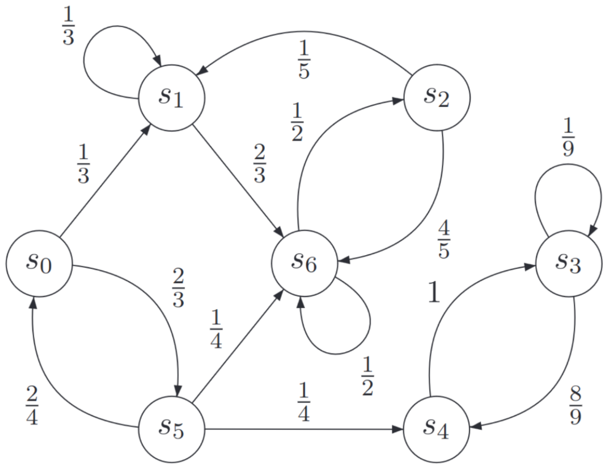
Model Checking: exercise set 4 - DTMCs

These exercises are from the *Principles of Model Checking* book.
Due date: February 27

10.1 Consider the Markov chain $\mathcal{M}$ shown below:



Let $C = \{s_0, s_1, s_4, s_6\}$ and $B = \{s_2, s_3\}$.

(a) Compute the probability measure of the union of the following cylinder sets:

$$Cyl(s_0s_1), Cyl(s_0s_5s_6), Cyl(s_0s_5s_4s_3), Cyl(s_0s_1s_6)$$

given that the initial distribution is given by $i_{init}(s_0) = 1$.

(b) Compute $\Pr(s_0 \models \lozenge B)$ using the least fixed point characterization.

(c) Compute $\Pr(s_0 \models C \,\mathsf{U}^{\leq 5} B)$ using:

(i) the least fixed point characterization;

(ii) transient state probabilities *(optional)*.

(d) Determine $\Pr(s_0 \models \lozenge \square D)$ with $D = \{s_3, s_4\}$.

10.3 Let $\mathcal{M} = (S, \mathbf{P}, i_{init}, AP, L)$ be a finite Markov chain, $s \in S$ and $C, B \subseteq S$ with $C \cap B = \emptyset$, and $n \in \mathbb{N}$ with $n \geq 1$. Let $C \,\mathsf{U}^{=n} B$ denote the event that a B-state will be entered after exactly n steps and all states that are visited before belong the C. That is, $s_0s_1s_2 \cdots \models C \,\mathsf{U}^{=n} B$ is and only if $s_n \in B$ and $s_i \in C$ for $0 \leq i < n$. The event $C \,\mathsf{U}^{\geq n} B$ denotes the union of the events $C \,\mathsf{U}^{=k} B$ where k ranges over all natural numbers $\geq n$. Provide an algorithm to compute:

(a) $\Pr(s \models C \,\mathsf{U}^{=n} B)$;

(b) $\Pr(s \models C \,\mathsf{U}^{\geq n} B)$.