Model Checking

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3.1

The traces are described by the following ω-regular expression:
$\{a\}(\emptyset + \{a\})(\{a, b\}\{a\})^\omega$

5.1

(a) $\{s_1, s_2, s_3, s_4\}$

(b) $\{s_1, s_2, s_3, s_4\}$

(c) $\emptyset$

(d) $\{s_1, s_2, s_3, s_4\}$

(e) $\{s_1, s_2, s_3, s_4\}$

(f) $\{s_1, s_2, s_3, s_4\}$

5.2

TS $\not\models \varphi_1$, let $\pi = s_1(s_3 s_4)^\omega$.
TS $\models \varphi_2$. All states reach strongly connected component $\{s_2, s_3, s_4, s_5\}$, and there are states in this SCC include c.
TS $\models \varphi_3$, because in the SCC a state where $\neg c$ is always followed by a state where c.
TS $\not\models \varphi_4$, take $\pi = s_1(s_3 s_4)^\omega$.
TS $\models \varphi_5$, because after getting out of s_1, you enter the SCC where always $b \vee c$.
TS $\not\models \varphi_6$, take $\pi = s_1(s_4 s_2)^\omega$.

5.4

(a) $\square(\neg\text{Peter.use} \vee \neg\text{Betsy.use})$

(b) $\square(\Diamond(\neg\text{Peter.use}) \vee \Diamond(\neg\text{Betsy.use}))$

(c) $\square(\text{user.request} \rightarrow \Diamond\text{user.use})$ for user $\in$ {Peter, Betsy}.

(d) Assumption: only one user an request at the same time. This means that they should not be allowed to request forever.
$\Box(\Diamond\neg\text{user.request})$ for user $\in$ {Peter, Betsy}.

(e) $\Box(\text{Peter.release} \rightarrow \circ\text{Betsy.use} \wedge \text{Peter.use} \rightarrow \circ\text{Betsy.release})$

5.6

(a) Equivalent.

$$\Box\varphi \rightarrow \Diamond\psi \equiv \neg\Box\varphi \vee \Diamond\psi$$

Suppose $\neg\Box\varphi$. Then it follows that there exists an $i \geq 0$ such that $\varphi \notin A_i$. Furthermore, A_i was preceded by a (possibly empty) sequence $A_0, \ldots A_{i-1}$ where $\forall j, 0 \leq j < n : \varphi \in A_j$. It follows that $\varphi \cup (\psi \cup \neg\varphi)$.

Suppose $\Diamond\varphi$. If $\neg\Box\varphi$ refer to previous case so consider cases where $\Box\varphi$. Then it follows that there exists an $i \geq 0$ such that $\psi \in A_i$. Furthermore, A_i was preceded by a (possibly empty sequence) $A_0, \ldots A_{i-1}$ where $\forall j, 0 \leq j < n : \varphi \in A_j$. It follows that $\varphi \cup (\psi \cup \neg\varphi)$ holds.

Suppose that $\Box\varphi$ and $\neg\Diamond\psi$. Then for all $i \geq 0$, $\varphi \in A_i$ and $\psi \notin A_i$. Then $\varphi \cup (\psi \cup \neg\varphi)$ clearly does not hold since $\psi \cup \neg\varphi$ will never be true.

(b) Equivalent. $\Diamond\Box\varphi \rightarrow \Box\Diamond\psi \equiv \Box\Diamond\neg\varphi \vee \Box\Diamond\varphi$.

Suppose that $\Box\Diamond\neg\varphi$. Then for every $i \geq 0$, there exists a $j \geq i$, s.t. $\varphi \notin A_j$. Then $\varphi \cup (\psi \vee \neg\varphi)$ holds for $A[i \ldots]$ because there exists a j s.t. $A[j \ldots]$ satisfies $(\psi \vee \neg\varphi)$. Since i was arbitrary, therefore $\Box\varphi \cup (\psi \vee \neg\varphi)$.

Suppose that $\Box\Diamond\psi$. Then for every $i \geq 0$, there exists a $j \geq i$, s.t. $\psi \in A_j$. If there is a k, $i \leq k < j$ for which $\varphi \notin A_k$, then apply the previous case. Otherwise, $\varphi \cup (\psi \vee \neg\varphi)$ holds for $A[i \ldots]$ because there exists a j s.t. $A[j \ldots]$ satisfies $(\psi \vee \neg\varphi)$. Since i was arbitrary, therefore $\Box\varphi \cup (\psi \vee \neg\varphi)$.

Suppose that $\Box\varphi$ and $\neg\Box\Diamond\varphi \equiv \Diamond\Box\neg\varphi$. Then there exists some $k \geq 0$ such that $\Box\neg\varphi$ holds for $A[k \ldots]$. Then $\varphi \cup (\psi \vee \neg\varphi)$ does not hold for $A[k \ldots]$. It follows that $\Box\varphi \cup (\psi \vee \neg\varphi)$ does not hold.

(c) $\Box\Box(\varphi \vee \neg\psi) \equiv \Box(\varphi \vee \neg\psi) \equiv \Box\neg(\varphi \wedge \psi) \equiv \neg\Diamond(\neg\varphi \wedge \psi)$

(d) Not equivalent. Consider $\emptyset \rightarrow \{\varphi\} \rightarrow \{\psi\}$ where the last state self-loops. Then $\Diamond\varphi \wedge \Diamond\psi$ is satisfied, while $\Diamond(\varphi \wedge \psi)$ is not.

(e) Equivalent because $\Box\varphi \rightarrow \circ\Diamond\varphi$.

(f) Not equivalent. Consider $\{\varphi\} \rightarrow \emptyset$ where the last state self-loops. Then $\Diamond\varphi$ is satisfied while $\Diamond\varphi \wedge \circ\Box\varphi$ is not.