**Assignment 1**

**Epidemic Modelling report**

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* **Introduction**

In this report, we will evaluate our implemented structures which are adjacency list, adjacency matrix, and incidence matrix in terms of their time complexities for the different operations and different use case scenarios. In each scenario, we will consider and recommend which implementation would be most appropriate. Lastly, we will show the outcomes of SIR epidemic model simulations.

* **Data and experiment setup**

To evaluate our data structures, we need to generate several graphs, so we decided to use Erdos-Renyi and Scale-free graphs which are automatic graph generators. For vertex degree, we decided to use 5, 20, and 50 because each number is more than double of previous number, so we could clearly see the difference of time complexities. For number of vertices, we decided to use 100, 200, and 400 because we noticed that in scenario 2, our laptop stopped running if we have over 500 vertices. We generated 3 graphs on each vertex degree and number of vertices so total 54 graphs are used. The table below lists the graphs of Erdos-Renyi and Scale-free.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Erdos-Renyi** | | **Vertex degree** | | |
|  |  | 5 | 20 | 50 |
| **Number of vertices** | 100 | ER\_V100D5\_1.net  ER\_V100D5\_2.net  ER\_V100D5\_3.net | ER\_V100D20\_1.net  ER\_V100D20\_2.net  ER\_V100D20\_3.net | ER\_V100D50\_1.net  ER\_V100D50\_2.net  ER\_V100D50\_3.net |
| 200 | ER\_V200D5\_1.net  ER\_V200D5\_2.net  ER\_V200D5\_3.net | ER\_V200D20\_1.net  ER\_V200D20\_2.net  ER\_V200D20\_3.net | ER\_V200D50\_1.net  ER\_V200D50\_2.net  ER\_V200D50\_3.net |
| 400 | ER\_V400D5\_1.net  ER\_V400D5\_2.net  ER\_V400D5\_3.net | ER\_V400D20\_1.net  ER\_V400D20\_2.net  ER\_V400D20\_3.net | ER\_V400D50\_1.net  ER\_V400D50\_2.net  ER\_V400D50\_3.net |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Scale-free** | | **Vertex degree** | | |
|  |  | 5 | 20 | 50 |
| **Number of vertices** | 100 | SF\_V100D5\_1.net  SF\_V100D5\_2.net  SF\_V100D5\_3.net | SF\_V100D20\_1.net  SF\_V100D20\_2.net  SF\_V100D20\_3.net | SF\_V100D50\_1.net  SF\_V100D50\_2.net  SF\_V100D50\_3.net |
| 200 | SF\_V200D5\_1.net  SF\_V200D5\_2.net  SF\_V200D5\_3.net | SF\_V200D20\_1.net  SF\_V200D20\_2.net  SF\_V200D20\_3.net | SF\_V200D50\_1.net  SF\_V200D50\_2.net  SF\_V200D50\_3.net |
| 400 | SF\_V400D5\_1.net  SF\_V400D5\_2.net  SF\_V400D5\_3.net | SF\_V400D20\_1.net  SF\_V400D20\_2.net  SF\_V400D20\_3.net | Sf\_V400D50\_1.net  SF\_V400D50\_2.net  SF\_V400D50\_3.net |

* **Use case scenarios**
* **Scenario 1 k-hop Neighbourhoods**

We decided to use 100 vertices for this scenario 1. The table below lists the times(seconds) of all of vertex’s k-hop neighbourhoods depend on size of k in adjacency list, adjacency matrix, and incidence matrix. We used both of Erdos-Renyi and Scale-free graphs for this scenario 1.

**Erdos-Renyi**

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | **Adjacency List** | | | | **Adjacency Matrix** | | | | **Incidence Matrix** | | | |
| **K** | First | Second | Third | Average | First | Second | Third | Average | First | Second | Third | Average |
|  | Degree 5 | | | | Degree 5 | | | | Degree 5 | | | |
| **3** | 0.105945 | 0.108925 | 0.124525 | 0.113132 | 0.084837 | 0.089869 | 0.088036 | 0.087581 | 0.167027 | 0.161263 | 0.159749 | 0.16268 |
| **10** | 0.129712 | 0.145878 | 0.137364 | 0.137651 | 0.123017 | 0.116049 | 0.115376 | 0.118147 | 0.258946 | 0.258996 | 0.257628 | 0.258523 |
| **25** | 0.154767 | 0.149092 | 0.153192 | 0.15235 | 0.129962 | 0.120649 | 0.125314 | 0.125308 | 0.269799 | 0.268799 | 0.265986 | 0.268195 |
|  | Degree 20 | | | | Degree 20 | | | | Degree 20 | | | |
| **3** | 0.173594 | 0.179942 | 0.174148 | 0.175895 | 0.110774 | 0.108834 | 0.115145 | 0.111584 | 0.509809 | 0.514442 | 0.513502 | 0.512584 |
| **10** | 0.180183 | 0.182342 | 0.179822 | 0.180782 | 0.130278 | 0.121130 | 0.119361 | 0.12359 | 0.539576 | 0.534681 | 0.522795 | 0.532351 |
| **25** | 0.185209 | 0.188447 | 0.183234 | 0.18563 | 0.130952 | 0.132574 | 0.128128 | 0.130551 | 0.544079 | 0.535796 | 0.540846 | 0.54024 |
|  | Degree 50 | | | | Degree 50 | | | | Degree 50 | | | |
| **3** | 0.265968 | 0.264703 | 0.269987 | 0.266886 | 0.122173 | 0.119973 | 0.125307 | 0.122484 | 0.833497 | 0.815988 | 0.852216 | 0.8339 |
| **10** | 0.269983 | 0.276835 | 0.277250 | 0.274689 | 0.125672 | 0.129153 | 0.130141 | 0.128322 | 0.906098 | 0.864765 | 0.893622 | 0.888162 |
| **25** | 0.295229 | 0.289343 | 0.289521 | 0.291364 | 0.126829 | 0.135923 | 0.139324 | 0.134025 | 0.949582 | 0.896224 | 0.931961 | 0.925922 |

**Scale-free**

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | **Adjacency List** | | | | **Adjacency Matrix** | | | | **Incidence Matrix** | | | |
| **K** | First | Second | Third | Average | First | Second | Third | Average | First | Second | Third | Average |
|  | Degree 5 | | | | Degree 5 | | | | Degree 5 | | | |
| **3** | 0.051676 | 0.053018 | 0.055041 | 0.053245 | 0.057376 | 0.068977 | 0.059845 | 0.062066 | 0.077556 | 0.081852 | 0.079657 | 0.079688 |
| **10** | 0.059549 | 0.060125 | 0.062533 | 0.060736 | 0.065868 | 0.070605 | 0.063944 | 0.066806 | 0.083243 | 0.085906 | 0.094976 | 0.088042 |
| **25** | 0.063027 | 0.067995 | 0.068092 | 0.066371 | 0.099508 | 0.075323 | 0.068739 | 0.08119 | 0.088117 | 0.093446 | 0.098134 | 0.093232 |
|  | Degree 20 | | | | Degree 20 | | | | Degree 20 | | | |
| **3** | 0.113201 | 0.113527 | 0.115795 | 0.114174 | 0.083313 | 0.082175 | 0.084492 | 0.083327 | 0.194403 | 0.197976 | 0.179677 | 0.190685 |
| **10** | 0.131369 | 0.128459 | 0.125843 | 0.128557 | 0.088562 | 0.084545 | 0.095764 | 0.089624 | 0.210518 | 0.209424 | 0.193544 | 0.204495 |
| **25** | 0.133828 | 0.138044 | 0.132032 | 0.134635 | 0.091339 | 0.098799 | 0.107227 | 0.099122 | 0.223187 | 0.211119 | 0.211068 | 0.215125 |
|  | Degree 50 | | | | Degree 50 | | | | Degree 50 | | | |
| **3** | 0.166205 | 0.156013 | 0.168567 | 0.163595 | 0.099107 | 0.089886 | 0.098688 | 0.095894 | 0.306954 | 0.280184 | 0.333832 | 0.30699 |
| **10** | 0.178598 | 0.173632 | 0.179835 | 0.177355 | 0.106341 | 0.099161 | 0.100629 | 0.102044 | 0.358003 | 0.311036 | 0.371904 | 0.346981 |
| **25** | 0.181879 | 0.179263 | 0.189662 | 0.183601 | 0.113484 | 0.116145 | 0.114555 | 0.114728 | 0.403437 | 0.339668 | 0.395831 | 0.379645 |

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| With Erdos-Renyi graphs, we can cleary see that time increases as vertex degree and k incresases. We can also see adjacency matrix is the most efficient graph and incidence matrix is the most inefficient graph. | | |
|  |  |  |

With Scale-free graphs, we can cleary see that time increases as vertex degree and k incresases. We can also see adjacency matrix is the most efficient graph and incidence matrix is the most inefficient graph.

* **Scenario 2 Dynamic Contact Conditions**

The table below lists the times(seconds) of non-existent edge additions and existent edges deletions depend on number of vertices and vertex degree in adjacency list, adjacency matrix, and incidence matrix. We used both of Erdos-Renyi and Scale-free graphs for this scenario 2.

**Erdos-Renyi (edge additions)**

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | **Adjacency List** | | | | **Adjacency Matrix** | | | | **Incidence Matrix** | | | |
| **V** | **First** | **Second** | **Third** | **Average** | **First** | **Second** | **Third** | **Average** | **First** | **Second** | **Third** | **Average** |
|  | Degree 5 | | | | Degree 5 | | | | Degree 5 | | | |
| **100** | 0.040029 | 0.042051 | 0.041132 | 0.041071 | 0.020275 | 0.021445 | 0.023556 | 0.021758 | 3.603995 | 3.992512 | 4.01123 | 3.869245 |
| **200** | 0.154132 | 0.161135 | 0.148223 | 0.154496 | 0.032213 | 0.035521 | 0.037729 | 0.035154 | 98.15533 | 97.33571 | 98.66154 | 98.05086 |
| **400** | 2.004663 | 2.023521 | 2.155332 | 2.061172 | 0.150833 | 0.144668 | 0.145663 | 0.147054 | 3053.929 | 3049.223 | 3055.663 | 3052.938 |
|  | Degree 20 | | | | Degree 20 | | | | Degree 20 | | | |
| **100** | 0.044023 | 0.040555 | 0.045552 | 0.043376 | 0.017162 | 0.015223 | 0.016633 | 0.016339 | 3.526291 | 3.475023 | 3.523511 | 3.508275 |
| **200** | 0.154218 | 0.163225 | 0.157003 | 0.158148 | 0.032115 | 0.031126 | 0.030882 | 0.031374 | 95.55213 | 94.77283 | 95.11203 | 95.14566 |
| **400** | 2.341528 | 2.315523 | 2.255317 | 2.304122 | 0.139422 | 0.142535 | 0.140552 | 0.140836 | 3003.223 | 3006.315 | 3009.713 | 3006.417 |
|  | Degree 50 | | | | Degree 50 | | | | Degree 50 | | | |
| **100** | 0.049112 | 0.043921 | 0.050223 | 0.047752 | 0.010691 | 0.012334 | 0.013552 | 0.012192 | 3.040098 | 3.12255 | 3.20096 | 3.121202 |
| **200** | 0.162153 | 0.165523 | 0.169552 | 0.165742 | 0.030552 | 0.029915 | 0.027715 | 0.029394 | 92.33851 | 91.88273 | 91.99532 | 92.07218 |
| **400** | 2.900091 | 2.912355 | 2.733532 | 2.848659 | 0.137235 | 0.129532 | 0.136523 | 0.13443 | 2952.209 | 2950.556 | 2947.617 | 2950.127 |

**Scale-free (edge additions)**

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | **Adjacency List** | | | | **Adjacency Matrix** | | | | **Incidence Matrix** | | | |
| **V** | **First** | **Second** | **Third** | **Average** | **First** | **Second** | **Third** | **Average** | **First** | **Second** | **Third** | **Average** |
|  | Degree 5 | | | | Degree 5 | | | | Degree 5 | | | |
| **100** | 0.036019 | 0.039015 | 0.038441 | 0.037825 | 0.019511 | 0.014892 | 0.015873 | 0.016758 | 3.512366 | 3.597725 | 3.578261 | 3.562784 |
| **200** | 0.152236 | 0.158823 | 0.150221 | 0.15376 | 0.028398 | 0.030186 | 0.027352 | 0.028645 | 96.16769 | 95.68264 | 96.22756 | 96.02596 |
| **400** | 1.969892 | 1.989732 | 2.007511 | 1.989045 | 0.147236 | 0.142612 | 0.148763 | 0.146203 | 3001.236 | 2998.435 | 3000.211 | 2999.961 |
|  | Degree 20 | | | | Degree 20 | | | | Degree 20 | | | |
| **100** | 0.042123 | 0.040216 | 0.036147 | 0.039495 | 0.015839 | 0.014162 | 0.014416 | 0.014805 | 3.525001 | 3.30016 | 3.211166 | 3.345442 |
| **200** | 0.156186 | 0.155198 | 0.153675 | 0.155019 | 0.025568 | 0.026018 | 0.023601 | 0.025062 | 92.58861 | 92.19605 | 92.66355 | 92.48273 |
| **400** | 2.105862 | 2.096632 | 2.208753 | 2.137082 | 0.140682 | 0.137856 | 0.137572 | 0.138703 | 2955.361 | 2952.981 | 2955.356 | 2954.566 |
|  | Degree 50 | | | | Degree 50 | | | | Degree 50 | | | |
| **100** | 0.045868 | 0.043175 | 0.045763 | 0.044935 | 0.011993 | 0.012236 | 0.011088 | 0.011772 | 3.398541 | 3.192648 | 3.145379 | 3.245522 |
| **200** | 0.159836 | 0.158752 | 0.156962 | 0.158516 | 0.021052 | 0.019965 | 0.018583 | 0.019866 | 89.11623 | 89.55816 | 90.37562 | 89.68333 |
| **400** | 2.736816 | 2.719756 | 2.708826 | 2.721799 | 0.106827 | 0.118967 | 0.097532 | 0.107775 | 2900.009 | 2901.664 | 2901.935 | 2901.202 |

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| With Erdos-Renyi graphs, we can clearly see that time increases as vertex and degree increases in adjacency list, but time increases as vertex increases and time decreases as degree increases in adjacency matrix and incidence matrix. We can also see adjacency matrix is the most efficient and incidence matrix is the most inefficient. | | |
|  |  |  |
| With Scale-free graphs, we can clearly see that time increases as vertex and degree increases in adjacency list, but time increases as vertex increases and time decreases as degree increases in adjacency matrix and incidence matrix. We can also see adjacency matrix is the most efficient and incidence matrix is the most inefficient. | | |

**Erdos-Renyi (edge deletions)**

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | **Adjacency List** | | | | **Adjacency Matrix** | | | | **Incidence Matrix** | | | |
| **V** | **First** | **Second** | **Third** | **Average** | **First** | **Second** | **Third** | **Average** | **First** | **Second** | **Third** | **Average** |
|  | Degree 5 | | | | Degree 5 | | | | Degree 5 | | | |
| **100** | 0.026715 | 0.020158 | 0.024872 | 0.023915 | 0.013643 | 0.014226 | 0.013032 | 0.013633 | 0.108641 | 0.107853 | 0.110862 | 0.109118 |
| **200** | 0.051132 | 0.050123 | 0.051003 | 0.050752 | 0.040238 | 0.045102 | 0.039901 | 0.041747 | 0.356918 | 0.349862 | 0.351152 | 0.352644 |
| **400** | 0.255015 | 0.260385 | 0.258331 | 0.257911 | 0.155321 | 0.149263 | 0.150362 | 0.151648 | 2.437728 | 2.581223 | 2.608635 | 2.542528 |
|  | Degree 20 | | | | Degree 20 | | | | Degree 20 | | | |
| **100** | 0.028192 | 0.027897 | 0.029085 | 0.028391 | 0.014763 | 0.014826 | 0.015077 | 0.014888 | 0.265847 | 0.276365 | 0.281746 | 0.274652 |
| **200** | 0.058993 | 0.061003 | 0.059766 | 0.059921 | 0.048173 | 0.049777 | 0.048871 | 0.048940 | 1.658839 | 1.705826 | 1.717775 | 1.694146 |
| **400** | 0.283452 | 0.269374 | 0.269337 | 0.274054 | 0.164335 | 0.161326 | 0.165523 | 0.163728 | 12.52632 | 11.55399 | 12.00864 | 12.02965 |
|  | Degree 50 | | | | Degree 50 | | | | Degree 50 | | | |
| **100** | 0.029985 | 0.038167 | 0.031156 | 0.033102 | 0.017395 | 0.016677 | 0.018636 | 0.017569 | 0.998321 | 1.087673 | 1.108635 | 1.064876 |
| **200** | 0.071773 | 0.077372 | 0.080611 | 0.076585 | 0.052237 | 0.050289 | 0.055372 | 0.052632 | 6.812352 | 6.836525 | 6.855213 | 6.834696 |
| **400** | 0.333871 | 0.323528 | 0.333127 | 0.330175 | 0.171738 | 0.185335 | 0.177727 | 0.178266 | 64.38723 | 64.99826 | 63.11382 | 64.16643 |

**Scale-free (edge deletions)**

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | **Adjacency List** | | | | **Adjacency Matrix** | | | | **Incidence Matrix** | | | |
| **V** | **First** | **Second** | **Third** | **Average** | **First** | **Second** | **Third** | **Average** | **First** | **Second** | **Third** | **Average** |
|  | Degree 5 | | | | Degree 5 | | | | Degree 5 | | | |
| **100** | 0.023051 | 0.021053 | 0.021113 | 0.021739 | 0.011976 | 0.011327 | 0.012005 | 0.011769 | 0.059228 | 0.067327 | 0.063216 | 0.063257 |
| **200** | 0.046923 | 0.046813 | 0.046933 | 0.046889 | 0.038922 | 0.039921 | 0.036128 | 0.038323 | 0.131892 | 0.127832 | 0.129336 | 0.129686 |
| **400** | 0.250135 | 0.248773 | 0.249111 | 0.249339 | 0.148775 | 0.145726 | 0.145558 | 0.146686 | 0.859912 | 0.922359 | 0.933651 | 0.905307 |
|  | Degree 20 | | | | Degree 20 | | | | Degree 20 | | | |
| **100** | 0.024634 | 0.025112 | 0.023995 | 0.024580 | 0.014155 | 0.012932 | 0.012633 | 0.01324 | 0.099239 | 0.102532 | 0.113066 | 0.104945 |
| **200** | 0.054113 | 0.054898 | 0.053882 | 0.054297 | 0.039983 | 0.041153 | 0.041262 | 0.040799 | 0.062603 | 0.071152 | 0.065512 | 0.066422 |
| **400** | 0.259332 | 0.265523 | 0.263316 | 0.262723 | 0.149555 | 0.150333 | 0.152553 | 0.150813 | 4.527381 | 5.022135 | 4.983621 | 4.844379 |
|  | Degree 50 | | | | Degree 50 | | | | Degree 50 | | | |
| **100** | 0.025593 | 0.025983 | 0.026643 | 0.026073 | 0.015663 | 0.016012 | 0.015266 | 0.015647 | 0.212102 | 0.200235 | 0.208853 | 0.207063 |
| **200** | 0.055113 | 0.059321 | 0.061123 | 0.058519 | 0.042733 | 0.044623 | 0.048765 | 0.045373 | 1.660552 | 1.677392 | 1.703321 | 1.680421 |
| **400** | 0.287371 | 0.295673 | 0.288831 | 0.290625 | 0.155531 | 0.156693 | 0.164321 | 0.158848 | 16.63523 | 15.99382 | 16.25599 | 16.29501 |

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| With Erdos-Renyi graphs, we can clearly see that time increases as vertex and degree increases. We can also see adjacency matrix is the most efficient and incidence matrix is the most inefficient. | | |
|  |  |  |
| With Scale-free graphs, we can clearly see that time increases as vertex and degree increases. We can also see adjacency matrix is the most efficient and incidence matrix is the most inefficient. | | |

* **Scenario 3 Dynamic People Tracing**

The table below lists the times(seconds) of non-existent vertex additions and existent vertex deletions depend on number of vertices and vertex degree in adjacency list, adjacency matrix, and incidence matrix. We used both of Erdos-Renyi and Scale-free graphs for this scenario 3.

**Erdos-Renyi (vertex additions)**

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | **Adjacency List** | | | | **Adjacency Matrix** | | | | **Incidence Matrix** | | | |
| **V** | **First** | **Second** | **Third** | **Average** | **First** | **Second** | **Third** | **Average** | **First** | **Second** | **Third** | **Average** |
|  | Degree 5 | | | | Degree 5 | | | | Degree 5 | | | |
| **100** | 0.001418 | 0.001315 | 0.001471 | 0.001401 | 0.023491 | 0.022123 | 0.020331 | 0.021981 | 0.020856 | 0.030331 | 0.029355 | 0.026847 |
| **200** | 0.001573 | 0.001587 | 0.001610 | 0.00159 | 0.083455 | 0.086331 | 0.091223 | 0.087003 | 0.041345 | 0.041002 | 0.041667 | 0.041338 |
| **400** | 0.003265 | 0.003156 | 0.003621 | 0.003347 | 0.456712 | 0.448828 | 0.436891 | 0.447477 | 0.263093 | 0.278832 | 0.265513 | 0.269146 |
|  | Degree 20 | | | | Degree 20 | | | | Degree 20 | | | |
| **100** | 0.001053 | 0.001321 | 0.001533 | 0.001302 | 0.021393 | 0.022354 | 0.020713 | 0.021486 | 0.022353 | 0.025332 | 0.024391 | 0.024025 |
| **200** | 0.001511 | 0.001487 | 0.001495 | 0.001497 | 0.085521 | 0.086245 | 0.091522 | 0.087762 | 0.085321 | 0.088292 | 0.092211 | 0.088608 |
| **400** | 0.003116 | 0.003898 | 0.003156 | 0.00339 | 0.444019 | 0.445883 | 0.458981 | 0.449627 | 0.752391 | 0.723769 | 0.808873 | 0.761677 |
|  | Degree 50 | | | | Degree 50 | | | | Degree 50 | | | |
| **100** | 0.001235 | 0.001311 | 0.001326 | 0.001290 | 0.023935 | 0.021223 | 0.022385 | 0.022514 | 0.098231 | 0.113626 | 0.107742 | 0.106533 |
| **200** | 0.001493 | 0.001522 | 0.001501 | 0.001505 | 0.085992 | 0.083112 | 0.095599 | 0.088234 | 0.203341 | 0.199235 | 0.200061 | 0.200879 |
| **400** | 0.003671 | 0.003701 | 0.003072 | 0.003481 | 0.441365 | 0.471362 | 0.411092 | 0.441273 | 1.793828 | 2.038964 | 1.977736 | 1.936842 |

**Scale-free (vertex additions)**

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | **Adjacency List** | | | | **Adjacency Matrix** | | | | **Incidence Matrix** | | | |
| **V** | **First** | **Second** | **Third** | **Average** | **First** | **Second** | **Third** | **Average** | **First** | **Second** | **Third** | **Average** |
|  | Degree 5 | | | | Degree 5 | | | | Degree 5 | | | |
| **100** | 0.000917 | 0.000957 | 0.000941 | 0.000938 | 0.021003 | 0.020607 | 0.021125 | 0.020911 | 0.021263 | 0.019553 | 0.020551 | 0.020455 |
| **200** | 0.001146 | 0.001151 | 0.001148 | 0.001148 | 0.082115 | 0.083189 | 0.081889 | 0.082397 | 0.035291 | 0.037993 | 0.036579 | 0.036621 |
| **400** | 0.003548 | 0.004168 | 0.003827 | 0.003847 | 0.437123 | 0.440162 | 0.439772 | 0.439019 | 0.265729 | 0.255271 | 0.261937 | 0.260979 |
|  | Degree 20 | | | | Degree 20 | | | | Degree 20 | | | |
| **100** | 0.000926 | 0.000933 | 0.000986 | 0.000948 | 0.020514 | 0.021902 | 0.020761 | 0.021059 | 0.023185 | 0.022371 | 0.021331 | 0.022295 |
| **200** | 0.001175 | 0.001149 | 0.001183 | 0.001169 | 0.082038 | 0.083116 | 0.080682 | 0.081945 | 0.078238 | 0.080128 | 0.076621 | 0.078329 |
| **400** | 0.003827 | 0.003916 | 0.004012 | 0.003918 | 0.439816 | 0.441725 | 0.440081 | 0.440540 | 0.492731 | 0.482538 | 0.491176 | 0.488815 |
|  | Degree 50 | | | | Degree 50 | | | | Degree 50 | | | |
| **100** | 0.000911 | 0.000981 | 0.000923 | 0.000938 | 0.020191 | 0.021883 | 0.021077 | 0.021050 | 0.039625 | 0.042059 | 0.048128 | 0.043270 |
| **200** | 0.001152 | 0.001117 | 0.001206 | 0.001158 | 0.083168 | 0.081169 | 0.081992 | 0.082109 | 0.115612 | 0.106389 | 0.119728 | 0.113909 |
| **400** | 0.003951 | 0.004091 | 0.003716 | 0.003919 | 0.437765 | 0.431758 | 0.440117 | 0.436546 | 1.205382 | 1.186352 | 1.217553 | 1.203095 |

|  |  |  |
| --- | --- | --- |
|  |  |  |
| With Erdos-Renyi graphs, we can clearly see that time increases as vertex and degree increases in incedence matrix, but time increases as vertex increases and time remains constant as degree increases in adjacency matrix and incidence matrix. We can also see adjacency list is the most efficient and incidence matrix is the most inefficient. | | |
|  |  |  |
| With Scale-free graphs, we can clearly see that time increases as vertex and degree increases in incedence matrix, but time increases as vertex increases and time remains constant as degree increases in adjacency matrix and incidence matrix. We can also see adjacency list is the most efficient and incidence matrix is the most inefficient. | | |

**Erdos-Renyi (vertex deletions)**

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | **Adjacency List** | | | | **Adjacency Matrix** | | | | **Incidence Matrix** | | | |
| **V** | **First** | **Second** | **Third** | **Average** | **First** | **Second** | **Third** | **Average** | **First** | **Second** | **Third** | **Average** |
|  | Degree 5 | | | | Degree 5 | | | | Degree 5 | | | |
| **100** | 0.003513 | 0.002538 | 0.003185 | 0.003078 | 0.010322 | 0.013929 | 0.012557 | 0.012269 | 0.044526 | 0.046816 | 0.045721 | 0.045687 |
| **200** | 0.008375 | 0.009736 | 0.007993 | 0.008701 | 0.035827 | 0.040887 | 0.041773 | 0.039495 | 0.090132 | 0.094736 | 0.087619 | 0.090829 |
| **400** | 0.019836 | 0.020572 | 0.024137 | 0.021515 | 0.077734 | 0.077583 | 0.074936 | 0.076751 | 0.557287 | 0.608625 | 0.631735 | 0.599215 |
|  | Degree 20 | | | | Degree 20 | | | | Degree 20 | | | |
| **100** | 0.009513 | 0.010552 | 0.009166 | 0.009743 | 0.012358 | 0.013216 | 0.013872 | 0.013148 | 0.185736 | 0.173827 | 0.176882 | 0.178815 |
| **200** | 0.012957 | 0.014882 | 0.013997 | 0.013945 | 0.042876 | 0.043882 | 0.041772 | 0.042843 | 0.960283 | 0.967271 | 0.948819 | 0.958791 |
| **400** | 0.043882 | 0.041867 | 0.045717 | 0.043822 | 0.079886 | 0.081635 | 0.083765 | 0.081762 | 6.876628 | 7.278691 | 7.189352 | 7.114890 |
|  | Degree 50 | | | | Degree 50 | | | | Degree 50 | | | |
| **100** | 0.012258 | 0.011732 | 0.010889 | 0.011626 | 0.014882 | 0.014521 | 0.014687 | 0.014696 | 0.793856 | 0.824658 | 0.786251 | 0.801588 |
| **200** | 0.021981 | 0.020875 | 0.019714 | 0.020856 | 0.044897 | 0.045776 | 0.044886 | 0.045186 | 5.672189 | 5.729947 | 5.620017 | 5.674051 |
| **400** | 0.087583 | 0.087652 | 0.091625 | 0.088953 | 0.084527 | 0.089372 | 0.087652 | 0.087183 | 45.50827 | 46.21883 | 45.87251 | 45.86653 |

**Scale-free (vertex deletions)**

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | **Adjacency List** | | | | **Adjacency Matrix** | | | | **Incidence Matrix** | | | |
| **V** | **First** | **Second** | **Third** | **Average** | **First** | **Second** | **Third** | **Average** | **First** | **Second** | **Third** | **Average** |
|  | Degree 5 | | | | Degree 5 | | | | Degree 5 | | | |
| **100** | 0.001161 | 0.001458 | 0.001358 | 0.001325 | 0.009251 | 0.009971 | 0.010557 | 0.009926 | 0.030238 | 0.027989 | 0.024172 | 0.027466 |
| **200** | 0.006238 | 0.005382 | 0.005751 | 0.005790 | 0.033286 | 0.034772 | 0.035152 | 0.034403 | 0.071372 | 0.080636 | 0.084561 | 0.078856 |
| **400** | 0.014836 | 0.013782 | 0.013997 | 0.014205 | 0.074192 | 0.071251 | 0.069928 | 0.071790 | 0.424838 | 0.442173 | 0.437613 | 0.434874 |
|  | Degree 20 | | | | Degree 20 | | | | Degree 20 | | | |
| **100** | 0.002313 | 0.002155 | 0.001972 | 0.002146 | 0.012931 | 0.012185 | 0.011959 | 0.012358 | 0.076172 | 0.082734 | 0.080726 | 0.079877 |
| **200** | 0.010327 | 0.011727 | 0.012007 | 0.011353 | 0.040581 | 0.039971 | 0.041667 | 0.040739 | 0.473816 | 0.483726 | 0.489162 | 0.482234 |
| **400** | 0.021893 | 0.022157 | 0.023115 | 0.022388 | 0.075836 | 0.075581 | 0.078921 | 0.076779 | 3.641623 | 3.573218 | 3.522178 | 3.579006 |
|  | Degree 50 | | | | Degree 50 | | | | Degree 50 | | | |
| **100** | 0.004957 | 0.004488 | 0.004571 | 0.004672 | 0.013896 | 0.013726 | 0.013266 | 0.013629 | 0.167382 | 0.176525 | 0.182185 | 0.175364 |
| **200** | 0.013867 | 0.014898 | 0.014266 | 0.014343 | 0.043183 | 0.039172 | 0.048551 | 0.043635 | 1.298367 | 1.326389 | 1.366321 | 1.330359 |
| **400** | 0.031258 | 0.032258 | 0.033118 | 0.032211 | 0.079112 | 0.081129 | 0.080612 | 0.080284 | 15.48762 | 14.99275 | 15.32816 | 15.26951 |

|  |  |  |
| --- | --- | --- |
|  |  |  |
| With Erdos-Renyi graphs, we can clearly see that time increases as vertex and degree increases. We can also see adjacency list is the most efficient and incidence matrix is the most inefficient. | | |
|  |  |  |
| With Scale-free graphs, we can clearly see that time increases as vertex and degree increases. We can also see adjacency list is the most efficient and incidence matrix is the most inefficient. | | |

* **SIR Model Epidemic Simulation**

To evaluate what effect the parameters of graph type, infection and recover probabilities have on the spread of the epidemic in the SIR model, we decided to use ER\_V200D20\_1.net and SF\_V200D20\_1.net graphs. We also decided to use adjacency matrix and incidence matrix. The table below lists the times(seconds) of SIR model simulation.

* **Seed Initialisation**

**Erdos-Renyi**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Data Structure** | **Seed Initialisation** | **Infection Probability** | **Recover Probability** | **Iteration** | **Time** |
| Adjacency Matrix | 1;10;20;30;40;50;60;70;80;90;100;110;120 | 0.9 | 0.1 | 12 | 0.0938724 |
| Adjacency Matrix | 1;10;20;30;40;50;60 | 0.9 | 0.1 | 10 | 0.0849762 |
| Adjacency Matrix | 1;10;20 | 0.9 | 0.1 | 8 | 0.0809861 |

**Scale-free**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Data Structure** | **Seed Initialisation** | **Infection Probability** | **Recover Probability** | **Iteration** | **Time** |
| Adjacency Matrix | 1;10;20;30;40;50;60;70;80;90;100;110;120 | 0.9 | 0.1 | 11 | 0.0899374 |
| Adjacency Matrix | 1;10;20;30;40;50;60 | 0.9 | 0.1 | 11 | 0.0735229 |
| Adjacency Matrix | 1;10;20 | 0.9 | 0.1 | 8 | 0.0722583 |

**Erdos-Renyi**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Data Structure** | **Seed Initialisation** | **Infection Probability** | **Recover Probability** | **Iteration** | **Time** |
| Incidence Matrix | 1;10;20;30;40;50;60;70;80;90;100;110;120 | 0.9 | 0.1 | 11 | 0.1482197 |
| Incidence Matrix | 1;10;20;30;40;50;60 | 0.9 | 0.1 | 10 | 0.1261612 |
| Incidence Matrix | 1;10;20 | 0.9 | 0.1 | 9 | 0.1108523 |

**Scale-free**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Data Structure** | **Seed Initialisation** | **Infection Probability** | **Recover Probability** | **Iteration** | **Time** |
| Incidence Matrix | 1;10;20;30;40;50;60;70;80;90;100;110;120 | 0.9 | 0.1 | 13 | 0.1062831 |
| Incidence Matrix | 1;10;20;30;40;50;60 | 0.9 | 0.1 | 9 | 0.0985572 |
| Incidence Matrix | 1;10;20 | 0.9 | 0.1 | 9 | 0.0815534 |

We can clearly see that time increases as number of seed initialisation increases. We can also see adjacency matrix is more efficient and scale-free graph type is more efficient.

* **Infection Probability**

**Erdos-Renyi**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Data Structure** | **Seed Initialisation** | **Infection Probability** | **Recover Probability** | **Iteration** | **Time** |
| Adjacency Matrix | 1;10;20;30;40;50;60;70;80;90;100;110;120 | 0.9 | 0.1 | 12 | 0.0938724 |
| Adjacency Matrix | 1;10;20;30;40;50;60;70;80;90;100;110;120 | 0.5 | 0.1 | 9 | 0.0888092 |
| Adjacency Matrix | 1;10;20;30;40;50;60;70;80;90;100;110;120 | 0.1 | 0.1 | 8 | 0.0811853 |

**Scale-free**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Data Structure** | **Seed Initialisation** | **Infection Probability** | **Recover Probability** | **Iteration** | **Time** |
| Adjacency Matrix | 1;10;20;30;40;50;60;70;80;90;100;110;120 | 0.9 | 0.1 | 11 | 0.0899374 |
| Adjacency Matrix | 1;10;20;30;40;50;60;70;80;90;100;110;120 | 0.5 | 0.1 | 9 | 0.0847815 |
| Adjacency Matrix | 1;10;20;30;40;50;60;70;80;90;100;110;120 | 0.1 | 0.1 | 8 | 0.0707726 |

**Erdos-Renyi**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Data Structure** | **Seed Initialisation** | **Infection Probability** | **Recover Probability** | **Iteration** | **Time** |
| Incidence Matrix | 1;10;20;30;40;50;60;70;80;90;100;110;120 | 0.9 | 0.1 | 11 | 0.1482197 |
| Incidence Matrix | 1;10;20;30;40;50;60;70;80;90;100;110;120 | 0.5 | 0.1 | 10 | 0.136763 |
| Incidence Matrix | 1;10;20;30;40;50;60;70;80;90;100;110;120 | 0.1 | 0.1 | 8 | 0.1186552 |

**Scale-free**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Data Structure** | **Seed Initialisation** | **Infection Probability** | **Recover Probability** | **Iteration** | **Time** |
| Incidence Matrix | 1;10;20;30;40;50;60;70;80;90;100;110;120 | 0.9 | 0.1 | 13 | 0.1062831 |
| Incidence Matrix | 1;10;20;30;40;50;60;70;80;90;100;110;120 | 0.5 | 0.1 | 11 | 0.101432 |
| Incidence Matrix | 1;10;20;30;40;50;60;70;80;90;100;110;120 | 0.1 | 0.1 | 8 | 0.099862 |

We can clearly see that time increases as infection probability increases. We can also see adjacency matrix is more efficient and scale-free graph type is more efficient.

* **Recover Probability**

**Erdos-Renyi**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Data Structure** | **Seed Initialisation** | **Infection Probability** | **Recover Probability** | **Iteration** | **Time** |
| Adjacency Matrix | 1;10;20;30;40;50;60;70;80;90;100;110;120 | 0.9 | 0.1 | 11 | 0.1482197 |
| Adjacency Matrix | 1;10;20;30;40;50;60;70;80;90;100;110;120 | 0.9 | 0.5 | 8 | 0.1160183 |
| Adjacency Matrix | 1;10;20;30;40;50;60;70;80;90;100;110;120 | 0.9 | 0.9 | 6 | 0.0967524 |

**Scale-free**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Data Structure** | **Seed Initialisation** | **Infection Probability** | **Recover Probability** | **Iteration** | **Time** |
| Adjacency Matrix | 1;10;20;30;40;50;60;70;80;90;100;110;120 | 0.9 | 0.1 | 13 | 0.1062831 |
| Adjacency Matrix | 1;10;20;30;40;50;60;70;80;90;100;110;120 | 0.9 | 0.5 | 7 | 0.0711635 |
| Adjacency Matrix | 1;10;20;30;40;50;60;70;80;90;100;110;120 | 0.9 | 0.9 | 5 | 0.0577827 |

**Erdos-Renyi**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Data Structure** | **Seed Initialisation** | **Infection Probability** | **Recover Probability** | **Iteration** | **Time** |
| Incidence Matrix | 1;10;20;30;40;50;60;70;80;90;100;110;120 | 0.9 | 0.1 | 11 | 0.1482197 |
| Incidence Matrix | 1;10;20;30;40;50;60;70;80;90;100;110;120 | 0.9 | 0.5 | 7 | 0.1008163 |
| Incidence Matrix | 1;10;20;30;40;50;60;70;80;90;100;110;120 | 0.9 | 0.9 | 6 | 0.0977362 |

**Scale-free**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Data Structure** | **Seed Initialisation** | **Infection Probability** | **Recover Probability** | **Iteration** | **Time** |
| Incidence Matrix | 1;10;20;30;40;50;60;70;80;90;100;110;120 | 0.9 | 0.1 | 13 | 0.1062831 |
| Incidence Matrix | 1;10;20;30;40;50;60;70;80;90;100;110;120 | 0.9 | 0.5 | 7 | 0.0872357 |
| Incidence Matrix | 1;10;20;30;40;50;60;70;80;90;100;110;120 | 0.9 | 0.9 | 6 | 0.0716356 |

We can clearly see that time decreases as recover probability increases. We can also see adjacency matrix is more efficient and scale-free graph type is more efficient.

* **Conclusion**

To evaluate our implemented structures in terms of their time complexities for the different operations and different use case scenarios, we used Erdos-Renyi and Scale-free graph generator. In scenario 1 which is k-hop neighbourhoods, with both of Erdos-Renyi and Scale-free graph types, we can see that time increases as vertex degree and k incresases. We recommend to use adjacency matrix because it is the most efficient data structure in every aspect in terms of time complexity. In scenario 2 which is dynamic contact conditions, with both of Erdos-Renyi and Scale-free graph types, we can see that time increases as vertex and degree increases in adjacency list, but time increases as vertex increases and time decreases as degree increases in adjacency matrix and incidence matrix. We recommend to use adjacency matrix because it is the most efficient data structure in every aspect in terms of time complexity. In scenario 3 which is dynamic people tracing, with both of Erdos-Renyi and Scale-free graph types, we can see that time increases as vertex and degree increases. We recommend to use adjacency list because it is the most efficient data structure in every aspect in terms of time complexity. Lastly, in SIR model epidemic simulation, we can see that time increases as number of seed initialisation increases, infection probability increases, and recover probability decreases. We recommend to use adjacency matrix and scale-free graph type because these are the most efficient data structure in every aspect in terms of time complexity.