Model Checking

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1

a

$$
\begin{aligned}
P(Cyl(s_0s_1) \cup Cyl(s_0s_5s_6) \cup Cyl(s_0s_5s_4s_3) \cup Cyl(s_0s_1s_6)) &=^1 \\
P(Cyl(s_0s_1) \cup Cyl(s_0s_5s_6) \cup Cyl(s_0s_5s_4s_3)) &= \\
\frac{1}{3} + \frac{2}{3} \cdot \frac{1}{4} + \frac{2}{3} \cdot \frac{1}{4} \cdot 1 &= \frac{2}{3}
\end{aligned}
$$

1. Because $Cyl(s_0s_1s_6) \subseteq Cyl(s_0s_1)$.

b

We can already observe that B is always reachable so the result is guaranteed to be 1. However, this is how you can apply the algorithm.

This yields the following equations. Let x_s denote $P_{\text{reach}}(s, B)$.

$$
\begin{aligned}
x_2 &= x_3 = 1 \\
x_4 &= x_3 \\
x_5 &= \frac{1}{4}x_4 + \frac{1}{4}x_6 + \frac{1}{2}x_0 \\
x_0 &= \frac{2}{3}x_5 + \frac{1}{3}x_1 \\
x_1 &= \frac{1}{3}x_1 + \frac{2}{3}x_6 \\
x_6 &= \frac{1}{2}x_6 + \frac{1}{2}x_2
\end{aligned}
$$

Simplifying yields $x_n = 1$ for $0 \leq n \leq 6$. Then $P(s_0 \Vdash \diamond B) = P_{\text{reach}}(s_0, B) = x_0 = 1$.

c (i)

In order to solve this, we need to modify bounded reachability slightly:

$$P_{\text{until}}(s,k,C,B) = \begin{cases} 1, & \text{if } s \in B \\ 0, & \text{if } s \notin C \text{ and } s \notin B \\ 0, & \text{if } k = 0 \text{ and } s \notin B \\ \sum_{s' \in S} P(s,s') \cdot P_{\text{until}}(s', k-1, C, B) & \text{otherwise} \end{cases}$$

Let $x_{s,k}$ denote $P_{\text{until}}(s,k,C,B)$. This gives the following equations for $0 \leq k < 5$.

$$\begin{aligned} x_{2,k+1} = x_{3,k+1} &= 1 \\ x_{4,k+1} &= x_{3,k} \\ s_{5,k+1} &= 0 \\ s_{2,k+1} &= 1 \\ s_{6,k+1} &= \frac{1}{2}s_{6,k} + \frac{1}{2}s_{2,k} \\ s_{1,k+1} &= \frac{2}{3}s_{6,k} + \frac{1}{3}s_{1,k} \\ s_{0,k+1} &= \frac{1}{3}s_{1,k} = \frac{2}{3}s_{5,k} \end{aligned}$$

Solving them yields $x_{6,3} = \frac{7}{8}$, $x_{1,4} = \frac{85}{108}$, and $x_{0,5} = \frac{85}{108}$.

d

$P(s_0 \Vdash \diamond\Box D) = P(s_0 \Vdash \diamond s_4)$, since we can always reach D, iff we enter s_4.

Then once again, we can make reachability equations. Let x_s denote $P_{\text{reach}}(s, D)$.

$$\begin{aligned} &x_1 = x_6 = x_2 = 0 \\ &x_3 = x_4 = 1 \\ &x_0 = \frac{2}{3}x_5 + \frac{1}{3}x_1 \\ &x_5 = \frac{1}{2}x_0 + \frac{1}{4}x_6 + \frac{1}{4}x_4 \end{aligned}$$

Simplifying yields $P(s_0 \Vdash \diamond\Box D) = x_0 = 0.4$.

3

a

$$A(s,k) = \begin{cases} 0 & \text{if } k = 0 \wedge s \notin B \\ 1 & \text{if } k = 0 \wedge s \in B \\ 0 & \text{if } k > 0 \wedge s \notin C \\ \sum_{s' \in S} P(s,s') \cdot A(s, k-1) & \text{otherwise} \end{cases}$$

b

$$B(s,n) = \begin{cases} 0 & \text{if } s \notin C \wedge n > 0 \\ \sum_{s' \in S} P(s,s') \cdot B(s', n-1) & \text{if } s \in C \wedge n > 0 \\ P(s \vDash C \cup B) & \text{otherwise} \end{cases}$$

where

$$P(s \vDash C \cup B) = \begin{cases} 0 & \text{if } B \text{ is not reachable from } s \\ 0 & \text{if } s \notin C \\ 1 & \text{if } s \in B \\ \sum_{s' \in S} P(s,s') \cdot P(s' \vDash C \cup B) & \text{otherwise} \end{cases}$$