

CS 3530: Assignment 3b

Fall 2014

Exercises

Exercise 2.9 (10 points)

Problem

Give a context-free grammar that generates the language

$$A = \{a^i b^j c^k : i = j \text{ or } j = k \text{ where } i, j, k \geq 0\}.$$

For all CFGs, describe the role that each rule performs as well as giving the actual rule.

Solution

$S \rightarrow \varepsilon \mid T \mid U$

We will need to first choose if $i=j$ or $j=k$

$T \rightarrow \varepsilon \mid TaTbT \mid TbTaT \mid TcT$

This will allow any combination of $i=j$ with any number of k

$U \rightarrow \varepsilon \mid UbUcU \mid UcUbU \mid UaU$

This will allow any combination of $j=k$ with any number of i

Problems

Problem 1.47 (10 points)

Problem

Let $\Sigma = \{1, \#\}$ and let

$$Y = \{w : w = x_1 \# x_2 \# \cdots \# x_k \text{ for } k \geq 0, \text{ each } x_i \in 1^*, \text{ and } x_i \neq x_j \text{ for } i \neq j\}$$

Prove that Y is not regular.

Solution

Assume Y is regular

string $s = 1^p \# 1^p$

split s into $s = xyz$ where:

1. $xy^iz \in Y$ (for each $i \geq 0$)
2. $|y| > 0$
3. $|xy| \leq p$

Any value of p would result in the same number of ones on both sides of the $\#$ any other format of string s could result in two different amounts of 1's but that would result in future $x \in Y$ to have the same number of 1's