CS 3530: Assignment 7d

Fall 2014

Problem 5.13 (20 points)

Problem

A *useless state* in a Turing machine is one that is never entered on any input string. Consider the problem of determining whether a Turing machine has any useless states. Formulate this problem as a language and show that it is undecidable.

Solution

We will Call the machine that determines if it has any usless states U_{TM}.

According to theorem 4.11: A_{TM} is undecidable where $A_{TM} = \{ \langle M, w \rangle \mid M$ is a TM that accepts $w \}$

So if we can do the reduction $A_{TM} \leq_m U_{TM}$ then we will know U_{TM} is undecidable.

To prove reducability we need a computable function f that takes input of the form $\langle M, w \rangle$ and returns output $\langle M', w' \rangle$ where $\langle M, w \rangle \in A_{TM}$ iff $\langle M', w' \rangle \in U_{TM}$.

R_{TM} = on input $\langle M, w \rangle$:
    1. Construct the following machine M'
        M' = on input x:
            1. Run M on x keeping track of all used states
            2. If M accepts, *accept*
            3. If M rejects, *reject*."
    2. If useless states are found *accept*.
    3. Else *reject*.