19/06/2012

1 SVOLTO $f(x,y) = \sqrt{(x+y)^2(y^2-x^2-1)} \quad \dfrac{x-y^2+1}{\ln(4x^2+y^2-1)}$

$$A = \{(x,y) \in \mathbb{R}^2 \mid (x+y)^2(y^2-x^2-1) \geq 0, \ \ln(4x^2+y^2-1) > 0, \ 4x^2+y^2-1 > 0\}$$

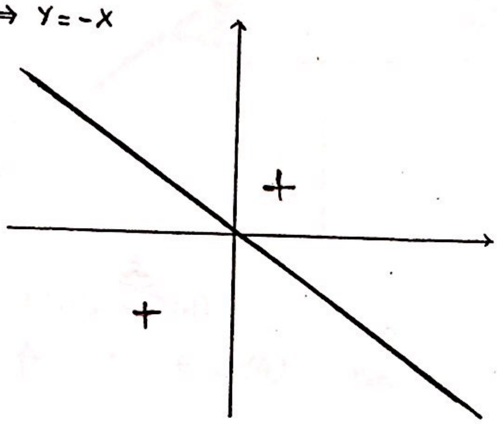
$f_1(x,y) = (x+y)^2$

$f_1 \in \mathcal{C}(\mathbb{R}^2)$ per CRITERIO COLLEGAMENTO

$\Rightarrow$ MRC

$x+y=0 \iff y=-x$

cercare come disegnare:
- ellisse
- iperbole
- parabole
- circonferenze



$f_2(x,y) = y^2-x^2-1$

$f: y \to y^2 \in \mathcal{C}(\mathbb{R})$ per CRITERIO COLLEGAMENTO $\in \mathcal{C}(\mathbb{R}^2)$

$f: x \to -x^2 \in \mathcal{C}(\mathbb{R})$ per CRITERIO COLLEGAMENTO $\in \mathcal{C}(\mathbb{R}^2)$

$f: x \to 1 \in \mathcal{C}(\mathbb{R})$ per CRITERIO COLLEGAMENTO $\in \mathcal{C}(\mathbb{R}^2)$

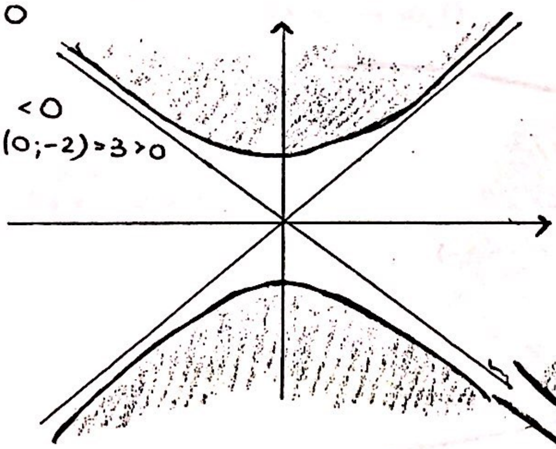
per CRITERIO SOMMA $f_2 \in \mathcal{C}(\mathbb{R}^2)$

$\Rightarrow$ MRC

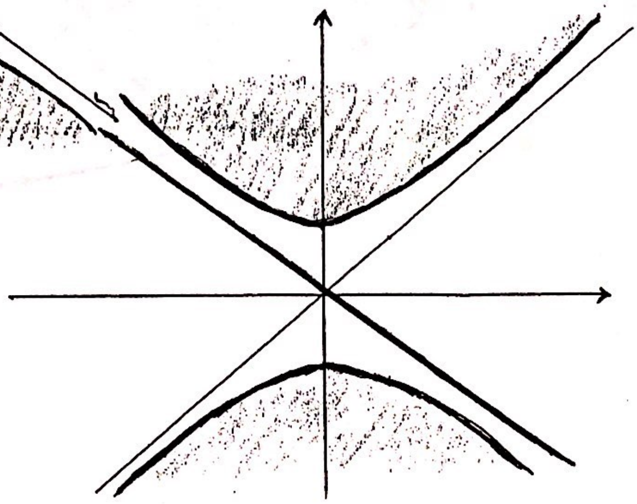
$y^2-x^2-1=0$

$f(0;0) = -1 < 0$

$f(0;2) = f(0;-2) = 3 > 0$



$f_1 \cdot f_2 \geq 0$

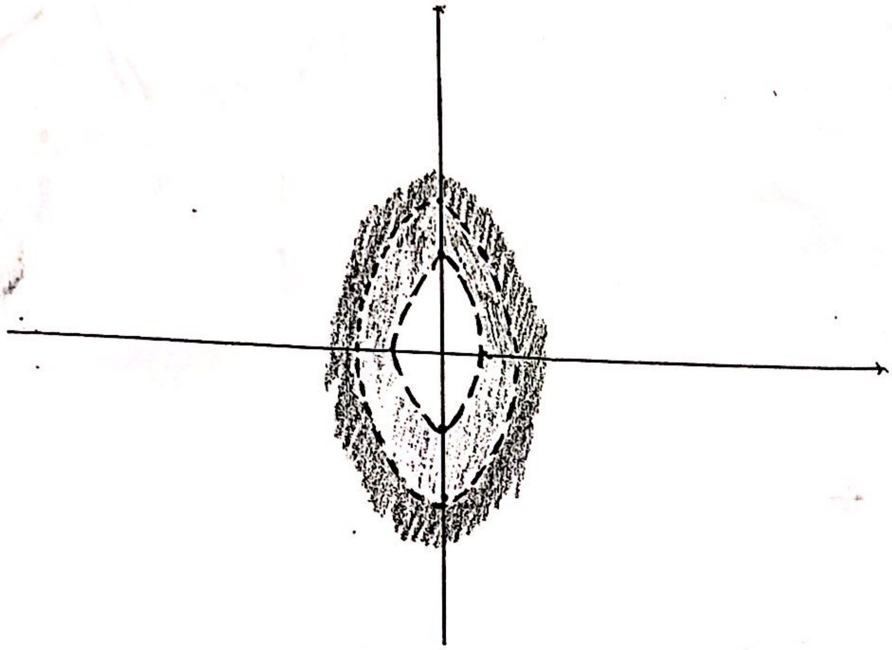


$f_3 = 4x^2 + y^2 - 1$

$f_3 \in \mathcal{C}(\mathbb{R}^2)$ per criterio COLLEGAMENTO - SOMMA

m.R.C.

$f_3 = 0 \iff 4x^2 + y^2 = 1$

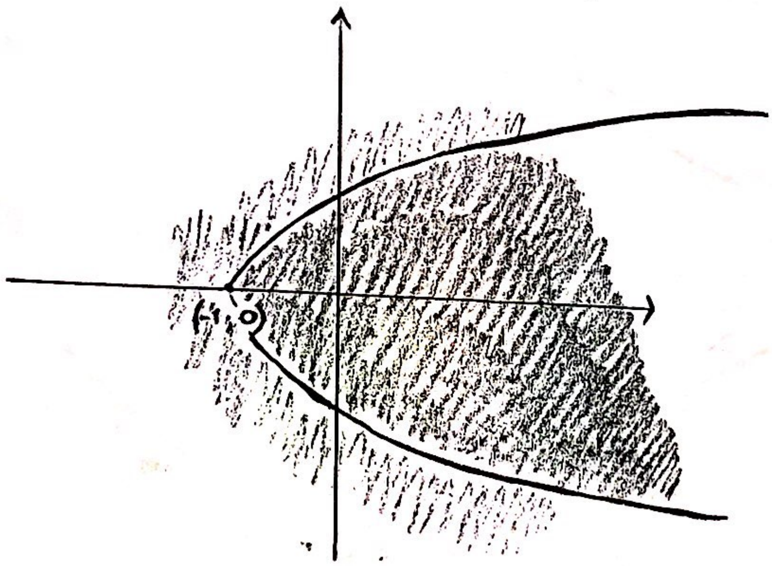


$f_4 = x - y^2 + 1$

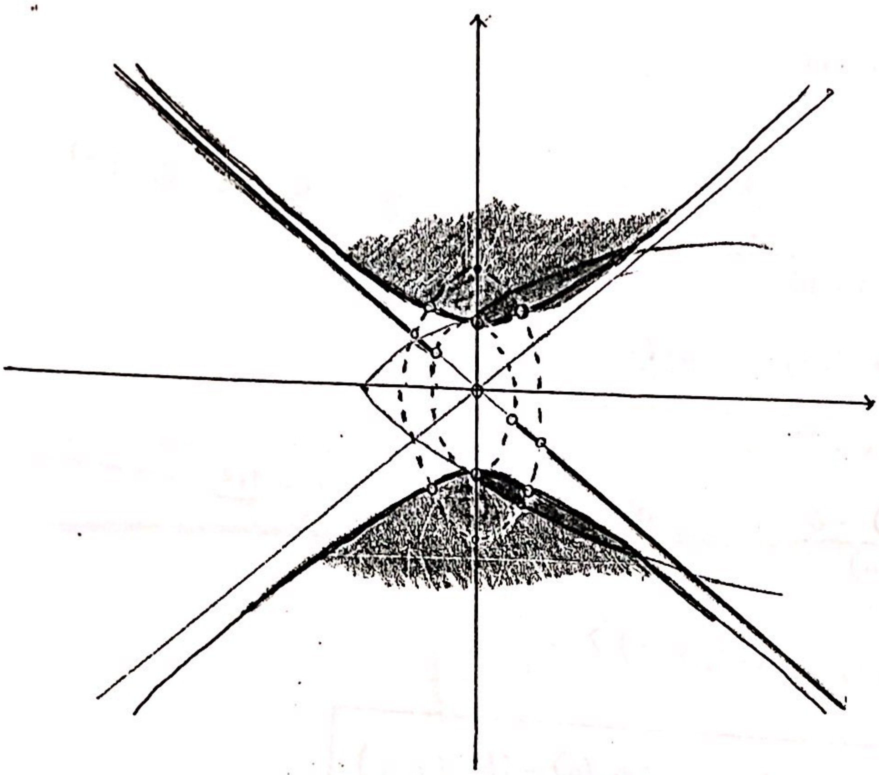
$f_4 \in \mathcal{C}(\mathbb{R}^2)$ per criterio SOMMA - COLLEGAMENTO

$\Rightarrow$ MRC

$x = y^2 - 1$



②



LIMITI: $(0;0) \notin D(A)$

$(-1/2;\,1/2) \in D(A)$ $\qquad \lim_{(-1/2;\,1/2)} f(x,y) = 0$

$(0;1) \in D(A)$ $\qquad \lim_{(x,y)\to(0;1)} \dfrac{\underbrace{\sqrt{(x+y)^2(y^2-x^2-1)}}_{\to 0 \text{ LIMITE COMPOSTA}}\;\;\overbrace{x-y^2+1}^{\to 0 \text{ per COMPOSTA}}}{\underbrace{\ln(\underbrace{4x^2+y^2-1}_{\to 0 \text{ per CONTINUITÀ}})}_{\to -\infty \text{ LIMITE COMPOSTA}}} = 0$

$(0;\sqrt{2}) \in D(A)$ $\qquad \lim_{(x,y)\to(0;\sqrt{2})} \dfrac{\underbrace{\sqrt{(x+y)^2(y^2-x^2-1)}}_{\to \sqrt{2} \text{ COMPOSTA}}\;\;\overbrace{x-y^2+1}^{\to -1 \text{ per CONTINUITÀ}}}{\underbrace{\ln(\underbrace{4x^2+y^2-1}_{\to 1 \text{ per CONTINUITÀ}})}_{\to 0 \text{ per LIMITE COMPOSTA}}} = -\infty$

$\left(\sqrt{\tfrac{1}{5}};\,\sqrt{\tfrac{6}{5}}\right) \in D(A)$

$\lim_{(x,y)\to\left(\sqrt{\frac{1}{5}};\,\sqrt{\frac{6}{5}}\right)} \dfrac{\underbrace{\sqrt{(x+y)^2(y^2-x^2-1)}}_{\to 0 \text{ COMPOSTA}}\;\;\overbrace{x-y^2+1}^{\to \sqrt{1/5}\,-\,1/5}}{\underbrace{\ln(\underbrace{4x^2+y^2-1}_{\to 1 \text{ CONTINUITÀ}})}_{\to 0}}$

IL LIMITE sulla restrizione è ZERO;
La Fz IN UN INTORNO NON è MAI LIMITATA $\longrightarrow$ $\nexists$ LIMITE

$$g(x,y) = \frac{x-y^2+1}{\ln(4x^2+y^2-1)} \qquad (1;1)$$

Th. DIFFERENZIALE TOTALE

$$g'_x = \frac{\ln(4x^2+y^2-1) - \dfrac{8x(x-y^2+1)}{4x^2+y^2-1}}{\ln^2(4x^2+y^2-1)} \qquad g'_x\,,\,g'_y \in \mathcal{C}^1(A)$$

$$g'_y = \frac{-2y\cdot\ln(4x^2+y^2-1) - \dfrac{2y(x-y^2+1)}{4x^2+y^2-1}}{\ln^2(4x^2+y^2-1)}$$

$$g'_x(1;1) = \frac{\ln(4)-2}{\ln^2(4)} \qquad g'_y(1;1) = \frac{-2\ln(4)-1/2}{\ln^2(4)}$$

$$\boxed{z\;=} \;\langle \nabla g(1,1),\,(x-1,\,y-1)\rangle =$$

$$\boxed{\frac{\ln(4)-2}{\ln^2(4)}(x-1) - \frac{2\ln(4)+1/2}{\ln^2(4)}(y-1)}$$

2)

$$\sum_{m=1}^{+\infty}\left(\frac{m+2}{m!}+\frac{1}{m}\right)\left(\frac{x+2}{3x+1}\right)^m$$

$$t = \frac{x+2}{3x+1}$$

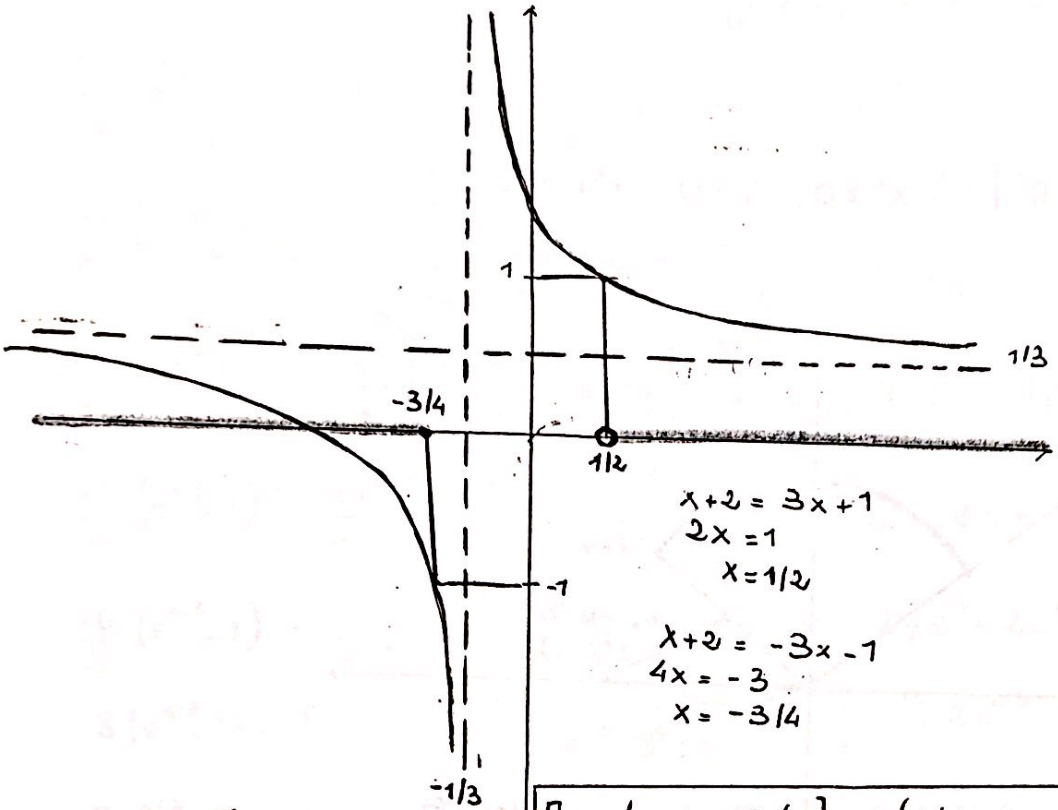
$$\lim_{m\to+\infty}\frac{\dfrac{m+3}{(m+1)!}+\dfrac{1}{(m+1)}}{\dfrac{m+2}{m!}+\dfrac{1}{m}} = \lim_{m\to+\infty}\frac{m}{m+1} = 1$$ per ORDINE di INFINITESIMO

$$(-1;1) \in \Gamma_t \in [-1;1]$$

per $t=-1$ TERMINI a SEGNI ALTERNI, TERMINI DECRESCENTI, TERMINE m-ESIMO INFINITESIMO — per LEIBNIZ CONVERGE

per $t=1$ DIVERGE

$$F = [-1;1)$$



$x+2 = 3x+1$

$2x = 1$

$x = 1/2$

$x+2 = -3x-1$

$4x = -3$

$x = -3/4$

$$\boxed{\Gamma_x = (-\infty ; -3/4] \cup (1/2 ; +\infty)}$$

$$S_t \quad \underbrace{\sum \left(\frac{n+2}{n!}\right) t^n}_{} + \underbrace{\sum \frac{1}{n} t^n}_{-\ln(1-t)}$$

$$= \sum \frac{(n+2)(n+1)\, t^n}{(n+1)!}$$

$$= \sum \frac{n t^n}{n!} + \sum \frac{2 t^n}{n!} = t \sum \frac{t^{n-1}}{(n-1)!} + 2 \sum \frac{t^n}{n!}$$

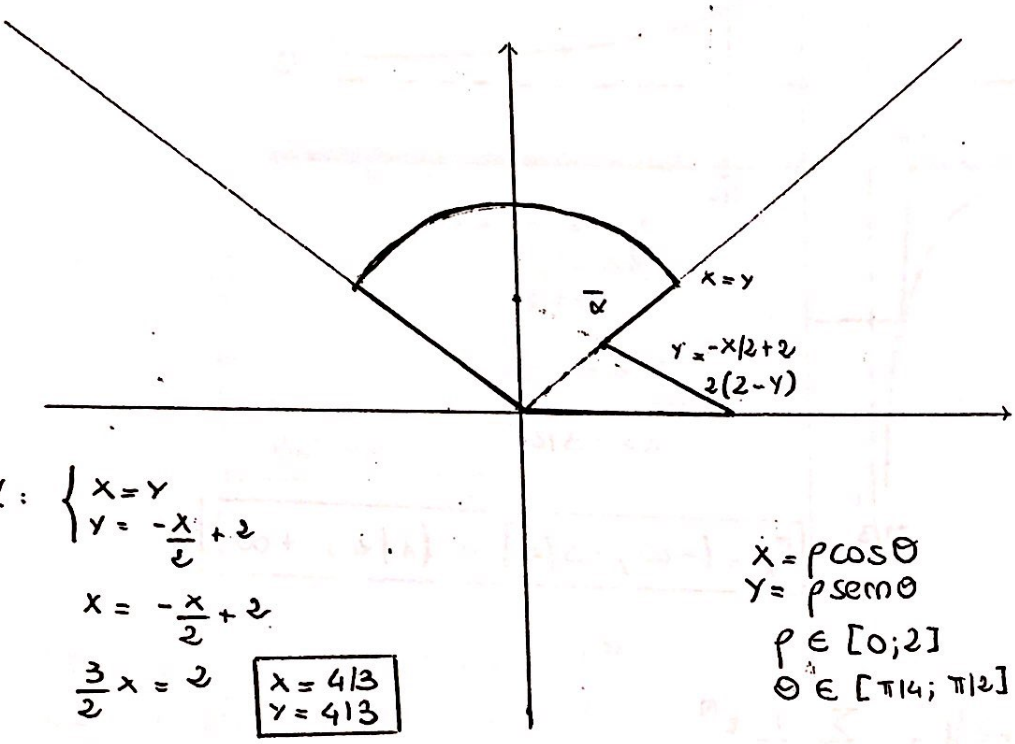
$$= t e^t + 2(e^t - 1)$$

$$\boxed{S_t = t e^t + 2(e^t - 1) - \ln(1-t)}$$

Scanned by CamScanner

③ $\iint_{E\cup F} x(e^y + xy + 1)\, dx\, dy$

$E = \{ (x,y) \in \mathbb{R}^2 \mid y^2 - x^2 \geq 0,\ y \geq 0,\ x^2 + y^2 \leq 4 \}$



$\bar{\alpha}: \begin{cases} x = y \\ y = -\frac{x}{2} + 2 \end{cases}$

$x = -\frac{x}{2} + 2$

$\frac{3}{2}x = 2$ $\boxed{\begin{matrix} x = 4/3 \\ y = 4/3 \end{matrix}}$

$x = \rho\cos\theta$
$y = \rho \operatorname{sen}\theta$
$\rho \in [0;2]$
$\theta \in [\pi/4; \pi/2]$

$$\iint_{E\cup F} xe^y\, dx\, dy + \iint_{E\cup F} x^2 y\, dx\, dy + \iint_{E\cup F} x\, dx\, dy =$$

$$\underbrace{\iint_{\triangle} xe^y\, dx\, dy}_{B} + \underbrace{\iint_{\triangledown} x^2 y\, dx\, dy}_{A} + \underbrace{\iint_{\triangle} x^2 y\, dx\, dy}_{C} + \underbrace{\iint_{\triangleright} x\, dx\, dy}_{D}$$

$$\underline{A}: \ 2\iint_{\triangledown} \rho^4 \cos^2\theta \operatorname{sen}\theta\, d\theta\, d\rho = 2\int_0^2 \rho^4 \left[-\frac{\cos^3\theta}{3} \right]_{\pi/4}^{\pi/2} d\rho = 2\int_0^2 \rho^4 \frac{\sqrt{2}}{12}\, d\rho$$

$$= \frac{\sqrt{2}}{6} \left[\frac{\rho^5}{5} \right]_0^2 = \frac{\sqrt{2}}{6} \cdot \frac{32}{5} = \boxed{\frac{16\sqrt{2}}{15}}$$

B: $$\int_0^{4/3} e^y \left[\int_y^{4-2y} x\,dx \right] dy = \int_0^{4/3} e^y \left[\frac{x^2}{2} \right]_y^{4-2y} dy =$$

$$\int_0^{4/3} \frac{e^y}{2} \left[(4-2y)^2 - y^2 \right] = \frac{1}{2} \int_0^{4/3} e^y (16 - 16y + 3y^2)\,dy =$$

$$\frac{16}{2} \int_0^{4/3} e^y\,dy - \frac{16}{2} \int_0^{4/3} y e^y\,dy + \frac{3}{2} \int_0^{4/3} e^y y^2\,dy =$$

$$\frac{16}{2} \left[e^y \right]_0^{4/3} - \frac{16}{2} \left[y e^y - \int e^y\,dy \right]_0^{4/3} + \frac{3}{2} \left[y^2 e^y - 2\int y e^y\,dy \right]_0^{4/3} =$$

$$\frac{16}{2} (e^{4/3} - 1) - \frac{16}{2} \left[y e^y - e^y \right]_0^{4/3} + \frac{3}{2} \left[y^2 e^y - 2 y e^y + 2 \int e^y\,dy \right]_0^{4/3} =$$

$$\frac{16}{2} (e^{4/3} - 1) - \frac{16}{2} \left(\frac{4}{3} e^{4/3} - e^{4/3} + 1 \right) + \frac{3}{2} \left[y^2 e^y - 2 y e^y + 2 e^y \right]_0^{4/3} =$$

$$8 (e^{4/3} - 1) - 8 \left(\frac{4}{3} e^{4/3} - e^{4/3} + 1 \right) + \frac{3}{2} \left[\frac{16}{9} e^{4/3} - \frac{8}{3} e^{4/3} + 2 e^{4/3} - 2 \right] =$$

$$8e^{4/3} - 8 - \frac{32}{3} e^{4/3} + 8e^{4/3} - 8 + \frac{8}{3} e^{4/3} - 4e^{4/3} + 3e^{4/3} - 3 = \boxed{7e^{4/3} - 19}$$

C: $$\int_0^{4/3} y \left[\int_y^{4-2y} x^2\,dx \right] dy = \int_0^{4/3} y \left[\frac{x^3}{3} \right]_y^{4-2y} dy = \int_0^{4/3} \frac{y}{3} \left[(4-2y)^3 - y^3 \right] dy =$$

$$\frac{1}{3} \int_0^{4/3} y \left(64 - 96y + 48y^2 - 9y^3 \right) dy = \frac{1}{3} \int_0^{4/3} 64y - 96y^2 + 48y^3 - 9y^4\,dy$$

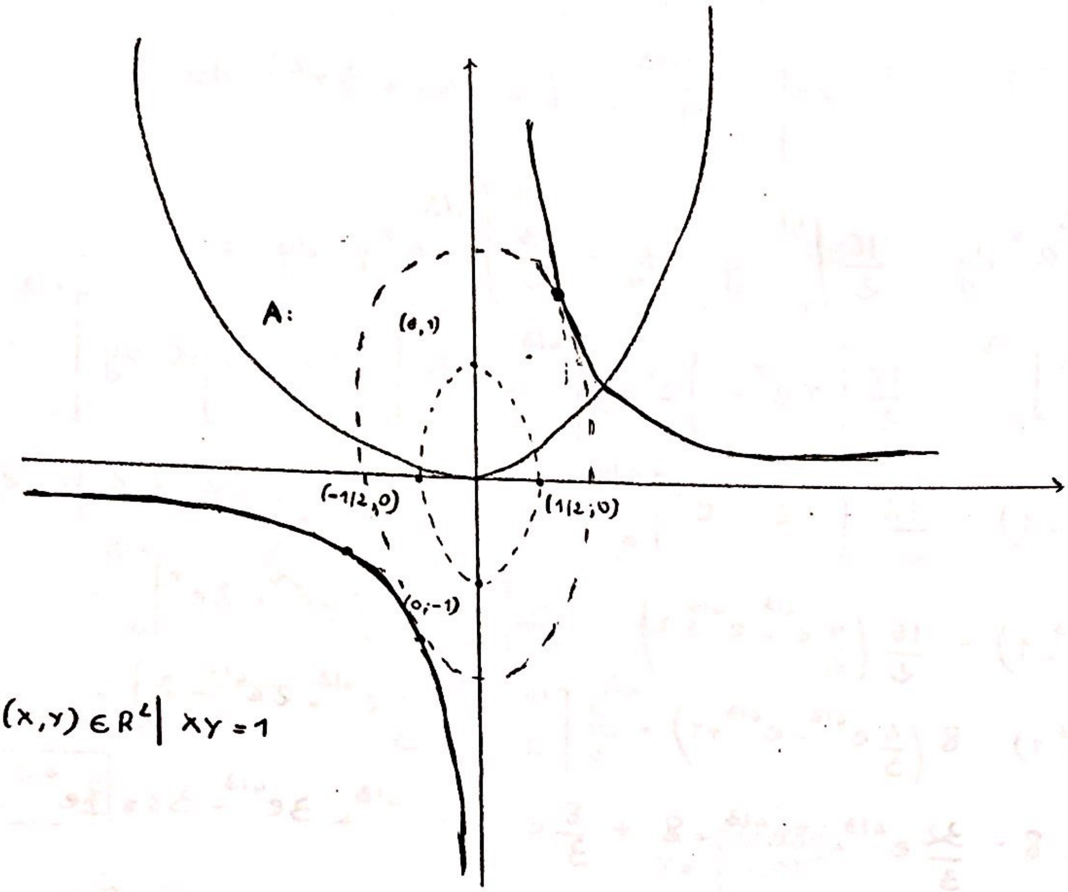
$$= \frac{1}{3} \left[\frac{64y^2}{2} - \frac{96y^3}{3} + \frac{48y^4}{4} - \frac{9y^5}{5} \right]_0^{4/3} = \boxed{\frac{512}{135}}$$

D: $$\int_0^{4/3} \left[\frac{x^2}{2} \right]_y^{4-2y} dy = \frac{1}{2} \int_0^{4/3} (4-2y)^2 - y^2\,dy = \frac{1}{2} \int_0^{4/3} 16 - 16y + 3y^2\,dy$$

$$= \frac{1}{2} \left[16y - \frac{16y^2}{2} + \frac{3y^3}{3} \right]_0^{4/3} = \boxed{\frac{128}{27}}$$

TOT: $\frac{16\sqrt{2}}{15} + 7e^{4/3} - 19 + \frac{512}{135} + \frac{128}{27} =$ [redacted]

④ $h(x,y) = \ln(4x^2+y^2-1)$



$G = \{(x,y) \in \mathbb{R}^2 \mid xy = 1$

$$\equiv \max_{\substack{\min \\ A\cap G}} 4x^2+y^2-1 \equiv \max_{\substack{\min \\ xy=1 \\ y=1/x}} 4x^2+\frac{1}{x^2}-1 \longrightarrow 8x - \frac{2}{x^3} = 0$$

$$\iff \frac{8x^4-2}{x^3} = 0$$

$$\iff x^4 = 1/4$$

$x = \sqrt[4]{1/4} = +\sqrt{1/2}$

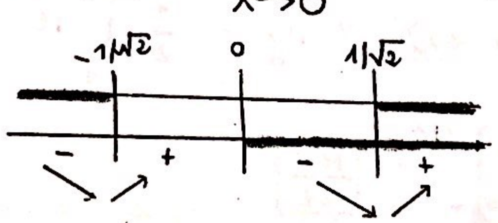
$x = -\sqrt{1/2}$

$\left(\frac{\sqrt{2}}{2}; \sqrt{2}\right)$

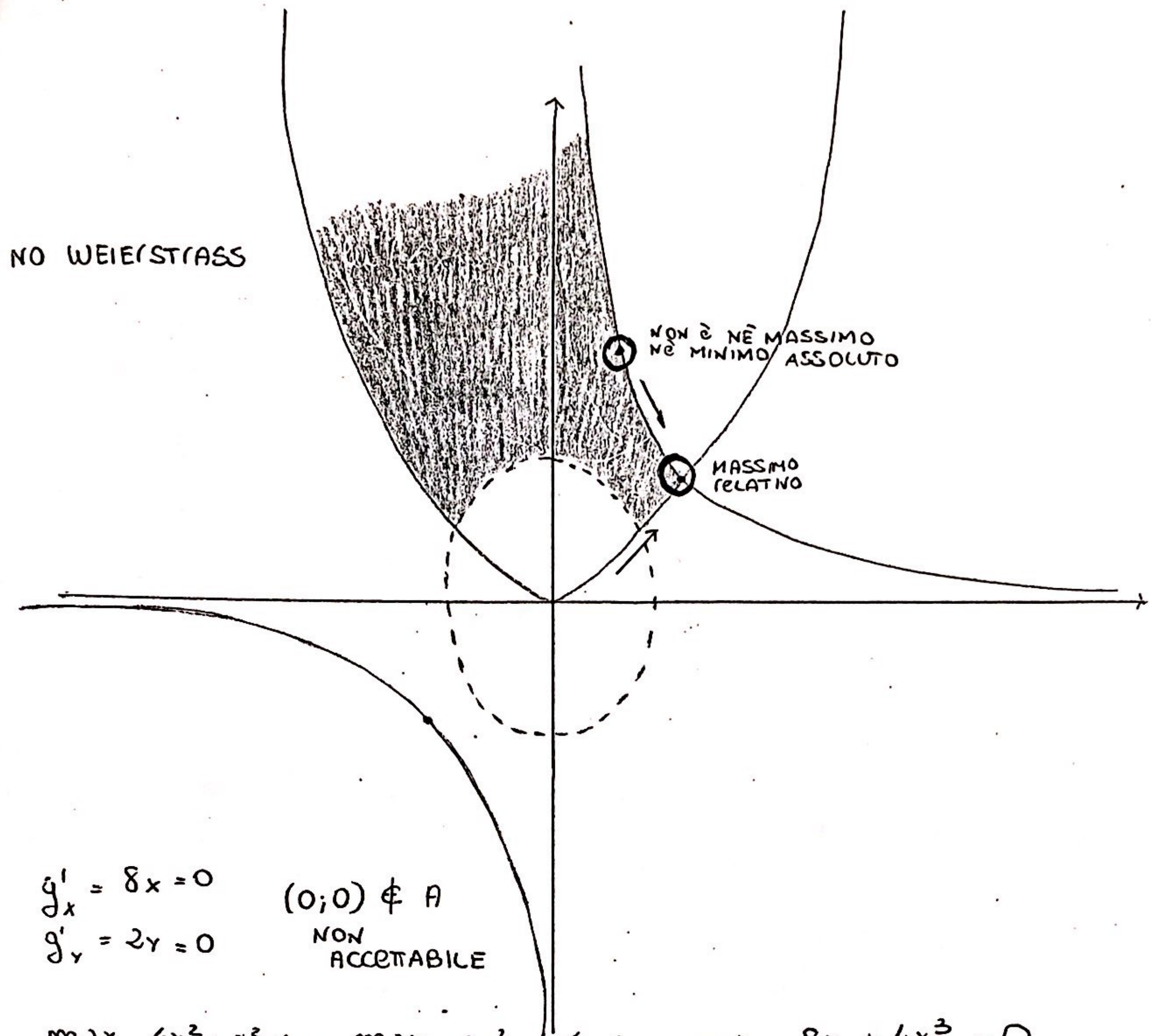
$\left(-\frac{\sqrt{2}}{2}; -\sqrt{2}\right)$

pti di MINIMO ASSOLUTO

$X' > 0 \iff \begin{array}{l} x^4 > 1/4 \\ x^3 > 0 \end{array} \begin{array}{l} \longrightarrow x > 1/\sqrt{2} \ , \ x < -1/\sqrt{2} \\ \longrightarrow x > 0 \end{array}$



∄ MASSIMO ASSOLUTI



$g'_x = 8x = 0$ $\quad (0;0) \notin A$

$g'_y = 2y = 0$ $\quad$ NON ACCETTABILE

$\max_{x^2=y}$ / $\min$ $\; 4x^2 + y^2 - 1 = \max / \min \; 4x^2 + x^4 - 1 \longrightarrow 8x + 4x^3 = 0$

$4x(2 + x^2) = 0$

$x = 0$
$y = 0$ N. A.

$\max_{y=x^2}$ / $\min$ $\; 4y + y^2 - 1 \longrightarrow 4 + 2y = 2 + y$

$8x + 4x^3 > 0 \qquad x(2+x^2) \qquad x > 0$