$\boxed{1}$ fatto in aula

SVOLTO

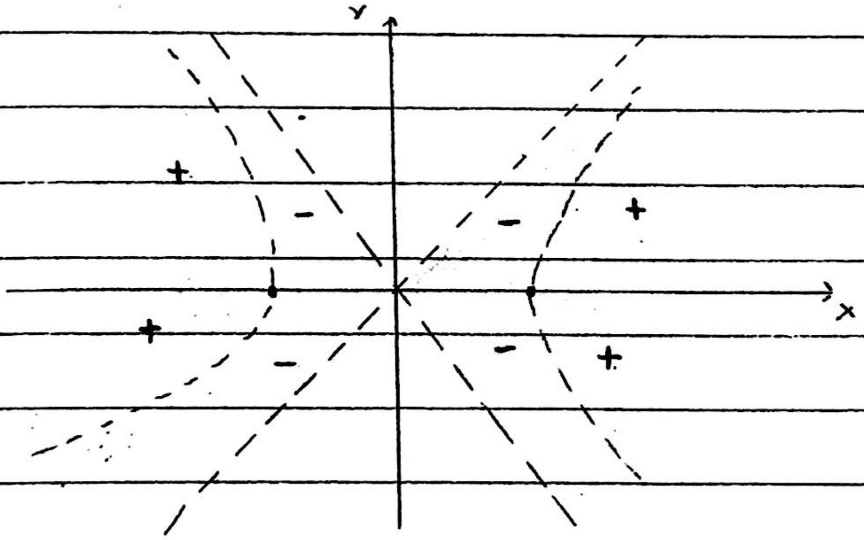
$$f(x,y) = \frac{x^2-4y^2}{\ln(x^2-y^2)} \sqrt{(x+2y)^2(x^2+4y^2-4)}$$

$$A = \{(x,y) \in \mathbb{R}^2 \mid x^2-y^2>0,\ x^2-y^2 \neq 1,\ (x+2y)^2(x^2+4y^2-4) \geq 0\}$$

$f_1 = \ln(x^2-y^2)$

$f: x \to x^2 \in \mathcal{C}(\mathbb{R})$ per criterio collegamento $f \in \mathcal{C}(\mathbb{R}^2)$
$f: y \to -y^2 \in \mathcal{C}(\mathbb{R})$ per criterio collegamento $f \in \mathcal{C}(\mathbb{R}^2)$
per criterio somma $f: (x,y) \to x^2-y^2 \in \mathcal{C}(\mathbb{R}^2)$
per criterio della composta $\ln(x^2-y^2)$ nel suo dominio
$\Longrightarrow$ MBC

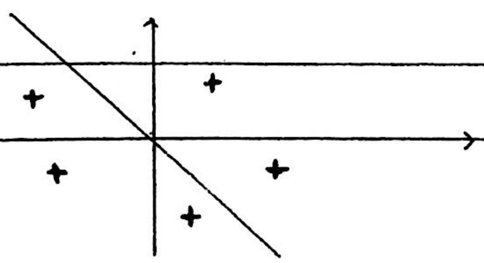
$\ln(x^2-y^2) = 0 \iff x^2-y^2 = 1$



$f(2;0) = f(-2;0) > 0$
$f(0.5;0) = f(-0.5;0) < 0$

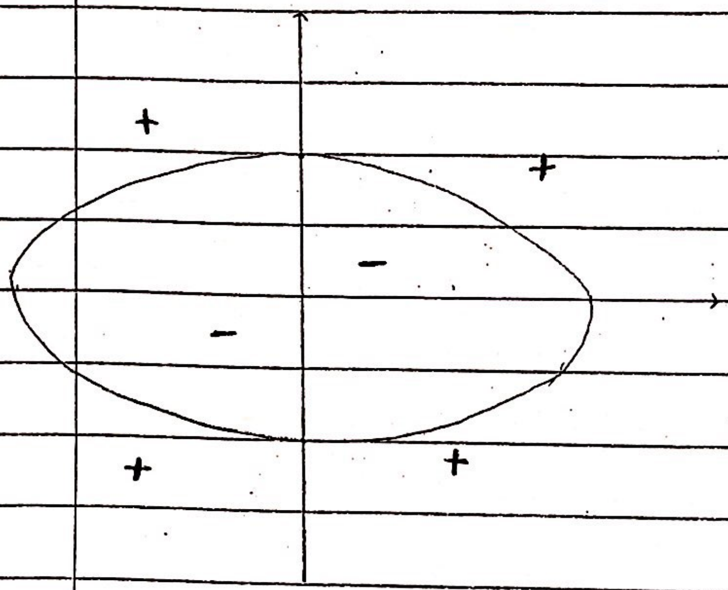
$f_3 = (x+2y)^2$ per criteri analoghi $f_3 \in \mathcal{C}(\mathbb{R}^2)$

$f_3 = 0 \iff y = -x/2$

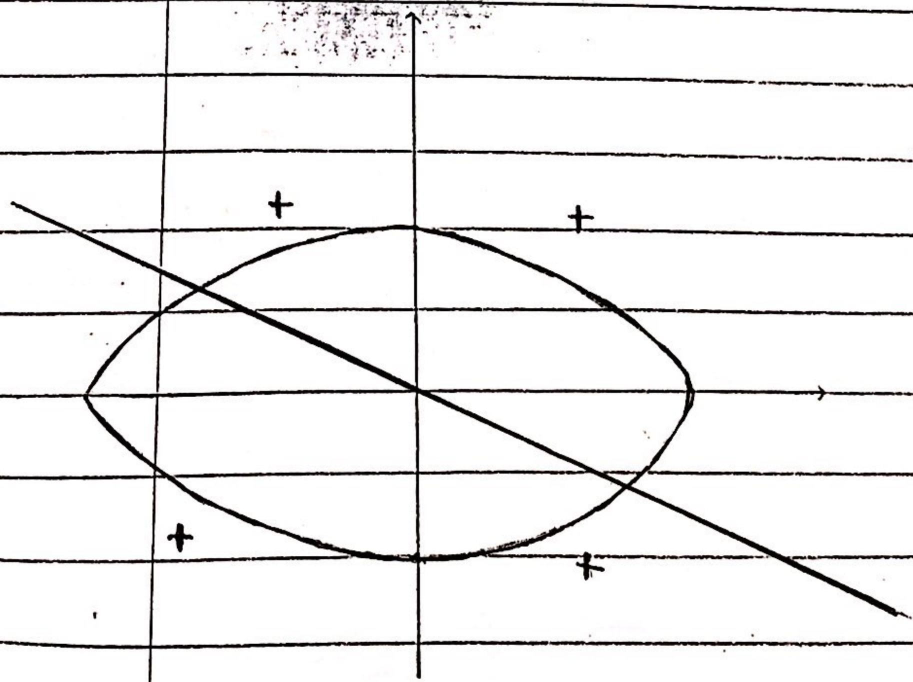


$f_4 = x^2 + 4y^2 - 4$

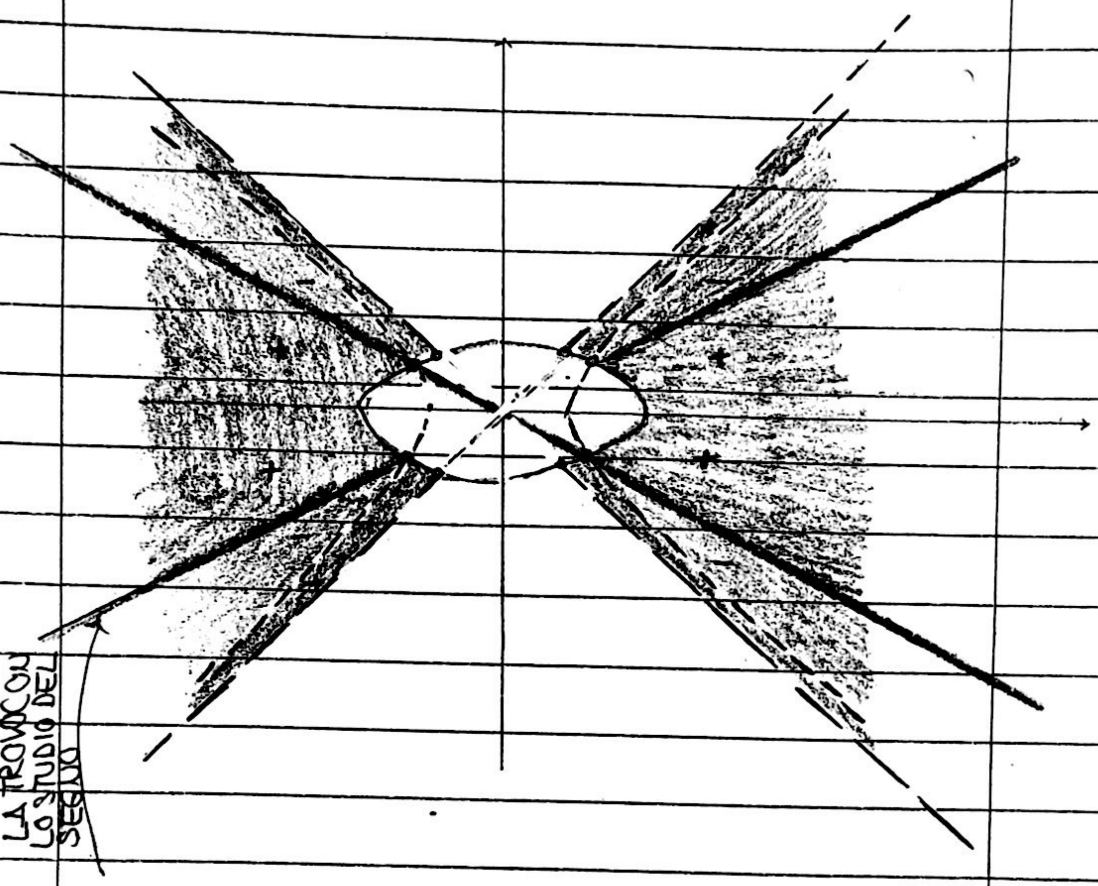
$f_4 \in C(\mathbb{R}^2) \Longrightarrow$ MRC $\quad x^2 + 4y^2 = 4$



$f_3 \cdot f_4 \geq 0$

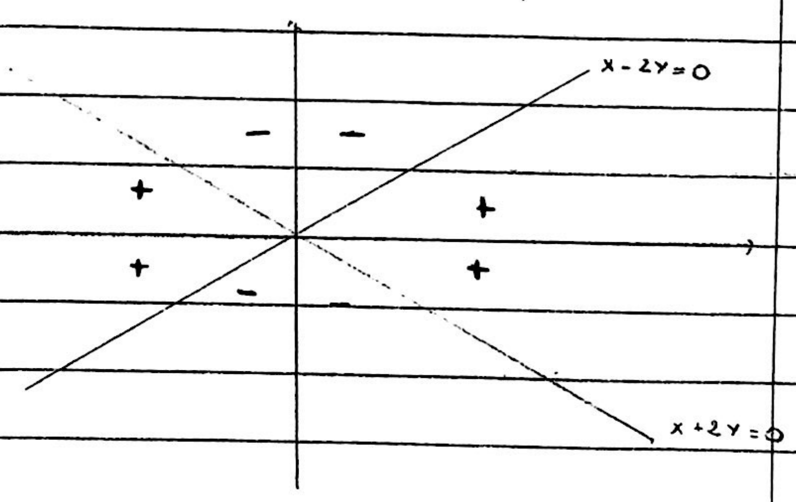


DOMINIO



$f_5 = x^2 - 4y^2 = (x+2y)(x-2y)$

$f_5 \in \mathcal{C}(\mathbb{R}^2) \Longrightarrow$ MBC $\quad f_5 = 0 \iff y = \pm x/2$



$f(1;0) = f(-1;0) = 1$

~~$f(0;1) = f(0;-1) = -4$~~

se ho $\frac{0}{0}$ trovo una restrizione → se il limite ∃ dovrebbe essere ma di solito $\frac{0}{0}$ → ∞

0 → solo perché tende a 0

(c) • $(0;0) \in D(A)$

$$\lim_{(x,y)\to(0;0)} \underbrace{\frac{\overbrace{x^2-4y^2}^{\to 0 \text{ CONTINUITÀ}}}{\ln(\underbrace{x^2-y^2}_{\to 0 \text{ CONTINUITÀ}})}}_{\to -\infty \text{ COMPOSTA}} \underbrace{\sqrt{(x+2y)^2\,(x^2+4y^2-4)}}_{\to 0 \text{ per CONTINUITÀ}} = 0$$

• $(1;\ 1/2) \notin D(A)$ NON ha senso

• $(1;\ -1/2)$

$$\lim_{(x,y)\to(1;-1/2)} f(x,y) = f(1;\ -1/2) = 0$$ POICHÈ f CONTINUA nel SUO DOMINIO

• $(2;\ \sqrt{3}) \in D(A)$

$$\lim_{(x,y)\to(2;\sqrt{3})} \frac{\overbrace{x^2-4y^2}^{\to -8 \text{ CONTINUITÀ LIMITATA}}}{\underbrace{\ln(x^2-y^2)}_{\to 0 \text{ COMPOSTA}}} \underbrace{\sqrt{(x+2y)^2\,(x^2+4y^2-4)}}_{\to \sqrt{\cdot} \text{ per CONTINUITÀ LIMITATA}} = \infty$$

• $\left(-\frac{2\sqrt{3}}{3};\ \frac{\sqrt{3}}{3}\right) \in D(A)$

$$\lim_{(x,y)\to\left(-\frac{2\sqrt{3}}{3};\ \frac{\sqrt{3}}{3}\right)} \frac{x^2-4y^2}{\ln(x^2-y^2)} \sqrt{(x+2y)^2\,(x^2+4y^2-4)} \ \nexists$$ per SOLITO MOTIVO

• $\left(\sqrt{8/5};\ \sqrt{3/5}\right) \in D(A)$ per MOTIVO ANALOGO a PRIMA $\nexists$ LIMITE

• $(2;2) \in D(A)$ $\lim = 0$

• $(\pi;0) \in D(A)$ $\lim = f(\pi, 0)$

(d) $g(x,y) = (x^2-3y^2)\sqrt{xy-1}$

$$g'_x = 2x\sqrt{xy-1} + \frac{y}{2\sqrt{xy-1}}(x^2-3y^2)$$

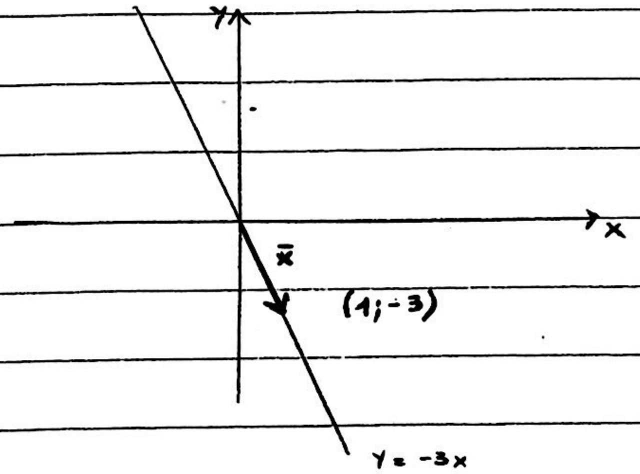
$$g'_y = -6y\sqrt{xy-1} - \frac{x}{2\sqrt{xy-1}}(x^2-3y^2)$$

$$\boxed{\nabla g(2,1) = (7/2\,;\ -7)}$$

$3x + y + 7 = 0$

è DIFFERENZIABILE? Th. DIFF TOTALE. VALGONO TUTTE le IPOTESI

$$\exists\ \frac{\partial g}{\partial \underline{v}}(2\,;\,1) = \langle \nabla g(2,1),\ \underline{v} \rangle$$



$$\underline{v} = \frac{\underline{x}}{\|\underline{x}\|} = \frac{(1,-3)}{\sqrt{1+9}} = \left(\sqrt{\frac{1}{10}}\,;\ -\frac{3}{\sqrt{10}}\right)$$

$$\frac{\partial g}{\partial \underline{v}} = \langle (7/2,\ -7),\ (\sqrt{1/10}\ ;\ -3/\sqrt{10}) \rangle =$$

$$\frac{7}{2\sqrt{10}} + \frac{21}{\sqrt{10}} = \boxed{\frac{49}{2\sqrt{10}}}$$

$$\langle \nabla g(2,1),\ [x-2,\ y-1] \rangle = 0$$

$$\boxed{\frac{7}{2}(x-2) - 7(y-1) = 0}$$

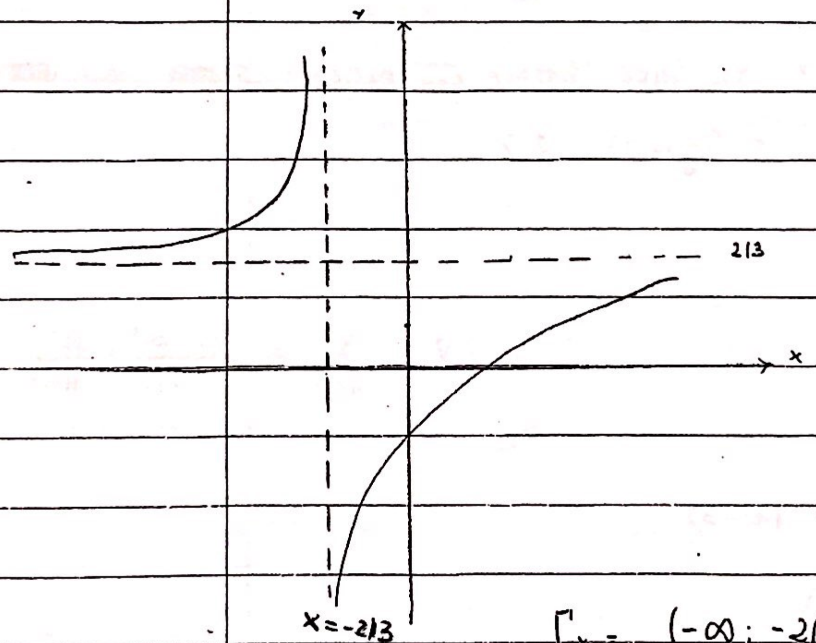
$\boxed{2}$

$$\sum_{n\geq 2} \frac{1}{n! - n^2}\left(\frac{2x-3}{3x+2}\right)^n$$

$$t = \frac{2x-3}{3x+2}$$

$$\lim_{n\to+\infty} \frac{n! - n^2}{(n+1)! - (n+1)^2} = \lim_{n\to+\infty} \frac{n! - n^2}{(n+1)\,(n! - (n+1))} = 0$$

$\Gamma_t = \mathbb{R}$



$\Gamma_x = (-\infty; -2/3) \cup (-2/3; +\infty)$

CONVERGENZA UNIFORME IN OGNI $[a,b] \subset \Gamma_x$
CONVERGENZA PUNTUALE IN OGNI pto di Γ_x

$$\sum_{n\geq 1}\left(n^2 + \frac{1}{n!}\right)x^n = \sum_{n\geq 1} n^2 x^n + \underbrace{\sum_{n\geq 1} \frac{x^n}{n!}}_{} \; e^x - 1$$

$$\sum_{n\geq 1} n(n-1)\,x^n + \sum_{n\geq 1} n x^n = x^2 \sum n(n-1)\,x^{n-2} + x \sum_{n\geq 1} n x^{n-1}$$

$$x^2 \frac{d''}{dx}\left(\sum_{m\geq 1} x^m\right) + x \frac{d}{dx}\left(\sum_{m\geq 1} x^m\right) =$$

$$x^2 \left(\frac{1}{1-x} - 1\right)'' + x \left(\frac{1}{1-x} - 1\right)' =$$

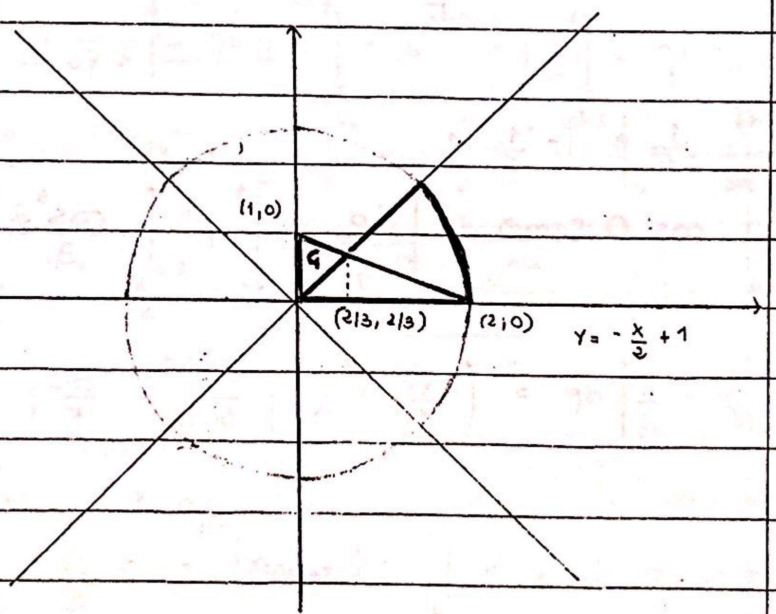
$$x^2 \left(\frac{1}{(1-x)^2}\right)' + \frac{x}{(1-x)^2} = \frac{2x^2}{(1-x)^3} + \frac{x}{(1-x)^2}$$

$$\boxed{S_x = e^x - 1 + \frac{2x^2}{(1-x)^3} + \frac{x}{(1-x)^2}}$$

$\boxed{3}$ $\displaystyle\iint_{E\cup F} x\,(e^{2y} + xy + 2)\,dx\,dy$

$E = \{(x,y) \in \mathbb{R}^2 \mid x^2 - y^2 \geq 0,\ y \geq 0,\ x^2 + y^2 \leq 4\}$

$F = \Delta\ (0;0)\ (0;1)\ (2;0)$



$$\iint_{E\cup F} f(x,y)\,dx\,dy = \iint_E f(x,y)\,dx\,dy + \iint_G f(x,y)\,dx\,dy$$

$$\iint_E f(x,y)\,dx\,dy$$

$$\begin{cases} x = \rho\cos\theta \\ y = \rho\,\mathrm{sen}\,\theta \end{cases} \qquad \begin{matrix} \rho \in [0;2] \\ \theta \in [0;\pi/4] \end{matrix}$$

$$= \iint_E x\,e^{2y}\,dx\,dy + \iint_E x^2 y\,dx\,dy + \iint_E 2x\,dx\,dy$$

$$\iint_E \rho^2\cos\theta\, e^{2\rho\,\mathrm{sen}\theta}\,d\rho\,d\theta + \iint_E \rho^4\cos^2\theta\,\mathrm{sen}\theta\,d\rho\,d\theta +$$

$$\iint_E 2\rho^2\cos\theta\,d\rho\,d\theta$$

manco il meno

$$\underline{\underline{A}}: \int_0^2 \frac{1}{2}\rho\left[\int_0^{\pi/4} 2\rho\cos\theta\, e^{2\rho\,\mathrm{sen}\theta}\,d\theta\right]d\rho = \frac{1}{2}\int_0 \rho\left[e^{2\rho\,\mathrm{sen}\theta}\right]_0^{\pi/4} d\rho =$$

$$\int_0^2 \frac{1}{2}\rho\,(e^{\rho\sqrt{2}}-1)\,d\rho = \int_0^2 \frac{1}{2}\rho\, e^{\rho\sqrt{2}}\,d\rho - \int_0^2 \frac{1}{2}\rho\,d\rho = \left|\frac{1}{2}\frac{1}{\sqrt{2}}\rho\, e^{\sqrt{2}\rho} - \int \frac{1}{2}\frac{1}{\sqrt{2}} e^{\sqrt{2}\rho}\,d\rho\right|_0^2 - 1$$

$$= \frac{e^{2\sqrt{2}}}{2} - \frac{1}{2\sqrt{2}}e^{2\sqrt{2}} + \frac{1}{2} - 1$$

$$\underline{\underline{B}}: \int_0^2 \rho^4\left[\int_0^{\pi/4} \cos^2\theta\,\mathrm{sen}\theta\,d\theta\right]d\rho = \int_0^2 \rho^4\left[-\frac{\cos^3\theta}{3}\right]_0^{\pi/4} d\rho$$

$$\int_0^2 \rho^4\left[-\frac{\sqrt{2}}{12}+\frac{1}{3}\right]d\rho = \left(-\frac{\sqrt{2}}{12}+\frac{1}{3}\right)\left|\frac{\rho^5}{5}\right|_0^2 = \frac{32}{5}\left(-\frac{\sqrt{2}}{12}+\frac{1}{3}\right)$$

$$\underline{\underline{C}}: 2\int_0^2 \rho^2\int_0^{\pi/4}\cos\theta\,d\theta\,d\rho = 2\int_0^2 \rho^2\left[\mathrm{sen}\theta\right]_0^{\pi/4} = 2\cdot\frac{\sqrt{2}}{2}\left[\frac{\rho^3}{3}\right]_0^2 = \frac{8\sqrt{2}}{3}$$

$$\iint_G f(x,y)\,dx\,dy$$

$$\int_0^{2/3}\left[\int_x^{-x/2+1} f(x,d)\,dy\right]dx$$

A

$$\int_0^{2/3} x\left[\int_x^{-x/2+1} e^{2y}\,dy\right]dx = \int_0^{2/3} x\left|\frac{e^{2y}}{2}\right|_x^{-x/2+1} dx$$

$$\int_0^{2/3} x\,\frac{e^{-x+2}}{2} - \frac{x\,e^{2x}}{2}\,dx$$

① $$\int_0^{2/3} \frac{x\,e^{-x+2}}{2}\,dx = \frac{1}{2}\left[-e^{-x+2}x + \int e^{-x+2}dx\right]_0^{2/3} =$$

$$\frac{1}{2}\left[-e^{-x+2}x - e^{-x+2}\right]_0^{2/3} = \frac{1}{2}\left(\frac{2}{3}e^{4/3} - e^{4/3} + e^{2}\right) =$$

$$\boxed{\frac{1}{3}e^{4/3} - \frac{e^{4/3}}{2} + \frac{e^2}{2}}$$

② $$\frac{1}{2}\int_0^{2/3} x\,e^{2x}\,dx = \frac{1}{2}\left[\frac{e^{2x}}{2}.x - \int\frac{e^{2x}}{2}dx\right]_0^{2/3} = \frac{1}{2}\left[\frac{e^{2x}}{2}.x - \frac{e^{2x}}{4}\right]_0^{2/3}$$

$$\frac{1}{2}\left(\frac{e^{4/3}}{3} - \frac{e^{4/3}}{4} + \frac{1}{4}\right) = \boxed{\frac{e^{4/3}}{6} - \frac{e^{4/3}}{8} + \frac{1}{8}}$$

B

$$\int_0^{2/3} x^2\int_x^{-x/2+1} y\,dy\,dx = \int_0^{2/3} x^2\left[\frac{y^2}{2}\right]_x^{-x/2+1} dx =$$

$$\int_0^{2/3}\frac{x^2}{2}\left(\frac{x^2}{4} - x + 1 - x^2\right)dx = \int_0^{2/3}\frac{x^4}{8} - \frac{x^3}{2} + \frac{x^2}{2} - \frac{x^4}{2}\,dx$$

$$= \left|\frac{x^5}{40} - \frac{x^4}{8} + \frac{x^3}{6} - \frac{x^5}{10}\right|_0^{2/3} = \boxed{\frac{2}{135}}$$

$$\int_0^{2/3} 2x \int_x^{-x/2+1} dy = \int_0^{2/3} 2x\left(-\frac{3x}{2}+1\right)dx = \int_0^{2/3} -3x^2+2x \; dx =$$

$$\left| -x^3+x^2 \right|_0^{2/3} = \boxed{\frac{4}{27}}$$

SOMMA le COSE $\square$

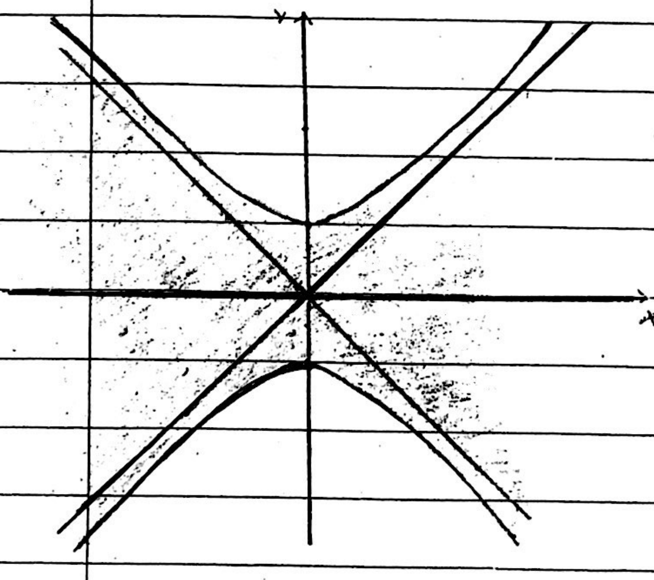
$\boxed{4}$

$h(x,y) = \sqrt{x^2-y^2+1}$

$G = \{(x,y)\in\mathbb{R}^2 \mid xy - y^2 = 0\}$

$y(x-y) = 0 \qquad \begin{cases} y = x \\ y = 0 \end{cases}$

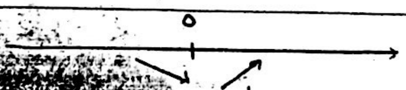
$A = \{(x,y)\in\mathbb{R}^2 \mid x^2-y^2+1 \geq 0\}$



$f(0;5) = f(0;-5) > 0$

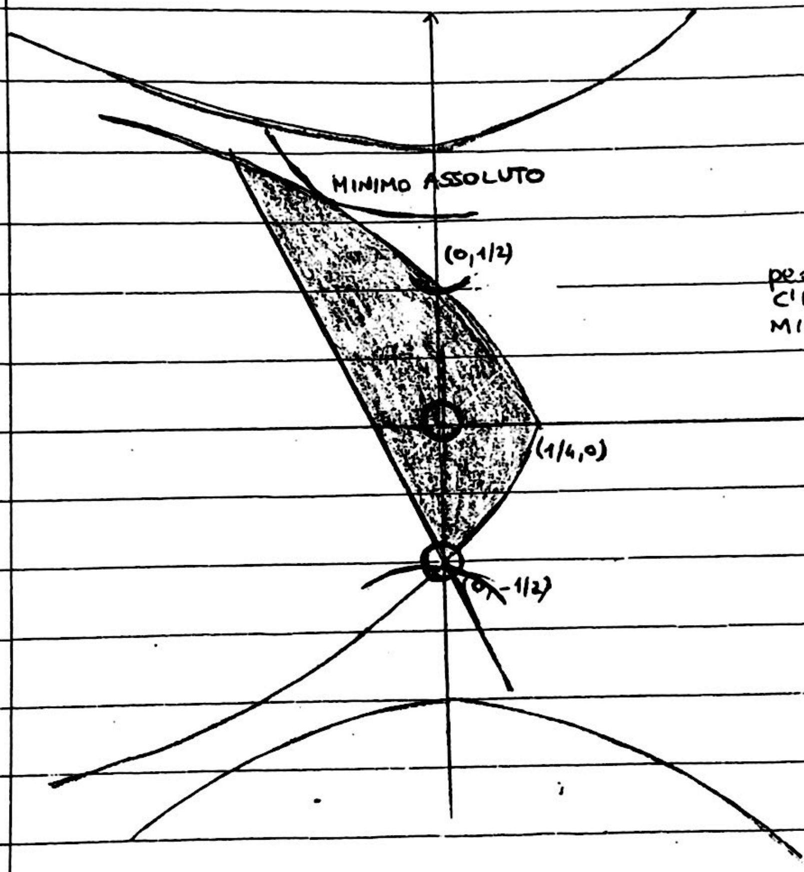
$\max_{\substack{\min \\ x=y}} x^2-y^2+1 = \max_{\min} 1$ NON ci SONO pti di OTTIMO

$\max_{\substack{\min \\ y=0}} x^2+1 \longrightarrow 2x = 0 \iff \begin{matrix} x=0 \\ y=0 \end{matrix}$ pto di MINIMO ASSOLUTO



f è ILLIMITATA^{sup} sulla restrizione [illegible]
$\Longrightarrow$ NON CI SONO pti ASSOLUTO

$H = \{(x,y) \in \mathbb{R}^2 \mid y^2 + x \leq 1/4 \ , \ x + y \geq -1/2\}$



per WEIERSTRASS
C'È pto di MASSIMO e di MINIMO ASSOLUTO

$\max_{\substack{\min \\ GnA}} \ x^2 - y^2 + 1$

$g'_x = 2x = 0 \iff x = 0$

$g'_y = -2y = 0 \iff y = 0$ NON è pto di OTTIMO

$\max_{\substack{\min \\ y = -1/2 - x}} \ x^2 - y^2 + 1 = \max_{\min} x^2 - \left(\frac{1}{4} + x^2 + x\right) + 1 = \max_{\min} -x + \frac{3}{4}$ NON CI SONO pti di OTTIMO

$\max_{\substack{\min \\ y^2 = 1/4 - x}} \ x^2 - y^2 + 1 = \max_{\min} \ x^2 + x + 3/4 \longrightarrow 2x + 1 = 0$

$\iff \boxed{\begin{array}{l} x = -1/2 \\ y = \sqrt{3}/2 \end{array}}$