PRIMI ESERCIZI 06/02/2014

1 $h(x,y) = \sqrt{2y - x}$

$D = \{ (x,y) \in \mathbb{R}^2 \mid y = \operatorname{sen} x \quad x \in [0; 2\pi] \}$

$E = \{ (x,y) \in \mathbb{R}^2 \mid y \geq 0 , \quad y - \operatorname{sen} x \leq 0 \}$

DOMINIO di h

$f : \mathbb{R} \to \mathbb{R} \quad y \to 2y$ è CONTINUA IN $\mathbb{R}$

per IL Th di COLLEGAMENTO $(x,y) \to 2y \in \mathcal{C}(\mathbb{R}^2)$

$g : \mathbb{R} \to \mathbb{R} \quad x \to x$ è CONTINUA IN $\mathbb{R}$

per IL Th di COLLEGAMENTO $(x,y) \to x \in \mathcal{C}(\mathbb{R}^2)$

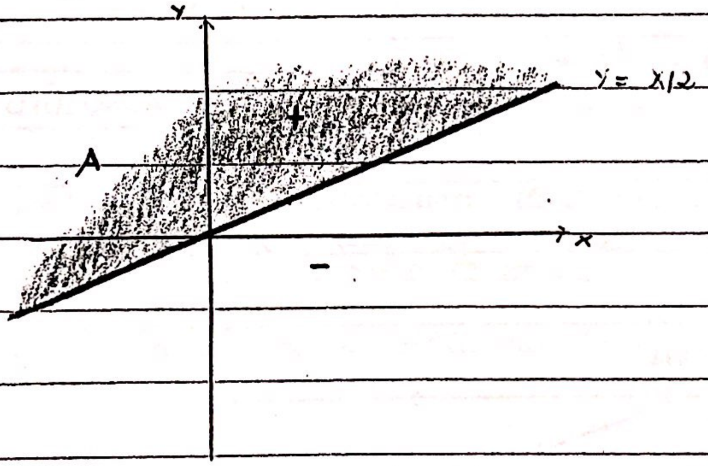
per IL criterio della SOMMA $2y - x \in \mathcal{C}(\mathbb{R}^2)$

per IL Th della COMPOSTA $\sqrt{2y-x} \in \mathcal{C}(\mathbb{R}^2)$

$\Longrightarrow$ POSSO APPLICARE MRC

$A = \{ (x,y) \in \mathbb{R}^2 \mid 2y - x \geq 0 \}$

$2y - x = 0 \iff y = x/2$

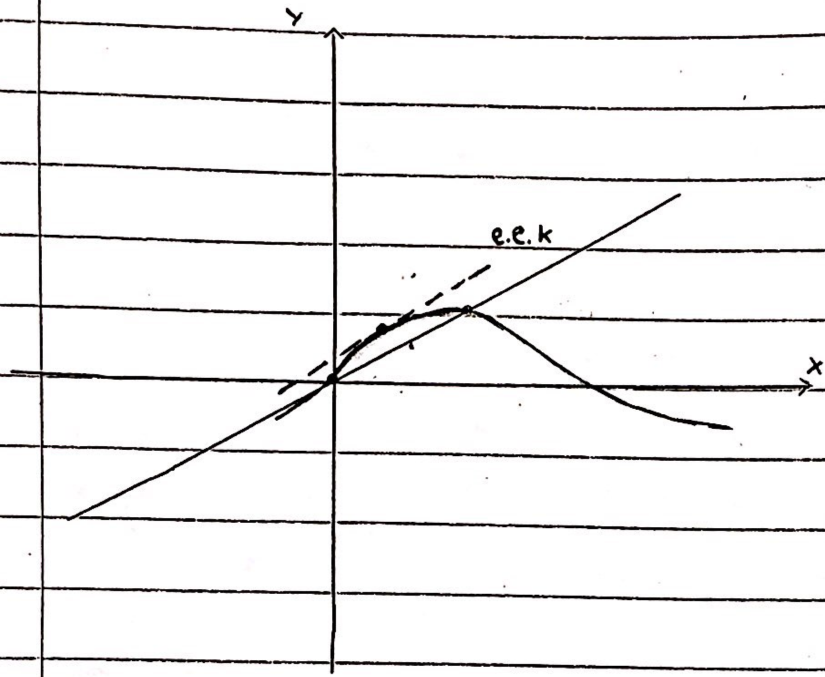


$f(1;1) = 1 > 0$

$f(-1;-1) = -1 < 0$

per la CRESCENZA di $\sqrt{\cdot}$

$$\max_{D}\min \sqrt{2y-x} \equiv \max\min \sqrt{2y-x}\Big|_{D\cap A} \equiv \max_{y=\operatorname{sen} x}\min 2y-x$$



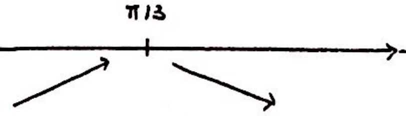
$(2\operatorname{sen}x - x)' \longrightarrow 2\cos x - 1 = 0 \iff \cos x = \frac{1}{2}$

$\iff x = \pi/3$

$y = \operatorname{sen}(\pi/3) = \sqrt{3}/2$

$\boxed{\text{MASSIMO ASSOLUTO } (\pi/3;\ \sqrt{3}/2)}$

$2\cos x - 1 > 0 \iff \cos x > 1/2 \iff x < \pi/3$



$(0;0)$ pto di MINIMO ASSOLUTO

$(\bar{\alpha}, \operatorname{sen}\bar{\alpha})$ pto di MINIMO ASSOLUTO

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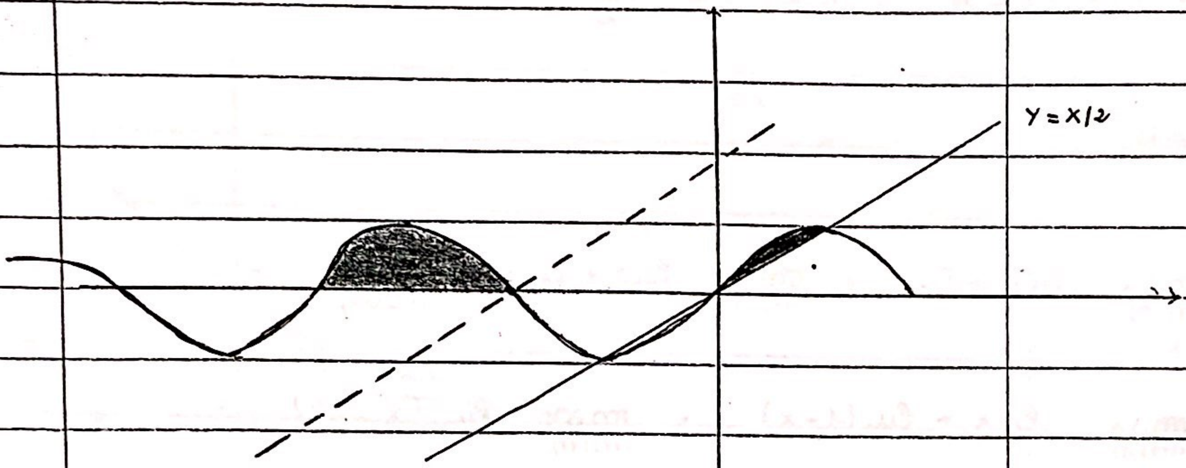
$$\max_{\substack{\min \\ E}} \sqrt{2y-x} = \max_{\min} 2y-x \Big|_{E\cap A}$$



$f(\pi/4;\ 0.5) < 0$
$f(\pi/4;\ 2) > 0$
$f(-\pi/4;\ -0{,}5) > 0$
$f(-\pi/4;\ -1) < 0$

$y \in \mathcal{C}(\mathbb{R}) \longrightarrow$ per Th di COLLEGAMENTO $y \in \mathcal{C}(\mathbb{R}^2)$
$\operatorname{sen} x \in \mathcal{C}(\mathbb{R}) \longrightarrow$ per Th di COLLEGAMENTO $\operatorname{sen} x \in \mathcal{C}(\mathbb{R}^2)$
per CRITERIO SOMMA $y - \operatorname{sen} x \in \mathcal{C}(\mathbb{R}^2)$
$\Rightarrow$ MAC $y - \operatorname{sen} x = 0 \iff y = \operatorname{sen} x$

TUTTI i pti sul SEGMENTO $\square$ SONO pti di MINIMO ASSOLUTO
$(0;0) \wedge (\bar{\alpha};\ \operatorname{sen}\bar{\alpha})$ pti di MINIMO ASSOLUTO

$(\pi/3;\ \sqrt{3}/2)$ pto di MASSIMO relativo

$(\pi\text{-}2\pi k;\ 0)$ pti di MINIMO relativo

$(\pi/3 - 2k\pi;\ \sqrt{3}/2)$ pti di MASSIMO relativo

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$\boxed{2}$ $\max_{\min} \ln x + \ln y$

$F = \{(x,y) \in \mathbb{R}^2 \mid x + y = 1\}$

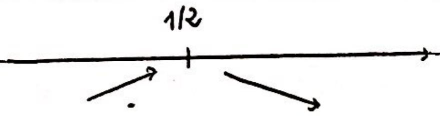
$G = \{(x,y) \in \mathbb{R}^2 \mid x + y \geq 1 \,,\; x^2 + \frac{3}{2} y^2 \leq 1\}$

$A = \mathbb{R}_+ \times \mathbb{R}_+$

$$\max_{\substack{\min \\ F}} \ln x + \ln y = \max_{\min} \ln x + \ln y \Big|_{F \cap DOM} =$$

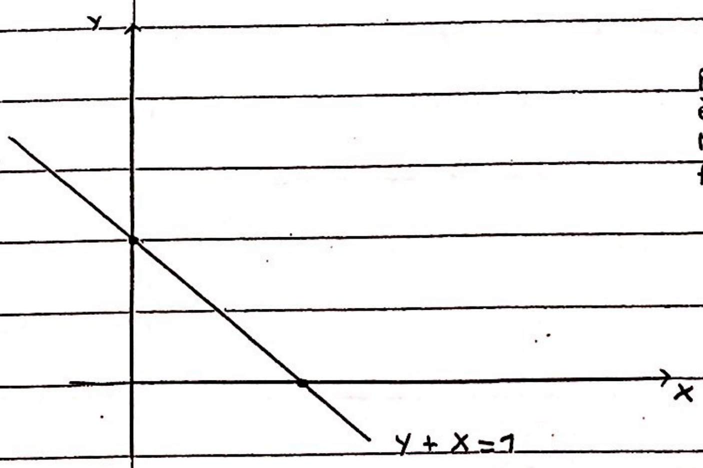
$$\max_{\min} \ln x + \ln(1-x) = \max_{\min} \ln(x - x^2) =$$

$$\max_{\min} x - x^2 \longrightarrow 1 - 2x = 0 \iff \begin{matrix} x = 1/2 \\ y = 1/2 \end{matrix}$$



$1 - 2x > 0 \iff x < 1/2$

$\boxed{(1/2;\ 1/2) \text{ pto di MASSIMO ASSOLUTO}}$



POICHÈ $\ln x + \ln y$ NON è INFERIORMENTE ILLIMITATA NON CI SONO pti di MINIMO ASSOLUTO

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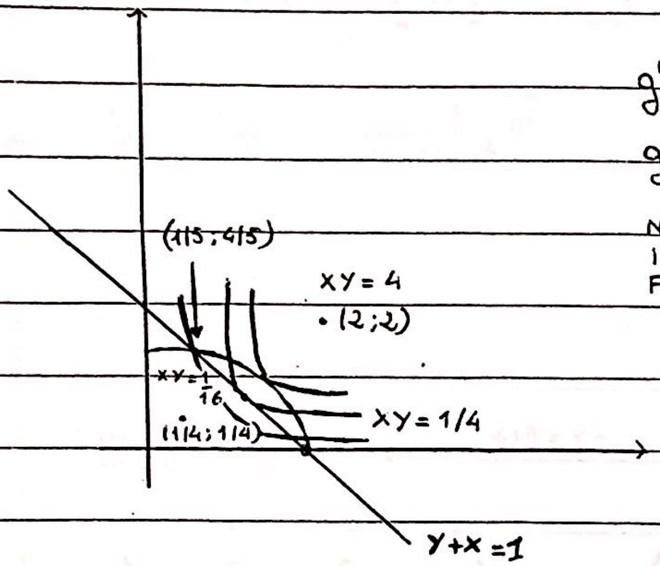
2)

???

$$\max_{\substack{\min \\ g}} \ln x + \ln y = \max_{\substack{\min \\ G \cap DOM}} \ln(x \cdot y) = \max_{\substack{\min \\ G \cap DOM}} xy$$



$g'_x = y = 0$

$g'_y = x = 0$ $\qquad (0;0) \notin G \cap dom$

NON CI SONO pti di OTTIMO INTERNI: GUARDIAMO la FRONTIERA

su $(1/2;1/2)$ NON è NÈ MASSIMO NÈ MINIMO

$$\max_{\substack{\min \\ x^2+\frac{3}{2}y^2=1}} xy = \max_{\substack{\min \\ y=\sqrt{\frac{2}{3}}\sqrt{1-x^2}}} xy = \max_{\min} x \cdot \sqrt{\frac{2}{3}} \cdot \sqrt{1-x^2} =$$

$$\max_{\min} x\sqrt{1-x^2} = \max_{\min} x - x^3 \longrightarrow 1-3x^2 = 0 \iff x = \pm\frac{1}{\sqrt{3}}$$

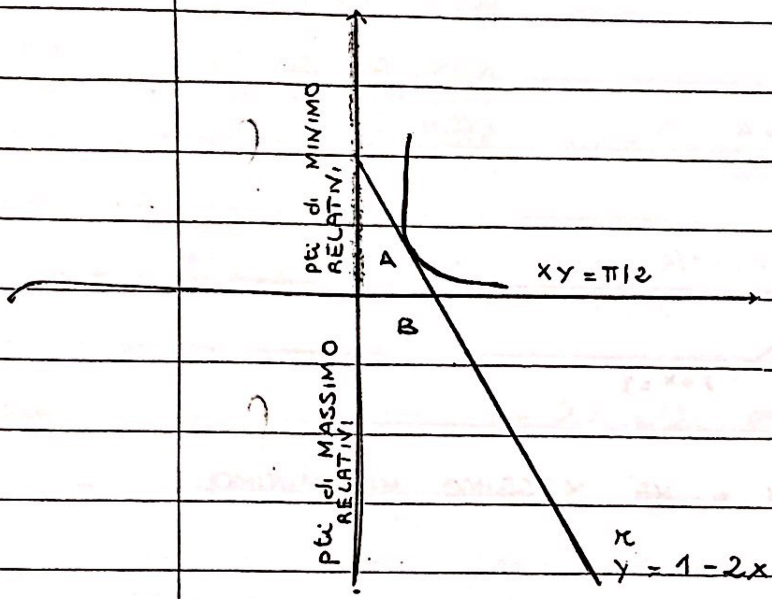
$(1/\sqrt{3}\,;\,2/3)$ pto di MASSIMO ASSOLUTO

$(1/5\,;\,4/5)$ pto di MINIMO relativo

$\boxed{3}$ $\max_{min} \operatorname{sen}(x \cdot y)$ Fatto anche in classe!

$E = \{ (x,y) \in \mathbb{R}^2 \mid 2x + y \leq 1 \ , x \geq 0 \}$

$dom = \mathbb{R}^2$



A chiuso e LIMITATO ; h CONTINUA $\longrightarrow$ ESISTONO MASSIMO e MINIMO ASSOLUTI

MASSIMI ASSOLUTI $\{ (x,y) \in E \mid x \cdot y = \frac{\pi}{2} + 2k\pi \quad k = 0, -1, -2 \ldots \}$

MINIMI ASSOLUTI $\{ (x,y) \in E \mid x \cdot y = -\pi/2 + 2k\pi \quad k = 0, -1, -2 \ldots \}$

$\max_{min} \operatorname{sen}(xy)\big|_r = \max_{min} xy\big|_r = \max_{min} x - 2x^2 \longrightarrow 1 - 4x = 0$ $\boxed{\begin{matrix} x = 1/4 \\ y = 1/2 \end{matrix}}$

MASSIMO ASSOLUTO

$\boxed{4}$ $$\sum_{n=1}^{+\infty} \left(\frac{1}{n} + \frac{1}{n(n+1)}\right)\left(\frac{x}{3x+1}\right)^n$$

PONIAMO $t = \dfrac{x}{3x+1}$

$$\Longrightarrow \sum_{n=1}^{+\infty} \left(\frac{1}{n} + \frac{1}{n(n+1)}\right) t^n$$

$$\lim_{n\to+\infty} \frac{\frac{1}{n+1} + \frac{1}{(n+1)(n+2)}}{\frac{1}{n} + \frac{1}{n(n+1)}} = \lim_{n\to+\infty} \frac{n\cancel{(n+1)}}{n+2} \cdot \frac{n+3}{\cancel{(n+1)}(n+2)}$$

$$= \lim_{n\to+\infty} \frac{n^2}{n^2} = 1$$

$$(-1;1) \subset \Gamma_t \subset [-1;1]$$

per $t=-1$ $\quad \sum \dfrac{1}{n} + \dfrac{1}{n(n+1)} (-1)^n$

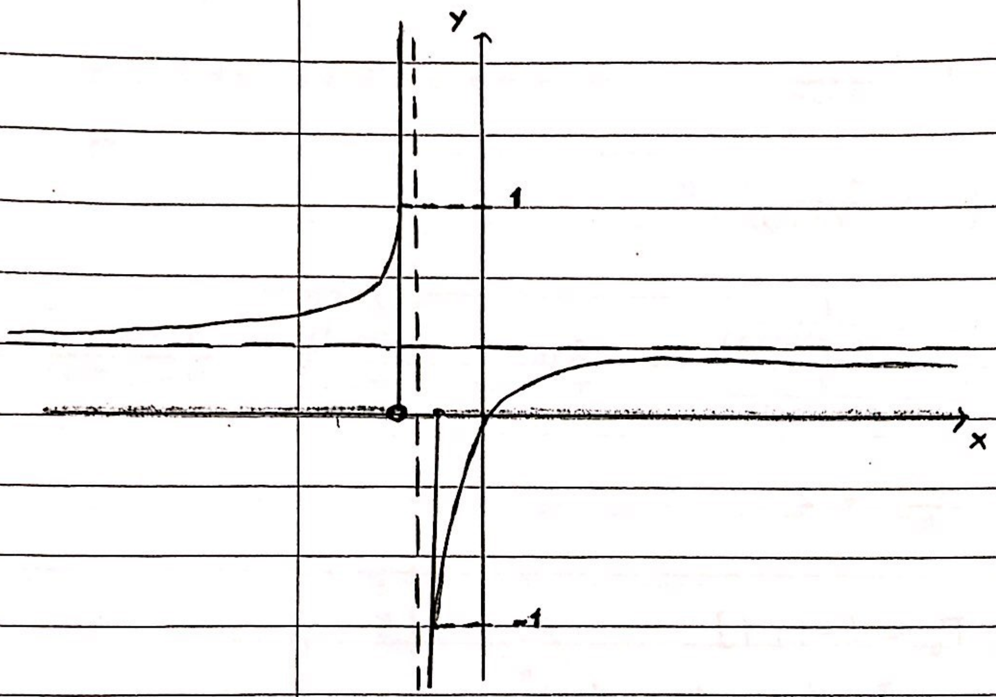
SERIE a TERMINI ALTERNATI
TERMINE n-ESIMO INFINITESIMO
TERMINI DECRESCENTI
} per LEIBNITZ CONVERGE

per $t=1$ $\quad \sum \dfrac{1}{n} + \dfrac{1}{n(n+1)} = \underbrace{\sum \dfrac{1}{n}}_{\text{DIVERGE}} + \sum \dfrac{1}{n(n+1)}$ DIVERGE

$$\longrightarrow \Gamma_t = [-1;1)$$

la SERIE IN t CONVERGE PUNTUALMENTE IN OGNI pto di Γ_t e CONVERGE UNIFORMEMENTE IN OGNI INTERVALLO CHIUSO $[a;b]$ TALE che $-1 \le a \le b < 1$

b- $\frac{x}{3x+1}$



$\frac{x}{3x+1} = -1 \qquad x = -3x - 1 \qquad 4x = -1 \qquad x = -1/4$

$\frac{x}{3x+1} = 1 \qquad x = 3x + 1 \qquad 2x = -1 \qquad x = -1/2$

$$\boxed{\Gamma_x = (-\infty; -1/2) \cup [-1/4; +\infty)}$$

SOMMA:

$$\sum_{n=1}^{+\infty}\left(\frac{1}{n}+\frac{1}{n(n+1)}\right)t^n = \sum_{n=1}^{+\infty}\left(\frac{1}{n}+\frac{1}{n}-\frac{1}{n+1}\right)t^n =$$

$$\sum_{n=1}^{+\infty}\left(\frac{2}{n}-\frac{1}{n+1}\right)t^n = \underbrace{2\sum\frac{t^n}{n}}_{A} - \underbrace{\sum\frac{t^n}{n+1}}_{B}$$

A. $\sum\frac{t^n}{n} = \sum\int_0^t u^{n-1}\,du = \int_0^t \sum_{n=1}^{+\infty} u^{n-1}\,du = \int_0^t \frac{1}{1-u}\,du$

$= -\ln(1-t)$

B: $\sum \frac{1}{t}\frac{t^{n+1}}{n+1} = \sum \int_0^t \frac{1}{t} u^n \, du = \frac{1}{t}\int_0^t \sum u^n \, du =$

$\frac{1}{t}\int_0^t \left(\frac{1}{1-u} - 1\right) du = \frac{1}{t}\left[-\ln(1-t) - t - 1\right] =$

$-\frac{\ln(1-t)}{t} - 1 - \frac{1}{t}$

$S_t = -2\ln(1-t) + \frac{\ln(1-t)}{t} + 1 + \frac{1}{t}$

$$\boxed{S(x) = -2\ln\left(1-\frac{x}{3x+1}\right) + \ln\left(1-\frac{x}{3x+1}\right)\frac{(3x+1)}{x} + 1 + \frac{3x+1}{x}}$$

$\boxed{5}$

$$\iint_E \frac{x^2}{y^2}\,dx\,dy \qquad E = \{(x,y)\in\mathbb{R}^2 \mid y \le x,\ xy \ge 1 \quad 0 \le x \le 2\}$$

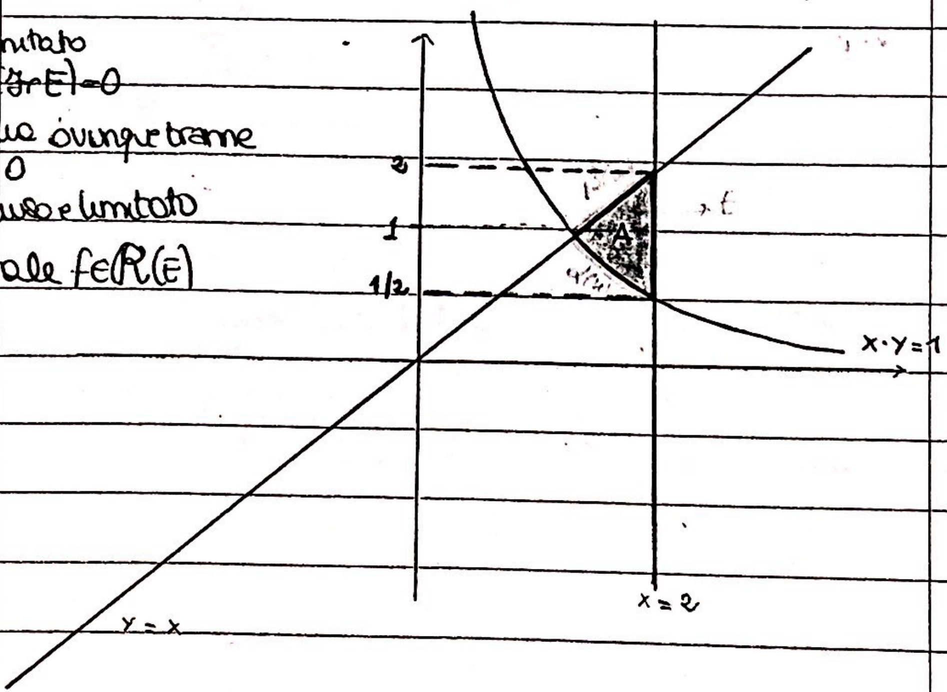
$E \in M$ normale $\to$ limitato
$\to m(\partial_r E) = 0$

la f è continua ovunque tranne che per $y \neq 0$

$f \in C(E)$ chiuso e limitato

$\Rightarrow \exists$ l'integrale $f \in R(E)$

$-y^{-4} = -1/y^4 \to \frac{-4}{y^5}$



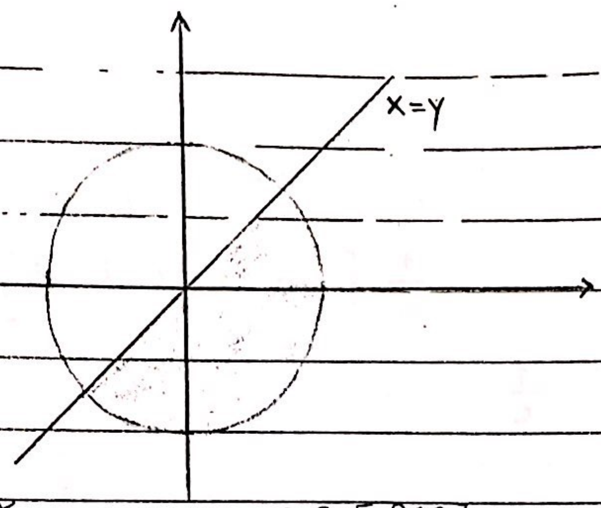
$$\int_1^2 x^2\left[\int_{1/x}^{x} y^{-2}\,dy\right]dx = \int_1^2 x^2\left[-\frac{1}{x} + x\right]dx = \left|-\frac{x^2}{2} + \frac{x^4}{4}\right|_1^2$$

$= \boxed{9/4}$

verifico se si può scambiare l'ordine di integrazione. Sì $\int_{1/2}^{2}\left[\int_{\alpha(x)}^{\beta(x)} dx\right]dy = \int_{1/2}^{1}\left[\int_{1/y}^{2}\frac{x^2}{y^2}dx\right]dy + \int_1^2\left[\int_y^2 \frac{x^2}{y^2}dx\right]dy$

$$\iint_E x^2y^2\,dx\,dy$$

$$E=\{(x,y)\in\mathbb{R}^2 \mid y\le x,\ x^2+y^2\le 4\}$$



$$\begin{cases} x=\rho\cos\theta \\ y=\rho\,\mathrm{sen}\,\theta \end{cases} \qquad \rho\in[0;2] \quad \theta\in[-3/4\pi;\ \pi/4]$$

$$\iint_E \rho^2\cos^2\theta\,\rho^2\,\mathrm{sen}^2\theta\,\rho\,d\rho\,d\theta = \iint_E \rho^5\cos^2\theta\,\mathrm{sen}^2\theta\,d\rho\,d\theta =$$

$$\int_0^2 \rho^5\int_{-3/4\pi}^{\pi/4}\cos^2\theta\,\mathrm{sen}^2\theta\,d\theta = \int_0^2\rho^5\int_{-3/4\pi}^{\pi/4}\frac{1-\cos^2 2\theta}{4}\,d\theta =$$

$$\cos 2\theta = \cos^2\theta-\mathrm{sen}^2\theta \qquad \cos^2\theta=\frac{1+\cos 2\theta}{2}$$

$$\mathrm{sen}^2\theta=\frac{1-\cos 2\theta}{2}$$

$$\int_0^2\rho^5\int_{-3/4\pi}^{\pi/4}\frac{2-1-\cos 4\theta}{8}\,d\theta =$$

$\int\cos 4\theta \quad \frac{\mathrm{sn}\,4\theta}{4}$

VII

$$\int_0^2\rho^5\int_{-3/4\pi}^{\pi/4}\frac{1}{8}-\frac{1}{8}(\cos 4\theta)\,d\theta = \int_0^2\rho^5\left[\frac{\theta}{8}-\frac{1}{32}\,\mathrm{sen}\,4\theta\right]_{-3/4\pi}^{\pi/4} =$$

$\to \frac{\pi}{4}$

$$=\int_0^2\frac{\rho^5}{8}\left|\theta-\frac{\mathrm{sen}\,4\theta}{4}\right|_{-3/4\pi}^{\pi/4}d\rho = \int_0^2\frac{\pi}{8}\rho^5\,d\rho =$$

$$\frac{\pi}{8}\left|\frac{\rho}{6}\right|_0^2 = \frac{32}{3}\cdot\frac{\pi}{8} = \boxed{\frac{4}{3}\pi}$$