CS 3530: Assignment 5c

Fall 2014

Exercises

Exercise 2.15 (5 points)

Problem

Give a counterexample to show that the following construction fails to prove that the class of context-free languages is closed under star. Let A be a CFL that is generated by the CFG $G = (V, \Sigma, R, S)$. Add the new rule $S \to SS$ and call the resulting grammar G'. This grammar is supposed to generate A^*.

Solution

Using a CFL: A = $\{a^n b^n : n \ \geq \ 0\}$
CFG G is generated: $S \to \varepsilon \mid aSb$
CFG G' is generated by adding the rule $S \to SS$ making $S \to \varepsilon \mid aSb \mid SS$
G' will result in strings in the form of a(ab)*ba(ab)*b which is not a part of the language A which demonstrates that the CFL isn't closed under star.

Additional problem

Let L be the language $\{w : w$ has equal numbers of as, bs, and cs$\}$

a. Prove that $\overline{L}$ is context free (7 points)

Solution

Assume $\overline{L}$ is context free
and there is a string $s \in \overline{L}$ of at least length p
then s may be divided int $s = uvxyz$ satisfying the following conditions
1. for each $i \geq 0, uv^i xy^i z \in \overline{L}$
2. $|vy| > 0$
3. $|vxy| \leq p$

if s is split into $s = a^{p+i} b^p c^{p+i}$ and $i > 0$ then the strings generated are found $\in \overline{L}$ and none of the rules are violated showing that our assumption of L being a CFL was true.

b. Prove that L is not context free (7 points)

Solution

Assume L is context free
and there is a string $s \in L$ of at least length p
then s may be divided int $s = uvxyz$ satisfying the following conditions
1. for each $i \geq 0, uv^i xy^i z \in L$
2. $|vy| > 0$
3. $|vxy| \leq p$

if s is split into $s = a^{p+i} b^p c^{p+i}$ and $i > 0$ then the string will contain more a's and c's than b's which is not in L. This violates the first condition which shows a condradiction which shows that our assumption of L being a CFL was false.

c. Conclude that CFLs are not closed under complement (1 point)

Solution

Parts a and b demonstrate the fact that part a is a CFL while part b shows by the pumping lemma that the complemt of the same language is not a CFL.