Equivalences
on Labelled Transition Systems

Models: Labelled Transition Systems

Labelled Transition System: $\langle\ S,\ L_I,\ L_U,\ T,\ s_0\ \rangle$

states

input labels

output labels

transitions

$T \subseteq S \times (L \cup \{\tau\}) \times S$

initial state

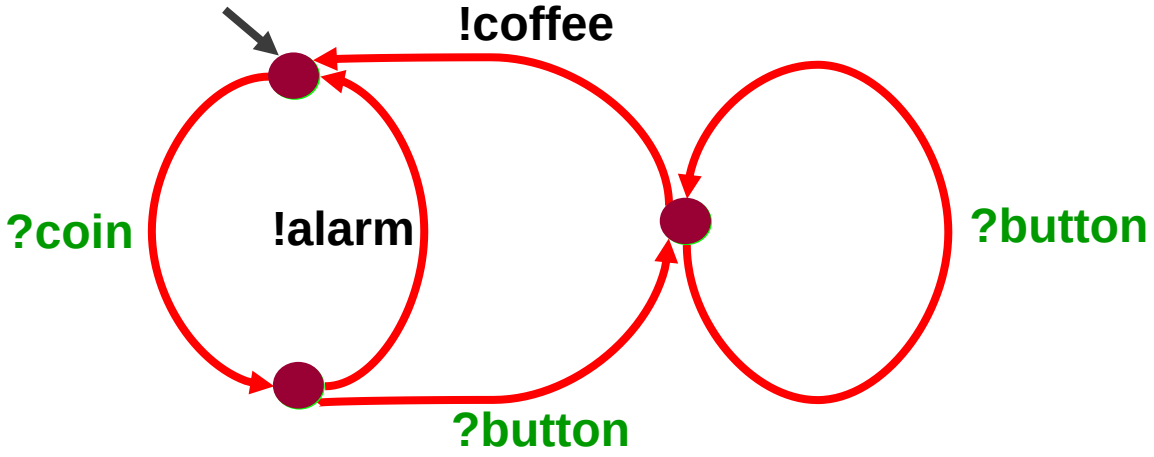
$s_0 \in S$

? = *input*

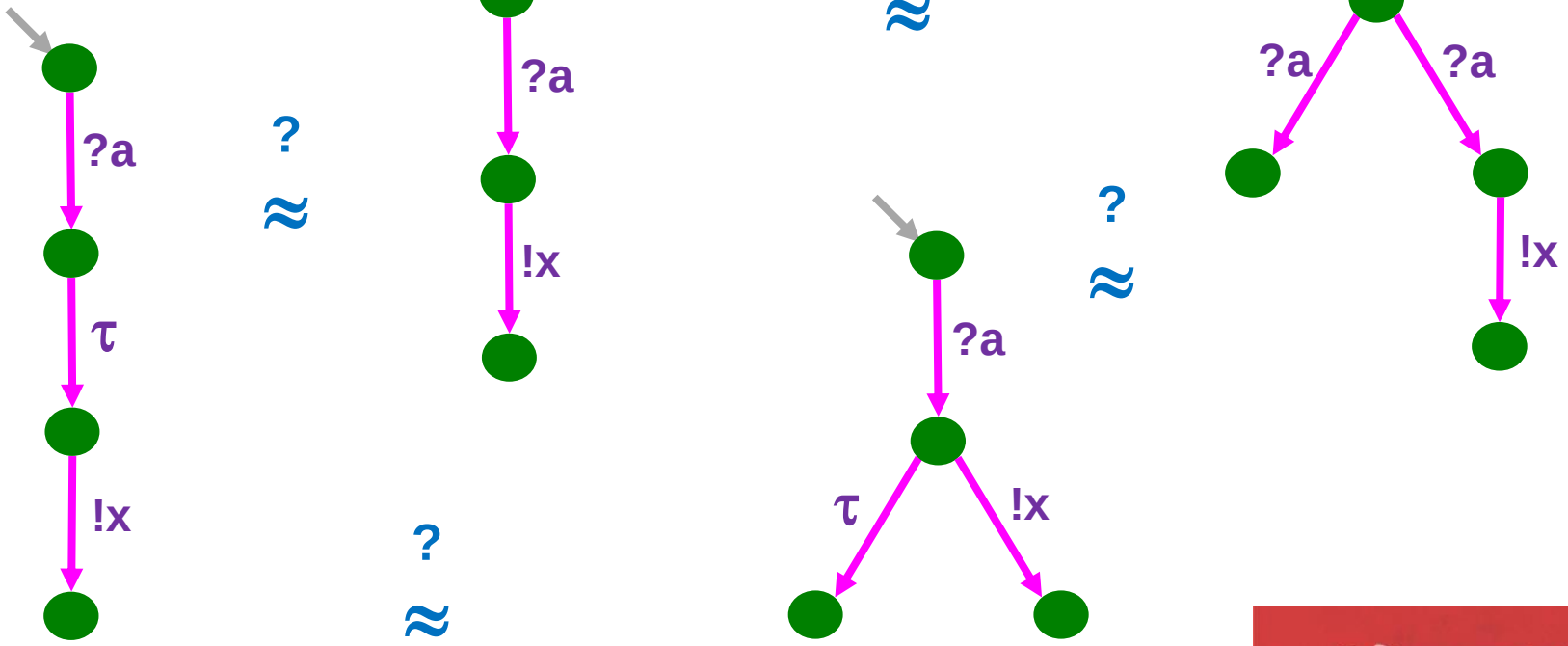
! = *output*

$L = L_I \cup L_U$

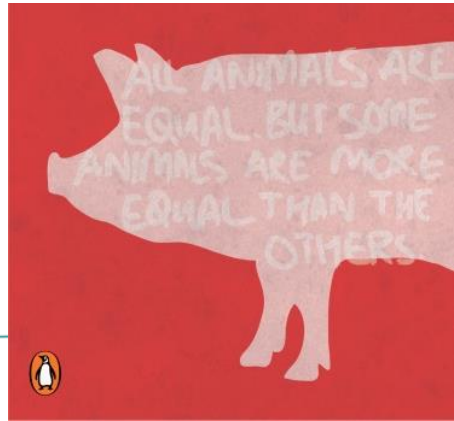
$L_I \cap L_U = \emptyset$



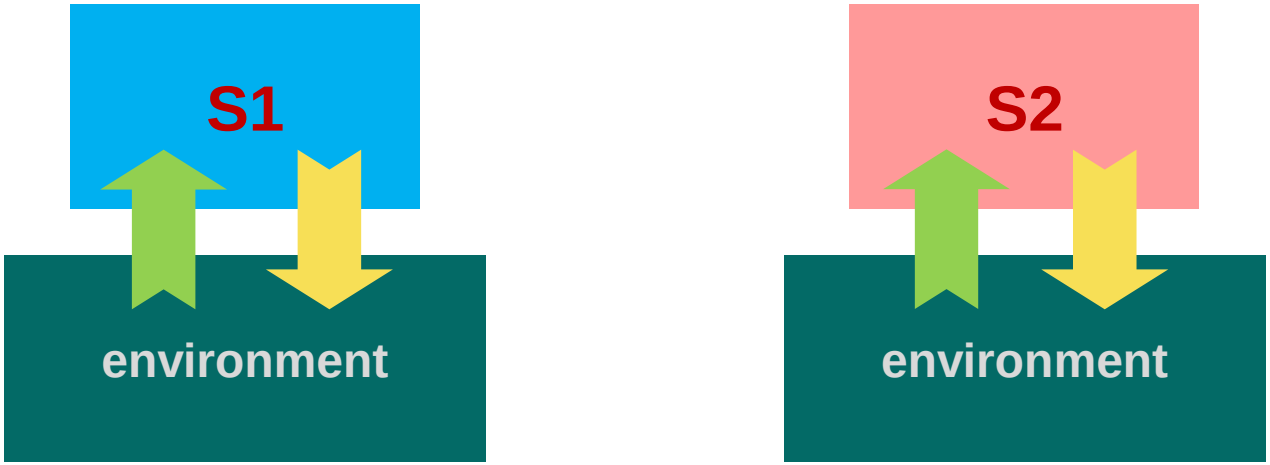
Observable Behaviour



“ Some transition systems are more equal than others “

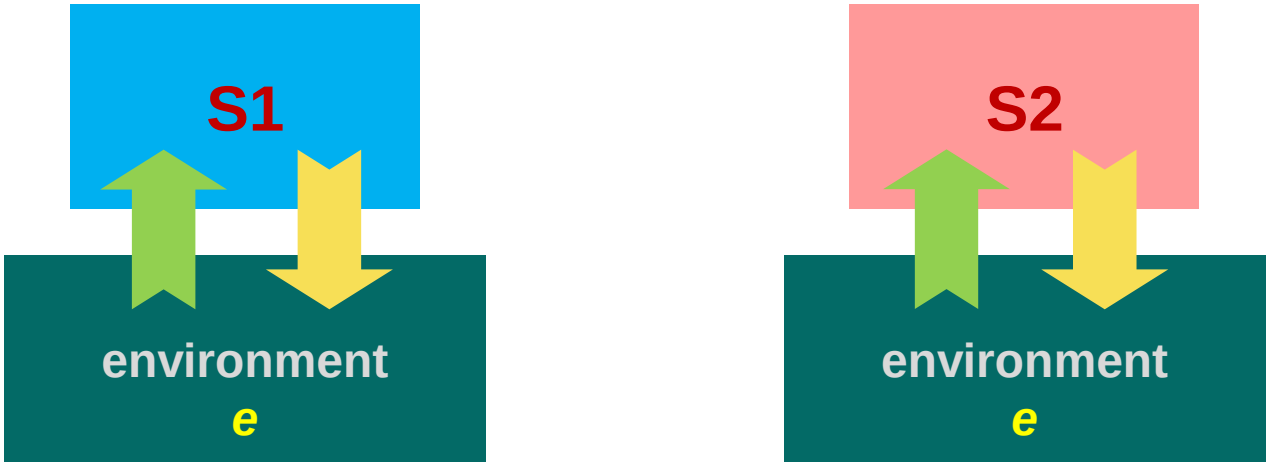


Comparing Transition Systems



- Suppose an environment interacts with the systems:
  - the environment tests the system as black box
    by observing and actively controlling it;
  - the environment acts as a tester;
- Two systems are equivalent iff they pass the same tests.

Comparing Transition Systems

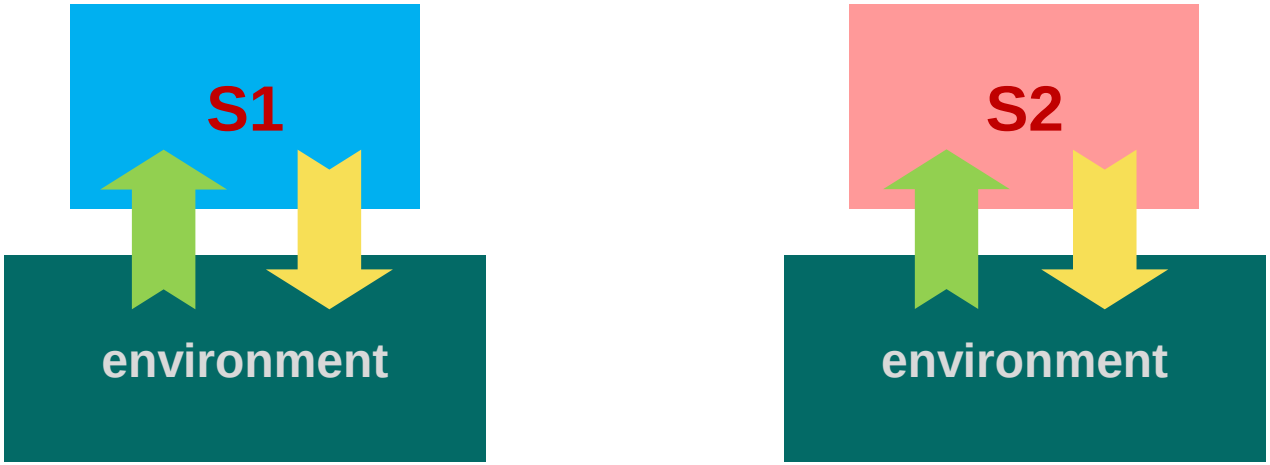


$$S1 \approx S2 \quad \Leftrightarrow \quad \forall\, t \in T.\ \ obs(t, S1) = obs(t, S2)$$

↓ ↓

? ?

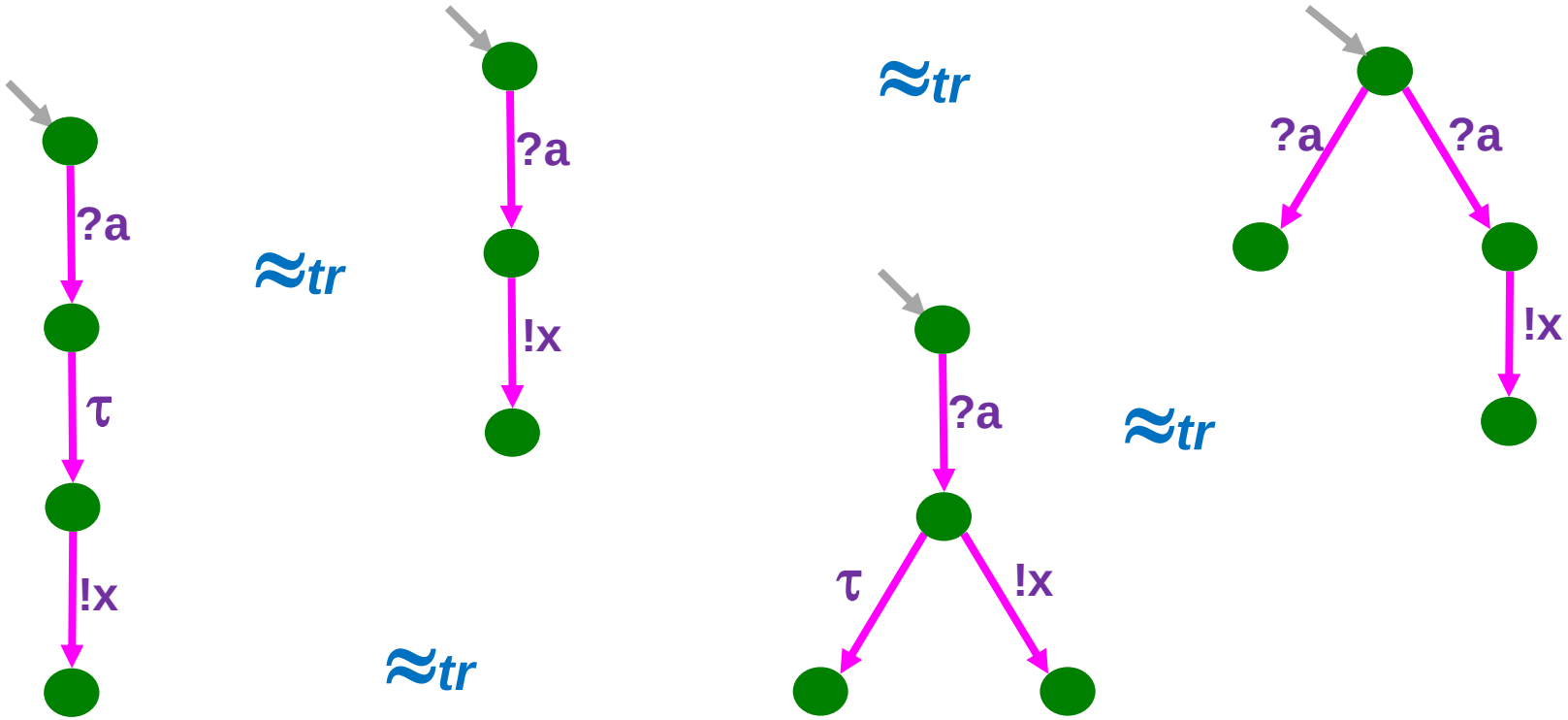
Comparing Transition Systems



$S1 \approx_{tr} S2 \Leftrightarrow traces(S1) = traces(S2)$

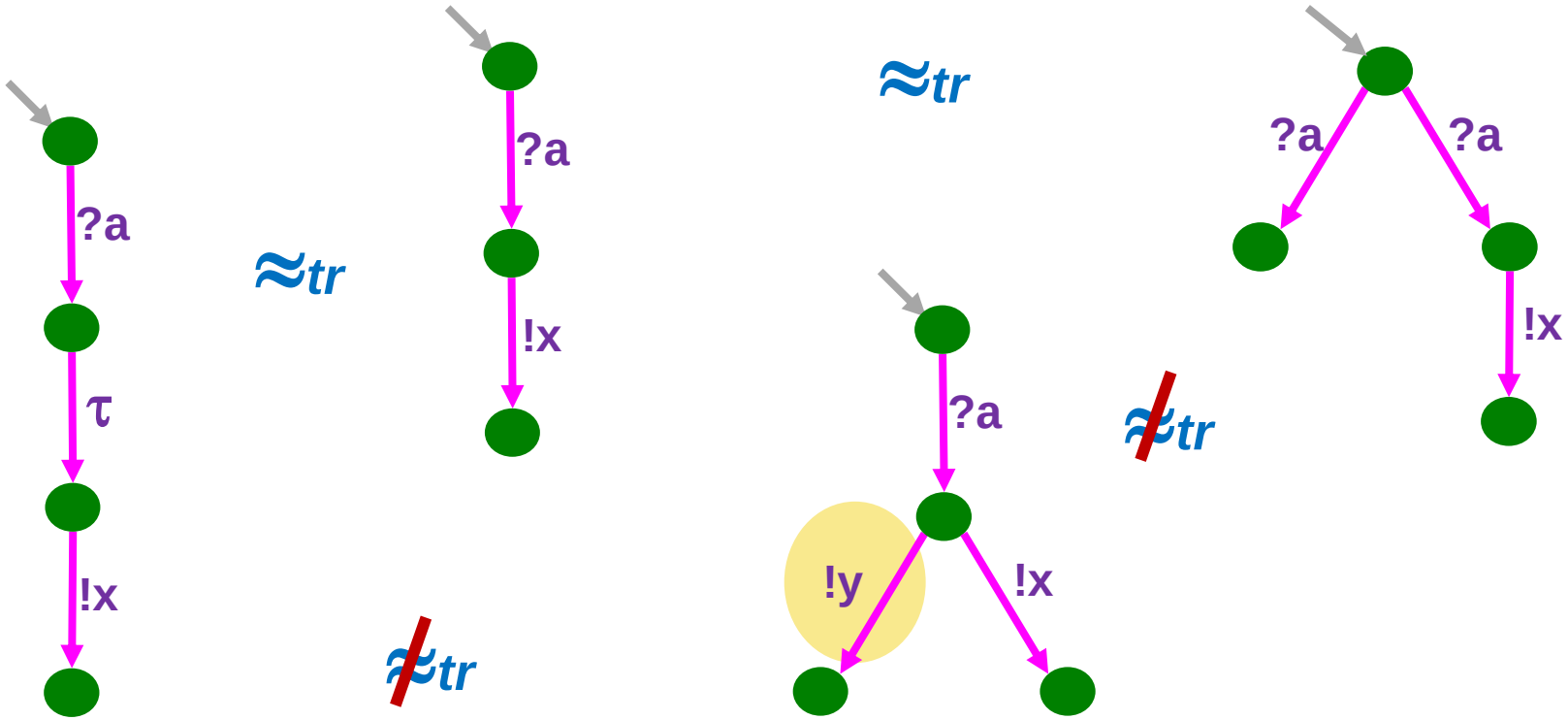
Traces: $traces(s) = \{ \sigma \in L^* \mid s \stackrel{\sigma}{\Longrightarrow} \}$

Trace Equivalence



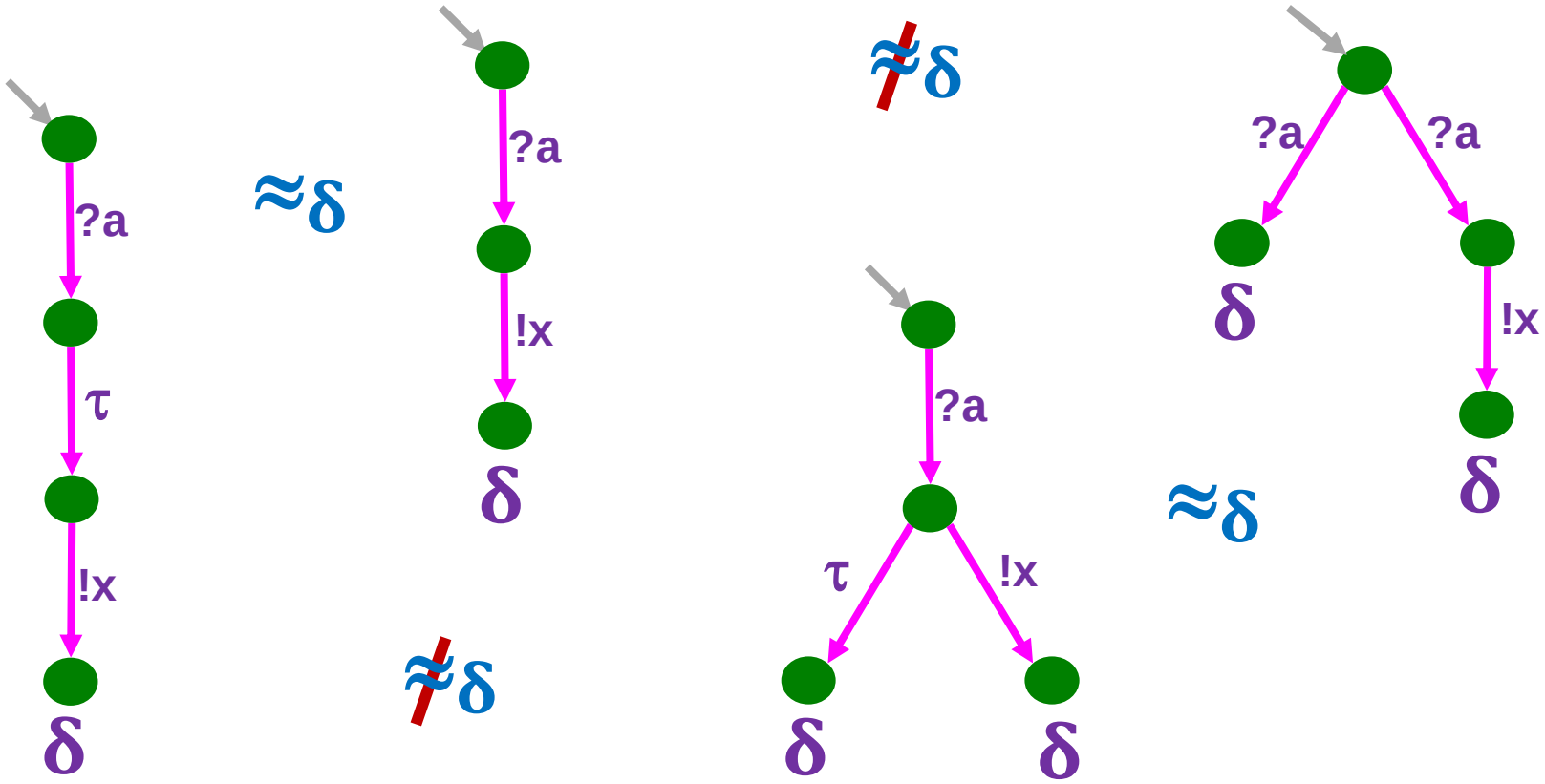
for all: ***traces*** **(.)** **=** **{ ε , ?a , ?a.!x }**

Trace Equivalence



?a. !y ∈ *traces* (S3)

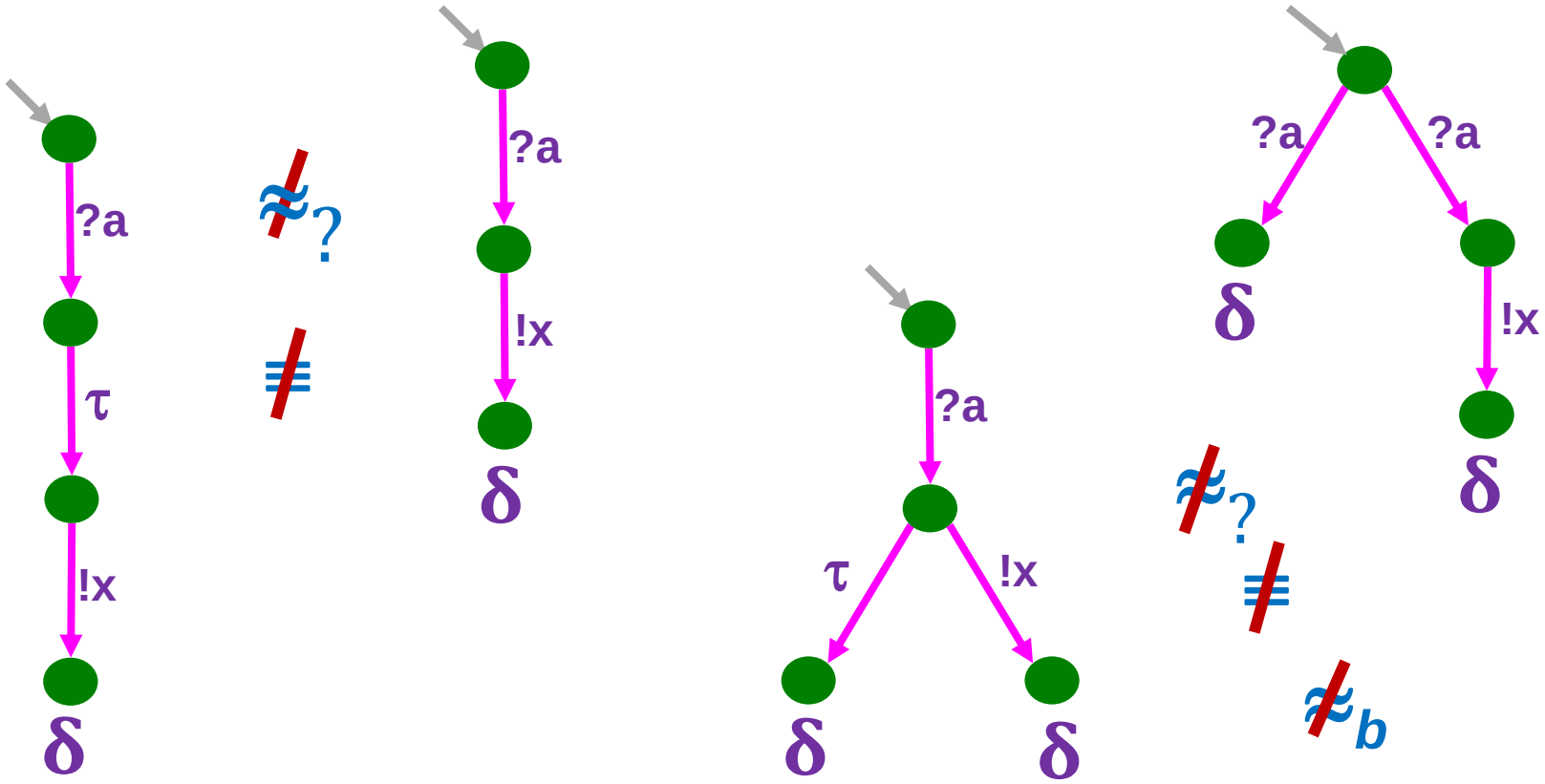
Equivalence with Quiescence δ



$$p \xrightarrow{\delta} p \quad = \quad \forall\ !x \in L_U \cup \{\tau\}\ .\ p \not\xrightarrow{!x}$$

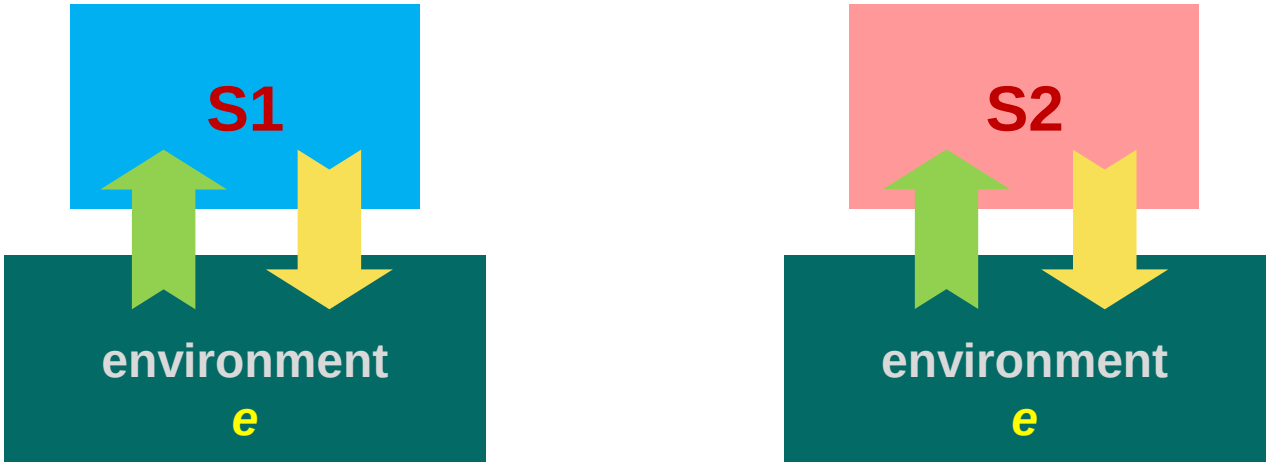
$$\mathit{Straces}(s) \quad = \quad \{\ \sigma \in (L \cup \{\delta\})^*\ \mid s \stackrel{\sigma}{\Longrightarrow}\ \}$$

Stronger Equivalences



≡	isomorphism
≈	bisimulation

Comparing Transition Systems



$$S1 \approx S2 \quad \Leftrightarrow \quad \forall\, t \in T.\ \ obs(t, S1) = obs(t, S2)$$

↓ ↓

? ?

MBT : Equivalences

1. *equivalent if they have the same behaviours*

$$S1 \approx S2 \Leftrightarrow traces(S1) = traces(S2)$$

$$S1 \approx S2 \Leftrightarrow Straces(S1) = Straces(S2)$$

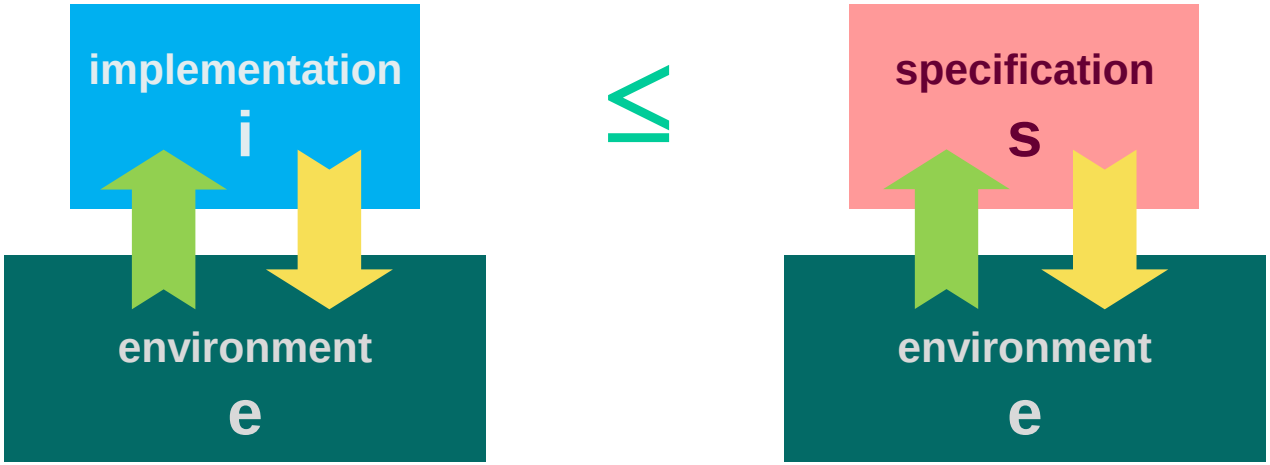
2. *equivalent if they pass the same tests*

$$S1 \approx S2 \Leftrightarrow \forall t \in \mathbf{T} . \; obs(t, S1) = obs(t, S2)$$

3. *equivalent if they have the same implementations*

$$S1 \approx S2 \Leftrightarrow Imp(S1) = Imp(S2)$$

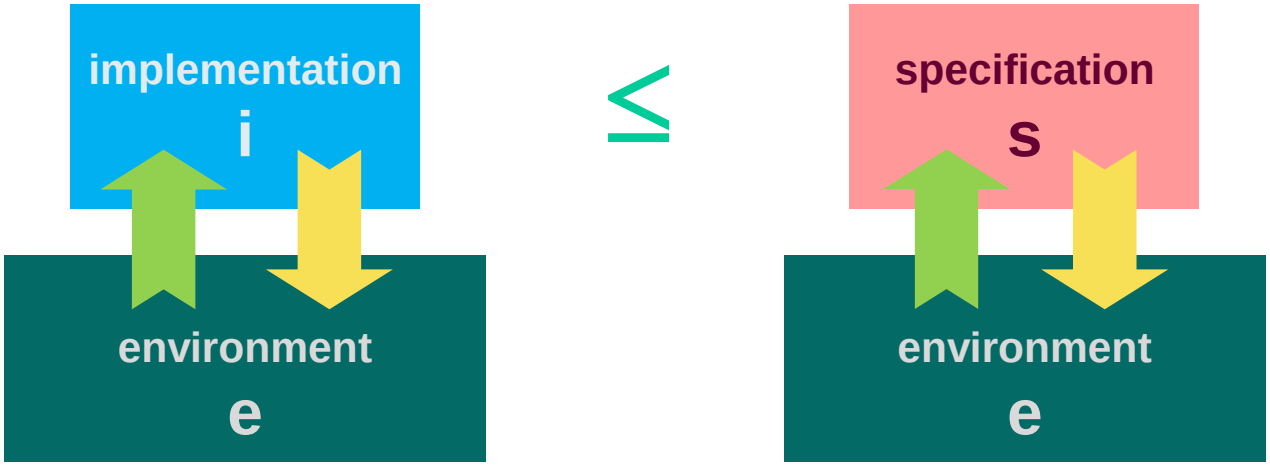
Comparing Transition Systems



- Suppose environment **e** interacts with implementation **i** and with the specification **s** :
  - **i** correctly implements **s**
    iff all observations of **i** can be related to observations of **s**

Implementation Relations on LTS

$i \in LTS$



$s \in LTS$

$$i \leq s \Leftrightarrow \forall t \in T . \; obs(t, i) \subseteq obs(t, s)$$

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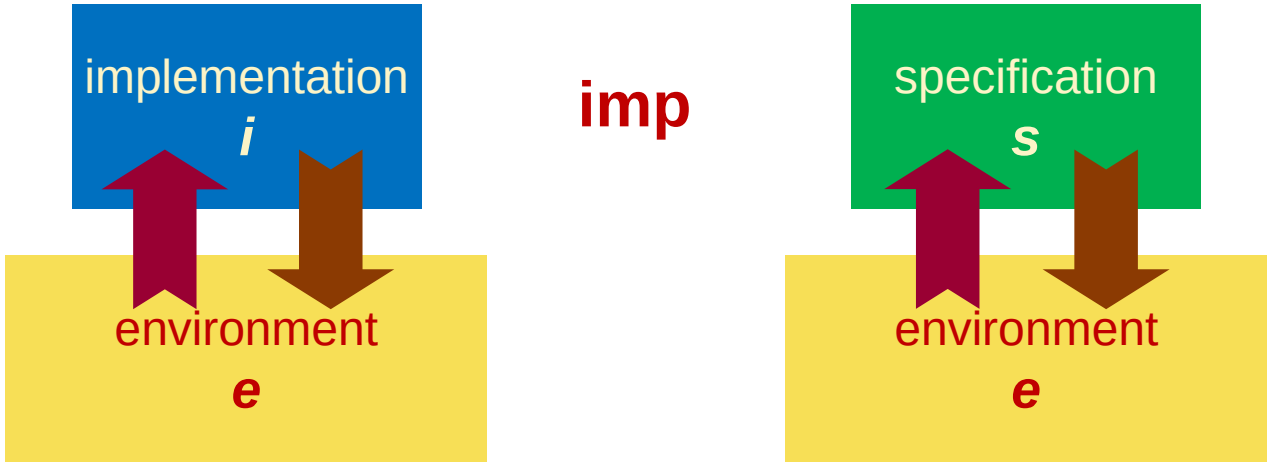
? ? ?

Implementation Relations for Input-Output Transition Systems



$i \in IOTS(L_I, L_U)$ $\quad$ $s \in LTS(L_I, L_U)$

$$\mathbf{imp} \subseteq IOTS(L_I, L_U) \times LTS(L_I, L_U)$$

i* imp *s

i is a conforming implementation of **s**

Implementation Relations on LTS

i imp s : implementation **i** implements specification **s**

• **imp** reflexive ?	**s imp s**	Yes
• **imp** symmetric ?	**i imp s** $\Rightarrow$ **s imp i**	No
• **imp** transitive ?	**i imp s**, **s imp t** $\Rightarrow$ **i imp t**	Yes
• **imp** anti-symmetric ?	**i imp s**, **s imp i** $\Rightarrow$ **i = s**	No
• **imp** linear ?	**i imp s** or **s imp i**	No

equivalence :	reflexive, symmetric, transitive
preorder :	reflexive, transitive
partial order :	anti-symmetric preorder
linear/total order :	linear partial order

$$Imp(s) = \{ i \mid i \text{ imp } s \}$$

MBT : *our choices*

$s \in LTS(L_I, L_U)$

$i \in IOTS(L_I, L_U)$

i* imp *s $\Leftrightarrow$ ***i* iuioco *s***

S1 $\approx$ **S2** $\Leftrightarrow$ *Ftraces* **(S1) =** *Ftraces* **(S2)**

Ftraces : input-failure traces over $\mathbf{L_I \cup L_U \cup \{ \delta \}}$

Test Cases

model of a test case

= labelled transition system

- labels in $L_I \cup L_U \cup \{ \theta \}$
- ‘quiescence’ / ‘time-out’ label θ
- tree-structured
- finite, deterministic
- sink states **pass** and **fail**
- from each state ≠ **pass**, **fail** :
  - either one input **!a** and all outputs **?x**
  - or all outputs **?x** and θ