Quantum-inspired Optimization and QAOA Validation for Cost-effective Fertilizer Allocation in Precision Agriculture

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***Abstract*—Effective fertilizer allocation enhances agricultural productivity and sustainability. Traditional heuristics often cause nutrient imbalance and excess cost. This study introduces a quantum-inspired Quadratic Unconstrained Binary Optimization (QUBO) model solved by the Compal GPU Annealer (CGA). Experiments on radish, cabbage, and rice demonstrated 8–20% cost savings over greedy heuristics. A small-scale Quantum Approximate Optimization Algorithm (QAOA) validation using IBM Qiskit confirmed quantum algorithmic compatibility.**

I. Introduction

Modern agriculture faces significant challenges in optimizing fertilizer application under constraints such as limited budgets, diverse nutrient requirements, and numerous fertilizer options [1]. Effective fertilization must balance economic efficiency with environmental sustainability, avoiding excess application that causes nutrient leaching and ecological harm.

Traditional heuristic methods often fail to address complex trade-offs between cost, nutrient adequacy, and practical constraints, frequently resulting in over-fertilization and environmental degradation. Addressing these issues requires a systematic optimization approach. A critical challenge is the combinatorial complexity introduced by varying fertilizer availability. Limited options may fail to meet nutrient targets under tight budgets, whereas extensive options drastically increase computational complexity.

We formulate fertilizer allocation as a Quadratic Unconstrained Binary Optimization (QUBO) problem [2], solved via a quantum-inspired Compal GPU Annealer (CGA). Experiments using realistic agricultural scenarios demonstrate that our QUBO-based model significantly outperforms conventional greedy heuristics, achieving substantial economic and environmental benefits.

To further emphasize quantum relevance, we performed a small-scale Quantum Approximate Optimization Algorithm (QAOA) experiment using IBM Qiskit [3], demonstrating the compatibility of our QUBO model with quantum computing approaches.

This study's key contributions include: (1) Introducing the first QUBO-based fertilizer allocation model; (2) Empirical validation using realistic agricultural scenarios; (3) Demonstrating substantial improvements in economic and ecological outcomes; (4) Validating quantum compatibility via small-scale QAOA simulation.

II. System Model and Problem Formulation

We propose a fertilizer optimization model considering crop nutrient demands, budget constraints, and fertilizer availability. The objective is minimizing fertilization cost while meeting nitrogen (N), phosphorus (P), and potassium (K) requirements without exceeding agronomic thresholds.

Define binary decision variables $X_{i,q}$, where $X_{i,q} = 1$ indicates selecting fertilizer i with dosage q (B kg bags); otherwise $X_{i,q} = 0$. Indices: i for fertilizer, and q for dosage levels up to M_i, determined by nutrient needs and a global maximum limit.

The optimization model is given as follows. The objective function is to minimize total fertilization cost:

$$\min Z = \sum_{i=1}^{m} \sum_{q=0}^{M_i} c_i \cdot q \cdot X_{i,q} \tag{1}$$

where c_i is cost per fertilizer bag.

In the following, the constraints for the optimization problem are introduced. For each nutrient:

$$DN_{\text{lower}} \leq \sum_{i=1}^{m} \sum_{q=0}^{M_i} n_i \cdot B \cdot q \cdot X_{i,q} + SN \leq DN_{\text{upper}} \tag{2}$$

$$DP_{\text{lower}} \leq \sum_{i=1}^{m} \sum_{q=0}^{M_i} p_i \cdot B \cdot q \cdot X_{i,q} + SP \leq DP_{\text{upper}} \tag{3}$$

$$DK_{\text{lower}} \leq \sum_{i=1}^{m} \sum_{q=0}^{M_i} k_i \cdot B \cdot q \cdot X_{i,q} + SK \leq DK_{\text{upper}} \tag{4}$$

DN, DP, and DK, with lower and upper bounds, represent the required nutrient ranges for nitrogen, phosphorus, and potassium, respectively. Here, n_i, p_i, k_i denote the nitrogen, phosphorus, and potassium content of fertilizer i, respectively. Initial soil nutrients (SN, SP, SK) are also considered.

Ensure expenditures remain within budget:

$$\sum_{i=1}^{m}\sum_{q=0}^{M_i} c_i \cdot q \cdot X_{i,q} \leq COST \tag{5}$$

Exactly one dosage per fertilizer :

$$\sum_{q=0}^{M_i} X_{i,q} = 1, \quad \forall i \in 1, \ldots, m \tag{6}$$

To facilitate quantum-inspired optimization, the constrained optimization problem is reformulated into a QUBO model by embedding all constraints as penalty terms within the objective function. This transformation enables effective exploration of feasible and cost-efficient fertilization plans using quantum-inspired solvers such as the CGA.

Simplified Model for Quantum Validation

We defined a simplified fertilizer allocation scenario suitable for quantum simulation, involving two fertilizers (A: \$200/bag, N=80, P=20, K=40; B: \$150/bag, N=40, P=40, K=80) and nutrient demands (N=40, P=40, K=45). Each fertilizer was limited to at most two bags, using slack variables and one-hot encoding to enforce nutrient constraints and dosage selection.

III. Experimental Evaluation and Results

We evaluated the proposed Quadratic Unconstrained Binary Optimization (QUBO) model solved by CGA against a greedy heuristic using datasets for radish, cabbage, and rice based on Taiwan's Ministry of Agriculture recommendations [4]–[6]. Fertilizer set sizes ranged from 6 to 21 types, each characterized by unique cost and nutrient (N-P-K) compositions per $B = 40$ kg bag. Each scenario was executed 10 times, recording minimum total cost and average runtime. The greedy heuristic iteratively selects fertilizers based on nutrient-to-cost efficiency until nutrient targets are met, providing rapid but potentially suboptimal solutions. We chose the greedy heuristic for performance comparison as other common heuristic methods, including Simulated Annealing (SA), Tabu Search (TS), and the D-Wave quantum annealer (Advantage2_system1.3), consistently failed to find feasible solutions under our defined problem constraints.

As shown in Fig. 1, CGA consistently achieves lower or comparable costs to the greedy heuristic across all crops, particularly notable for cabbage (cost savings of 8–20%). Figure 2 indicates the greedy heuristic executes instantly, while CGA takes approximately 26–32 seconds, still practical for real-world applications. Unlike the greedy heuristic's local decisions, CGA comprehensively addresses nutrient and budget constraints, delivering economically efficient and environmentally sustainable plans.

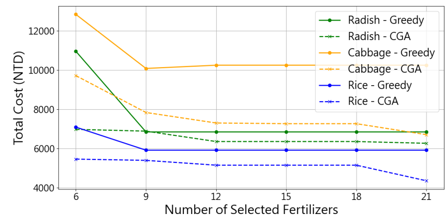

Fig. 1. Total Fertilization Cost: Greedy vs. CGA across Crops and Fertilizer Set Sizes.

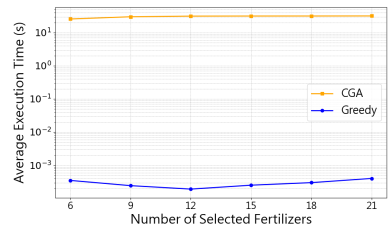

Fig. 2. Average Execution Time: Greedy vs. CGA across Fertilizer Set Sizes.

Quantum Validation Experiment

We validated quantum feasibility by comparing QAOA (IBM Qiskit Aer simulator) and CGA using the simplified scenario (Sec. II). Both methods yielded identical optimal results (cost \$150), but QAOA required much longer execution (242.2 s vs. CGA's 13.72 s) and had a lower success rate (20% vs. CGA's 100%). Additionally, the D-Wave quantum annealer (Advantage2_system1.3) solved the same scenario optimally with a significantly shorter runtime (about 3 s) and a 100% success rate, indicating its strong potential as an effective alternative solution for QUBO-formulated problems.

IV. Quantum Computing Feasibility and Future Directions

Our small-scale QAOA experiment demonstrated the feasibility of encoding fertilizer allocation problems for quantum algorithms. Although current quantum hardware and simulation capabilities cannot yet address full-scale real-world scenarios, these preliminary results highlight significant promise. Future quantum hardware advancements and algorithm improvements may enable practical quantum solutions for large-scale agricultural optimization. In the interim, hybrid quantum-classical methods can serve as practical transitional solutions.

References

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