CS 3530: Assignment 7a

Fall 2014

Problems

Problem 3.19 (20 points)

Problem

Show that every infinite Turing-recognizable language has an infinite decidable subset.
hint: think about enumerators

Solution

If language L is recognizable it must have an enumerator E. We will then set up the enumerator E' as having a counter c that keeps track of the size of strings.

So as E' simulates E it processes individual strings w.
If the length of w is greater than c, then print w and set c equal to the length of w.
Else do nothing with w.

So E' will print strings if they are longer than the preceding ones(in lexicographic order) so E' will only print strings in L. Since L is infinite E' will print an infinite subset of language L.

Theorem 3.21: L is Turing-recognizable iff some enumerator enumerates it.
Theorem (S42): L is Turing-recognizable iff L is enumerated by some TM.
Theorem (S44): L is decidable iff L is enumerable in lexicographic order (lexicographic order has shorter strings before longer, and alphabetic order among strings of the same length).