

Comment on “Risk Preferences Are Not Time Preferences”: Balancing on a Budget Line[†]

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In a recent experimental study of intertemporal risky decision making, Andreoni and Sprenger (2012) find that subjects exhibit a preference for intertemporal diversification, which is inconsistent with discounted expected utility theory. It was claimed that their results are also at odds with models involving probability weighting, such as rank-dependent utility and cumulative prospect theory. Here we demonstrate, however, that rank-dependent probability weighting explains intertemporal diversification if decision makers care about portfolio risk. Moreover, we provide a unified account of all of Andreoni and Sprenger’s key findings. (JEL C91, D81, D91)

In a recent paper, Andreoni and Sprenger (2012)—henceforth, AS—investigate decision making when both risk and delay are present. Their experiment is based on the convex time budget (CTB) method by which subjects choose a portfolio of potentially risky payments at two different future dates. This feature generates a richer environment than conventional choices between smaller sooner and larger later payments as it enables subjects to diversify their earnings across time. In their study, AS focus on the common-ratio property of discounted expected utility. For example, if a sooner payment will materialize 100 percent of the time and a later payment 80 percent of the time, then scaling down the probabilities to 50 percent and 40 percent, respectively, should not affect individuals’ intertemporal allocations. AS find that subjects exhibit common-ratio violations¹ in their intertemporal allocations, consistent with the large body of evidence on atemporal risky decisions (Allais 1953; Conlisk 1989; Starmer 2000; Fehr-Duda and Epper 2012). However, they also detect a pattern of behavior that, *prima vista*, seems to be at odds with models involving probability weighting, such as rank-dependent utility (RDU) (Quiggin 1982) and cumulative prospect theory (Tversky and Kahneman 1992). When all payments are risky, their experimental design produces states of nature in which only one of the payments arrives. Faced with this possibility, many subjects

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¹When, as in this example, one of the choice pairs features a certainty, the term “certainty effect” is typically used for common-ratio violations.

exhibit a preference for intertemporal diversification: they shy away from extreme allocations because they might end up with no payment at all and, therefore, prefer a more evenly balanced portfolio than in the case when all payments are guaranteed.

This evidence of intertemporal diversification triggered successive experimental work by Cheung (2015) and Miao and Zhong (2015) who examine the robustness of the AS findings. The upshot of these investigations is that, by and large, subjects' behavior depends significantly on whether diversification opportunities are present or not.

In this paper we demonstrate that probability weighting in rank-dependent models is able to explain subjects' preference for intertemporal diversification as well as their proneness to intertemporal common-ratio violations and, therefore, all the major AS findings. The method by which decision makers evaluate future prospects is key for our results. In our analysis, they care about the risk of their *entire* portfolios, consisting of two potential payments at different future dates, and not about the isolated risks of the individual payments.² Consequently, as a representative RDU decision maker evaluates a payment occurring in conjunction with another payment more favorably than the same payment occurring in isolation, i.e., when the other payment fails to materialize, she prefers a more evenly balanced portfolio. For such a decision maker it would be irrational to choose extreme allocations because she would forgo the utility gains from diversification. If there are no opportunities to diversify, however, risk does not affect her intertemporal allocations. Our analysis shows that rank-dependent models perform well not only in their original domain of atemporal decisions but also in intertemporal settings.

The remainder of the paper is structured as follows. First, we briefly review the specifics of the CTB design. In Section II, we show that rank-dependent models are able to accommodate all the characteristic pattern in the AS data. We conclude with a brief discussion in Section III where we argue that rank-dependent models are superior to other approaches suggested in the literature. Additional materials covering these alternatives are presented in the online Appendix.

I. The Convex Time Budget Experiment

In the AS experiment, subjects are endowed with 100 tokens that can be allocated in any integer combination $(n, 100 - n)$ to a sooner payment at time t and a later payment at $t + k$.³ Tokens allocated to the later payment are exchanged for cash at a constant rate⁴ a_{t+k} , whereas tokens allocated to the sooner payment are exchanged at a rate a_t that varies with an interest rate r such that $a_t = (a_{t+k})/(1 + r)$. In accordance with AS, we will henceforth label experimental earnings "consumption." Allocating n tokens to the sooner payment yields a, potentially risky, consumption stream $(c_t, c_{t+k}) = (na_t, (100 - n)a_{t+k})$. Hence, on a given budget line $(1 + r)c_t + c_{t+k} = 100a_{t+k}$ holds. Since a_{t+k} is constant throughout the experiment, changes in the budget constraint are governed by the interest rate r , which takes on values from

²In online Appendix A1 we investigate two alternative evaluation methods, the separable and the recursive methods. Neither one of them provides a coherent explanation for all the patterns in the AS data.

³Sooner payments are always made in $t = 7$ days after the experiment. There are two different time delays $k \in \{28, 56\}$ days. For the purpose of this paper, changes in k are of no concern.

⁴In the experiment $a_{t+k} = \text{USD } 0.2$.

0 percent up to 2,116.6 percent per annum. Additionally, AS assume that subjects discount the future at a constant rate, which implies an exponentially declining discount function $\delta(t)$.⁵

Furthermore, all the uncertainty concerning sooner and later payments is resolved simultaneously at the end of the experimental session, i.e., subjects know by then which consumption stream will materialize. Finally, and most importantly, sooner and later payments arrive *independently* with probabilities p_1 and p_2 , respectively. As mentioned in the introduction, decision makers care about portfolio risk and, therefore, about the joint distribution of the random variables \tilde{c}_t and \tilde{c}_{t+k} . Consequently, the independent arrival of the payments generates four distinct states of nature: either both payments arrive, only the sooner payment arrives, only the later payment arrives, or no payment at all arrives. These four states of nature define the prospect over the possible consumptions streams,

$$P(p_1, p_2)$$

$$= ((c_t, c_{t+k}), p_1 p_2; (c_t, 0), p_1(1 - p_2); (0, c_{t+k}), (1 - p_1)p_2; (0, 0), (1 - p_1)(1 - p_2)).$$

In line with the previous literature (e.g., Loewenstein and Prelec 1992), we represent preferences over consumption streams by an additive and separable functional, such that the discounted utility of $(\tilde{c}_t, \tilde{c}_{t+k})$ equals $v(\tilde{c}_t)\delta(t) + v(\tilde{c}_{t+k})\delta(t + k)$, where the random variable \tilde{c}_t (\tilde{c}_{t+k}) takes on the values 0 or c_t (0 or c_{t+k}), and v satisfies $v(0) = 0$, $v' > 0$ and $v'' < 0$.

Under discounted expected utility the prospect $P(p_1, p_2)$ is evaluated as

$$\begin{aligned} (1) \quad V_{DEU}(P(p_1, p_2)) &= p_1 p_2 (v(c_t)\delta(t) + v(c_{t+k})\delta(t + k)) \\ &\quad + p_1(1 - p_2)v(c_t)\delta(t) + (1 - p_1)p_2 v(c_{t+k})\delta(t + k) \\ &= p_1 v(c_t)\delta(t) + p_2 v(c_{t+k})\delta(t + k), \end{aligned}$$

leading to the first-order condition (FOC)

$$(2) \quad FOC_{DEU}(P(p_1, p_2)): \frac{v'(c_t)}{v'(c_{t+k})} = (1 + r) \frac{\delta(t + k)}{\delta(t)} \frac{p_2}{p_1}.$$

Recall that subjects' decision variable is the number of tokens n allocated to the sooner payment. Because $(c_t, c_{t+k}) = (na_t, (100 - n)a_{t+k})$, equation (2) determines the optimal level of n , $n^*(r)$, which in turn determines the optimal consumption

⁵Our analysis does not depend on the assumption of constant discounting.

levels $c_t^*(r)$ and $c_{t+k}^*(r)$. In equilibrium, any increase in the value of the right-hand side of equation (2) has to be balanced by a commensurate increase in the ratio of the marginal utilities on the left-hand side in order to maintain optimality. Under strictly concave utility, i.e., decreasing marginal utility, rebalancing c_t^* and c_{t+k}^* on the budget line entails decreasing c_t^* and thereby automatically increasing c_{t+k}^* . Therefore, c_t^* decreases in r and p_2/p_1 and any other factor that might appear on the right-hand side of equation (2).

The AS experiment involves three different common-ratio situations with varying manifestations of certainty: a baseline pair where $P(1, 1)$ is combined with its scaled-down counterpart $P(\lambda, \lambda)$, $0 < \lambda < 1$, and two differential risk conditions where $P(1, q)$ and $P(q, 1)$ are paired with $P(\lambda, \lambda q)$ and $P(\lambda q, \lambda)$, respectively. In the experiment, the scaling factor λ amounts to 0.5 and the probability q is fixed at 0.8. Discounted expected utility theory makes clear predictions for all three pairs of prospects: allocations should not change whenever p_1 and p_2 are scaled down by a common factor. However, AS find that scaling down the probabilities by $\lambda = 0.5$ always results in significant shifts in budget shares. Moreover, subjects' behavior differs distinctly between baseline and differential risk conditions.

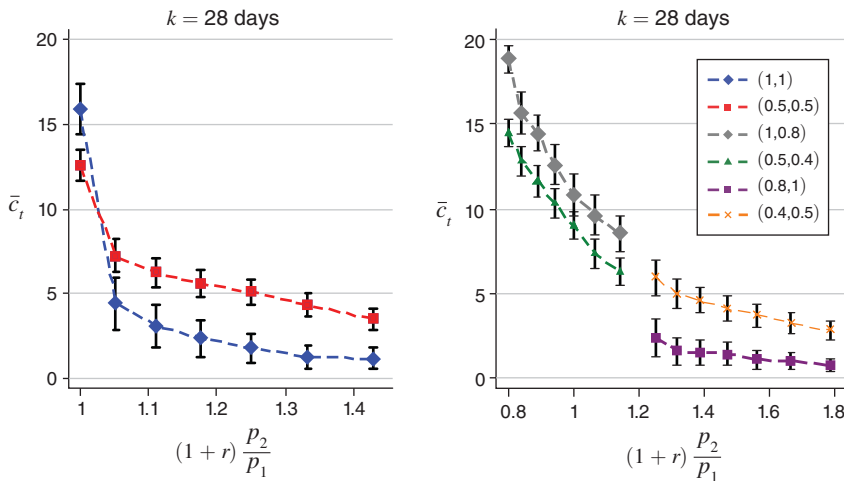
Concerning the baseline pair, average sooner consumption levels in the certain $(1, 1)$ -condition are initially greater than in the risky $(0.5, 0.5)$ -condition and drop more steeply with increasing interest rates r , as depicted in the left panel of Figure 1: AS's original graph shows a crossover of the $(1, 1)$ -curve and the $(0.5, 0.5)$ -curve, a result that was recently replicated by Cheung (2015) and Miao and Zhong (2015). When both payments are guaranteed and the interest rate is zero, subjects allocate the greatest part of their budget to the sooner payment. Since interest rates rise substantially,⁶ allocations to the sooner payment become increasingly expensive and, hence, drop steeply. In contrast, when there is the possibility that only one of the payments will arrive, subjects choose a more evenly balanced portfolio of sooner and later payments than in the certain case, which is consistent with a diversification motive. In fact, AS report that in the $(1, 1)$ -condition, 80.7 percent of allocations are at one or the other budget corners while only 26.1 percent are corner solutions in the $(0.5, 0.5)$ -condition.

Contrary to the baseline crossover result, scaling down the probabilities in the differential risk conditions results in consistently smaller or larger allocations, evident in the right panel of Figure 1. Clearly, both patterns of behavior are incompatible with the additively separable discounted-expected-utility model. But how can these findings be reconciled with probability weighting in rank-dependent models, such as RDU or cumulative prospect theory?

II. Rank-Dependent Prospect Valuation

Rank-dependent models depart from expected utility theory in that consequences are ordered by magnitude and their utilities are weighted by decision weights (Quiggin 1982; Wakker 1994). They feature a potentially nonlinear probability weighting function $g(p)$ obeying $g' > 0$, $g(0) = 0$, $g(1) = 1$, which provides the

⁶ Annual interest rates underlying the data in Figure 1 amount to 0, 85.7, 226.3, 449.7, 796.0, 1,323.4, and 2116.6 percent.

FIGURE 1. AVERAGE CONSUMPTION CURVES $\bar{c}_t(p_1, p_2)$

Notes: Left panel (baseline conditions): The figure presents the observed mean sooner consumption levels, $\bar{c}_t(1,1)$ and $\bar{c}_t(0.5,0.5)$, as functions of $(1+r)p_2/p_1 = 1+r$ for the certain (diamonds) and the risky (squares) conditions when later consumption is delayed by another 28 days. Right panel (differential risk conditions): The graphs correspond to observed mean sooner consumption levels $\bar{c}_t(1,0.8)$ (diamonds) and $\bar{c}_t(0.5,0.4)$ (triangles) as well as $\bar{c}_t(0.8,1)$ (squares) and $\bar{c}_t(0.4,0.5)$ (crosses) as functions of $(1+r)p_2/p_1$ when later consumption is delayed by another 28 days.

Source: Left panel: Andreoni and Sprenger (2012, p. 3,367, Figure 2). Right panel: Andreoni and Sprenger (2012, p. 3,370, Figure 4).

basis for calculating the decision weights. Decision weights involve cumulative probabilities, i.e., probabilities for receiving a particular outcome or anything better. Therefore, only the decision weight of the best outcome is equal to its simple probability weight. In the following, we assume that decision makers' preferences conform to RDU.⁷ Details on the special case of Halevy's (2008) model of time-dependent probability weighting are delegated to online Appendix A2 (see also Epper and Fehr-Duda 2012).

As $P(p_1, p_2)$ is a prospect over four potential consumption streams, these have to be rank-ordered according to the principles of RDU. Clearly, the materialization of both payments (c_t, c_{t+k}) yields the highest utility, and no payment the lowest. But which consumption stream is ranked second and third depends on whether the discounted utility of sooner consumption is greater than the discounted utility of later consumption or vice versa. For brevity, we introduce the following notation: $c_t \succ (\prec) c_{t+k} \Leftrightarrow v(c_t)\delta(t) > (<) v(c_{t+k})\delta(t+k)$. Decision weights for $P(p_1, p_2)$ are summarized in Table 1.⁸

⁷Cumulative prospect theory coincides with RDU when only gains are involved.

⁸In the case of indifference, the prospect involves only three outcomes with the intermediate decision weight amounting to $g(p_1 + p_2 - p_1 p_2) - g(p_1 p_2)$.

TABLE 1—RDU DECISION WEIGHTS

Consumption stream	Probability	Decision weight	
		$c_t \succ c_{t+k}$	$c_t \prec c_{t+k}$
(c_t, c_{t+k})	$p_1 p_2$	$g(p_1 p_2)$	
$(c_t, 0)$	$p_1(1 - p_2)$	$g(p_1) - g(p_1 p_2)$	$g(p_1 + p_2 - p_1 p_2) - g(p_2)$
$(0, c_{t+k})$	$(1 - p_1)p_2$	$g(p_1 + p_2 - p_1 p_2) - g(p_1)$	$g(p_2) - g(p_1 p_2)$
$(0, 0)$	$(1 - p_1)(1 - p_2)$	$1 - g(p_1 + p_2 - p_1 p_2)$	

The respective first-order conditions are given by

$$(3) \quad FOC_{RDU}(P(p_1, p_2)) : \frac{v'(c_t)}{v'(c_{t+k})}$$

$$= \begin{cases} (1 + r) \frac{\delta(t+k)}{\delta(t)} \frac{g(p_1 p_2) + g(p_1 + p_2 - p_1 p_2) - g(p_1)}{g(p_1)} & \text{if } c_t \succ c_{t+k} \\ (1 + r) \frac{\delta(t+k)}{\delta(t)} \frac{g(p_2)}{g(p_1 p_2) + g(p_1 + p_2 - p_1 p_2) - g(p_2)} & \text{if } c_t \prec c_{t+k} \end{cases}$$

The effects of a change in ranks of c_t and c_{t+k} on the allocation decision depend on the characteristics of the probability weighting function g . There is a large body of evidence that the representative probability weighting function is inverse S-shaped,⁹ characterized by a concave segment over small probabilities, such that the function intersects the diagonal from above at some probability smaller than 0.5, and by a convex segment over medium and large probabilities (see Gonzalez and Wu 1999 and Bruhin, Fehr-Duda, and Epper 2010 among others). The latter region is characterized by Jensen's inequality $g(\mu p_1 + (1 - \mu)p_2) < \mu g(p_1) + (1 - \mu)g(p_2)$ for all $0 < \mu < 1$.¹⁰ The typical function increases steeply in the vicinities of zero and one, but is relatively flat otherwise. Therefore, it is almost linear over a considerable range of probabilities and can be approximated by $\hat{g}(p) = d + sp$ with $0 < d, s < 1$ (Tversky and Fox 1995; Tversky and Wakker 1995). Over that range, probability weights grow subproportionally in p , i.e., $\hat{g}(p_1)/\hat{g}(p_2) < \hat{g}(\lambda p_1)/\hat{g}(\lambda p_2)$ for $p_1 < p_2$ and $0 < \lambda < 1$. Subproportionality, at least over some range of probabilities, is a general feature of probability weighting functions that maps the certainty effect and general common-ratio violations (Prelec 1998).

In the following, we examine the implications of a representative probability weighting function for the three prototypical common-ratio pairs of prospects $P(1, 1)$

⁹This observation applies both to the majority of individuals and to aggregate behavior (Fehr-Duda and Epper 2012).

¹⁰Note that in conjunction with a concave utility function v convexity of g implies risk aversion.

TABLE 2—DECISION WEIGHT RATIOS

Condition	Ratio	Value
(1, 1)	$\frac{g(1)}{g(1)}$	1
$(0.5, 0.5), c_t \succ c_{t+k}$	$\frac{g(0.25) + g(0.75) - g(0.5)}{g(0.5)}$	1.053
$(0.5, 0.5), c_t \prec c_{t+k}$	$\frac{g(0.5)}{g(0.25) + g(0.75) - g(0.5)}$	0.950
(1, 0.8)	$\frac{g(0.8)}{g(1)}$	0.624
$(0.5, 0.4), c_t \succ c_{t+k}$	$\frac{g(0.2) + g(0.7) - g(0.5)}{g(0.5)}$	0.912
$(0.5, 0.4), c_t \prec c_{t+k}$	$\frac{g(0.4)}{g(0.2) + g(0.7) - g(0.4)}$	0.858
(0.8, 1)	$\frac{g(1)}{g(0.8)}$	1.604
$(0.4, 0.5), c_t \succ c_{t+k}$	$\frac{g(0.2) + g(0.7) - g(0.4)}{g(0.4)}$	1.166
$(0.4, 0.5), c_t \prec c_{t+k}$	$\frac{g(0.5)}{g(0.2) + g(0.7) - g(0.5)}$	1.097

Note: Values derived from Prelec's (1998) probability weighting function $g(p) = \exp(-\beta(-\ln p)^\alpha)$, with $\alpha = 0.5$ and $\beta = 1$.

versus $P(\lambda, \lambda)$, $P(1, q)$ versus $P(\lambda, \lambda q)$, and $P(q, 1)$ versus $P(\lambda q, \lambda)$. The ratios of decision weights in the first-order conditions for $\lambda = 0.5$ and $q = 0.8$ are displayed in Table 2. For illustrative purposes, the right-most column of Table 2 presents a numerical example of representative decision weight ratios based on Prelec's (1998) probability weighting function.

The baseline conditions: $P(1, 1)$ versus $P(\lambda, \lambda)$. Clearly, the ratio of decision weights in the first-order condition, equation (3), equals 1 in the certain case. Prospect valuation in the scaled-down condition yields

$$(4) \quad V_{RDU}(P(\lambda, \lambda)) = \begin{cases} g(\lambda)v(c_t)\delta(t) + (g(\lambda^2) + g(2\lambda - \lambda^2) - g(\lambda))v(c_{t+k})\delta(t+k) & \text{if } c_t \succ c_{t+k} \\ (g(\lambda^2) + g(2\lambda - \lambda^2) - g(\lambda))v(c_t)\delta(t) + g(\lambda)v(c_{t+k})\delta(t+k) & \text{if } c_t \prec c_{t+k}. \end{cases}$$

If strict convexity holds for the range of probabilities under consideration, the following inequality is satisfied:

$$(5) \quad g(\lambda) - g(\lambda^2) < g(2\lambda - \lambda^2) - g(\lambda),$$

entailing for the ratios of decision weights in the first-order condition pertaining to $c_t \succ c_{t+k}$ and $c_t \prec c_{t+k}$

$$(6) \quad \frac{g(\lambda^2) + g(2\lambda - \lambda^2) - g(\lambda)}{g(\lambda)} > 1 > \frac{g(\lambda)}{g(\lambda^2) + g(2\lambda - \lambda^2) - g(\lambda)}.$$

Recall that the optimum level of sooner consumption decreases with the magnitude of the ratio of decision weights. Therefore, sooner consumption is lower in the risky situation than in the certain one when $c_t \succ c_{t+k}$, and higher when $c_t \prec c_{t+k}$. Hence, RDU generally predicts a crossover of the $(1, 1)$ –and the (λ, λ) –consumption curves if probability weights are convex, which is usually the case over the range of probabilities under consideration here, i.e., $[0.25, 1]$, resulting from $p_1 = p_2 = 0.5$.¹¹ In that situation, all the possible consumption streams are equally likely and, therefore, discounted expected utility assigns the same weight to c_t and c_{t+k} irrespective of their materializing in conjunction or in isolation.¹² RDU with an inverse S-shaped probability weighting function, however, entails a pronounced overweighting of the extreme consumption streams (c_t, c_{t+k}) and $(0, 0)$ while the intermediate ones $(c_t, 0)$ and $(0, c_{t+k})$ are underweighted relative to their objective probabilities (Wakker 2010; Fehr-Duda and Epper 2012). This characteristic of decision weights maps people's dislike of the situation when only one of the payments arrives and represents their preference for intertemporal diversification.

Our approach captures the specific characteristics of the decision situation in the CTB task: subjects choose a portfolio of future payments, the arrival of which is determined independently by two random draws. Since subjects can choose any convex combination of sooner and later payments, this decision environment produces opportunities for portfolio diversification. Now suppose that a single draw controls both payments, generating the prospect $P_{SINGLE}(\lambda) = ((c_t, c_{t+k}), \lambda; (0, 0), 1 - \lambda)$. Clearly, in that case, diversification is not possible, and the same decision weight, $g(\lambda)$, is assigned to both c_t and c_{t+k} . A comparison of the respective decision weights shows that, in total, positive consumption utilities carry a larger weight in the independent-draw situation than in the single-draw one. In the independent case, decision weights assigned to c_t and c_{t+k} amount to $g(\lambda)$ or $g(\lambda^2) + g(2\lambda - \lambda^2) - g(\lambda)$. As $g(\lambda) < g(\lambda^2) + g(2\lambda - \lambda^2) - g(\lambda)$ holds over the convex region of g (see equation (5)), a risk averse RDU maximizer always prefers two independent draws to a single draw determining both payments, i.e., she prefers a situation with diversification opportunities to a situation with a lack thereof. Consequently, we predict behavior to differ systematically between these types of situations, which was recently corroborated by the experimental study of Miao and Zhong (2015).¹³

¹¹ Compare, for example, with Figure 1 in Fehr-Duda and Epper (2012, p. C-1), which shows parametric as well as nonparametric estimates of probability weights for a student and a broad population sample.

¹² In this context, discounted expected utility theory's failure of accommodating intertemporal diversification has long been identified as one of its main weaknesses (Richard 1975; Kihlstrom and Mirman 1974; Epstein and Tanny 1980). To remedy this problem, some authors proposed an intertemporal utility function on top of instantaneous (atemporal) utility, see online Appendix A4 for details.

¹³ Cheung (2015) also studied behavior in CTBs under positively correlated payments. Contrary to Miao and Zhong (2015), where the average token allocation in the certain condition practically coincides with the allocation in the correlated condition, Cheung (2015) finds a crossover of the curves for the certain and the correlated-risk

The differential risk conditions: $P(1, q)$ versus $P(\lambda, \lambda q)$ and $P(q, 1)$ versus $P(\lambda q, \lambda)$.

The prospects involving one certain payment constitute binary prospects and, therefore, contrary to the scaled-down cases there is no problem with changing ranks. As noted above, the representative probability weighting function is near linear away from the boundaries. Using the linear approximation $\hat{g}(p) = d + sp$, introduced above, yields the following relationships for the first pair:

$$(7) \quad \frac{g(\lambda^2 q) + g(\lambda + \lambda q - \lambda^2 q) - g(\lambda)}{g(\lambda)} \approx \frac{\hat{g}(\lambda q)}{\hat{g}(\lambda)} \approx \frac{g(\lambda q)}{g(\lambda^2 q) + g(\lambda + \lambda q - \lambda^2 q) - g(\lambda q)}.$$

Therefore, the ratios of decision weights are quite similar even though the rank order of c_t and c_{t+k} changes. Moreover, since \hat{g} is subproportional,

$$(8) \quad \frac{g(\lambda q)}{g(\lambda)} \approx \frac{\hat{g}(\lambda q)}{\hat{g}(\lambda)} > \hat{g}(q) \approx g(q)$$

holds.¹⁴ This result is an immediate consequence of the certainty effect captured by subproportional probability weights: reducing a certainty to a probability of $q < 1$ has a much greater impact on behavior than reducing λ to λq has. Hence, for suitable values of λ and q , the sooner-consumption curve for $P(\lambda, \lambda q)$ does not cross the respective curve for $P(1, q)$ and always lies below it. This is exactly the pattern found by AS, depicted in Figure 1. Given the typical shape of probability weights, a linear approximation works well over the range under consideration here, i.e., $[0.2, 0.8]$, which is clearly bounded away from zero and one.¹⁵ It is straightforward to show that, when the probabilities are flipped to produce the reverse pair of prospects, $P(q, 1)$ and $P(\lambda q, \lambda)$, the opposite result holds. Therefore, probability weighting predicts behavior in the differential risk conditions as found by AS.

III. Discussion

We have demonstrated that rank-dependent probability weighting correctly predicts behavior in the baseline as well as the differential risk conditions, accounting for both the intertemporal certainty effect and the preference for diversification evident in subjects' behavior. Hence, the family of rank-dependent models provides a unified explanation for all of the key AS findings. However, rank-dependent models are not the only approaches that can accommodate the evidence of the CTB experiment. AS mention the so-called $u - v$ model as another promising candidate.¹⁶ The

conditions, albeit less strongly pronounced than in the independent-risk case. We surmise that subjects may have tried hedging across conditions, possibly because of difficulties of understanding the experimental instructions. See Cheung (2015), footnote 21 and his Appendix D.

¹⁴ See also the numerical calibrations in Table 2.

¹⁵ Refer, for example, to Figure 1 in Fehr-Duda and Epper (2012, p. C-1). Under conventional parametric specifications of $g(p)$ and representative parameter values, a crossover is predicted only for q very close to one or λ very close to zero.

¹⁶ See also their discussion in Andreoni and Sprenger (2010).

$u - v$ model adheres to expected utility but assumes that the utility of uncertain outcomes u is distinct from the utility of certain outcomes v , with u being more concave than v . While $u - v$ preferences can, in principle, account for the CTB findings when sooner and later payments arrive independently of each other, the model has a number of unattractive features. First, it generates a discontinuity and a violation of first-order stochastic dominance at certainty (Schmidt 1998). Second, the $u - v$ model can only handle the certainty effect but not more general boundary effects which have been shown to emerge when there are more than two prospective outcomes (Camerer 1992; Harless and Camerer 1994). Furthermore, it predicts identical behavior in situations where the arrival of the payments is determined by two independent draws or by a single one.

Several other models, such as Kihlstrom and Mirman's (1974) model of intertemporal risk aversion, can accommodate a preference for intertemporal diversification as well. In online Appendix A3 we take up the Kihlstrom-Mirman approach and show that it predicts the AS baseline findings when assuming expected utility and positing a concave intertemporal transformation of the instantaneous utilities of consumption. However, this model is incapable of mapping intertemporal common-ratio violations. The greatest weakness of such an approach, however, is its reliance on expected utility for atemporal decisions.

Overall, RDU can explain all of the major findings in CTB experiments and provides the most convincing explanation of the evidence. The model respects first-order stochastic dominance, it can handle general boundary effects aside from the certainty effect, and correctly predicts behavior under different correlation structures. Thus, RDU and its cousins are an attractive modeling choice not only in atemporal, but also in intertemporal situations.

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