Exam Automated Reasoning

Radboud University

17/01/2024

- This exam contains 12 questions on 6 pages.
- The maximum number of points is 100.
  For each question, the number of points is given in parentheses.
- Concise answers suffice.
- *Good luck!*

1 The Tseitin Transformation (10)

Use the Tseitin Transformation to translate

$$(\neg y \leftrightarrow z) \vee (y \wedge \neg(x \wedge \neg z))$$

to CNF. Make it clear how you got to this CNF (for instance by indicating the formulas $A \leftrightarrow \varphi$ that you are using).

2 The DPLL algorithm (12)

A) Consider the CNF φ

$$\underbrace{(x_1 \lor \neg x_2)}_{c_1} \land \underbrace{(\neg x_2 \lor \neg x_3 \lor x_4)}_{c_2} \land \underbrace{(x_1 \lor x_2 \lor x_5)}_{c_3} \land \underbrace{(x_1 \lor \neg x_2 \lor x_4)}_{c_4}$$

and the partial assignment $\alpha : x_1 \mapsto \texttt{false}, \quad x_2 \mapsto \texttt{true}$.

For each clause, decide whether the clause is unassigned, satisfied, unsatisfiable or unit (multiple answers are possible).

B) Consider the CNF φ

$$\underbrace{(\neg x_1 \lor x_4 \lor x_5)}_{c_1} \land \underbrace{(x_2 \lor \neg x_3)}_{c_2} \land \underbrace{(\neg x_2 \lor \neg x_3 \lor \neg x_5)}_{c_3} \land \underbrace{(x_1 \lor \neg x_2 \lor \neg x_5)}_{c_4}$$

and the partial assignment α: $x_1 \mapsto \texttt{false}, \quad x_3 \mapsto \texttt{true}$

Apply unit propagation repeatedly until you either

- find a satisfiable assignment, or
- find a conflict.

Give the variables/assignments that you obtain and which clauses you use.

C) Consider the CNF

$$\underbrace{(x_3 \lor x_4 \lor x_5)}_{c_1} \land \underbrace{(\neg x_2 \lor \neg x_3 \lor \neg x_5 \lor x_6)}_{c_2} \land \underbrace{(x_1 \lor \neg x_2 \lor x_5)}_{c_3} \land \underbrace{(x_1 \lor x_2 \lor x_6)}_{c_4}$$

and the following decisions

$$x_1 \mapsto \texttt{false}, \quad x_6 \mapsto \texttt{false}. \quad x_4 \mapsto \texttt{false}$$

A CDCL solver detects a conflict. Give a conflict clause.

D) Consider the following clause:

$$u \lor \neg v \lor w \lor \neg x \lor \neg y \lor z$$

and the partial assignment

$$u \mapsto \texttt{false}, v \mapsto \texttt{true}$$

The watched literals are $w, \neg x$.

What are the watched literals after the following (**independent**) operations (Thus, operations executed in part i) are not relevant for ii)).

i) assign $w \mapsto \texttt{true}$

ii) assign $w \mapsto \texttt{false}$

iii) unassign u then assign $u \mapsto \texttt{true}$

3 Resolution rules (6)

Consider the CNF φ:

$$\underbrace{(x_1 \vee x_2 \vee x_4)}_{c_1} \wedge \underbrace{(\neg x_1 \vee x_2 \vee x_3)}_{c_2} \wedge \underbrace{(x_1 \vee \neg x_3)}_{c_3} \wedge \underbrace{(\neg x_2)}_{c_4} \wedge \underbrace{(\neg x_1 \vee \neg x_3)}_{c_5} \wedge \underbrace{(\ x_1\ )}_{c_6}$$

Apply **resolution** on φ to establish that the formula is unsatisfiable.

4 Eager SMT (6)

For the following SMT formula over real-valued a, b, c, d,

$$\underbrace{(a = b)}_{t_1} \wedge \underbrace{(b = c)}_{t_2} \wedge (\underbrace{a \neq c}_{t_3} \vee \underbrace{a \neq d}_{t_4})$$

the *Boolean skeleton* is:

$$t_1 \wedge t_2 \wedge (t_3 \vee t_4)$$

A) Provide a satisfying solution (a model) to the Boolean skeleton that does not correspond to a satisfying solution of the SMT formula. That is, pick a solution of $t_1, \ldots t_4$ such that one cannot pick variables $a, b, c.d$ that satisfy the corresponding terms in the SMT formula.

B) Add a missing clause (a disjunction of Boolean literals) to the Boolean skeleton to ensure that the resulting SAT formula is a correct (eager) encoding of the SMT formula. Give a brief justification.

5 Lazy SMT (6)

Consider the formula

$$a = 0 \wedge (a < 0 \vee b > 0) \wedge (a + b = 0 \vee c = 4) \wedge (c = c - 1 \vee b + c < 0)$$

with real-valued a, b, c, d. Consider a **lazy** CDCL(T) SMT-solver that will iteratively invoke a theory-solver. Give the first three invocations of a theory-solver and how it updates the SAT-solver.

Hint 1: multiple answers are correct, but working from left-to-right when in doubt is helpful.
Hint 2: You do not have to explain the internals of the theory-solver and you may assume it yields minimal infeasible subsets.

6 Predicate logic (8)

Prove that

$$(\ \forall x[\mathtt{P}(x) \to \mathtt{Q}(x)] \wedge \exists x[\mathtt{P}(x)]\ ) \to \exists x[\mathtt{Q}(x)]$$

is a valid statement in predicate logic, using resolution. Explain all your steps.
Hint: the actual resolution is only worth 1 point; the other 7 points are from the steps before!

7 Ordered resolution (6)

Suppose you are doing **ordered** resolution, using a lexicographic path ordering with $\mathtt{P} > \mathtt{R} > \mathtt{Q} > \mathtt{f} > \mathtt{a} > \mathtt{b}$.

Please indicate which steps you are allowed to take in the following situation.
Hint: you do not have to do followup steps or complete the resolution; just state the steps which you are allowed to do for the current CNF, and what the resulting clause for that step is. (There are at most 5.)

1. $\mathtt{P}(x) \vee \mathtt{P}(\mathtt{f}(x)) \vee \mathtt{R}(x)$
2. $\neg\mathtt{R}(x)$
3. $\neg\mathtt{P}(\mathtt{a}) \vee \mathtt{Q}(\mathtt{a})$
4. $\mathtt{R}(\mathtt{a}) \vee \mathtt{Q}(\mathtt{b})$
5. $\neg\mathtt{P}(\mathtt{f}(\mathtt{f}(x))) \vee \neg\mathtt{Q}(x)$

8 Monotonic algebras (12)

Consider the TRS with a single rule

$$\mathtt{f}(\mathtt{h}(x), y) \to \mathtt{h}(\mathtt{h}(\mathtt{f}(x, y)))$$

A) Prove that this TRS is terminating using monotonic algebras (that is, find suitable interpretation functions for $\mathtt{f}$ and $\mathtt{h}$). (6)

B) Explain how a computer program could find this solution using the help of an SMT solver. (You do not need to explain how the SMT solver operates.) (6)

Hint: both answers can be given in one go by using a systematic approach with interpretation shapes and absolute positiveness to find a suitable interpretation function.

9 Critical pairs (10)

List the critical pairs of the following TRS:

$$\begin{array}{lrcl} 1. & \mathtt{add}(x, \mathtt{0}) & \rightarrow & x \\ 2. & \mathtt{add}(x, \mathtt{s}(y)) & \rightarrow & \mathtt{s}(\mathtt{add}(x, y)) \\ 3. & \mathtt{add}(\mathtt{add}(x, y), z) & \rightarrow & \mathtt{add}(x, \mathtt{add}(y, z)) \end{array}$$

You may omit trivial critical pairs.

10 Confluence (4)

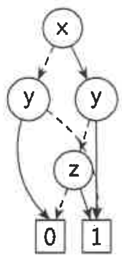
Please describe the steps you would need to do to determine if a **terminating** TRS is confluent.

11 BDDs (11)

Recall that dashed lines represent negative assignments and solid lines positive assignments. An ordering $x < y < z$ means that x nodes come before y nodes.

A) For $(x \wedge y) \vee (y \wedge \neg z)$ and variable order $x < y < z$, give the ROBDD.

B) The ROBDD below encodes a function $f(x, y, z)$.



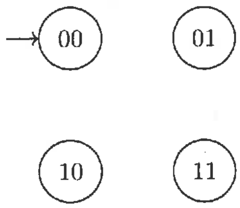
Create an ROBDD for $u \vee f(x, y, z)$ with ordering $u < x < y < z$.

C) Give the co-factor $f_{|x=1}$ as ROBDD using f as above.

D) Give a propositional formula encoding the co-factor $f_{|y=0}$ using f as above.

12 Reachability (9)

Consider the symbolic transition system with Boolean variables x, y and their primed copies x', y'. As in the lecture, we use 01 to encode the state $\langle x \mapsto$ $\texttt{false}, y \mapsto \texttt{true}\rangle$.



A) In the drawing above (copy this to your solution sheet), draw the transitions for $T_1 = \neg x \wedge \neg y \wedge x'$.

B) For the transition system with Boolean variables x, y, as above: How many transitions has the graph with the symbolic transition function $T_2 = y'$?

C) Consider a transition system with nodes reflecting an integer value n between 0 and 15 (if you prefer to think about Boolean variables: we can use 4 Boolean variables). The initial state is $n = 0$. The transition relation is the set of pairs $\{(n, n') \mid n' = n + 1 \text{ and } n \text{ is even}\}$. Give the smallest *inductive* set of states.

This is the last page.