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Quiz

1. Indicate for each of the following statements whether it is **true**, **untrue** or **unknown**.

   (a) SAT is in P.
   Answer: unknown.

   (b) SAT is in NP.
   Answer: true.

   (c) SAT is NP-complete.
   Answer: true.

   (d) P $\neq$ NP.
   Answer: unknown.

2. How can you use a SAT solver to prove that the statement

   $$\textit{if } A \to B \textit{ and } B \to C \textit{ then } A \to C$$

   is a tautology?

   Answer:

   (a) Translate the statement to proposition logic:

   $$((A \to B) \wedge (B \to C)) \to (A \to C)$$

   (b) *Negate* this statement

   $$\neg(\ ((A \to B) \wedge (B \to C)) \to (A \to C)\ )$$

   (c) Use standard logic transformations to pull the negation inwards, and replace implication by disjunction:

   $$\begin{gathered}\neg(\ ((A \to B) \wedge (B \to C)) \to (A \to C)\ ) \\ \equiv \\ (A \to B) \wedge (B \to C) \wedge \neg(A \to C) \\ \equiv \\ (\neg A \vee B) \wedge (\neg B \vee C) \wedge A \wedge \neg C\end{gathered}$$

   (d) This formula is in CNF, so we feed it into a SAT-solver.
   The SAT-solver should return **UNSAT**.
   This shows that the *negation* of the original formula is unsatisfiable, so that all valuations make the original formula *true*; that is, that the original formula is indeed a tautology.
   (Note: if the formula had not yet been in CNF format in this step, we would need further transformations of the formula, such as the Tseitin transformation that we will discuss next week.)