Model Checking

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Let DTMC$(M, \sigma) = (S^M, s_0^M, P', L^M)$, where
$P'(s, s') = \sum_{(p,a)\in\sigma(s)} p \cdot P^M(s, a, s')$

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In order to solve these, we first need to calculate qualitative probabilistic reachability $S^{\max=0}$. We do this using the algorithm from the lecture.

$$\{s_4\}$$
$$\{s_4, s_3, s_2\}$$
$$\{s_4, s_3, s_2, s_0, s_1\}$$

It turns out that $S^{\max=0} = \{s_5\}$. Then

$$P_{\max}(s, s \vDash \diamond s_4) = \begin{cases} 1, & \text{if } s \vDash \diamond s_4 \\ 0, & \text{if } s \in \{s_5\} \\ \max\{\sum_{s'\in S} P(s, \alpha, s') \cdot x'_s \mid \alpha \in \text{Act}(s)\} & \text{otherwise} \end{cases}$$

a

Solving the problem using linear equations gives:
Let $x_i = P_{\max}(s_i, \diamond s_4)$.
$x_4 = 1$ (since x_4 satisfies the condition)
$x_5 = 0$ (since it can never reach x_4).
Then for the rest: $(S^? = \{s_0 + s_1 + s_2 + s_3\}$

Minimize $s_0 + s_1 + s_2 + s_3$, such that:

$$\begin{aligned}
x_0 &\geq 0.5 \cdot x_1 + 0.5 \cdot x_3 && (a)\\
x_0 &\geq 0.25 \cdot x_2 + 0.75 \cdot x_3 && (b)\\
x_1 &\geq 0.5 \cdot x_2 + 0.5 \cdot x_3 && (a)\\
x_2 &\geq 0.5 \cdot x_4 && (a)\\
x_2 &\geq 0.25 \cdot x_4 + 0.5 \cdot x_3 && (b)\\
x_3 &\geq 0.75 \cdot x_3 + 0.25 \cdot x_4 && (a)
\end{aligned}$$

Simplifying yields:

$$\begin{aligned}
x_0 &= 0.9375\\
x_1 &= 0.875\\
x_2 &= 0.75\\
x_3 &= 1
\end{aligned}$$

b

For value iteration, the equations translate to

$$P_{\max}(s, s \vDash \diamond s_4)^{(n+1)} = \begin{cases} 1, & \text{if } s \vDash \diamond s_4 \\ 0, & \text{if } s \in \{s_5\} \\ \max\{\sum_{s' \in S} P(s, \alpha, s') \cdot x_s^{(n)} \mid \alpha \in \mathrm{Act}(s)\} & \text{otherwise} \end{cases}$$

and

$$x_s^{(0)} = \begin{cases} 1 & \text{if } s \vDash \diamond s_4 (\text{equivalent to } s = s_4 \text{ for 0 iterations}) \\ 0 & \text{otherwise} \end{cases}$$

Now let $x_i^{(n)} = P_{\max}(s_i, \diamond s_4)^{(n)}$.

0	1	2	3	4	5
0	0	0	0	1	0
0	0	0.5	0.25	1	0
0.3125	0.375	0.5	0.5	1	0

c

We use the Theorem that states that for every MDP, there exists a memoryless scheduler that yields the maximum probability. This means that we don't need to account for the path.

Let a memoryless scheduler be defined as a function $\forall s : p : s \to \mathrm{Act}(s)$.

Start with an arbitrary scheduler p_0, such that:

$$
\begin{aligned}
p_0(s_0) &= a \\
p_0(s_1) &= a \\
p_0(s_2) &= a \\
p_0(s_3) &= a \\
p_0(s_4) &= a \\
p_0(s_5) &= a
\end{aligned}
$$

p_1. In state s_2, taking b is beneficial over a, since a yields 0.5 and b yields 0.75. Hence, $p_1(s_2) = b$. On the other hand, in state s_0 it does not matter if we take a or b, hence $p_1(s_0) = a$.

p_2. In state s_0, taking a still yields the same as taking b, so we leave p_2 unchanged, we are done. The optimal scheduler is.

$$
\begin{aligned}
p(s_0) &= a \quad (\text{or } p(s_0) = b) \\
p(s_1) &= a \\
p(s_2) &= a \\
p(s_3) &= b \\
p(s_4) &= a \\
p(s_5) &= a
\end{aligned}
$$