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Definition (Frequentist learning)

1. Take N samples of (s, a),
2. Count how many times we see successor state s_i, call this $\#(s, a, s_i)$,
3. We estimate $\tilde{P}(s, a, s_i) = \frac{\#(s,a,s_i)}{N}$.

Frequentist learning - why does this work?

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$\tilde{P}(s, a, \cdot)$ forms a valid probability distribution:

$$N = \sum_j \#(s, a, s_j) \implies \sum_i \tilde{P}(s, a, s_i) = \sum_i \frac{\#(s, a, s_i)}{\sum_j \#(s, a, s_j)} = 1.$$

Key problem in frequentist learning

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What to do if we know that this transition exists, i.e., $P(s, a, s_i) > 0$?

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Suppose we have a state-action (s, a) pair with m successor states, and want to learn the probabilities

$$P(s, a, s_1), \ldots, P(s, a, s_m).$$

Again we take $N = \#(s, a)$ samples and count how many times we see s_i: $k_i = \#(s, a, s_i)$.

Updating distributions

These counts have a multinomial likelihood

$$Mn(k_1, \ldots, k_m \mid P(s,a,\cdot)) \propto \prod_{i=1}^{m} P(s,a,s_i)^{k_i}.$$

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The Dirichlet distribution is a conjugate prior to the multinomial likelihood:

$$Dir(P(s, a, \cdot) \mid \alpha_1, \ldots, \alpha_m) \propto \prod_{i=1}^{m} P(s, a, s_i)^{\alpha_i - 1}.$$

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Given a prior Dirichlet distribution and a multinomial likelihood, we can update the prior to a posterior Dirichlet distribution with

$$Dir(P(s, a, \cdot) \mid \alpha_1 + k_1, \ldots, \alpha_m + k_m).$$

MAP estimation

After computing the posterior distribution $Dir(P(s, a, \cdot) \mid \alpha_1, \ldots, \alpha_m)$, we derive point estimates via the mode:

$$\tilde{P}(s, a, s_i) = \frac{\alpha_i - 1}{\left(\sum_{j=1}^{m} \alpha_j\right) - m}.$$

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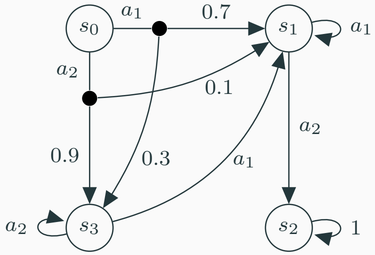
A Dirichlet distribution with $\alpha_i = \alpha_j$ for all i, j yields a uniform distribution.
The higher the values for α_i, the more data you need to shift away from the prior.

Depending on the specific situation, better choices may exist!

Example

Suppose we want to learn (s_0, a_1) in the MDP:



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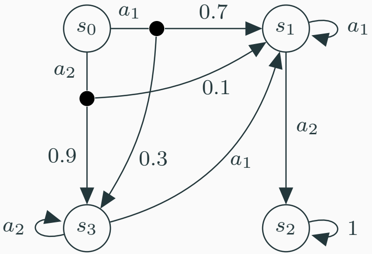

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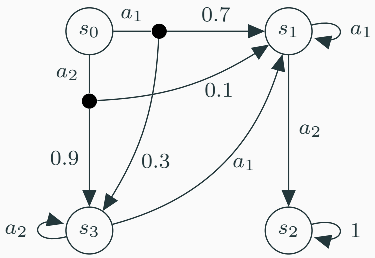

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- Bayesian: Assume prior Dirichlet distribution with $\alpha_1 = \alpha_3 = 10$.

  Posterior: $\alpha_1 = 10 + 13$, $\alpha_3 = 10 + 7$.

  MAP-estimation:

  $\tilde{P}(s_0, a_1, s_1) = \frac{22}{38} = 0.579$,
  $\tilde{P}(s_0, a_1, s_3) = \frac{16}{38} = 0.421$.

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Then with probability of at least $1 - \epsilon$ the true MDP M is contained in the IMDP $\mathcal{M}$:

$$\Pr(M \in \mathcal{M}) \geq 1 - \epsilon.$$

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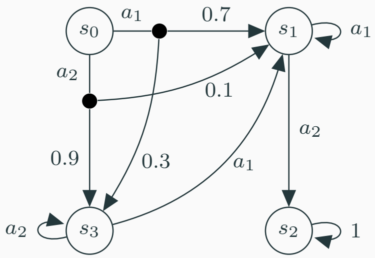

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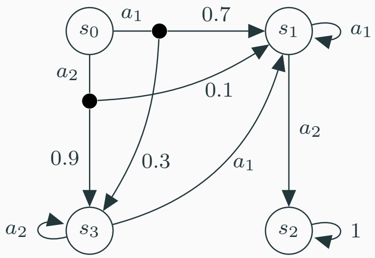

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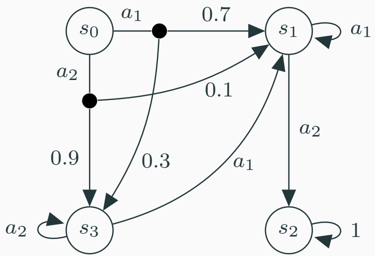

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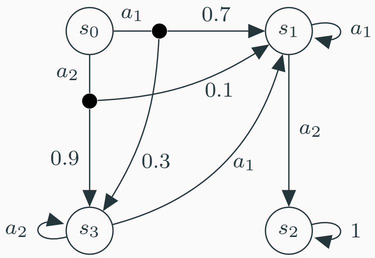

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- $\underline{P}(s_0, a_1, s_1) = 0.65 - 0.409 = 0.241$,
- $\overline{P}(s_0, a_1, s_1) = 0.65 + 0.409 = 1.059 \equiv 1.0$,
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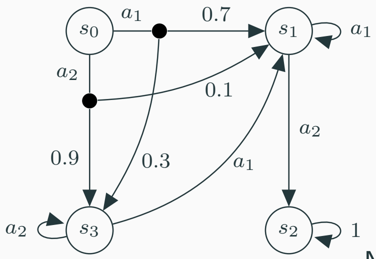

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Note that values are forced into the $[0, 1]$ interval.

Key problems in PAC learning

1. The amount of data required for useful guarantees is enormous,
2. PAC learning assumes the underlying distribution(s) are fixed.

Linearly Updating Intervals

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We assume two intervals for each transition:

1. An interval of prior transition probabilities $[\underline{P}(s,a,s'), \overline{P}(s,a,s')]$,
2. A strength interval $[\underline{n}(s,a,s'), \overline{n}(s,a,s')]$.

(1) Serves as prior that will be updated,
(2) Controls how much data we need.

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2. Update lower bound:

$$\underline{P}(s, a, s_i)' = \begin{cases} \frac{\overline{n}(s,a,s_i)\underline{P}(s,a,s_i)+k_i}{\overline{n}(s,a,s_i)+N} & \text{if } \forall j. \frac{k_j}{N} \geq \underline{P}(s, a, s_j) \text{ (prior-data agreement),} \\ \frac{\underline{n}(s,a,s_i)\underline{P}(s,a,s_i)+k_i}{\underline{n}(s,a,s_i)+N} & \text{if } \exists j. \frac{k_j}{N} < \underline{P}(s, a, s_j) \text{ (prior-data conflict).} \end{cases}$$

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4. Return updated transitions $[\underline{P}(s, a, \cdot)', \overline{P}(s, a, \cdot)']$
   and strengths $[\underline{n}(s, a, \cdot) + N, \overline{n}(s, a, \cdot) + N]$.

Example (single interval)

Prior	strength	estimate	posterior	strength
$[0.0, 1.0]$	$[0, 10]$	$\frac{1}{2}$	$[0.083, 0.917]$	$[2, 12]$
$[0.0, 1.0]$	$[0, 10]$	$\frac{50}{100}$	$[0.45, 0.55]$	$[100, 110]$
$[0.0, 1.0]$	$[0, 1000]$	$\frac{50}{100}$	$[0.045, 0.95]$	$[100, 1100]$
$[0.4, 0.6]$	$[0, 10]$	$\frac{1}{1}$	$[0.45, 1.0]$	$[1, 11]$
$[0.4, 0.6]$	$[10, 100]$	$\frac{1}{1}$	$[0.406, 0.636]$	$[11, 101]$

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Initialize: set confidence parameter $\delta \in (0,1)$ and time counter $t = 1$.

1. Build L_1 MDP with

$$\tilde{P}(s,a,s') = \frac{\#(s,a,s')}{\max\{1,\#(s,a)\}}, \quad d(s,a) = \sqrt{\frac{14|S|\log(|A|t_k/\delta)}{\max\{1,\#(s,a)\}}},$$

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4. Repeat.

Solving the optimistic inner problem efficiently (L_1 MDPs)

For UCRL2 we need to compute the optimistic value and policy:

$$V_{n+1}(s) = \max_{a \in A} \left\{ R(s,a) + \gamma \sup_{P(s,a) \in \mathcal{P}(s,a)} \left\{ \sum_{s' \in S} P(s,a)(s') V_n(s') \right\} \right\}$$

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3. $l = m$,
4. While $\sum_j P(s_j) > 1$:
    - $P(s_l) = \max\{0, 1 - \sum_{j \neq l} P(s_j)\}$,
    - $l = l - 1$,

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$$V_{n+1}(s) = \max_{a \in A} \left\{ R(s,a) + \gamma \sup_{P(s,a) \in \mathcal{P}(s,a)} \left\{ \sum_{s' \in S} P(s,a)(s') V_n(s') \right\} \right\}$$

To do so, we have a similar algorithm as for IMDPs:

1. Order $s_1, \ldots, s_m$ such that $V_n(s_1) \geq \cdots \geq V_n(s_m)$,
2. Set $P(s_1) = \min\{1, \tilde{P}(s_1) + d/2\}$ and for $j > 1$: $P(s_j) = \tilde{P}(s_j)$,
3. $l = m$,
4. While $\sum_j P(s_j) > 1$:
   - $P(s_l) = \max\{0, 1 - \sum_{j \neq l} P(s_j)\}$,
   - $l = l - 1$,
5. Return P.

UCRL2 - full algorithm

Set $\delta \in (0,1)$, $t = 1$, $\#(s,a) = 0$, $\#(s,a,s') = 0$,

For episode $k = 1, 2, \ldots,$ do:

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1. Build L_1 MDP at episode k:
   1.1 $t_k = t$,
   1.2 $\tilde{P}(s,a,s') = \frac{\#(s,a,s')}{\max\{1,\#(s,a)\}}$, $d(s,a) = \sqrt{\frac{14|S|\log(|A|t_k/\delta)}{\max\{1,\#(s,a)\}}}$

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   1.3 Compute optimistic policy π_k in L_1 MDP $(S, A, \tilde{P}, d, R, \gamma)$,

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2. Sampling:
   2.1 Set local counters $\forall(s,a,s') : v_k(s,a) = 0, v_k(s,a,s') = 0$,

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   2.2 While $v_k(s,\pi_k(s)) < \max\{1, \#(s,\pi_k(s))\}$:
      - Execute action $a = \pi_k(s)$, update counter $v_k(s,a) = v_k(s,a) + 1$

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      - Execute action $a = \pi_k(s)$, update counter $v_k(s,a) = v_k(s,a) + 1$
      - Observe successor state s', update counter $v_k(s,a,s') = v_k(s,a,s') + 1$,

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For episode $k = 1, 2, \dots,$ do:

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   - Set s' as the current state: $s = s'$, update $t = t + 1$,

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   - Observe successor state s', update counter $v_k(s,a,s') = v_k(s,a,s') + 1$,
   - Set s' as the current state: $s = s'$, update $t = t + 1$,

   2.3 End episode k, update global counters $\#(s,a) \mathrel{+}= v_k(s,a)$, $\#(s,a,s') \mathrel{+}= v_k(s,a,s')$