Model Checking: exercise set 12 - Dynamic fault trees

Due date: June 5

1. Prove that the exponential probability distribution is memoryless, i.e., that

$$\Pr(X > t + t' \mid X > t') = \Pr(X > t).$$

(The proof is in the article on Brightspace, but try it yourself first).

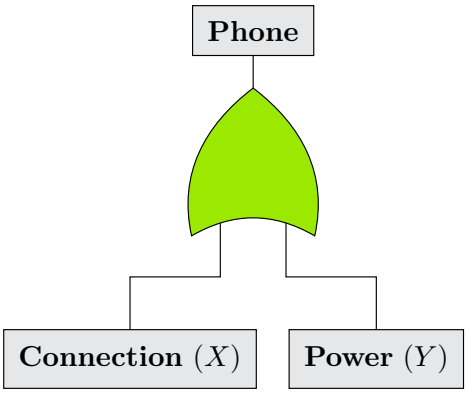
2. You are waiting for a package to arrive from the webshop *Web-1*. People have figured out that, for this webshop, the delivery time X is exponentially distributed with parameter $\frac{1}{15}$ days (notation: $X \sim Exp(\frac{1}{15})$ days).
   (a) What is the expected amount you have to wait?
   (b) What is the probability that you have to wait:
      i. Less than 5 days?
      ii. More than 10 days?
      iii. Exactly 10 days?
      iv. Between 10 and 25 days?
   (c) You have been waiting for 15 days. What is the probability that from now on, you have to wait:
      i. Less than 5 days?
      ii. More than 10 days?
      iii. Exactly 10 days?
      iv. Between 10 and 25 days?

3. You are ordering, on the same day, a product from webshop *Web-1* and a product from webshop *Web-2*. The delivery time X for *Web-1* is still exponentially distributed with parameter $\frac{1}{15}$ days ($X \sim Exp(\frac{1}{15})$ days). The delivery time Y for *Web-2* is also exponentially distributed with parameter $\frac{1}{5}$ days ($Y \sim Exp(\frac{1}{5})$ days).
   (a) What is the probability that both packages will arrive at exactly the same moment?
   (b) Give the delivery distribution for the first package to arrive, i.e., determine the probability of your package arriving within time t?
   (c) What is the expected delivery time for the first package?
   (d) What is the probability for the *Web-1* package to arrive first?

   Hint: Look at p181-182 of the paper on exponentials, as well as Example (a) on p183.

   Reformulation: A completely different formulation of the same problem:

   Your phone fails if either your connection fails or you have no power. The probability that your connetion fails X is exponentially distributed with parameter $\frac{1}{15}$ days ($X \sim Exp(\frac{1}{15})$ days). The probability that you have no power Y is exponentially distributed with parameter $\frac{1}{5}$ days ($Y \sim Exp(\frac{1}{5})$ days). See the fault tree on the next page:



(e) Consider the following four questions and explain why these answers are the same as for (a)-(d). Give one argument; there is no need to go into each question separately.

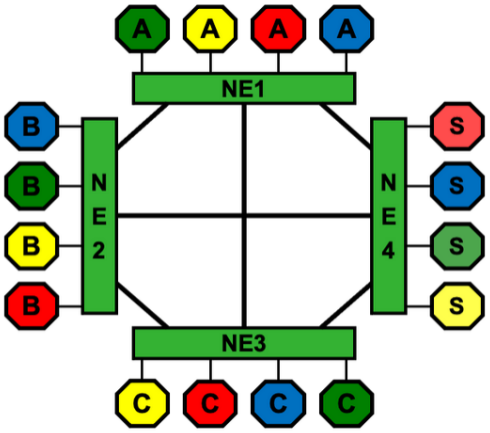
(a') What is the probability that your connection and power fail at the same time?

(b') Give the failure distribution for your phone, i.e., determine the probability for your phone to fail within time t?

(c') What is the expected time for your phone to fail?

(d') What is the probability that your phone fails due to your connection, i.e., your connection fails before you are out of power?

4. Consider a fault-tolerant system, as in the figure below.



- There are 4 types of processors: *red, blue, yellow,* and *green.*
- For each processor type (=color) there are 4 redundant copies, called A, B, C, and S, where S acts as a spare.
- The processors are connected via 4 network elements (NE), where each network element connects 4 processors, being one processor of each type.
- If the network element fails, then all elements connected to it also fail.

Requirements:

- For the system to be up, at least 2 processors must be available for each type, as well as their associated network elements.

Construct a dynamic fault tree for this system. Use the dynamic spare and FDEP gates.