Compiler Construction

Week 2: Parsing I

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Radboud University

Recap

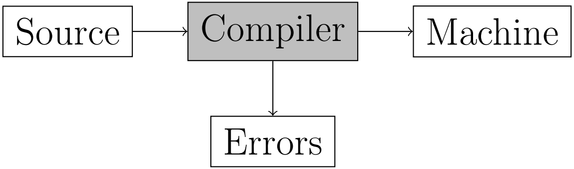
What was a compiler again?

Abstract view on a compiler

Compiler

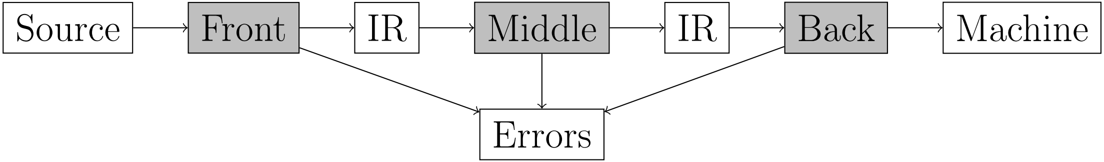
What was a compiler again?

Abstract view on a compiler

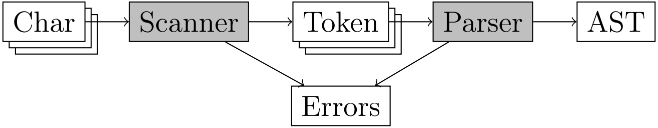


What was a compiler again?

Three pass compiler



Frontend



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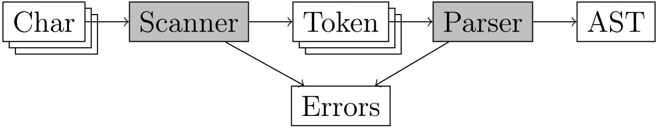
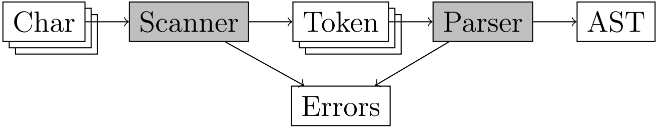


Duty of the frontend

- Recognise the (context-free) syntax
- Produce IR
- Report errors

Frontend



Duty of the frontend

- Recognise the (context-free) syntax
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Duty of the scanner

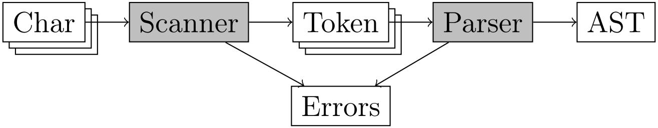
- Translate source code:

```
x = val + 42;
```

- to regular tokens

```
<id,x> <sym,=> ...
```

Frontend



Duty of the frontend

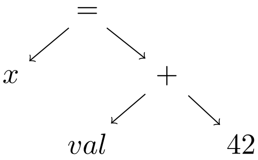
- Recognise the (context-free) syntax
- Produce IR
- Report errors

Duty of the scanner

- Translate source code:
  `x = val + 42;`
- to regular tokens
  `<id,x> <sym,=>` ...

Duty of the parser

- Translate tokens:
  `<id,x> <sym,=>` ...
- To an abstract syntax tree:



Scanning or parsing?

Where to draw the line

Scanning

Parsing

Scanning or parsing?

Where to draw the line

Scanning

- Words

Parsing

- Sentences (structure)

Scanning or parsing?

Where to draw the line

Scanning

- Words
- Regular grammar

Parsing

- Sentences (structure)
- Context free grammar

Scanning or parsing?

Where to draw the line

Scanning

- Words
- Regular grammar
- No recursion in the rules

Parsing

- Sentences (structure)
- Context free grammar
- Recursion allowed

Scanning or parsing?

Where to draw the line

Scanning

- Words
- Regular grammar
- No recursion in the rules
- Efficient

Parsing

- Sentences (structure)
- Context free grammar
- Recursion allowed
- Inefficient

Scanning or parsing?

Problems

- The lexer solves the easy problems

Scanning or parsing?

Problems

- The lexer solves the easy problems
- The parser solves the hard problems

Scanning or parsing?

Problems

- The lexer solves the easy problems
- The parser solves the hard problems
- Ambiguity

Scanning or parsing?
Problems

- The lexer solves the easy problems
- The parser solves the hard problems
- Ambiguity
  - Precendence

Scanning or parsing?

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- The lexer solves the easy problems
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Scanning or parsing?

Problems

- The lexer solves the easy problems
- The parser solves the hard problems
- Ambiguity
  - Precendence
  - Associativity
  - Dangling else

Context-free grammars

Context-free grammar (CFG)

and Backus-Naur form (BNF)

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and Backus-Naur form (BNF)

A CFG G is a 4-tuple

$$(V_t, V_n, S, P)$$

Context-free grammar (CFG)

and Backus-Naur form (BNF)

A CFG G is a 4-tuple

$$(V_t, V_n, S, P)$$

where

$$\begin{aligned} V_t &\quad \text{terminals} \\ V_n &\quad \text{non-terminals} \\ S &\quad \text{start } (S \in V_n) \\ P &\quad \text{productions} \end{aligned}$$

Context-free grammar (CFG)

and Backus-Naur form (BNF)

A CFG G is a 4-tuple

$$(V_t, V_n, S, P)$$

where

V_t	terminals
V_n	non-terminals
S	start ($S \in V_n$)
P	productions

Notation for grammars

$$
\begin{array}{lclccc}
goal & ::= & expr & & & \\
expr & ::= & expr & op & expr & \\
 & \mid & (& expr &) & \\
 & \mid & int & & & \\
 & \mid & id & & & \\
op & ::= & + & & & \\
 & \mid & * & & &
\end{array}
$$

terminal ($\in V_t$)

Context-free grammar (CFG)

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Notation for grammars

$$\begin{array}{rcl} \mathit{goal} & ::= & \mathit{expr} \\ \mathit{expr} & ::= & \mathit{expr} \quad \mathit{op} \quad \mathit{expr} \\ & \mid & (\quad \mathit{expr} \quad) \\ & \mid & \mathit{int} \\ & \mid & \mathit{id} \\ \mathit{op} & ::= & + \\ & \mid & * \end{array}$$

terminal ($\in V_t$)

non-terminal ($\in V_n$)

Context-free grammar (CFG)

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start (S)

terminal ($\in V_t$)

non-terminal ($\in V_n$)

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A CFG G is a 4-tuple

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V_t	terminals
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Notation for grammars

$$
\begin{array}{rcllll}
\boxed{goal & ::= & expr} & & & \text{production } (\in P) \\
expr & ::= & expr & op & expr & \\
\text{start } (S) & | & (& expr &) & \\
 & | & int & & & \text{terminal } (\in V_t) \\
 & | & id & & & \\
op & ::= & + & & & \\
 & | & * & & & \\
\text{non-terminal } (\in V_n) & & & & &
\end{array}
$$

Derivations

Example

```
x + 42 * 2
<id,x> <op,+> <int,42> <op,*> <int,2>
```

Rewriting

goal

Derivations

Example

```
x + 42 * 2
<id,x> <op,+> <int,42> <op,*> <int,2>
```

Rewriting

$$goal \Rightarrow \mathbf{expr}$$

Derivations

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Rewriting

$$goal \Rightarrow expr$$

Derivations

Example

```
x + 42 * 2
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Rewriting

$$\begin{aligned} goal &\Rightarrow expr \\ &\Rightarrow \mathbf{expr\ op\ expr} \end{aligned}$$

Derivations

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$$\begin{aligned} goal &\Rightarrow expr \\ &\Rightarrow expr\ op\ expr \\ &\Rightarrow id, x\ op\ expr \\ &\Rightarrow id, x\ op, +\ \mathbf{expr} \end{aligned}$$

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`x + 42 * 2`

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Rewriting

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\begin{aligned}
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&\Rightarrow id, x\ op, +\ int, 42\ op, *\ expr \\
&\Rightarrow id, x\ op, +\ int, 42\ op, *\ int, 2
\end{aligned}
$$

Rewriting grammars

- Confluent?

Rewriting grammars

- Confluent?
- Referentially transparent?

Rewriting grammars

- Confluent?
- Referentially transparent?
- One grammar specifies all valid sentences

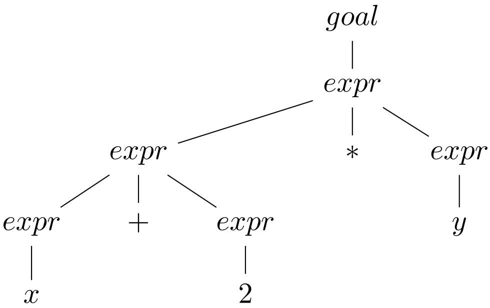
Using CFG's is problematic

Using CFG's is problematic, but not necessarily

Solving problems

Operator precedence

```
x + 2 * y
<id,x> <op,+> <int,2> <op,*> <id,y>
```



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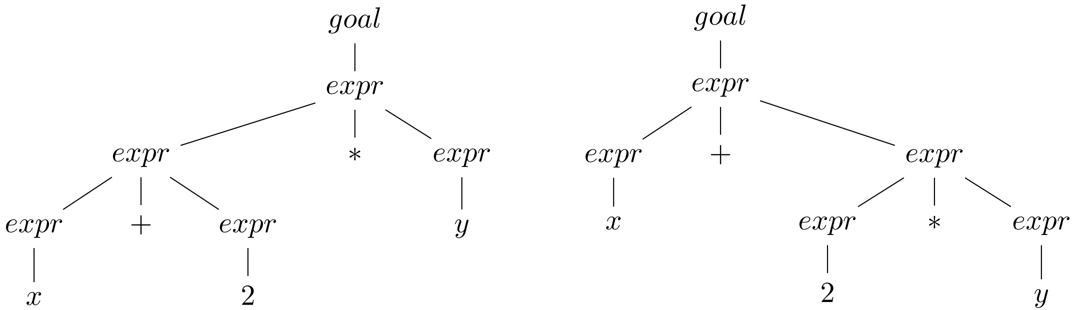

Solving problems

Operator precedence: Solutions

- Preprocess and insert brackets[2]

[2] https://en.wikipedia.org/wiki/Operator-precedence_parser#Alternative_methods

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Solving problems

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Encode in the grammar

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Solving problems

Operator precedence: Removing precedence ambiguity

Intuition

Top down, so parse from weakest to strongest precedence

In practice

$$
\begin{aligned}
goal &::= \\
sum &::= \\
fact &::= \\
basic &::=
\end{aligned}
$$

Solving problems

Operator precedence: Removing precedence ambiguity

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&\quad \mid \; basic \\
basic &::= int \\
&\quad \mid \; id
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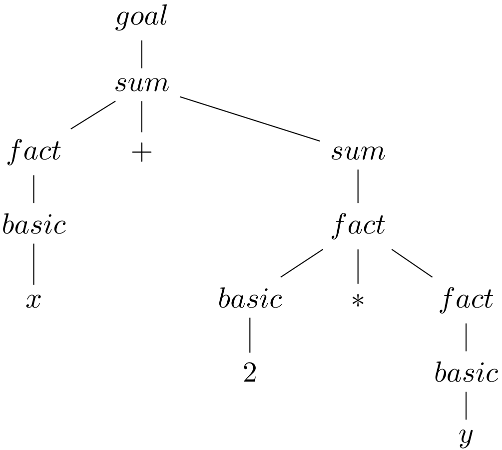

Solving problems

Operator associativity

```
1-2-3
<int,1> <op,-> <int,2> <op,-> <int,3>
```

Solving problems

Operator associativity

`1-2-3`

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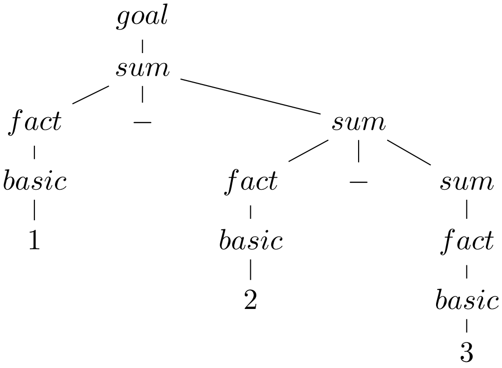

Solving problems

Operator associativity

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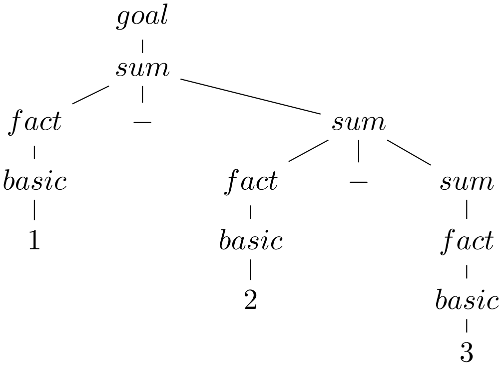

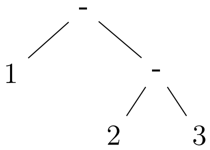

Solving problems

Operator associativity

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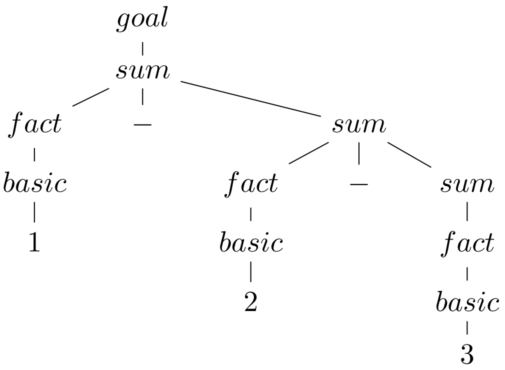

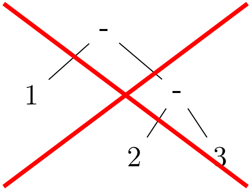

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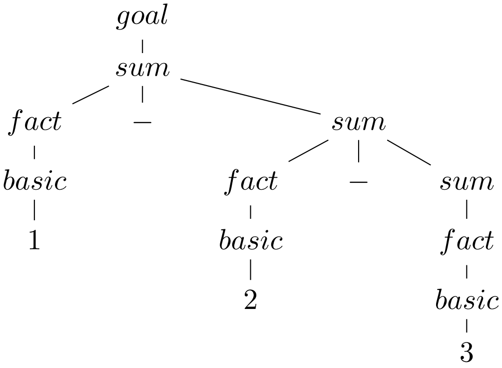

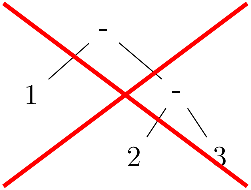

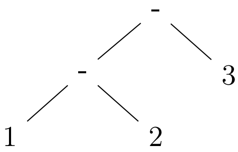

Solving problems
Operator associativity

- Postprocess and fix precedence
- Shunting yard
- Precedence climbing
- or…

Encode in the grammar

Solving problems

Operator associativity: Solutions

Change order right?

Change order right?

$$
\begin{aligned}
goal &::= sum \\
sum &::= \color{red}{sum} \quad [+-]\ fact \\
&\quad\ \mid\ fact \\
fact &::= fact \quad * \quad basic \\
&\quad\ \mid\ basic \\
basic &::= int \\
&\quad\ \mid\ id
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Solving problems

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Tail recursion

Parse left associative operators greedy

Solving problems

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\begin{aligned}
goal &::= sum \\
sum &::= fact \quad \{-\ fact\ \}^* \\
fact &::= fact \quad * \quad basic \\
&\quad |\ basic \\
basic &::= int \\
&\quad |\ id
\end{aligned}
$$

Change order right?

$$
\begin{array}{lcl}
goal & ::= & sum \\
sum & ::= & sum \quad [+-]\ fact \\
 & | & fact \\
fact & ::= & fact \quad * \quad basic \\
 & | & basic \\
basic & ::= & int \\
 & | & id
\end{array}
$$

Tail recursion

Parse left associative operators greedy

$$
\begin{array}{lcl}
goal & ::= & sum \\
sum & ::= & fact \quad \{-\ fact\ \}^* \\
fact & ::= & basic \quad \{*\ basic\ \}^* \\
basic & ::= & int \\
 & | & id
\end{array}
$$

Solving problems

Dangling else

$$\begin{aligned} stmt \ ::= & \ if \ \ expr \ \ then \ \ expr \\ | & \ \ if \ \ expr \ \ then \ \ expr \ \ else \ \ else \\ | & \ \ \ldots \end{aligned}$$

Solving problems

Dangling else

$$\begin{aligned} stmt \; ::= \; & if \;\; expr \;\; then \;\; expr \\ \mid \; & if \;\; expr \;\; then \;\; expr \;\; else \;\; else \\ \mid \; & \ldots \end{aligned}$$

```
if E1 then if E2 then S1 else S2
```

Solving problems

Dangling else

$$
\begin{aligned}
stmt \ ::= \ & if \ \ expr \ \ then \ \ expr \\
\mid \ & if \ \ expr \ \ then \ \ expr \ \ else \ \ else \\
\mid \ & \ldots
\end{aligned}
$$

```
if E1 then if E2 then S1 else S2
```

```
if E1 then (if E2 then S1) else S2
```

Solving problems

Dangling else

$$\begin{aligned} stmt \ ::= & \ if \ \ expr \ \ then \ \ expr \\ | & \ if \ \ expr \ \ then \ \ expr \ \ else \ \ else \\ | & \ \dots \end{aligned}$$

```
if E1 then if E2 then S1 else S2

if E1 then (if E2 then S1) else S2

if E1 then (if E2 then S1 else S2)
```

Solving problems

Dangling else

$$
\begin{aligned}
stmt \ ::= \ & if \ \ expr \ \ then \ \ expr \\
| \ \ & if \ \ expr \ \ then \ \ expr \ \ else \ \ else \\
| \ \ & \ldots
\end{aligned}
$$

```
if E1 then if E2 then S1 else S2

if E1 then (if E2 then S1) else S2

if E1 then (if E2 then S1 else S2)
```

Solution: endif, brackets, innermost else, refactor grammar

Intermezzo: Lexer hack for ANSI C

```
void fun()
{
  T (x);
}
```

Intermezzo: Lexer hack for ANSI C

```
void fun()
{
  T (x);
}
```

Function call or cast?

Intermezzo: Lexer hack for ANSI C

```
void fun()
{
  T (x);
}
```

```
void fun()
{
  T * x;
}
```

Function call or cast?

Intermezzo: Lexer hack for ANSI C

```
void fun()
{
  T (x);
}
```

Function call or cast?

```
void fun()
{
  T * x;
}
```

Multiplication or pointer declaration?

Left recursion

- Top down parsers cannot handle left recursion

Definition

Grammar G is left recursive if

$$\exists A \in V_n \text{ such that } A \Rightarrow^{+} A\alpha \text{ for some } \alpha$$

Left recursion

- Top down parsers cannot handle left recursion
- Infinite recursion

Definition

Grammar G is left recursive if

$$\exists A \in V_n \text{ such that } A \Rightarrow^{+} A\alpha \text{ for some } \alpha$$

Remove left recursion

Grammar

$$\begin{aligned} foo \ ::= & \ foo \ \ a \\ | & \ \ b \end{aligned}$$

Remove left recursion

Grammar

$$
\begin{aligned}
foo \ &::= foo \ \ a \\
&\ \ | \ \ b
\end{aligned}
$$

Language: ba^*

Remove left recursion

Grammar

$$
\begin{aligned}
foo &::= foo \;\; a \\
&\mid \;\; b
\end{aligned}
$$

Language: ba^*

Rewritten grammar

$$
\begin{aligned}
foo &::= b \;\; bar \\
bar &::= a \;\; bar \\
&\mid \;\; \epsilon
\end{aligned}
$$

Tricky

Removing immediate left recursion

Removing immediate left recursion

$$A ::= A\alpha_1 \mid \cdots \mid A\alpha_m \mid \beta_1 \mid \cdots \mid \beta_n$$

Removing immediate left recursion

$$A ::= A\alpha_1 \mid \cdots \mid A\alpha_m \mid \beta_1 \mid \cdots \mid \beta_n$$

$$\begin{aligned} A &::= \beta_1 A' \mid \cdots \mid \beta_n A' \\ A' &::= \alpha_1 A' \mid \cdots \mid \alpha_m A' \mid \epsilon \end{aligned}$$

Example I

Initial grammar

$$
\begin{aligned}
expr \;\; ::= &\; expr\ [+ - */\hat{}\,]expr \\
\mid &\; '(' \quad expr \quad ')' \\
\mid &\; int \\
\mid &\; id
\end{aligned}
$$

Example II

Fix precedence and associativity

$$
\begin{aligned}
expr &::= expr\ [+-]fact \\
&\ \mid\ \ fact \\
fact &::= fact\ [*/]pow \\
&\ \mid\ \ pow \\
pow &::= basic\ [\hat{}\ ]pow \\
&\ \mid\ \ basic \\
basic &::= '('expr\ ')' \\
&\ \mid\ \ int \\
&\ \mid\ \ id
\end{aligned}
$$

Example III

Apply direct left recursion elimination

$$
\begin{aligned}
expr \;&::=\; fact\ expr' \\
expr' \;&::=\; [+-] fact\ expr' \\
&\quad\mid\ \epsilon \\
fact \;&::=\; pow\ fact' \\
fact' \;&::=\; [*/] pow\ fact' \\
&\quad\mid\ \epsilon \\
pow \;&::=\; basic\ \hat{}\ pow \\
&\quad\mid\ basic \\
basic \;&::=\; '('expr\ ')' \\
&\quad\mid\ int \\
&\quad\mid\ id
\end{aligned}
$$

Example IV

Or alternatively

$$
\begin{aligned}
expr &::= fact\ \{[+-]fact\ \}^* \\
fact &::= pow\ \{[*/]pow\ \}^* \\
pow &::= basic\ \hat{}\,pow \\
&\ \ \mid\ \ basic \\
basic &::= '('expr\ ')' \\
&\ \ \mid\ \ int \\
&\ \ \mid\ \ id
\end{aligned}
$$

Parsing strategies

Parsing in practice

Parsing in practice

- Given a grammar and a sentence

Parsing in practice

- Given a grammar and a sentence
- Find a derivation

Parsing in practice

- Given a grammar and a sentence
- Find a derivation

Parsing in practice

- Given a grammar and a sentence
- Find a derivation, or an error
- Two main strategies

Parsing in practice

- Given a grammar and a sentence
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Parsing in practice

- Given a grammar and a sentence
- Find a derivation, or an error
- Two main strategies

1. Top-down

Parsing in practice

- Given a grammar and a sentence
- Find a derivation, or an error
- Two main strategies

1. Top-down
2. Bottom-up

Parsing in practice

Top-down parsers

- Start at root (S)
- Pick a production and try to match input
- Backtracking may be required
- Some grammars are predictive

Parsing in practice

Top-down parsers

- Start at root (S)
- Pick a production and try to match input
- Backtracking may be required
- Some grammars are predictive

Bottom-up parsers

- Start at the leafs
- Start in a valid state for the first token
- Consume, change state, encode possibilities
- Store state and sentential forms on a stack
- Topic of next week

Top-down parsing

Top-down parsers

Top-down parsers

- Start at root (S)

Top-down parsers

Top-down parsers

- Start at root (S)
- Pick an alternative and try to match input

Top-down parsers

Top-down parsers

- Start at root (S)
- Pick an alternative and try to match input
- When the terminal doesn't match: backtrack (lookahead)

Top-down parsers

Top-down parsers

- Start at root (S)
- Pick an alternative and try to match input
- When the terminal doesn't match: backtrack (lookahead)
- Find the next node

How to make a top-down parser

- Recursive descent parser

How to make a top-down parser

- Recursive descent parser
  - Function

How to make a top-down parser

- Recursive descent parser
  - Function
  - Recursive

How to make a top-down parser

- Recursive descent parser
  - Function
  - Recursive
  - Descent

How to make a top-down parser

- Recursive descent parser
  - Function
  - Recursive
  - Descent
- Combinators

How to make a top-down parser

- Recursive descent parser
  - Function
  - Recursive
  - Descent
- Combinators
- Libraries, bake your own

Let's jump right in!

Grammar

$$
\begin{aligned}
stmt \ ::= &\ id \ = expr \ ; \\
\mid &\ \{ \quad stmt\ ^* \ \}
\end{aligned}
$$

Let’s jump right in!

Grammar

$$
\begin{aligned}
stmt \;::=\;& id \;=\; expr \;\; ; \\
\mid\;& \{ \;\; stmt^{\,*} \;\; \}
\end{aligned}
$$

Haskell

```
data Stmt = Assignment IdToken Expr | Stmts [Stmt]

pStmt =  Assignment <$> satisfy isIdToken <* symbol '=' *> pExpr <*
    symbol ';'
     <|> Stmts      <$> symbol '{' *> many pStmt <* symbol '}'
```

What's powering this

P. Wadler, *How to replace failure by a list of successes a method for exception handling, backtracking, and pattern matching in lazy functional languages* (1985)

What's powering this

P. Wadler, *How to replace failure by a list of successes a method for exception handling, backtracking, and pattern matching in lazy functional languages* (1985)

failure	$[]$
deterministic	$[rs]$
nondeterministic	$[rs_1, \ldots, rs_n]$

What's powering this

P. Wadler, *How to replace failure by a list of successes a method for exception handling, backtracking, and pattern matching in lazy functional languages* (1985)

failure	$[]$
deterministic	$[rs]$
nondeterministic	$[rs_1, \ldots, rs_n]$

```
type Parser s a = [s] → [(a, [s])]
```

What's powering this

P. Wadler, *How to replace failure by a list of successes a method for exception handling, backtracking, and pattern matching in lazy functional languages* (1985)

failure	$[]$
deterministic	$[rs]$
nondeterministic	$[rs_1, \ldots, rs_n]$

```
type Parser s a = [s] → [(a, [s])]
newtype Parser s a = Parser {runParser :: [s] → [(a, [s])]}
```

Satisfy (`g ::= t`)

Recognize a symbol given a predicate

Satisfy (`g ::= t`)

Recognize a symbol given a predicate

```
satisfy :: (s → Bool) → Parser s s
```

Satisfy (`g ::= t`)

Recognize a symbol given a predicate

```
satisfy :: (s → Bool) → Parser s s
satisfy f = Parser $ \input → case input of
    (s:rest) | f s → [(s, rest)]
    _              → []
```

Satisfy (`g ::= t`)

Examples

```
runParser (satisfy isDigit) "123"
```

Satisfy (`g ::= t`)

Examples

```
runParser (satisfy isDigit) "123" -- [('1',"23")]
```

Satisfy (`g ::= t`)

Examples

```
runParser (satisfy isDigit) "123"
runParser (satisfy isDigit) "abc"
```

— $[('1', "23")]$

Satisfy (`g ::= t`)

Examples

```
runParser (satisfy isDigit) "123" -- [('1',"23")]
runParser (satisfy isDigit) "abc" -- []
```

Satisfy (`g ::= t`)

Examples

```
runParser (satisfy isDigit) "123" -- [('1',"23")]
runParser (satisfy isDigit) "abc" -- []
runParser (satisfy isDigit) "1"
```

Satisfy (`g ::= t`)

Examples

```
runParser (satisfy isDigit) "123" -- [('1',"23")]
runParser (satisfy isDigit) "abc" -- []
runParser (satisfy isDigit) "1"   -- [('1',"")]
```

Satisfy (`g ::= t`)

Examples

```
runParser (satisfy isDigit) "123" -- [('1',"23")]
runParser (satisfy isDigit) "abc" -- []
runParser (satisfy isDigit) "1"   -- [('1',"")]
runParser (satisfy isDigit) ""
```

Satisfy (g ::= t)

Examples

```
runParser (satisfy isDigit) "123" -- [('1',"23")]
runParser (satisfy isDigit) "abc" -- []
runParser (satisfy isDigit) "1"   -- [('1',"")]
runParser (satisfy isDigit) ""    -- []
```

Satisfy (`g ::= t`)

Examples

```
runParser (satisfy isDigit) "123" -- [('1',"23")]
runParser (satisfy isDigit) "abc" -- []
runParser (satisfy isDigit) "1"   -- [('1',"")]
runParser (satisfy isDigit) ""    -- []
```

Abbreviations

Satisfy (`g ::= t`)

Examples

```
runParser (satisfy isDigit) "123" -- [('1',"23")]
runParser (satisfy isDigit) "abc" -- []
runParser (satisfy isDigit) "1"   -- [('1',"")]
runParser (satisfy isDigit) ""    -- []
```

Abbreviations

```
symbol s = satisfy (==s)
```

Satisfy (`g ::= t`)

Examples

```
runParser (satisfy isDigit) "123" -- [('1',"23")]
runParser (satisfy isDigit) "abc" -- []
runParser (satisfy isDigit) "1"   -- [('1',"")]
runParser (satisfy isDigit) ""    -- []
```

Abbreviations

```
symbol s = satisfy (==s)
top = satisfy (\_ → True)
```

Satisfy (g ::= t)

Examples

```
runParser (satisfy isDigit) "123" -- [('1',"23")]
runParser (satisfy isDigit) "abc" -- []
runParser (satisfy isDigit) "1"   -- [('1',"")]
runParser (satisfy isDigit) ""    -- []
```

Abbreviations

```
symbol s = satisfy (==s)
top = satisfy (\_ -> True)

runParser (symbol '.') "abc"
```

Satisfy (g ::= t)

Examples

```
runParser (satisfy isDigit) "123" -- [('1',"23")]
runParser (satisfy isDigit) "abc" -- []
runParser (satisfy isDigit) "1"   -- [('1',"")]
runParser (satisfy isDigit) ""    -- []
```

Abbreviations

```
symbol s = satisfy (==s)
top = satisfy (\_ → True)

runParser (symbol '.') "abc"                -- []
```

Satisfy (g ::= t)

Examples

```
runParser (satisfy isDigit) "123" -- [('1',"23")]
runParser (satisfy isDigit) "abc" -- []
runParser (satisfy isDigit) "1"   -- [('1',"")]
runParser (satisfy isDigit) ""    -- []
```

Abbreviations

```
symbol s = satisfy (==s)
top = satisfy (\_ → True)

runParser (symbol '.') "abc"                    -- []
runParser (symbol '.') ".bc"
```

Satisfy (`g ::= t`)

Examples

```
runParser (satisfy isDigit) "123" -- [('1',"23")]
runParser (satisfy isDigit) "abc" -- []
runParser (satisfy isDigit) "1"   -- [('1',"")]
runParser (satisfy isDigit) ""    -- []
```

Abbreviations

```
symbol s = satisfy (==s)
top = satisfy (\_ → True)

runParser (symbol '.') "abc"                   -- []
runParser (symbol '.') ".bc"                   -- [('.',"bc")]
```

Satisfy (`g ::= t`)

Examples

```
runParser (satisfy isDigit) "123" -- [('1',"23")]
runParser (satisfy isDigit) "abc" -- []
runParser (satisfy isDigit) "1"   -- [('1',"")]
runParser (satisfy isDigit) ""    -- []
```

Abbreviations

```
symbol s = satisfy (==s)
top = satisfy (\_ → True)

runParser (symbol '.') "abc"                    -- []
runParser (symbol '.') ".bc"                    -- [('.',"bc")]
runParser (symbol "hello") ["hello", "world"]
```

Satisfy (g ::= t)

Examples

```
runParser (satisfy isDigit) "123" -- [('1',"23")]
runParser (satisfy isDigit) "abc" -- []
runParser (satisfy isDigit) "1"   -- [('1',"")]
runParser (satisfy isDigit) ""    -- []
```

Abbreviations

```
symbol s = satisfy (==s)
top = satisfy (\_ → True)

runParser (symbol '.') "abc"                  -- []
runParser (symbol '.') ".bc"                  -- [('.',"bc")]
runParser (symbol "hello") ["hello", "world"] -- [("hello",["world"])]
```

Choice (`g ::= p1 | p2`)

Choose one or the other parser

Choice (`g ::= p1 | p2`)

Choose one or the other parser

```
infixl 3 <|>
(<|>) :: Parser s a → Parser s a → Parser s a
```

Choice (`g ::= p1 | p2`)

Choose one or the other parser

```
infixl 3 <|>
(<|>) :: Parser s a → Parser s a → Parser s a
x <|> y = Parser $ \t→runParser x t ++ runParser y t
```

Choice (`g :== p1 | p2`)

Examples

```
pLayout :: Parser Char Char
pLayout = symbol ' ' <|> symbol '\n' <|> symbol '\t'
```

Choice (g ::= p1 | p2)

Examples

```
pLayout :: Parser Char Char
pLayout = symbol ' ' <|> symbol '\n' <|> symbol '\t'

runParser pLayout " 123"
```

Choice (`g :== p1 | p2`)

Examples

```
pLayout :: Parser Char Char
pLayout = symbol ' ' <|> symbol '\n' <|> symbol '\t'

runParser pLayout " 123" -- [(' ',"123")]
```

```
pDigit = satisfy isDigit
pAlpha = satisfy isAlpha
pAlnum = pDigit <|> pAlpha
```

Choice (`g :== p1 | p2`)

Examples

```
pLayout :: Parser Char Char
pLayout = symbol ' ' <|> symbol '\n' <|> symbol '\t'
```

`runParser pLayout " 123"` — *[(' ',"123")]*

```
pDigit = satisfy isDigit
pAlpha = satisfy isAlpha
pAlnum = pDigit <|> pAlpha
```

`runParser (pDigit <|> pAlpha <|> pAlnum) "123"`

Choice (g :== p1 | p2)

Examples

```
pLayout :: Parser Char Char
pLayout = symbol ' ' <|> symbol '\n' <|> symbol '\t'
```

runParser pLayout " 123" — *[(' ',"123")]*

```
pDigit = satisfy isDigit
pAlpha = satisfy isAlpha
pAlnum = pDigit <|> pAlpha
```

runParser (pDigit <|> pAlpha <|> pAlnum) "123" — *[('1',"23"),('1',"23")]*

Sequence (`g ::= p q`)

Recognize multiple (non)terminals

Sequence (g ::= p q)

Recognize multiple (non)terminals

```
infixl 4 <*>
(<*>) :: Parser s (a → b) → Parser s a → Parser b
```

Sequence (`g ::= p q`)

Recognize multiple (non)terminals

```
infixl 4 <*>
(<*>) :: Parser s (a → b) → Parser s a → Parser b
l <*> r = Parser $ \t →
    [ (fa a, ts')
    | (fa, ts ) ← runParser l t
    , ( a, ts') ← runParser r ts
    ]
```

Sequence (`g ::= p q`)

Recognize multiple (non)terminals

```
infixl 4 <*>
(<*>) :: Parser s (a → b) → Parser s a → Parser b
l <*> r = Parser $ \t→
    [ (fa a, ts')
    | (fa, ts ) ← runParser l t
    , ( a, ts') ← runParser r ts
    ]
```

Return constant values

```
pure :: a → Parser s a
pure a = Parser $ \t→[(a, t)]
```

Sequence: bind

Right-hand determined by left-hand result

Sequence: bind

Right-hand determined by left-hand result

```
infixl 1 >>=
(>>=) :: Parser s a → (a → Parser s b) → Parser s b
```

Sequence: bind

Right-hand determined by left-hand result

```
infixl 1 >>=
(>>=) :: Parser s a → (a → Parser s b) → Parser s b
ma >>= a2mb = Parser $ \t→concat
    [ runParser (a2mb a) ts
    | (a, ts) ← runParser ma t
    ]
```

Sequence (`g :== p1 p2`)

Examples

```
pAlphaAndDigit = pure (,) <*> pAlpha <*> pDigit

runParser pAlphaAndDigit "a1b"
```

Sequence (`g :== p1 p2`)

Examples

```
pAlphaAndDigit = pure (,) <*> pAlpha <*> pDigit

runParser pAlphaAndDigit "a1b" -- [((a,1),"b")]
```

Sequence (`g :== p1 p2`)

Examples

```
pAlphaAndDigit = pure (,) <*> pAlpha <*> pDigit

runParser pAlphaAndDigit "a1b" -- [((a,1),"b")]

pAlphaAndDigit = pAlpha >>= \x → pDigit >>= \y → pure (x, y)
```

Expressive power `<*>` vs. `>>=`

Which has more expressive power?

Expressive power <*> vs. »=

Which has more expressive power?

```
pTwice :: Parser Char Char
pTwice = pAlpha >>= symbol
```

Expressive power <*> vs. »=

Which has more expressive power?

```
pTwice :: Parser Char Char
pTwice = pAlpha >>= symbol
```

Context sensitive!

Expressive power <*> vs. »=

Which has more expressive power?

```
pTwice :: Parser Char Char
pTwice = pAlpha >>= symbol
```

Context sensitive!

```
runParser pTwiceA "aabc"
```

Expressive power `<*>` vs. `»=`

Which has more expressive power?

```
pTwice :: Parser Char Char
pTwice = pAlpha >>= symbol
```

Context sensitive!

```
runParser pTwiceA "aabc" -- [('a',"bc")]
```

Expressive power `<*>` vs. `>>=`

Which has more expressive power?

```
pTwice :: Parser Char Char
pTwice = pAlpha >>= symbol
```

Context sensitive!

```
runParser pTwiceA "aabc" -- [('a',"bc")]
runParser pTwiceA "abc"
```

Expressive power <*> vs. »=

Which has more expressive power?

```
pTwice :: Parser Char Char
pTwice = pAlpha >>= symbol
```

Context sensitive!

```
runParser pTwiceA "aabc" -- [('a',"bc")]
runParser pTwiceA "abc"  -- []
```

Functor

Transform the result of a parser

Functor

Transform the result of a parser

```
infixr 7 <$>
(<$>) :: (a → b) → Parser s a → Parser s b
```

Functor

Transform the result of a parser

```
infixr 7 <$>
(<$>) :: (a → b) → Parser s a → Parser s b
f <$> p = Parser $ \t→[(f a, ts) | (a, ts)←runParser p t]
```

Functor

Transform the result of a parser

```
infixr 7 <$>
(<$>) :: (a → b) → Parser s a → Parser s b
f <$> p = Parser $ \t → [(f a, ts) | (a, ts) ← runParser p t]

runParser (digitToInt <$> pDigit) "1ab"
```

Functor

Transform the result of a parser

```
infixr 7 <$>
(<$>) :: (a → b) → Parser s a → Parser s b
f <$> p = Parser $ \t→[(f a, ts) | (a, ts)←runParser p t]

runParser (digitToInt <$> pDigit) "1ab" — [(1,"ab")]
```

Functor

Transform the result of a parser

```
infixr 7 <$>
(<$>) :: (a → b) → Parser s a → Parser s b
f <$> p = Parser $ \t → [(f a, ts) | (a, ts) ← runParser p t]

runParser (digitToInt <$> pDigit) "1ab" -- [(1,"ab")]

pAlphaAndDigit =
```

Functor

Transform the result of a parser

```
infixr 7 <$>
(<$>) :: (a → b) → Parser s a → Parser s b
f <$> p = Parser $ \t→[(f a, ts) | (a, ts)←runParser p t]

runParser (digitToInt <$> pDigit) "1ab" -- [(1,"ab")]

pAlphaAndDigit = (,) <$> pAlpha <*> pDigit
```

Forgetful sequence (`g ::= '(' t ')'`)

Parse but do not save

Forgetful sequence (`g ::= '(' t ')'`)

Parse but do not save

```
infixl 6 <*, *>
(<*) :: Parser s a → Parser s b → Parser s a

(*>) :: Parser s a → Parser s b → Parser s b
```

Forgetful sequence (`g ::= '(' t ')'`)

Parse but do not save

```
infixl 6 <*, *>
(<*) :: Parser s a → Parser s b → Parser s a
x <* y = (\x y→x) <$> x <*> y

(*>) :: Parser s a → Parser s b → Parser s b
x *> y = (\x y→y) <$> x <*> y
```

Forgetful sequence (`g ::= '(' t ')'`)

Parse but do not save

```
infixl 6 <*, *>
(<*) :: Parser s a → Parser s b → Parser s a
x <* y = (\x y→x) <$> x <*> y

(*>) :: Parser s a → Parser s b → Parser s b
x *> y = (\x y→y) <$> x <*> y

parens p = symbol '(' *> p <* symbol ')'
```

Forgetful sequence (g ::= '(' t ')')

Parse but do not save

```
infixl 6 <*, *>
(<*) :: Parser s a → Parser s b → Parser s a
x <* y = (\x y→x) <$> x <*> y

(*>) :: Parser s a → Parser s b → Parser s b
x *> y = (\x y→y) <$> x <*> y

parens p = symbol '(' *> p <* symbol ')'

runParser (parens pDigit) "(1)ab"
```

Forgetful sequence (g ::= '(' t ')')

Parse but do not save

```
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runParser (parens pDigit) "(1)ab" -- [('1',"ab")]
```

Counting parenthesis depth

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```
parens  =  (+1) <$> (symbol '(' *> parens <* symbol ')')
       <|> pure 0

runParser parens "((()))"
```

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```
parens  =  (+1) <$> (symbol '(' *> parens <* symbol ')')
       <|> pure 0

runParser parens "((()))" -- [(3,""),(0,"((()))")]
```

Collecting results

Save results in a list

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infixr 6 <:>
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```

Abbreviations, Kleene star

```
many :: Parser s r → Parser s [r]
many p = p <:> many p <|> pure []

some :: Parser s r → Parser s [r]
some p = p <:> many p
```

Collecting results

Handle with care

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```
runParser ((read :: String → Int) <$> many pDigit) "123."
```

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runParser ((read :: String → Int) <$> many pDigit) "123."
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```
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  — [(123,"."), (12,"3."), (1,"23."),
```

Collecting results

Handle with care

```
runParser ((read :: String → Int) <$> many pDigit) "123."
  -- [(123,"."), (12,"3."), (1,"23."), (*** Exception: Prelude.read:
     no parse
```

The better choice

Only parse the second if the first fails

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(<<|>) :: (Parser s r) → (Parser s r) → Parser s r
```

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Only parse the second if the first fails

```
infixr 4 <<|>
(<<|>) :: (Parser s r) → (Parser s r) → Parser s r
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    []  → runParser y input
    res → res
```

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```

Lookahead

- Arbitrary lookahead

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- Combinators manage

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Predictive parsing

Intuition

Which production to expand

Predictive parsing

Intuition

Which production to expand

Definition

For RHS $\alpha \in G$, define $FIRST(\alpha)$.

$$w \in V_t, w \in FIRST(\alpha) \text{ iff } \alpha \Rightarrow^{+} w\gamma.$$

Predictive parsing

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For any two productions $A \rightarrow \alpha$ and $A \rightarrow \beta$

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Lookahead of 1!

Left factoring

Can we transform grammars to have this property?

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Sometimes

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- For each non-terminal A

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Sometimes

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- Replace all of the A productions

$$A \to \alpha\beta_1 \mid \alpha\beta_2 \mid \cdots \mid \alpha\beta_n$$

with

$$\begin{aligned} A &\to \alpha A' \\ A' &\to \beta_1 \mid \beta_2 \mid \cdots \mid \beta_n \end{aligned}$$

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- Repeat

Left factoring in general

Question

By left factoring and eliminating left-recursion, can we transform an arbitrary context-free grammar to a form where it can be predictively parsed with a single token look ahead?

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Left factoring in general

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By left factoring and eliminating left-recursion, can we transform an arbitrary context-free grammar to a form where it can be predictively parsed with a single token look ahead?

Answer

Undecidably hard

Left factoring in general

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By left factoring and eliminating left-recursion, can we transform an arbitrary context-free grammar to a form where it can be predictively parsed with a single token look ahead?

Answer

Undecidably hard

Example

$$\{a^n 0 b^n | n \geq 1\} \bigcup \{a^n 1 b^{2n} | n \geq 1\}$$

Why left factoring?

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  Parser combinators can handle arbitrary backtracking
- Requirement
  Recursive descent parsers require a store of arbitrary size

Conclusion

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Conclusion

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- Problem? Change the grammar
  - Left recursion
  - Precedence
  - Associativity
  - Left factoring
- Parser combinators
  - Easy translations
  - Hides input passing

Next week

Homework

- Start implementing your parser

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- Assignment is on Brightspace

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- You may use parser combinator libraries or parser generators

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- Bottom-up parsers
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- Scannerless parsers