Model Checking

Linear Temporal Logic

[Baier & Katoen, Chapter 5.1]

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Radboud University, 2025

Credit to the slides: Prof. Dr. Dr.h.c. Joost-Pieter Katoen

Is There a Proper Logic to Define Properties?

Who would tell an engineer to write regular expressions for bad prefixes?

Overview

Overview

Recall: LT Properties

- An LT property is a set of infinite traces over AP
- Specifying such sets explicitly is often inconvenient
- Mutual exclusion is specified over $AP = \{ c_1, c_2 \}$ by

$$E_{mutex} = \text{ set of infinite words } A_0\, A_1 \ldots \text{ with } \{ c_1, c_2 \} \not\subseteq A_i \text{ for all } 0 \leq i$$

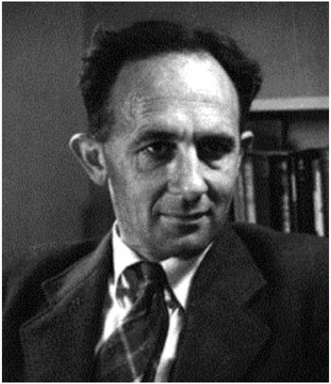
- Starvation freedom is specified over $AP = \{ c_1, w_1, c_2, w_2 \}$ by

$$E_{nostarve} = \text{ set of infinite words } A_0\, A_1 \ldots \text{such that:}$$

$$\left(\overset{\infty}{\exists}\, j.\ w_1 \in A_j\right) \Rightarrow \left(\overset{\infty}{\exists}\, j.\ c_1 \in A_j\right) \wedge \left(\overset{\infty}{\exists}\, j.\ w_2 \in A_j\right) \Rightarrow \left(\overset{\infty}{\exists}\, j.\ c_2 \in A_j\right)$$

Such properties can be specified much more succinctly using logic
(or using ω-regular expressions)

Linear Temporal Logic



Arthur Norman Prior
(1914–†1969)

Amir Pnueli
(1941–†2009)

LTL Syntax

Definition: LTL syntax
BNF grammar for LTL formulas with proposition $a \in AP$:

$$\varphi ::= \text{true} \;\Big|\; a \;\Big|\; \varphi_1 \wedge \varphi_2 \;\Big|\; \neg\varphi \;\Big|\; \underbrace{\bigcirc\varphi}_{\text{next}} \;\Big|\; \underbrace{\varphi_1 \,\mathsf{U}\, \varphi_2}_{\text{until}}$$

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$\{a, b\}$

S_0

$a \wedge b$

- Propositional logic
  - $a \in AP$ — atomic proposition
  - $\neg\varphi$ and $\varphi \wedge \psi$ — negation and conjunction

- Temporal modalities
  - $\bigcirc\varphi$ — neXt state fulfills φ
  - $\varphi \,\mathsf{U}\, \psi$ — φ holds Until a ψ-state is reached

Linear Temporal Logic (LTL) is a logic to describe LT properties

$$\varphi \vee \psi \equiv \neg(\neg\varphi \wedge \neg\psi)$$

$$\varphi \Rightarrow \psi \equiv \neg\varphi \vee \psi$$

$$\varphi \Leftrightarrow \psi \equiv (\varphi \Rightarrow \psi) \wedge (\psi \Rightarrow \varphi)$$

$$\varphi \oplus \psi \equiv$$

$$\text{true} \equiv \neg\,\text{false}$$

$$\text{false} \equiv \neg\,\text{true}$$

Derived Operators

$$
\begin{aligned}
\varphi \vee \psi &\equiv \neg(\neg\varphi \wedge \neg\psi) \\
\varphi \Rightarrow \psi &\equiv \\
\varphi \Leftrightarrow \psi &\equiv \\
\varphi \oplus \psi &\equiv \\
\textsf{true} &\equiv \\
\textsf{false} &\equiv \\
\Diamond \varphi &\equiv \qquad \text{“some time in the future”} \\
\Box \varphi &\equiv \qquad \text{“from now on forever”}
\end{aligned}
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precedence order: the unary operators bind stronger than the binary ones.

$\neg$ and $\bigcirc$ bind equally strong. U takes precedence over $\wedge$, $\vee$, and $\Rightarrow$

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\varphi \lor \psi &\equiv \neg(\neg\varphi \land \neg\psi) \\
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$$\varphi \Rightarrow \psi \equiv \neg\varphi \lor \psi$$

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$$\varphi \oplus \psi \equiv (\varphi \land \neg\psi) \lor (\neg\varphi \land \psi)$$

$$\text{true} \equiv \varphi \lor \neg\varphi$$

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\Diamond\varphi &\equiv \mathsf{true}\ \mathsf{U}\ \varphi \quad \text{"some time in the future"} \\
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\end{aligned}$$

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$$\varphi \Leftrightarrow \psi \equiv (\varphi \Rightarrow \psi) \wedge (\psi \Rightarrow \varphi)$$

$$\varphi \oplus \psi \equiv (\varphi \wedge \neg\psi) \vee (\neg\varphi \wedge \psi)$$

$$\mathsf{true} \equiv \varphi \vee \neg\varphi$$

$$\mathsf{false} \equiv \neg\,\mathsf{true}$$

$$\Diamond\varphi \equiv \mathsf{true}\,\mathsf{U}\,\varphi \qquad \text{“some time in the future”}$$

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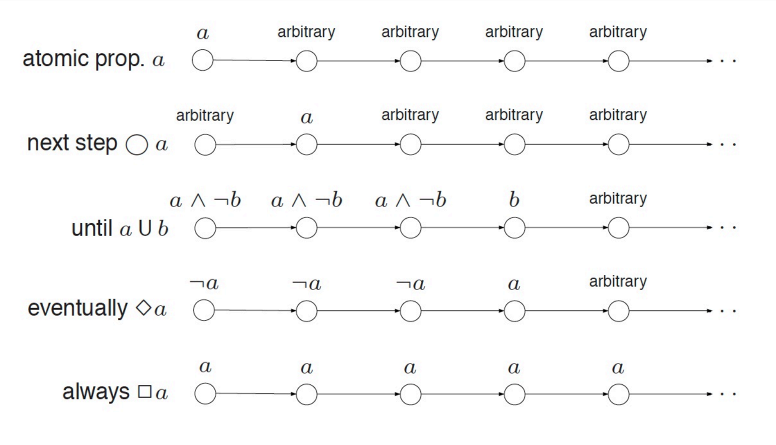
Intuitive Semantics



Example: Traffic Light Properties

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Example: Traffic Light Properties

- The traffic light becomes green eventually: $\Diamond\, green$
- Once red, the light cannot become green immediately:

$$\Box\,(red \;\Rightarrow\; \neg \bigcirc green)$$

- Once red, the light becomes green eventually: $\Box\,(red \;\Rightarrow\; \Diamond\, green)$

- Once red, the light always becomes green eventually after being yellow for some time inbetween:

$$\Box(red \;\Rightarrow\; \bigcirc\,(red \;\mathrm{U}\; (yellow \wedge \bigcirc\,(yellow \;\mathrm{U}\; green))))$$

Example Properties in LTL

- Reachability
  - negated reachability $\Diamond \neg \psi$
  - conditional reachability $\varphi \,\mathsf{U}\, \psi$
  - reachability from any state not expressible

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- Reachability
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  - conditional reachability $\varphi \,\mathsf{U}\, \psi$
  - reachability from any state not expressible
- Safety
  - simple safety $\Box \neg\varphi$
  - conditional safety $(\varphi \,\mathsf{U}\, \psi) \vee \Box \varphi$

Example Properties in LTL

- Reachability
  - negated reachability — $\Diamond \neg \psi$
  - conditional reachability — $\varphi \, \mathsf{U} \, \psi$
  - reachability from any state — not expressible
- Safety
  - simple safety — $\Box \neg \varphi$
  - conditional safety — $(\varphi \, \mathsf{U} \, \psi) \vee \Box \varphi$
- Liveness — $\Box (\varphi \Rightarrow \Diamond \psi)$ and others

Overview

Semantics Over Words

Definition: LTL semantics over infinite words
The LT-property induced by LTL formula φ over AP is:

$$Words(\varphi) \;=\; \left\{ \sigma \in \left(2^{AP}\right)^{\omega} \mid \sigma \vDash \varphi \right\}, \text{where } \vDash \text{ is the smallest relation with:}$$

$$\begin{array}{llll}
\sigma & \vDash & \text{true} & \\
\sigma & \vDash & a & \text{iff} \quad a \in A_0 \quad (\text{i.e., } A_0 \vDash a) \\
\sigma & \vDash & \varphi_1 \wedge \varphi_2 & \text{iff} \quad \sigma \vDash \varphi_1 \text{ and } \sigma \vDash \varphi_2 \\
\sigma & \vDash & \neg\,\varphi & \text{iff} \quad \sigma \nvDash \varphi \\
\sigma & \vDash & \bigcirc\,\varphi & \text{iff} \quad \sigma[1..] = A_1 A_2 A_3 \ldots \vDash \varphi \\
\sigma & \vDash & \varphi_1 \,\mathsf{U}\, \varphi_2 & \text{iff} \quad \exists j \geq 0.\ \sigma[j..] \vDash \varphi_2 \;\text{ and }\; \sigma[i..] \vDash \varphi_1,\ 0 \leq i < j
\end{array}$$

$$\begin{array}{ccccc}
\varphi_1 & \varphi_1 & \varphi_1 & & \varphi_2 \\
A_0 & A_1 & A_2 & \ldots & A_j \\
i & i & i & & j
\end{array}$$

for $\sigma = A_0 A_1 A_2 \ldots$, let $\sigma[i..] = A_i A_{i+1} A_{i+2} \ldots$ be the suffix of σ from index i on.

Semantics of $\Box$, $\Diamond$, $\Box\Diamond$ and $\Diamond\Box$

$$\sigma \quad \models \quad \Diamond\,\varphi \qquad \text{iff} \quad \exists j \geq 0.\ \sigma[j..] \models \varphi$$

Semantics of $\Box$, $\Diamond$, $\Box\Diamond$ and $\Diamond\Box$

$$\sigma \models \Diamond\varphi \quad \text{iff} \quad \exists j \geq 0.\ \sigma[j..] \models \varphi$$

$$\sigma \models \Box\varphi \quad \text{iff} \quad \forall j \geq 0.\ \sigma[j..] \models \varphi$$

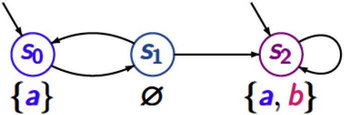
Semantics of $\Box$, $\Diamond$, $\Box\Diamond$ and $\Diamond\Box$

$$\begin{array}{lllll}
\sigma & \models & \Diamond\,\varphi & \text{iff} & \exists j \geq 0.\ \sigma[j..] \models \varphi \\
\sigma & \models & \Box\,\varphi & \text{iff} & \forall j \geq 0.\ \sigma[j..] \models \varphi \\
\sigma & \models & \Box\Diamond\,\varphi & \text{iff} & \forall j \geq 0.\ \exists i \geq j.\ \sigma[i\ldots] \models \varphi
\end{array}$$

Semantics of $\square$, $\Diamond$, $\square\Diamond$ and $\Diamond\square$

$$
\begin{array}{llll}
\sigma & \models & \Diamond\,\varphi & \text{iff} \quad \exists j \geq 0.\ \sigma[j..] \models \varphi \\
\sigma & \models & \square\,\varphi & \text{iff} \quad \forall j \geq 0.\ \sigma[j..] \models \varphi \\
\sigma & \models & \square\Diamond\,\varphi & \text{iff} \quad \forall j \geq 0.\, \exists i \geq j.\ \sigma[i\ldots] \models \varphi \\
\sigma & \models & \Diamond\square\,\varphi & \text{iff} \quad \exists j \geq 0.\forall i \geq j.\ \sigma[i\ldots] \models \varphi
\end{array}
$$

Example



$AP = \{a, b\}$

path $\pi = s_0\, s_1\, s_2\, s_2\, s_2\, s_2 \ldots$ $\quad$ $\mathit{trace}(\pi) = \{a\}\, \varnothing\, \{a, b\}^\omega$

$\pi \models a$, but $\pi \not\models b$	as $L(s_0) = \{a\}$
$\pi \models \bigcirc(\neg a \wedge \neg b)$	as $L(s_1) = \varnothing$
$\pi \models \bigcirc \bigcirc (a \wedge b)$	as $L(s_2) = \{a, b\}$
$\pi \models (\neg b) \mathsf{U} (a \wedge b)$	as $s_0, s_1 \models \neg b$
$\pi \models (\neg b) \mathsf{U} \Box(a \wedge b)$	and $s_2 \models a \wedge b$

Semantics over Paths and States

Let $TS = (S, Act, \rightarrow, I, AP, L)$ be a transition system and φ be an LTL-formula over AP.

- For infinite path fragment π of TS:

$$\pi \vDash \varphi \qquad \text{iff} \qquad trace(\pi) \vDash \varphi$$

- For state $s \in S$:

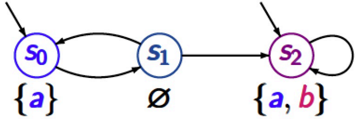
$$s \vDash \varphi \qquad \text{iff} \qquad \forall \pi \in Paths(s).\ \pi \vDash \varphi$$

- For transition system TS:

$$TS \vDash \varphi \qquad \text{iff} \qquad Traces(TS) \subseteq Words(\varphi) \qquad \text{iff} \qquad \forall s \in I.\ s \vDash \varphi$$

Example

stop the lecture



$AP = \{a, b\}$

$\mathcal{T} \models a$ as $s_0 \models a$ and $s_2 \models a$

$\mathcal{T} \not\models \Diamond\Box a$ as $s_0\, s_1\, s_0\, s_1 \ldots \not\models \Diamond\Box a$

$\mathcal{T} \models \Diamond\Box b \vee \Box\Diamond(\neg a \wedge \neg b)$ as $s_2 \models b$, $s_1 \not\models a, b$

$\mathcal{T} \models \Box(a \rightarrow (\bigcirc\neg a \vee b))$ as $s_2 \models b$, $s_0 \models \bigcirc\neg a$

On The Semantics of Negation

For paths, it holds $\pi \vDash \varphi$ if and only if $\pi \nvDash \neg\varphi$ since:

$$Words(\neg\varphi) = \left(2^{AP}\right)^{\omega} \setminus Words(\varphi) \quad .$$

On The Semantics of Negation

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$$Words(\neg\varphi) = \left(2^{AP}\right)^{\omega} \setminus Words(\varphi) \quad .$$

But: $TS \nvDash \varphi$ and $TS \vDash \neg\varphi$ are *not* equivalent in general

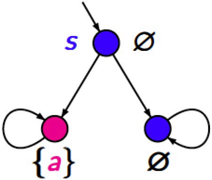
It holds: $TS \vDash \neg\varphi$ implies $TS \nvDash \varphi$. Not always the reverse!

Note that:

$$\begin{aligned} TS \nvDash \varphi \quad & \text{iff} \quad Traces(TS) \nsubseteq Words(\varphi) \\ & \text{iff} \quad Traces(TS) \setminus Words(\varphi) \neq \varnothing \\ & \text{iff} \quad Traces(TS) \cap Words(\neg\varphi) \neq \varnothing \quad . \end{aligned}$$

TS neither satisfies φ nor $\neg\varphi$ if there are paths π_1 and π_2 in TS such that $\pi_1 \vDash \varphi$ and $\pi_2 \vDash \neg\varphi$

Example



$$s \not\models \Diamond a \quad \text{and} \quad s \not\models \neg\Diamond a$$

LTL Formulas for LT Properties

Provide LTL formulas over $AP = \{ a, b \}$ for the LT properties:

- set of all words $A_0 \, A_1 \ldots$ over $\left(2^{AP}\right)^{\omega}$ such that:

$$\begin{aligned} & \forall i \geq 0. \, (a \in A_i \; \Rightarrow \; i > 0 \; \wedge \; b \in A_{i-1}) \\ \equiv \; & \forall j \geq 0. \, \left(b \in A_j \; \vee \; a \notin A_{j+1}\right) \\ \equiv \; & \mathit{Words}(\, \square \, (b \; \vee \; \neg \bigcirc a)\,) \end{aligned}$$

LTL Formulas for LT Properties

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$$\equiv\ \forall j \geq 0.\ \left(b \in A_j \ \vee\ a \notin A_{j+1}\right)$$
$$\equiv\ Words(\,\Box\,(b \vee \neg\bigcirc a)\,)$$

- set of all words of the form

$$\{ b \}^{n_1} \{ a \} \{ b \}^{n_2} \{ a \} \{ b \}^{n_3} \{ a \} \ldots$$

where $n_i \geq 0$. This is captured by

$$Words(\,\Box\,((b \wedge \neg a)\,\mathsf{U}\,(a \wedge \neg b))\,)$$

Overview

LTL Equivalence

Definition: LTL equivalence
LTL formulas φ, ψ (both over AP) are equivalent:

$$\varphi \equiv_{LTL} \psi \quad \text{if and only if} \quad \mathit{Words}(\varphi) = \mathit{Words}(\psi).$$

If it is clear from the context that we deal with LTL-formulas, we simply write $\varphi \equiv \psi$.

Equivalently:

$$\varphi \equiv_{LTL} \psi \text{ iff } \big(\text{ for all transition systems } TS : TS \models \varphi \text{ iff } TS \models \psi \ \big).$$

Duality and Idempotence

Duality:

$$\begin{aligned} \neg \Box \varphi &\equiv \Diamond \neg \varphi \\ \neg \Diamond \varphi &\equiv \Box \neg \varphi \\ \neg \bigcirc \varphi &\equiv \bigcirc \neg \varphi \end{aligned}$$

Duality and Idempotence

Duality:

$$
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\neg \Box \varphi &\equiv \Diamond \neg \varphi \\
\neg \Diamond \varphi &\equiv \Box \neg \varphi \\
\neg \bigcirc \varphi &\equiv \bigcirc \neg \varphi
\end{aligned}
$$

Idempotence:

$$
\begin{aligned}
\Box \Box \varphi &\equiv \Box \varphi \\
\Diamond \Diamond \varphi &\equiv \Diamond \varphi \\
\varphi \,\mathsf{U}\, (\varphi \,\mathsf{U}\, \psi) &\equiv \varphi \,\mathsf{U}\, \psi \\
(\varphi \,\mathsf{U}\, \psi) \,\mathsf{U}\, \psi &\equiv \varphi \,\mathsf{U}\, \psi
\end{aligned}
$$

Absorption and Distributive

Absorption: $\Diamond \Box \Diamond \varphi \equiv \Box \Diamond \varphi$

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Distributive: $\bigcirc (\varphi \, \mathsf{U} \, \psi) \equiv (\bigcirc \varphi) \, \mathsf{U} \, (\bigcirc \psi)$

$\Diamond(\varphi \vee \psi) \equiv \Diamond\varphi \vee \Diamond \psi$

$\Box(\varphi \wedge \psi) \equiv \Box\varphi \wedge \Box \psi$

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$\Diamond(\varphi \vee \psi) \equiv \Diamond\varphi \vee \Diamond \psi$

$\Box(\varphi \wedge \psi) \equiv \Box\varphi \wedge \Box \psi$

but ......: $\Box(\varphi \,\mathsf{U}\, \psi) \not\equiv (\Box\varphi) \,\mathsf{U}\, (\Box\psi)$

$\Diamond(\varphi \wedge \psi) \not\equiv \Diamond\varphi \wedge \Diamond \psi$

$\Box(\varphi \vee \psi) \not\equiv \Box\varphi \vee \Box \psi$

Weak Until

Definition: the weak-until-operator
The weak-until (or: unless) operator is defined by

$$\varphi \, \mathsf{W} \, \psi \;=\; (\varphi \, \mathsf{U} \, \psi) \;\vee\; \Box\varphi.$$

In contrast to until, weak until does not require to establish ψ eventually

Weak Until

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The weak-until (or: unless) operator is defined by

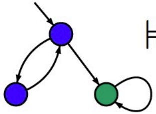
$$\varphi \,\mathsf{W}\, \psi \;=\; (\varphi \,\mathsf{U}\, \psi) \;\lor\; \Box\varphi.$$

In contrast to until, weak until does not require to establish ψ eventually

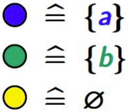
Until U and weak until W are dual:

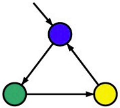
$$\begin{aligned} \neg(\varphi \,\mathsf{U}\, \psi) &\equiv (\varphi \land \neg\psi) \,\mathsf{W}\, (\neg\varphi \land \neg\psi) \\ \neg(\varphi \,\mathsf{W}\, \psi) &\equiv (\varphi \land \neg\psi) \,\mathsf{U}\, (\neg\varphi \land \neg\psi) \end{aligned}$$

Example

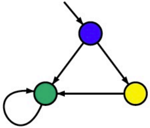


$\models a \,\mathsf{W}\, b$





$\models a \,\mathsf{W}\, b$ (even $a \,\mathsf{U}\, b$)

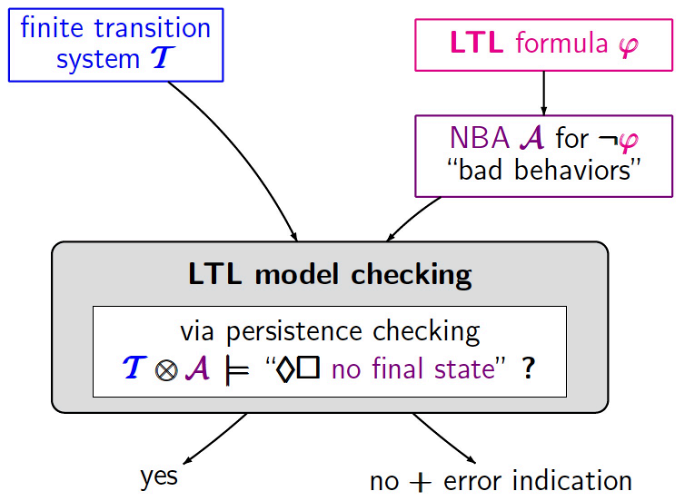


$\not\models a \,\mathsf{W}\, b$

Overview

Can We Do LTL Model Checking?

Automata-Based LTL Model Checking



Overview

Summary

- Linear temporal logic (LTL) is a logic to succinctly describe LT properties
- LTL-formulas are equivalent iff they describe the same LT properties