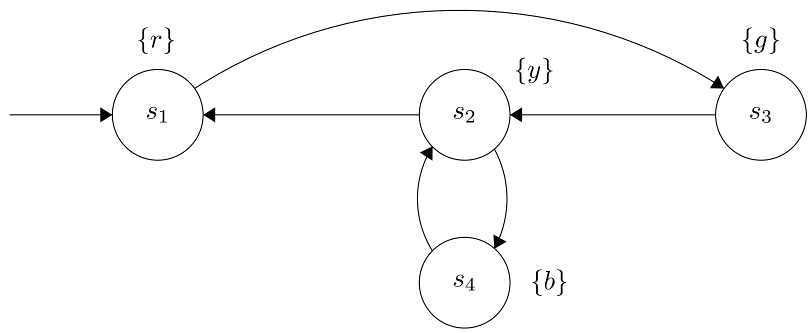
Model Checking: exercise set 2 - CTL

These exercises are from the *Principles of Model Checking* book.
Due date: February 13

6.1 Consider the following transition system over $AP = \{b, g, r, y\}$:



The following atomic propositions are used: r (red), y (yellow), g (green), and b (black). The model is intended to describe a traffic light that is able to blink yellow. You are requested to indicate for each of the following CTL formulae the set of states for which these formulae hold:

(a) $\forall\Diamond\, y$

(b) $\forall\Box\, y$

(c) $\forall\Box\,\forall\Diamond\, y$

(d) $\exists\Diamond\, g$

(e) $\forall(g \,\mathsf{U}\, \forall(y \,\mathsf{U}\, r))$

6.3 Which of the following assertions are correct? Provide a proof or a counterexample.

(a) If $s \models \exists\Box\, a$, then $s \models \forall\Box\, a$.

(b) If $s \models \forall\Box\, a$, then $s \models \exists\Box\, a$.

(c) If $s \models \forall\Diamond\, a \vee \forall\Diamond\, b$, then $s \models \forall\Diamond\, (a \vee b)$.

6.4 Let Φ be an arbitrary CTL formula. Which of the following equivalences for CTL formulae are correct? No proofs needed.

(a) $\forall\bigcirc\,\forall\Diamond\,\Phi \equiv \forall\Diamond\,\forall\bigcirc\,\Phi$

(b) $\exists\bigcirc\,\exists\Diamond\,\Phi \equiv \exists\Diamond\,\exists\bigcirc\,\Phi$

(c) $\forall\bigcirc\,\forall\Box\,\Phi \equiv \forall\Box\,\forall\bigcirc\,\Phi$

(d) $\forall\Box\,\Phi \wedge (\neg\Phi \vee \exists\bigcirc\,\exists\Diamond\,\neg\Phi) \equiv \exists\bigcirc\,\neg\Phi \wedge \forall\bigcirc\,\Phi$

(e) $\forall\Box\,\forall\Diamond\,\Phi \equiv \Phi \wedge (\forall\bigcirc\,\forall\Box\,\forall\Diamond\,\Phi) \vee \forall\bigcirc\,(\forall\Diamond\,\Phi \wedge \forall\Box\,\forall\Diamond\,\Phi)$

6.15 Prove, using Theorem 6.18, that there does not exist an equivalent LTL formula for the CTL formula:

$$\Phi = \forall\Diamond\,(a \wedge \exists\bigcirc\, a).$$