mini hw-2

In the D&C #6: Selection Problem, the input elements are divided into groups of 5. Will the algorithm work in linear time if they are divided into groups of 9? Please justify your answer by analyzing the time complexity.

If the input elements are divided into groups of 9, then:

$$T(n) \leq \begin{cases} a, \text{ if } n = 1 \\ T(\frac{n}{9}) + T(\frac{5n}{18}) + b \cdot n \text{ , if } n > 1 \end{cases}$$

Induction hypothesis: $T(n) \leq c \cdot n$

$n = 1, a > c$

$n > 1,$
$T(n) \leq T(n/9) + T(5n/18) + b \cdot n$
$\leq \frac{1}{9}cn + \frac{5}{18}cn + bn = \frac{7}{18}cn + bn = cn - (\frac{11}{18}cn - bn) \leftarrow \text{select } \frac{11}{18}c > b$
$\leq cn$

Thus, $T(n) = O(n)$.

Because we should at leas go through all data, $T(n) = \Omega(n)$.

In conclusion, $T(n) = \Theta(n)$.