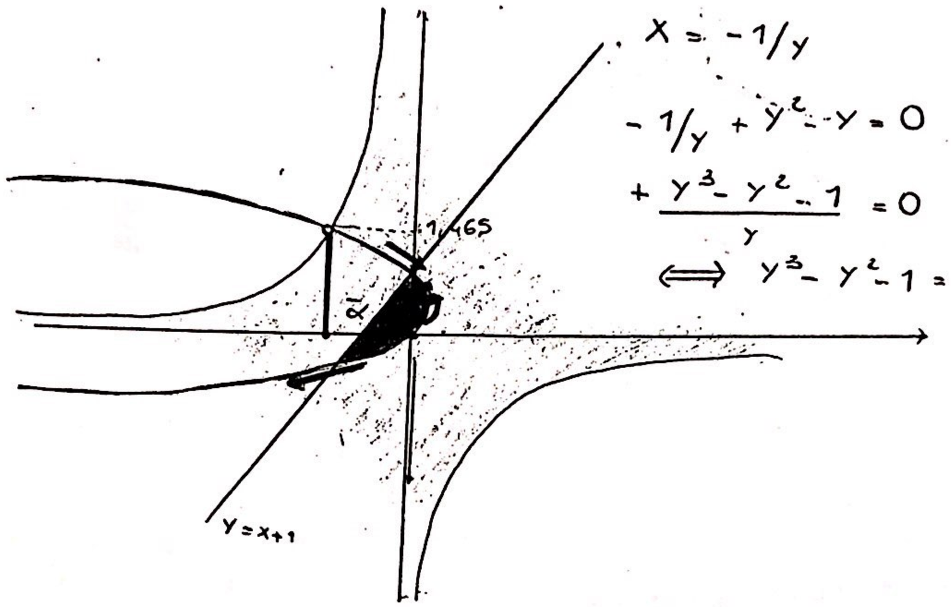
$$\bar{\alpha}: \begin{cases} x + y^2 - y = 0 \\ xy + 1 = 0 \end{cases}$$

$x = -1/y$

$-1/y + y^2 - y = 0$

$+\dfrac{y^3 - y^2 - 1}{y} = 0$

$\iff y^3 - y^2 - 1 = 0$

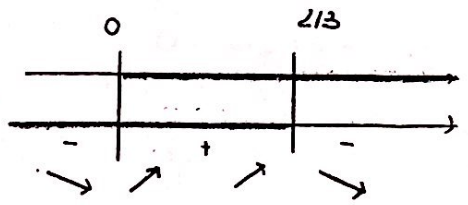


$$\max_{G}\min f(x,y) \equiv \max_{G\cap A}\min\ xy+1 \equiv \max_{\substack{x=-y^2+y \\ x\geq\bar{\alpha}}}\min\ xy+1$$

$$\equiv \max_{x\geq\bar{\alpha}}\min\ -y^3+y^2+1 \longrightarrow -3y^2+2y=0 \iff y(-3y+2)=0$$

$\iff \boxed{\begin{matrix} y=0 \\ x=0 \end{matrix}}$

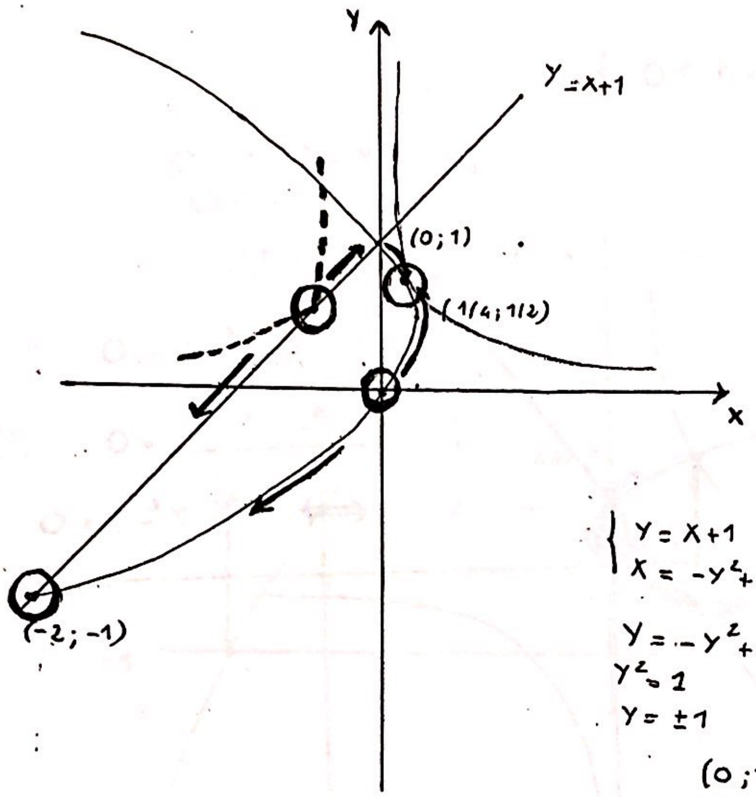
$\iff \boxed{\begin{matrix} y=2/3 \\ x=2/9 \end{matrix}}$



FZ ILLIMITATA SIA SUPERIORMENTE CHE INFERIORMENTE: NON CI SONO PTI DI OTTIMO ASSOLUTO

$(0;0)$ pto di MINIMO RELATIVO

$(2/3;2/9)$ pto di MASSIMO RELATIVO



$$\begin{cases} y = x+1 \\ x = -y^2+y \end{cases}$$

$y = -y^2 + y + 1$

$y^2 = 1$

$y = \pm 1$

(0;1)

(-2;-1)

CHIUSO e LIMITATO: CI SONO MASSIMO e MINIMO ASSOLUTO

$g'_x = y$

$g'_y = x$

(0;0) NON ACCETABILE: pto di NULLA

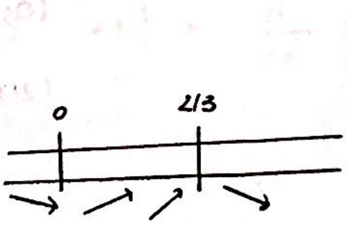
$\underset{y=x+1}{\max\ \min}\ xy+1 = \underset{-2\le x\le 0}{\max\ \min}\ x^2+x+1 \longrightarrow 2x+1=0 \iff \boxed{x=-1/2 \quad y=1/2}$

pto di MINIMO ASSOLUTO

$\underset{x=-y^2+y}{\max\ \min}\ xy+1 = \max\ \min\ -y^3+y^2+1 \longrightarrow -3y^2+2y=0$

$\iff y(-3y+2)=0$

$\iff \boxed{y=0 \quad x=0}$ N.A. $\quad \boxed{y=2/3 \quad x=2/9}$ pto di MASSIMO RELATIVO



$\boxed{(-2;-1)}$

pto di MASSIMO ASSOLUTO