Model Checking

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1

$$V_{n+1}(s) = \begin{cases} 1 & \text{if } s \in T \\ 0 & \text{if } s \in S^{max=0} \\ \max_{a \in A} \left\{ R(s,a) + \gamma \inf_{P(s,a) \in D(s,a)} \left\{ \sum_{s' \in S} P(s,a)(s') V_n(s') \right\} \right\} \end{cases}$$

2 a) Best Case instance: $P(s_0, a_1)(s_1) = 0.7$,

$P(s_0, a_1)(s_3) = 0.3$

Worst case instance: $P(s_0, a_2, s_1) = 0.6$

$P(s_0, a_2, s_3) = 0.4$

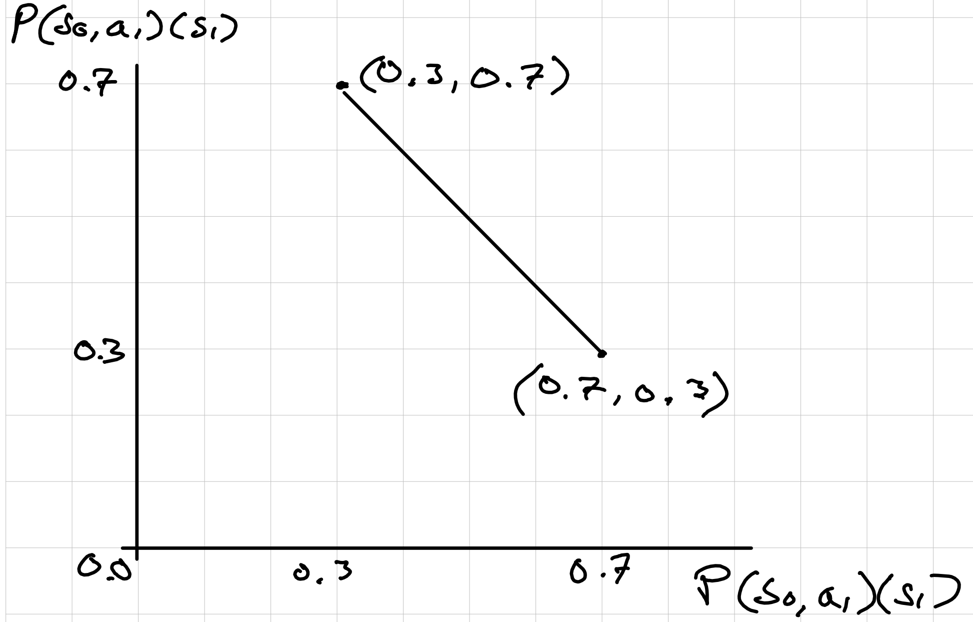
b) $D(s_0, a_1) =$

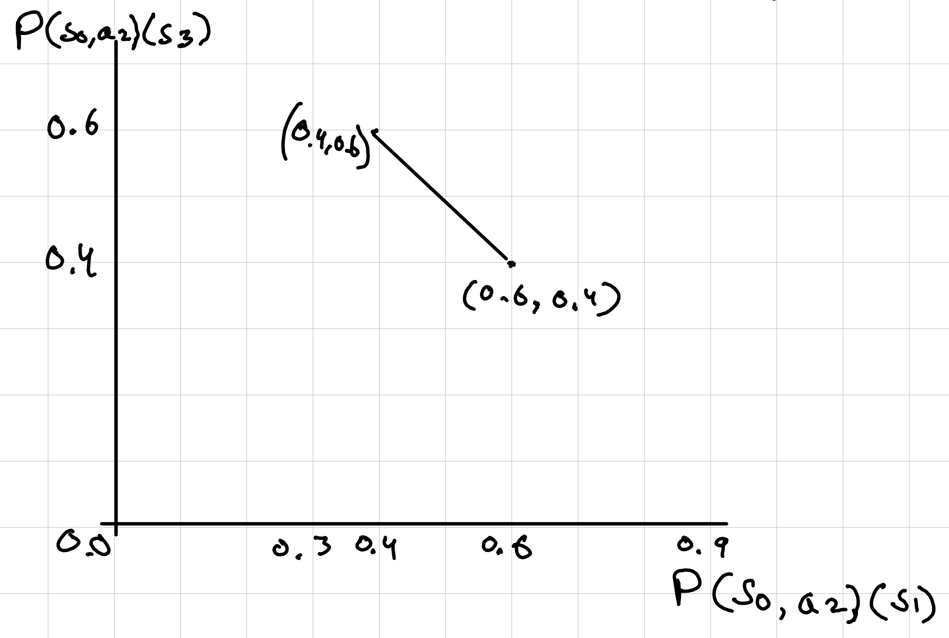
$$\left\{ P \in D(\{s_1, s_3\}) \mid P(s_1) \in [0.3, 0.7] \wedge P(s_3) \in [0.3, 0.7] \right\}$$

$D(s_0, a_2) =$

$$\left\{ P \in D(\{s_1, s_3\}) \mid P(s_1) \in [0.4, 0.6] \wedge P(s_3) \in [0.4, 0.6] \right\}$$

c)





d) For $P(s_0, a_1)$: $(0.3, 0.7)$, $(0.7, 0.3)$

For $P(s_0, a_2)$: $(0.4, 0.6)$, $(0.6, 0.4)$

3

Order the states $s_1, s_2 \dots s_n$ such that: $V_n(s_1) \leq V_n(s_2) \leq \dots \leq V_n(s_m)$.
Then find index j such that

- All states indexed $< s_j$ get the lower bound as transition value,
- All states indexed $> s_j$ get the upper bound as transition value,
- State s_j gets a value in $[P(s_j), P(s_j)]$ such that we have a valid distribution.

In the *inner maximization problem*, nature is cooperative and gives us the transition function with the highest probablity of going to a desired states. States are ordered according to 'desirability'.

4

a

$$
\begin{aligned}
P(s, a, s_1) &= \frac{1}{6} \\
P(s, a, s_2) &= \frac{1}{2} \\
P(s, a, s_3) &= 0 \\
P(s, a, s_4) &= \frac{1}{3}
\end{aligned}
$$

b

$$\epsilon = 0.01$$
$$\epsilon_M = 0.01/8 = 0.00125$$
$$\delta_M = \sqrt{\frac{\log(\frac{2}{0.00125})}{2 \cdot 12}} \approx 0.6660$$
$$\underline{P}(s, a, s_1) = \frac{1}{6} - \delta_M \equiv 0$$
$$\bar{P}(s, a, s_1) = \frac{1}{6} + \delta_M \approx 0.8326$$
$$\underline{P}(s, a, s_1) = \frac{1}{2} - \delta_M \equiv 0$$
$$\bar{P}(s, a, s_1) = \frac{1}{2} + \delta_M \equiv 1$$
$$\underline{P}(s, a, s_1) = 0 - \delta_M \equiv 0$$
$$\bar{P}(s, a, s_1) = 0 + \delta_M \approx 0.6660$$
$$\underline{P}(s, a, s_1) = \frac{1}{3} - \delta_M \equiv 0$$
$$\bar{P}(s, a, s_1) = \frac{1}{3} + \delta_M \approx 0.9993$$