# 4 Cell Model

marcus lai

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## Model Set Up

We represent the 4-cell model with 4 differential equations, one for each cell. Our initial model considers a spring force between each adjacent cell to keep the system intact, a frictional force between right cells and left cells and between the cells and the EMS, and drag. We assume that the mass of the cells are negligible.

To set up the system of differential equations, we enumerate the cells, letting the cell 1 be ABal, cell 2 be ABar, cell 3 be ABpr, cell 4 be ABpl. We let the positive y region denote the left side of the system, negative y denote the right side. We let the positive x region denote the anterior of the system and negative x denote the posterior region.

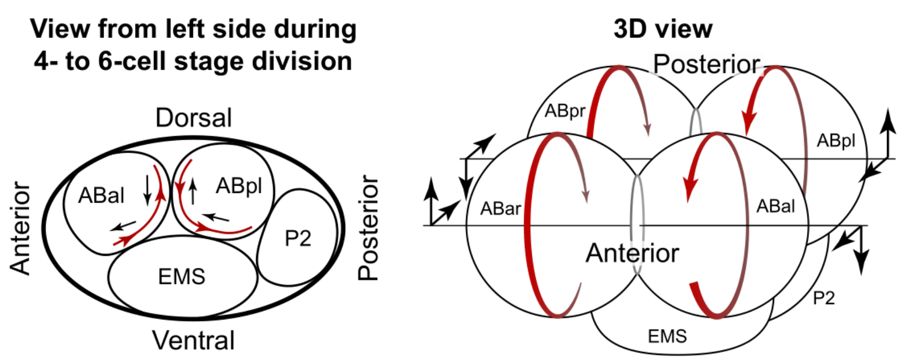
We also let $k$ be the spring constant, $l$ be the initial rest length of the spring (also one cell diameter), $d$ be the friction variable, and $c$ be the drag variable. This yields:

$$0 = k(||\bar{p_1} - \bar{p_2}|| - l)\widehat{u_{12}} + k(||\bar{p_1} - \bar{p_4}|| - l)\widehat{u_{14}} - k(p_{1_z})\hat{k} + d(\widehat{u_{41}} \times \widehat{u_{43}} - \widehat{u_{14}} \times \widehat{u_{12}}) + d(-\widehat{u_{12}} \times \hat{k}) - cp_1'$$

$$0 = k(||\bar{p_2} - \bar{p_1}|| - l)\widehat{u_{21}} + k(||\bar{p_2} - \bar{p_3}|| - l)\widehat{u_{23}} - k(p_{2_z})\hat{k} + d(\widehat{u_{32}} \times \widehat{u_{34}} - \widehat{u_{23}} \times \widehat{u_{21}}) + d(-\widehat{u_{21}} \times \hat{k}) - cp_2'$$

$$0 = k(||\bar{p_3} - \bar{p_2}|| - l)\widehat{u_{32}} + k(||\bar{p_3} - \bar{p_4}|| - l)\widehat{u_{34}} - k(p_{3_z})\hat{k} + d(\hat{u}_{23} \times \widehat{u_{21}} - \hat{u}_{32} \times \hat{u}_{34}) + d(-\hat{u}_{34} \times \hat{k}) - cp_3'$$

$$0 = k(||\bar{p_4} - \bar{p_1}|| - l)\widehat{u_{41}} + k(||\bar{p_4} - \bar{p_3}|| - l)\widehat{u_{43}} - k(p_{4_z})\hat{k} + d(\widehat{u_{14}} \times \widehat{u_{12}} - \widehat{u_{41}} \times \widehat{u_{43}}) + d(-\widehat{u_{43}} \times \hat{k}) - cp_4'$$



Where $\hat{u_{xy}} = \frac{p_y - p_x}{||p_y - p_x||}$ is a unit vector from $x$ to $y$. In our model, the EMS is assumed to be the entire $z = -0.5$ plane. We obtain the friction direction by taking the negation of the relative displacement of the contact point between two cells of interest. We rearrange the equation to obtain:

$$\frac{dp_1}{dt} = \frac{k}{c}(||\bar{p_1}-\bar{p_2}||-l)\hat{u_{12}}+\frac{k}{c}(||\bar{p_1}-\bar{p_4}||-l)\hat{u_{14}}-\frac{k}{c}(p_{1_z})\hat{k}+\frac{d}{c}(\hat{u_{41}}\times\hat{u_{43}}-\hat{u_{14}}\times\hat{u_{12}})+\frac{d}{c}(-\hat{u_{12}}\times\hat{k})$$

$$\frac{dp_2}{dt} = \frac{k}{c}(||\bar{p_2}-\bar{p_1}||-l)\hat{u_{21}}+\frac{k}{c}(||\bar{p_2}-\bar{p_3}||-l)\hat{u_{23}}-\frac{k}{c}(p_{2_z})\hat{k}+\frac{d}{c}(\hat{u_{32}}\times\hat{u_{34}}-\hat{u_{23}}\times\hat{u_{21}})+\frac{d}{c}(-\hat{u_{21}}\times\hat{k})$$

$$\frac{dp_3}{dt} = \frac{k}{c}(||\bar{p_3}-\bar{p_2}||-l)\hat{u_{32}}+\frac{k}{c}(||\bar{p_3}-\bar{p_4}||-l)\hat{u_{34}}-\frac{k}{c}(p_{3_z})\hat{k}+\frac{d}{c}(\hat{u}_{23}\times\hat{u_{21}}-\hat{u}_{32}\times\hat{u}_{34})+\frac{d}{c}(-\hat{u}_{34}\times\hat{k})$$

$$\frac{dp_4}{dt} = \frac{k}{c}(||\bar{p_4}-\bar{p_1}||-l)\hat{u_{41}}+\frac{k}{c}(||\bar{p_4}-\bar{p_3}||-l)\hat{u_{43}}-\frac{k}{c}(p_{4_z})\hat{k}+\frac{d}{c}(\hat{u_{14}}\times\hat{u_{12}}-\hat{u_{41}}\times\hat{u_{43}})+\frac{d}{c}(-\hat{u_{43}}\times\hat{k})$$

We nondimensionalize time by the duration of our experiment and position by number of cell diameters.

$$\tau = \frac{t}{t_{final}}$$

$$\bar{q_i} = \frac{\bar{p_i}}{l},\ i = 1,2,3,4$$

Then by chain rule, our system becomes:

$$\frac{d\bar{q_1}}{d\tau} = t_{final}\big[\frac{k}{c}(||\bar{q_1}-\bar{q_2}||-1)\hat{u_{12}}+\frac{k}{c}(||\bar{q_1}-\bar{q_4}||-1)\hat{u_{14}}-\frac{k}{c}(q_{1_z})\hat{k}+\frac{d}{cl}(\hat{u_{41}}\times\hat{u_{43}}-\hat{u_{14}}\times\hat{u_{12}})+\frac{d}{cl}(-\hat{u_{12}}\times\hat{k})\big]$$

$$\frac{d\bar{q_2}}{d\tau} = t_{final}\big[\frac{k}{c}(||\bar{q_2}-\bar{q_1}||-1)\hat{u_{21}}+\frac{k}{c}(||\bar{q_2}-\bar{q_3}||-1)\hat{u_{23}}-\frac{k}{c}(q_{2_z})\hat{k}+\frac{d}{cl}(\hat{u_{32}}\times\hat{u_{34}}-\hat{u_{23}}\times\hat{u_{21}})+\frac{d}{cl}(-\hat{u_{21}}\times\hat{k})\big]$$

$$\frac{d\bar{q_3}}{d\tau} = t_{final}\big[\frac{k}{c}(||\bar{q_3}-\bar{q_2}||-1)\hat{u_{32}}+\frac{k}{c}(||\bar{q_3}-\bar{q_4}||-1)\hat{u_{34}}-\frac{k}{c}(q_{3_z})\hat{k}+\frac{d}{cl}(\hat{u}_{23}\times\hat{u_{21}}-\hat{u}_{32}\times\hat{u}_{34})+\frac{d}{cl}(-\hat{u}_{34}\times\hat{k})\big]$$

$$\frac{d\bar{q_4}}{d\tau} = t_{final}\big[\frac{k}{c}(||\bar{q_4}-\bar{q_1}||-1)\hat{u_{41}}+\frac{k}{c}(||\bar{q_4}-\bar{q_3}||-1)\hat{u_{43}}-\frac{k}{c}(q_{4_z})\hat{k}+\frac{d}{cl}(\hat{u_{14}}\times\hat{u_{12}}-\hat{u_{41}}\times\hat{u_{43}})+\frac{d}{cl}(-\hat{u_{43}}\times\hat{k})\big]$$

We let $B = \frac{k}{c}$ and $A = \frac{d}{cl}$. Note there is no significant meaning to the choice of variable name, but this choice allows consistency with the code.

$$\frac{d\bar{q_1}}{d\tau} = t_{final}\big[B\big((||\bar{q_1}-\bar{q_2}||-1)\hat{u_{12}}+(||\bar{q_1}-\bar{q_4}||-1)\hat{u_{14}}-(q_{1_z})\hat{k}\big)+A\big(\hat{u_{41}}\times\hat{u_{43}}-\hat{u_{14}}\times\hat{u_{12}}-\hat{u_{12}}\times\hat{k}\big)\big]$$

$$\frac{d\bar{q_2}}{d\tau} = t_{final}\big[B\big((||\bar{q_2}-\bar{q_1}||-1)\hat{u_{21}}+(||\bar{q_2}-\bar{q_3}||-1)\hat{u_{23}}-(q_{2_z})\hat{k}\big)+A\big(\hat{u_{32}}\times\hat{u_{34}}-\hat{u_{23}}\times\hat{u_{21}}-\hat{u_{21}}\times\hat{k}\big)\big]$$

$$\frac{d\bar{q_3}}{d\tau} = t_{final}\big[B\big((||\bar{q_3}-\bar{q_2}||-1)\hat{u_{32}}+(||\bar{q_3}-\bar{q_4}||-1)\hat{u_{34}}-(q_{3_z})\hat{k}\big)+A\big(\hat{u}_{23}\times\hat{u_{21}}-\hat{u}_{32}\times\hat{u}_{34}-\hat{u}_{34}\times\hat{k}\big)\big]$$

$$\frac{d\bar{q_4}}{d\tau} = t_{final}\big[B\big((||\bar{q_4}-\bar{q_1}||-1)\hat{u_{41}}+(||\bar{q_4}-\bar{q_3}||-1)\hat{u_{43}}-(q_{4_z})\hat{k}\big)+A\big(\hat{u_{14}}\times\hat{u_{12}}-\hat{u_{41}}\times\hat{u_{43}}-\hat{u_{43}}\times\hat{k}\big)\big]$$

In addition to these forces, a conditional spring force is triggered if two diagonal cells get within 1 diameter of each other. Similar to a regular adjacent cell spring.

Note to self: to finish 1 iteration of the experiment, the sum of all time steps must total 1.

## Fitting Cortical Flow

We use experimental data to fit cortical flow to the functional form $f(t) = \alpha t e^{\lambda t}$ using scipy.optimize.fmin. We chose to fit the cortical separately for each side. The data was not non-dimensionalized so we use $t_{final} \cdot \tau$ as our functional input when performing Euler and Runge Kutta. We see from our model derivation that the relative displacement of contact points depends linearly on the cell's rotational velocity, and hence the cortical flow velocity. Without considering a variable coritcal flow, fitting our model to the data is the equivalent of assuming a constant rotational velocity which is indirectly considered into $A$ (in $d$). We replace $A$ by $\bar{A}(\tau) = f(t_{final} \cdot \tau) \cdot A$ in our experiment where $f$ is the fitted cortical flow function.

## A Second Model

We propose a second model that simulates the elongation from mitosis during the transition from the 4 cell to 6 cell phase. We assume a relation between the observed strength of cortical flow and the rate of elongation. Thus, we let the resting length become a function of time, taking the integral of our approximated cortical flow function, normalizing it to reach 1 at $t_{final}$, and multiplying by 1.5 to represent the increase of the rest length to 1.5 cell diameters by the end of the experiment.

Our second model also considers the effects of the embryo egg shell by including a linear force (or van der waals forces?) that "pushes away" the cells from the shell surface from the closest point on the surface . This is implemented by finding the surface point that minimizes the norm squared between the cell position and the egg-shell surface. Then, we take the negation of it to obtain the force direction. Then magnitude of the force is then dependent on the minimum value.

We demonstrate the effects of rotation on the system by a comparison of the system with and without the friction force.