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Quiz

1. What is the difference between weak and strong normalisation?

   Answer: weak normalisation means that all terms can be reduced to a normal form. Strong normalisation (aka termination) means that there is no infinite reduction. Put differently, if we define a finished reduction sequence starting in s as either a finite sequence $s \Rightarrow s_1 \Rightarrow s_2 \Rightarrow \dots \Rightarrow s_n$ with $n \geq 0$ and s_n a normal form, or an infinite sequence $s \Rightarrow s_1 \Rightarrow s_2 \Rightarrow \dots$, then:

   Weak normalisation: for all terms s, **there exists** a finished reduction sequence starting in s that is finite

   Strong normalisation: for all terms s, **all** finished reduction sequences starting in s are finite

2. What is the difference between local confluence and general confluence?

   Answer: local confluence means that if $s \Rightarrow t$ and $s \Rightarrow q$ then there is some u such that both $t \Rightarrow^* u$ and $q \Rightarrow^* u$. General confluence means that if $s \Rightarrow^* t$ and $s \Rightarrow^* q$ then there is some u such that both $t \Rightarrow^* u$ and $q \Rightarrow^* u$. Put differently:

   Local confluence: If s reduces to both t and q in **one step**, then t and q can be reduced to the same term.

   Confluence: If s reduces to both t and q in **an arbitrary number of steps**, then t and q can be reduced to the same term.

3. Use the lexicographic path ordering (by hand) to prove termination of:

$$\begin{array}{rcl} f(g(x), g(b)) & \Rightarrow & f(x, x) \\ g(a) & \Rightarrow & b \\ b & \Rightarrow & a \end{array}$$

   Answer: note that the third rule forces us to choose $b \rhd a$, and therefore the second rule forces us to choose $g \rhd b$. So we choose $g \rhd b \rhd a$; it turns out not to matter how f falls in the precedence. We end up with the following derivation:

   (a) $f(g(x), g(b)) \succ f(x, x)$ by **(lex)**, (3d), (3e), (3e)
   (b) $g(a) \succ b$ by **(copy)** since $g \rhd b$
   (c) $b \succ a$ by **(copy)** since $b \rhd a$
   (d) $g(x) \succ x$ by **(sub)** since $x \succeq x$
   (e) $f(g(x), g(b)) \succ x$ by **(sub)**, 3d

4. What properties should a relation $\succ$ satisfy to be a reduction order?

   Answer: it should be **stable** and **monotonic**. That is:

   stable If $s \succ t$ and σ is a substitution, then also $s\sigma \succ t\sigma$.

   monotonic If $s_i \succ t_i$ then $f(s_1, \dots, s_i, \dots, s_n) \succ f(s_1, \dots, t_i, \dots, s_n)$.