

CS 3530: Assignment 8b

Fall 2014

Problems

Problem 7.34 (20 points)

Problem

A subset of the nodes of a graph G is a **dominating-set** if every other node of G is adjacent to some node in the subset. Let

$$\text{DOMINATING-SET} = \{\langle G, k \rangle : G \text{ has a dominating set with } k \text{ nodes}\}.$$

Show that it is NP-complete by giving a reduction from VERTEX-COVER.

Solution

Theorem 7.44:

VERTEX-COVER = $\{\langle G, k \rangle \mid G \text{ is an undirected graph that has a } k\text{-node vertex cover}\}$ is NP-complete.

To show that they are equivalent we will have to do reductions to show the following:

1. If there is a VERTEX-COVER then there is a DOMINATING-SET.
2. If there is a DOMINATING-SET then there is a VERTEX-COVER

1. Reducing from a VERTEX-COVER to a DOMINATING-SET

Given a vertex cover graph $G(V, E)$ with a cover size of k construct a graph $G'(V, E)$ with dominating set of size k by adding to each edge a new node that has edges connected to the original edge nodes. If there is a vertex cover for G then there is a dominating set for G' .

2. Reducing from a DOMINATING-SET to a VERTEX-COVER

The reverse of the above reduction should be possible by combining nodes into one of its neighbors.