LEARNING A FOUR STATE MEALY MACHINE WITH $L^{\#}$

We show how the $L^{\#}$ algorithm learns a Mealy machine that outputs 1 if the number of both a's and b's that have occurred is even, and 0 otherwise.

The $L^{\#}$ algorithm constructs the observation tree $\mathcal{T}$ of Figure 5 by performing the steps below. Note that this is just one possible run of the algorithm as the rules may be applied in different orders and the teacher may provide different counterexamples.

(1) The initial observation tree has a single state 1, which constitutes the basis.
(2) Rule (R2) is applied twice to explore the outgoing transitions of state 1, for inputs a and b, leading to new frontier states 2 and 3, respectively.
(3) Since the frontier has no isolated states and the basis is complete, the learner applies rule (R4) and submits a first hypothesis $\mathcal{H}_1$ to the teacher:

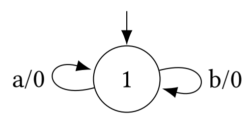

Fig. 1. Hypothesis $\mathcal{H}_1$

Suppose the teacher returns the (informative) counterexample $a\ a\ b\ b\ a\ b\ a\ b$. Then the observation tree is extended with a corresponding path with new states 4 to 10. Since the first conflict already occurs on the frontier, $a \vdash 1\#2$, counterexample processing finishes immediately. The remainder of the counterexample is ignored (for the moment).

(4) Rule (R1) is applied to add state 2 to the basis.
(5) Rule (R2) is applied to explore the outgoing transition of state 2 for inputs b, leading to new frontier state 11.
(6) Rule (R3) is applied with witness a to identify frontier states 3, 4 and 11, leading to new states 12, 13 and 14, respectively.
(7) Rule (R4) is applied with the hypothesis $\mathcal{H}_2$ from Figure 2.

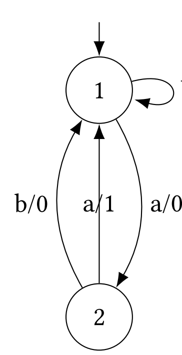

Fig. 2. Hypothesis $\mathcal{H}_2$

The learner observes that $\mathcal{H}_2$ is not consistent with the observation tree: input sequence $a\ a\ b\ b$ provides a counterexample. The learner can analyze this counterexample and there is no need to forward $\mathcal{H}_2$ to the teacher! After $a\ a$ the frontier is reached, and after $a\ a\ b$ there is a conflict. Therefore, the learner performs an output query $b\ b$, and a new outgoing

b transition is added from state 3 to the new state 15. Since the first conflict already occurs on the frontier, $b \vdash 1\#2$, counterexample processing finishes.

(8) Rule (R1) is applied to add state 3 to the basis.

(9) In order to identify the frontier states, the learner applies witnesses a and b in all frontier states, leading to new states 16 to 20.

(10) Rule (R4) is applied with hypothesis $\mathcal{H}_3$ from Figure 3:

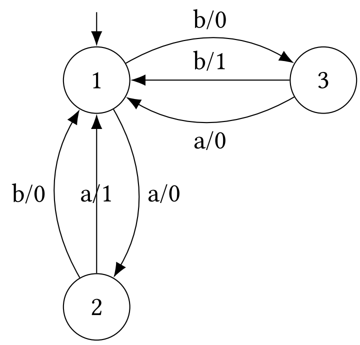

Fig. 3. Hypothesis $\mathcal{H}_3$

The learner observes that also $\mathcal{H}_3$ is not consistent with the observation tree: input sequence $a\ a\ b\ b\ a\ b\ a\ b$ provides a counterexample. The learner analyzes this counterexample and does not forward $\mathcal{H}_3$ to the teacher. After $a\ a$ the frontier is reached, and after $a\ a\ b\ b\ a\ b\ a$ there is a conflict. Therefore, the learner performs an output query $a\ b\ a\ b$. A new outgoing b-transition is added to state 14, leading to the new state 21. Since the first conflict already occurs on the frontier, $a\ b \vdash 1\#11$, counterexample processing finishes.

(11) Rule (R1) is applied to add state 11 to the basis.

(12) Rule (R3) is applied to identify the frontier states using witnesses $a\ b$ and b.

(13) Rule (R4) is applied with hypothesis $\mathcal{H}_4$ from Figure 4. The hypothesis is sent to the teacher,

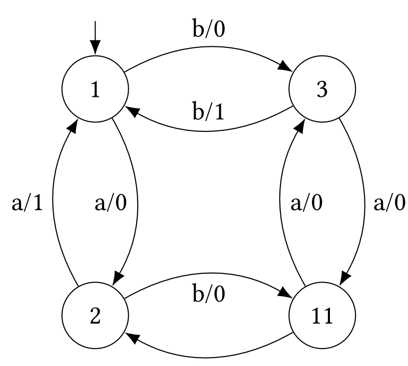

Fig. 4. Hypothesis $\mathcal{H}_4$

who confirms that the hypothesis is correct and compliments the learner for the good work!

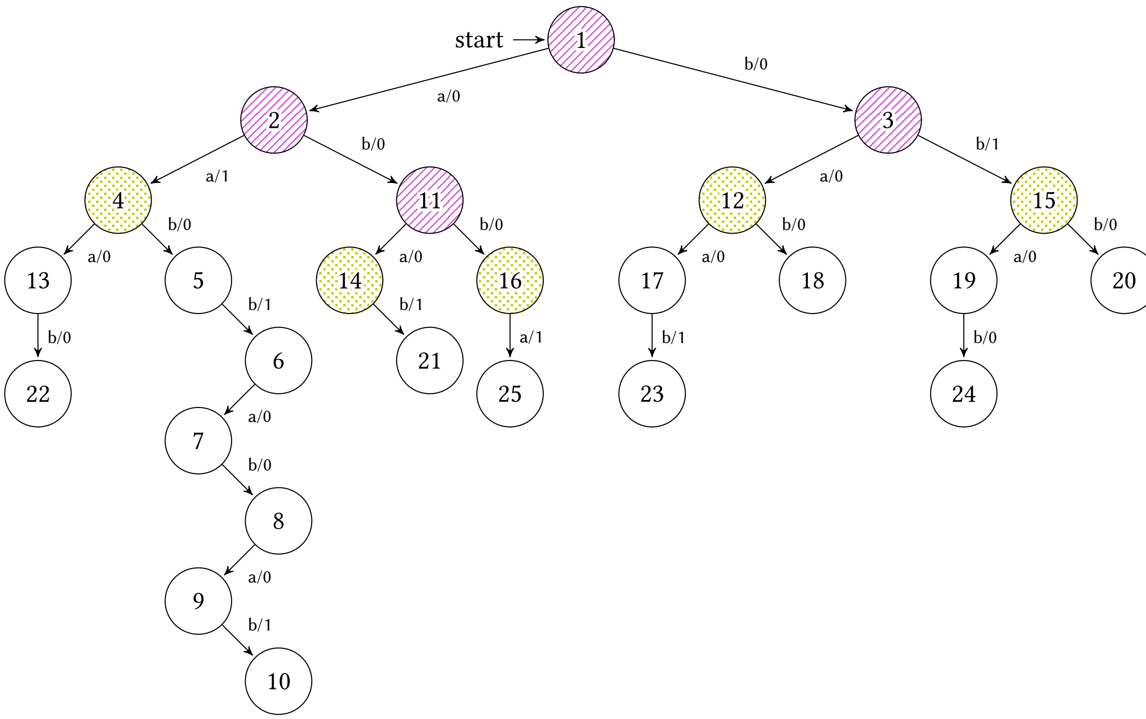

Fig. 5. Final observation tree